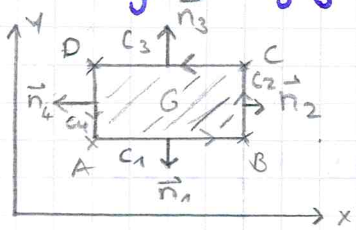
Lösung 8 Punkte

$\vec{V} = x^2 y\,\vec{i} - x^3 y^2 \vec{j}$



(vollst. Skizze) 2

a) $W = \oint_C \vec{V}\, d\vec{r}$

$C = C_1 + C_2 + C_3 + C_4$

$C_1: \quad \vec{r}_1 = t\vec{i} + \vec{j}, \quad d\vec{r}_1 = \vec{i}\,dt, \quad 1 \le t \le 3$

$C_2: \quad \vec{r}_2 = 3\vec{i} + t\vec{j}, \quad d\vec{r}_2 = \vec{j}\,dt, \quad 1 \le t \le 2$

$C_3: \quad \vec{r}_3 = t\vec{i} + 2\vec{j}, \quad d\vec{r}_3 = \vec{i}\,dt, \quad 3 \ge t \ge 1$

1 $C_4: \quad \vec{r}_4 = \vec{i} + t\vec{j}, \quad d\vec{r}_4 = \vec{j}\,dt, \quad 2 \ge t \ge 1$

$$W = \int_1^3 t^2\,dt + \int_1^2 -27 \cdot t^2\,dt + \int_3^1 2 \cdot t^2\,dt + \int_2^1 -t^2\,dt$$

$$= \left[\tfrac{1}{3}t^3\right]_1^3 - 9\left[t^3\right]_1^2 - \frac{2}{3}\left[t^3\right]_1^3 + \frac{1}{3}\left[t^3\right]_1^2$$

$$= \left(9 - \frac{1}{3}\right) - 9(8-1) - \frac{2}{3}(27-1) + \frac{1}{3}(8-1)$$

$$= \frac{26}{3} - 63 - 18 + \frac{2}{3} + \frac{7}{3}$$

$$= 9 - \tfrac{1}{3} - 81 + 3$$

1 $$= -69\tfrac{1}{3}$$

4 $$W = \int_{y=1}^{2} \int_{x=1}^{3} \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\,dx\,dy = -\int_{y=1}^{2} \int_{x=1}^{3} x^2(3y^2+1)\,dx\,dy$$

$$= -\left[\tfrac{1}{3}x^3\right]_1^3 \cdot \left[y^3 + y\right]_1^2$$

1 $$= -\frac{1}{3}(27-1) \cdot \left[(8+2) - (1+1)\right] = -\frac{26}{3} \cdot 8 = -\frac{208}{3} = -69\tfrac{1}{3}$$

$\Rightarrow$ Übereinstimmung

b) $F = \oint V \cdot \vec{n} \; ds \quad , ds = |d\vec{r}|$

Weg ändert sich nicht, $|d\vec{r}_{1,2}| = dt \quad , |d\vec{r}_{3,4}| = -dt \quad$ da $dt < 0$

$\vec{n}_1 = -\vec{j} \quad , \vec{n}_2 = \vec{i} \quad , \vec{n}_3 = \vec{j} \, , \vec{n}_4 = -\vec{i}$ 1

$$F = \int_1^3 t^3 \cdot 1 \, dt + \int_1^2 9 \cdot t \, dt + \int_3^1 + t^3 \cdot 4 \, dt + \int_2^1 t \, dt$$

$$= \int_1^3 t^3 \, dt + 9 \int_1^2 t \, dt - 4 \int_1^3 t^3 \, dt - \int_1^2 t \, dt$$

$$= -3 \left[\frac{1}{4} t^4 \right]_1^3 + 8 \left[\frac{1}{2} t^2 \right]_1^2$$

$$= \frac{-3}{4} (81 - 1) + 4 (4 - 1)$$

$$= -3 \cdot 20 + 12$$

$$= -48$$

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$$F = \iint_G \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} \, dx\, dy = \int_{y=1}^{2} \int_{x=1}^{3} 2xy - 2x^3 y \, dx\, dy$$

$$= \int_{y=1}^{2} 2y \, dy \cdot \int_{x=1}^{3} x - x^3 \, dx = \left[y^2 \right]_1^2 \cdot \left[\frac{1}{2} x^2 - \frac{1}{4} x^4 \right]_1^3$$

$$= (4-1) \cdot \left[\left(\frac{9}{2} - \frac{81}{4} \right) - \left(\frac{1}{2} - \frac{1}{4} \right) \right]$$

$$= 3 \cdot (4 - 20)$$

$$= -48$$

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