NETFLIX

Restricted Boltzmann Machines

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Outline

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Problem

Problem: Collaborative Filtering – Movie Ratings

	Star Trek	The Matrix	Van Helsing	Harry Potter	The Hobbit
James T. Kirk	1	1	$\times$	0	$\times$
Trinity	$\times$	1	0	1	1
Anna Valerious	$\times$	$\times$	1	$\times$	0
Severus Snape	0	1	0	1	0
Thorin Oakenshield	1	1	1	$\times$	0

Problem: Collaborative Filtering – Movie Ratings

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Thorin Oakenshield	1	1	1	×	0

Goal:

Problem: Collaborative Filtering – Movie Ratings

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Goal:

- approximately represent a complex probability distribution

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- learn probability distribution based on given samples

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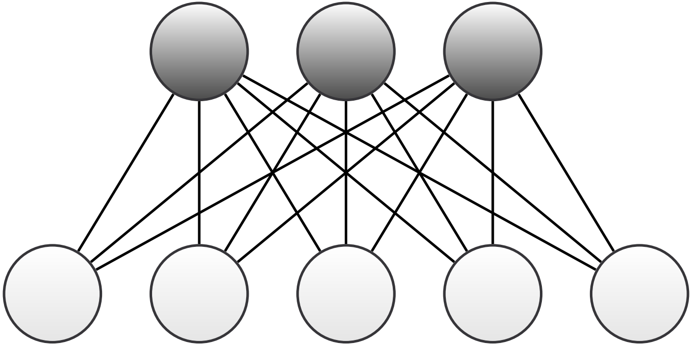
Goal:

- approximately represent a complex probability distribution
- learn probability distribution based on given samples
- make predictions based on learned parameters

The Model

The Model: Idea



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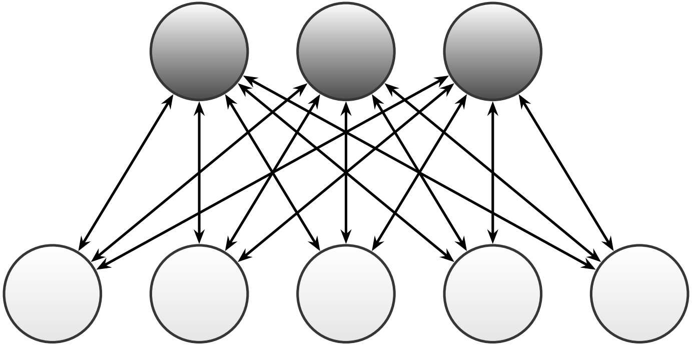

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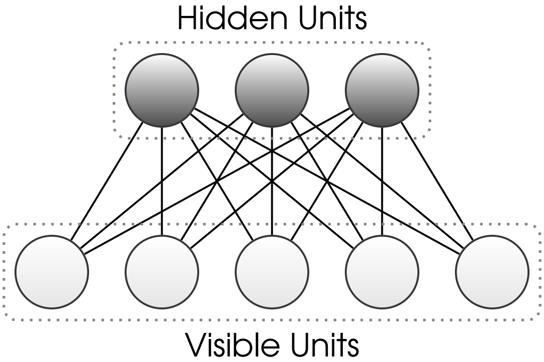

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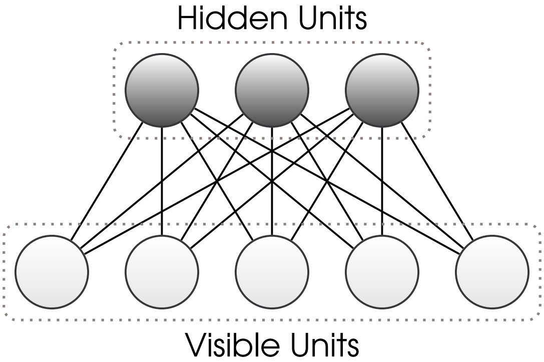
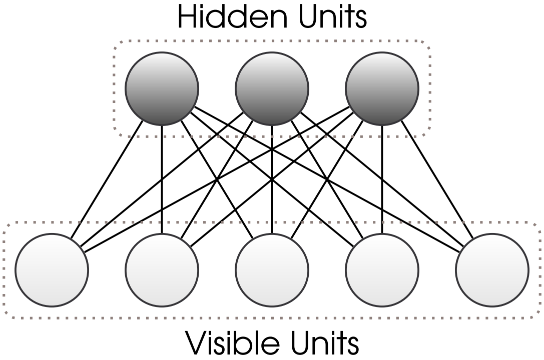


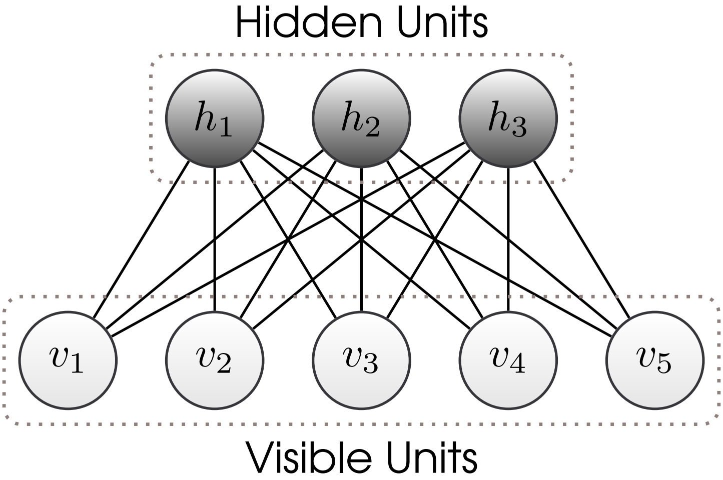
- units are divided into two subsets

The Model: Idea

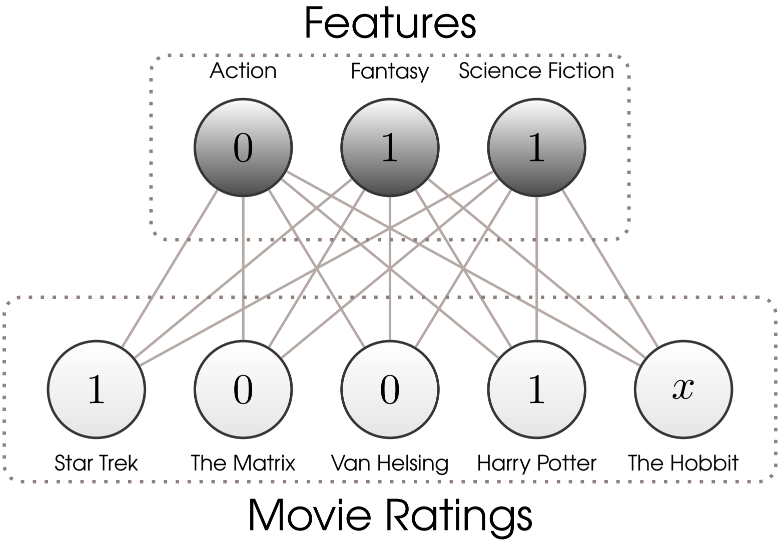


- units are divided into two subsets
- only connections between hidden and visible units are allowed

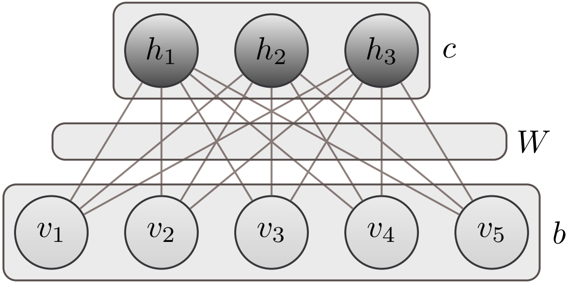
The Model: Idea – Inputs



The Model: Idea – Example



The Model: Parameters



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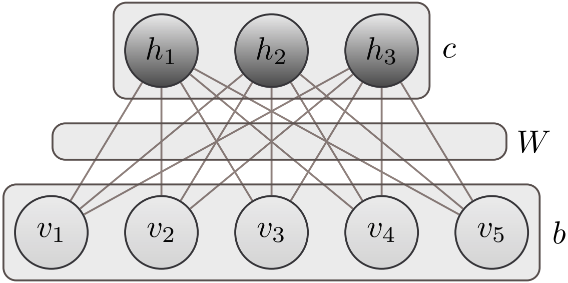

$$v \in V := \{0,1\}^n \qquad h \in H := \{0,1\}^m \qquad \vartheta := (W,b,c) \in \mathbb{R}^{(n \times m)+n+m}$$

The Model: Probability Distribution and Energy

$$p[\vartheta]\colon V \times H \to [0,1] \qquad p[\vartheta](v,h) := \frac{e^{-E[\vartheta](v,h)}}{Z(\vartheta)}$$

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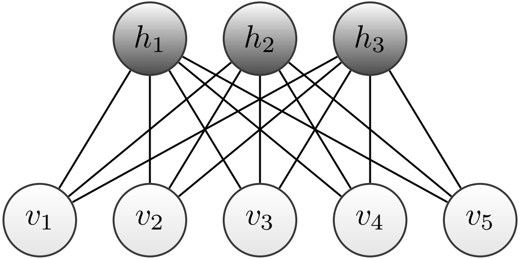
$$E[\vartheta]\colon V \times H \to \mathbb{R} \qquad E[\vartheta](v,h) := -v^{\mathrm{T}}Wh - v^{\mathrm{T}}b - h^{\mathrm{T}}c$$

$$Z(\vartheta) := \sum_{v\in V}\sum_{h\in H} e^{-E[\vartheta](v,h)}$$

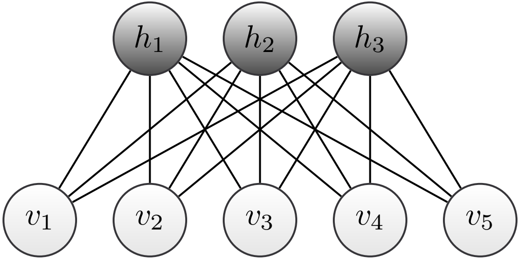
The Model: Probability Distribution for Visible Units

$$p[\vartheta]\colon V \to [0,1] \qquad p[\vartheta](v) \coloneqq \sum_{h\in H} p[\vartheta](v,h)$$

The Model: Posterior Probability



The Model: Posterior Probability



$$p[\vartheta](h|v) = \prod_{j=1}^{m} p[\vartheta]\left(h_j = 1|v\right)$$

Learning

Learning: Maximum Likelihood Estimation

$$\mathcal{S} \in V^s \qquad \mathcal{L}[\mathcal{S}] \colon \mathbb{R}^{n \times m + n + m} \to \mathbb{R} \qquad \mathcal{L}[\mathcal{S}](\vartheta) := \frac{1}{s} \sum_{k=1}^{s} \ln p[\vartheta]\,(\mathcal{S}_k)$$

- maximize the product of probabilities of given samples
- equivalent to maximizing log-likelihood function

Learning: Gradient Ascent

$$\nabla_W \mathcal{L}[\mathcal{S}](\vartheta) = \frac{1}{s}\sum_{k=1}^{s} \mathbb{E}_\vartheta\left[\mathcal{V}\mathcal{H}^{\mathrm{T}} \middle| \mathcal{S}_k\right] - \mathbb{E}_\vartheta\left[\mathcal{V}\mathcal{H}^{\mathrm{T}}\right]$$

- use stochastic gradient ascent with minibatches

Learning: Gradient Ascent

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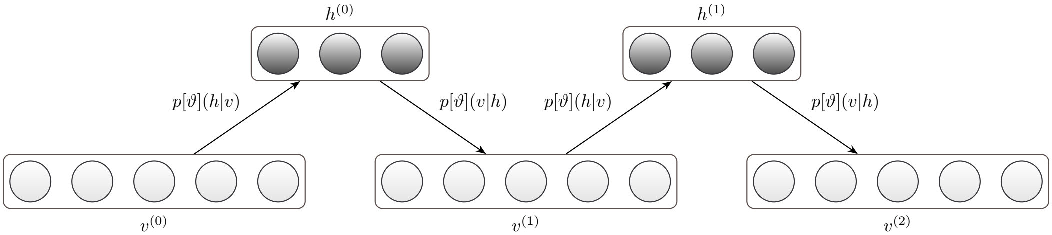
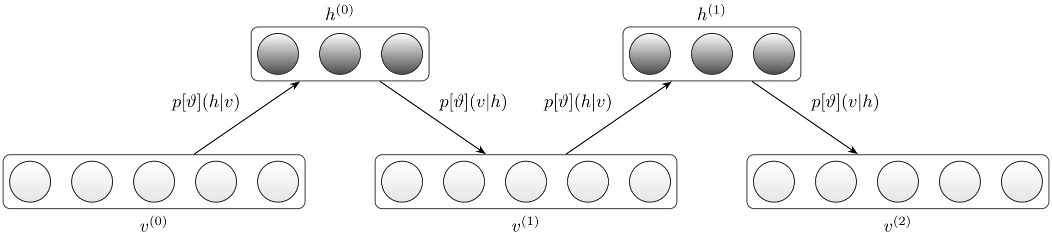
- use stochastic gradient ascent with minibatches
- evaluating the gradient introduces problems

Learning: Gibbs Sampling



- to estimate $\mathbb{E}_{\vartheta}\left[\mathcal{V}\mathcal{H}^{\mathrm{T}}\right]$ perform Gibbs sampling

Learning: Gibbs Sampling



- to estimate $\mathbb{E}_\vartheta\left[\mathcal{V}\mathcal{H}^{\mathrm{T}}\right]$ perform Gibbs sampling
- slow because it has to reach equilibrium

Learning: Contrastive Divergence

- abort Gibbs Sampling after $v^{(k)}$ and $h^{(k)}$ are computed

Learning: Contrastive Divergence

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Learning: Contrastive Divergence

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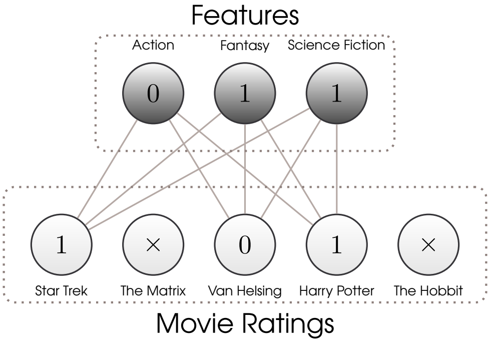
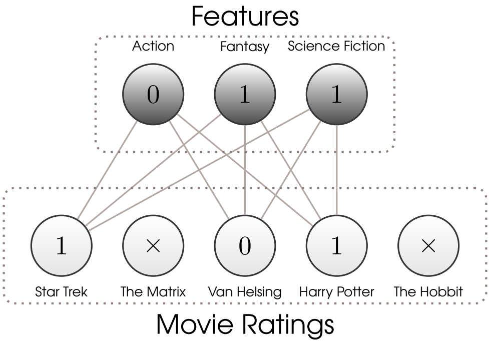
$$\mathbb{E}_{\vartheta}\left[\mathcal{V}\mathcal{H}^{\mathrm{T}}\right] \approx v^{(k)} {h^{(k)}}^{\mathrm{T}}$$

Learning: Example

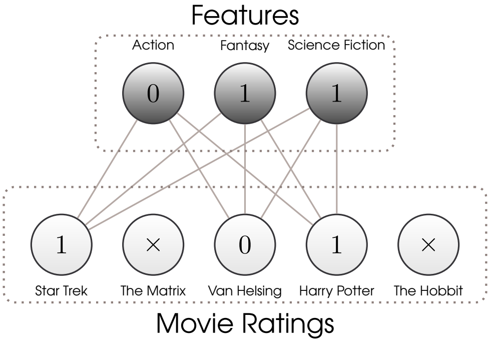


Learning: Example



- one RBM for every user with connections for rated movies

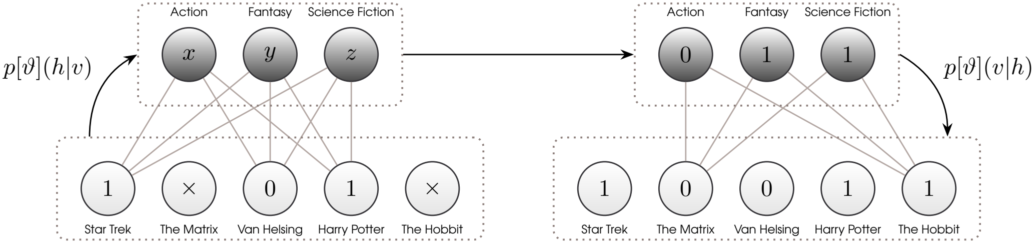
Learning: Example



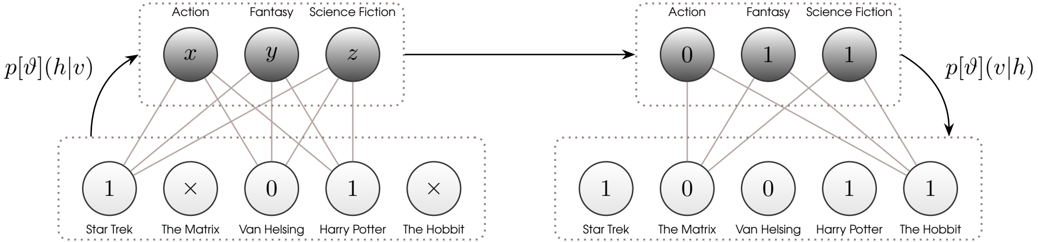
- one RBM for every user with connections for rated movies
- weights and biases off all RBM are tied together

Inference

Inference: Example

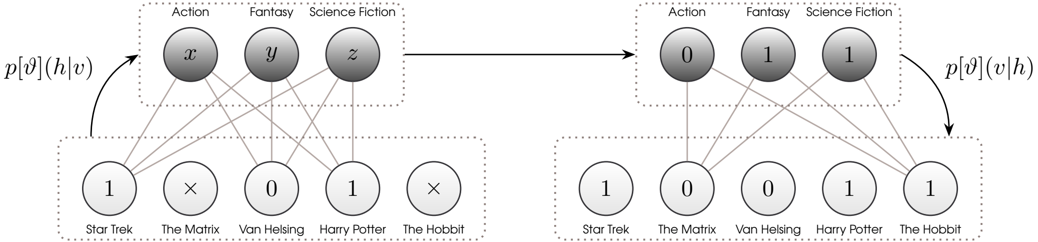


Inference: Example



- compute hidden values only for rated movies

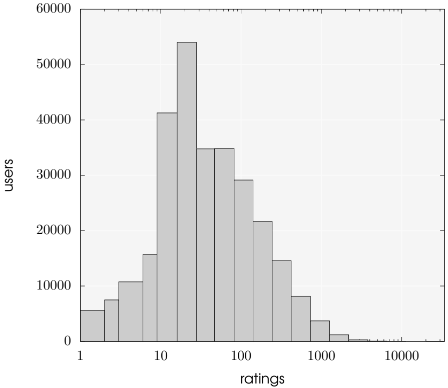
Inference: Example

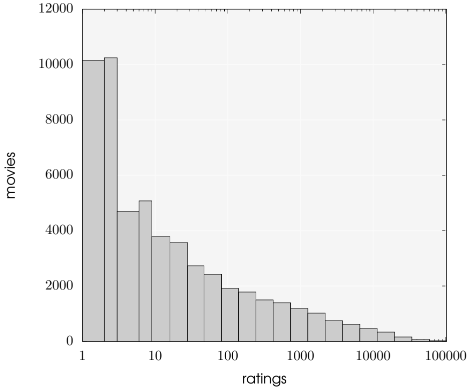


- compute hidden values only for rated movies
- compute visible values of unrated movies based on hidden values

Results

Results: MovieLens Dataset by GroupLens





- $\sim$ 27,000,000 ratings, $\sim$ 58,000 movies, $\sim$ 280,000 users
- current implementation not completely tested
  ($\sim 80\%$ correct predictions)

Going Further

Going Further: Tweak the Learning

- Contrastive Divergence Variants
- Momentum
- Weight Decay
- Different types of units

Going Further: Applications

- language modeling and document retrieval
- classification
- reducing dimensionality of data

References

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