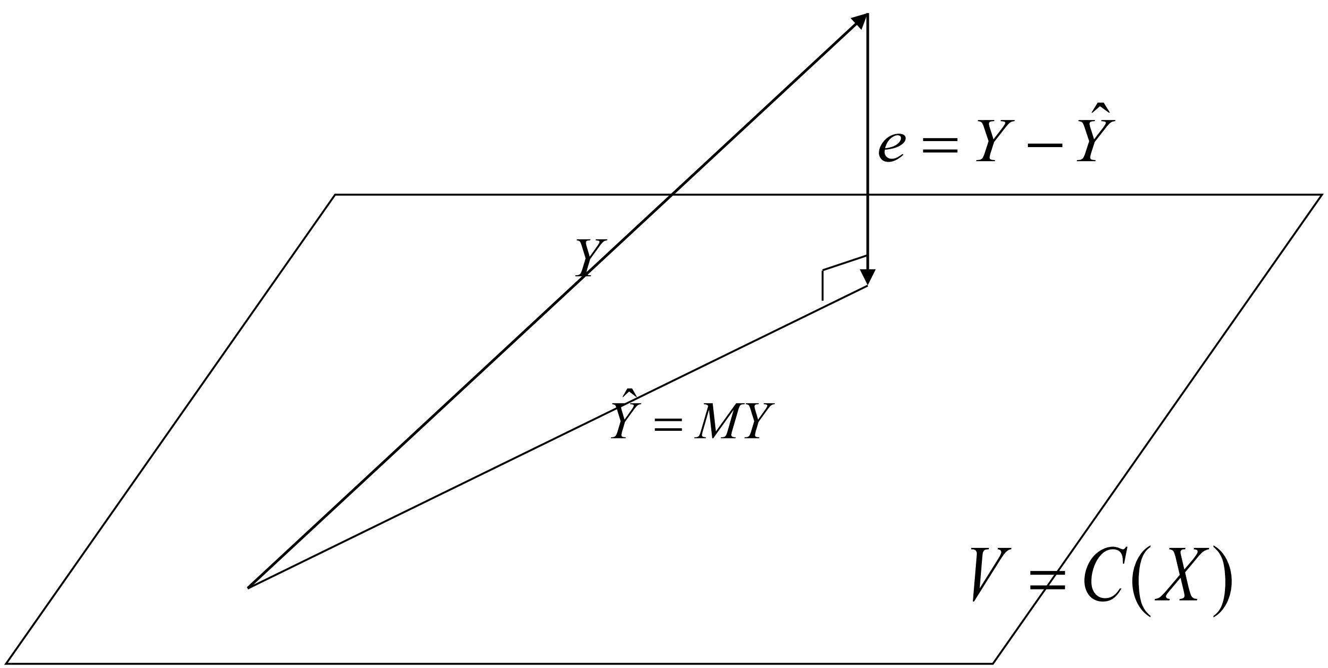
Orthogonal Projection of Y onto $V = C(X)$

$V = C(X) =$ Space spanned by columns of X

$\hat{Y} = MY =$ Orthogonal Projection of Y onto C(X)

$M = X(X^T X)^{-} X^T$

$e = Y - \hat{Y} =$ Residual Vector



Orthogonal Projection of Y onto $C(X_0) \subset C(X)$

$\hat{Y} = MY =$ Orthogonal Projection of Y onto C(X)

$\hat{Y}_0 = M_0 Y =$ Orthogonal Projection of Y onto C(X_0)

$M = X(X^T X)^- X^T$ and $M_0 = X_0(X_0^T X_0)^- X_0^T$

$e = Y - \hat{Y} = (I - M)Y$ and $e_0 = Y - \hat{Y}_0 = (I - M_0)Y$

$v = \hat{Y} - \hat{Y}_0 = (M - M_0)Y = (I - M_0)Y - (I - M)Y = e_0 - e \quad \in C(M - M_0) = C(X_0)^{\perp}{}_{C(X)}$