Lab #7 : Context-free Grammars

Exercise 1

Given the context-free grammar G:

$$\begin{array}{lcl} S & \longrightarrow & aB \mid bA \\ A & \longrightarrow & a \mid aS \mid bAA \\ B & \longrightarrow & b \mid bS \mid aBB \end{array}$$

- give a left derivation to produce the string $aaabbabbba$
- give a right derivation to produce the string $aaabbabbba$
- give a parse tree to produce the string $aaabbabbba$
- describe $L(G)$, the language produced by G. Is G a regular language?

Exercise 2

Given the context-free grammar G:

$$\begin{array}{lcl} S & \longrightarrow & aAa \\ A & \longrightarrow & Sb \mid bBB \\ B & \longrightarrow & abb \mid aC \\ C & \longrightarrow & aCA \end{array}$$

- simplify G
- convert G into Chomsky normal form
- give a parse tree to produce the string $aababbabbaba$

Exercise 3

Given the context-free grammar G:

$$\begin{array}{lcl} S & \longrightarrow & aB \mid ab \\ A & \longrightarrow & aAB \mid a \\ B & \longrightarrow & ABb \mid b \end{array}$$

Prove that the grammar G is ambiguous.

Exercise 4

For each grammar below, prove that the grammar is ambiguous and find an equivalent unambiguous grammar:

$$\begin{array}{ll} G_1: & S \longrightarrow SS \mid a \mid b \\ G_2: & S \longrightarrow ABA,\ A \longrightarrow aA,\ B \longrightarrow bB \\ G_3: & S \longrightarrow aSb \mid aaSb \mid \epsilon \\ G_4: & S \longrightarrow aSb \mid abS \mid \epsilon \end{array}$$

Exercise 5

Find context-free grammars for the following languages:

- $L_1 = \{w \in \{a, b, c\}^* \mid w \text{ starts and ends with two different symbols}\}$
- $L_2 = \{w \in \{0, 1\}^* \mid \text{every substring of } w \text{ of length 5 has at least one } 0\}$
- $L_3 = \{w \in \{a, b\}^* \mid |w| \text{ is odd and the middle symbol is } a\}$
- $L_4 = \{w \in \{a, b\}^* \mid |w| \text{ is odd and the first, middle and last symbols are all the same}\}$

Tell which of L_1, L_2, L_3 and L_4 are regular languages.

Exercise 6

Find context-free grammars for the following languages:

- $L_1 = \{w \in \{a, b, c\}^* \mid w = a^n b^n c^m,\ n, m > 0\}$
- $L_2 = \{w \in \{0, 1\}^* \mid w = zz',\ z' \neq \overline{z},\ |z| = |z'|\}$
- $L_3 = \{w \in \{a, b, c\}^* \mid w = a^i b^j c^k,\ i = j \text{ or } j = k\}$

Exercise 7

Find a context-free grammar for the language L of *true* arithmetic expressions over the alphabet $\Sigma = \{1, 2, +, =\}$. For example, the following strings are in L:

- $1 + 1 = 2$
- $1 + 2 = 1 + 2$
- $1 + 2 + 1 = 2 + 2$

Exercise 8

Prove that the set of all context-free languages is closed under the union, the concatenation and the star operations.

Exercise 9

Given G the grammar of exercise 1, prove by induction on the length of the strings that $L(G) = \{w \in \{a, b\}^* \mid |w| \geq 2, |w|_a = |w|_b\}$

Hint: you should prove the following three properties by induction on the length of w:

- $S \overset{*}{\Longrightarrow} w \iff w$ has the same number of a's and b's
- $A \overset{*}{\Longrightarrow} w \iff w$ has one more a than b's
- $B \overset{*}{\Longrightarrow} w \iff w$ has one more b than a's