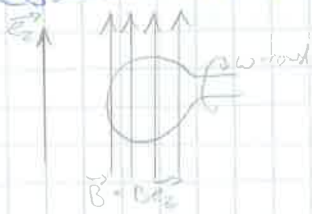
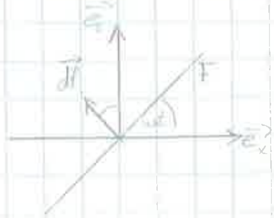
Aufgabe 24: Leiterschleife (Fläche F, Widerstand R, Selbstinduktiv. L)





$\vec{df} = \cos(\omega t)\vec{e_z} - \sin(\omega t)\vec{e_x}$

$RJ(t) = U(t) = U_B(t) + U_S(t)$

$U_B(t) = -\frac{d}{dt}\int \vec{B}\,\vec{df} = -\frac{d}{dt}(BF\cos\omega t)$

$U_B(t) = \omega BF\sin(\omega t)$

$\cdot\ U_S(t) = -L\dot{J}$

$\rightsquigarrow \dot{J} + \frac{R}{L}J = \frac{BF\omega}{L}\sin(\omega t) = f(t)$

hom. Gl.: $\dot{J} + \frac{R}{L}J = 0$

$\rightsquigarrow \frac{dJ}{J} = -\frac{R}{L}dt \rightsquigarrow \ln J = -\frac{R}{L}t + \tilde{c}$

$\rightsquigarrow J_h(t) = Ce^{-\frac{R}{L}t}$

Ansatz für die inhomog. Gl.: $J(t) = C(t)e^{-\frac{R}{L}t}$
(Variation der Konstanten)

$\dot{J} = \dot{C}e^{-\frac{R}{L}t} - C\frac{R}{L}e^{-\frac{R}{L}t}$

$\dot{J} + \frac{R}{L}J = \dot{C}e^{-\frac{R}{L}t} \overset{!}{=} f(t)$

$\rightsquigarrow \dot{C} = f(t)e^{\frac{R}{L}t}$

$\rightsquigarrow C(t) = \frac{BF\omega}{L}\int \sin(\omega t)e^{\frac{R}{L}t}dt$

$\rightsquigarrow C(t) = \frac{BF\omega}{L}\frac{\frac{R}{L}\sin(\omega t) - \omega\cos(\omega t)}{\omega^2 + (\frac{R}{L})^2}e^{\frac{R}{L}t} + D$

$\rightsquigarrow J(t) = De^{-\frac{R}{L}t} + \frac{BF\omega}{\omega^2L^2 + R^2}(R\sin(\omega t) - \omega L\cos(\omega t))$

Falls $J(0) = 0$

$0 = D + \frac{BF\omega}{\omega^2L^2 + R^2}(-\omega L)$

$\rightsquigarrow D = \frac{BF\omega^2 L}{\omega^2L^2 + R^2}$

Aufgabe 25:

(r, φ, z)

$\vec{J} = (0, 0, J_z(r, t))$

quasi-stationäre Maxwell-Gln.

$$\left.\begin{array}{ll} \text{rot}\,\vec{H} = \vec{J}, & \text{rot}\,\vec{E} = -\dot{\vec{B}} \\ \vec{J} = \sigma\vec{E} & \vec{B} = \mu_0\vec{H} \end{array}\right\} \text{rot rot}\,\vec{J} = -\mu_0\sigma\dot{\vec{J}}$$

$\frac{1}{\sigma}\text{rot}\,\vec{J} = -\dot{\vec{B}} = -\mu_0\dot{\vec{H}}$
$\text{rot rot}\,\vec{J} = -\mu_0\sigma(\text{rot}\,\dot{\vec{H}}) = -\mu_0\sigma\dot{\vec{J}}$

Ansatz: $J_z(r, t) = \text{Re}(a(r)e^{-i\omega t})$

$\rightsquigarrow \frac{1}{r}\frac{d}{dr}r\frac{da}{dr} = -i\omega\mu_0\sigma a \qquad x := \sqrt{i\omega\mu_0\sigma}\,r$

$\rightsquigarrow a'' + \frac{a'}{x} + a = 0 \qquad$ | Besselsche DGl: $a'' + \frac{a'}{x} + (1 - \frac{n}{x^2})a = 0$ (hier $n=0$)

(a endlich bei $r=0$ ⇝ $B=0$) J_n Besselfkt. N_n Neumannsche Fkt.

also: $\vec{j} = j_z \vec{e}_z$ mit $j_z = \mathrm{Re}[A J_0(\sqrt{i\omega\mu_0\sigma}\, r) e^{-i\omega t}]$

$$J(t) = \int_0^{r_0} j_z 2\pi r\, dr = \mathrm{Re}[J_0 e^{-i\omega t}]$$

$$\rightsquigarrow A = \frac{J_0}{2\pi \int J_0(\sqrt{i\omega\mu_0\sigma}\, r) r\, dr}$$

o.B.d.A. J_0 reell ⇝ $J(t) = J_0 \cos\omega t$

außen: $\vec{H} = H_\varphi \vec{e}_\varphi$, $\vec{B} = \mu_0 \vec{H}$

$H_\varphi = H_\varphi(r,t)$

$$J(t) = \int \vec{j}\, d\vec{f} = \int \mathrm{rot}\, \vec{H}\, d\vec{f} = \oint \vec{H}\, d\vec{r} = \underline{H_\varphi 2\pi r}$$

$$\rightsquigarrow H_\varphi(r) = \frac{J(t)}{2\pi r} = \frac{J_0 \cos(\omega t)}{2\pi r}$$

$$\vec{E} = E_z \vec{e}_z \qquad \mathrm{rot}\, \vec{E} = -\dot{\vec{B}} \qquad E_z = E_z(r,t)$$

$$\mathrm{rot}\, \vec{E} = -\frac{\partial E_z}{\partial r} \vec{e}_\varphi = \frac{\mu_0 J_0 \omega \sin(\omega t)}{2\pi r} \vec{e}_\varphi$$

$$\rightsquigarrow E_z = \frac{\mu_0 \omega J}{2\pi} \ln\frac{r_0}{r} \sin(\omega t) + \mathrm{Re}(C e^{-i\omega t})$$

· Bestimmung von C am Rand bei $r = r_0$

$$E_z = \frac{1}{\sigma} j_z = \frac{1}{\sigma} \mathrm{Re}[A J_0(\sqrt{i\omega\mu_0\sigma}\, r_0) e^{-i\omega t})]$$

$$\rightsquigarrow \underline{\underline{C = \frac{J_0\, J_0(\sqrt{i\omega\mu_0\sigma}\, r_0)}{2\pi\sigma \int J_0(\sqrt{i\omega\mu_0\sigma}\, r) r\, dr}}}$$

Aufgabe 26

$$W_{magn.} = \frac{1}{2} \int \vec{H}\vec{B}\, d^3\vec{r} = \frac{1}{2\mu_0} \int \vec{B}^2 d^3\vec{r}$$

$$\frac{\partial}{\partial t} W_{magn.} = \frac{1}{2\mu_0} \frac{\partial}{\partial t} \int \vec{B}^2 d^3\vec{r} = \frac{1}{2\mu_0} \int 2\dot{\vec{B}}\vec{B}\, d^3\vec{r} = -\frac{1}{\mu_0} \int \mathrm{rot}\, \vec{E}\vec{B}\, d^3\vec{r}$$

Vektoridentität: $\mathrm{div}(\vec{F} \times \vec{G}) = \vec{F}\, \mathrm{rot}\, \vec{G} - \vec{G}\, \mathrm{rot}\, \vec{F}$

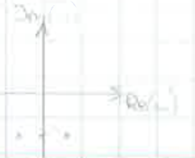
$$= \frac{1}{\mu_0} \int (\underbrace{\mathrm{div}(\vec{B} \times \vec{E})}_{\text{Gaußscher Satz über sehr großes Volumen} \to 0} - \vec{E}\, \mathrm{rot}\, \vec{B})\, d^3\vec{r} = -\frac{1}{\mu_0} \int \vec{E}\, \mathrm{rot}\, \vec{B}\, d^3\vec{r}$$

$$= -\frac{1}{\mu_0} \mu_0 \int \vec{E}\, \mathrm{rot}\, \vec{H}\, d^3\vec{r} = -\int \vec{E}\vec{j}\, d^3\vec{r} \overset{\vec{E} = \frac{1}{\sigma}\vec{j}}{=} \underline{\underline{-\frac{1}{\sigma} \underbrace{\int \vec{j}^2 d^3\vec{r}}_{>0}}}$$

→ negative Energieänderung ⇒ Energieabnahme der magn. Energie
⇒ $\vec{B}$ nimmt ab ($\& \vec{H}$) ⇒ $\vec{j}$ nimmt ab: $j \to 0$ für $t \to \infty$

05.02.15 **Aufgabe 29**

b) $$\chi_e = \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\gamma\omega} \qquad (\omega_p, \omega_0, \gamma > 0)$$



Nullstellen des Nenners (Polstellen von χ_e): $\omega_{\pm} = \frac{1}{2}\left(-i\gamma \pm \sqrt{4\omega_0^2 - \gamma^2}\right)$ liegen in d. unteren Halbebene

$$\chi_e = \frac{-\omega_p^2}{(\omega-\omega_+)(\omega-\omega_-)}$$
$$= \frac{-\omega_p^2}{\omega_+-\omega_-}\left(\frac{1}{\omega-\omega_+} - \frac{1}{\omega-\omega_-}\right)$$

$$G(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \chi_e(\omega) e^{-i\omega t} d\omega \quad (\tau \text{ reell})$$

12.02.15

$\omega = \omega_R + i\omega_I$ (R-Realteil, I-Imaginärteil)

$-i\omega\tau = -i\tau(\omega_R + i\omega_I) = \tau\omega_I - i\tau\omega_R$

① $\tau > 0$: $e^{-i\omega\tau} = e^{\tau\omega_I - i\tau\omega_R} \xrightarrow[|\omega|\to\infty]{} 0$
falls $\omega_I = \mathrm{Im}(\omega) < 0$ $(\sim \tau\omega_I < 0)$

② $\tau < 0$: $e^{-i\omega\tau} = e^{\tau\omega_I - i\tau\omega_R} \xrightarrow[|\omega|\to\infty]{} 0$
falls $\omega_I = \mathrm{Im}(\omega) > 0$

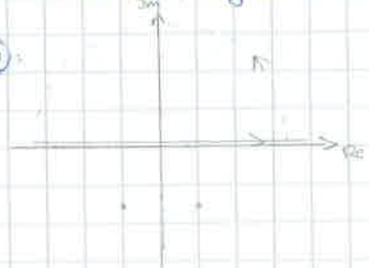
zu ①: $G(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \chi_e(\omega) e^{-i\omega\tau} d\omega$

Residuensatz: $= \underbrace{-}_{\text{Umlaufsinn}}\frac{2\pi i}{2\pi}\left(\frac{-\omega_p^2 e^{-i\omega_+\tau}}{\omega_+-\omega_-} + \frac{\omega_p^2 e^{-i\omega_-\tau}}{\omega_+-\omega_-}\right) = \frac{i\omega_p^2}{\sqrt{4\omega_0^2-\gamma}}\left(e^{-\frac{\tau}{2}(\gamma - i\sqrt{4\omega_0^2-\gamma^2})} - e^{-\frac{\tau}{2}(\gamma+i\sqrt{\ })}\right)$

$$= -\frac{i\omega_p^2 e^{-\frac{\tau\gamma}{2}}}{\sqrt{4\omega_0^2-\gamma^2}}\, 2i \sin\left(\frac{\tau}{2}\sqrt{4\omega_0^2-\gamma^2}\right)$$
$$= \underline{\underline{\frac{2\omega_p^2 e^{-\frac{\tau\gamma}{2}}}{\sqrt{4\omega_0^2-\gamma^2}} \sin\left(\frac{\tau}{2}\sqrt{4\omega_0^2-\gamma^2}\right)}}$$

Bem.: $G(\tau)$ ist in jedem Fall reell (wegen $\sin(ix) = i\sinh(x)$, $\sqrt{\ } = i\cdot$ reelle Größe)

zu ②:



$\rightsquigarrow \underline{G(\tau<0) = 0}$