1 Introduction

We choose the following.

$$x = r\cos\varphi \qquad y = r\sin\varphi$$

Then we do get

$$r = \sqrt{x^2 + y^2} \qquad \varphi = \arctan\frac{y}{x}$$

The result is given by

$$\begin{aligned}\int_{-\infty}^{\infty} e^{-x^2}\,\mathrm{d}x &= \sqrt{\int_0^{\infty}\int_0^{2\pi} e^{-r^2} r\,\mathrm{d}\varphi\,\mathrm{d}r} \\ &= \sqrt{\pi}\end{aligned}$$