Aufgabe 4: Lösung 4 ~~5~~ Punkte

a) $y = x^3$, Flächenmassendichte σ_0

$\sigma = \sigma_0$, gesucht: $\mu(x,y,z)$, Ansatz: $\mu(x,y,z) = \delta(y-x^3)\cdot f(x,y,z)$ 1

$$\iint \sigma \, df = \iiint \mu \, dV$$

$$df = \underbrace{\sqrt{1+\left(\frac{\partial y}{\partial x}\right)^2+\left(\frac{\partial y}{\partial z}\right)^2}\; dx\,dz}_{} = \sqrt{1+(3x^2)^2+0}\;\overset{dxdz}{=} \sqrt{1+9x^4}\; dx\,dz$$ 1/2

$$\Rightarrow \iint \sigma \, df = \sigma_0 \iint \sqrt{1+9x^4}\, dx\,dz$$

$$\iiint \mu \, dV = \iiint \mu \, dx\,dy\,dz = \iint \left(\int \mu \, dy\right) dx\,dz \overset{!}{=} \sigma_0 \iint \sqrt{1+9x^4}\, dx\,dz$$

$$\Rightarrow \int \mu \, dy = \sigma_0 \sqrt{1+9x^4}$$

$$\Rightarrow \mu(x,y,z) = \sigma_0 \sqrt{1+9x^4}\; \delta(y-x^3)$$

oder: $\mu(x,y,z) = \sigma_0 \sqrt{1+9y^{4/3}}\; \delta(y-x^3)$ 1/2

b) $g(x) = -\Theta(-3-x) + (x+2)\left[\Theta(3+x) - \Theta(1+x)\right]$

$\quad + x^2\left[\Theta(1+x) - \Theta(x-1)\right] + (2-x)\,\Theta(x-1)$

$= -\Theta(-3-x) + (x+2)\,\Theta(3+x) + \Theta(1+x)\left[x^2-x-2\right] + \Theta(x-1)\left[2-x-x^2\right]$

$= -\Theta(-3-x) + (x+2)\,\Theta(3+x)\cdot\Theta(-1-x) + x^2\,\Theta(1+x)\,\Theta(1-x) + (2-x)\,\Theta(x-1)$

$= -\Theta(-3-x) + (x+2)\,\Theta(-3-4x-x^2) + x^2\,\Theta(1-x^2) + (2-x)\,\Theta(x-1)$

(weitere Varianten sind möglich) 2