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Winter term 2017/18

Problems Advanced Quantum Mechanics

Blatt 1

Aufgabe 1: Relativistic Effects

1+4 = 5 points

This problem relates to the stationary perturbation theory taught in introductory Quantum Mechanics. Please consult your notes or a text book to recall this chapter of Quantum Theory.

The Hamilton operator of a non-relativistic electron with mass m in the spherically Coulomb potential reads

$$H_0 = \frac{p^2}{2m} - \frac{e^2}{r}\,.$$

Now we include relativistic effects in the most simple fashion by considering the relativistic expression for the kinetic energy, which leads to

$$H' = \sqrt{m^2c^4 + p^2c^2} - \frac{e^2}{r}\,.$$

- Expand the kinitic term $\sqrt{m^2c^4 + p^2c^2}$ in powers of $x = p^2/(m^2c^2)$ up to second order and consider the terms not appearing in the non-relativistic Hamiltonian as perturbation (hint: neglect constant terms).
- Calculate the change of the ground state energy in first order perturbation theory (hint: use your knowledge about the non-relativistic hydrogen atom).

Aufgabe 2: Perturbation of harmonic oscillator

1+3+2 = 6 points

Now we consider a harmonic oscillator with Hamiltonian

$$H_0 = \frac{1}{2m}p^2 + \frac{m\omega^2}{2}x^2 = \hbar\omega\Big(a^\dagger a + \frac{1}{2}\Big)\,.$$

We perturb the oscillator with an attractive force $-4\lambda x^3$ with is derived from a quartic potential

$$V(x) = \lambda x^4 = \frac{\lambda}{16\zeta^4}\Big(a + a^\dagger\Big)^4 \equiv \frac{\lambda}{16\zeta^4}\Delta\,.$$

- Determine the constant ζ?
- Multiply out the quartic polynomial $\Delta = (a + a^\dagger)^4$ and collect terms which change the occupation number $N = a^\dagger a$ by the same amount. With the help of $aa^\dagger = a^\dagger a + 1 = N + 1$ bring the operator Δ into the form

$$\Delta = P_1(N) + aP_2(N)a + a^\dagger P_2(N)a^\dagger + a^4 + a^{\dagger 4}$$

Determine the terms P_1 und P_2.

- Calculate the change of energies $E_n = \hbar\omega(n+1/2)$ of the harmonic oscillator in first order perturbation theory.

Submission date: Tuesday, 24. October 2017, before the lecture begins