

# Problems in Advanced Quantum Theory

## Sheet 5

### Problem 8: Coupling to an electric field in perturbation theory

3+3+1 points

A rod of length  $d$  and uniform mass distribution is pivoted at its center and constrained to rotate on a two-dimensional plane. The rod has mass  $M$  and pointlike charges  $+e$  and  $-e$  at its endpoints.

1. Describe this system in quantum mechanics, finding its Hamiltonian operator, eigenfunctions and eigenvalues.
2. A constant and uniform electric field  $\mathbf{E}$ , lying in the rotation plane of the rod, is then applied to the system. We treat the effect of the electric field as a perturbation. What are the new eigenfunctions of the Hamiltonian and the new energies at first order in  $|\mathbf{E}|$ ?
3. What are the new energies at second order in  $|\mathbf{E}|$ ?

### Problem 9: Relativistic corrections to the Hydrogen atom

1+4 points

The nonrelativistic Hamiltonian for the electron in the Hydrogen atom reads

$$H_0 = \frac{p^2}{2m} - \frac{e^2}{r}. \quad (1)$$

The relativistic version of this is instead

$$H' = \sqrt{m^2 c^4 + p^2 c^2} - \frac{e^2}{r}. \quad (2)$$

1. Expand the relativistic kinetic term of  $H'$  in powers of  $x = p^2/(m^2 c^2)$  to second order and consider the deviation from  $H_0$  as perturbation. (Neglect an irrelevant  $p$ -independent term.)
2. Compute the change in the energy of the ground state at first order in perturbation theory.

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