

# Photic Extremum Lines

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## Abstract

*In the field of illustrative visualization, feature lines are essential for conveying the shape of a given object. Photic extremum lines (PELs) are a type of feature line which are, besides normal and view position, dependent on the illumination. quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.*

**Keywords:** Non-Photorealistic Rendering, Feature Lines, View-Dependent Object-Space Algorithm, Contours, Silhouettes, Suggestive Contours, Photic Extremum Lines, Illumination, Interactive

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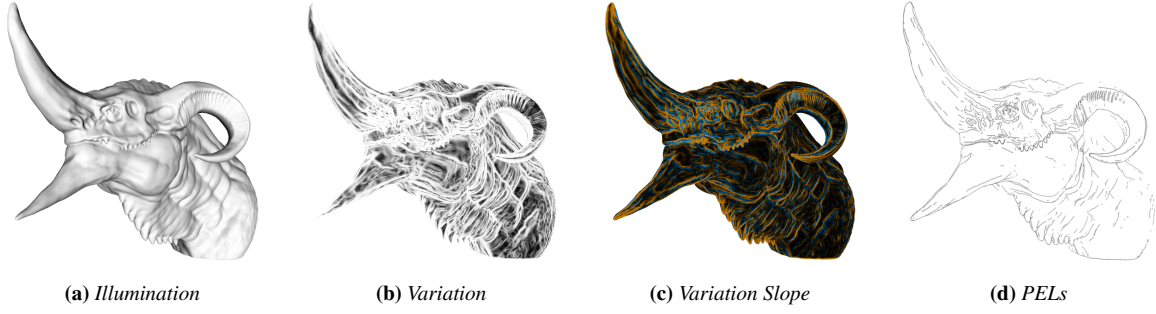
## 1 Introduction

Illustrative visualization is the science and art of effectively communicating known aspects of scientific data in an accurate and intuitive way. Especially for the rendering of volumetric datasets in medicine, it is a valuable tool to reduce a vast amount of complex information to its essence given a focus and a context. It is well-known that photorealism is not really able to provide an efficient depiction of features of interest. Our knowledge of human cognition shows that, artistic drawings or paintings seem to be more suitable for communication and more pleasing in visual experience in comparison to a photograph of the same scene. Therefore non-photorealistic rendering (NPR) techniques, typically inspired by artistic styles, are used to create such illustrations. NPR is able to show features that may not be seen by using a global illumination algorithm.

One of the most important tools used in illustrative vi-

sualization for conveying the shape of an object are feature lines. By taking advantage of human visual acuity, line drawings can express a large amount of information which makes them more expressive than photographs. There are many different types of commonly-used feature lines, such as contours (silhouettes), suggestive contours, or ridge-valley lines. Typically, they are only dependent on the surface geometry, such as normal and curvature, and/or the view position. However, human perception is highly sensitive to high luminance variations. As a consequence, conveying the shape of objects by using feature lines should also depend on the illumination of an object.

In this report we present photic extremum lines (PELs) created by. Strongly inspired by the edge detection techniques for 2D images, PELs characterize a sudden change of illumination on 3D shapes in object space. This enables further processing such as line stylization.



**Figure 1: Short Summary Part**

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Additionally, we will talk about an implementation.

## 2 Related Work

The main references of this report are

## 3 Mathematical Preliminaries

### DEFINITION 3.1: Mesh Function

$$f: S \rightarrow \mathbb{R}$$

### DEFINITION 3.2: (First Fundamental Form Triangle)

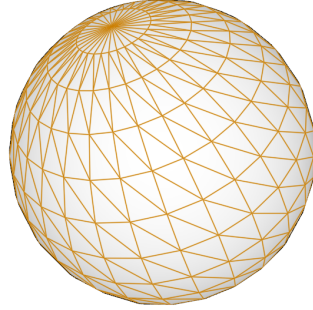
$$I_{uv} := \begin{pmatrix} \|u\|^2 & \langle u | v \rangle \\ \langle u | v \rangle & \|v\|^2 \end{pmatrix}$$

$$I_{uv}^{-1} = \frac{\text{adj } I_{uv}}{\det I_{uv}} = \frac{1}{\|u\|^2 \|v\|^2 - |\langle u | v \rangle|^2} \begin{pmatrix} \|v\|^2 & -\langle u | v \rangle \\ -\langle u | v \rangle & \|u\|^2 \end{pmatrix}$$

### DEFINITION 3.3: (Gradient Triangle)

$$[\nabla f]_{uv} = I_{uv}^{-1} \begin{pmatrix} \Delta_u f \\ \Delta_v f \end{pmatrix}$$

$$\nabla f = \begin{pmatrix} u & v \end{pmatrix} [\nabla f]_{uv}$$



**Figure 2: Triangulated Meshes**

### DEFINITION 3.4:

$$\partial_w f(x) := \langle \nabla f(x) | w \rangle$$

$$\mathcal{D}_f g(x) := \left\langle \nabla g(x) \left| \frac{\nabla f(x)}{\|\nabla f(x)\|} \right. \right\rangle$$

## 4 Photic Extremum Lines

### DEFINITION 4.1: (Photic Extremum Lines)

Let  $S$  be a smooth surface patch and  $\varphi: S \rightarrow \mathbb{R}$  three-times continuously differentiable scalar illumination function. The set of photic extremums over  $S$  with respect to  $\varphi$  consists of all points  $x \in S$  where the variation of illumination in the direction of its gradient reaches a local maximum. In other

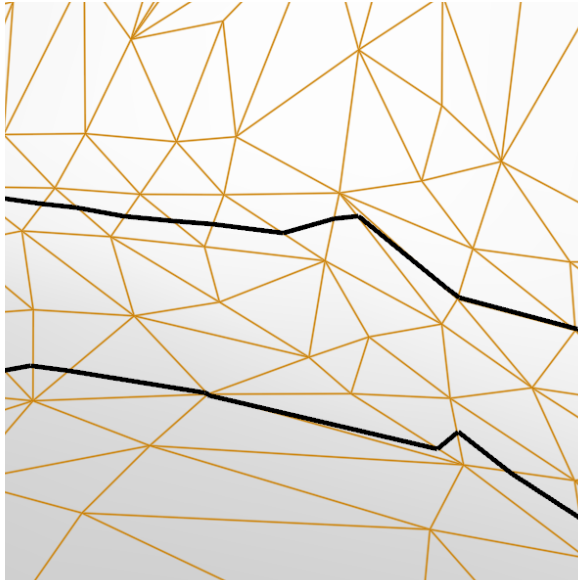


Figure 3: Sub-Polygon Feature Lines

words, such that the following holds.

$$\mathcal{D}_\varphi \|\nabla \varphi\| (x) = 0 \quad \mathcal{D}_\varphi^2 \|\nabla \varphi\| (x) < 0$$

## 5 Algorithm

### Algorithm

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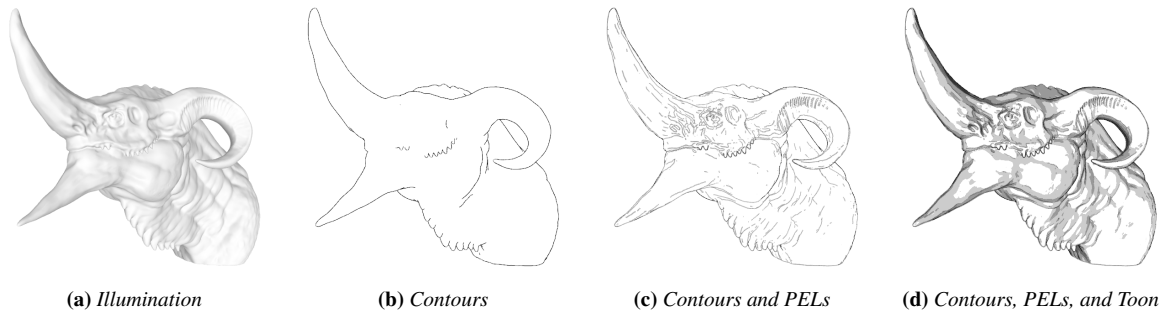
## 6 Implementation

## 7 Results and Comparison

## 8 Conclusions

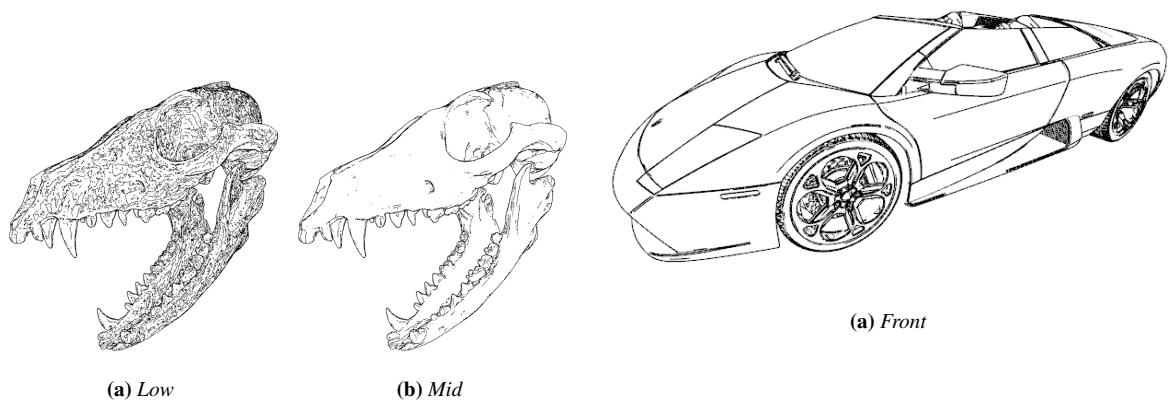
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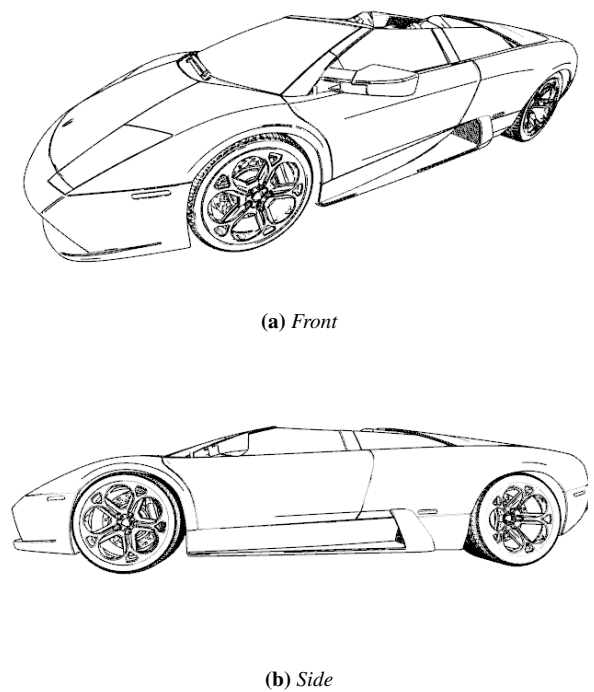


**Figure 4: Short Summary Part**

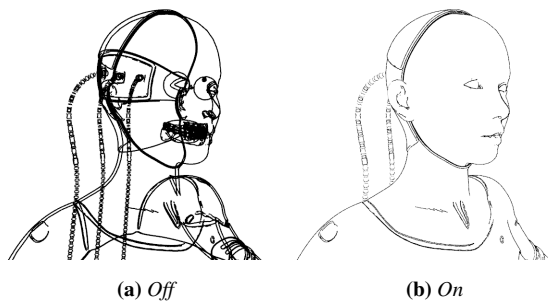
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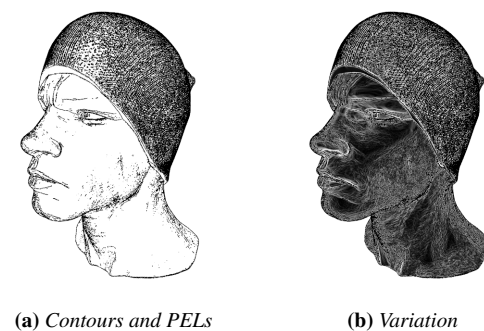
**Figure 5: Effect of thresholding**



**Figure 7: Nearly Perfect Line Extraction for Smooth Objects**



**Figure 6: Two-Pass Rendering for Hidden Line Removal**



**Figure 8: Erroneous Line Extraction for Noisy Objects**