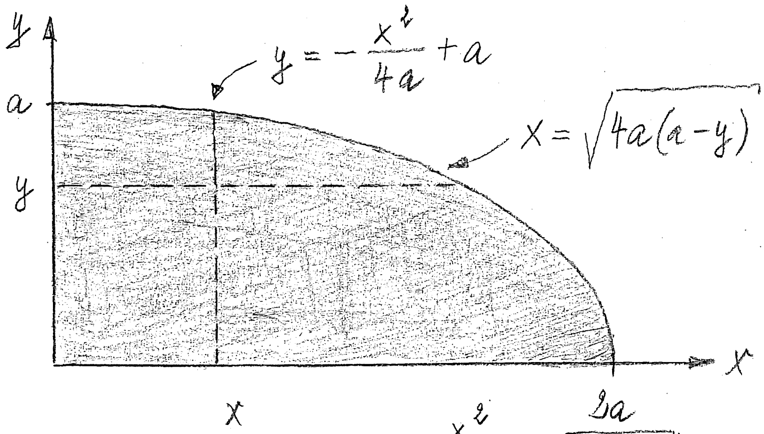
Aufgabe 1   Umkehrung der Integrations-Reihenfolge   Anna Wölfl   1/1

a)

$$I = \int_0^a dy \int_0^{\sqrt{4a^2-4ay}} f(x,y)\,dx$$

$$0 \le y \le a : \quad x^2 = 4a(a-y), \quad \text{Parabel } y = \frac{1}{4a}\left(-x^2+4a^2\right) = -\frac{x^2}{4a}+a$$



lesen ab:

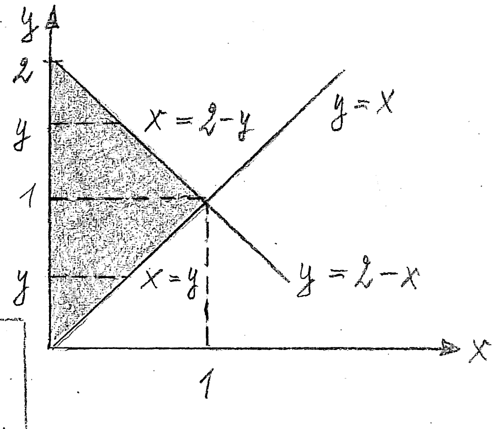
$$\boxed{\begin{array}{l} 0 \le x \le 2a \\ 0 \le y \le -\frac{x^2}{4a}+a \end{array}}$$

Resultat:

$$\boxed{I = \int_0^{2a} dx \int_0^{-\frac{x^2}{4a}+a} f(x,y)\,dy}$$

b) (i) $\int_0^1 x\,dx \int_x^{2-x} \frac{dy}{y}$

$$0 \le x \le 1 : \quad x \le y \le 2-x$$



lesen ab für $\boxed{\begin{array}{l} 0 \le y \le 1 : \quad 0 \le x \le y \\ 1 \le y \le 2 : \quad 0 \le x \le 2-y \end{array}}$

$$\int_0^1 \frac{dy}{y}\int_0^y x\,dx + \int_1^2 \frac{dy}{y}\int_0^{2-y} x\,dx = \int_0^1 \frac{dy}{y}\cdot\frac{y^2}{2} + \int_1^2 \frac{dy}{y}\,\frac{(2-y)^2}{2}$$

$$= \int_0^1 \frac{1}{2}y\,dy + \int_1^2 \frac{1}{2}\left(\frac{4}{y}-4+y\right) = \frac{1}{4}y^2\Big|_0^1 + \left(2\ln y - 2y + \frac{1}{4}y^2\right)\Big|_1^2$$

$$= \cancel{\frac{1}{4}} + 2\ln 2 - 4 + 2 + 1 - \cancel{\frac{1}{4}} = \underline{\underline{2\ln 2 - 1}}$$

(1/2) • ursprünglich: $\displaystyle\int_0^1 dx\; x \cdot \ln y \Big|_x^{2-x} = \underbrace{\int_0^1 dx\; x \ln(2-x)} - \int_0^1 dx\; x \ln x$

Substitution: $\xi = 2 - x \;;\; x = 2 - \xi$

$d\xi = -dx$

$x = 0 : \xi = 2$

$x = 1 : \xi = 1$

$$= -\int_{\xi=2}^{1} d\xi\,(2-\xi)\ln\xi - \int_{x=0}^{1} dx\; x \ln x$$

$$= +\int_1^2 2\ln\xi\, d\xi - \underbrace{\int_1^2 \xi\ln\xi\, d\xi - \int_0^1 x\ln x\, dx}$$

$$= 2(\xi\ln\xi - \xi)\Big|_1^2 - \left[\xi^2\left(\frac{1}{2}\ln\xi - \frac{1}{4}\right)\right]_0^2$$

Bronstein/S., S. 328; № 471

$$= 2(2\ln 2 - 2) - 2 - 4\left(\frac{1}{2}\ln 2 - \frac{1}{4}\right)$$

$$= 4\ln 2 - 4 + 2 - 2\ln 2 + 1$$

$$= \underline{\underline{2\ln 2 - 1}}$$

denn: $\lim\limits_{\xi\to 0}(\xi^2 \ln\xi)$

$$= \lim_{\xi\to 0}\frac{\ln\xi}{\frac{1}{\xi^2}} = \lim_{\xi\to 0}\frac{1/\xi}{-2/\xi^3}$$

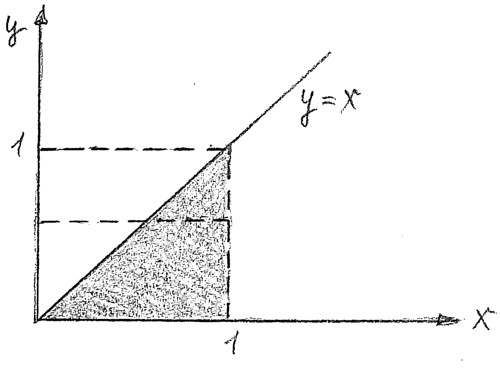
$$= \lim_{\xi\to 0}\left(-\frac{1}{2}\xi^2\right) = 0$$

Übereinstimmung, aber komplizierter

(ii) $\displaystyle\int_0^1 dx \int_0^x \sqrt{y(2-y)}\, dy$

$0 \le x \le 1 : \quad 0 \le y \le x$

lesen ab $\boxed{0 \le y \le 1 : \quad y \le x \le 1}$



$$\int_0^1 \sqrt{y(2-y)}\, dy \int_y^1 dx = \int_0^1 (1-y)\sqrt{y(2-y)}\, dy$$

Substitution: $\eta = 2y - y^2 = y(2-y)$

$d\eta = 2(1-y)\,dy$

$y = 0 : \eta = 0 \;;\; y = 1 : \eta = 1$

$$= \int_{\eta=0}^{1} \frac{1}{2} d\eta \cdot \eta^{1/2} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} \eta^{3/2} \Big|_0^1 = \underline{\underline{\frac{1}{3}}}$$

• <u>ursprünglich:</u> $\int_0^x \sqrt{2y - y^2}\, dy$, Bronstein/S., S. 313, № 245
mit $a = -1,\ b = 2,\ c = 0$
$\Delta = -4,\ k = 1$

$$= \frac{2 - 2y}{-4} \sqrt{2y - y^2} \Big|_0^x + \frac{1}{2} \int_0^x \frac{dy}{\sqrt{2y - y^2}} \quad \text{, ebenda, № 241}$$

$$= \frac{1}{2}(x-1)\sqrt{2x - x^2} + \frac{1}{2} \frac{-1}{1} \arcsin \frac{2 - 2y}{2} \Big|_0^x$$

$$= \frac{1}{2}(x-1)\sqrt{2x - x^2} - \frac{1}{2}\left[\arcsin(1-x) - \arcsin 1\right]$$

$$= \frac{1}{2}(x-1)\sqrt{2x - x^2} - \frac{1}{2}\arcsin(1-x) + \frac{1}{2} \cdot \frac{\pi}{2} \qquad \Big| \int_0^1 dx$$

$$\int_0^1 dx \int_0^x \sqrt{2y - y^2}\, dy = \frac{1}{2} \underbrace{\int_0^1 (x-1)\sqrt{2x - x^2}\, dx}_{\text{siehe oben (Vorzeichen!)}} - \frac{1}{2} \int_0^1 \arcsin(1-x)\, dx + \frac{\pi}{4} \int_0^1 dx$$

$$= -\frac{1}{2} \cdot \frac{1}{3} - \frac{1}{2} \underbrace{\int_0^1 \arcsin(1-x)\, dx}_{} + \frac{\pi}{4}$$

<u>Substitution:</u> $\xi = 1 - x$; $x = 1 - \xi$
$d\xi = -dx$
$x = 0 : \xi = 1$
$x = 1 : \xi = 0$

$$= -\frac{1}{6} + \frac{1}{2} \int_1^0 \arcsin \xi \, d\xi + \frac{\pi}{4}$$

$$= -\frac{1}{6} - \frac{1}{2}\left[\xi \arcsin \xi + \sqrt{1 - \xi^2}\right]_0^1 + \frac{\pi}{4} \quad \text{, Bronstein/S., S. 329, № 488}$$

$$= -\frac{1}{6} - \frac{1}{2}\left(\frac{\pi}{2} - 1\right) + \frac{\pi}{4} = -\frac{1}{6} - \cancel{\frac{\pi}{4}} + \frac{1}{2} + \cancel{\frac{\pi}{4}}$$

$= \underline{\underline{\frac{1}{3}}}$; <u>Übereinstimmung</u>, aber viel komplizierter