

Exercise 2

Problem 2.1 (Ring as subgraph of hypercube). Show that a hypercube with $n = 2^d$ nodes whose dimension d is larger than or equal to 2 contains a ring with n nodes as a subgraph.

Problem 2.2 (Characteristics of Interconnection Networks). An undirected graph can be used to describe a static interconnection network. The nodes and edges of the graph represent the processors and their interconnections, respectively. The following criteria characterize the cost and the performance of an interconnection network:

- The *diameter* of a network is the maximum distance between any two processors. Here, the distance between two processors is defined as the minimum number of edges between them, i.e., the shortest path in the graph between the two nodes representing the two processors.
- The *edge connectivity* of a network is the minimum number of edges that must be removed from the network to break it into two disconnected networks.
- The *bisection width* of a network is the minimum number of edges that have to be removed from the network to partition the network into two equal halves.

Complete the following table containing the diameter d , the edge connectivity e , and the bisection width b of a completely-connected network (CCC), a star, a ring, a two-dimensional mesh with wraparound (2D w), a two-dimensional mesh without wraparound (2D wo), a three-dimensional mesh with wraparound (3D w), a three-dimensional mesh without wraparound (3D wo), and a hypercube (cube). Let p denote the number of processors.

	CCC	star	ring	2D w	2D wo	3D w	3D wo	cube
d								
e								
b								

What is described by these network criteria? What is the connection between the diameter of a network and a scalar reduction operation executed on this network?

Conclusion: *Criteria of interconnection networks may give a hint on execution times of basic communication operations.*