Lab #6 : The pumping lemma

Exercise 1

Prove that the language L on $\Sigma = \{a, b\}$ defined by

$$L = \{w \in \Sigma^* \mid |w|_a < |w|_b\}$$

is not regular

Exercise 2

Prove that the language L on $\Sigma = \{a, b\}$ defined by

$$L = \{w \in \Sigma^* \mid w = (ab)^n a^k \ n > k, k \geq 0\}$$

is not regular

Exercise 3

Prove that the language L on $\Sigma = \{a, b, c\}$ defined by

$$L = \{w \in \Sigma^* \mid w = a^n b^k c^{n+k} \ n \geq 0, k \geq 0\}$$

is not regular

Exercise 4

Prove that the language L on $\Sigma = \{a, b\}$ defined by

$$L = \{w \in \Sigma^* \mid \exists u \in \Sigma^*, \ w = uu^R\}$$

is not regular

Exercise 5

The language L on $\Sigma = \{a, b, c\}$ is defined as follow:

- $L_1 = \{a^{2i} b^j c^j \mid i \geq 1, j \geq 0\}$
- $L_2 = b^* c^*$
- $L = L_1 \bigcup L_2$

Prove that:

- the language L verify the Pumping Lemma
- the language L is not regular

What can we conclude about the Pumping Lemma?

Exercise 6

Given L_1 and L_2 the following languages:

- $L_1 = \{a^i \mid i \text{ is a perfect square } \}$
- $L_2 = \{w \in \{a, b, c\}^* \mid |w|_a + |w|_b \text{ is a perfect square}\}$

Show that neither L_1 nor L_2 are rational.

Exercise 7

We proved in class that the language $L = \{w \in \{0, 1\}^* \mid w = 0^n 1^n,\ n \geq 0\}$ was not regular. Prove that the language

$$L' = \{w \in \{0, 1\}^* \mid w = 0^n 1^m,\ n \neq m\}$$

is not regular.