Lab #2 : DFAs

Exercise 1

For $\Sigma = \{0, 1\}$, give DFA's for the languages on Σ^* consisting of:

1. all strings with no 0
2. all strings with exactly one 0
3. all strings with at least one 0
4. all strings with no more than three 0

Exercise 2

For $\Sigma = \{a, b, c\}$, give DFA's for the languages:

1. $L = \{w \in \Sigma^* : |w|_a \text{ is even}\}$
2. $L = \{w \in \Sigma^* : |w|_a \text{ is even and } |w|_b \text{ is odd}\}$
3. $L = \{w \in \Sigma^* : \ baba \text{ is a suffix of } w\}$
4. $L = \{w \in \Sigma^* : \ baba \text{ is a factor of } w\}$

Exercise 3

For $\Sigma = \{a, b, c\}$, give DFA's for the languages:

- $L_1 = \{w \in \Sigma^* : \ bac \text{ is a prefix of } w\}$
- $L_2 = \{w \in \Sigma^* : \ bac \text{ is not a prefix of } w\}$

What can you conclude?

Exercise 4

For $\Sigma = \{0, 1\}$, give a DFA for the language on Σ^* of strings that have a 1 in every even-numbered position. Positions are numbered starting at 1. For example, the strings 0101 and 01011 are in this language.

Generalize and give a DFA for the language of strings that have a 1 in every k^{th} position, starting with position number k ($k \geq 1$).

Exercise 5

For $\Sigma = \{a, b\}$, give a DFA for the language

$$L = \{w \in \Sigma^* : w \text{ has exactly two } a\text{'s and more than two } b\text{'s}\}$$

Justify the number of states of your DFA. Explain how we can generalize this construction to a language over Σ^* of the strings which have exactly n a's and more than m b's.

Exercise 6

For $\Sigma = \{a, b, c\}$, give a **minimal** DFA for the language

$$L = \{w \in \Sigma^* : \text{ the two } \textit{last} \text{ symbols of } w \text{ are different}\}$$

Explain why your DFA is **minimal**.

Exercise 7

For $\Sigma = \{0, 1\}$, give a DFA for the language

$$L = \{w \in \Sigma^* : \text{ the } \textit{third} \text{ symbol from the right of } w \text{ is } 0\}$$

For example, 011011, 00000 and 101010 are in L although 00, 11100 and 000111 are not.

What is the number of states of the previous DFA? What is the number of states of the DFA for the language

$$L = \{w \in \Sigma^* : \text{ the } k^{th} \text{ symbol from the right of } w \text{ is } 0\}$$

Explain your answer!

Exercise 8

Given an alphabet $\Sigma = \{a_1, a_2, .., a_n\}$, give the formal definition of a DFA for the language L of strings over Σ^* which start and end with the same symbol.

(hint: describe, in function of $n = |\Sigma|$, the quintuple $M = (Q, \Sigma, \delta, q_0, F)$ such that $L(M) = L$)

Exercise 9

Let us define an operation T (like Truncate) which removes the rightmost symbol from any string. For example, $T(aaaba)$ is $aaab$. The operation can be extended to languages by

$$T(L) = \{T(w) : w \in L\}$$

Show how, given a DFA for any language L over an alphabet Σ, one can construct a DFA for $T(L)$. Give examples using some of the DFA from previous exercises.

Exercise 10

Construct a DFA that accepts strings on the alphabet $\Sigma = \{0, 1\}$ if and only if the value of the string, interpreted as a binary representation of an integer, is zero modulo five. For example, 0101 and 1111, representing the integers 5 and 15, respectively, are to be accepted.

Exercise 11

A run in a string is a substring of length at least two, as long as possible and consisting entirely of the same symbol. For instance, the string $abbbaab$ contains a run of b's of length three and a run of a's of length two. Find a DFA for the following language on the alphabet $\Sigma = \{a, b\}$:

$$L = \{w \in \Sigma^* : w \text{ contains no runs of length less than four }\}$$