5 Punkte
(3+2)

Lösung: Gauß'sche Integrale

a) gesucht: $I = \int_0^\infty e^{-x^2} dx$

1/2 über $I = \frac{1}{2}\sqrt{J}$ mit $J = \left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)^2$

$$J = \int_{-\infty}^{\infty} e^{-x^2} dx \cdot \int_{-\infty}^{\infty} e^{-y^2} dy = \iint_{-\infty}^{\infty} e^{-(x^2+y^2)} dx\, dy$$

1 $\xrightarrow{\text{Polarkoordinaten}}$ $= \int_{r=0}^{\infty} \int_{\varphi=0}^{2\pi} e^{-r^2} r\, dr\, d\varphi = 2\pi \cdot \int_{r=0}^{\infty} r e^{-r^2} dr$

Substituiere $u = r^2 \rightsquigarrow du = 2r\,dr$

1 $= 2\pi \cdot \frac{1}{2} \int_{u=0}^{\infty} e^{-u} du = \pi \cdot \left[-e^{-u}\right]_0^\infty = \pi\left[0 - (-1)\right] = \pi$ //

1/2 $\Rightarrow \underline{I = \sqrt{\pi}/2}$

b) gesucht: $K = \int_{-\infty}^{\infty} e^{-ax^2+bx} dx$

1 quadratische Ergänzung: $ax^2 + bx = a(x^2 + \frac{b}{a}x) = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right]$

$$K = \int_{-\infty}^{\infty} e^{-a(x+b/2a)^2} \cdot e^{b^2/4a} dx$$

Substituiere $u = \sqrt{a}\,(x + \frac{b}{2a}) \rightsquigarrow du = \sqrt{a}\,dx$

1 $= \underbrace{\int_{-\infty}^{\infty} e^{-u^2} du}_{\sqrt{\pi}} \cdot \frac{1}{\sqrt{a}} \cdot e^{b^2/4a} = \underline{\sqrt{\frac{\pi}{a}}\, e^{b^2/4a}}$