STA6171: Statistical Computing for DS 1

Combinatorial Optimization

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Introduction to Combinatorial Optimization

Optimization for Combination.

- Let us assume that we are seeking the maximum of $f(\theta)$ w.r.t $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_p)$, where $\boldsymbol{\theta} = \boldsymbol{\Theta}$ and $\boldsymbol{\Theta}$ consists of N elements for a finite positive integer N.

$\theta_1, \cdots, \theta_p$ $\quad l(\theta_1, \cdots, \theta_p)$

$\theta_1, \theta_2 \rightarrow$ maximum $\Rightarrow \theta_2$ fix.

fix

- Example: Profile Likelihood
  - In statistical applications, it is common for a likelihood function to depend on configuration parameters that describe the form of a statistical model and for which there are many discrete choices, as well as a small number of other parameters that could be easily optimized if the best configuration were known.
  - View $f(\boldsymbol{\theta})$ as the log profile likelihood of a configuration, $\boldsymbol{\theta}$, that is, the highest likelihood attainable using that configuration.

Salesman Traveling Problem

(Eg) 9 cities

Seoul Chuncheon Kangleung Chongju Daejeon Jeonju Daegu Kwangju Pusan

Travel all 9 cities with shortest distance (Order)

=> Make visiting orders of cities

make order

S → Chun → Kangl → Chang

S → Dae → Jeon → Kwang →

9! = 362,880. 9×8

from all possible orders find one order that has the shortest distance

=> Combinatorial Optimization

=> Not easy to find global optimum.

=> Find several local optimums. Among them, find optimal solution

=> Forget to find a global optimum,

Regression with variable selection

$\Rightarrow$ small n large p

$\theta_1, \cdots, \theta_p$

$x_1, \cdots, x_p$

We need to ~~implement~~ consider variable selection

Variable Selection include in model 1
exclude in model 0
binary

Total possible models : $2^p - 1$

find the best model from $2^p - 1$ cases

$2^{1000} - 1$

need a combinatorial optimization

Profile likelihood Case

$\theta_1, \underbrace{\theta_2, \cdots, \theta_{10}}_{\text{fix}}$ we ~~θ_2~~ choose 5 points from $\underbrace{\theta_2, \cdots, \theta_{10}}_{9}$

possible combination 5^9

need to find that maximize likelihood with θ_1

Hard Optimization Problems

- Hard optimization problems are generally combinatorial in nature.
- p items may be combined or sequenced in a very large number of ways. and each choice corresponds to one element in the space of possible solutions.

  order
- Maximization requires a search of this very large space.
- Example: Traveling Salesman Problem
  - Suppose sales man must visit each of p cities exactly once and return to his point of origin using the shortest total travel distance.
  - Seek to minimize the total travel distance over all possible routes.
  - If the distance between two cities does not depend on the direction traveled between them, then there are $(p-1)!/2$ possible routes.
  - The difficulty of optimization depends on p.