16.4.14

Frage: Wie lauten die $a^{i'}$ in Abhängigkeit von $\ddot{x}^{i'}$ und $\dot{x}^{i'}$?

Analyse: $x = \rho\cos\varphi \Rightarrow \dot x = \dot\rho\cos\varphi - \rho\sin\varphi\cdot\dot\varphi$

$$\ddot x = (\ddot\rho - \rho\dot\varphi^2)\cos\varphi - (2\dot\rho\dot\varphi + \rho\ddot\varphi)\sin\varphi$$

$$y = \rho\sin\varphi \Rightarrow \dot y = \dot\rho\sin\varphi + \rho\cos\varphi\cdot\dot\varphi$$

$$\ddot y = (\ddot\rho - \rho\dot\varphi^2)\sin\varphi + (2\dot\rho\dot\varphi + \rho\ddot\varphi)\cos\varphi$$

$$z = z' \Rightarrow \dot z = \dot z'$$

$$\ddot z = \ddot z'$$

~~RA~~ (a)

Nun: $\vec a = \ddot x\vec b_x + \ddot y\vec b_y + \ddot z\vec b_z \overset{(a)}{=} (\ddot\rho - \rho\dot\varphi^2)\underbrace{(\vec b_x\cos\varphi + \vec b_y\sin\varphi)}_{\vec b_\rho} + (\rho\ddot\varphi + 2\dot\rho\dot\varphi)\underbrace{(\vec b_y\cos\varphi - \vec b_x\sin\varphi)}_{1/\rho\ \vec b_\varphi} + \ddot z\,\vec b_z$

Nun: (S. 15 Skript) $\vec b_\rho = \vec b_x\cos\varphi + \vec b_y\sin\varphi$

$$\vec b_\varphi = \rho(-b_x\sin\varphi + \vec b_y\cos\varphi)$$

$$\vec b_{z'} = \vec b_z$$

$$\Rightarrow \vec a = \underbrace{(\ddot\rho - \rho\dot\varphi^2)}_{a^\rho = a^{1'}}\vec b_\rho + \underbrace{\left(\ddot\varphi + \frac{2\dot\rho}{\rho}\dot\varphi\right)}_{a^\varphi = a^{2'}}\vec b_\varphi + \underbrace{\ddot z'}_{a^{z'} = a^{3'}}\vec b_{z'} = \sum_{i'=1}^3 a^{i'}\vec b_{i'}$$

> Übergang zu einem bel. System $\underset{(x^i)}{S} \longrightarrow \underset{(x^{i'})}{\Sigma'}$

$$\vec a = \sum_{i=1}^3 \ddot x^i\vec b_i = \sum_{i=1}^3\sum_{j'=1}^3 \ddot x^i \underbrace{\frac{\partial x^{j'}}{\partial x^i} b_{j'}}_{\sum_{j'} \vec b_i} = \sum_{j'=1}^3 \underbrace{\left(\sum_{i=1}^3 \ddot x^i \frac{\partial x^{j'}}{\partial x^i}\right)}_{a^{j'}} \vec b_{j'}$$

$$a^{j'} = \sum_{i=1}^3 \frac{\partial x^{j'}}{\partial x^i}\frac{d^2}{dt^2}\left[x^i(x^{k'}(t))\right] = \sum_{i=1}^3 \frac{\partial x^i}{\partial x^{j'}}\frac{d}{dt}\left(\sum_{k'=1}^3 \frac{\partial x^i}{\partial x^{k'}}\dot x^{k'}\right)$$

$$= \sum_{i=1}^3 \frac{\partial x^{j'}}{\partial x^i}\sum_{k'=1}^3\left(\frac{\partial x^i}{\partial x^{k'}}\ddot x^{k'} + \sum_{l'=1}^3 \frac{\partial^2 x^i}{\partial x^{k'}\partial x^{l'}}\dot x^{l'}\dot x^{k'}\right)$$

$$\sum_{k'=1}^3 \underbrace{\left(\sum_{j'=1}^3 \frac{\partial x^{j'}}{\partial x^i}\frac{\partial x^i}{\partial x^{k'}}\right)}_{\delta^{j'}_{k'}} \ddot x^{k'} = \ddot x^{j'}$$

$$\Rightarrow \boxed{a^{j'} = \ddot x^{j'} + \sum_{k'=1}^3\sum_{l'=1}^3 \Gamma^{j'}_{k'l'}\dot x^{k'}\dot x^{l'}}$$

$$\Gamma^{j'}_{k'l'} = \sum_{i=1}^3 \frac{\partial x^{j'}}{\partial x^i}\frac{\partial^2 x^i}{\partial x^{k'}\partial x^{l'}}$$ Christoffel-Symbol