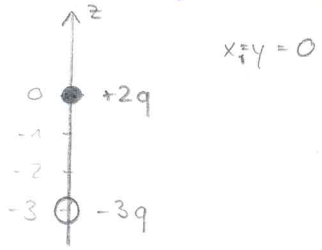
Lösung: Jonenkristalle 6 Punkte

a) $\rho(x,y,z) = q\,\delta(x)\,\delta(y)\,[2\,\delta(z) - 3\,\delta(z+3)]$



$\dim[\rho] = \frac{As}{m^3} \rightsquigarrow [q] = As$

1

in Zylinderkoordinaten: $r=0,\ \varphi=0$ o.B.d.A., $z_1 = 0,\ z_2 = -3$

$$\rho(r,\varphi,z) = \frac{q}{r}\,\delta(r)\,\delta(\varphi)\,[2\,\delta(z) - 3\,\delta(z+3)]$$ 1

$$\left([\rho] = [q]\cdot\left[\frac{1}{r}\right]\cdot[\delta(r)]\cdot[\delta(\varphi)]\cdot[(2\delta(z)-3\delta(z+3))] \right.$$
$$= As\cdot\frac{1}{m}\cdot\frac{1}{m}\cdot 1\cdot\frac{1}{m}$$
$$\left. = \frac{As}{m^3}\ \checkmark \right)$$

b) Flächenladungsdichte:

$$\sigma(x,y) = \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} [(-1)^{i+j}\cdot q]\,\delta(x - i\cdot a)\,\delta(y - j\cdot b)$$ 1

$$\rho(x,y,z) = \delta(z)\cdot\sum\sum\ \ldots$$ 1

c) $$\rho(x,y,z) = \sum_{i=0}^{1}\sum_{j=0}^{1} (-1)^{i+j}\,q\,\delta(x-ia)\,\delta(y-jb)\cdot\delta(z)$$

$$Q = \iiint \rho\,dx\,dy\,dz = \iint \sigma(x,y)\,dx\,dy\cdot\underbrace{\int_{-\infty}^{\infty}\delta(z)\,dz}_{=1}$$
$$= q\iint \delta(x)\delta(y) - \delta(x-a)\delta(y) - \delta(x)\delta(y-b) + \delta(x-a)\delta(y-b)\,dx\,dy$$
$$= q\,[1 - 1 - 1 + 1] = 0$$ 1

$$\vec{p} = \iiint \vec{r}\,\rho\,dx\,dy\,dz = \iiint \begin{pmatrix} x\cdot\sigma(x,y)\,\delta(z) \\ y\cdot\sigma(x,y)\,\delta(z) \\ z\cdot\sigma(x,y)\,\delta(z) \end{pmatrix} dx\,dy\,dz$$

$$= q\Big[\iint x\,[\delta(x)\delta(y) - \delta(x-a)\delta(y) - \delta(x)\delta(y-b) + \delta(x-a)\delta(y-b)]\,dx\,dy\cdot\vec{i}$$
$$+ \iint y\,[\delta(x)\delta(y) - \delta(x-a)\delta(y) - \delta(x)\delta(y-b) + \delta(x-a)\delta(y-b)]\,dx\,dy\cdot\vec{j}\Big]$$
$$= q\,[(0 - a - 0 + a)\,\vec{i} + (0 - 0 - b + b)\,\vec{j}]$$
$$= \vec{0}$$ 1