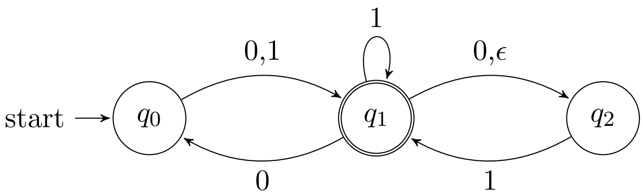
Lab #3 : NFAs

Exercise 1

Which of the strings 00, 01001, 10010, 000, 0000 are accepted by the following NFA?



Exercise 2

For $\Sigma = \{a, b, c\}$, give NFA's for the languages:

- $L_1 = \{w \in \Sigma^* : \; bac \text{ is a prefix of } w\}$
- $L_2 = \{w \in \Sigma^* : \; bac \text{ is a suffix of } w\}$
- $L_3 = \{w \in \Sigma^* : \; bac \text{ is a factor of } w\}$

Compare the NFA's obtained with the DFA's from lab #2.

Exercise 3

For $\Sigma = \{a, b, c\}$, we define the language L as follow:

$$L = \{ab, abc\}^*$$

- give a DFA for the language L
- design an ϵNFA with exactly **three** states for the language L
- remove the ϵ-transition from the previous ϵNFA and compare the resulting NFA with the previous DFA

Exercise 4

For $\Sigma = \{a, b\}$, we define the language L as follow:

$$L = \{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$$

- give an NFA with no more than **five** states for L
- give a DFA for L
- construct a DFA for L from the previous NFA using the NFA-to-DFA algorithm reviewed in class
- compare the two DFA's

Exercise 5

Given an alphabet $\Sigma = \{a_1, a_2, .., a_n\}$, we define the language L_n of strings over Σ^* as follow:

$$L_n = \{w \in \Sigma^* : \text{ the two } \textit{last} \text{ symbols of } w \text{ are different}\}$$

- give an NFA for L_3 with $\Sigma = \{a, b, c\}$
- construct a DFA for L_3 from the previous NFA using the NFA-to-DFA algorithm reviewed in class and compare this DFA with the one from exercice 6 of lab #2
- give the formal definition of a NFA for L_n for any n

Exercise 6

Given an alphabet $\Sigma = \{a_1, a_2, .., a_n\}$, an integer $k \geq 1$ and $a \in \Sigma$ a symbol of the alphabet, we define the language $L_{k,a}$ of strings over Σ^* as follow:

$$L_{k,a} = \{w \in \Sigma^* : \text{ the } k^{th} \text{ symbol from the right of } w \text{ is } a\}$$

- give an NFA for $L_{3,0}$ with $\Sigma = \{0, 1\}$
- construct a DFA for $L_{3,0}$ from the previous NFA using the NFA-to-DFA algorithm reviewed in class and compare this DFA with the one from exercice 7 of lab #2
- give the formal definition of a NFA for $L_{k,a}$ for any $\Sigma = \{a_1, a_2, .., a_n\},\ k \geq 1,\ a \in \Sigma$

Exercise 7

For $\Sigma = \{a, b, c\}$, we define the language L as follow:

$$L = \{w \in \Sigma^* : w \text{ does not contain the factor } bac\}$$

- give a DFA M for L
- define formally the language L^R
- give a ϵNFA N_R for the language L^R and give convincing arguments to show that $L(N_R) = L^R$
- remove the ϵ-transition from N_R to get the NFA N'_R
- construct a DFA M_R for L^R from N'_R using the NFA-to-DFA algorithm reviewed in class and compare M and M_R
- give convincing arguments to show that $L(M)^R = L(M_R)$

Exercise 8

From a language L over an alphabet Σ we create a new language $C_k(L)$ for some $k \geq 1$ by removing the k leftmost symbols of every string in L. Specifically,

$$C_k(L) = \{w \in \Sigma^* : vw \in L, \text{ with } |v| = k\}$$

Show that if L is regular, then $C_k(L)$ is also regular. Give a detailed example with $\Sigma = \{a, b\}$, $k = 2$ and L being the following language:

$$L = \{w \in \Sigma^* : w \text{ has the suffix } aaa\}$$