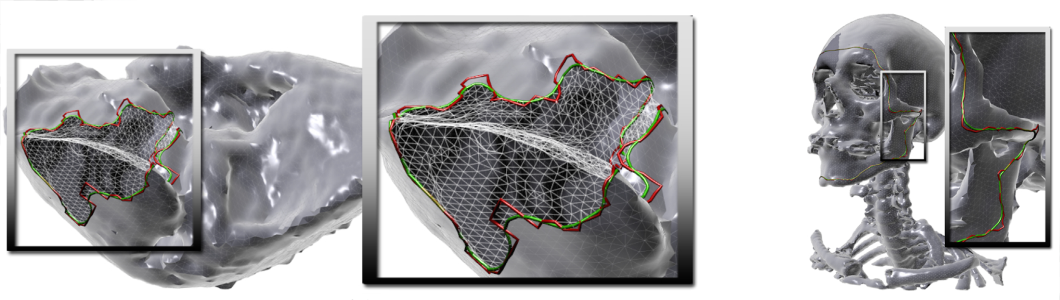
Curve Smoothing on Surface Meshes

Markus Pawellek

August 21, 2023

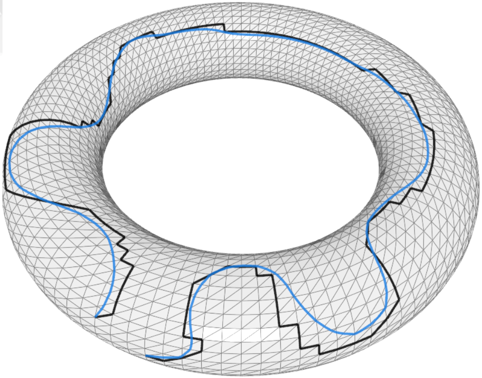
Introduction and Background

Introduction and Background: Reference



Kai Lawonn, Rocco Gasteiger, Christian Rössl, and Bernhard Preim. "Adaptive and Robust Curve Smoothing on Surface Meshes". In: *Computers & Graphics* 40 (2014), pp. 22–35. DOI: 10.1016/j.cag.2014.01.004

Introduction and Background: Goals



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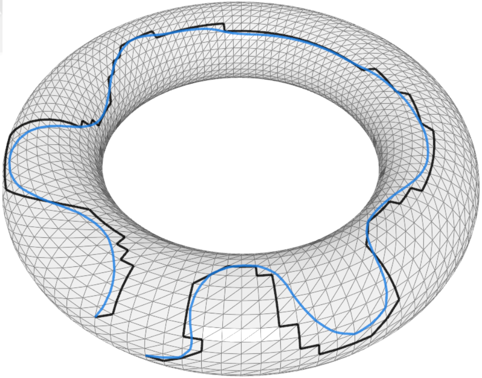

- Closeness to Initial Curve

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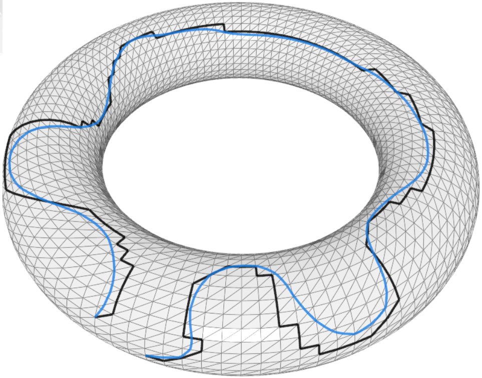
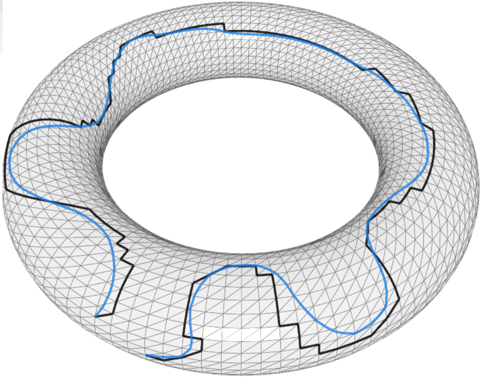


- Closeness to Initial Curve
- Balancing Closeness and Smoothness by Parameter



- Closeness to Initial Curve
- Balancing Closeness and Smoothness by Parameter
- Proof of Convergence and Smoothness

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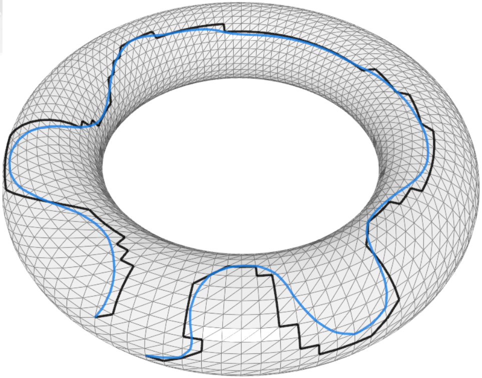
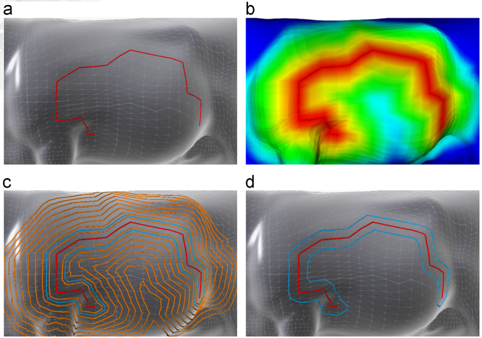


- Closeness to Initial Curve
- Balancing Closeness and Smoothness by Parameter
- Proof of Convergence and Smoothness
- Robust toward Geometric and Parametric Noise

Introduction and Background: Implementation



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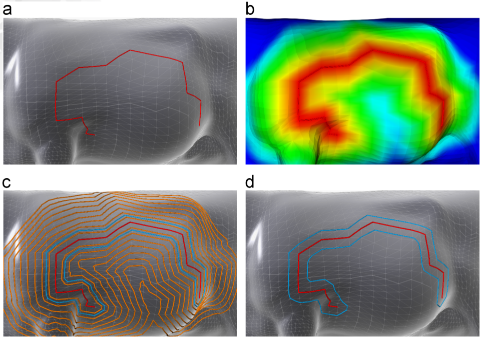

- Distance Envelope

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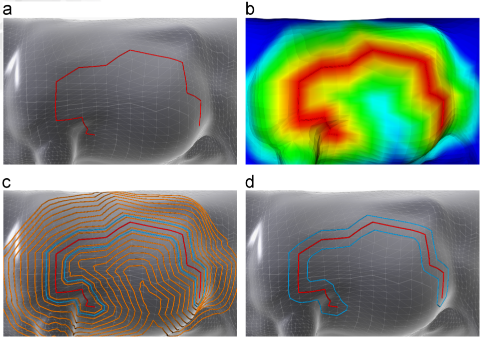
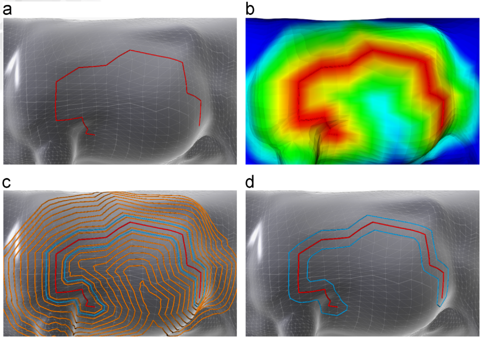


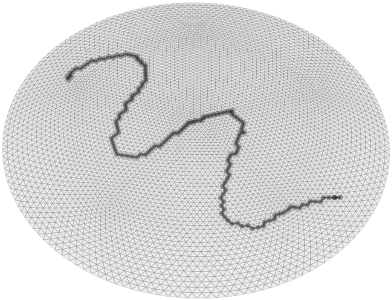
- Distance Envelope
- Weighted Relaxation of Geodesic Curvature to Desired Geodesic Curvature



- Distance Envelope
- Weighted Relaxation of Geodesic Curvature to Desired Geodesic Curvature
- Convergence based on Length Reduction

The Idea

The Idea: Surfaces in 2D



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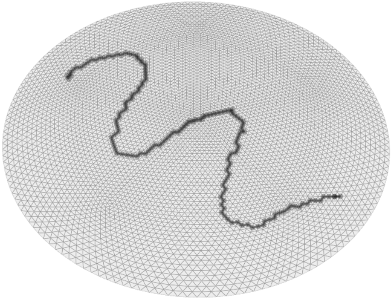

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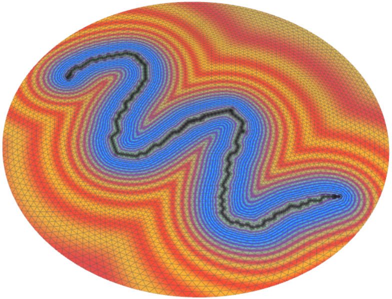

1. Start with initial curve
2. Generate distance field and penalty potential

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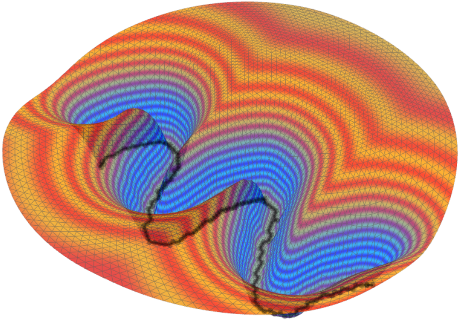

1. Start with initial curve
2. Generate distance field and penalty potential
3. Lift surface to 3D by adding potential to coordinates

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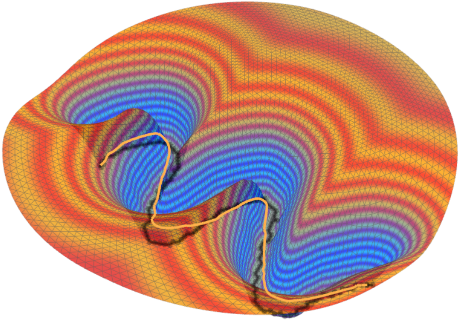

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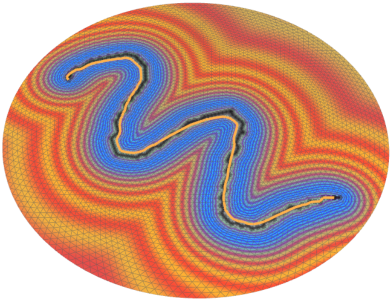

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5. Project geodesic back to original surface

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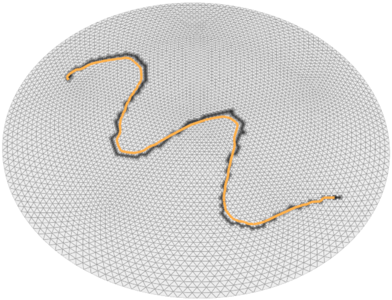

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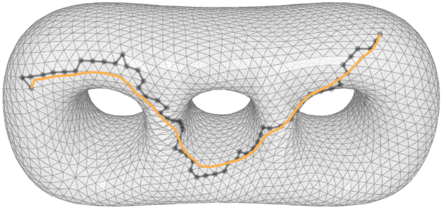

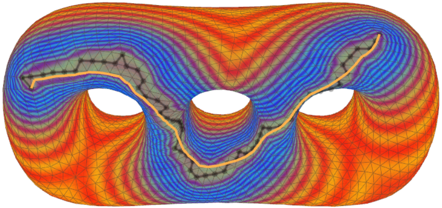

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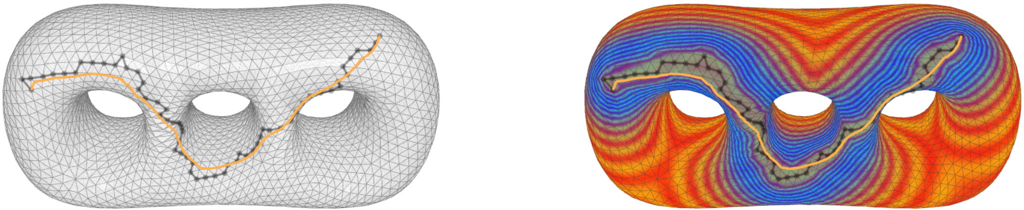

- Surfaces embedded in 3D are lifted into 4D by appending potential to fourth coordinate of vertex positions

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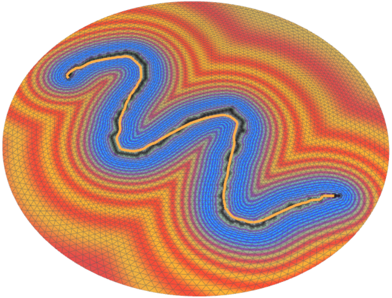

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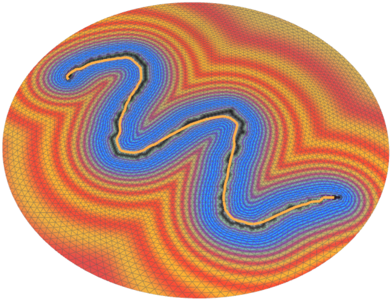

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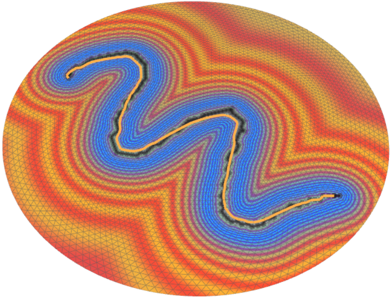

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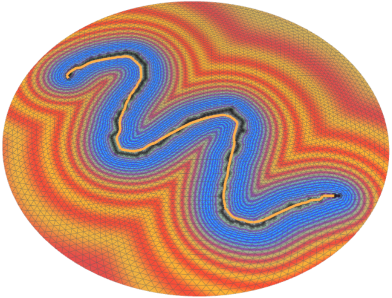

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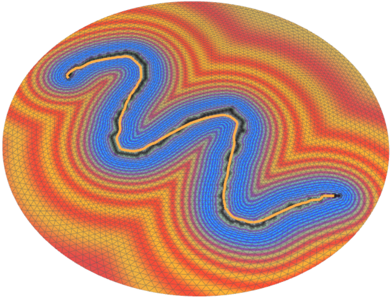

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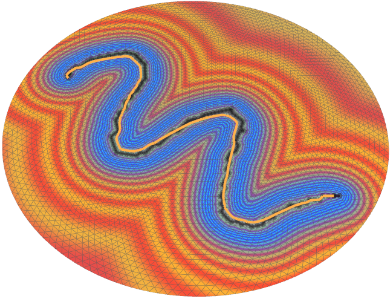

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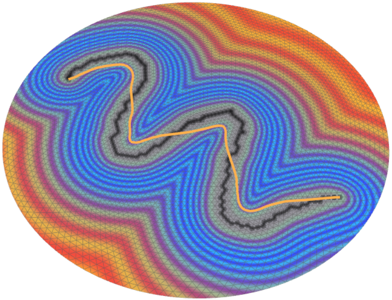

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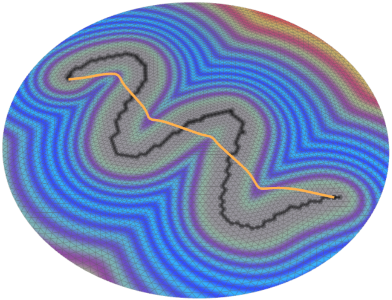

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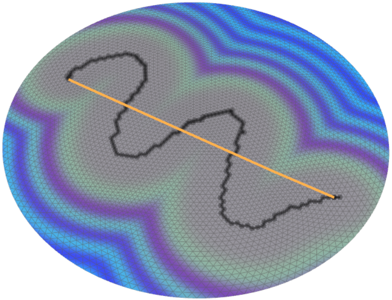

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Properties, Limitations, and Future Work

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- Preserves closeness by definition
- Other potentials allow for more general movement
- Arbitrarily many parameters
- Potentials may also depend on other surface characteristics, such as mean curvature

Limitations and Future Work

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- Scaling of potential

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- Other References: To best of my knowledge, it has not been done.