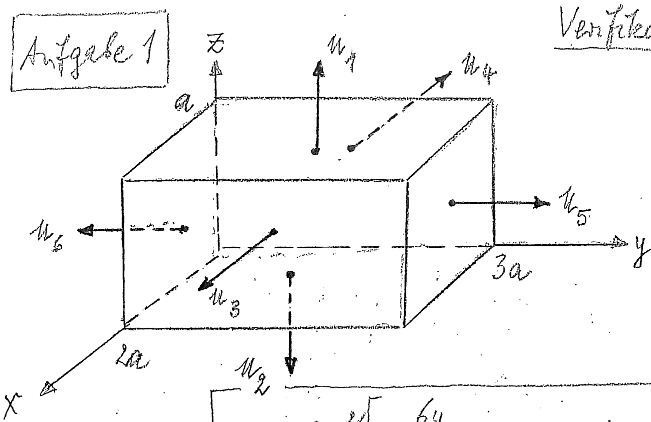
Aufgabe 1

Verifikation des Gaußschen Satzes



$n_1 = \vec{k} = -n_2 \quad , \quad dx\,dy$

$n_3 = \vec{i} = -n_4 \quad , \quad dy\,dz$

$n_5 = \vec{j} = -n_6 \quad , \quad dx\,dz$

$$\boxed{\mathfrak{a} = ka^2\left[\frac{6y}{\sqrt{x^2+y^2+a^2}}\vec{i} + \frac{3z}{\sqrt{y^2+z^2+4a^2}}\vec{j} + \frac{2x}{\sqrt{x^2+z^2+9a^2}}\vec{k}\right]}$$

: Fluß: $$\oiint_S \mathfrak{a}\,d\vec{f} = \sum_{i=1}^{6}\iint \mathfrak{a}\,d\vec{f}_i = \iint_{\substack{A_1\\(z=a)}} \frac{2x}{\sqrt{x^2+10a^2}}dx\,dy - \iint_{\substack{A_2\\(z=0)}} \frac{2x}{\sqrt{x^2+9a^2}}dx\,dy$$

(hier ohne Faktor ka^2)

$$+\iint_{\substack{A_3\\(x=2a)}} \frac{6y}{\sqrt{y^2+5a^2}}dy\,dz - \iint_{\substack{A_4\\(x=0)}} \frac{6y}{\sqrt{y^2+a^2}}dy\,dz$$

$$+\iint_{\substack{A_5\\(y=3a)}} \frac{3z\,dx\,dz}{\sqrt{z^2+13a^2}} - \iint_{\substack{A_6\\(y=0)}} \frac{3z}{\sqrt{z^2+4a^2}}dx\,dz$$

1

+12P

Integrale vom Typ $\int \frac{u}{\sqrt{u^2+c^2}}du = \sqrt{u^2+c^2}$, Bronstein, S. / S. 309, № 193 (wenn von Hand, sonst Null)

$$\oiint_S \mathfrak{a}\,d\vec{f} = 2\int_0^{3a}dy\int_0^{2a}dx\,\frac{x}{\sqrt{x^2+10a^2}} - 2\int_0^{3a}dy\int_0^{2a}dx\,\frac{x}{\sqrt{x^2+9a^2}}$$

$$+6\int_0^{a}dz\int_0^{3a}dy\,\frac{y}{\sqrt{y^2+5a^2}} - 6\int_0^{a}dz\int_0^{3a}\frac{y}{\sqrt{y^2+a^2}}dy$$

$$+3\int_0^{2a}dx\int_0^{a}dz\,\frac{z}{\sqrt{z^2+13a^2}} - 3\int_0^{2a}dx\int_0^{a}dz\,\frac{z}{\sqrt{z^2+4a^2}}$$

(42)

$$= 6a\left[\sqrt{x^2+10a^2} - \sqrt{x^2+9a^2}\right]_0^{2a} + 6a\left[\sqrt{y^2+5a^2} - \sqrt{y^2+a^2}\right]_0^{3a}$$

$$+ 6a\left[\sqrt{z^2+13a^2} - \sqrt{z^2+4a^2}\right]_0^{a}$$

$$= 6a\left[a\sqrt{14} - a\sqrt{10} - a\sqrt{13} + 3a\right] + 6a\left[a\sqrt{14} - a\sqrt{5} - a\sqrt{10} + a\right]$$

$$+ 6a\left[a\sqrt{14} - a\sqrt{13} - a\sqrt{5} + 2a\right]$$

$$= 6a^2\left[3\sqrt{14} - 2\sqrt{10} - 2\sqrt{13} + 6 - 2\sqrt{5}\right]$$

mit Faktor ka^2: $$\boxed{\oiint_S \mathfrak{A}\, d\vec{f} = 6ka^4\left[6 - 2\sqrt{5} - 2\sqrt{10} - 2\sqrt{13} + 3\sqrt{14}\right]}$$ 2 (falls bis zum Ende umgeformt)

: Gaußscher Satz: $$\boxed{\operatorname{div} \mathfrak{A} = ka^2\left[\frac{-6xy}{(x^2+y^2+a^2)^{3/2}} - \frac{3yz}{(y^2+z^2+4a^2)^{3/2}} - \frac{2xz}{(x^2+z^2+9a^2)^{3/2}}\right]}$$ 1

zu berechnen $$\int_0^{2a} dx \int_0^{3a} dy \int_0^{a} dz \, \operatorname{div} \mathfrak{A}$$

bei v-Integration u wie Konstante behandelt; Bronstein/S., S. 310, No 207

es ist $$\int_0^{u_0} du \int_0^{v_0} dv \frac{uv}{(u^2+v^2+c^2)^{3/2}} = \int_0^{u_0} du\, u \cdot \left[-\frac{1}{\sqrt{u^2+v^2+c^2}}\right]_{v=0}^{v_0}$$ +1 ZP

$$= -\int_0^{u_0} du \frac{u}{\sqrt{u^2+v_0^2+c^2}} + \int_0^{u_0} du \frac{u}{\sqrt{u^2+c^2}}$$, Bronstein/S., S. 309, No 193

$$= -\sqrt{u^2+v_0^2+c^2}\Big|_0^{u_0} + \sqrt{u^2+c^2}\Big|_0^{u_0} = -\sqrt{u_0^2+v_0^2+c^2} + \sqrt{v_0^2+c^2}$$

$$+ \sqrt{u_0^2+c^2} + c$$ +1 ZP

damit $$-6\int_0^a dz \cdot \int_0^{2a} dx \int_0^{3a} dy \frac{xy}{(x^2+y^2+a^2)^{3/2}} - 3\int_0^{2a} dx \cdot \int_0^{3a} dy \int_0^a dz \frac{yz}{(y^2+z^2+4a^2)^{3/2}}$$

$$-2\int_0^{3a} dy \cdot \int_0^{2a} dx \int_0^a dz \frac{xz}{(x^2+z^2+9a^2)^{3/2}}$$

(4/3)

$$= -6a\left[-\sqrt{14a^2} + \sqrt{10a^2} + \sqrt{5a^2} + a\right] - 6a\left[-\sqrt{14a^2} + \sqrt{5a^2} + \sqrt{13a^2} + 2a\right]$$

$$- 6a\left[-\sqrt{14a^2} + \sqrt{10a^2} + \sqrt{13a^2} + 3a\right]$$

mit Faktor ka^2:

$$\boxed{\iiint_V \operatorname{div} \mathfrak{A}\, dV = -6ka^4\left[-3\sqrt{14} + 2\sqrt{10} + 2\sqrt{5} + 6 + 2\sqrt{13}\right]}$$

$$\boxed{= 6ka^4\left[6 - 2\sqrt{5} - 2\sqrt{10} - 2\sqrt{13} + 3\sqrt{14}\right]}$$, Übereinstimmung

1

5+3 ZP