Lösung: Aufgabe 1

4 Punkte

- Bestimmung der Integrationsgrenzen:

$0 \le y \le 25$

$\frac{y}{a} - \frac{b}{a} \le x \le \sqrt{y}$

Bestimmung der Konstanten a & b: $y = ax + b$

P1 (-10, 0), P2 (-15, 25) einsetzen:

$0 = -10a + b$

$-(25 = -15a + b)$

$-25 = 5a \Rightarrow a = -5 \Rightarrow b = -50$ $y = -5x - 50$

$\Rightarrow -\frac{y}{5} - 10 \le x \le \sqrt{y}$ 1

- Integration:

$$V = \int_{y=0}^{25} \int_{x=-\frac{y}{5}-10}^{\sqrt{y}} z(x,y)\, dx\, dy = \int_{y=0}^{25} \int_{x=-\frac{y}{5}-10}^{\sqrt{y}} \frac{y}{10}\, dx\, dy = \frac{1}{10} \int_{y=0}^{25} y\, [x]_{-\frac{1}{5}y-10}^{\sqrt{y}}\, dy$$ 1

$$= \frac{1}{10} \int_{y=0}^{25} y^{3/2} + \frac{1}{5} y^2 + 10y\, dy = \frac{1}{10} \left[\frac{2}{5} y^{5/2} + \frac{1}{15} y^3 + 5y^2 \right]_0^{25}$$ 1

$$= \frac{1}{10} \left[\frac{2}{5} \cdot 5^5 + \frac{1}{15} \cdot 25^3 + 5 \cdot 25^2 \right] = \frac{1}{10} \left[2 \cdot 25^2 + \frac{5}{3} \cdot 25^2 + 5 \cdot 25^2 \right]$$

$$= \frac{25^2}{10} \left[\frac{6 + 5 + 15}{3} \right] = \frac{5 \cdot 25}{6} \cdot 26 = \frac{5^3 \cdot 13}{3} = \frac{1625}{3} \,/\!/$$ 1