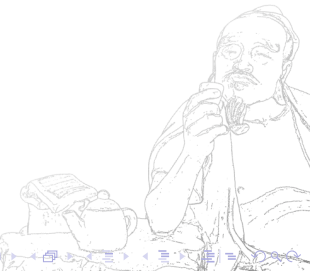
Illustrative Visualization: Photic Extremum Lines

Markus Pawellek

January 10, 2022



Outline

Related Work

Related Work

Tools

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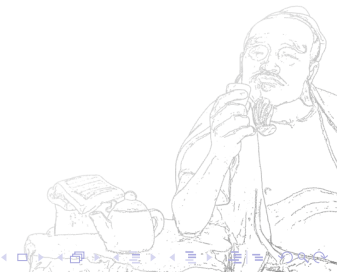
2003 Isenberg et al. "A Developer's Guide to Silhouette Algorithms for Polygonal Models"



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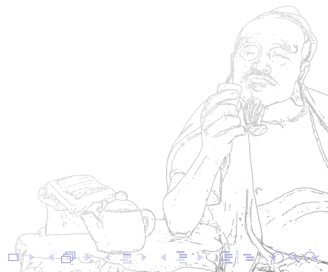

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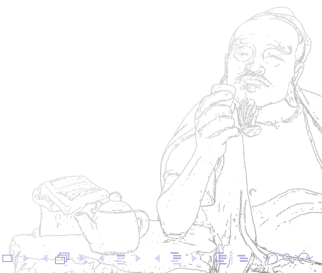

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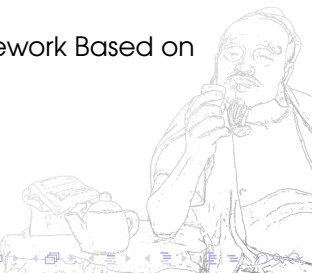

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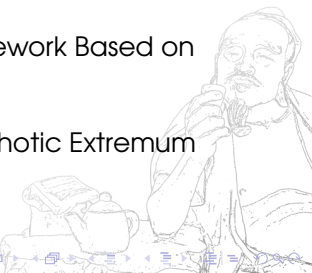
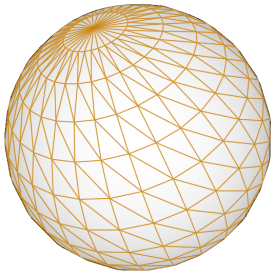
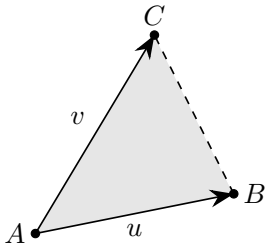
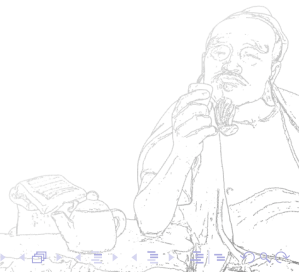


Mathematical Preliminaries

Mathematical Preliminaries: Mesh Function







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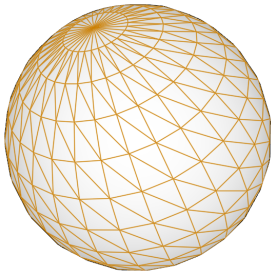

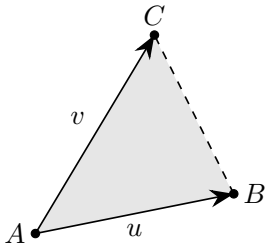

- $f : S \to \mathbb{R}$ on mesh S characterized by its values at vertices

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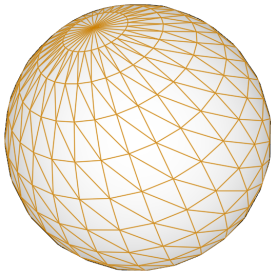
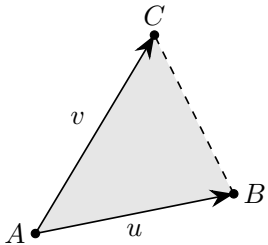
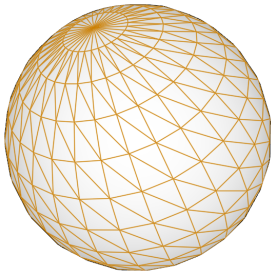
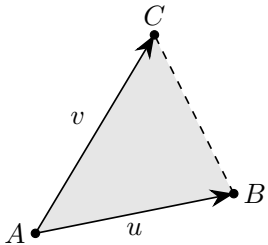


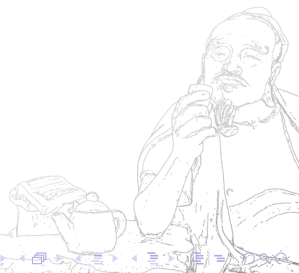


- $f : S \to \mathbb{R}$ on mesh S characterized by its values at vertices
- For interiors of faces, use barycentric interpolation

Mathematical Preliminaries: Mesh Function Gradient







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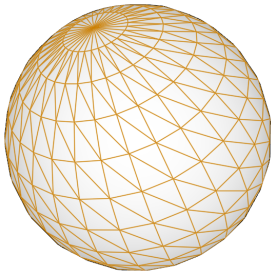
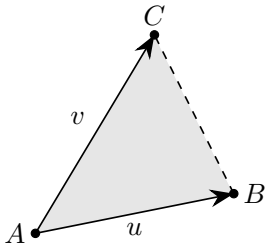
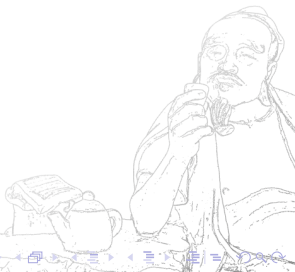




- Compute ∇f for each face



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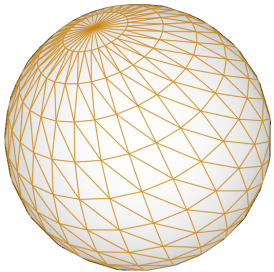
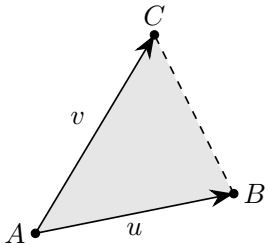
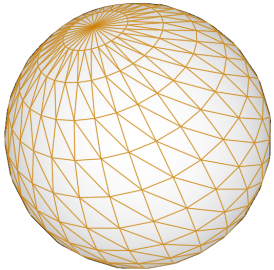
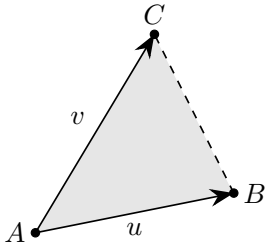




- Compute ∇f for each face
- For each vertex, accumulate weighted gradients for adjacent faces

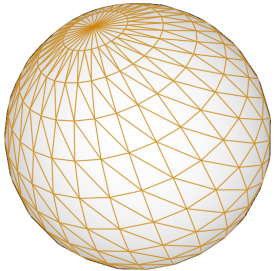
Mathematical Preliminaries: Mesh Function Gradient

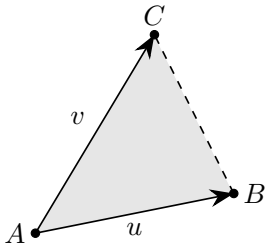




$$\mathrm{I}_{uv} \coloneqq \begin{pmatrix} \|u\|^2 & \langle u, v\rangle \\ \langle u, v\rangle & \|v\|^2 \end{pmatrix} \qquad \nabla f = \begin{pmatrix} u & v \end{pmatrix} \mathrm{I}_{uv}^{-1} \begin{pmatrix} f(B) - f(A) \\ f(C) - f(A) \end{pmatrix}$$

Mathematical Preliminaries: Directional Derivatives



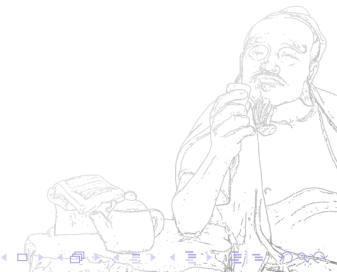


$$\partial_w f(x) = \langle \nabla f(x), w \rangle \qquad \mathcal{D}_f\, g(x) \coloneqq \left\langle \nabla g(x), \frac{\nabla f(x)}{\|\nabla f(x)\|} \right\rangle$$

Photic Extremum Lines

Photic Extremum Lines: Definition

- Scalar illumination function
  $\varphi\colon S \to \mathbb{R}$ on mesh S
  (e.g. directional light source)



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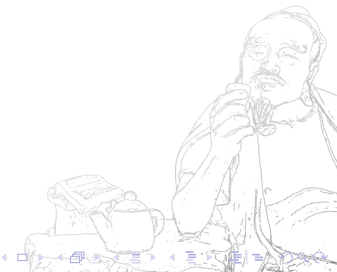

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Illumination variation in the direction its gradient reaches local maximum

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Illumination variation in the direction its gradient reaches local maximum

$$\mathcal{D}_{\varphi}\|\nabla\varphi\|(x) = 0 \qquad \mathcal{D}^2_{\varphi}\|\nabla\varphi\|(x) < 0$$

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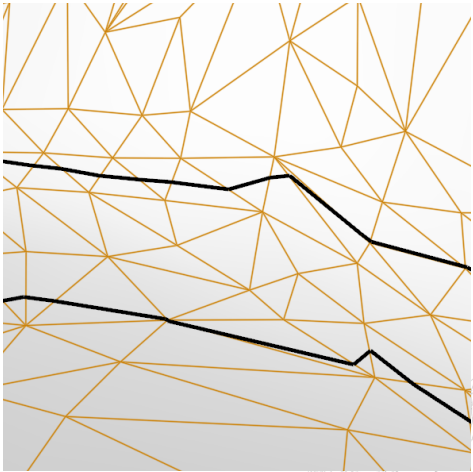
5. Detect line vertices on edges by testing for photic extremums
6. Trace and filter out lines by using a threshold
7. Render visible lines

Algorithm: Line Detection and Tracing

- For each edge $[v, w] \subset S$, check zero-crossing:

$$h(x) := \mathcal{D}_{\varphi} \|\nabla \varphi\| (x)$$

$$h(v)h(w) < 0$$



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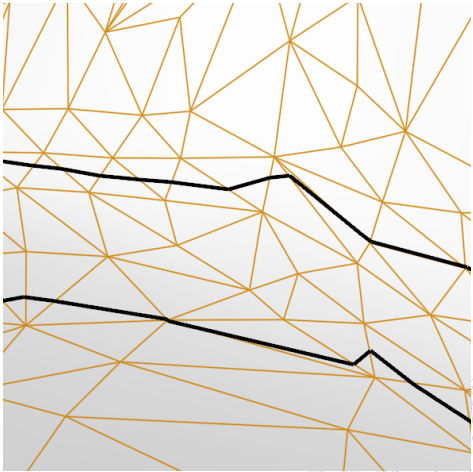
$$h(v)h(w) < 0$$

- Approximate zero-crossing:

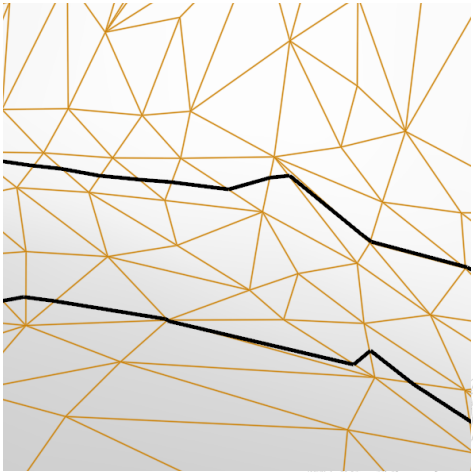
$$p := \frac{|h(w)|\, v + |h(v)|\, w}{|h(v)| + |h(w)|}$$



Algorithm: Line Detection and Tracing

- Check maximum condition:

$$\mathcal{D}^2_{\varphi} \|\nabla\varphi\| (p) < 0$$

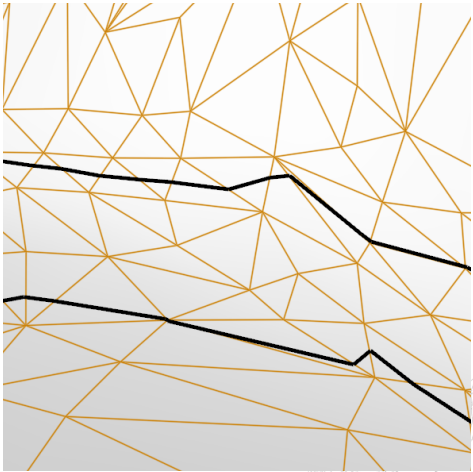


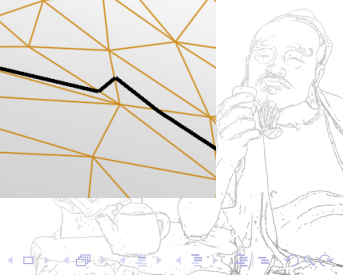
Algorithm: Line Detection and Tracing

- Check maximum condition:

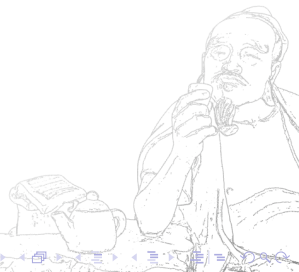
$$\mathcal{D}^2_{\varphi} \|\nabla\varphi\| (p) < 0$$

- For each triangle, connect valid zero-crossings of adjacent edges to segments





Algorithm: Threshold Filter



Algorithm: Threshold Filter

Strength S of photic extremum or strength $\mathcal{S}$ of photic extremum line L:

$$S(x) = \|\nabla\varphi(x)\| > T \qquad \text{or} \qquad \mathcal{S}(L) \coloneqq \int_L \|\nabla\varphi(s)\| \, \mathrm{d}s > T$$

Results

Photic Extremum Lines: Properties

- Object-space method
- View- and light-dependent
- Third- and fourth-order derivatives
-

Problems

preprocessing and good meshes

Conclusions

Thank you for Your Attention!

References

(1) Tobias Isenberg et al. "A Developer's Guide to Silhouette Algorithms for Polygonal Models". In: *Computer Graphics and Applications, IEEE* 23 (August 2003), pp. 28–37. DOI: 10.1109/MCG.2003.1210862.

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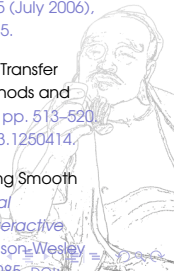

Previous Work