

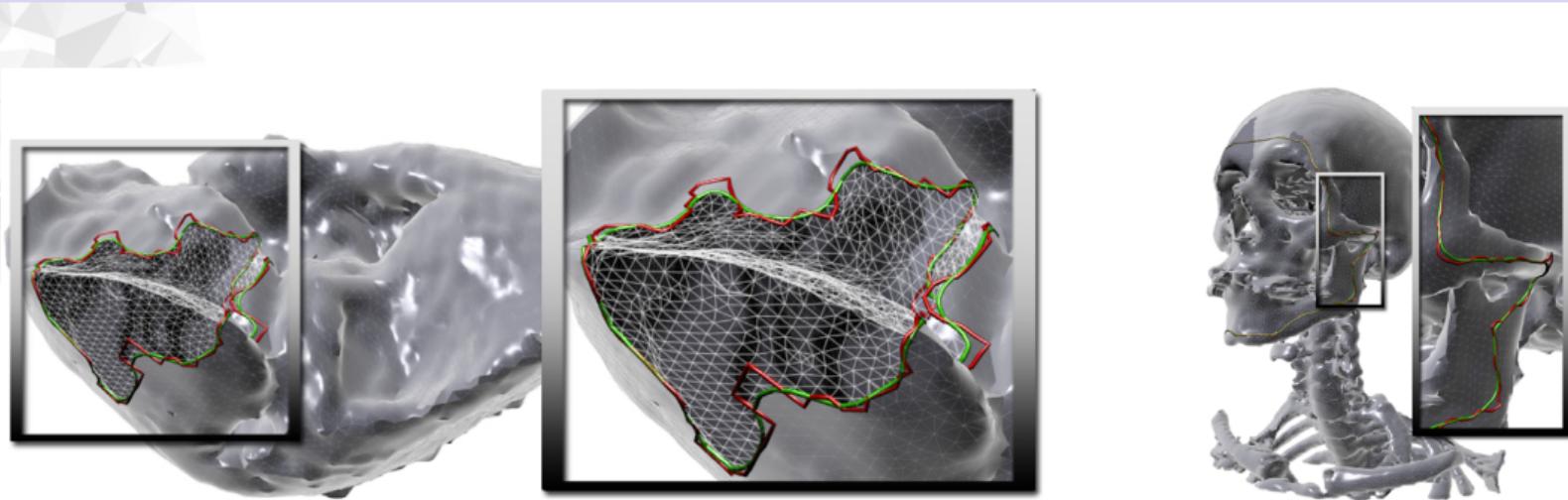
Curve Smoothing on Surface Meshes

Markus Pawellek

August 19, 2023

Introduction and Background

Introduction and Background: Reference

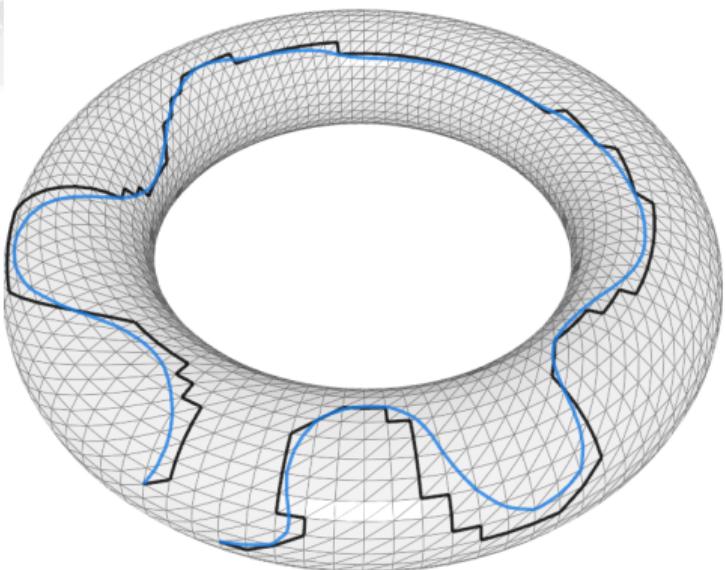


Kai Lawonn, Rocco Gasteiger, Christian Rössl, and Bernhard Preim.

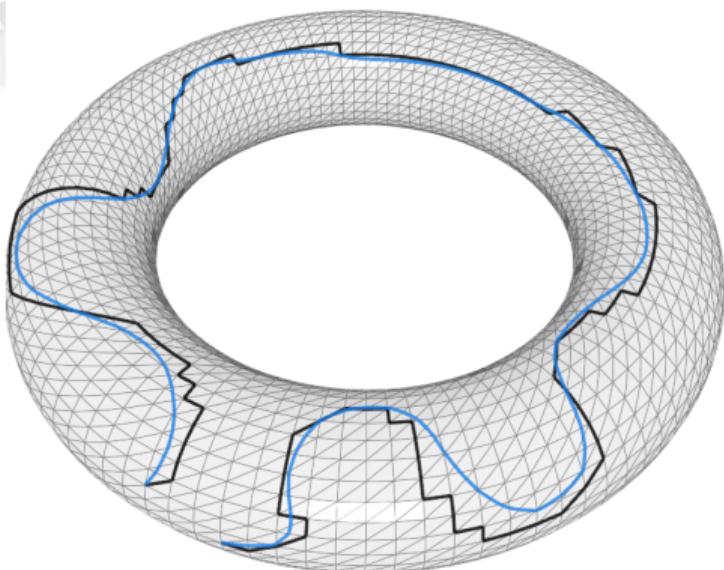
“Adaptive and Robust Curve Smoothing on Surface Meshes”. In:

Computers & Graphics 40 (2014), pp. 22–35. DOI: 10.1016/j.cag.2014.01.004

Introduction and Background: Goals

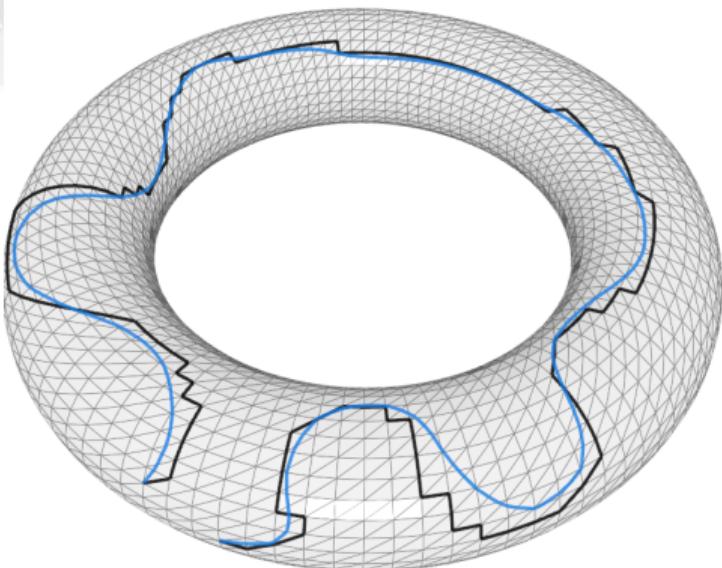


Introduction and Background: Goals



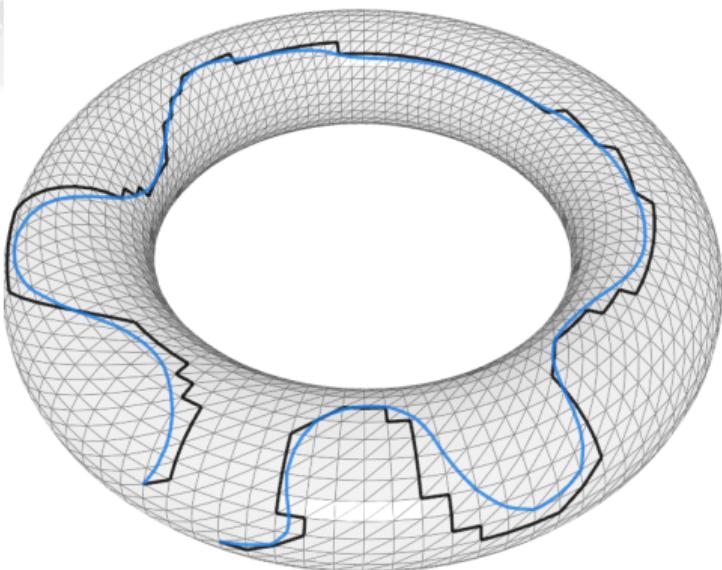
- ▶ Closeness to Initial Curve

Introduction and Background: Goals



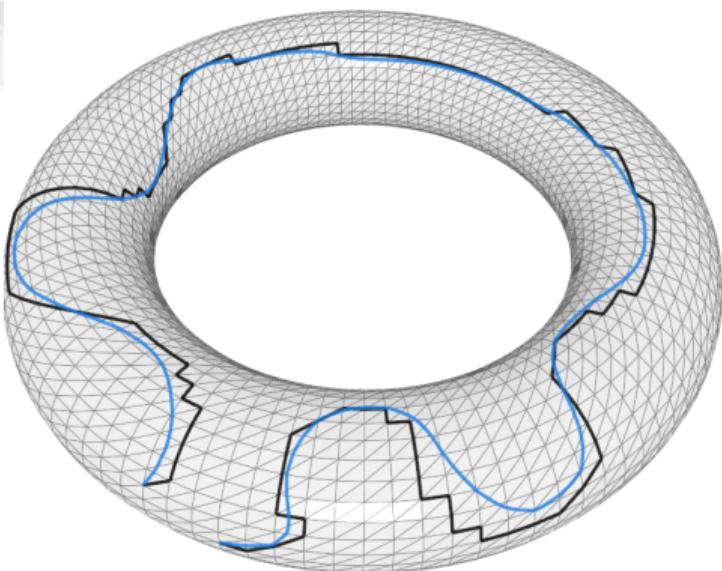
- ▶ Closeness to Initial Curve
- ▶ Balancing Closeness and Smoothness by Parameter

Introduction and Background: Goals



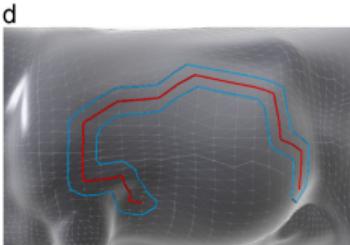
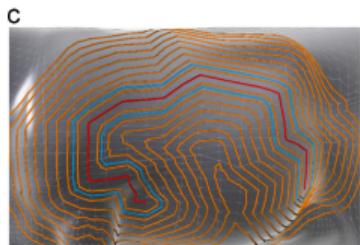
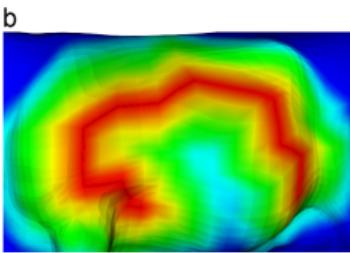
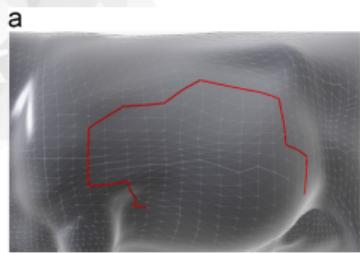
- ▶ Closeness to Initial Curve
- ▶ Balancing Closeness and Smoothness by Parameter
- ▶ Proof of Convergence and Smoothness

Introduction and Background: Goals

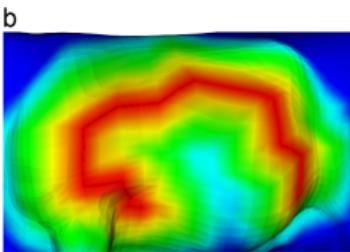
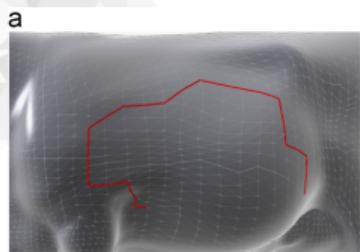


- ▶ Closeness to Initial Curve
- ▶ Balancing Closeness and Smoothness by Parameter
- ▶ Proof of Convergence and Smoothness
- ▶ Robust toward Geometric and Parametric Noise

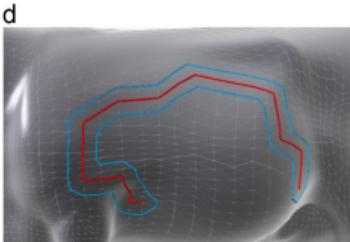
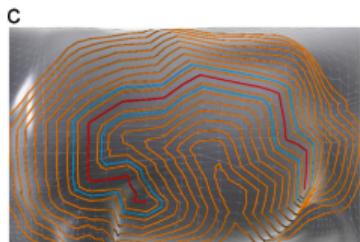
Introduction and Background: Implementation



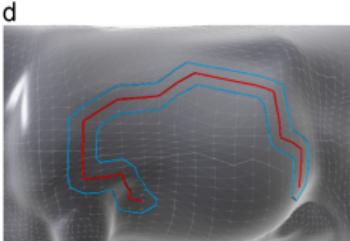
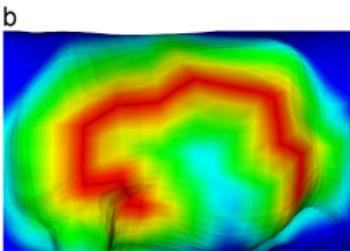
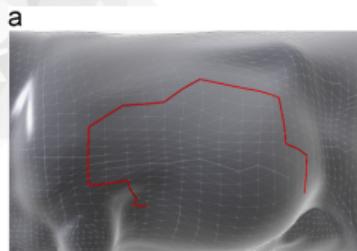
Introduction and Background: Implementation



► Distance Envelope

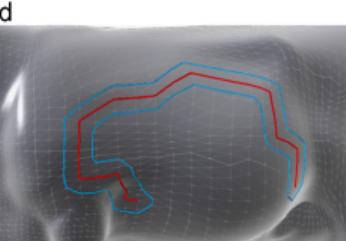
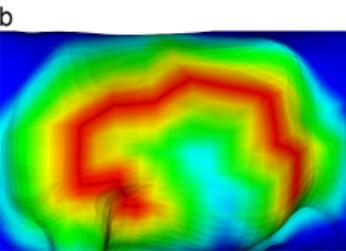
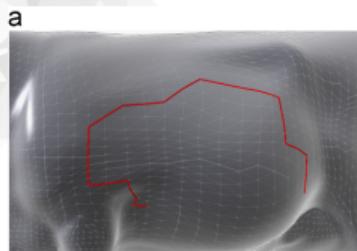


Introduction and Background: Implementation



- ▶ Distance Envelope
- ▶ Weighted Relaxation of Geodesic Curvature to Desired Geodesic Curvature

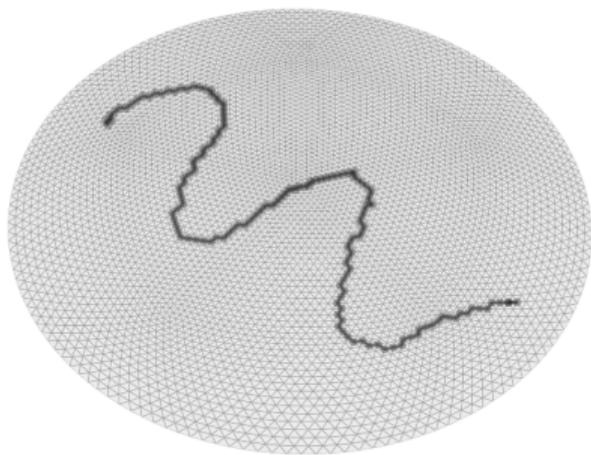
Introduction and Background: Implementation



- ▶ Distance Envelope
- ▶ Weighted Relaxation of Geodesic Curvature to Desired Geodesic Curvature
- ▶ Convergence based on Length Reduction

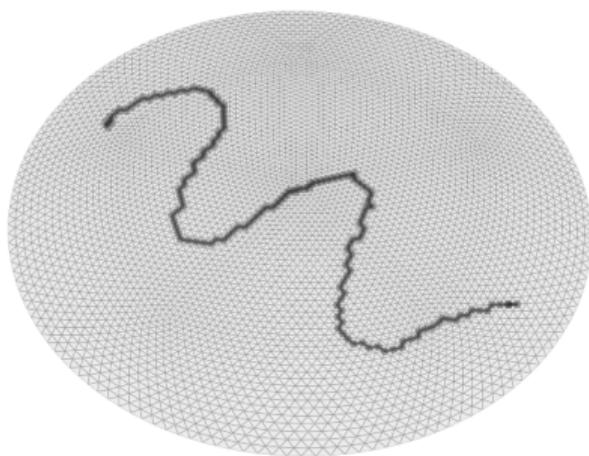
The Idea

The Idea: Surfaces in 2D

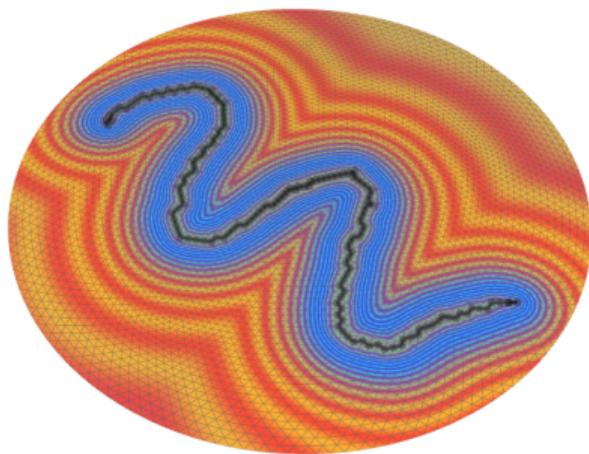


The Idea: Surfaces in 2D

1. Start with initial curve

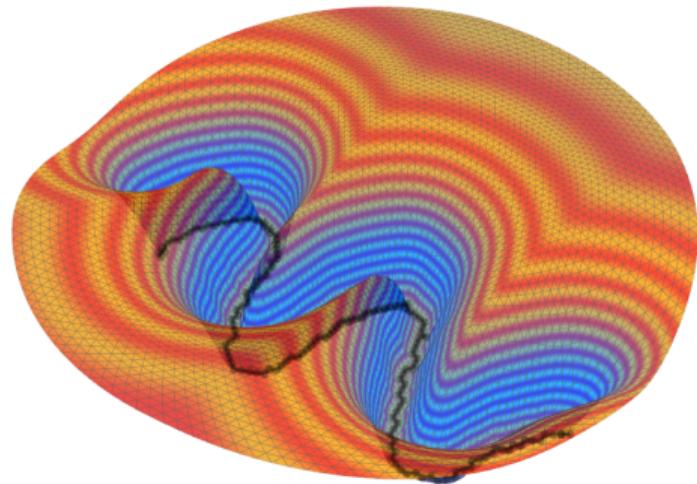


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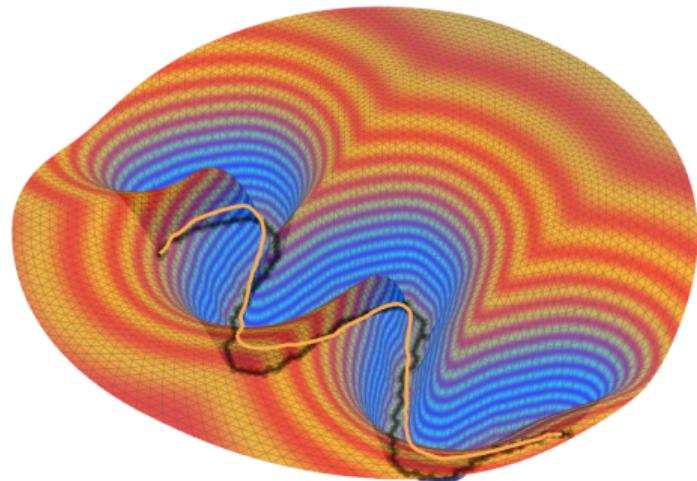
1. Start with initial curve
2. Generate distance field
and penalty potential

The Idea: Surfaces in 2D



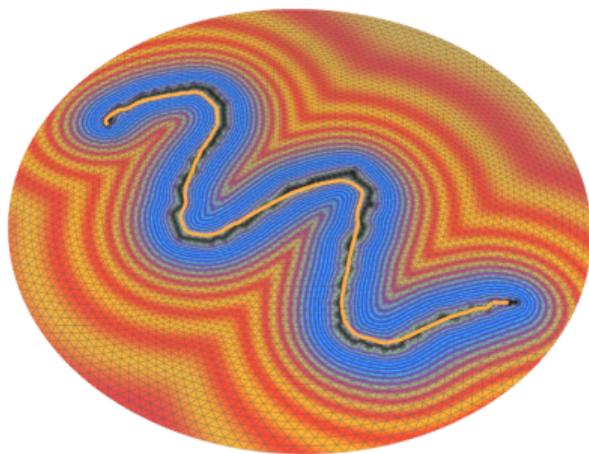
1. Start with initial curve
2. Generate distance field and penalty potential
3. Lift surface to 3D by adding potential to coordinates

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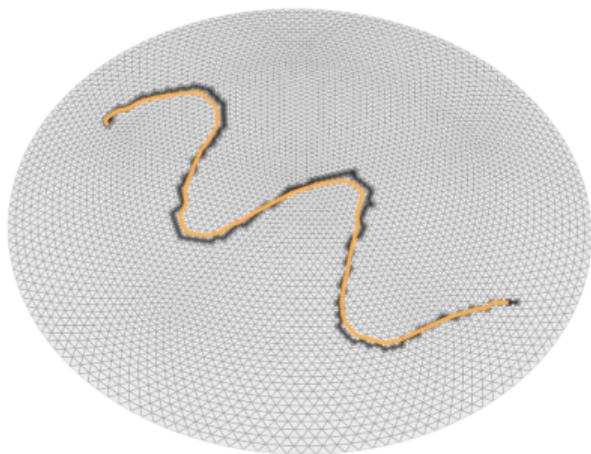
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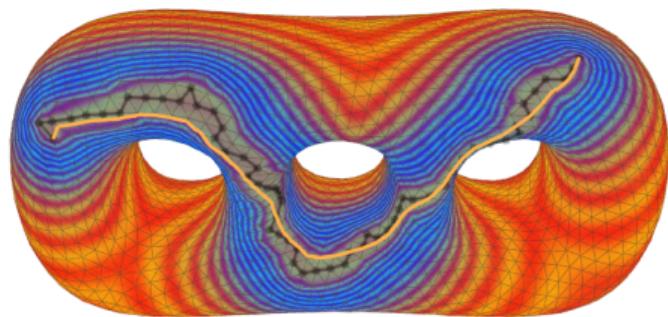
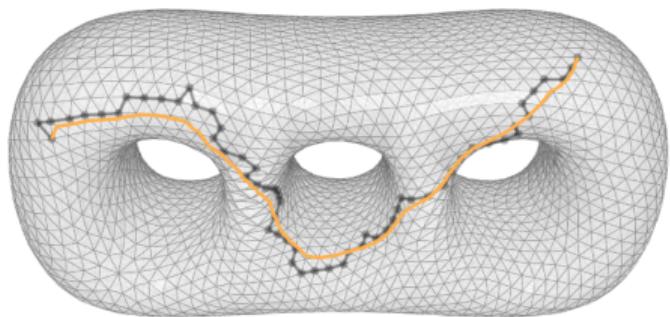
1. Start with initial curve
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4. Generate geodesic for lifted surface
5. Project geodesic back to original surface

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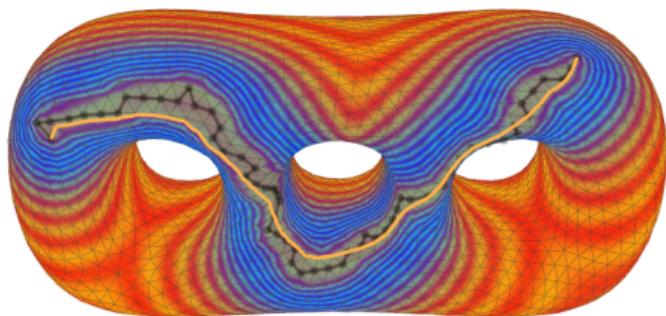
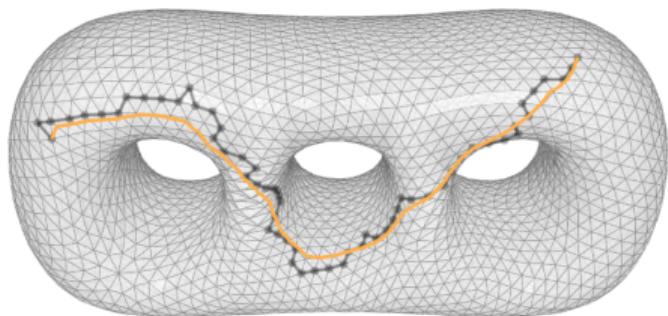


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The Idea: Surfaces in 3D

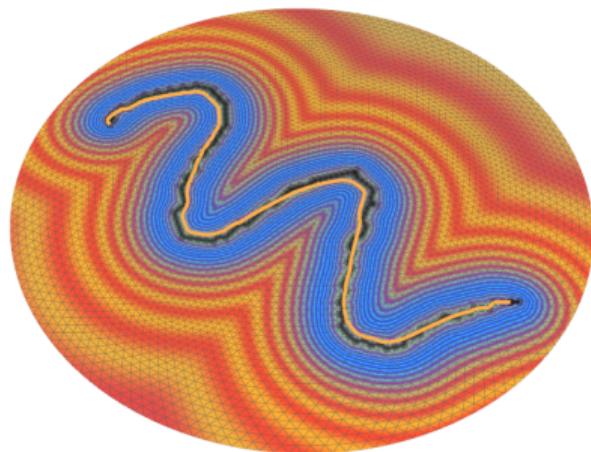


The Idea: Surfaces in 3D



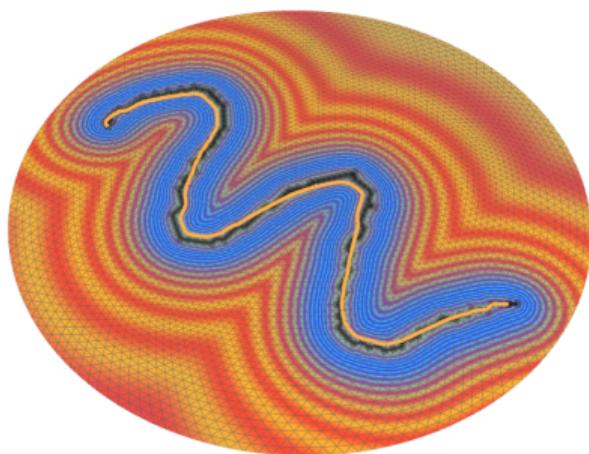
- ▶ Surfaces embedded in 3D are lifted into 4D by appending potential to fourth coordinate of vertex positions

The Idea: Balancing Closeness and Smoothness

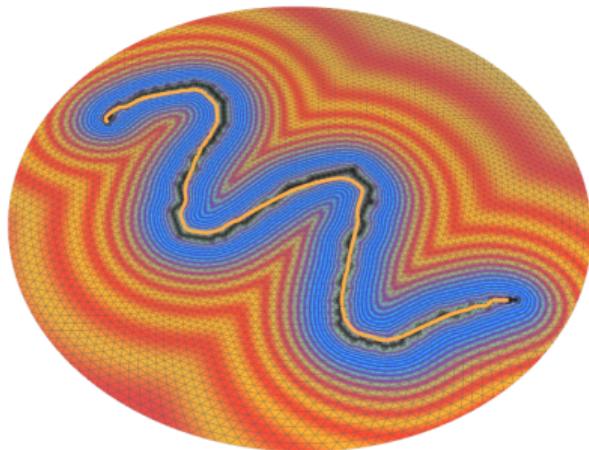


The Idea: Balancing Closeness and Smoothness

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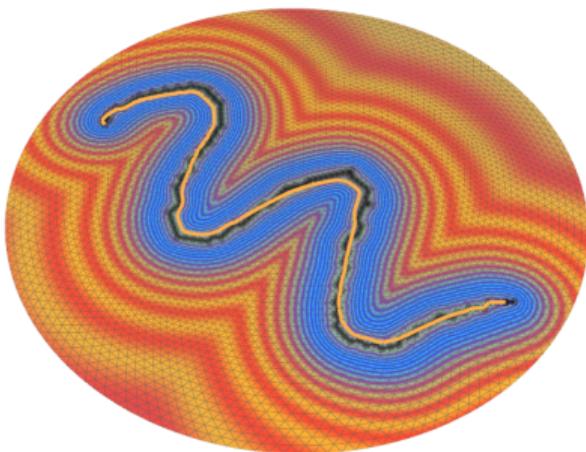


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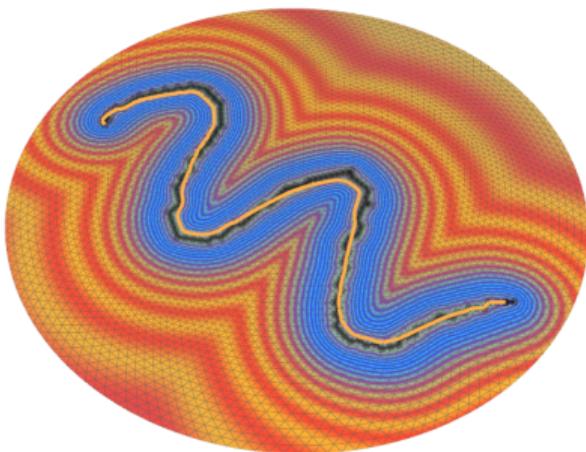
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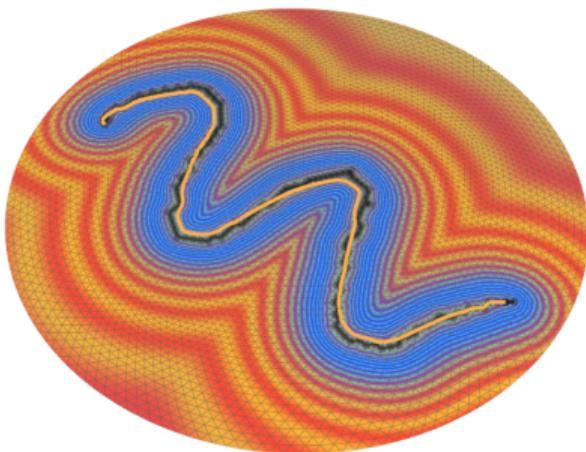
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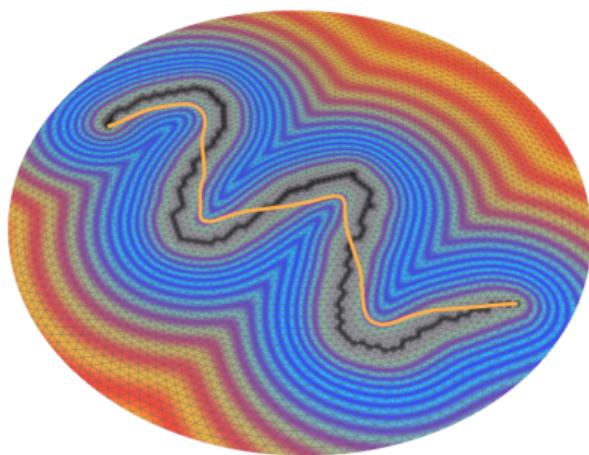


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$$\varphi_{\alpha,\tau} : [0, \infty) \rightarrow \mathbb{R}, \quad \alpha, \tau \in (0, \infty)$$

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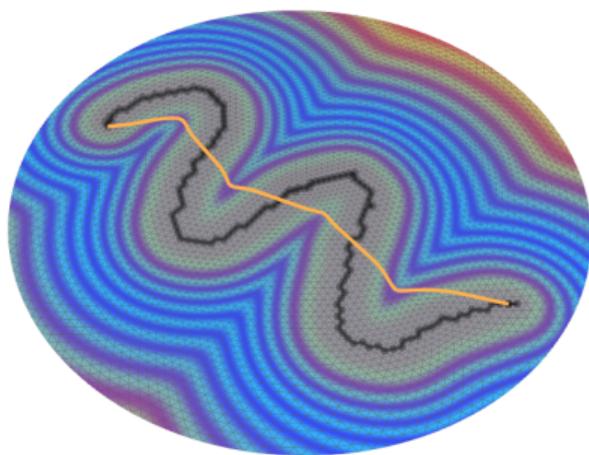


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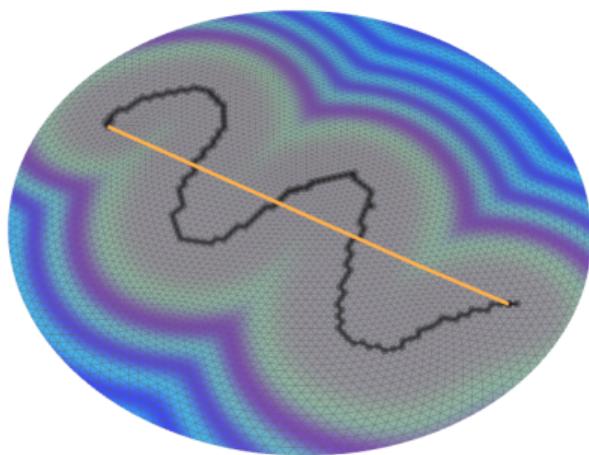


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Properties, Limitations, and Future Work

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- ▶ Other potentials allow for more general movement
- ▶ Arbitrarily many parameters
- ▶ Potentials may also depend on other surface characteristics, such as mean curvature

Limitations and Future Work

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- ▶ Good family of penalty potentials

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- ▶ Other References: To best of my knowledge, it has not been done.