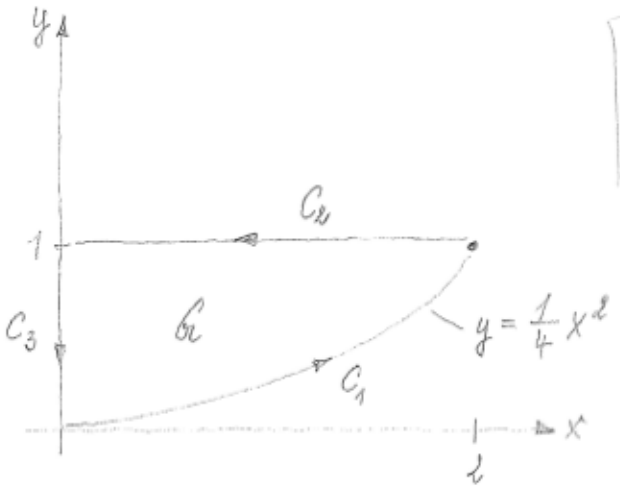
(2/4) **Aufgabe 4** Kraft und Arbeit **5 Punkte**

(nur Weg b) war gefragt)

$$\vec{f} = xy\,\vec{i} - y^2\,\vec{j}$$



2

a) Arbeit: $A = \oint_C \vec{f}\,d\vec{r} = \oint_C (xy\,dx - y^2\,dy)$

C_1: $y = \frac{1}{4}x^2$, $dy = \frac{1}{2}x\,dx$

$0 \le x \le 2$

$$\int_{C_1} \vec{f}\,d\vec{r} = \int_0^2 \left(\frac{1}{4}x^3 - \frac{x^4}{16}\cdot\frac{x}{2}\right)dx$$

$$= \frac{1}{16}x^4\Big|_0^2 - \frac{1}{32}\cdot\frac{x^6}{6}\Big|_0^2$$

$$= 1 - \frac{64}{32\cdot 6} = 1 - \frac{1}{3} = \underline{\underline{\frac{2}{3}}}$$

C_2: $y = 1$, $dy = 0$; $2 \ge x \ge 0$

$$\int_{C_2} \vec{f}\,d\vec{r} = \int_2^0 x\,dx = \frac{x^2}{2}\Big|_2^0 = \underline{\underline{-2}}$$

C_3: $x = 0$, $dx = 0$; $1 \ge y \ge 0$

$$\int_{C_3} \vec{f}\,d\vec{r} = -\int_1^0 y^2\,dy = -\frac{1}{3}y^3\Big|_1^0 = \underline{\underline{\frac{1}{3}}}$$

- Zusammen: $\boxed{\oint \vec{f}\,d\vec{r} = \frac{2}{3} - 2 + \frac{1}{3} = -1}$

war nicht gefragt

b) Greenscher Satz: $P = xy$ $\quad Q = -y^2$

$$\frac{\partial P}{\partial y} = x \qquad \frac{\partial Q}{\partial x} = 0$$

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$$\iint_G \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dx\,dy = \int_0^2 dx \int_{x^2/4}^1 dy\,(-x) = -\int_0^2 dx\, x\cdot\left(1 - \frac{x^2}{4}\right)$$

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$$= -\int_0^2 dx\, x + \frac{1}{4}\int_0^2 dx\, x^3 = -\frac{x^2}{2}\Big|_0^2 + \frac{1}{4}\frac{x^4}{4}\Big|_0^2 = -2 + 1 = \boxed{-1}$$

1

Übereinstimmung