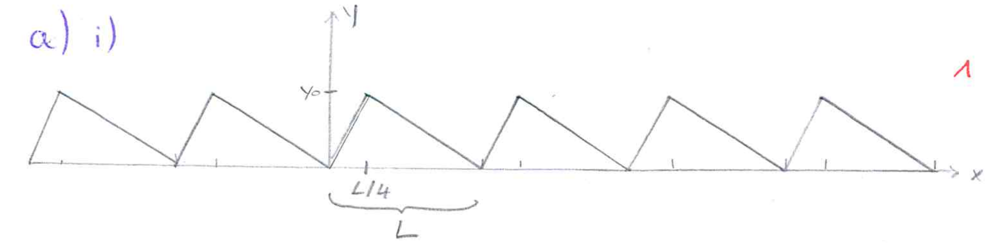
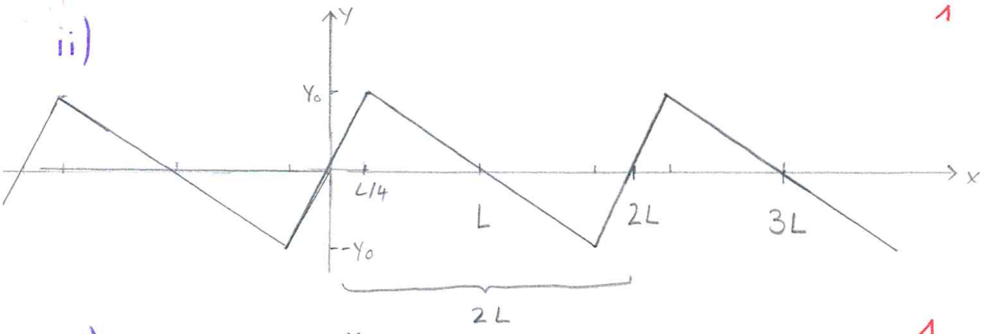
Lösung: Auslenkung einer Saite 9 Punkte [3 + 2 + 4]

a) i) 1



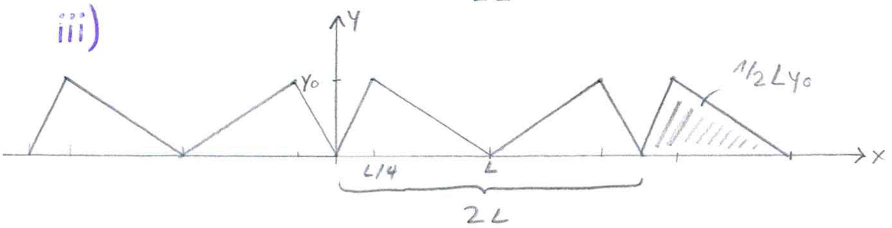
Periode $L_1 = L$

ii) 1



b) Periode $L_2 = 2L$ 1/2

iii) 1



Periode $L_3 = 2L$ 1/2

a_0: Mittelwert der Funktion über eine Periode

$a_0 = \frac{1}{2L} \cdot 2 \cdot \left(\frac{1}{2} L y_0\right) = \frac{1}{2} y_0$ 1

nur Cosinus Glieder: symmetrisch bei Spiegelung an y-Achse

c) Fourier-Reihe für (ii): antisymmetrisch ⇝ ungerade Funktion → Sinus-Reihe

1 $\underline{a_0 = a_n = 0}$

1 $$f(x) = \begin{cases} 4 y_0 \, x/L & \text{für } 0 \le x \le \frac{L}{4} \\ \frac{4}{3} y_0 \left(1 - \frac{x}{L}\right) & \text{für } \frac{L}{4} \le x \le \frac{7}{4} L \\ 4 y_0 \left(\frac{x}{L} - 2\right) & \text{für } \frac{7}{4} L \le x \le 2L \end{cases}$$

$$b_n = \frac{2}{2L} \int_0^{2L} f(x) \sin\left(\frac{2\pi n x}{2L}\right) dx$$

1/2 $$= \frac{1}{L}\left[4y_0 \int_0^{L/4} \frac{x}{L} \sin\left(\frac{\pi n x}{L}\right) dx + \frac{4}{3} y_0 \int_{L/4}^{\frac{7}{4}L} \left(1 - \frac{x}{L}\right) \sin\left(\frac{\pi n x}{L}\right) dx + 4 y_0 \int_{\frac{7}{4}L}^{2L} \left(\frac{x}{L} - 2\right) \sin\left(\frac{\pi n x}{L}\right) dx\right]$$

NR: $\int \frac{x}{L} \sin\left(\frac{\pi n x}{L}\right) dx = -\frac{x}{\pi n} \cos\left(\frac{\pi n x}{L}\right) + \int \frac{1}{\pi n} \cos\left(\frac{\pi n x}{L}\right) dx$

$u = x/L \quad u' = 1/L$

$v' = \sin\left(\frac{\pi n x}{L}\right) \quad v = \frac{-L}{\pi n} \cos\left(\frac{\pi n x}{L}\right)$

$= -\frac{x}{\pi n} \cos\left(\frac{\pi n x}{L}\right) + \frac{L}{\pi^2 n^2} \sin\left(\frac{\pi n x}{L}\right)$

$$= \frac{4y_0}{L}\left(\left[\frac{L}{\pi^2 n^2} \sin\left(\frac{\pi n x}{L}\right) - \frac{x}{\pi n} \cos\left(\frac{\pi n x}{L}\right)\right]_0^{L/4} + \frac{1}{3}\left[\frac{-L}{\pi n} \cos\left(\frac{\pi n x}{L}\right) + \frac{x}{\pi n} \cos\left(\frac{\pi n x}{L}\right) - \frac{L}{\pi^2 n^2} \sin\left(\frac{\pi n x}{L}\right)\right]_{L/4}^{\frac{7}{4}L} + \left[\frac{L}{\pi^2 n^2} \sin\left(\frac{\pi n x}{L}\right) - \frac{x}{\pi n} \cos\left(\frac{\pi n x}{L}\right) + \frac{2L}{\pi n} \cos\left(\frac{\pi n x}{L}\right)\right]_{\frac{7}{4}L}^{2L}\right)$$

$$= \frac{\cdots}{L}\left(\left[\frac{\cdots}{\pi^2 n^2}\sin\left(\frac{\cdots}{4}\right) - \frac{\cdots}{4\pi n}\cos\left(\frac{\cdots}{4}\right)\right] - [0 - 0]\right.$$

$$+ \frac{1}{3}\left(\left[-\frac{L}{\pi n}\cos\left(\frac{\pi n \cdot 7}{4}\right) + \frac{7L}{4\pi n}\cos\left(\frac{7\pi n}{4}\right) - \frac{L}{n^2\pi^2}\sin\left(\frac{7\pi n}{4}\right)\right]\right.$$

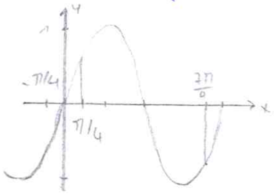
$$\left. - \left[-\frac{L}{\pi n}\cos\left(\frac{\pi n}{4}\right) + \frac{L}{4\pi n}\cos\left(\frac{\pi n}{4}\right) - \frac{L}{n^2\pi^2}\sin\left(\frac{\pi n}{4}\right)\right]\right)$$

$$+ \left[\frac{L}{\pi^2 n^2}\overset{=0}{\sin(2\pi n)} \cancel{- \frac{2L}{\pi n}\underbrace{\cos(2\pi n)}_{=1}} \cancel{+ \frac{2L}{\pi n}\cos(2\pi n)}\right]$$

$$- \left[\frac{L}{\pi^2 n^2}\sin\left(\frac{7\pi n}{4}\right) - \frac{7L}{4\pi n}\cos\left(\frac{7\pi n}{4}\right) + \frac{2L}{2\pi n}\cos\left(\frac{7\pi n}{4}\right)\right]$$ 1/2

Überlegungen:

$$\sin\left(\frac{7n\pi}{4}\right) = \sin\left(2n\pi - \frac{\pi n}{4}\right) = \underbrace{\sin(2\pi n)}_{=0}\cdot\cos\left(\frac{n\pi}{4}\right) - \underbrace{\cos(2\pi n)}_{=1}\sin\left(\frac{n\pi}{4}\right)$$

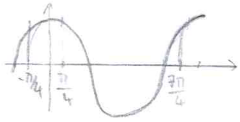


$$= -\sin\left(\frac{n\pi}{4}\right)$$

oder: $\sin\left(\frac{7n\pi}{4}\right) = \sin\left(-\frac{n\pi}{4}\right) = -\sin\left(\frac{n\pi}{4}\right)$ da sin. antisymm.

falls man keine Additionstheoreme kennt ...

$$\cos\left(\frac{7n\pi}{4}\right) = \cos\left(2n\pi - \frac{n\pi}{4}\right) = \underbrace{\sin(2\pi n)}_{=0}\sin\left(\frac{n\pi}{4}\right) + \underbrace{\cos(2\pi n)}_{=1}\cos\left(\frac{n\pi}{4}\right) = \cos\left(\frac{n\pi}{4}\right)$$



oder: $\cos\left(\frac{7n\pi}{4}\right) = \cos\left(-\frac{n\pi}{4}\right) = \cos\left(\frac{n\pi}{4}\right)$ da cos symm.

$$= \frac{4y_0}{\pi^2 n^2}\left[\sin\left(\frac{\pi n}{4}\right) + \frac{1}{3}\sin\left(\frac{n\pi}{4}\right) + \frac{1}{3}\sin\left(\frac{\pi n}{4}\right) + \sin\left(\frac{\pi n}{4}\right)\right]$$

$$+ \frac{4y_0}{\pi n}\cos\left(\frac{n\pi}{4}\right)\left[-\frac{1}{4} - \cancel{\frac{1}{3}} + \frac{7}{12} + \cancel{\frac{1}{3}} - \frac{1}{12} + \frac{7}{4} - 2\right]$$

$$= \frac{4y_0}{\pi^2 n^2}\sin\left(\frac{\pi n}{4}\right)\underbrace{\left[1 + \frac{2}{3} + 1\right]}_{8/3} + \frac{4y_0}{\pi n}\cos\left(\frac{n\pi}{4}\right)\underbrace{\left[\frac{3}{2} + \frac{1}{2} - 2\right]}_{=0}$$

$$= \frac{32y_0}{3\pi^2 n^2}\sin\left(\frac{\pi n}{4}\right) \;//$$ 1/2

$$\Rightarrow b_n = \frac{32y_0}{3\pi^2 n^2}\sin\left(\frac{\pi n}{4}\right) \quad \Rightarrow f(x) = \frac{32y_0}{3\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\left(\frac{\pi n}{4}\right)\sin\left(\frac{\pi n x}{L}\right) \;//$$ 1/2