Friedrich-Schiller-Universität Jena
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Winter term 2017/18

Problems in Advanced Quantum Mechanics

Blatt 2

Problem 3: Permutations 2 points

Consider the Hamiltonian of a system of two spinless particles of equal mass,

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V(x_1, x_2) \quad \text{with} \quad V(x_1, x_2) = V(x_2, x_1)$$

Show that if $\psi(x_1, x_2)$ is an eigenfunction of H with energy E, then either $\psi(x_2, x_1) = \psi(x_1, x_2)$ or $\psi(x_2, x_1) = -\psi(x_1, x_2)$. (non - degenerate)

Problem 4: Symmetry and time-evolution 1 point

Explain, why a initially completely symmetric or anti-symmetric wave function describing a system of identical particles remains symmetric or anti-symmetric at later times.
Remark: problem from an earlier Klausur

Problem 5: Non-interacting identical particles 1+3+2=5 points

Three non-interacting identical particles with spin $s = 0$ are in a infinitely deep one-dimensional potential well

$$V(x) = \begin{cases} 0 & \text{for} \quad |x| \leq a \\ \infty & \text{for} \quad |x| > a\,. \end{cases}$$

1. What is the Hilbert space of the total system?
2. Assume that $|n\rangle$ with $n = 1, 2, \ldots$ is an orthonormal eigenbasis of the one-particle Hamiltonian $H^{(1)}$. Construct the eigenbasis of the total Hamiltonian. Distinguish between the cases when the quantum numbers n_1, n_2, n_3 of the three particles are all equal, two are equal or all three are different.
3. Find the five orthonormal eigenstates of the total Hamiltonian with lowest energies.

Hint: the eigenbasis is not just given by the tensor products $|n_1\rangle \otimes |n_2\rangle \otimes |n_3\rangle$.

Problem 6: Fermions on a sphere 3 points

Several identical and non-interacting particles with spin 1/2 and mass m are confined to the surface of ball of radius R. How many particles are in the system if the energy of the ground state is $E = 42\hbar^2/2mR^2$?

Submission date: Tuesday, 31. October 2017, before the lecture begins