Friedrich-Schiller-Universität Jena
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Restricted Boltzmann Machines for Collaborative Filtering

REPORT

for the lecture "Computational Physics III - Machine Learning"

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Jena, February 11, 2019

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RESTRICTED BOLTZMANN MACHINES FOR COLLABORATIVE FILTERING

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Abstract

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1 Introduction

On October 2, 2006 Netflix announced the start of the so-called "Netflix Prize" competition. It was an open competition aimed to find the best algorithm for collaborative filtering to predict user ratings for movies based on previous ratings without any other information about the users or the movies. Netflix's current algorithm "Cinematch" introduced the threshold that had to be bested. For this Netflix provided a training dataset with over 100,000,000 ratings that more than 480,000 users gave about 17,000 movies. The complete competition lasted over three years and included two "Progress Prizes" in the years 2007 and 2008. Finally on September 18, 2009 Netflix announced the winner-team of the $\$\,1,000,000$ "Grand Prize" with its last submission 24 minutes before the conclusion of the contest. The solution of the winner-team "BellKor's Pragmatic Chaos" was based on the work of [13] which used restricted Boltzmann machines (RBMs) to efficiently predict the user ratings. With this algorithm the winner-team was able to achieve an improvement over Netflix's current algorithm by $10.05\,\%$. [9, 10, 13]

In [13] different RBMs were applied to the task of collaborative filtering for the first time. Even with the basic ideas presented in this paper one could achieve an error rate that was well over $6\,\%$ better than the score of Netflix's own system. Additionally, in comparison to other proposed solutions RBMs were able to deal much more efficiently with big datasets, like the ones given by Netflix for the competition. [13]

RBMs have found their application in other machine learning topics as well. In [8] an RBM is introduced in topic modelling as a more precise alternative to the common latent Dirichlet allocation. In [6] discriminative RBMs are used for classification in a self-contained framework and in [4] RBMs are even used as basic building blocks in much bigger deep neural networks (DNNs) to efficiently reduce the dimensionality of some given data. According to [7] RBMs play an important role in the mathematical theory of machine learning and are not fully investigated, yet.

RBMs in general consists of a basic and simple structure with good mathematical properties for which one can find efficient learning algorithms. The successful application of RBMs in these different topics of machine learning make them an interesting subject to study and understand. In the next sections we will talk about the details of RBMs used for the topic of collaborative filtering and especially about how to predict user ratings for movies as a direct application to a real world problem.

2 The Problem

Collaborative filtering as seen in a modern narrow sense basically can be described as a method to make automatic predictions about interests of users by collecting

Table 1: The table shows examples of binary ratings for movies made by some imaginary users. Every row represents a user and every column a movie. The number 0 is used to point out that the user does not like the movie and 1 for the opposite. The symbol $\times$ is used if there was no rating for the movie by this user.

	Star Trek	The Matrix	Van Helsing	Harry Potter	The Hobbit
James T. Kirk	1	1	$\times$	0	$\times$
Trinity	$\times$	1	0	1	1
Anna Valerious	$\times$	$\times$	1	$\times$	0
Severus Snape	0	1	0	1	0
Thorin Oakenshield	1	1	1	$\times$	0

the preferences or tastes of many users. The underlying assumption of the collaborative filtering approach is that if a person A has the same opinion as a person B on an issue, A is more likely to have B's opinion on a different issue than that of a randomly chosen person.

To understand the statement of the problem we will use the prediction of movie ratings as done for the "Netflix Prize" competition. In table 1 one can see some examples for these ratings. The entries with 0 and 1 are already known and shall be used to predict the unknown values with $\times$. In the example the users "James T. Kirk" and "Thorin Oakenshield" both like the movies "Star Trek" and "The Matrix". So one could assume that the user "James T. Kirk" likes the movie "Van Helsing" and that the user "Thorin Oakenshield" does not like the movie "Harry Potter".

The abstract goals to achieve this can be described as follows.

- Approximately represent probability distributions over ratings
- Learn a probability distribution based on some given ratings.
- Make predictions for unrated movies for learned parameters.

3 Background

Before we go into the details of the model of an RBM let us consider some fundamentals in stochastics and statistics to better understand the approach taken by the RBM.

4 The Model

To understand the model of an RBM let us first consider the general idea by looking at the left part of figure 1. It shows a simple schematic example of an RBM. At first sight there seems to be no real difference to a standard feed-forward neural network (FFNN) with two layers. For an RBM every edge is undirected. Therefore the influence of one neuron to another cannot be computed directly as one maybe used to be in the FFNN.

Apart from this subtle difference there are two obvious properties which are defining the RBM. First, the neurons in the RBM are separated into two subsets, the hidden units and the visible units. And second, connections between neurons via edges are only allowed between those two subsets.

Based on these properties there is no real obvious interpretation for values of these units. But we have to remember that we want to model a probability distribution. So we will interpret every unit as input value for our RBM. Looking at the right part of figure 1 it is shown applied on the movie ratings from a user. Every rating from one user can be seen as a visible value. Based on these visible values an RBM can assign some hidden values based on some learned features, like movie genres "Action" or "Fantasy". These hidden values for example would describe if the user likes the movie genre.

Because we want to model the probability distribution with the given undirected bipartite graph, we first have to define some parameters. Here we can choose the standard approach of introducing biases for every

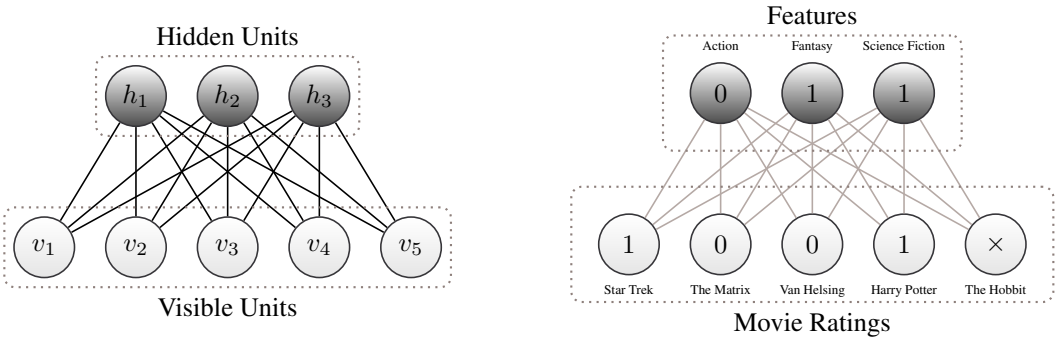

Figure 1: Figure for scheme and example!

neuron and weights for every edge. Therefore we get two bias vectors for the hidden and visible values and one weight matrix. Figure 2 shows schematically.

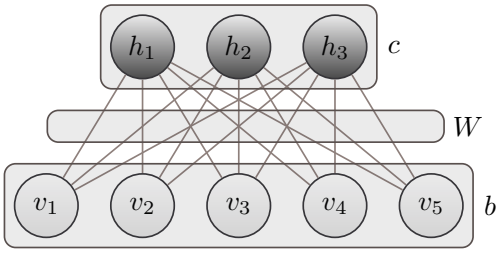

Figure 2: The figure shows the basic scheme of an RBM with the weight matrix W and bias vectors b and c as parameters describing the probability distribution modelled by the RBM.

To be precise, we will first define the sets for the input parameters.

$$v \in V \coloneqq \{0,1\}^n$$

$$h \in H \coloneqq \{0,1\}^m$$

The parameters describing the RBM are taken to be real. We will abbreviate the weight matrix and the two bias vectors as one parameter. This will make the following formulas much more readable.

$$\vartheta \coloneqq (W,b,c) \in \mathbb{R}^{(n\times m)+n+m}$$

Now we will define the probability distribution the RBM is modelling with respect to its parameters. This is a definition and not a corollary. Of course, we could choose another approach. But this would not be a RBM.

$$p[\vartheta]\colon V \times H \to [0,1]$$

$$p[\vartheta](v,h) \coloneqq \frac{e^{-E[\vartheta](v,h)}}{Z(\vartheta)}$$

Here we define $E[\vartheta]$ to be the so called energy function of the RBM. We have already introduced non-linearity by the exponential function. Therefore one chooses $E[\vartheta]$ as simple as possible.

$$E[\vartheta]\colon V \times H \to \mathbb{R}$$

$$E[\vartheta](v,h) \coloneqq -v^{\mathrm{T}}Wh - v^{\mathrm{T}}b - h^{\mathrm{T}}c$$

For completeness the normalization will be supplied. The definition for $Z(\vartheta)$ is straightforward and not important for the learning or inference processes.

$$Z(\vartheta) \coloneqq \sum_{v\in V}\sum_{h\in H} e^{-E[\vartheta](v,h)}$$

At this point one could think, that major problem in our model. If we cannot observe hidden values then how should we able to model a probability distribution over them. We can omit this problem by defining the probability distribution for the visible values.

$$p[\vartheta]\colon V \to [0,1]$$

$$p[\vartheta](v) \coloneqq \sum_{h\in H} p[\vartheta](v,h)$$

This is the complete description of our model. But because of the two main properties of an RBM we can derive an important equation which will enable us to learn an RBM and to do some inference.

$$p[\vartheta](h|v) = \prod_{j=1}^{m} p[\vartheta]\left(h_j = 1|v\right)$$

5 Learning

The learning procedure for an RBM is rather easy due to its basic structure and good properties. First, we need some scalar potential to optimize. Learning is always about optimizing some sort of function. Because we want to learn a probability distribution on the data set we will use the maximum-likelihood estimation and will try to find a maximum.

$$\mathcal{S} \in V^s$$

As always we will not use the maximum-likelihood function but the log-likelihood function which simplifies the process of computing derivatives and gives us an equivalent optimization condition.

$$\mathcal{L}[\mathcal{S}] \colon \mathbb{R}^{n \times m + n + m} \to \mathbb{R}$$

$$\mathcal{L}[\mathcal{S}](\vartheta) \coloneqq \frac{1}{s} \sum_{k=1}^{s} \ln p[\vartheta]\left(\mathcal{S}_k\right)$$

We will take one of the simples algorithms to maximize this function. "Gradient Ascent" works exactly like "Gradient Descent" but finds the maximum instead of the minimum. For this we need the gradients of the log-likelihood function with respect to the weight matrix and the bias vectors.

$$\nabla_W \mathcal{L}[\mathcal{S}](\vartheta) = \frac{1}{s} \sum_{k=1}^{s} \mathbb{E}_\vartheta \left[\mathcal{V}\mathcal{H}^{\mathrm{T}} \middle| \mathcal{S}_k\right] - \mathbb{E}_\vartheta \left[\mathcal{V}\mathcal{H}^{\mathrm{T}}\right]$$

At first sight this formula seems to be complicated. But the left part can be easily computed. The right part is much more difficult. The typical method of finding the expectation of the model itself one has to do Gibbs sampling. Figure 3 demonstrates this method schematically.

Using Gibbs sampling for the right part of the gradient is mathematically ideal but fails when applied to reality because the algorithm is slow. The typical procedure to make things good again is to abort the series after some given integer. Then one can approximate the expectation as follows. This is called "Contrastive Divergence".

$$\mathbb{E}_\vartheta \left[\mathcal{V}\mathcal{H}^{\mathrm{T}}\right] \approx v^{(k)} {h^{(k)}}^{\mathrm{T}}$$

The algorithm is shown in the following example listing.

Now one has to talk about the application of the algorithm to collaborative filtering. For this every user will get its own RBM which learns only based on the rated and not the unrated movies. To not have a set of independent RBMs trained with only one sample one connects the weights and biases of each RBM. This means that if two users have rated the same movie then for this movie the same weights and biases will be used. Figure 4 shows the application of this algorithm to the movie ratings by a user again.

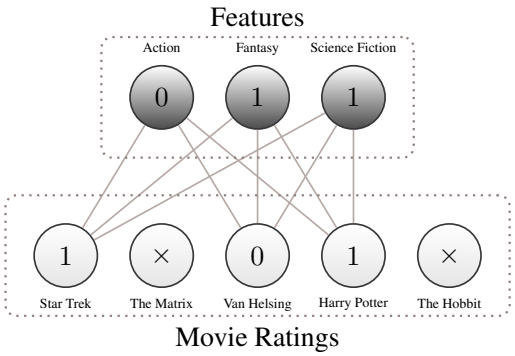

Figure 4: The figure shows the application of the learning algorithm to the movie ratings for some user.

6 Inference

The general inference for an RBM can be done by using the Gibbs sampling method explained in the last chapter. Here we will explicitly talk about the inference for the collaborative filtering problem. Figure 5 shows such an application in a schematic example.

First, we get the vector of rated and unrated movies from a given user. We then have to sample the hidden values by using only the values for the rated movies via the a posterior probability. After this we are now able to sample values for the unrated movies again by using the a posterior probability. The values sampled are then the predictions of the user ratings.

7 Implementation

8 Conclusion

RBMs have a simple structure and can be trained with CD which is an efficient algorithm. Based on SOURCE they seem to be one of the best known methods for

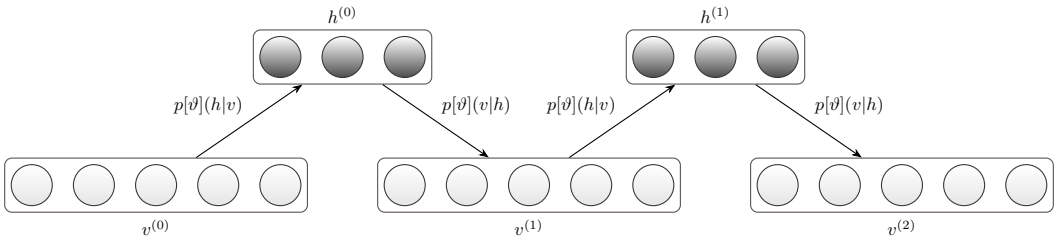

Figure 3: The figure shows the basic scheme of Gibbs sampling.

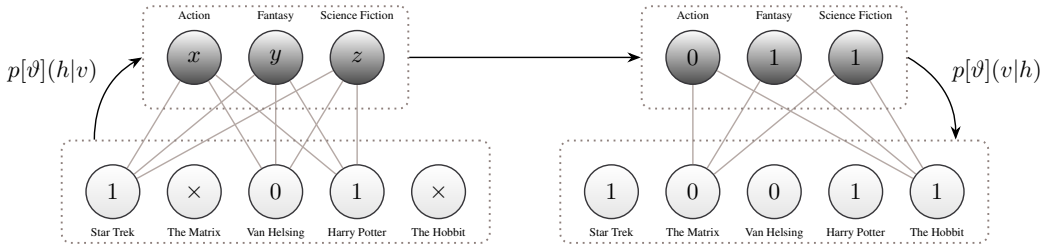

Figure 5: The figure shows the application of inference to the prediction of movie ratings for users.

collaborative filtering. As said in the introduction this is not the only application. RBMs should be used as basic building blocks. They are a powerful tool. Use them if there is a simple connection to hidden features in your data and if you want to predict something. It may be a good idea to insert these into your DNNs as dimensionality reduction.

Of course one should consider to tweak the explained ideas and algorithms. One can use momentum, weight decay and different types of units. There are some variants of the contrastive divergence as well. According the PAPER even mathematics has not completed the topic of RBMs. They seem to be promising in explaining the connection of quantum theory and deep neural networks.

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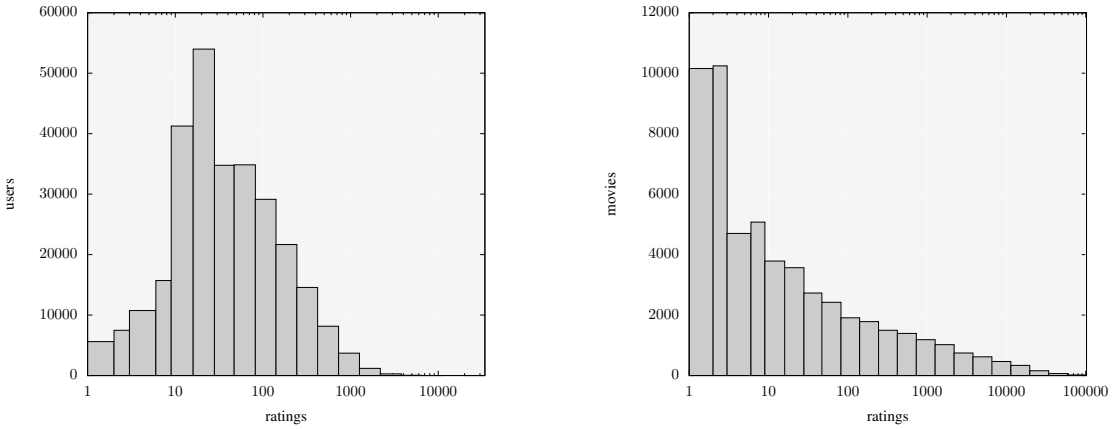

Figure 6: The figure shows the two histograms for rating counts over users and movies.

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