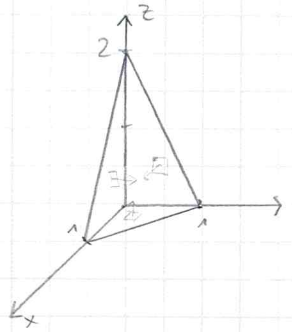
## Aufgabe 4 4 Punkte

$$\vec{\Phi} = xy\,\vec{i} + xz\,\vec{j} + yz\,\vec{k}$$

$$\operatorname{rot}\vec{\Phi} = \left(\frac{\partial \Phi_z}{\partial y} - \frac{\partial \Phi_y}{\partial z}\right)\vec{i} + \left(\frac{\partial \Phi_x}{\partial z} - \frac{\partial \Phi_z}{\partial x}\right)\vec{j} + \left(\frac{\partial \Phi_y}{\partial x} - \frac{\partial \Phi_x}{\partial y}\right)\vec{k}$$

$$= (z-x)\vec{i} + (0-0)\vec{j} + (z-x)\vec{k}$$




1/2 $\quad d\vec{f}_1 = \vec{e}_x\,dy\,dz \quad , x=0, \quad 0 \le z \le 2-2y, \quad 0 \le y \le 1$

1/2 $\quad d\vec{f}_2 = \vec{e}_y\,dx\,dz \quad , y=0, \quad 0 \le z \le 2-2x, \quad 0 \le x \le 1$

1/2 $\quad d\vec{f}_3 = \vec{e}_z\,dy\,dx \quad , z=0, \quad 0 \le y \le 1-x, \quad 0 \le x \le 1$

$$\iint_F \operatorname{rot}\vec{\Phi}\,d\vec{f} = \iint_{F_1} \operatorname{rot}\vec{\Phi}\,d\vec{f}_1 + \iint_{F_2} \operatorname{rot}\vec{\Phi}\,d\vec{f}_2 + \iint_{F_3} \operatorname{rot}\vec{\Phi}\,d\vec{f}_3$$

$$= \int_{y=0}^{1} \int_{z=0}^{2-2y} z\,dy\,dz + \int_{x=0}^{1} \int_{z=0}^{2-2x} 0\,dx\,dz$$

(Aufstellen Integrale) 1

$$+ \int_{x=0}^{1} \int_{y=0}^{1-x} (-x)\,dy\,dx$$

$$= \int_{y=0}^{1} \left[\frac{1}{2}z^2\right]_0^{2(1-y)} dy + \int_{x=0}^{1} -x\left[y\right]_0^{1-x} dx$$

$$= \int_{y=0}^{1} 2(1-y)^2\,dy - \int_{x=0}^{1} x(1-x)\,dx$$

$$= -\frac{2}{3}\left[(1-y)^3\right]_0^1 - \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1$$

(auf Rechnung verteilt) 1,5

$$= \frac{2}{3} - \frac{1}{2} + \frac{1}{3} = 1 - \frac{1}{2} = \frac{1}{2}$$

**Zum Vergleich:**

$$\int_C \vec{\Phi}\,d\vec{r}$$

$C_1: \quad \vec{r}_1 = t\,\vec{i} + (1-t)\vec{j} \quad , d\vec{r}_1 = (\vec{i} - \vec{j})\,dt \quad , 1 \ge t \ge 0.$

$C_2: \quad \vec{r}_2 = t\,\vec{j} + 2(1-t)\vec{k} \quad , d\vec{r}_2 = (\vec{j} - 2\vec{k})\,dt \quad , 1 \ge t \ge 0$

$C_3: \quad \vec{r}_3 = t\,\vec{i} + 2(1-t)\vec{k} \quad , d\vec{r}_3 = (\vec{i} - 2\vec{k})\,dt \quad , 0 \le t \le 1$

$$\int_C \vec{\Phi}\,d\vec{r} = \int_1^0 t\cdot(1-t)\,dt + \int_1^0 t\cdot 2(1-t)\cdot(-2)\,dt + 0$$

$$= -3 \int_1^0 t(1-t)\,dt = 3\left[\frac{1}{2}t^2 - \frac{1}{3}t^3\right]_0^1 = \frac{3}{2} - 1 = \frac{1}{2}$$