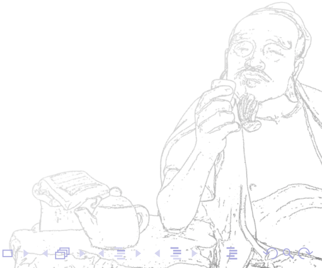


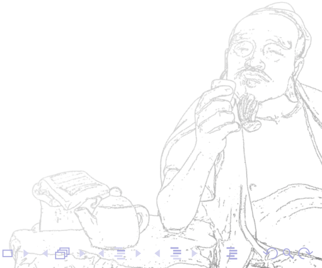

Illustrative Visualization: Photic Extremum Lines

Markus Pawellek

January 11, 2022



Outline



Related Work

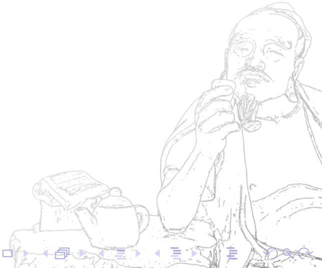
Related Work

Tools

Related Work

Tools

2003 Isenberg et al. “A Developer’s Guide to Silhouette Algorithms for Polygonal Models”



Related Work

Tools

2003 Isenberg et al. “A Developer’s Guide to Silhouette Algorithms for Polygonal Models”

2004 Rusinkiewicz “Estimating Curvatures and Their Derivatives on Triangle Meshes”

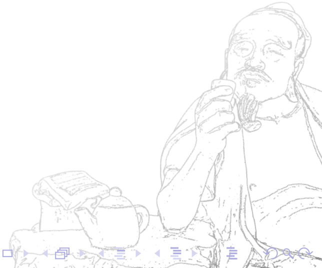
Related Work

Tools

2003 Isenberg et al. “A Developer’s Guide to Silhouette Algorithms for Polygonal Models”

2004 Rusinkiewicz “Estimating Curvatures and Their Derivatives on Triangle Meshes”

Algorithm



Related Work

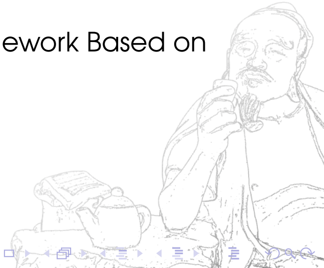
Tools

2003 Isenberg et al. "A Developer's Guide to Silhouette Algorithms for Polygonal Models"

2004 Rusinkiewicz "Estimating Curvatures and Their Derivatives on Triangle Meshes"

Algorithm

2007 Xie et al. "An Effective Illustrative Visualization Framework Based on Photic Extremum Lines (PELs)"



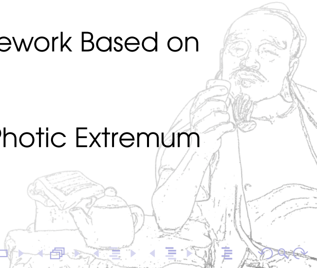
Related Work

Tools

- 2003 Isenberg et al. “A Developer’s Guide to Silhouette Algorithms for Polygonal Models”
- 2004 Rusinkiewicz “Estimating Curvatures and Their Derivatives on Triangle Meshes”

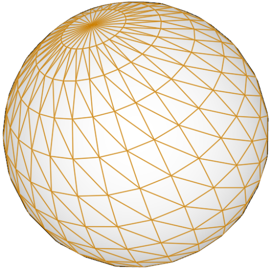
Algorithm

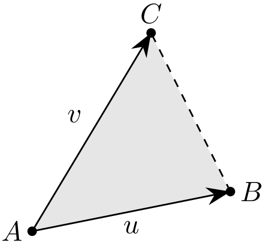
- 2007 Xie et al. “An Effective Illustrative Visualization Framework Based on Photic Extremum Lines (PELs)”
- 2010 Zhang, He, and Seah “Real-Time Computation of Photic Extremum Lines (PELs)”



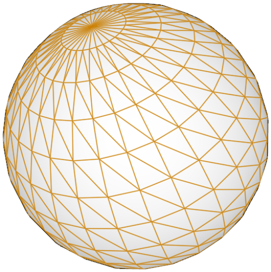
Mathematical Preliminaries

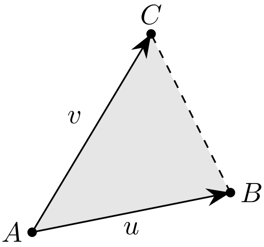
Mathematical Preliminaries: Mesh Function





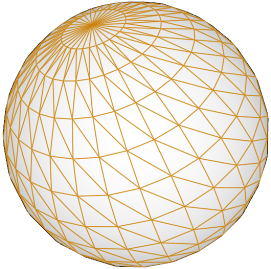
Mathematical Preliminaries: Mesh Function

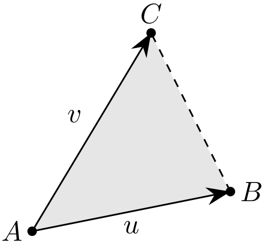




- $f : S \to \mathbb{R}$ on mesh S characterized by its values at vertices

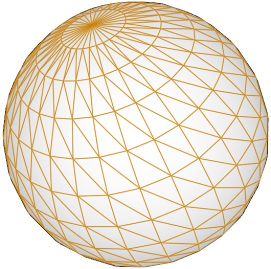
Mathematical Preliminaries: Mesh Function

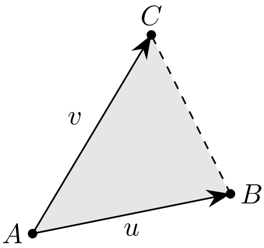




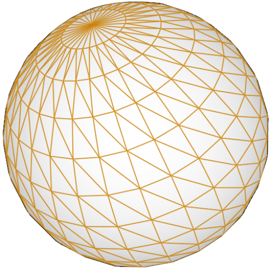
- $f : S \to \mathbb{R}$ on mesh S characterized by its values at vertices
- For interiors of faces, use barycentric interpolation

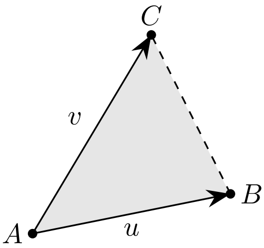
Mathematical Preliminaries: Mesh Function Gradient





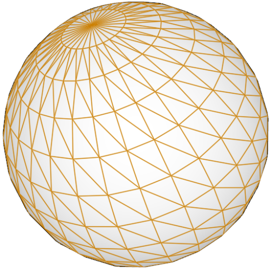
Mathematical Preliminaries: Mesh Function Gradient

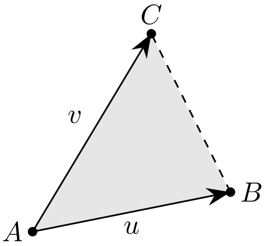




- Compute ∇f for each face

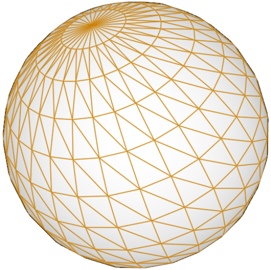
Mathematical Preliminaries: Mesh Function Gradient

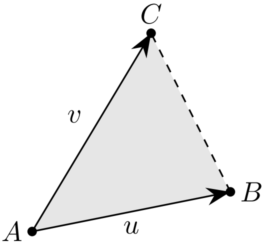




- Compute ∇f for each face
- For each vertex, accumulate weighted and rotated gradients for adjacent faces

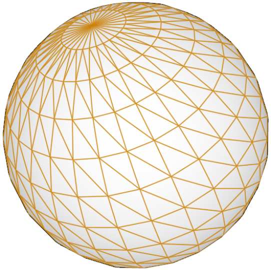
Mathematical Preliminaries: Mesh Function Gradient

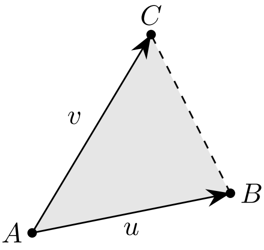




$$\mathrm{I}_{uv} := \begin{pmatrix} \|u\|^2 & \langle u, v\rangle \\ \langle u, v\rangle & \|v\|^2 \end{pmatrix} \qquad \nabla f = \begin{pmatrix} u & v \end{pmatrix} \mathrm{I}_{uv}^{-1} \begin{pmatrix} f(B) - f(A) \\ f(C) - f(A) \end{pmatrix}$$

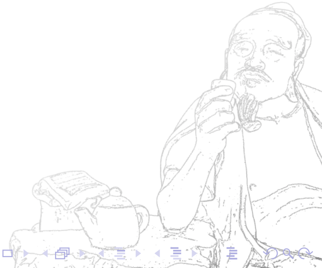
Mathematical Preliminaries: Directional Derivatives





$$\partial_w f(x) = \langle \nabla f(x), w \rangle \qquad \mathcal{D}_f\, g(x) \coloneqq \left\langle \nabla g(x), \frac{\nabla f(x)}{\|\nabla f(x)\|} \right\rangle$$

Photic Extremum Lines



Photic Extremum Lines

- Scalar illumination function $\varphi\colon S \to \mathbb{R}$ on mesh S (e.g. directional light source)

Photic Extremum Lines

- Scalar illumination function $\varphi\colon S \to \mathbb{R}$ on mesh S (e.g. directional light source)
- Variation of illumination $\|\nabla\varphi\|$

Photic Extremum Lines

- Scalar illumination function $\varphi\colon S \to \mathbb{R}$ on mesh S (e.g. directional light source)
- Variation of illumination $\|\nabla\varphi\|$

Photic Extremum

Illumination variation in the direction its gradient reaches local maximum

Photic Extremum Lines

- Scalar illumination function $\varphi\colon S \to \mathbb{R}$ on mesh S (e.g. directional light source)
- Variation of illumination $\|\nabla\varphi\|$

Photic Extremum

Illumination variation in the direction its gradient reaches local maximum

$$\mathcal{D}_\varphi \|\nabla\varphi\|(x) = 0 \qquad \mathcal{D}^2_\varphi \|\nabla\varphi\|(x) < 0$$

Photic Extremum Lines

- Scalar illumination function $\varphi \colon S \to \mathbb{R}$ on mesh S (e.g. directional light source)
- Variation of illumination $\|\nabla \varphi\|$

Photic Extremum

Illumination variation in the direction its gradient reaches local maximum

$$\mathcal{D}_{\varphi} \left\|\nabla \varphi\right\|(x) = 0 \qquad \mathcal{D}^2_{\varphi} \left\|\nabla \varphi\right\|(x) < 0$$

Photic Extremum Lines

- Scalar illumination function $\varphi\colon S \to \mathbb{R}$ on mesh S (e.g. directional light source)
- Variation of illumination $\|\nabla\varphi\|$

Photic Extremum

Illumination variation in the direction its gradient reaches local maximum

$$\mathcal{D}_{\varphi} \|\nabla\varphi\| (x) = 0 \qquad \mathcal{D}^2_{\varphi} \|\nabla\varphi\| (x) < 0$$

Photic Extremum Lines

- Scalar illumination function $\varphi\colon S \to \mathbb{R}$ on mesh S (e.g. directional light source)
- Variation of illumination $\|\nabla\varphi\|$

Photic Extremum

Illumination variation in the direction its gradient reaches local maximum

$$\mathcal{D}_{\varphi}\,\|\nabla\varphi\|\,(x) = 0 \qquad \mathcal{D}^2_{\varphi}\,\|\nabla\varphi\|\,(x) < 0$$

Algorithm

Algorithm: Overview

Algorithm: Overview

1. Compute φ

Algorithm: Overview

1. Compute φ
2. Compute $\frac{\nabla\varphi}{\|\nabla\varphi\|}$ and $\|\nabla\varphi\|$

Algorithm: Overview

1. Compute φ
2. Compute $\frac{\nabla\varphi}{\|\nabla\varphi\|}$ and $\|\nabla\varphi\|$
3. Compute $\mathcal{D}_\varphi \|\nabla\varphi\|$

Algorithm: Overview

1. Compute φ
2. Compute $\frac{\nabla \varphi}{\|\nabla \varphi\|}$ and $\|\nabla \varphi\|$
3. Compute $\mathcal{D}_{\varphi} \|\nabla \varphi\|$
4. Compute $\mathcal{D}_{\varphi}^2 \|\nabla \varphi\|$

Algorithm: Overview

1. Compute φ
2. Compute $\frac{\nabla \varphi}{\|\nabla \varphi\|}$ and $\|\nabla \varphi\|$
3. Compute $\mathcal{D}_{\varphi} \|\nabla \varphi\|$
4. Compute $\mathcal{D}_{\varphi}^{2} \|\nabla \varphi\|$
5. Detect line vertices on edges by testing for photic extremums

Algorithm: Overview

1. Compute φ
2. Compute $\frac{\nabla\varphi}{\|\nabla\varphi\|}$ and $\|\nabla\varphi\|$
3. Compute $\mathcal{D}_{\varphi}\,\|\nabla\varphi\|$
4. Compute $\mathcal{D}^2_{\varphi}\,\|\nabla\varphi\|$
5. Detect line vertices on edges by testing for photic extremums
6. Trace and filter out lines by using a threshold

Algorithm: Overview

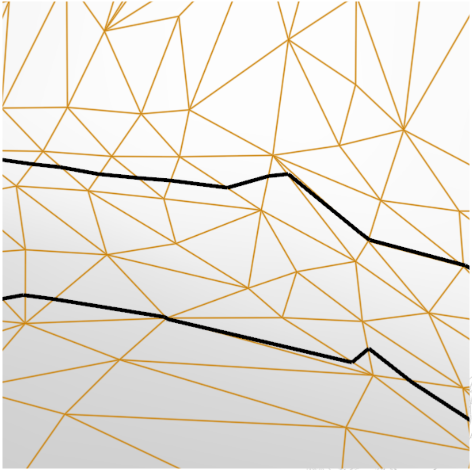
1. Compute φ
2. Compute $\frac{\nabla\varphi}{\|\nabla\varphi\|}$ and $\|\nabla\varphi\|$
3. Compute $\mathcal{D}_{\varphi}\,\|\nabla\varphi\|$
4. Compute $\mathcal{D}^2_{\varphi}\,\|\nabla\varphi\|$
5. Detect line vertices on edges by testing for photic extremums
6. Trace and filter out lines by using a threshold
7. Render visible lines

Algorithm: Line Detection and Tracing

- For each edge $[v, w] \subset S$, check zero-crossing:

$$h(x) := \mathcal{D}_{\varphi} \|\nabla \varphi\| (x)$$

$$h(v)h(w) < 0$$



Algorithm: Line Detection and Tracing

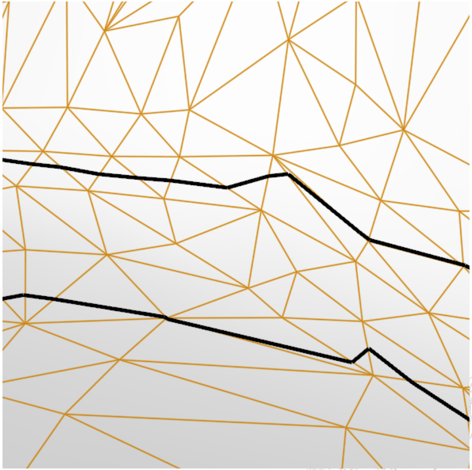
- For each edge $[v, w] \subset S$, check zero-crossing:

$$h(x) := \mathcal{D}_{\varphi} \left\| \nabla \varphi \right\| (x)$$

$$h(v)h(w) < 0$$

- Approximate zero-crossing:

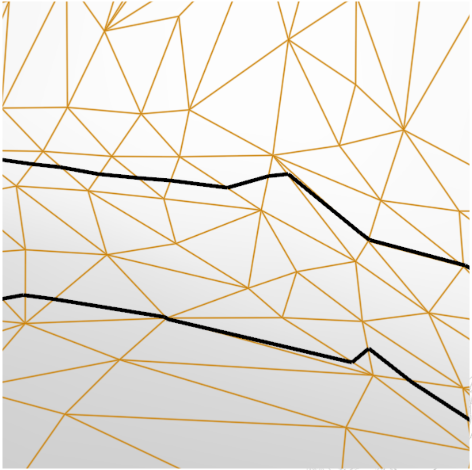
$$p := \frac{|h(w)|\, v + |h(v)|\, w}{|h(v)| + |h(w)|}$$



Algorithm: Line Detection and Tracing

- Check maximum condition:

$$\mathcal{D}^2_\varphi \left\| \nabla \varphi \right\| (p) < 0$$

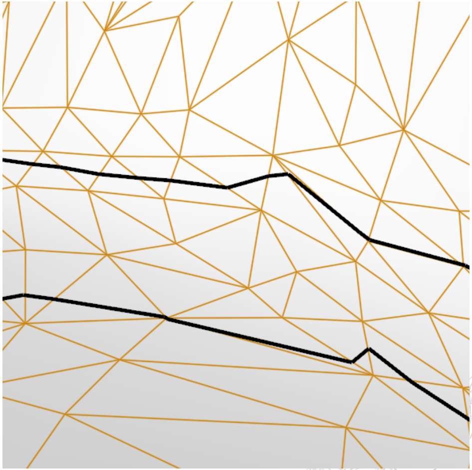


Algorithm: Line Detection and Tracing

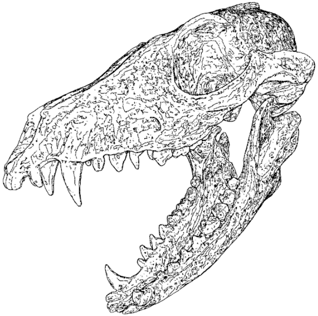
- Check maximum condition:

$$\mathcal{D}^2_{\varphi} \|\nabla\varphi\| (p) < 0$$

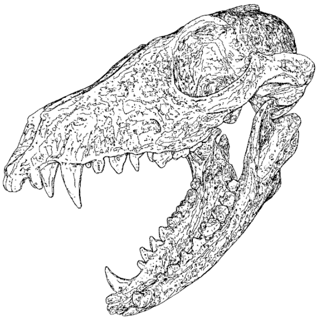
- For each triangle, connect valid zero-crossings of adjacent edges to segments



Algorithm: Threshold Filter



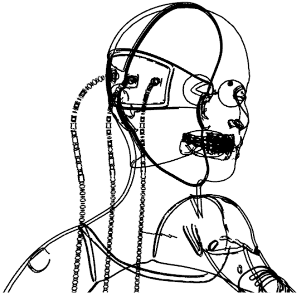
Algorithm: Threshold Filter

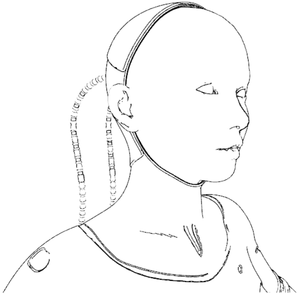


Strength S of photic extremum or strength $\mathcal{S}$ of photic extremum line L:

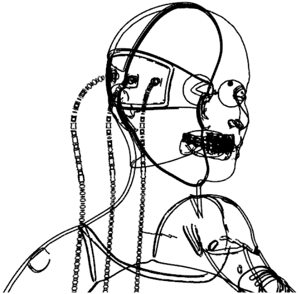
$$S(x) = \|\nabla\varphi(x)\| > T \qquad \text{or} \qquad \mathcal{S}(L) := \int_L \|\nabla\varphi(s)\| \,\mathrm{d}s > T$$

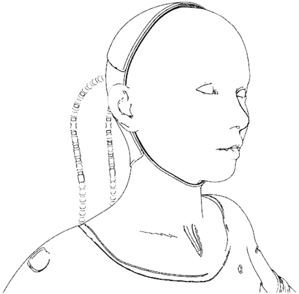
Algorithm: Hidden Line Removal





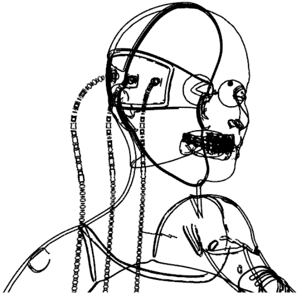
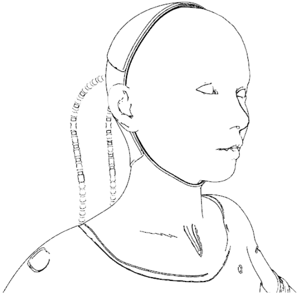
Algorithm: Hidden Line Removal





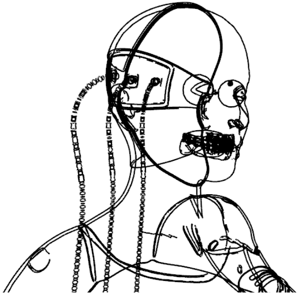
- Use z-buffer in a two-pass rendering approach

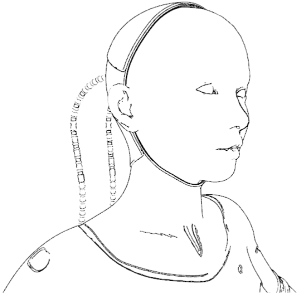
Algorithm: Hidden Line Removal

- Use z-buffer in a two-pass rendering approach
- Render the shape with a custom shader for its fragments

Algorithm: Hidden Line Removal





- Use z-buffer in a two-pass rendering approach
- Render the shape with a custom shader for its fragments
- Render visible feature lines by using depth testing

Results

Results: General Properties

Results: General Properties

- Able to render convex and concave edges simultaneously

Results: General Properties

- Able to render convex and concave edges simultaneously
- Applicable to isosurfaces of volumetric datasets

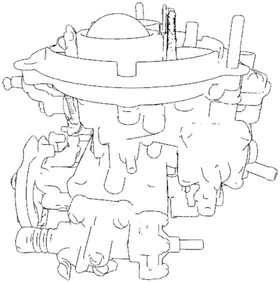
Results: General Properties

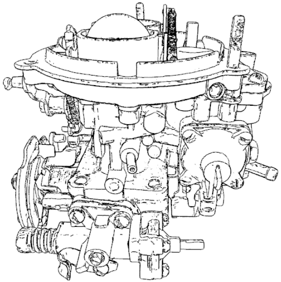
- Able to render convex and concave edges simultaneously
- Applicable to isosurfaces of volumetric datasets
- Highly dependent on scalar illumination function

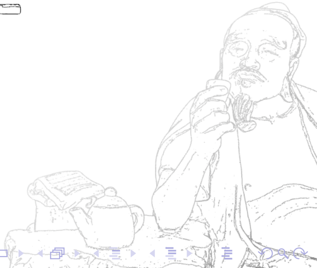
Results: General Properties

- Able to render convex and concave edges simultaneously
- Applicable to isosurfaces of volumetric datasets
- Highly dependent on scalar illumination function
- Computationally expensive: interactive on CPU, real-time capable on GPU

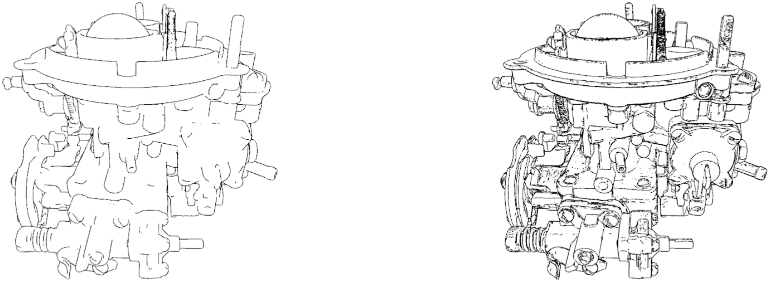
Results: Contours vs. Photic Extremum Lines





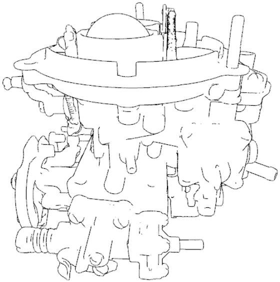


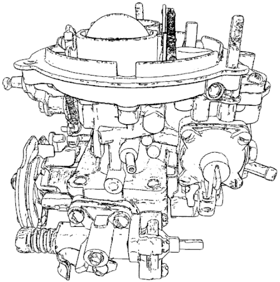
Results: Contours vs. Photic Extremum Lines



- Contours lack details, but are strongest for overall shape

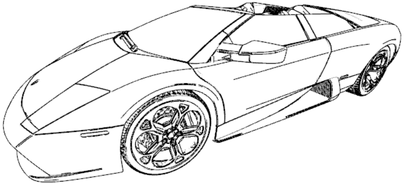
Results: Contours vs. Photic Extremum Lines

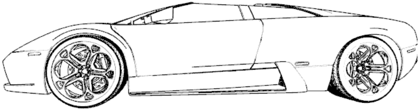




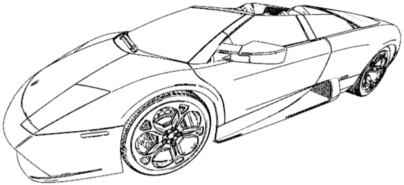
- Contours lack details, but are strongest for overall shape
- Photic extremum lines convey additional structure

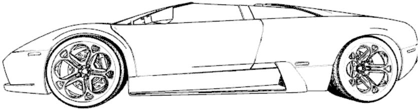
Results: Normal Denoising





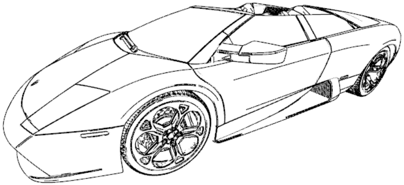
Results: Normal Denoising

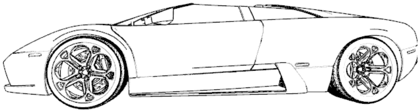




- Scanned models with many triangles often provide noisy normals

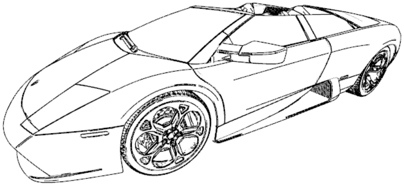
Results: Normal Denoising

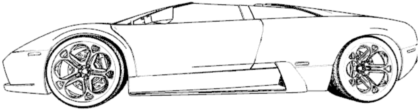




- Scanned models with many triangles often provide noisy normals
- Such noise leads to small feature line artifacts

Results: Normal Denoising





- Scanned models with many triangles often provide noisy normals
- Such noise leads to small feature line artifacts
- Bilateral normal filtering should be applied

Conclusions

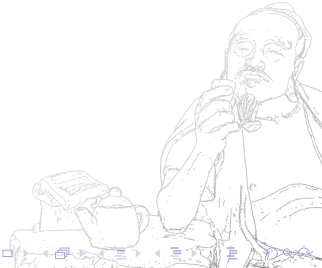
Conclusions

Summary

Conclusions

Summary

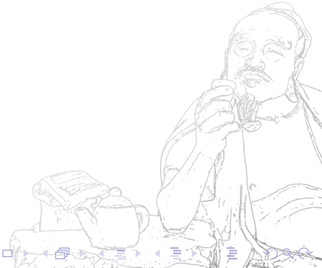
- Photic Extremums: $\mathcal{D}_{\varphi} \left\| \nabla \varphi \right\| (x) = 0 \ , \quad \mathcal{D}^2_{\varphi} \left\| \nabla \varphi \right\| (x) < 0$



Conclusions

Summary

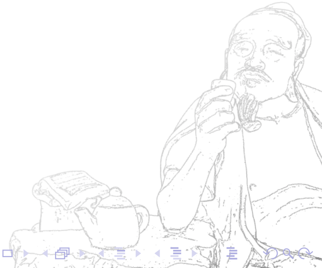
- Photic Extremums: $\mathcal{D}_{\varphi} \left\| \nabla \varphi \right\| (x) = 0 \;, \quad \mathcal{D}_{\varphi}^{2} \left\| \nabla \varphi \right\| (x) < 0$
- View- and light-dependent object-space feature line method



Conclusions

Summary

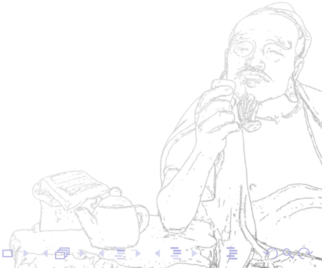
- Photic Extremums: $\mathcal{D}_{\varphi} \left\| \nabla \varphi \right\| (x) = 0 \ , \quad \mathcal{D}^2_{\varphi} \left\| \nabla \varphi \right\| (x) < 0$
- View- and light-dependent object-space feature line method
- Computationally expensive third- to fourth-order derivatives



Conclusions

Summary

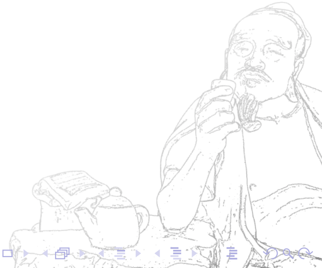
- Photic Extremums: $\mathcal{D}_{\varphi} \left\|\nabla \varphi\right\| (x) = 0 \,, \quad \mathcal{D}^2_{\varphi} \left\|\nabla \varphi\right\| (x) < 0$
- View- and light-dependent object-space feature line method
- Computationally expensive third- to fourth-order derivatives
- Convey shapes similar to human perception



Conclusions

Summary

- Photic Extremums: $\mathcal{D}_{\varphi} \|\nabla\varphi\| (x) = 0 \; , \quad \mathcal{D}^2_{\varphi} \|\nabla\varphi\| (x) < 0$
- View- and light-dependent object-space feature line method
- Computationally expensive third- to fourth-order derivatives
- Convey shapes similar to human perception



Conclusions

Summary

- Photic Extremums: $\mathcal{D}_{\varphi} \left\| \nabla \varphi \right\| (x) = 0 \ , \quad \mathcal{D}^2_{\varphi} \left\| \nabla \varphi \right\| (x) < 0$
- View- and light-dependent object-space feature line method
- Computationally expensive third- to fourth-order derivatives
- Convey shapes similar to human perception

Future Work

- Faster GPU-based implementation even for volumetric datasets

Conclusions

Summary

- Photic Extremums: $\mathcal{D}_{\varphi} \left\| \nabla \varphi \right\| (x) = 0 \; , \quad \mathcal{D}^2_{\varphi} \left\| \nabla \varphi \right\| (x) < 0$
- View- and light-dependent object-space feature line method
- Computationally expensive third- to fourth-order derivatives
- Convey shapes similar to human perception

Future Work

- Faster GPU-based implementation even for volumetric datasets
- Robustness: Exaggerated and mean curvature illumination

Conclusions

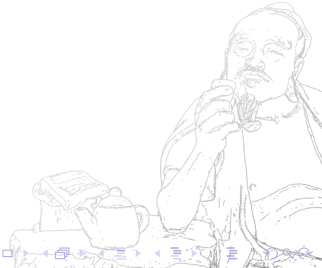
Summary

- Photic Extremums: $\mathcal{D}_\varphi \|\nabla\varphi\|(x) = 0\,, \quad \mathcal{D}^2_\varphi \|\nabla\varphi\|(x) < 0$
- View- and light-dependent object-space feature line method
- Computationally expensive third- to fourth-order derivatives
- Convey shapes similar to human perception

Future Work

- Faster GPU-based implementation even for volumetric datasets
- Robustness: Exaggerated and mean curvature illumination
- Robustness: Automatic thresholding and noise filtering

Thank you for Your Attention!



References

(1) Tobias Isenberg et al. "A Developer's Guide to Silhouette Algorithms for Polygonal Models". In: *Computer Graphics and Applications, IEEE* 23 (August 2003), pp. 28 –37. DOI: 10.1109/MCG.2003.1210862.

(2) Szymon Rusinkiewicz. "Estimating Curvatures and Their Derivatives on Triangle Meshes". In: October 2004, pp. 486–493. ISBN: 0-7695-2223-8. DOI: 10.1109/TDPVT.2004.1335277.

(3) Xuexiang Xie et al. "An Effective Illustrative Visualization Framework Based on Photic Extremum Lines (PELs)". In: *IEEE transactions on visualization and computer graphics* 13 (November 2007), pp. 1328–1335. DOI: 10.1109/TVCG.2007.70538.

(4) Long Zhang, Ying He, and Hock Seah. "Real-Time Computation of Photic Extremum Lines (PELs)". In: *The Visual Computer* 26 (June 2010), pp. 399–407. DOI: 10.1007/s00371-010-0454-x.

(5) Douglas DeCarlo et al. "Suggestive Contours for Conveying Shape". In: *ACM Trans. Graph.* 22 (July 2003), pp. 848–855. DOI: 10.1145/1201775.882354.

(6) Michael Kolomenkin, Ilan Shimshoni, and Ayellet Tal. "Demarcating Curves for Shape Illustration". In: *ACM Trans. Graph.* 27 (December 2008), p. 157. DOI: 10.1145/1457515.1409110.

(7) Szymon Rusinkiewicz, Michael Burns, and Douglas DeCarlo. "Exaggerated Shading for Depicting Shape and Detail". In: *ACM Trans. Graph.* 25 (July 2006), pp. 1199–1205. DOI: 10.1145/1179352.1142015.

(8) Mark Meyer et al. "Discrete Differential-Geometry Operators for Triangulated 2-Manifolds". In: *Proceedings of Visualization and Mathematics* 3 (November 2001). DOI: 10.1007/978-3-662-05105-4_2.

(9) Gordon Kindlmann et al. "Curvature-Based Transfer Functions for Direct Volume Rendering: Methods and Applications". In: vol. 2003. November 2003, pp. 513–520. ISBN: 0-7803-8120-3. DOI: 10.1109/VISUAL.2003.1250414.