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Problems in Advanced Quantum Theory

Sheet 7

Problem 12: Time-dependent perturbation theory 2+4+4 points

An atomic system is subject to an external oscillating electric field

$$\mathbf{E}(t) = \mathbf{E}_0 \sin \omega t \,. \tag{1}$$

To first order in the electromagnetic interaction, and neglecting magnetic dipoles, we can describe the effect of the external field as a time-dependent perturbation

$$V(t) = \theta(t) \left[\hat{F}^\dagger e^{\mathrm{i}\omega t} + \hat{F} e^{-\mathrm{i}\omega t} \right] , \tag{2}$$

where $\theta(t)$ is a Heaviside step function and

$$\hat{F} = -\frac{1}{2} \hat{\mathbf{d}} \cdot \mathbf{E}_0 \,. \tag{3}$$

Here $\hat{\mathbf{d}}$ is the self-adjoint electric dipole operator.

Call $|n^{(0)}\rangle$ the state of the system for $t < 0$, and $\hbar\omega_n$ the corresponding energy. Call

$$|n(t)\rangle = |n^{(0)}\rangle + |n^{(1)}(t)\rangle \tag{4}$$

the state for $t > 0$ at first order in perturbation theory, where

$$|n^{(1)}(t)\rangle = \sum_m c_m^{(1)}(t) |m^{(0)}\rangle \,. \tag{5}$$

Finally denote

$$\omega_{nm} = \omega_n - \omega_m \,. \tag{6}$$

1. Express the expectation value of the interaction-picture dipole operator

$$\mathbf{D}(t) = \langle n(t) | \hat{\mathbf{d}}_I(t) | n(t) \rangle \,, \tag{7}$$

at *first* order in the perturbation, in terms of the coefficients $c_m^{(1)}(t)$, of ω_{mn} and of the matrix elements of the Schrödinger-picture dipole operator

$$\mathbf{d}_{nm} = \langle n^{(0)} | \hat{\mathbf{d}} | m^{(0)} \rangle \,. \tag{8}$$

You can assume that $\mathbf{d}_{nn} = 0$.

2. Using the explicit formula for $c_m^{(1)}(t)$ as a functional of $V(t)$ provided by time-dependent perturbation theory, write $\mathbf{D}(t)$ as a function of t, $\mathbf{d}_{nm}$ and $\mathbf{E}_0$.

3. Show that in the limit $\omega_{mn}/\omega \to 0$, i.e. when $|\omega_{mn}|$ is negligible in comparison to ω, one can write

$$\mathrm{D}_i(t) = \sum_{j=1}^{3} \sigma_{ij}\, \mathrm{E}_j(t)\,, \tag{9}$$

where $i, j = 1, 2, 3$ label the spatial components of the vectors, and σ_{ij} is called polarization tensor. Express the latter in terms of ω and $(\mathrm{d}_{i/j})_{mn}$.

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