Lab #5 : Regular expressions and regular languages

Exercise 1

Prove or disprove that:

$$(a+b)^* = (b^*a^*)^*$$

Exercise 2

Find regular expressions to denote the following languages:

- $L_1 = \{w \in \{a,b\}^* : w = a^n b^m, n \geq 3, m \text{ is even } \}$
- $L_2 = \{w \in \{a,b\}^* : w = a^n b^m, (n+m) \text{ is even } \}$
- $L_3 = \{w \in \{0,1\}^* : w \text{ has no pair of consecutive zeros } \}$
- $L_4 = \{w \in \{0,1\}^* : w \text{ has exactly one pair of consecutive zeros } \}$
- $L_5 = \{w \in \{a,b\}^* : |w| \text{ is a multiple of } 3\}$
- $L_6 = \{w \in \{a,b\}^* : |w|_a \text{ is a multiple of } 3\}$

Exercise 3

Describe in English the regular language denoted by the following regular expression:

$$(aa)^*b(aa)^* + a(aa)^*ba(aa)^*$$

Exercise 4

Construct a DFA for each of the following languages:

- $L_1 = (aa)^* \bigcap ((aaa)^*(a+aa))$
- $L_2 = (0+11)^* \bigcap (01+10)^*$
- $L_3 = (0+11)^* - (01+10)^*$

Exercise 5

The *nor* and the *cor* operations of two languages are defined by:

$$nor(L_1, L_2) = \{w \ : \ w \notin L_1 \text{ and } w \notin L_2\}$$

$$cor(L_1, L_2) = \{w \ : \ w \in \overline{L_1} \text{ or } w \in \overline{L_2}\}$$

Show that the family of regular languages is closed under the *nor* and the *cor* operations.

Exercise 6

If L is a regular language, prove that the following languages are regular:

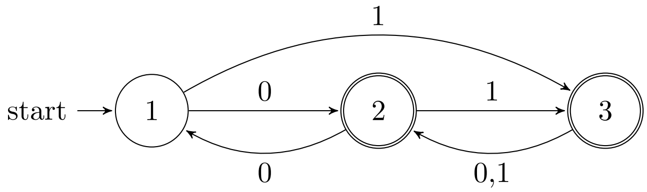
- $L_1 = \{uv \ : \ u \in L, \ |v| = 2\}$
- $L_2 = \{uv \ : \ u \in L, \ v \in L^R\}$

Exercise 7

Show that if the statement "If L_1 is regular and $L_1 \bigcup L_2$ is also regular, then L_2 must be regular" were true for all L_1 and L_2, then all languages would be regular.

Exercise 8

Let M be the following DFA over $\{0, 1\}^*$:



Give a regular expression to denote $L(M)$.