

$$(6) \quad ds^2 = a^2(t) [d\chi^2 + r^2(\chi) (d\vartheta^2 + \sin^2\vartheta d\varphi^2)] - c^2 dt^2$$

$$r = \chi \quad : \quad \epsilon = 0$$

$$r = \sin \chi \quad : \quad \epsilon = 1$$

$$r = \sinh \chi \quad : \quad \epsilon = -1$$

$$\chi = \text{const}, \quad t = t_e = \text{const}, \quad \Delta l \ll a(t)$$

$$\Rightarrow ds^2 = dl^2 = a^2(t_e) r^2(\chi) (d\vartheta^2 + \sin^2\vartheta d\varphi^2) \\ = a^2(t_e) r^2(\chi) d\vartheta^2$$

$$\Rightarrow \Delta\vartheta = \frac{\Delta l}{a(t_e) r(\chi)}$$

$$\text{case } \epsilon = 0: \quad a(t_e) \chi = D(t_e), \quad \Delta\vartheta = \frac{\Delta l}{D(t_e)}$$

$$\text{case } \epsilon = 1: \quad r(\chi) = \sin \chi = \sin \frac{D}{a} \Rightarrow \Delta\vartheta = \frac{\Delta l}{a \sin \frac{D}{a}}$$

$$D \ll a: \quad \Delta\vartheta \approx \frac{\Delta l}{D(t_e)} \quad (\text{reduces to } \epsilon = 0)$$

$$D = \pi a: \quad \Delta\vartheta = \infty$$

$$D < \frac{\pi a}{2}: \quad \text{Galaxy becomes smaller when more far away}$$

$$D > \frac{\pi a}{2}: \quad \text{Galaxy becomes bigger again}$$

$$\text{case } \epsilon = -1: \quad r(\chi) = \sinh(\chi) = \sinh \frac{D}{a}, \quad \Delta\vartheta = \frac{\Delta l}{a \sinh(\frac{D}{a})}$$

$$D \ll a: \quad \text{analog to } \epsilon = 0$$

$$D \gg a: \quad \Delta\vartheta \approx \frac{2\Delta l}{a} e^{-\frac{D}{a}}$$

(becomes smaller really fast)  $\rightarrow$  form of point

$$\Rightarrow D_H \lesssim a \quad \text{because we do not observe the other cases}$$