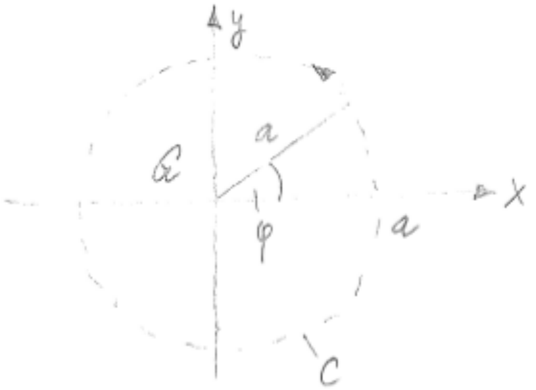
Aufgabe 3 — Ein Kurvenintegral (II) — 5 Punkte

$$\oint_C \vec{w}\, d\vec{r} \quad \text{mit} \quad \vec{w} = (x^2+y^2)\,y\,\vec{i} - (x^2+y^2)\,x\,\vec{j} + (a^3+z^3)\,\vec{k}$$

C in (x,y)-Ebene: $x^2+y^2=a^2$

1



Polarkoordinaten:

$$x = a\cos\varphi \qquad y = a\sin\varphi$$

$$dx = -a\sin\varphi\, d\varphi \qquad dy = a\cos\varphi\, d\varphi$$

0,5

a) Kurvenintegral

$$d\vec{r} = dx\,\vec{i} + dy\,\vec{j}\,, \quad dz = 0$$

$$\boxed{\oint_C \left[(x^2+y^2)\,y\,dx - (x^2+y^2)\,x\,dy\right]}$$

$$= \int_0^{2\pi} \left[a^2\cdot a\sin\varphi\,(-a\sin\varphi) - a^2\cdot a\cos\varphi\,(a\cos\varphi)\right] d\varphi$$ 0,5

$$= -a^4 \int_0^{2\pi} \underbrace{(\sin^2\varphi + \cos^2\varphi)}_{1}\, d\varphi$$

$$\boxed{= -2\pi a^4}$$ 1

b) Greenscher Satz

$$P = x^2 y + y^3 \qquad Q = -x^3 - xy^2$$

$$\frac{\partial P}{\partial y} = x^2 + 3y^2 \qquad \frac{\partial Q}{\partial x} = -3x^2 - y^2$$ 1

$$\boxed{\iint_G \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy} = \iint_G (-4x^2 - 4y^2)\,dx\,dy$$

$$= -4 \iint_G \underbrace{(x^2+y^2)}_{r^2}\,dx\,dy = -4\int_0^a r\,dr \int_0^{2\pi} d\varphi \cdot r^2 = -8\pi \int_0^a r^3\,dr$$

(nicht a^2!)

$$= -\cancel{8}^{\,2}\pi\, \frac{r^4}{\cancel{4}}\Bigg|_0^a = \boxed{-2\pi a^4}$$ 1

Übereinstimmung