Presentation: Simulating the Evolution of Binary Black Holes
Outline

Inhaltsverzeichnis

1 Introduction

- show video of simulation
- What is numerical relativity?
- Why do we need it?

Why?

- evolution and merging of compact binaries
- predict and explain astronomical measurements

2 Background

- spacetime, manifolds, metrics
- tensors, curvature, geodesics
- Einstein field equations
- analytical solutions
  - Schwarzschild black holes
  - Kerr black holes

3 Basic Idea

- What do we have to do for the simulation?
- Why do we have to do this?
- 3+1 decomposition
- construction of initial data
- choosing coordinates
- matter sources
- locating black hole horizons
- recasting the evolution equations
- numerical methods
- binary black hole evolution

4 $3+1$ Decomposition of Einstein's Equations

Use the directly the notion of basis vectors.

- Why do we have to decompose Einstein's equations?
- foliations of spacetime
- extrinsic curvature
- equations of Gauss, Codazzi and Ricci
- constraint and evolution equations
- ADM equations

What?
Why?

- Einstein equations are complicated and do not want to be solved: contain time derivatives of second order, cannot be categorized to find solution method
- simulating 4 dimensions will need crazy amount of computation power and storage because dimension appears in exponent of grid point count
- separating time will result in time evolution (without saving many time slices)
- spatial equations can be categorized with better conditions

4.1 Foliation of Spacetime

- carving (M, g_{ab}) into stack of spatial (spacelike) 3-dimensional slices Σ
- construct spatial metric γ_{ab} and time normal Ω_a (together with lapse function α)
- find method to project spacetime tensors to their spatial and timelike part
- define spatial covariant derivative
- construct spatial Riemann tensor (describes intrinsic curvature of spatial slice)
- describe extrinsic curvature K_{ab}
- conditions for γ_{ab} and K_{ab}: equations of Gauss, Codazzi and Ricci

4.2 The Constraint and Evolution Equations

- use Einstein field equations together with Gauss and Codazzi equation
- derive Hamiltonian and momentum constraint
- define shift vector β^a to create natural time derivative
- use equations of Einstein, Gauss and Ricci to derive evolution equation of extrinsic curvature
- derive evolution equation for spatial metric

4.3 Choosing Basis Vectors

- formulate ADM equations based on α and β^i (sketch at blackboard)
- examples: Schwarzschild and Kerr black holes
- two comments

5 Constructing initial data

- conformal transformations
- example methods: conformal transverse-traceless decomposition, conformal thin-sandwich decomposition
- conformal thin-sandwich decomposition
- realistic data modelling

Why?

- physical realistic conditions for binary black holes
- initial values to simulate time evolution from a starting point
- initial values have to fulfill constraint equations
- $12-4=8$ degrees of freedom, 4 degrees for spacetime translations, leaving 4 degrees of freedom for characterizing gravitational field
- separation of longitudinal and transversal part not possible

5.1 Conformal transformations

- construct conformally related metric $\bar{\gamma}_{ij}$ based on conformal factor ψ
- separate extrinsic curvature K_{ij} into trace K and traceless part A_{ij}
- formulate constrain equations (for BBH set matter sources to zero)
- construct head-on collision BBH and equilibrium state BBH through CTF approach

6 Choosing coordinates

- gauge variables: lapse and shift
- geodesic slicing
- example methods: maximal slicing and singularity avoidance, harmonic coordinates

Why?

- gauge variables
- well-behaved, long-time evolution
- avoiding singularities

6.1 Geodesic Slicing

- easy: $\alpha = 1, \beta^i = 0$
- does not avoid singularities

6.2 Maximal Slicing

- set mean curvature and time derivative to zero
- compute constraint equations

7 Recasting the Evolution Equations

- instability problem of ADM evolution equations
- example methods: generalized harmonic coordinates, first-order symmetric hyperbolic formulations, BSSN formulation
- BSSN formulation

8 Numerical Methods

- classification of partial differential equations
- finite difference methods

9 Binary Black Hole Evolution and Results

- show some videos
- gravitational lensing
- inspiral, merge and ringdown
- 3 phases: quasi-circular inspiral, plunge/merger, ringdown

10 Conclusion and References

SXS: Gravitational Lensing of GW150914

NASA: Colliding Neutron Stars create Black Hole and Gamma-ray Burst

Literatur

[1] *Colliding Neutron Stars Create Black Hole and Gamma-ray Burst*, April 2011. https://www.nasa.gov/topics/universe/features/gamma-ray-engines.html#.WA4phz_VTZE.

[2] *SXS Project: Simulating Extreme Spacetimes*, Juli 2018. https://www.black-holes.org/explore/movies.

[3] Baumgarte, Thomas W. and Stuart L. Shapiro: *On the numerical integration of einstein's field equations*. Physical Review D, 59(2):024007, 1998.

[4] Baumgarte, Thomas W. and Stuart L. Shapiro: *Numerical Relativity: Solving Einstein's Equations on the Computer*. Cambridge University Press, 2010.

[5] Hehl, F. W., R. A. Puntigam, and H. Ruder: *Relativity and Scientific Computing: Computer Algebra, Numerics, Visualization*. Springer, 1996.

[6] Meinel, Reinhard: *Spezielle und allgemeine Relativitätstheorie für Bachelorstudenten*. Springer Spektrum, 2016.

[7] Oloff, Rainer: *Geometrie der Raumzeit: Eine mathematische Einführung in die Relativitätstheorie*. Vieweg-Teubner, 5. Auflage, 2010.