FRACTAL ANALYSIS

Revisiting Pollock's drip paintings

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We investigate the contentions that Jackson Pollock's drip paintings are fractals produced by the artist's Lévy distributed motion and that fractal analysis may be used to authenticate works of uncertain provenance[1–5]. We find that the paintings exhibit fractal characteristics over too small a range to be usefully considered as fractal; their limited fractal characteristics are easily generated without Lévy motion, both by freehand drawing and gaussian random motion. Several problems must therefore be addressed before fractal analysis can be used to authenticate paintings[1].

An image is considered fractal if, when covered with a grid of square boxes of size L (refs 1, 6), the number of filled boxes $N \propto L^D$, where D, the fractal dimension, is a non-integer. It is generally accepted[7,8] that, to establish power-law behaviour, the range of box sizes must span more than one or two orders of magnitude. For a Pollock drip painting, the range is typically limited to three orders of magnitude by the ratio of the canvas size to the size of the smallest feature. For all Pollock paintings examined so far, the box-counting curve, a plot of N versus L, is found to be a broken power law[5] ($N \propto L^{D_L}$ for $L > l_T$, and $N \propto L^{D_D}$ for $L < l_T$, where L_T is a characteristic length). Thus, there are two power laws and less than two orders of magnitude to establish each. Some pitfalls of inferring fractal behaviour from such a limited range are demonstrated below.

For multicoloured, multilayered drip paintings (a large part of Pollock's oeuvre), it has been claimed[4] that the visible part of each separate layer and the composite are all fractals. This inference is probably an artefact of the limited range of data used, because it is not a mathematical possibility if a model in which the individual layers are ideal fractals is considered[6] — as, for example, in Cantor dusts or Sierpinski carpets. Calculation shows that the box-counting curves for the visible parts of each layer and the composite painting are not power laws, even though the individual layers are perfect fractals (Fig. 1).

Taylor *et al.* claim that the defining visual character of Pollock's drip paintings is their fractal nature[4] and that fractals arise from the specific pouring technique developed by Pollock[9]. To test this, we drew a number of freehand sketches: Fig. 2 shows one we dub *Untitled 5*, and its box-counting curve. By the criteria espoused by Taylor and colleagues[2–5], it is a high-quality fractal; however, Fig. 2 undermines their claim[2–5,9] that Pollock's works derive their artistic merit from their (limited) fractal content.

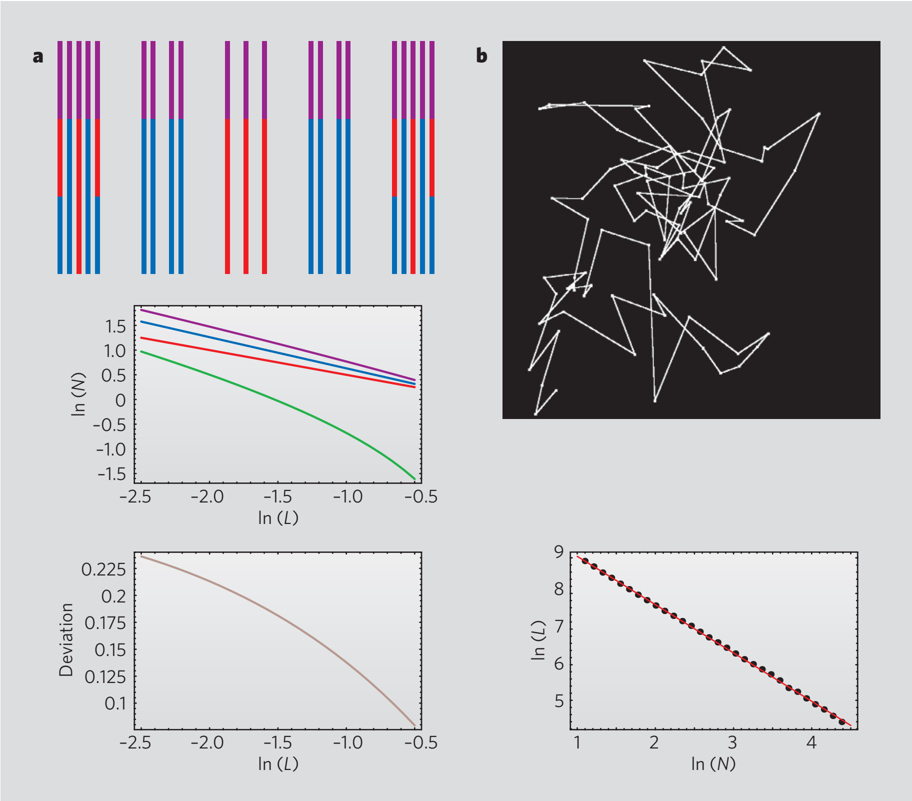

Figure 1 | Fractal barcode and gaussian walk. a, Top, a middle-third Cantor dust anchor layer (blue), overlaid with a second Cantor dust (red). Half-blue/half-red bars correspond to the intersection of the dusts; purple corresponds to their union. Centre, box-counting curves: blue dust (shown in blue), red dust (red), the uncovered part of the blue dust (green) and the composite (purple). The curvature of the traces indicates that the uncovered portion of the blue layer and the composite are not true fractals. To highlight the curvature of the composite, the lower graph shows the difference of the (rigorously linear) blue and purple curves. **b**, A linear fit to the box-counting curve of a 100-step gaussian walk has a slope of 1.35, with standard deviation $\chi = 0.025$.
Methods. For **a**, the blue dust is obtained by repeatedly dividing segments into three parts and retaining only the first and third; the red dust is obtained by dividing into nine parts and retaining the first, fifth and ninth parts. The 'fractal barcode' shows the appearance of the dusts after four iterations. For **b**, step size is calculated as 0.09 × frame width. Smallest box size, 3 pixels. Sizes range over 1.4 orders of magnitude, with magnification $C = 1.12$ (see Fig. 2 for definition).

There are insufficient data[10,11] to determine directly whether Pollock's motion while painting constituted a Lévy flight. Lévy motion has been inferred[2–5] from the known result that Lévy flights leave a fractal trail[6]. However, the box-counting data of a simulated gaussian random walk of 100 steps, examined over only one-and-a-half orders of magnitude, are well described as a fractal ($D = 1.35$ in our simulation; Fig. 1) — although sufficiently long gaussian random walks are rigorously known to be non-fractal, with dimension $D = 2$. Thus, the limited fractal content of Pollock's work does not require Lévy flights for its explanation. Furthermore, Lévy motion has no natural length scales, whereas Pollock's paintings and motion seem to have many[4].

Box-counting authentication techniques assume that the parameters, l_T, D_D and D_L and χ_2(standard deviation; Fig. 2, legend) show characteristic trends that distinguish Pollock's paintings from those of his imitators[5,9]. For multifractals[12] and non-fractals, χ_2 is not an intrinsic characteristic of the image but depends on the magnification, C (Fig. 2, legend). If Pollock's paintings are multifractal[13], it would be imprudent to use χ_2 as a characteristic parameter.

Also, we question the presumption that parameters such as l_T for Pollock's works are essentially random variables, circumscribed by a range determined[9] by examination of just 17

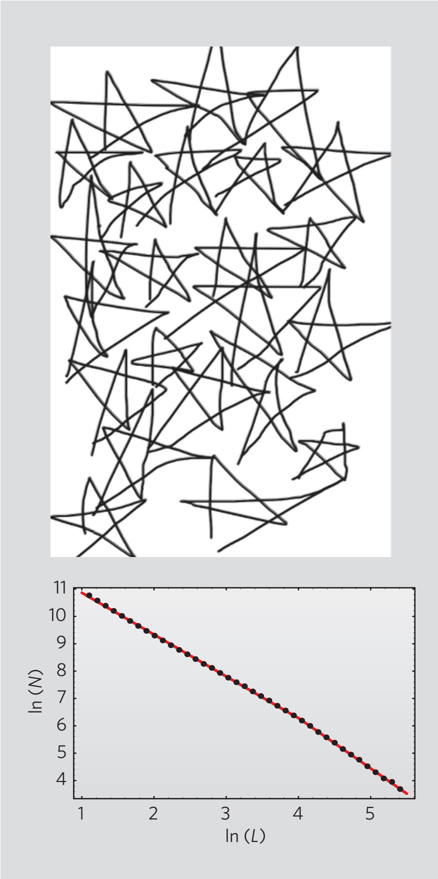

Figure 2 | *Untitled 5* and its box-counting curve. The best broken power-law fit to these data corresponds to slopes of $D_D = 1.53$ and $D_L = 1.84$. The break occurs at $\ln(L) \approx 4$; the standard deviation of the data from the fit, χ, is 0.022. *Untitled 5* (top) fulfils all the criteria used in box-counting authentication that have been made public: it shows a broken power-law behaviour with $D_D < D_L$ and, for a magnification factor C similar to that used by Taylor *et al.*, has a χ_2 value in the 'permissible' range of $0.009 < \chi < 0.025$. **Methods.** All our sketches, including *Untitled 5*, are freehand drawings made in Adobe Photoshop using a 14-point Adobe Photoshop 'paintbrush'. The paintbrush leaves a mark when dragged continuously across the 'canvas' by a computer mouse. Although not drip paintings, these patterns are human- and not computer-generated, in the sense that these terms are commonly understood and as used by Taylor *et al.*[4]. The box sizes were chosen to be hC^n, where $n = 1,2,3,\ldots M$. Here $h = 3$ pixels is the smallest box size, the magnification factor $C = 1.12$, the cutoff M determines the biggest box size, and the box sizes range over about 1.9 orders of magnitude.

drip paintings (out of about 180). Further study of systematic effects is needed. As the values measured by Taylor *et al.*[9] are not available, a consensus on the limiting range, if there is one, can emerge only after other groups have replicated and extended the box-counting data set. Finally, we note that *Untitled 5* fulfils all the criteria used in box-counting authentication that have so far been made public.

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Our use[1] of the term 'fractal'[2] is consistent with that by the research community. In dismissing Pollock's fractals[1,3] because of their limited magnification range, Jones-Smith and Mathur[4] would also dismiss half the published investigations of physical fractals[5]. On the basis of previous debates on limited-range fractals[5,6], a fractal description is particularly appropriate for Pollock's patterns because it is physically reasonable and because it is useful for condensing the description of a complex geometry, as we now describe.

Fractal description is physically reasonable because Pollock's technique involved a motion-dominated process at large length scales and a paint-dominated process at small scales, with a transition between the two expected at several centimetres. The fractal behaviours observed in the scaling plots above and below this transition are physically reasonable — many physiological processes[7], including human motion[8,9], are fractal, and falling liquid can itself be fractal[10].

However, for *Untitled 5* there is no physical reason to expect a transition between two processes at $\ln(L) \approx 4$. In the absence of this artificial transition, the scaling plot does not fit fractal behaviour. Furthermore, when we generate 'drawings' similar to *Untitled 5*, some plots curve down and some curve up (Fig. 1a), in contrast to the consistent behaviour observed in Pollock's poured paintings. Combined, these observations demonstrate that the "freehand sketches" of Jones-Smith and Mathur[4] are not fractal. Hence, *Untitled 5* does not undermine any claim that we have made about Pollock's work.

As to the usefulness of a fractal description, Pollock's fractals satisfy all the criteria for usefulness proposed for limited-range fractals[5]. Furthermore, our fractal description of Pollock's patterns is useful across disciplines, for example in psychology experiments (showing that Pollock's fractals induce the same perceptual responses as mathematical fractals[11]), and in applications of fractal geometry to distinguish paintings by different artists[3,12].

We disagree with the claim by Jones-Smith and Mathur[4] that Pollock's fractal characteristics are easily generated by gaussian random motion. Our simulations generate scaling plots with variable curvature (Fig. 1b), in contrast to the consistent behaviour shown in Pollock's paintings[12].

It is both mathematically and physically possible for two exposed patterns and their composite all to be fractal. This depends on the relative densities of the exposed patterns and the scaling behaviour of boxes containing both patterns. Jones-Smith and Mathur's Cantor dust does not apply to Pollock paintings, where the overlap of layers is considerably more complicated. Furthermore, their Fig. 1a features box counts of less than one (in reality, a box is either filled or not). The gradient of their green trace becomes greater than one (impossible for an object with $0 < D < 1$). Their Fig. 1a also misleads because it concentrates on large scales — the deviation of the blue and composite box counts becomes insignificant at finer scales (Fig. 1c).

We performed an actual box count on the dust analysed by Jones-Smith and Mathur (Fig. 1d) in order to show that their analytical simulations are flawed. Their expressions are evaluated at each iteration, instead of correctly box counting after fully generating the pattern. Their green curve is generated by using an expression that holds only for specific cases in which a 'box' exactly matches the width of a Cantor dust segment: it is therefore invalid for most of the continuous range the authors display.

Finally, we keep the value of the magnification factor C constant in recent authenticity procedures because χ_2 is dependent on C (ref. 12, which also gives our authenticity criteria, which are not confidential). We encourage further research.

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