Lösung: Die Heaviside'sche Sprungfunktion    3 Punkte


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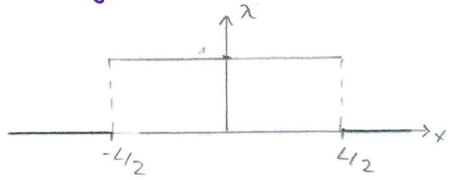
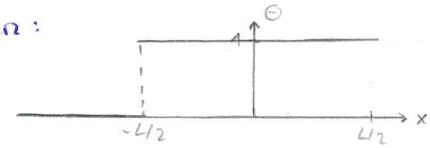
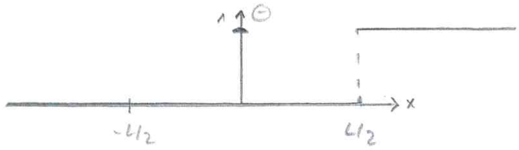



a) Überlegungen:

$\Theta(x+L/2)$




$\Theta(x-L/2)$




$$\Rightarrow \quad \lambda(x) = \frac{Q}{L}\left[\Theta(x+L/2) - \Theta(x-L/2)\right]$$

Außerdem:

$$\Theta(x+L/2) - \Theta(x-L/2) = \begin{cases} 1 & \text{für } -L/2 < x < L/2 \\ 0 & \text{sonst} \end{cases}$$

$$= \Theta(x+L/2)\cdot\Theta\left(\frac{L}{2}-x\right) = \begin{cases} 1 & \text{für } x > -L/2 \vee x < L/2 \\ 0 & \text{sonst} \end{cases}$$

$$\Rightarrow \lambda(x) = \frac{Q}{L}\,\Theta(x+L/2)\cdot\Theta\left(\frac{L}{2}-x\right)$$ Linienladungsdichte    1

Raumladungsdichte: $\rho(x,y,z) = \frac{Q}{L}\,\Theta(x+L/2)\,\Theta\left(\frac{L}{2}-x\right)\,\delta(y)\,\delta(z)$    1

b) Zylinderkoordinaten: $0 \leq r \leq \frac{L}{2}$, $\varphi = 0$, $\varphi = \pi$, $z = 0$

$$\rho(r,\varphi,z) = \frac{Q}{L}\,\Theta\left(\frac{L}{2}-r\right)\cdot\frac{\delta(\varphi)+\delta(\varphi-\pi)}{r}\cdot\delta(z)$$    1

[Probe: $\iiint \rho(r,\varphi,z)\, r\, dr\, d\varphi\, dz = \frac{Q}{L}\underbrace{\int_{-\infty}^{\infty}\delta(z)\,dz}_{=1}\cdot\int_{0}^{\infty}\Theta\left(\frac{L}{2}-r\right)dr\cdot\underbrace{\int_{0}^{2\pi}\delta(\varphi)+\delta(\varphi-\pi)\,d\varphi}_{=1+1=2}$

$$= \frac{Q}{L}\cdot\int_0^{L/2} dr\cdot 2$$

$$= \frac{Q}{L}\cdot\frac{L}{2}\cdot 2 = Q \;\checkmark$$ ]