# Lab #1 : Languages

## Exercise 1

Given an alphabet $\Sigma$, a *substring* of a string $u \in \Sigma^*$ is a string $w \in \Sigma^*$ such that $\exists x, y \in \Sigma^*$ with $u = xwy$. If $|\Sigma| = 3$, what is the mininimum and maximum number of substrings of a string of length 7?

## Exercise 2

Given an alphabet $\Sigma$ such that $|\Sigma| = n$ and $x \in \Sigma$, calculate:

1. the number of strings of lenght $p$ $(p > 0)$
2. the number of strings of lenght $p$ $(p > 0)$ with at least one occurence of $x$
3. the number of strings of lenght $p$ $(p > 0)$ with exactly one occurence of $x$
4. the number of strings of lenght $p$ $(p > 0)$ with exactly $q$ occurences of $x$ $(q > 0)$

## Exercise 3

Prove that, for three languages $A$, $B$ and $C$ over some alphabet $\Sigma$, $A.(B \cup C) = A.B \cup A.C$

Find three languages $A$, $B$ and $C$ over some alphabet $\Sigma$ such that $A.(B \cap C) \neq A.B \cap A.C$

## Exercise 4

Is there any language $L$ over some alphabet such that $L^*$ is finite?

## Exercise 5

Prove that, for any language $L$ over some alphabet $\Sigma$, $L^*$ and $(\overline{L})^*$ cannot be **both** finite.

## Exercise 6

An infinite set is said to be *countable* if it has a bijection with the natural numbers. Given an alphabet $\Sigma$, prove that $\Sigma^*$ is countable (*hint: any infinite subset of the natural numbers is countable*).

## Exercise 7

Given the alphabet $\Sigma = \{0, 1\}$ and $L = \{00, 01, 10, 11\}$ over $\Sigma$, prove that $L^* = \{w \in \Sigma^* : |w| \text{ is even}\}$.

Could we find a language $X$ over some alphabet $\Sigma$ such that $X^*$ is the language of all strings of $\Sigma^*$ with odd length?