Aufgabe 1

Linienelement: $\quad \mathrm{d}s^2 = g_1^2 \mathrm{d}x_2^2 + g_2^2 \mathrm{d}x_2^2 + g_3^2 \mathrm{d}x_3^2$

Beispiele:

	x_1	x_2	x_3	g_1	g_2	g_3
kartes.	x	y	z	1	1	1
Zylinder	r	ϕ	z	1	r	1
Kugel	r	θ	ϕ	1	r	$r\sin\theta$

Volumenelement: ds $\quad V = g_1 g_2 g_3 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3$

Gradient: ds $\quad \operatorname{grad} U = \left(\frac{1}{g_1} U x_1, \frac{1}{g_2} U x_2, \frac{1}{g_2} U x_2\right)$

$\operatorname{rot}\operatorname{grad} U = 0$

$\operatorname{div}\operatorname{rot}\vec{A} = 0$

$\operatorname{rot}\operatorname{rot}\vec{A} = \operatorname{grad}\operatorname{div}\vec{A} - \Delta\vec{A} \quad$ (kartesische Koordinaten)

$\operatorname{div}(\vec{A}\times\vec{B}) = \vec{B}\operatorname{rot}\vec{A} - \vec{A}\operatorname{rot}\vec{B}$

$\operatorname{grad}(UV) = U\operatorname{grad}V + V\operatorname{grad}U$

$\operatorname{rot}(\lambda\vec{A}) = \lambda\operatorname{rot}\vec{A} + (\operatorname{grad}\lambda)\times\vec{A}$

$\operatorname{div}(\lambda\vec{A}) = \lambda\operatorname{div}\vec{A} + \vec{A}\operatorname{grad}\lambda$

$\operatorname{rot}(\vec{A}\times\vec{B}) = (\vec{B}\operatorname{grad})\vec{A} - (\vec{A}\operatorname{grad})\vec{B} + \vec{A}\operatorname{div}\vec{B} - \vec{B}\operatorname{div}\vec{A}$

$\operatorname{grad}(\vec{A}\vec{B}) = (\vec{B}\operatorname{grad})\vec{A} + (\vec{A}\operatorname{grad})\vec{B} + \vec{A}\times\operatorname{rot}\vec{B} + \vec{B}\times\operatorname{rot}\vec{A}$

a) Zylinderkoordinaten

Linienelement: $\quad \mathrm{d}s^2 = \mathrm{d}r^2 + r^2\mathrm{d}\phi^2 + \mathrm{d}z^2$

Volumenelement: $\quad \mathrm{d}V = r\mathrm{d}r\mathrm{d}\phi\mathrm{d}z$

Gradient: $\quad \operatorname{grad}U = \frac{\partial U}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial U}{\partial \phi}\vec{e}_\phi + \frac{\partial U}{\partial z}\vec{e}_z$

Divergenz: $\quad \operatorname{div}\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$

Rotation: $\quad \operatorname{rot}\vec{A} = \left[\frac{1}{r}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right]\vec{e}_r + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right]\vec{e}_\phi + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi}\right]\vec{e}_z$

Laplace-Operator: $\quad \Delta U = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 U}{\partial\phi^2} + \frac{\partial^2 U}{\partial z^2}$

b) Kugelkoordinaten

Linienelement: $\quad \mathrm{d}s^2 = \mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2$

Volumenelement: $\quad \mathrm{d}V = r^2\sin\theta\mathrm{d}r\mathrm{d}\theta\mathrm{d}\phi$

bitte wenden

Gradient: $\quad \operatorname{grad} U = \frac{\partial U}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial U}{\partial \theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial U}{\partial \phi}\vec{e}_\phi$

Divergenz: $\quad \operatorname{div}\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{\partial A_\phi}{\partial\phi}\right]$

Rotation:

$$\operatorname{rot}\vec{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial\phi}\right]\vec{e}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\vec{e}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\vec{e}_\phi$$

Laplace-Operator:

$$\Delta U = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial U}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 U}{\partial\phi^2}$$