

# Photic Extremum Lines

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## Abstract

*In the field of illustrative visualization, feature lines are essential for conveying the shape of a given object. Photic extremum lines (PELs) are a type of feature line which are, besides normal and view position, dependent on the illumination. quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.*

**Keywords:** Non-Photorealistic Rendering, Feature Lines, View-Dependent Object-Space Algorithm, Contours, Silhouettes, Suggestive Contours, Photic Extremum Lines, Illumination, Interactive

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## 1 Introduction

Illustrative visualization is the science and art of effectively communicating known aspects of scientific data in an accurate and intuitive way. Especially for the rendering of volumetric data sets in medicine, it is a valuable tool to reduce a vast amount of complex information to its essence. In this respect, photorealistic rendering techniques are suboptimal because they are not able to efficiently depict features of interest. Our knowledge of human cognition shows that, artistic drawings or paintings, in comparison to a photograph of the same scene, seem to be more suitable for communication and more pleasing in visual experience (Xie et al. 2007). Therefore non-photorealistic rendering techniques, typically inspired by artistic styles, are used to create such illustrations.

Feature lines represent a given data set as a line drawing. In such a way, a large amount of information can be communicated in a succinct manner by taking ad-

vantage of human visual acuity. Used as a tool in illustrative visualization, feature lines can convey the shape of objects much more efficiently compared to a photograph.

There are many different types of commonly-used feature lines, such as contours (Isenberg et al. 2003), suggestive contours (DeCarlo et al. 2003), or ridge-valley lines. Typically, these are only dependent on the view position and the surface geometry. However, human perception is highly sensitive to high variations in illumination. As a consequence, for conveying the shape of objects according to human perception, feature lines should also depend on the lighting of an object.

In this report, we present photic extremum lines (PELs) first introduced in Xie et al. (2007). Strongly inspired by the edge detection techniques for 2D images, PELs characterize a sudden change of illumination on 3D shapes in object space. This makes them flexible and enables further processing such as line stylization.

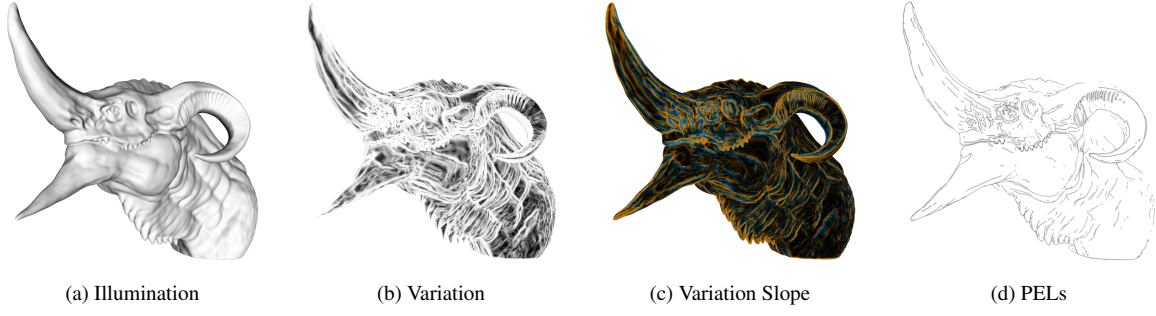


Figure 1: **Short Summary Part**

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Additionally, we will talk about an implementation.

## 2 Related Work

The main references of this report are

## 3 Mathematical Preliminaries

### DEFINITION 3.1: Mesh Function

$$f: S \rightarrow \mathbb{R}$$

### DEFINITION 3.2: (First Fundamental Form Triangle)

$$I_{uv} := \begin{pmatrix} \|u\|^2 & \langle u | v \rangle \\ \langle u | v \rangle & \|v\|^2 \end{pmatrix}$$

$$I_{uv}^{-1} = \frac{\text{adj } I_{uv}}{\det I_{uv}} = \frac{1}{\|u\|^2 \|v\|^2 - |\langle u | v \rangle|^2} \begin{pmatrix} \|v\|^2 & -\langle u | v \rangle \\ -\langle u | v \rangle & \|u\|^2 \end{pmatrix}$$

### DEFINITION 3.3: (Gradient Triangle)

$$[\nabla f]_{uv} = I_{uv}^{-1} \begin{pmatrix} \Delta_u f \\ \Delta_v f \end{pmatrix}$$

$$\nabla f = \begin{pmatrix} u & v \end{pmatrix} [\nabla f]_{uv}$$

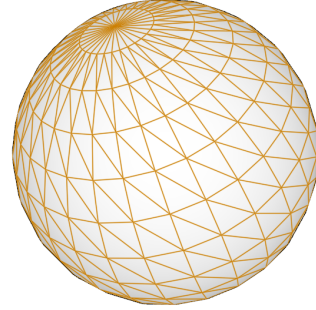


Figure 2: Triangulated Meshes

### DEFINITION 3.4:

$$\partial_w f(x) := \langle \nabla f(x) | w \rangle$$

$$\mathcal{D}_f g(x) := \left\langle \nabla g(x) \left| \frac{\nabla f(x)}{\|\nabla f(x)\|} \right. \right\rangle$$

$$-\langle u | v \rangle$$

## 4 Photic Extremum Lines

### DEFINITION 4.1: (Photic Extremum Lines)

Let  $S$  be a smooth surface patch and  $\varphi: S \rightarrow \mathbb{R}$  three-times continuously differentiable scalar illumination function. The set of photic extremums over  $S$  with respect to  $\varphi$  consists of all points  $x \in S$  where the variation of illumination in the direction of its gradient reaches a local maximum. In other

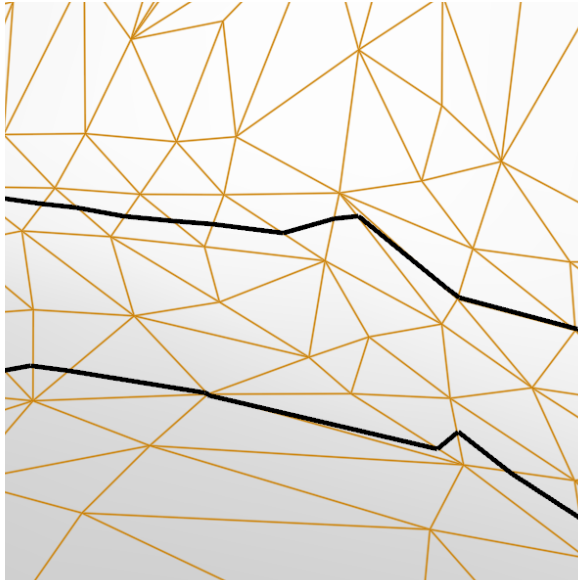


Figure 3: Sub-Polygon Feature Lines

words, such that the following holds.

$$\mathcal{D}_\varphi \|\nabla \varphi\| (x) = 0 \quad \mathcal{D}_\varphi^2 \|\nabla \varphi\| (x) < 0$$

## 5 Algorithm

### Algorithm

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## 6 Implementation

## 7 Results and Comparison

## 8 Conclusions

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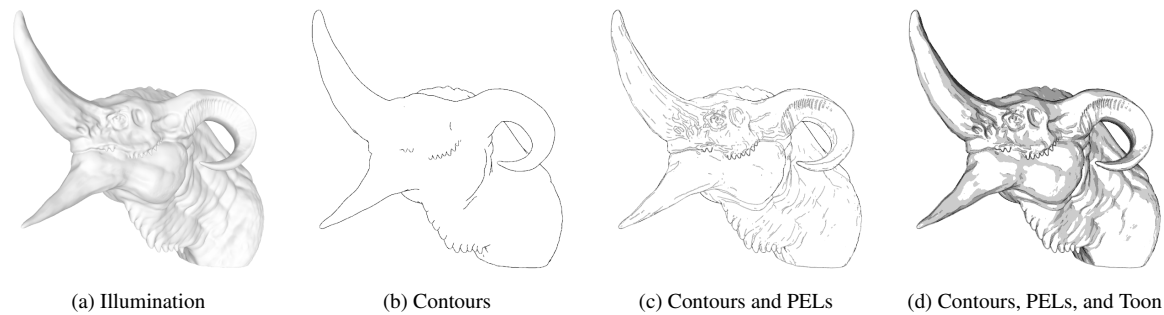


Figure 4: **Short Summary Part**

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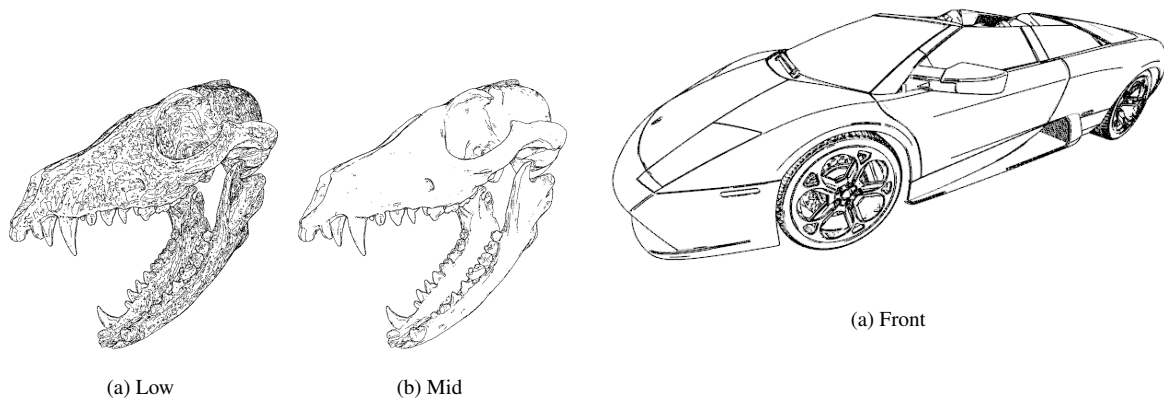


Figure 5: Effect of thresholding

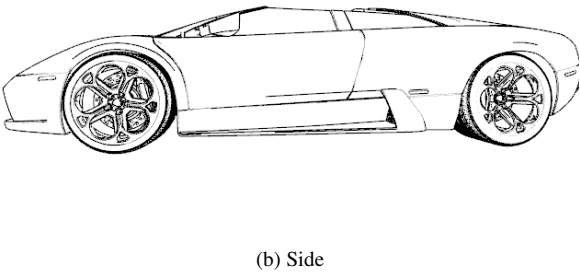


Figure 7: Nearly Perfect Line Extraction for Smooth Objects

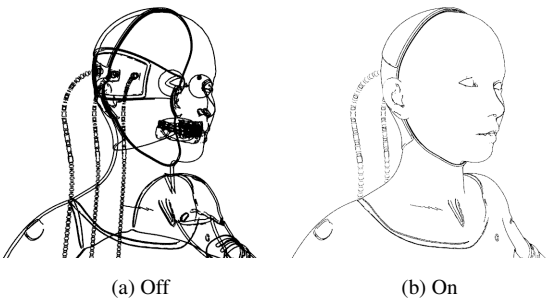


Figure 6: Two-Pass Rendering for Hidden Line Removal

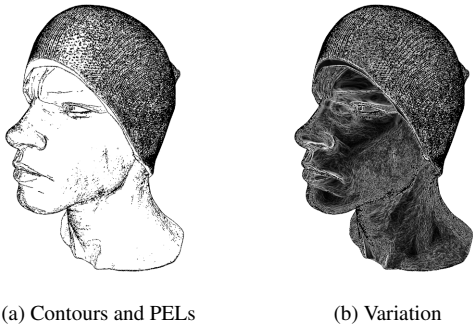


Figure 8: Erroneous Line Extraction for Noisy Objects