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Fractal analysis of Jackson Pollock's painting evolution

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ABSTRACT

By mid-1940s, Jackson Pollock, one of the most influential artists in the twentieth century plastic art, developed a unique painting technique (pouring and dripping) to generate complex non-figurative patterns. It has been argued that methods from fractality theory are suitable for characterizing these complex patterns. In fact, recent studies have shown that, indeed, Pollock's drip paintings are fractal, multifractal, multiscaling and evolved to higher complexity from 1945 to 1950. However, the development of this drip and pouring technique was neither instantaneous nor spontaneous, but influenced by diverse cultural and personal factors along of Pollock's life. The aim of this work is to study the evolution of some fractality indices of Pollock's paintings for the period from 1930 to 1955 and, in this form, detect changes in this painting technique and relate them to major cultural influences. To this end, about 30 paintings are analyzed by applying a two-dimensional detrended fluctuation analysis (DFA). Results indicate two large shifts in the fractality indices. One transition involves a change in the correlations dimension by 1937, while a second transition implicates a shift in the short-scale Hurst exponent by 1945-1946. Based on descriptions from Pollock's biographers, it is postulated that the first change may be strongly influenced by Mexican muralist Siqueiros and the second one by the moving of Jackson Pollock and Lee Krasner for living in the natural landscapes at Springs, Long Island. Our findings in this work support these claims.

1. Introduction

Jackson Pollock was a prominent painter in the 1940s American artistic scenario. The main Pollock's contribution to the painting vanguard was the development of a pouring and dripping painting technique with the use of sticks, hardened brushes and basting syringes as paint applicators [6,24]. This painting technique resulted in a nonfigurative, highly abstract plastic art where the aesthetic features result from the complexity of the color and luminance patterns, rather than of specific geometric arrangements commonly found in figurative expressions. Fig. 1 shows the famous Pollock's painting *Eyes in the Heat* (1946, Peggy Guggenheim Collection, Venice) in both original color and gray-scale formats. Figures are completely absent while the perception of aesthetic attributes could depend on the subjectivity of the observer.

The decades following Pollock's death by 1953 witnessed the surge of exciting discussions and controversies on the real contribution of Pollock's work to the American and even worldwide plastic art history. The point is that Pollock's painting challenged the analysis from the traditional framework of the figurative expressions. Given the lack of a minimal figurative structure, it has been postulated that fractal analysis is the suitable framework to characterize the complex patterns emerging from Pollock's drip paintings. Taylor and coworkers pioneered this effort [27–30] by studying the black-pigment (blobs) and multiple color patterns contained in the structure of Pollock's drip paintings. The use of box-counting methods showed that the structure of blob patterns shares some similarity with those formed in nature by trees, clouds and coastlines (which are some of the most typical

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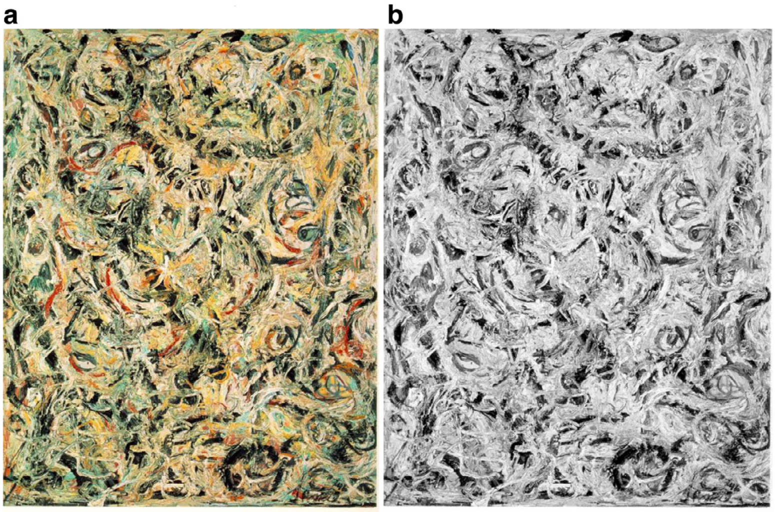

Fig. 1. Sample of a Pollock's drip painting – *Eyes in the Heat* (1946) – in original color (a) and gray-scale (b) formats.

and very first examples of fractality in nature). An interesting result indicated that blob structures can be characterized almost uniformly with a fractal dimension of about 1.7. The complex geometry of Pollock's drip paintings was also considered by Mureika et al. [15,16], who extended the analysis to other artists of the abstract expressionist school (Quebec-based group Les Automatistes).

The aforementioned studies showed a good agreement with those by Taylor and collaborators in the sense that one can associate an effective fractal dimension in the range 1.6–1.8 with the blobs patterns on the paintings. It was also concluded that the fractal dimension is not sufficient to distinguish between different artists when only large size scales are considered [15]. In this way, a final difference between Pollock's drip paintings and the paintings of other contemporaneous artists cannot be established only from a fractality standpoint. Along this line, it has been also suggested that fractal properties of patterns formed by specific colors could be used as an authentication method for artists and their paintings [16,31]. However, additional fractal studies [7,8] have challenged this approach by arguing that Pollock's drip paintings exhibit fractal characteristics over a very small range to be usefully considered as fractal. Indeed, fractality of this type can be generated without the use of, say, Levy motion processes. These studies considered color blobs for characterizing fractality of nonfigurative visual art. In principle, the aesthetic structure in Pollock's drip paintings is a consequence of the whole color structure, and not only of large blobs and color gradients. In this way, in contrast to previous studies that focused on the fractality of blobs and contours, Alvarez-Ramirez et al. [1] applied two-dimensional DFA [3] on gray-scale images to characterize the fractal structure of luminance patterns for Pollock's drip paintings. It was shown that fractality in Pollock's drip painting can be associated with 1/f-noise structures. For spatial scales below 30 cm, it was found that Pollock's drip paintings are organized as 1/f-noise structures, which presumably are linked to pleasant (i.e., aesthetic) perception [20,33].

The evolution of Pollock artistic formation has been extensively documented by several authors [4,17,24]. By 1930, Thomas Hart Benton introduced Pollock into systemic studies of painting techniques at the Art Students League of New York. This phase in the Pollock formation was quasi-figurative and was influenced by native American culture and Benton's rural American. An important event in Pollock's formation occurred at 1936 when he attended an experimental workshop on the use of liquid paint by Mexican muralist David Alfaro Siqueiros. Pollock acquired the painting pouring technique as a central component for the makeup of canvases of the early 1940s. Pollock married the American painter Lee Krasner in October 1945, and in November they moved to Springs on Long Island, New York. In his stance at Long Island, Pollock refined the painting technique of working freely with liquid paint. During this rather long period that lasted from 1945 to 1952, Pollock abandoned the traditional easel layout to begin painting his canvases laid out on the floor. It is apparent that this change allowed Pollock to develop his characteristic drip-and-splash style with some abruptness in 1947. In the last part of this period, Pollock returned to figurative or quasi-figurative by producing black and white works [24].

In principle, signatures of the different artistic phases should be reflected in the fractal structure of Pollock's paintings. Taylor et al. [29] found some evidence of this by showing that, for the drip painting period 1945–1953, the blobs fractal dimension increased with respect to the year the opus was painted. In turn, this suggested that Pollock improved his drip-and-pouring technique through the years, which is reflected as an increase of the box-counting fractal dimension D_{BC} from about 1.12 in his early attempts in 1945 to 1.72–1.89 at his peak phase by 1950–1952.

In this regard, the aim of the present work is to provide further analyses of the evolution of Pollock's painting technique as reflected in the fractal structure of the intricate blobs and color geometry. To this end, the two-dimensional detrended fluctuation analysis (DFA) was applied for studying the fractality of 30 Pollock's paintings belonging to the period from 1930 to 1955. As done in our previous study [1], we consider luminance patterns rather than individual color blobs and contours. By doing so, the fractality of the whole painting composition is estimated, giving a closer view of the evolution of Pollock's technique.

Although still incomplete, the luminance pattern provides relevant insights into the color composition contained in a painting. For instance, the corresponding fractal dimension, being necessarily larger than two, suggests the degree of three-dimensional effect produced by the luminance pattern. In contrast, the fractal dimension of color blobs provides some information on the covering of a particular color structure on the plane. It should be pointed out that neither color blobs nor gray-scale patterns are enough by themselves for characterizing the complexity of a painting, but these approaches should be used jointly for obtaining complementary information.

The application of the two-dimensional DFA to the luminance pattern of the Pollock's paintings indicated the years 1937-1938 and 1945-1946 as two important transitions. Based on arguments and information from Pollock's biographers and painting critics [17,24], it is suggested that the first one could be caused by the use of liquid paint motivated by the Mexican muralist Siqueiros. On the other hand, the second transition year could be related to the start of the Springs (New York) period that led Pollock to develop his dripping and pouring technique. The results are contrasted with previous analysis that used fractality to characterize Pollock's painting patterns [1,7,8,15,16,27–30].

2. Fractal analysis method

The majority of the images considered in this study are digital scans of Pollock's works from Robertson [21], Spring [25] and the book for Pollock's exhibition in Museo Correr-Piazza San Marco, the Main Civic Museum of Venice (2002). The resolution of the scans was chosen as 300 dpi, creating images roughly 1000 pixels in length for the longest side. The scanned images were converted into a luminance pattern by transforming them into gray-scale images represented as a two-dimensional matrix array by means of the Matlab software. Some of Pollock's paintings studied in this work are *Untitled - Selfportrait* (1931–1935), *Cotton Pickers* (1934), *Landscape with Tree to Right* (1936), *Kneeling Figure* (1934), *Naked Man* (Tate Gallery, London, 1938), *Composition with Serpent Mask* (1938), *The Moon Woman* (Peggy Guggenheim Collection, 1942), *Composition with Pouring 11* (Hirshhorn Musem and Sculpture Garden, Washington, 1943), *Water Birds* (1943), *Untitled* (1945), *Free Form* (1946), *Lucifer* (The Anderson Collection at Stanford University, 1947), *Full Fathom Five* (MoMa, New York, 1947), *Number 14, 1948* (1948), *Figure* (1948), *Number 23, 1948* (1948), *Number 8, 1949* (1949), *Number 27, 1950* (Whitney Museum of Art, 1950), *Number 32, 1950* (Kunstsammlung Nordrhein-Westfale, Düsseldorf, 1950), *Autumn Rhythm: Number 30* (MoMa New York, 1950), *Unknown* (1950), *Untitled* (National Gallery of Art, Washington, 1951), *Blue Poles: Number 11, 1952* (National Gallery of Australia in Canberra, 1952), *Convergence: Number 10, 1952* (Albright – Knox Art Gallery, Buffalo, NY, 1952), *Autumn Rhythm: Number 30, 1950* (1950), *Blue Poles: Number 11, 1952* (1950), *Reflections of the Big Dipper* (1947), *Number 1* (Museum of Contemporary Art, Los Angeles, 1949), *Number 28* (National Gallery of Canada, 1950), and *Lavender Mist: Number 1, 1950* (National Gallery of Art, Washington, 1950). Fig. 1 shows the drip painting *Eyes in the Heat* (Peggy Guggenheim Collection, Venice, 1947) in both original color and gray-scale formats. Note that the gray-scale retains the luminance pattern associated to the color distribution on the painting.

2.1. Fractal dimension

Color blobs are fractal structures that do not cover the plane. Topologically, color blobs in Pollock's drip paintings [15,16, 27–30] are not dense in the plane, meaning that one can find large lacunarities. In this case, methods based on the characterization of fractal "curves" (e.g., box-counting methods) are suitable for estimating fractality features. In particular, the low-density of color blobs in the Euclidean plane implies that the fractal dimension is necessarily smaller than two [13]. However, as illustrated in Fig. 1 of the present work, luminance or gray-scale patterns are dense in the plane, although with irregularities that resemble a complex material structure [3]. Nowadays, the estimation of the fractal dimension with box-counting methods is a standard procedure available in many software packages. The basic procedure to estimate the fractal dimension with box-counting methods is as follows ([14,23,26]:

- **Step 1**. Consider an image of size $M \times N$ as a three-dimensional spatial surface with (x, y) denoting the pixel position on the plane of the image, and the third (vertical) coordinate z pixel gray level. The (x, y) plane is partitioned into non-overlapping blocks of size $s \times s$.
- **Step 2**. The spatial scale of each block is taken as the integer s, where $\min\{M, N\}/2 \geq s > 1$.
- **Step 3.** Each block contains a column of boxes of size $s \times s \times s'$, where s' is the height of the block, with s' meeting $G/s' = \min\{M, N\}/s$ and G is the total number of gray levels.
- **Step 4.** Let the minimum and maximum gray level in the (i, j)th block be into the kth and lth boxes, respectively. The number of boxes covering this block is counted as $n(i, j; s) = l - k + 1$.

The contributions from all blocks is given by

$$N(s) = \sum_{i}^{M} \sum_{j}^{N} n(i, j; s) \tag{1}$$

Typically, $N(s)$ follows a power-law of the form

$$N(s) = K_{BC} s^{-D_{BC}} \tag{2}$$

where K_{BH} is a positive constant and D_{BC} is the box-counting fractal dimension. The fractal dimension D_{BC} can be estimated from standard least-squares linear fit of $\log(N(s))$ versus $\log(s)$.Improvements of the method described above include efficient algorithms for accounting for sharp edges and boundaries [10].

2.2. Detrended fluctuation analysis (DFA)

Although the fractal dimension provides valuable information on the fractality of the painting complex pattern, it should be recognized that gray-scale patterns are quite complex structures for which a sole parameter, like the fractal dimension, is not sufficient to give a full characterization. In this way, we propose the usage of a scaling method for characterizing high-dimensional, presumably fractal, sets. For one-dimensional sequences, the DFA method [9,19] has proven to be efficient and accurate for detecting scaling, fractality and multifractality for data arising from diverse science and engineering fields, from physiology, physics, finance, mechanics and many more (see www.physionet.org for a detailed list of about 800 publications reporting significant applications of the DFA).

The DFA method was extended for high-dimensional sets [3], with potential applications for the characterization of scaling and fractality of images obtained from SEM, astronomy, complex materials, etc. [3, 11,18]. An advantage of DFA methods is the stability of its statistics, giving a robust estimate of scaling parameters even in the presence of trends, additive white noise, etc. [5]. The numerical results by Gu and Zhou [3] have proven the ability of the DFA to extract the fractality degree from an image by contrasting the results with synthetic two-dimensional fractals generated with well-accepted methods.

In the following, a brief description of the DFA as reported by Gu and Zhou [3] will be given. In a preliminary step, the images are stored as two-dimensional matrix arrays. Subsequently, the following steps are followed:

- **Step 1**. A two-dimensional array $X(i, j)$, $i = 1, \ldots, M$ and $j = 1, \ldots, N$, is partitioned into $M_s \times N_s$ disjoint square segments of the same size $s \times s$, where $M_s = [M/s]$ and $N_s = [N/s]$. Each segment is denoted by $X_{v,w}(i, j; s) = X(l_1 + i, l_2 + j; s)$ for $1 \leq i, j \leq s$, where $l_1 = (v - 1)s$ and $l_2 = (w - 1)s$.
- **Step 2**. For each segment $X_{v,w}(i, j)$, the cumulative sum $u_{v,w}(i, j; s)$ is calculated as

$$u_{v,w}(i, j; s) = \sum_{k_1=1}^{i} \sum_{k_2=1}^{j} X_{v,w}(k_1, k_2; s) \tag{3}$$

- **Step 3**. The trend of the constructed surface $u_{v,w}$ can be computed by least-square fitting it with a bivariate polynomial function $p_{v,w}(i, j; s)$. The detrended fluctuation function $F_{v,w}(s)$ of the segment $X_{v,w}$ is given by the sample variance as

$$\varphi_{v,w}(s)^2 = \frac{1}{s^2} \sum_{i=1}^{s} \sum_{j=1}^{s} [u_{v,w}(i, j; s) - p_{v,w}(i, j; s)]^2 \tag{4}$$

- **Step 4**. The total detrended fluctuation is computed by averaging over all the segments as follows:

$$F(s) = \left(\frac{1}{M_s N_s} \sum_{v=1}^{M_s} \sum_{w=1}^{N_s} \varphi_{v,w}(s)^2 \right)^{1/2} \tag{5}$$

- **Step 5**. By varying the value of s in the range from $s_{\min} \approx 10$ to $s_{\max} \approx \min(M, N)/5$, one can determine the scaling relation between the detrended fluctuation function $F(s)$and the spatial scale s as follows:

$$F(s) = K_H s^H \tag{6}$$

where the exponent H, known as the Hurst index, is an indicator of the roughness of the two-dimensional array. The value $H = 0.5$ reflects that long-term correlations are absent and the roughness achieves a maximum. In contrast, values $H > 0.5$ indicate the presence of correlations in the array conformation. Large values of H are associated to smoother, less fractal, surfaces.

It has been shown that for non-intersecting self-affine surfaces the Hurst index is related to the box-counting fractal dimension of surface $H = 3 - D_{BC}$ as [13]. That is, the lower is the Hurst index, the more invasive is the surface. The relationship $H = 3 - D_{BC}$ suggests that a simple application of box-counting methods for estimating the fractal dimension D_{BC} should suffice for estimating the Hurst index H. However, for complex morphologies there is not guarantee that the fractal surface is non-intersecting. In fact, the morphology exhibited in Fig. 1 shows a complex pattern, with intersecting clusters and fractionated structures suggesting a three-dimensional effect. This indicates that the morphologies showed in Fig. 1 display different fractal dimension and Hurst effects [2].

3. Results

In the sequel, the scale s will be expressed relative to the horizontal physical length (in centimeters) of the painting. This is done by using a simple relationship between the number of pixels of the image and the real horizontal length as reported in Pollock's painting catalogue.

3.1. Fractal dimension

Fig. 2 exhibits the results of the box-counting method applied to two illustrative cases of Pollock paintings. For the

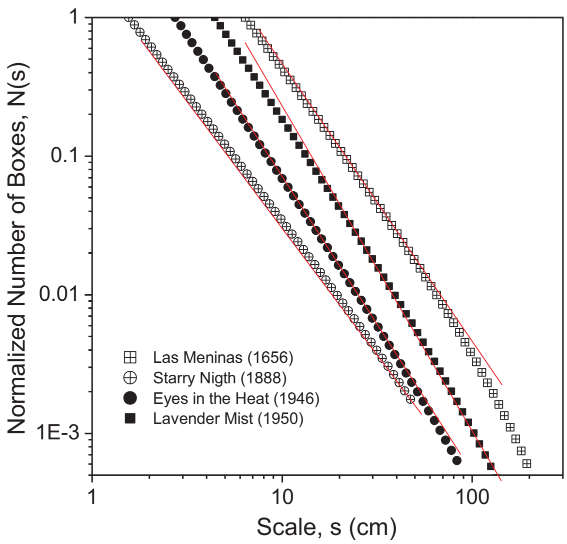

Fig. 2. Scaling pattern resulting from the box-counting method for estimating the fractal dimension. For Pollock's paintings, the resulting fractal dimension is 2.67 and 2.73 for *Eyes in the Heat* and *Lavender Mist*, respectively. In contrast, the fractal dimension is 2.42 and 2.53 for Velazquez's *Las Meninas* and van Gogh's *Starry Night* paintings, respectively.

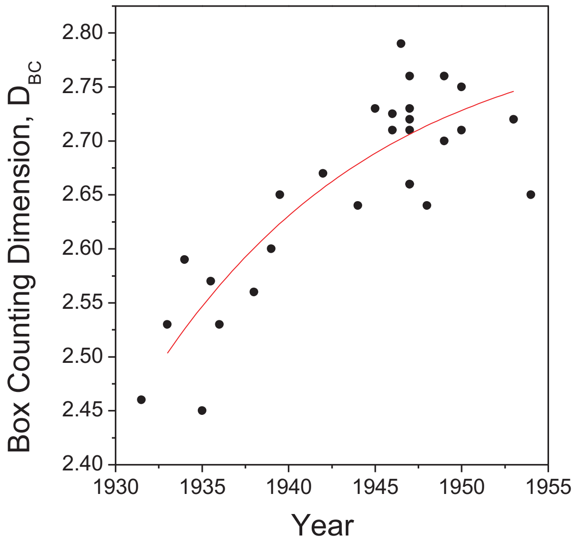

Fig. 3. Evolution of the box-counting dimension D_{BC} for Pollock's paintings in the period from 1930 to 1955. Note the gradual increment of D_{BC}, modeled as an exponential growth function, suggesting that Pollock introduced more and more sophistication in his painting technique.

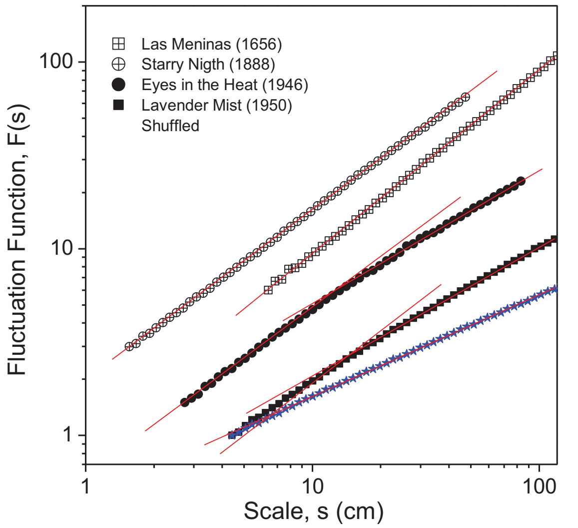

Fig. 4. DFA results for two samples of Pollock's paintings. Note the multi-scaling behavior with a crossover at about s_{cr} separating two different scaling behaviors. For Pollock's paintings, the short- and large-scale Hurst exponents are 0.82 and 0.67 for *Lavender Mist*; 0.89 and 0.70 for *Eyes in the Heat*. Velazquez's *Las Meninas* and van Gogh's *Starry Night* paintings exhibited monoscaling behavior with Hurst exponent given by 0.98 and 0.91, respectively. The scaling behavior is consequence of the intrinsic painting structure, as indicated by the lower plot corresponding to shuffled gray-scale data. In this case, the Hurst exponent $H \approx 0.5$ that corresponds to uncorrelated scaling behavior. The DFA results for the figurative Velazquez's *Las Meninas* and van Gogh's *Starry Night* paintings do not exhibit the multiscaling behavior displayed by Pollock's paintings.

sake of comparison with other cases, the number of covering boxes was normalized by the respective number for the smallest spatial scale. Fig. 2 also shows the cases of Velazquez's Las Meninas and van Gogh's Starry Night, which correspond to figurative and quasi-figurative styles, respectively. The slope of the log–log plot of the box counting results corresponds with the fractal dimension D_{BC}. It is noted that the nonfigurative Pollock's paintings exhibit a greater fractal dimension than the figurative and quasi-figurative counterparts. Fig. 3 presents the fractal dimension D_{BC} values plotted against the year the paintings were performed. It is shown that the fractal dimension increased with the years, indicating that Pollock's technique evolved to explore more fractal random painting structures. The evolution of the fractal dimension can be described with an exponential function with an asymptotic value at about $D_{BC} = 2.7$. Interestingly, this stable value was achieved by 1945 when Pollock moved to Springs, Long Island. Taylor et al. [32] have indicated that such transition could be attributed to the contact of Pollock with an environment surrounded by complex patterns of nature. The fractal dimension of Pollock's paintings evolved to complex patterns as figurative style was gradually abandoned to pursue a more erratic paint distribution by means of pouring paint blobs on the canvas lying down on the studio flour. In fact, the result in Fig. 3 suggests that Pollock's technique evolved continuously from a quasi-figurative style with relatively low fractal dimension to an abstract structure with high fractal dimension.

3.2. Hurst exponent

Fig. 4 presents the DFA results for two famous paintings of the dripping period. The painting *Eyes in the Heat* (1946) corresponds to the first year of the period and Lavender Mist (1950) was painted when Pollock had mastered the dripping technique. The two-dimensional DFA reveals that Pollock's paintings exhibit scaling behavior for a wide range of scales. In terms of physical dimensions, this range corresponds to scales from centimeters to meters. It is noted that the paintings contain two distinct scaling behaviors at different scale domains, with a crossover s_{cr} separating short- and large-scale scaling behavior. The Hurst exponent for short scales is larger than the Hurst exponent for large scales, a result already reported by Taylor et al. [29]. Although only two cases are displayed in Fig. 4, this scaling pattern was also found for Pollock's paintings of all the periods. It is apparent that this is a distinctive feature of Pollock's technique arising from intrinsic painting structure. In fact, Fig. 4 also shows the DFA result when the gray-scale luminance is randomly shuffled, showing a scaling behavior with $H \approx 0.51$ corresponding to uncorrelated patterns. This result shows that the scaling behavior in Pollock's paintings is induced by the intrinsic array of colors, reflected in the luminance, and not to the frequency distribution of gray-scale levels.

The DFA applied to the gray-scale patterns provides relevant information, in terms of a fractal scaling exponent, about the internal organization of the paintings. Following Taylor et al. [29], it is apparent that the different scaling behavior for short and large scales resulted from the interaction of two distinct physical processes. The large-scale patterns resulted from Pollock's Levy flights (a Levy flight is a combination of discrete, random jumps coupled with local fractal Brownian motion) across the canvas and the short-scale structure can be attributed to the painting technique that is largely dependent on the tools (e.g., brush, easel, etc.) and

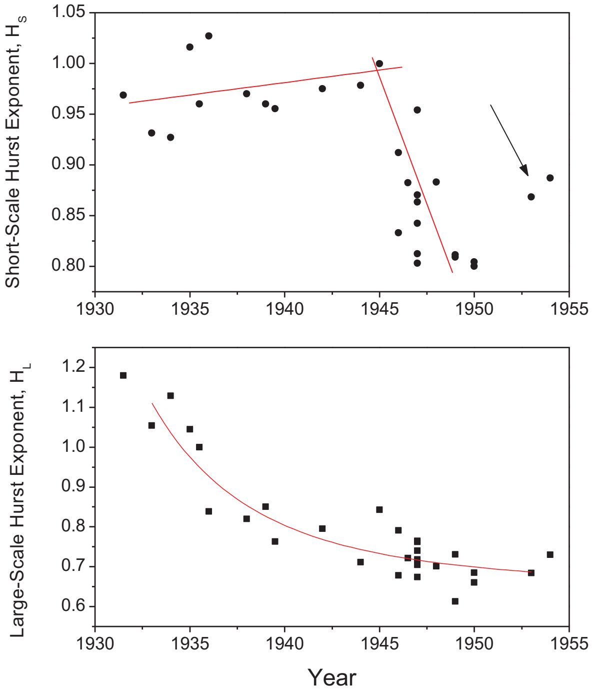

Fig. 5. Short- and large-scale Hurst exponents for the Pollock's paintings. It is apparent that the large-scale Hurst exponent decays gradually to achieve an asymptotic value of about $H_L = 0.69$. In contrast, the short-scale Hurst exponent is not monotonous, showing an important shift from increasing to abrupt decreasing at 1945-1946.

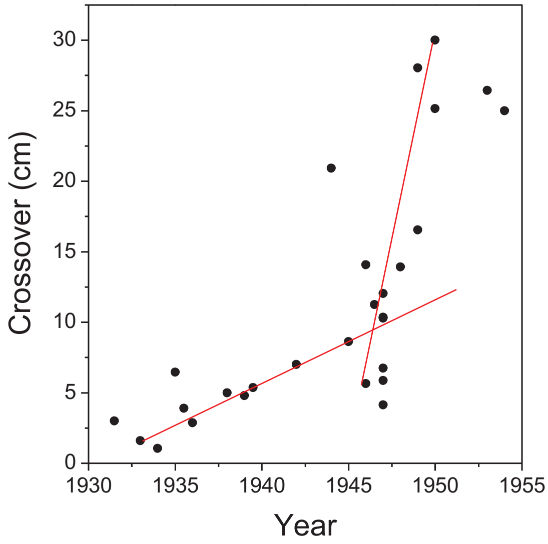

Fig. 6. Evolution of the crossover that separates small- and large-scale fractal behaviors. A shift located at about 1945-1946 is observed.

the physical characteristics of the painting material (viscosity, absorption into the canvas, etc.).

Fig. 4 also shows the DFA results for two famous paintings; namely, Velazquez's *Las Meninas* (1656) and van Gogh's *Starry Night* (1888). The first painting corresponds to the Spanish Golden Age and the second one to the European Post–impressionism Movement. Fig. 4 shows that *Las Meninas* and *Starry Night* paintings are more spatially correlated than Pollock's paintings, with about $H = 1.1$ for the former and about $H = 0.65$ for the latter in the large-scale range. This suggests that, at least for two figurative cases, Pollock's paintings are more complex in terms of spatial patterns than figurative and quasi-figurative paintings. It is apparent that weakly correlations, reflecting a quasi-random fractal pattern, and multiscaling, represented by a change in the scaling behavior at about $s_{cr} = 10 - 15$ cm, are two important features of Pollock's paintings. Weak short- and large-scale correlations and the crossover value will be used to track the evolution of Pollock's painting along the period from 1930 to 1955.

Fig. 5 presents the short- and large-scale Hurst exponent as a function of the year. As already noted, the short-scale exponent is larger than the large-scale exponent, meaning that large-scale patterns are more random than short-scale patterns [29]. This effect could be attributed to the some regularity of paint blobs, which when ensemble into a large-scale pattern are randomly distributed. The large-scale Hurst exponent can be described as an exponentially decreasing function with asymptotic value at about $H_L = 0.69$. On the other hand, the short-scale Hurst exponent is not monotonous, showing a transition from increasing to decreasing by 1945. Note the drastic decrement of H_S after this transition time, going from 1.0 to 0.8 in about two years. Also note the increment, indicated by an arrow, of H_S in the last years, coinciding with Pollock's return to a quasi-figurative style. It should be remarked that a similar increment of the fractal dimension around 1945 was previously detected for black blobs [29, 32]. In fact, it was shown (see Fig. 3a by Taylor et al. [29] and Fig. 3 by Taylor et al. [32]) an increment from about 1.1 to 1.7 after Pollock moved to Long Island. Fig. 6 exhibits the crossover value in physical dimensions, showing an increasing behavior with a transition at 1945-1946 when the crossover jumped from about 5 cm to about 30 cm by 1950. A weak decrement is observed for the last Pollock's years. While the result by Taylor et al. [32] was based on black paint blobs, our result was based on the entire luminance pattern. This explains the fact that the shift showed in Fig. 5 of our work is sharper than the shift reported in Fig. 3 by Taylor et al. [32]. On the other hand, the fact that the two approaches, namely back paint blobs and luminance pattern, led to the same result indicate that the 1945 fractality transition is really linked to a significant evolution of Pollock's drip painting.

It has been shown that for non-intersecting self-affine surfaces the Hurst index is related to the box-counting fractal dimension D_{BC} as $H = 3 - D_{BC}$ [13]. Deviations from this equality indicate deviations from a non-intersecting fractal surface. To asses these deviations, a DFA correlations fractal dimension was defined as $D_{DFA} = 3 - H_L$ and the results are shown in Fig. 7a. As in the case of the box-counting fractal dimension, the DFA dimension is an increasing function of time and can be described with an exponential function. However, important differences with respect to the box-counting dimension are observed, as shown in Fig. 7b. Note that the difference $D_{BC} - D_{DFA}$ is a decreasing function for the period from 1930 to about 1937. Afterwards, the fractal dimension difference stabilized at about 0.4.

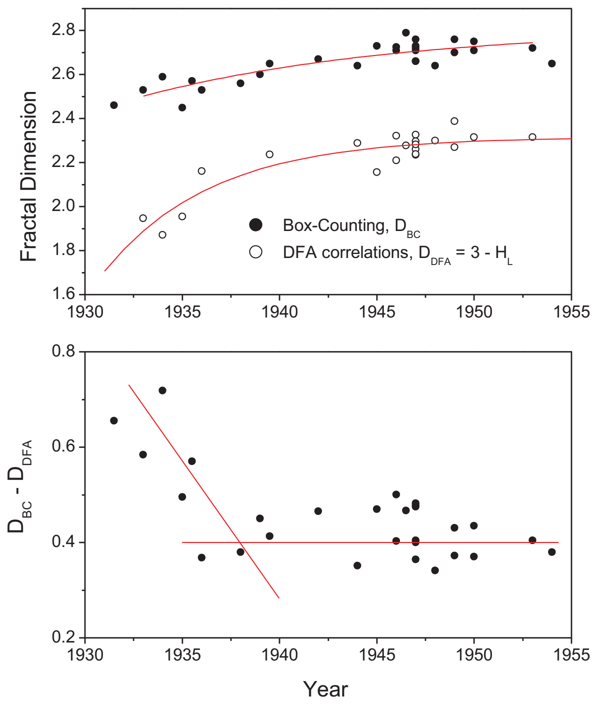

Fig. 7. (a) Box-counting and DFA correlations fractal dimensions exhibiting a monotonous increasing behavior. Note that $D_{BC} > D_{DFA}$, suggesting that Pollock's painting have self-intersecting effects. (b) The difference $D_{BC} - D_{DFA}$ decreased from 1930 to about 1937. After 1937, it is apparent that this difference stabilized at about 0.4.

4. Discussion

The DFA results of Pollock's paintings showed that the fractality indices and parameters are not constant, but changed along the period from 1930 to 1955. Some changes are gradual, indicating a continuous learning and adjusting of his painting technique. Other changes are more abrupt, reflected as shifts of the Hurst exponent and the crossover dividing different scaling behavior. Is it possible to relate these changes to different events in Pollock's painting career? In the following, we will make an effort to relate the changes in fractality with some major events as related by art historians and biographers [4,17,24].

- Fig. 7b showed that the fractal dimension difference $D_{BC} - D_{DFA}$ decreased from 1930 to about 1937. Afterwards, this difference stabilized at about 0.4. As mentioned before, for non-intersecting self-affine surfaces the Hurst exponent is related to the box-counting fractal dimension of surface D_{BC} as $H = 3 - D_{BC}$. Hence, for this type of fractal surfaces, one should have $D_{BC} - D_{DFA} = 0$. The inequality $D_{BC} - D_{DFA} > 0$ indicates that paintings are intersecting surfaces. In fact, fractal dimension is a local property, and long-term spatial dependence detected by DFA is a global characteristic. If $D_{BC} \neq D_{DFA}$, local properties are not necessarily reflected in the global ones [13]. Large values of the difference $D_{BC} - D_{DFA}$ indicate that the local features are less important than global features within the painting structure. In contrast, small values of $D_{BC} - D_{DFA}$ reflect a widespread influence of local features in the global painting composition - the global is constructed from the local and vice versa. The continuous decrement in the fractal difference $D_{BC} - D_{DFA}$ from 1930 to 1937 suggests that Pollock was not stacked in a well-defined painting style, but continuously experimented with forms, structures and techniques. Maybe, the continuous experimentation was motivated by Thomas Hart Benton influence that shaped Pollock's technique in his rhythmic use of paint. However, was there an event for explaining the stabilization of $D_{BC} - D_{DFA}$ by 1937? Pollock was introduced to the use of liquid paint in 1936, at an experimental workshop operated in New York City by the Mexican muralist David Alfaro Siqueiros. Physically, the application of liquid paint allowed a more easy dispersion of colors which, in turn, provoked a complex entanglement of local structures to induce a global surface morphology. In this form, it can be conjectured that the usage of liquid paint as an important technical element stabilized the intrinsic luminance correlations in Pollock's paintings.
- Fig. 5 showed that the short-scale Hurst exponent increased slightly from 1930 to about 1945. In 1945, the Hurst exponent H_S started a rapid decrement as Pollock refined his technique to achieve the maximum flourishing in 1950. This is in agreement with previous Taylor et al.'s results (see Fig. 3a by Taylor et al. [29] and Fig. 3 by Taylor et al. [32]) that showed an important increment in the fractal dimension of color blobs. In fact, it was shown that the fractal dimension, estimated with box-counting methods, of the black blobs increased from about 1.1 to 1.7 by 1945. Our results, together with previous ones [32], indicate that the fractal dimension change by 1945 is not only due to particular color structures, but also to the effect of a change in the composition of the whole painting structure. The decrement of the short- and large-scale Hurst exponents reflects that Pollock refined his drip technique by reducing the correlations at both local and global structures. It is apparent that one aim of Pollock's painting technique, via Levy flight movements [27,28], was to link the local and the global structures within a unifying structure. It is commonly accepted that the spontaneous change from quasi-figurative into a full non-figurative painting style was induced by the move of Pollock and Krasner to the natural landscapes in Springs, Long Island by 1945. It was there that he perfected the technique of working spontaneously with liquid paint by using hardened brushes, sticks, and even basting syringes as paint applicators. The pouring of large masses of paint without following a traditional figurative pattern led Pollock to generate structures resembling that of the nature scenarios. Blobs [32] and luminance patterns were conjugated within a singular nonfigurative technique giving complex aesthetic geometric arrangements.
- Fig. 6 shows that the crossover also suffered a drastic increment starting in 1945. This means that, by refining his dripping technique, Pollock was able to extend the local fractal structures into larger surfaces. This was possible by abandoning the convention of painting on an upright surface and moving on the floor surface, which allowed the application of paint from all directions in canvas of large dimensions from 2 to 3 meters. As remarked by Taylor et al. [27,28], Pollock obtained this effect by moving

literally in all canvas directions following a Levy flight dance and using his whole body to paint.

- At the peak of his fame, Pollock abruptly abandoned the drip style. Pollock's work after 1951 was darker in color, including a collection painted in black on unprimed canvases. This was followed by a return to color [24], and he reintroduced figurative elements. This return to a more figurative painting is reflected by an increment of the short-term Hurst exponent, indicated by an arrow in Fig. 5, and a decrement of the crossover value that reflects the usage of traditional painting techniques.

In the recent years, the fractality of Pollock's paintings has motivated diverse studies on complexity of visual patterns in art and landscapes. Zheng et al. [34] proposed a layered approach to implement a computer model Pollock's dripping style. Machado et al. [12] used machine learning techniques to predict the average visual complexity of images. Taylor et al. [30] conducted a 10 years study on the perceptual and physiological responses to Pollock's fractals, concluding that art might have deep effects in the perceptual, physiological and neurological condition of the observer. Shamir [22] found that the uniqueness of Pollock's drip painting style is not reflected merely by fractals, but also by other numerical image content descriptors that reflect the visual content. Overall, these reports showed that the deep complexity of Pollock's painting technique is still motivating studies on aesthetic perception, computation algorithms for visual art and brain dynamics.

5. Conclusions

The main conclusions drawn from the fractality study of Pollock's paintings in this work are the following: (a) The analysis of luminance patterns, rather than of color blobs, corroborate the fractality of Pollock's paintings, with increased fractality indices during the dripping and pouring phase in the late 1940s. (b) All Pollock's paintings are multiscaling, including those from the period from 1930 to 1945. The multiscaling degree was more important in the drip phase. (c) The results suggest that the evolution of Pollock's painting technique was formed by continuous learning, cultural and technical influences (e.g., the use of liquid painting by Mexican muralist Siqueiros) and personal events (e.g., the move to Springs on Long Island). (d) As previously observed [31], it is apparent that large-scale fractality indices are not enough for authenticating Pollock's drip paintings as similar values are shared by non-Pollock's paintings (e.g., Les Automatistes group). Overall, our results provide more scientific evidence of the complex and fascinating structured of Pollock's artistic work.

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