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Comparison of Particle-In-Cell and Fokker-Planck Methods as Applied to the Modeling of Auxiliary-Heated Mirror Plasmas

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1 Introduction

The transport and confinement of charged particles in an auxiliary-heated mirror plasma is modeled via the bounce-averaged Fokker-Planck (F-P) code SMOKE [2] and the direct-implicit particle-in-cell (PIC) code TESS [1]. The test case studied is that of a tandem mirror end plug which is heated by the injection of ECRH at both the fundamental and second harmonic of the electron gyrofrequency ($\omega_{RF,1} = \omega_{ce}$ and $\omega_{RF,2} = 2\omega_{ce}$). Figure 1 shows the magnetic field and potential profiles that are prescribed for the test case. Both electron-electron and electron-ion collisions are included. Each code employs a relativistic description of the electron dynamics in one spatial dimension. Each code follows the system to equilibrium, where the loss of particles (resulting from collisional detrapping of the loss-cone distribution) balances the trapping of particles (resulting from the preferential increase in the magnetic moment of the particle due to the resonant absorption of RF wave energy).

The modeling comparison is divided into three sections. The first entails benchmarking the

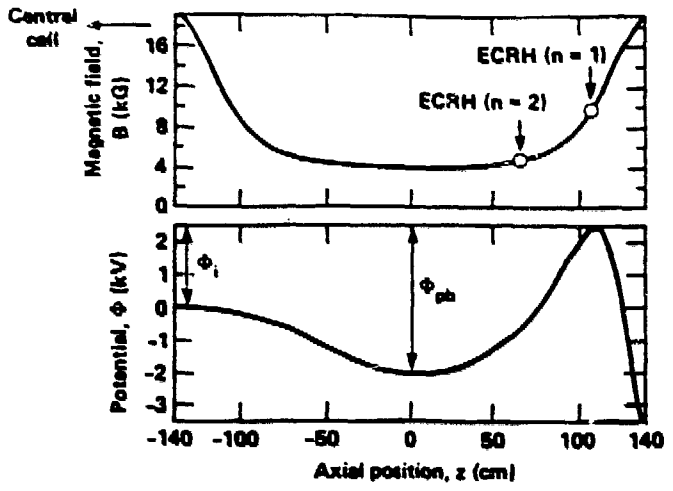

Figure 1: Magnetic field and potential profiles in the end plug of a tandem mirror plasma. The central cell is to the left, and the end wall is to the right.

physics results from the PIC code against those from the F-P code. The test case determines the confinement and transport of electrons *only*. The magnetic and electrostatic fields are prescribed quantities. Comparison is made of the electron velocity-space density contours at different axial locations, as well as the electron density and kinetic energy profiles. In order to determine the effect of particle counting statistics on these quantities, the PIC code is run with varying numbers of simulation electrons. The PIC code is also run including the ions, and a self-consistent calculation of the electrostatic potential. This last run allows us to determine the accuracy of the assumed potential profile used in the other runs. Next, a computational cost analysis is performed on each code. In addition to comparing the total run time for each of the codes, various code dependent costs are provided, such as the cost to push a particle each time step in the PIC code and the cost to "invert" the phase-space matrices in the F-P code. Finally, the advantages and disadvantages of each code are discussed in the context of the chosen test case, including a comparison of the effort involved in setting up the input deck for each code.

2 The Multiregion Bounce-Averaged Fokker-Planck Code

The SMOKE code determines the evolution of a particle distribution function f resulting from Coulomb collisional and RF-induced velocity space diffusion. The code numerically solves the

relativistic Fokker-Planck kinetic equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{\partial}{\partial \mathbf{p}} \cdot \left[q \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) f + \mathbf{\Gamma}_{coll} + \mathbf{\Gamma}_{rf} \right] = 0, \tag{1}$$

where t is the time, $\mathbf{x}$ is the position, $\mathbf{v}$ is the velocity, $\mathbf{p} \equiv \gamma m \mathbf{v}$ is the momentum, $\gamma \equiv (1 - v^2/c^2)^{-1/2}$ is the relativistic factor, m is the rest mass, c is the speed of light, $\mathbf{E}$ and $\mathbf{B}$ are of the electric and magnetic fields (static and RF components), and $\mathbf{\Gamma}_{coll}$ and $\mathbf{\Gamma}_{rf}$ are the fluxes arising from Coulomb collisional and RF wave interactions respectively. Since the bounce time of particles in the mirror (τ_b) is much smaller than the collisional or RF diffusion times (τ_{coll}, τ_{rf}), a bounce-averaged version of the F-P equation [3] is solved in (ε, μ) phase space

$$\tau_b \frac{\partial f}{\partial t} = \frac{\partial}{\partial \varepsilon} \left(D_{\varepsilon\varepsilon} \frac{\partial f}{\partial \varepsilon} + D_{\varepsilon\mu} \frac{\partial f}{\partial \mu} + D_\varepsilon f \right) + \frac{\partial}{\partial \mu} \left(D_{\mu\varepsilon} \frac{\partial f}{\partial \varepsilon} + D_{\mu\mu} \frac{\partial f}{\partial \mu} + D_\mu f \right), \tag{2}$$

where $\varepsilon \equiv (\gamma - 1)mc^2 + q\Phi$ is the total energy, $\mu \equiv p_\perp^2 / 2mB_0$ is the magnetic moment and $\tau_b \equiv \oint dz / |v_\parallel|$ is the particle bounce time. Each of the coefficient $D_{\alpha\beta}$ have contributions from Coulomb collisional and RF wave interactions. The quasilinear model of Bernstein and Baxter [3] is used to model the RF-induced diffusion.

The bounce-averaged F-P equation (2) is solved for the various regions (trapped-particle populations) via a two-step process. First, the (ε, μ) phase space is mapped into a rectangular (Cartesian coordinate) region (x, y). Second, a Galerkin finite-element method is used to solve (2) in the (x, y) phase space. (The same procedure is used to solve Poisson's equation for the Rosenbluth potentials.)

3 The Direct-Implicit Particle-In-Cell Code

The TESS code computes the trajectories of individual charged particles in either a prescribed or self-consistent electrostatic potential, and a prescribed magnetic field. The implicit nature of the code allows one to simulate long-wavelength, low-frequency phenomena, without having to resolve high-frequency effects, such as electron plasma oscillations. A relativistic guiding center formulation of the equations of motion in one spatial dimension ($\hat{z}$) is used

$$\begin{aligned} \frac{dz}{dt} &= \frac{p_z}{\gamma m} \\ \frac{dp_z}{dt} &= qE_z - \frac{\mu}{\gamma} \nabla B_z + \frac{dp_z}{dt}\Big)_{coll} + \frac{dp_z}{dt}\Big)_{rf} \\ \frac{d\mu}{dt} &= \frac{d\mu}{dt}\Big)_{coll} + \frac{d\mu}{dt}\Big)_{rf}. \end{aligned} \tag{3}$$

In general, the code simulates the transport and confinement of both ions and electrons. The self-consistent ambipolar potential is then calculated via the direct-implicit form of Poisson's equation

$$-\nabla\cdot\left(1+\sum_s \chi_s\right)\nabla\Phi(z) = 4\pi e\left(Zn_i(z)-n_e(z)\right), \tag{4}$$

where the summation is over species s (ions and electrons), n_s is the free-streaming or explicit density of species s and χ_s is the implicit susceptibility which accounts for the implicit correction to the free-streaming density. The equations of motion (3) and modified Poisson equation (4) are solved via standard finite difference techniques. A self-consistent, relativistic, Monte Carlo binary collision model [4] and a quasilinear RF diffusion model [5] (after Bernstein and Baxter [3], as implemented by Rognlien [6]) are also included.

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