

# Photic Extremum Lines

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## Abstract

*In the field of illustrative visualization, feature lines are essential for conveying the shape of a given object. Photic extremum lines (PELs) are a type of feature line which are, besides normal and view position, dependent on the illumination. quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.*

**Keywords:** Non-Photorealistic Rendering, Feature Lines, View-Dependent Object-Space Algorithm, Contours, Silhouettes, Suggestive Contours, Photic Extremum Lines, Illumination

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## 1 Introduction

Illustrative visualization is the science and art of effectively communicating known aspects of scientific data in an accurate and intuitive way. Especially for the rendering of volumetric data sets in medicine, it is a valuable tool to reduce a vast amount of complex information to its essence. In this respect, photorealistic rendering techniques are suboptimal because they are not able to efficiently depict features of interest. Our knowledge of human cognition shows that, artistic drawings or paintings, in comparison to a photograph of the same scene, seem to be more suitable for communication and more pleasing in visual experience (Xie et al. 2007). Therefore non-photorealistic rendering techniques, typically inspired by artistic styles, are used to create such illustrations. (Viola et al. 2005)<sup>1</sup>

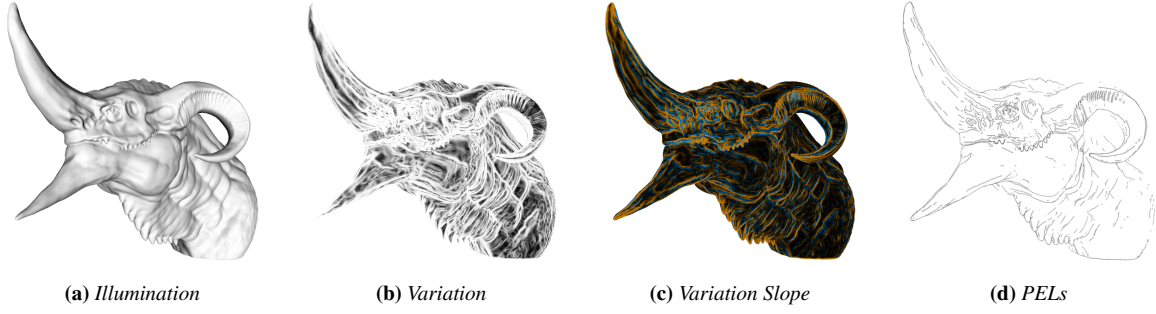
Feature lines represent a given data set as a line draw-

ing to mimic hand-drawn illustrations. In such a way, a large amount of information can be communicated in a succinct manner by taking advantage of human visual acuity. Used as an abstraction tool in illustrative visualization, feature lines convey the shape of objects much more efficiently compared to a photograph. (Isenberg et al. 2003; Viola et al. 2005; Xie et al. 2007)

There are many different types of commonly-used feature lines, such as contours (Isenberg et al. 2003), suggestive contours (DeCarlo et al. 2003), ridge-valley lines (Ohtake, Belyaev, and Seidel 2004), apparent ridges (Judd, Durand, and Adelson 2007), and demarcating curves (Kolomenkin, Shimshoni, and Tal 2008). Typically, these only depend on the surface geometry, such as normal and curvature, and possibly the view position. However, human perception is highly sensitive to high variations in illumination. As a consequence, for conveying the shape of objects according to human perception, feature lines should also depend on

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<sup>1</sup>In this report, citations concerning more than one sentence are given at the end of the respective paragraph.



**Figure 1: Short Summary Part**

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the lighting of an object. (Xie et al. 2007; Zhang et al. 2011)

In this report, we present the concept and implementation of photic extremum lines (PELs), one of the first types of feature lines exhibiting a dependency on illumination. PELs have been first introduced in Xie et al. (2007) and further developed in Zhang, He, and Seah (2010). Strongly inspired by the edge detection techniques for 2D images, they are characterized by a sudden change of illumination on the surface of a 3D object. Since their computation is taken out in object space, PELs are flexible and enable further post-processing such as line stylization and shading (Isenberg et al. 2003). Furthermore, by manipulating the illumination of an object, the user can take full control to adjust the rendering output and achieve desired illustration results. Implementations for PELs can be done for the CPU and GPU, nowadays, achieving real-time performance. (Xie et al. 2007; Zhang, He, and Seah 2010)

**DEFINITION 3.2:** (First Fundamental Form Triangle)

$$I_{uv} := \begin{pmatrix} \|u\|^2 & \langle u | v \rangle \\ \langle u | v \rangle & \|v\|^2 \end{pmatrix}$$

$$I_{uv}^{-1} = \frac{\text{adj } I_{uv}}{\det I_{uv}} = \frac{1}{\|u\|^2 \|v\|^2 - |\langle u | v \rangle|^2} \begin{pmatrix} \|v\|^2 & -\langle u | v \rangle \\ -\langle u | v \rangle & \|u\|^2 \end{pmatrix}$$

**DEFINITION 3.3:** (Gradient Triangle)

$$[\nabla f]_{uv} = I_{uv}^{-1} \begin{pmatrix} \Delta_u f \\ \Delta_v f \end{pmatrix}$$

$$\nabla f = \begin{pmatrix} u & v \end{pmatrix} [\nabla f]_{uv}$$

## 2 Related Work

The main references of this report are

## 3 Mathematical Preliminaries

**DEFINITION 3.1:** Mesh Function

$$f: S \rightarrow \mathbb{R}$$

**DEFINITION 3.4:**

$$\partial_w f(x) := \langle \nabla f(x) | w \rangle$$

$$\mathcal{D}_f g(x) := \left\langle \nabla g(x) \left| \frac{\nabla f(x)}{\|\nabla f(x)\|} \right. \right\rangle$$

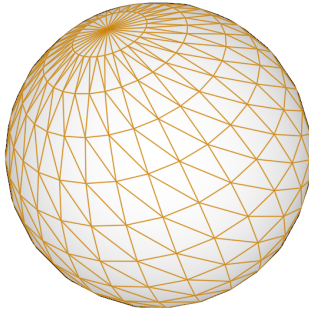


Figure 2: Triangulated Meshes

#### 4 Photoc Extremum Lines

##### DEFINITION 4.1: (Photoc Extremum Lines)

Let  $S$  be a smooth surface patch and  $\varphi: S \rightarrow \mathbb{R}$  three-times continuously differentiable scalar illumination function. The set of photoc extremums over  $S$  with respect to  $\varphi$  consists of all points  $x \in S$  where the variation of illumination in the direction of its gradient reaches a local maximum. In other words, such that the following holds.

$$\mathcal{D}_\varphi \|\nabla \varphi\| (x) = 0 \quad \mathcal{D}_\varphi^2 \|\nabla \varphi\| (x) < 0$$

#### 5 Algorithm

##### Algorithm

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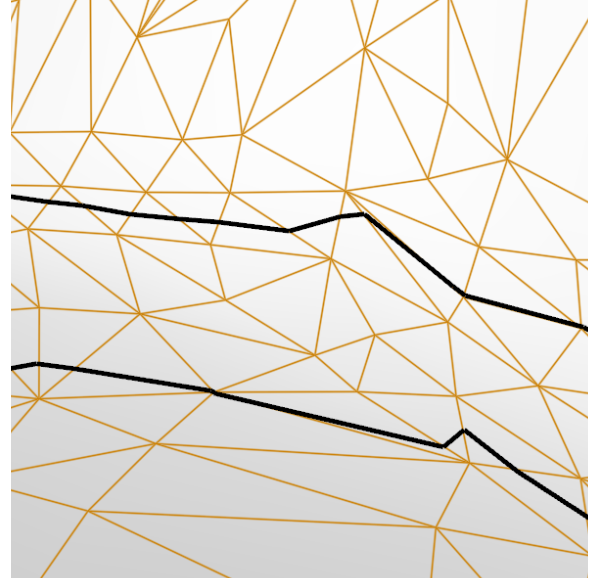


Figure 3: Sub-Polygon Feature Lines

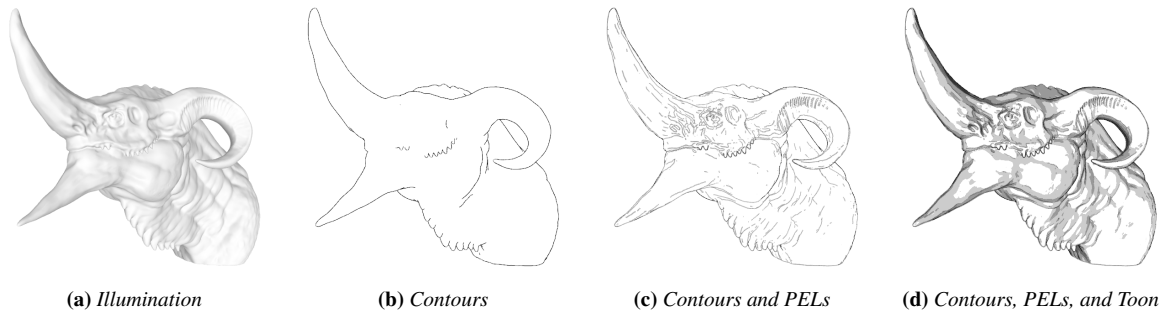
#### 6 Implementation

#### 7 Results and Comparison

#### 8 Conclusions

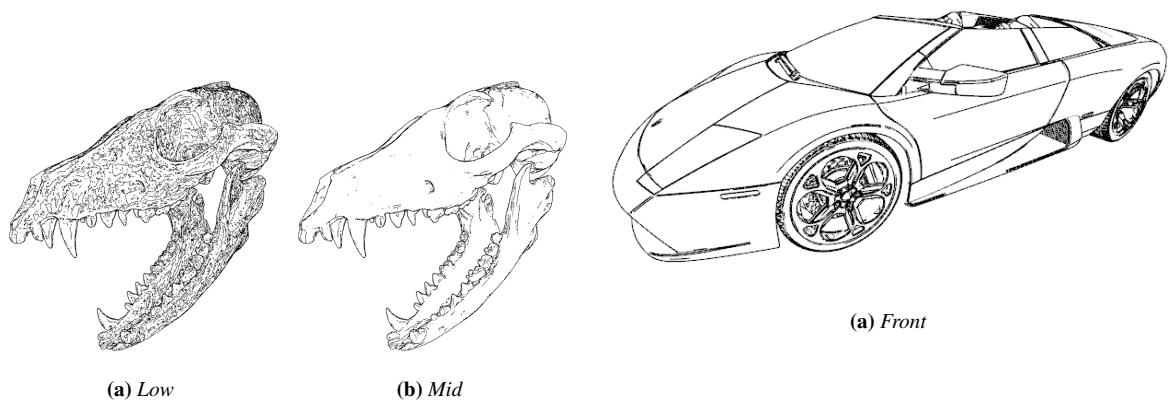
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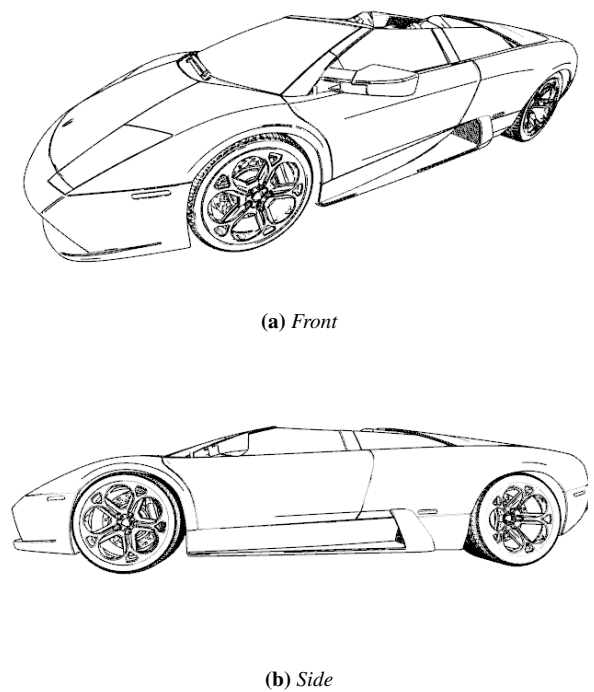


**Figure 4: Short Summary Part**

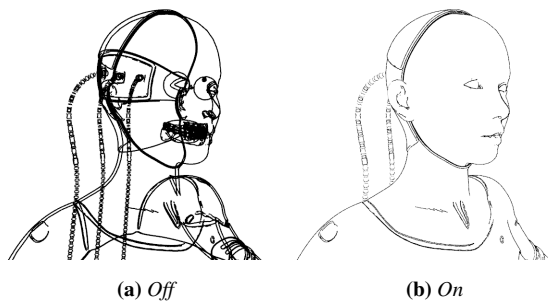
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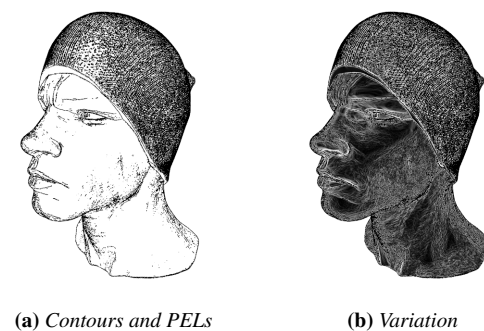
**Figure 5: Effect of thresholding**



**Figure 7: Nearly Perfect Line Extraction for Smooth Objects**



**Figure 6: Two-Pass Rendering for Hidden Line Removal**



**Figure 8: Erroneous Line Extraction for Noisy Objects**

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