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Problems in Advanced Quantum Theory

Sheet 10

Problem 15: Partial Wave Analysis of Scattering 4+2+2 points

A nonrelativistic particle of mass m and energy E is scattered by a central potential

$$U(r) = \frac{2mV(r)}{\hbar^2} = -2\left(\frac{\lambda}{\cosh(\lambda r)}\right)^2 . \tag{1}$$

This is called a Pöschl-Teller potential, and it is well known since the equation

$$\frac{\mathrm{d}^2\phi(r)}{\mathrm{d}r^2} + k^2\phi(r) = U(r)\phi(r)\,, \tag{2}$$

with constant k, can be exactly solved. Its general solution reads

$$\phi(r) = \alpha\,(\lambda\tanh(\lambda r) - \mathrm{i}k)\,e^{\mathrm{i}kr} + \beta\,(\lambda\tanh(\lambda r) + \mathrm{i}k)\,e^{-\mathrm{i}kr}\,, \tag{3}$$

where α and β are integration constants. For very small E the cross section is dominated by the $l = 0$ ($s-$wave) partial wave amplitude.

1. Compute the phase shift $\delta_{l=0}$ for this channel.
2. Write the corresponding total cross section.
3. Analyse the $E \to 0$ limit of this total cross section.

Hint: it is useful to study the equation (descending from the Schrödinger equation) fulfilled by

$$u_0(r) = rA_0(r)\,, \tag{4}$$

where $A_l(r)$ are the radial coefficients in the partial wave amplitude of the out-state wave function. One then needs to impose appropriate boundary conditions on $u_0(r)$.

Problem 16: ***(Bonus!) Cross Sections in the Born Approximation*** (3+2+3) points

Consider a Gaussian radial potential

$$V(r) = V_0 e^{-\alpha r^2}\,, \qquad \alpha > 0\,, \tag{5}$$

scattering a nonrelativistic particle of mass m and energy E. Within the first-order of the Born approximation, compute:

1. *the scattering amplitude $f(\theta)$;*
2. *the differential cross section $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta)$;*
3. *the total cross section σ.*

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