Institute for Computer Science, Friedrich Schiller University Jena ***fmi.uni-jena.de***
H. Martin Bücker, Ralf Seidler ***{martin.buecker,ralf.seidler}@uni-jena.de***

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Exercise 5

Problem 5.1 (n-body Problem Using a Direct Method)**.** Consider the gravitational forces between n particles of varying mass in two space dimensions. To every particle i, we associate a mass m_i as well as a location and velocity denoted by

$$\mathbf{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \quad \text{and} \quad \mathbf{v}_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix},$$

respectively. The gravitational force on mass m_i by a single mass m_j is then given by

$$\frac{G m_i m_j (\mathbf{x}_i - \mathbf{x}_j)}{||\mathbf{x}_i - \mathbf{x}_j||^3},$$

where G is the gravitational constant. The product of mass and acceleration of particle i is obtained by summing the gravitational forces over all particles leading to

$$m_i \frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = \sum_{\substack{i=1 \\ i \neq j}}^{n} \frac{G m_i m_j (\mathbf{x}_i - \mathbf{x}_j)}{||\mathbf{x}_i - \mathbf{x}_j||^3}. \tag{5.1}$$

Furthermore, the position and velocity of particle i are related by

$$\frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}t} = \mathbf{v}_i. \tag{5.2}$$

The explicit Euler method is among the simplest approaches to integrate equations of the form (5.1) or (5.2). Starting at some time t_0, this method discretizes time on an equidistant grid

$$t^{(k)} = t_0 + k\Delta t, \quad k = 0, 1, 2, \ldots,$$

where Δt is the size of the time step and the superscript denotes the index of the time step. Using the explicit Euler method, the velocity at the next time step is given by

$$\mathbf{v}_i^{(k+1)} = \mathbf{v}_i^{(k)} + \Delta t \sum_{\substack{i=1 \\ i \neq j}}^{n} \frac{G m_j (\mathbf{x}_i - \mathbf{x}_j)}{||\mathbf{x}_i - \mathbf{x}_j||^3} \tag{5.3}$$

and the postion at the next time step is given by

$$\mathbf{x}_i^{(k+1)} = \mathbf{x}_i^{(k)} + \Delta t \mathbf{v}_i^{(k)}. \tag{5.4}$$

A so-called *direct method* consists of a straightforward sequential algorithm that considers all pairs of particles in each time step. It computes the contributions to the velocity of all particles by summing over (5.3). Likewise, the contributions to the location follows from summing over (5.4). Thus, a direct methods needs $O(n^2)$ operations per time step.
Implement a direct method for the solution of the n-body problem and parallelize this program using OpenMP.

Conclusion: *Shared-memory parallelism simplifies the management of irregular memory accesses.*