Lösung: Dreiecksschwingung 7 Punkte + 2 Zusatz auf d)

a) $V(t) = \begin{cases} \frac{2t}{\pi} & \text{für } -\frac{\pi}{2} \le t \le \frac{\pi}{2} \\ 2 - \frac{2}{\pi} t & \text{für } \frac{\pi}{2} \le t \le \frac{3\pi}{2} \end{cases}$ 1

- periodisch mit $L = 2\pi$ ✓
- eindeutig ✓
  stetig ~~außer an $\pm\frac{n\pi}{2}$, $n \in \mathbb{N}_0$~~ ✓

(auf sinnvolle Überlegungen, müssen nicht vollständig sein) 1

- endliche Anzahl von Extremwerten im Intervall 2π: 2 ✓
- $\int_{-\pi/2}^{3\pi/2} |V(t)|\, dt = \int_{-\pi/2}^{\pi/2} \left|\frac{2t}{\pi}\right| dt + \int_{\pi/2}^{3\pi/2} \left|2 - \frac{2}{\pi}t\right| dt$

$$= \int_{-\pi/2}^{0} -\frac{2t}{\pi}\, dt + \int_{0}^{\pi/2} \frac{2t}{\pi}\, dt + \int_{\pi/2}^{\pi} 2 - \frac{2}{\pi}t\, dt - \int_{\pi}^{3\pi/2} 2 - \frac{2}{\pi}t\, dt$$

$$= \left[\frac{t^2}{\pi}\right]_0^{-\pi/2} + \left[\frac{t^2}{\pi}\right]_0^{\pi/2} + \left[2t - \frac{t^2}{\pi}\right]_{\pi/2}^{\pi} - \left[2t - \frac{t^2}{\pi}\right]_{\pi}^{3\pi/2}$$

$$= \frac{\pi}{4} + \frac{\pi}{4} + (2\pi - \pi) - (\pi - \frac{\pi}{4}) - (3\pi - \frac{9\pi}{4}) + (2\pi - \pi)$$

$$= \frac{3\pi}{4} + \frac{9\pi}{4} - 2\pi = 3\pi - 2\pi = \pi \rightarrow \text{konvergiert}$$ ✓

b) $a_n = 0 \quad \forall n \in \mathbb{N}_0$, da ungerade Funktion 1

$$b_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \frac{2}{\pi} t \cdot \sin(nt)\, dt + \frac{1}{\pi}\int_{\pi/2}^{3\pi/2} 2\left(1 - \frac{t}{\pi}\right)\sin(nt)\, dt$$ 1/2

NR: $\int t \cdot \sin(nt)\, dt = -\frac{t}{n}\cos(nt) + \int \frac{1}{n}\cos(nt)\, dt = -\frac{t}{n}\cos(nt) + \frac{1}{n^2}\sin(nt)$ + 1 ZP

$u = t \rightsquigarrow u' = 1$
$v' = \sin(nt) \rightsquigarrow v = -\frac{1}{n}\cos(nt)$

$$\Rightarrow b_n = \frac{2}{\pi^2}\left[-\frac{t}{n}\cos(nt) + \frac{1}{n^2}\sin(nt)\right]_{-\pi/2}^{\pi/2} - \frac{2}{\pi n}\left[\cos(nt)\right]_{\pi/2}^{3\pi/2}$$

$$- \frac{2}{\pi^2}\left[-\frac{t}{n}\cos(nt) + \frac{1}{n^2}\sin(nt)\right]_{\pi/2}^{3\pi/2}$$ 1/2

$$= \frac{2}{\pi^2 n^2}\left[\sin\left(\frac{n\pi}{2}\right) - \sin\left(-\frac{n\pi}{2}\right) - \sin\left(\frac{3n\pi}{2}\right) + \sin\left(\frac{n\pi}{2}\right)\right]$$

$$+ \frac{2}{\pi n}\left[-\cancel{\frac{1}{2}\cos\left(\frac{n\pi}{2}\right)} + -\cancel{\frac{1}{2}\cos\left(\frac{n\pi}{2}\right)} - \cos\left(\frac{3\pi n}{2}\right) + \cancel{\cos\left(\frac{\pi n}{2}\right)} + \frac{3}{2}\cos\left(\frac{3n\pi}{2}\right) - \frac{1}{2}\cos\left(\frac{n\pi}{2}\right)\right]$$

$$= \frac{6}{\pi^2 n^2}\sin\left(\frac{n\pi}{2}\right) - \frac{2}{\pi^2 n^2}\sin\left(\frac{3n\pi}{2}\right) - \frac{1}{\pi n}\cos\left(\frac{n\pi}{2}\right) + \frac{1}{\pi n}\cos\left(\frac{3\pi n}{2}\right)$$

Es gilt: $\sin\left(\frac{3\pi n}{2}\right) = \sin\left(\frac{\pi n}{2} + \pi n\right) = \sin\left(\frac{\pi n}{2}\right)\cdot\cos(n\pi)$ (da $\sin(n\pi) = 0$)

$\cos\left(\frac{3\pi n}{2}\right) = \cos\left(\frac{\pi n}{2} + \pi n\right) = \cos\left(\frac{\pi n}{2}\right)\cdot\cos(\pi n)$

$$\Rightarrow b_n = \frac{2}{n^2\pi^2} \sin\left(\frac{\pi n}{2}\right)\Big[3 - \underbrace{\cos(n\pi)}_{=(-1)^n}\Big] + \frac{1}{n\pi}\cos\left(\frac{\pi n}{2}\right)\Big[-1 + \underbrace{\cos(n\pi)}_{=(-1)^n}\Big]$$ 1

Fallunterscheidung:

i) n gerade, d.h. $n = 2k$, $k \in \mathbb{N}_0$

$$b_{2k} = \frac{2}{4k^2\pi^2}\underbrace{\sin(\pi \cdot k)}_{=0}\Big[3 - (-1)^{2k}\Big] + \frac{1}{2k\pi}\cos(k\cdot\pi)\underbrace{\Big[-1+1\Big]}_{=0} = 0 \quad //$$ 1/2

ii) n ungerade, d.h. $n = 2k+1$ $k \in \mathbb{N}_0$

$$b_{2k+1} = \frac{2}{(2k+1)^2\pi^2}\sin((k+1/2)\pi)\underbrace{\Big[3-(-1)\Big]}_{=4} + \frac{1}{(2k+1)\pi}\underbrace{\cos((k+1/2)\pi)}_{=0}\Big[-1-1\Big]$$

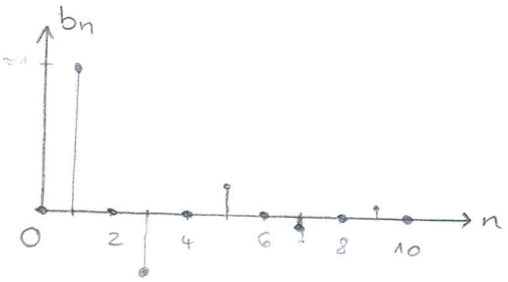
$$= \frac{8}{(2k+1)^2\pi^2}\Big[\sin(k\pi)\underbrace{\cos(\pi/2)}_{=0} + \underbrace{\cos(k\pi)}_{=(-1)^k}\underbrace{\sin(\pi/2)}_{=1}\Big]$$

$$= \frac{8}{(2k+1)^2\pi^2}(-1)^k$$ 1/2

//

k	0	1	2	3	...
$n = 2k+1$	1	3	5	7	...
b_n	$8/\pi^2$	$\frac{-8}{9\pi^2}$	$\frac{8}{25\pi^2}$	$\frac{-8}{49\pi^2}$	...

c)



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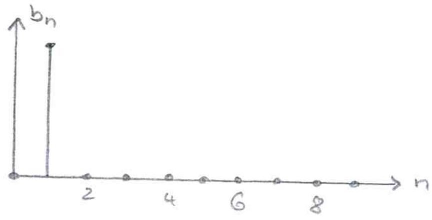
d) Zusatz

$$\sin(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{2\pi n t}{L}\right) + b_n \sin\left(\frac{2\pi n t}{L}\right)\right) \qquad L = 2\pi$$

$$\Rightarrow a_n = a_0 = 0$$

$$\Rightarrow b_n = 0 \quad \forall\, n \geq 2, \quad b_1 = 1$$



=> Dreiecksschwingung ähnelt Sinusschwingung
–> in Fourierreihe reicht erstes Glied für gute Approximation
=> Unterschied: Dreiecksschwingung enthält unendlich viele Frequenzen

+ 2 ZP