Friedrich-Schiller-Universität Jena Winter term 2018/19
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Problems in Advanced Quantum Theory

Sheet 3

Problem 5: Spin coupling between two electrons 2+2+2+2 points

Consider two electrons whose Coulomb interaction is completely screened and which are kept at fixed positions in space. They interact with each other, as well as with an external magnetic field, through their magnetic moments, that is by means of their spins. We model this interaction with the following Hamiltonian:

$$H = a\,\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + b\,(S_{1z} + S_{2z})\,, \tag{1}$$

with a, $b \in \mathbb{R}$. Find the eigenvalues and an orthonormal basis of eigenvectors of H.

Problem 6: Coupling of three angular momenta 8 points

A system consists of three spin-1/2 particles. Construct an orthonormal basis $|S,\, S_z\rangle$ of eigenvectors of $\boldsymbol{S}^2$ and S_z, where

$$\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2 + \boldsymbol{S}_3 \tag{2}$$

is the total spin of the system.

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