Friedrich Schiller University Jena
Faculty of Mathematics and Computer Science

Design and Implementation of High-Performance, Adaptive, and Robust Curve Smoothing on Surface Meshes and its Application to Medical Visualization

MASTER'S THESIS

for obtaining the academic degree

Master of Science (M.Sc.) in Mathematics

submitted by Markus Pawellek

born on May 7th, 1995 in Meiningen
Student Number: 144645

Primary Supervisor:	Kai Lawonn
Secondary Supervisor:	Noeska Smit

Bergen, July 13, 2022

Abstract

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.

Zusammenfassung

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.

Acknowledgments

I am grateful to Kai Lawonn and Noeska Smit for their supervising, their helpful suggestions, and for our interesting discussions. Also great thanks to Ann Sommerfeld for assisting in writing the thesis and proofreading.

Contents

List of Figures

List of Tables

List of Definitions and Theorems

List of Code

List of Abbreviations and Acronyms

Abbreviation	Definition
iid	Independently and Identically Distributed
CDF	Cumulative Distribution Function
SLLN	Strong Law of Large Numbers
LTE	Light Transport Equation
API	Application Programming Interface
RAII	Resource Acquisition is Initialization
SFINAE	Specialization Failure is not an Error
STL	Standard Template Library

Symbol Table

Symbol	Definition
Logic	
$\exists \ldots : \ldots$	There exists . . ., such that . . .
$a \coloneqq b$	a is defined by b.
Set Theory	
$\{\ldots\}$	Set Definition
$\{\ldots \mid \ldots\}$	Set Definition with Condition
$x \in A$	x is an element of the set A.
$A \subset B$	The set A is a subset of the set B.
$A \cap B$	Intersection — $\{x \mid x \in A \text{ and } x \in B\}$ for sets A, B
$A \cup B$	Union — $\{x \mid x \in A \text{ or } x \in B\}$ for sets A, B
$A \setminus B$	Relative Complement — $\{x \in A \mid x \notin B\}$ for sets A, B
$A \times B$	Cartesian Product — $\{(x, y) \mid x \in A, y \in B\}$ for sets A and B
A^n	n-fold Cartesian Product of Set A
$\emptyset$	Empty set — $\{\}$.
$\#A$	Number of Elements in the Set A
$\mathcal{P}(A)$	Power Set of Set A
Special Sets	
$\mathbb{N}$	Set of Natural Numbers
$\mathbb{N}_0$	$\mathbb{N} \cup \{0\}$
$\mathbb{P}$	Set of Prime Numbers
$\mathbb{Z}$	Set of Integers
$\mathbb{Z}_n$	Set of Integers Modulo n
$\mathbb{F}_m$	Finite Field with $m \in \mathbb{P}$ Elements
$\mathbb{F}_m^{p \times q}$	Set of $p \times q$-Matrices over Finite Field $\mathbb{F}_m$
$\mathbb{F}_2$	Finite Field of Bits
$\mathbb{F}_2^n$	Set of n-bit Words
$\mathbb{R}$	Set of Real Numbers
$\mathbb{R}^n$	Set of n-dimensional Real Vectors
$\mathbb{S}^2$	Set of Directions — $\left\{x \in \mathbb{R}^3 \mid \|x\| = 1\right\}$
Functions	
$f \colon X \to Y$	f is a function with domain X and range Y.
id_X	Identity Function over the Set X
$f \circ g$	Composition of Functions f and g
f^{-1}	Inverse Image of Function f
f^n	n-fold Composition of Function f
Bit Arithmetic	
$x_{n-1} \ldots x_1 x_0$	n-bit Word x of Set $\mathbb{F}_2^n$
$x \leftarrow a$	Left Shift of all Bits in x by a
$x \rightarrow a$	Right Shift of all Bits in x by a
$x \circlearrowleft a$	Circular Left Shift of all Bits in x by a
$x \oplus y$	Bit-Wise Addition of x and y
$x \odot y$	Bit-Wise Multiplication of x and y
$x \mid y$	Bit-Wise Or of x and y

Symbol	Definition
Probability Theory	
$\mathcal{B}(\mathbb{R})$	Borel σ-Algebra over $\mathbb{R}$
$(\Sigma, \mathcal{A})$	Measurable Space over Σ with σ-Algebra $\mathcal{A}$
λ	Lebesgue Measure
$\int_U f \,\mathrm{d}\lambda$	Lebesgue Integral of f over U
$\mathrm{L}^2(U, \lambda)$	Set of Square-Integrable Functions over the Set U with Respect to the Lebesgue Measure λ
$(\Omega, \mathcal{F}, P)$	Probability Space over Ω with σ-Algebra $\mathcal{A}$ and Probability Measure P
$\int_\Omega X \,\mathrm{d}P$	Integral of Random Variable X with respect to Probability Space $(\Omega, \mathcal{A}, P)$
$\int_\Omega X(\omega) \,\mathrm{d}P(\omega)$	$\int_\Omega X \,\mathrm{d}P$
P_X	Distribution of Random Variable X
$\mathbb{E}\, X$	Expectation Value of Random Variable X
$\mathrm{var}\, X$	Variance of Random Variable X
$\sigma(X)$	Standard Deviation of Random Variable X
$\mathbb{1}_A$	Characteristic Function of Set A
δ_ω	Dirac Delta Distribution over $\mathbb{S}^2$ with respect to $\omega \in \mathbb{S}^2$
$\bigotimes_{n \in I} P_n$	Product Measure of Measures P_n Indexed by the Set I
Miscellaneous	
$(x_n)_{n \in I}$	Sequence of Values x_n with Index Set I
$\lvert x \rvert$	Absolute Value of x
$\lVert x \rVert$	Norm of Vector x
$x \mod y$	x Modulo y
$\gcd(\rho, k)$	Greatest Common Divisor of ρ and k
$\max(x, y)$	Maximum of x and y
$\lim_{n \to \infty} x_n$	Limit of Sequence $(x_n)_{n \in \mathbb{N}}$
$\sum_{k=1}^{n} x_k$	Sum over Values x_k for $k \in \mathbb{N}$ with $k \leq n$
$\dim X$	Dimension of X
$\lceil x \rceil$	Ceiling Function
$\langle x \mid y \rangle$	Scalar Product
$[a, b]$	$\{x \in \mathbb{R} \mid a \leq x \leq b\}$
(a, b)	$\{x \in \mathbb{R} \mid a < x < b\}$
$[a, b)$	$\{x \in \mathbb{R} \mid a \leq x < b\}$
Constants	
∞	Infinity
π	$3.1415926535\ldots$ — Pi
Units	
$1\,\mathrm{B}$	$1\,\mathrm{Byte} = 8\,\mathrm{bit}$
$1\,\mathrm{GiB}$	$2^{30}\,\mathrm{B}$
$1\,\mathrm{s}$	$1\,\mathrm{Seconds}$
$1\,\mathrm{min}$	$1\,\mathrm{Minutes} = 60\,\mathrm{s}$
$1\,\mathrm{GHz}$	$1\,\mathrm{Gigahertz} = 10^9\,\mathrm{Hertz}$

1 Introduction

2 Preliminaries

3 Previous Work

4 Implementation

5 Evaluation and Results

6 Conclusions and Future Work

References

Lawonn, Kai et al. (2014). “Adaptive and Robust Curve Smoothing on Surface Meshes”. In: *Computers & Graphics* 40, pp. 22–35. DOI: 10.1016/j.cag.2014.01.004.

A Mathematical Proofs

B Further Code

Statutory Declaration

I declare that I have developed and written the enclosed Master's thesis completely by myself, and have not used sources or means without declaration in the text. Any thoughts from others or literal quotations are clearly marked. The Master's thesis was not used in the same or in a similar version to achieve an academic grading or is being published elsewhere.
On the part of the author, there are no objections to the provision of this Master's thesis for public use.

Bergen, July 13, 2022

Markus Pawellek