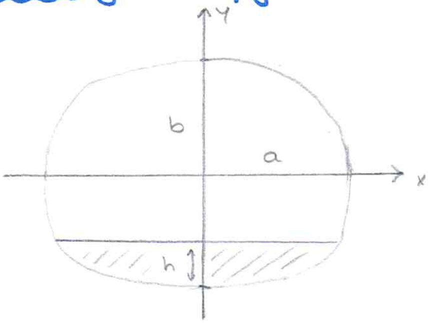
Lösung: Aufgabe 1 4 ~~5~~ Punkte



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\tilde{h} = \frac{h}{b}$$

$$V = \iiint dx\,dy\,dz$$

$$= \int_{y=-b}^{h-b} \int_{x=-a\sqrt{1-y^2/b^2}}^{a\sqrt{1-y^2/b^2}} \int_{z=0}^{L} dx\,dy\,dz \qquad 1$$

$$= L \cdot \int_{y=-b}^{h-b} 2a\sqrt{1-y^2/b^2}\; dy$$

Subst.:
$\tilde{y} = y/b$
$dy = b \cdot d\tilde{y}$ 1/2

$$= 2abL \int_{\tilde{y}=-1}^{-1+\tilde{h}} \sqrt{1-\tilde{y}^2}\, d\tilde{y}$$

Substituiere:
$\tilde{y} = \sin u$
$d\tilde{y} = \cos u \, du$
.1/2

$$= 2abL \int_{\underset{-\frac{\pi}{2}}{\cancel{3\pi/2}}}^{\arcsin(\tilde{h}-1)} \cos^2 u \, du$$

Nebenrechnung: (partielle Integration)

$$\int \cos^2 u\, du = \int \cos u \cdot \cos u \, du = \sin u \cos u + \int \sin^2 u \, du$$

$$= \sin u \cos u + u - \int \cos^2 u \, du$$

$$\Rightarrow \int \cos^2 u \, du = \frac{1}{2}(u + \sin u \cos u)$$

$$V = 2abL \cdot \frac{1}{2}\left[u + \sin u \cos u\right]_{3\pi/2}^{\arcsin(\tilde{h}-1)} \qquad 1$$

$$= abL\left[\arcsin(\tilde{h}-1) + (\tilde{h}-1)\sqrt{1-(\tilde{h}-1)^2} \overbrace{- \frac{3\pi}{2}}^{+\frac{\pi}{2}}\right] \qquad 1$$

$$= abL\left[(\tilde{h}-1)\sqrt{2\tilde{h}-\tilde{h}^2} + \underbrace{\left(-\frac{3\pi}{2} - \arcsin(1-\tilde{h})\right)}_{=\arccos(1-\tilde{h})}\right]$$

$$= abL\left[(\tilde{h}-1)\sqrt{2\tilde{h}-\tilde{h}^2} + \arccos(1-\tilde{h})\right] \quad //$$

(nur andere Formulierung des Ergebnisses)