

Aufgabe 1 *Formelblatt*

a) Zylinderkoordinaten

Linienelement: $ds^2 = dr^2 + r^2 d\phi^2 + dz^2$

Volumenelement: $dV = r dr d\phi dz$

Gradient: $\text{grad } U = \frac{\partial U}{\partial r} \vec{e}_r + \frac{1}{r} \frac{\partial U}{\partial \phi} \vec{e}_\phi + \frac{\partial U}{\partial z} \vec{e}_z$

Divergenz: $\text{div } \vec{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$

Rotation: $\text{rot } \vec{A} = \left[\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] \vec{e}_r + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right] \vec{e}_\phi + \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\phi) - \frac{\partial A_r}{\partial \phi} \right] \vec{e}_z$

Laplace-Operator: $\Delta U = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 U}{\partial \phi^2} + \frac{\partial^2 U}{\partial z^2}$

b) Kugelkoordinaten

Linienelement: $ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$

Volumenelement: $dV = r^2 \sin \theta dr d\theta d\phi$

Gradient: $\text{grad } U = \frac{\partial U}{\partial r} \vec{e}_r + \frac{1}{r} \frac{\partial U}{\partial \theta} \vec{e}_\theta + \frac{1}{r \sin \theta} \frac{\partial U}{\partial \phi} \vec{e}_\phi$

Divergenz: $\text{div } \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{\partial A_\phi}{\partial \phi} \right]$

Rotation:

$$\text{rot } \vec{A} = \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta A_\phi) - \frac{\partial A_\theta}{\partial \phi} \right] \vec{e}_r + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r A_\phi) \right] \vec{e}_\theta + \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \vec{e}_\phi$$

Laplace-Operator:

$$\Delta U = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial U}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial U}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 U}{\partial \phi^2}$$