1 Preliminaries

1.1 Metric Spaces

- Definition: Metric and Metric Space
- Definition: Induced Metric
- Examples: d_1, d_∞, d_2 for $\mathbb{R}^n$ and $C(I)$
- Definition: Convergent Sequences and Limits
- Lemma: Limit is Unique
- Definition: Cauchy Sequence
- Proposition: Convergent Sequence is Cauchy Sequence
- Remark: d_1 restricted to $\mathbb{Q}$ is not dense.
- Definition: Complete Metric Spaces
- Definition: Dense Subsets
- Definition: Continuity of Functions
- Proposition: Characterization of Continuity
- Definition: Isometry and Isometric Metric Spaces
- Theorem: Unique Completion of Isometric Dense Metric Subspaces
- Definition: Open and Closed Ball, Open Sets, Neighborhood and Interior Points
- Definition: Set of Limit Points, Closed Sets and Closures
- Lemma: Characterization of Limit Points and Closed Sets
- Lemma: Closed Subspaces of Complete Spaces are Complete
- Remark: Characterization of Density of Subspace
- Theorem: Rules for Open and Closed Sets
- Theorem: Characterization of Continuous Functions
- Theorem: Product Metric

1.2 Normed Vector Spaces

- Definition: Linear Operator
- Definition: Norm, Seminorm and Normed Spaces
- Remark: Normed Spaces Induce Metric Spaces
- Example: Minkowski-Norm for $\mathbb{R}^n$ and $C(I)$
- Lemma: Young's Inequality
- Theorem: Hölder's Inequalities
- Theorem: Minkowski's Inequalities
- Theorem: Second Version of Triangle Inequality
- Definition: Equivalent Norms
- Remark: Finite Dimensional Norms are Equivalent, Indistinguishability
- Remark: Product Metric
- Proposition: $+$ and $\cdot$ of Normed Spaces are Continuous
- Definition: Bound linear Transformations
- Lemma: Normed Space of Bounded Linear Operators and Norm Characterizations
- Theorem: Characterization of Bounded Linear Operators
- Definition: Banach Space
- Theorem: Completion of Normed Vector Spaces
- Remark: $L^p(I)$ Spaces
- Theorem: BLT-Theorem
- Remark: Lebesgue-Integral as Extension of $C_c(\mathbb{R}^n)$
- Theorem: Characterization of Completeness

2 Measure Theory

2.1 Measure Theory

- Measurable Spaces and Measure Spaces
  - Definition: Algebra, σ-Algebra, Measurable Space, Measurable Sets
  - Example: Trivial σ-Algebras
  - Lemma: Properties of Algebras and σ-Algebras
  - Definition: Generator of σ-Algebra
  - Remark: Existence and Uniqueness of Smallest σ-Algebra
  - Definition: Borel-σ-Algebra
  - Lemma: Monotonicity of Generator
  - Example: Generators of Borel-σ-Algebra
  - Example, Proposition, Lemma, Corollary: Generator of Product Borel-σ-Algebra
  - Definition: Product σ-Algebra
  - Proposition, Example: Elementary Family Induces Algebra
  - Definition: Measure, Measure Spaces, finite and σ-finite
  - Example: Dirac Point Measure
  - Theorem: Monotonicity, Subadditivity, Continuity from Below and Above
  - Definition: Null Set, Almost Everywhere Holding Statements
  - Definition: Complete Measures
  - Theorem, Definition: Completion of Measure Space
- Lebesgue-Measure
  - Definition: Lebesgue Outer Measure
  - Remark: Lebesgue Outer Measure is well defined
  - Theorem: Restriction of Lebesgue Outer Measure to Borel-σ-Algebra is a Measure
  - Definition: Lebesgue Measure and Lebesgue-measurable Sets
  - Theorem: Measurable Sets
  - Theorem: Characterization Lebesgue-Measurability
- Construction of Measures from Outer Measures
  - Definition: Outer Measure
  - Lemma: Construction of Outer Measure
  - Example: Lebesgue Outer Measure is Outer Measure
  - Definition, Remark: Measurable Set with Respect to Outer Measure
  - Theorem: Caratheodory, Existence of Measures
  - Lemma: Open Intervals are outer-measurable
  - Theorem: Characterization of Lebesgue-Measurability
- Uniqueness of Measures and Monotone Classes
  - Definition: Monotone Class and Generator
  - Theorem: Monotone Class Theorem
  - Theorem: Uniqueness of Measures

2.2 Integration Theory

- Integration
  - Measurable Functions
  - Lemma: Characterization, Composition, Continuity
  - Proposition: Measurable Functions on Product Spaces
  - Corollary: Coordinates, Sum and Product are Measurable
  - Theorem: sup, inf, limsup, liminf, lim are Measurable
  - Definition: Simple Functions
  - Lemma: Simple Function Formulation with Linear Combination, Characterization Measurability
  - Theorem: Monotone Convergence of Simple Functions to Measurable Functions
  - Definition: Integral of Simple Functions
  - Proposition: Measure, Linearity and Monotonicity of Positive Simple Function Integrals
  - Definition: Lebesgue Integral for Non-negative Functions
  - Remark: Properties of Lebesgue Integral
  - Theorem: Monotone Convergence Theorem by Lebesgue
  - Corollary: Linearity, Interchange of Sum and Integral
  - Theorem: Lemma by Fatou
  - Definition: Lebesgue Integral
  - Proposition, Definition: L^1 Space
  - Theorem: Dominated Convergence
  - Theorem: Parameter dependent Integrals
- Product Measures
  - Proposition: Section Property
  - Theorem: Product Measure
  - Theorem: Fubini-Tonelli
- L^p Spaces
  - Definition: L^p Spaces
  - Lemma: L^p is a Vector Space
  - Theorem: Hölder's Inequality
  - Theorem: Minkowski's Inequality
  - Theorem: L^p-Norm Implies almost everywhere Pointwise Convergence of Subsequence
  - Corollary: Cauchy Sequence
  - Theorem: Set of Simple Functions is Dense in L^p
  - Corollary: L^p has countable dense subset, $C_c(\mathbb{R}^n)$ is Dense in $L^p(\mathbb{R}^n)$
  - Definition: Essential Supremum and L^∞
  - Theorem: Properties of L^∞