Lösung: Fourier-Transformationen 6 Punkte [2+4]

a) $f(x) = \Theta(x-a)\, e^{-bx}$

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx = \frac{1}{\sqrt{2\pi}} \int_{a}^{\infty} e^{-bx} e^{-ikx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_a^{\infty} e^{-x(b+ik)} dx = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{-(b+ik)} \left[e^{-x(b+ik)} \right]_a^{\infty}$$ 1

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{b+ik} e^{-a(b+ik)}$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{b-ik}{b^2+k^2} e^{-ab} \cdot e^{-iak} \,//$$ 1

(nur wenn Nenner reell gemacht wurde!)

b) $f(t) = \begin{cases} t^3 & 0<t<1 \\ 0 & \text{sonst} \end{cases}$

$$\tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = \frac{1}{\sqrt{2\pi}} \int_0^1 t^3 e^{-i\omega t} dt$$

$u = t^3$, $u' = 3t^2$, $v' = e^{-i\omega t}$, $v = \frac{-1}{i\omega} e^{-i\omega t} = \frac{i}{\omega} e^{-i\omega t}$

$$= \frac{1}{\sqrt{2\pi}} \left[\left[\frac{it^3}{\omega} e^{-i\omega t} \right]_0^1 - \frac{3i}{\omega} \int_0^1 t^2 e^{-i\omega t} dt \right]$$

$u = t^2$, $u' = 2t$, $v' = e^{-i\omega t}$, $v = \frac{i}{\omega} e^{-i\omega t}$

$$= \frac{1}{\sqrt{2\pi}} \frac{i}{\omega} e^{-i\omega} - \frac{3i}{\sqrt{2\pi}\,\omega} \left(\left[\frac{it^2}{\omega} e^{-i\omega t} \right]_0^1 - 2 \int_0^1 \frac{ti}{\omega} e^{-i\omega t} dt \right)$$

$$= \frac{e^{-i\omega}}{\sqrt{2\pi}} \left[\frac{i}{\omega} + \frac{3}{\omega^2} \right] - \frac{6}{\sqrt{2\pi}\,\omega^2} \cdot \int_0^1 t e^{-i\omega t} dt$$

$u = t$, $u' = 1$, $v' = e^{-i\omega t}$, $v = \frac{i}{\omega} e^{-i\omega t}$

$$= \frac{e^{-i\omega}}{\sqrt{2\pi}} \left[\frac{i}{\omega} + \frac{3}{\omega^2} \right] - \frac{6}{\sqrt{2\pi}\,\omega^2} \left(\left[\frac{i}{\omega} t e^{-i\omega t} \right]_0^1 - \frac{i}{\omega} \int_0^1 e^{-i\omega t} dt \right)$$

$$= \frac{e^{-i\omega}}{\sqrt{2\pi}} \left[\frac{i}{\omega} + \frac{3}{\omega^2} - \frac{6i}{\omega^3} \right] + \frac{6i}{\sqrt{2\pi}\,\omega^3} \left[\frac{i}{\omega} e^{-i\omega t} \right]_0^1$$

$$= \frac{e^{-i\omega}}{\sqrt{2\pi}} \left[\frac{i}{\omega} + \frac{3}{\omega^2} - \frac{6i}{\omega^3} \right] - \frac{6}{\sqrt{2\pi}\,\omega^4} \left[e^{-i\omega} - 1 \right]$$

$$= \frac{1}{\sqrt{2\pi}\,\omega^4} \left[e^{-i\omega} \left[i\omega^3 + 3\omega^2 - 6i\omega - 6 \right] + 6 \right] \,//$$

Anmerkung: Aufgabenstellung forderte explizit dreifache partielle Integration, wenn mit CAS gelöst keine Punkte geben!