RCS 103

Introduction

Lecture 1

1. Numbers
2. Numbers and real line
3. Counting, Natural, Integers, Real numbers,
4. Rational Numbers
5. Intervals
6. Inequalities and Equations
7. Absolute Value

Definitions

Number:

A number is a mathematical object used to count, measure and label

There exist different type or categories of numbers

Namely: *Integers*
Counting
Natural
Rational/Irrational
Real Numbers

Definitions

Integers:

Are whole number or a number that is not a fraction (something complete in itself).

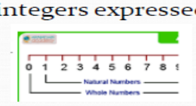
Definitions

Counting numbers:

Are positive integers expressed as 1, 2, 3, …

Natural numbers:

Are positive integers expressed as 0, 1, 2, 3, …



Definitions

Rational numbers:

A **rational number** is any **number** that can be expressed as the quotient or fraction p/q of two integers, a numerator p and a non-zero denominator q. Since q may be equal to 1, every integer is a **rational number.**

Definitions

Irrational numbers:

An **irrational number** cannot be expressed as a ratio between two **numbers** and it cannot be written as a simple fraction because there is not a finite **number** of **numbers** when written as a decimal. Instead, the **numbers** in the decimal would go on forever, without repeating.

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Definitions

Real Numbers:

The **real numbers** is a number that can be expressed as a/b where b is not zero

They include all the rational **numbers**, such as the integer −5 and the fraction 4/3, and all the irrational **numbers**, such as √2

Included within the irrationals are the transcendental **numbers**, such as π (3.14159265...).

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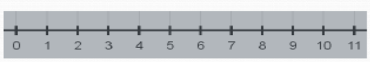
Definitions

Real Numbers:

The **real numbers** are also said to be numbers expressed on a number line



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Interval

An Interval :

A subset of the real line (number line) is called an interval if it contains at least two numbers and also contains all real numbers between any two of its elements

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Interval

If a and b are real numbers and $a < b$, we often refer to

(i) the **open interval** from a to b, denoted by (a, b), consisting of all real numbers x satisfying $a < x < b$.

(ii) the **closed interval** from a to b, denoted by $[a, b]$, consisting of all real numbers x satisfying $a \le x \le b$.

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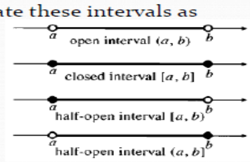
Interval

(iii) the **half-open interval** $[a, b)$, consisting of all real numbers x satisfying $a \le x < b$.

(iv) the **half-open interval** $(a, b]$, consisting of all real numbers x satisfying $a < x \le b$.

We illustrate these intervals as



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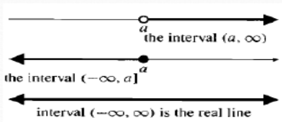
Interval

Note we could also have these infinite intervals:



Inequalities

The relation between two expressions that are not equal, employing a sign such as ≠ "not equal to," > "greater than," or < "less than."

Example:

Solve the following inequalities. Express the solution sets in terms of intervals and graph them.

(a) $2x - 1 > x + 3$

(b) $-\frac{x}{3} \geq 2x - 1$

(c) $\frac{2}{x-1} \geq 5$

Inequalities

Solution:

(a) $2x - 1 > x + 3$	Add 1 to both sides.
$2x > x + 4$	Subtract x from both sides.
$x > 4$	The solution set is the interval $(4, \infty)$.

Inequalities

Solution:

(b) $-\frac{x}{3} \geq 2x - 1$	Multiply both sides by -3.
$x \leq -6x + 3$	Add $6x$ to both sides.
$7x \leq 3$	Divide both sides by 7.
$x \leq \frac{3}{7}$	The solution set is the interval $(-\infty, 3/7]$.

Inequalities

Solution:

(c) We transpose the 5 to the left side and simplify to rewrite the given inequality in an equivalent form:

$$\frac{2}{x-1} - 5 \geq 0 \iff \frac{2 - 5(x-1)}{x-1} \geq 0 \iff \frac{7-5x}{x-1} \geq 0.$$

The fraction $\frac{7-5x}{x-1}$ is undefined at $x = 1$ and is 0 at $x = 7/5$. Between these numbers it is positive if the numerator and denominator have the same sign, and negative if they have opposite sign. It is easiest to organize this sign information in a chart:

Inequalities

x		1		7/5	
$7 - 5x$	+	+	+	0	−
$x - 1$	−	0	+	+	+
$(7 - 5x)/(x - 1)$	−	undef	+	0	−

Thus the solution set of the given inequality is the interval $(1, 7/5]$.

Inequalities

Exercise: 1

Solve the following system of Inequalities

(a) $3 \leq 2x + 1 \leq 5$

(b) $3x - 1 < 5x + 3 \leq 2x + 15.$

Exercise: 2

Solve the following system of Quadratic Inequalities

(a) $x^2 - 5x + 6 < 0$ (b) $2x^2 + 1 > 4x.$

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Absolute value

The **absolute value**, or **magnitude**, of a number x, denoted $|x|$ (read "the absolute value of x"), is defined by the formula

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

It is important to remember that $\sqrt{a^2} = |a|$. Do not write $\sqrt{a^2} = a$ unless you already know that $a \geq 0$.

Absolute value

The absolute value function has the following properties:

1. $|-a| = |a|$. A number and its negative have the same absolute value.
2. $|ab| = |a||b|$ and $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$. The absolute value of a product (or quotient) of two numbers is the product (or quotient) of their absolute values.
3. $|a \pm b| \leq |a| + |b|$ (the **triangle inequality**). The absolute value of a sum of or difference between numbers is less than or equal to the sum of their absolute values.

Absolute value

Example 1

Solve: (a) $|2x + 5| = 3$ (b) $|3x - 2| \leq 1.$

Absolute value

Solution (a)

$|2x + 5| = 3 \iff 2x + 5 = \pm 3.$

Thus, either $2x = -3 - 5 = -8$ or

$2x = 3 - 5 = -2.$

The solutions are $x = -4$ and $x = -1$.

Absolute value

Solution (b)

$|3x - 2| \leq 1 \iff -1 \leq 3x - 2 \leq 1.$

We solve this pair of inequalities:

$$\left\{\begin{array}{r} -1 \leq 3x - 2 \\ -1 + 2 \leq 3x \\ 1/3 \leq x \end{array}\right\} \quad \text{and} \quad \left\{\begin{array}{l} 3x - 2 \leq 1 \\ 3x \leq 1 + 2 \\ x \leq 1 \end{array}\right\}.$$

Thus the solutions lie in the interval $[1/3, 1]$.

Absolute value

Exercise 1

1. Solve the equation $|x + 1| = |x - 3|$.

2. What values of x satisfy the inequality $\left|5 - \frac{2}{x}\right| < 3$?

END