AE-410 GPS (Assignment-1 Fundamentals of GPS)

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In [ ]:
```
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```

Some important Relations / Properties

- $C_i(\eta - \tau)$ is the PRN code (similar to a vector/array with 1023 dimensions/elements) for $i-th$ satellite, delayed by τ chips
- $C_i(\eta) = C_i(\eta + 1023)$ i.e. PRN codes are periodic with period = 1023 chips = 1 millisecond
- $\frac{C_i(\eta) \oplus C_j(\eta-\tau)}{1023} = 1$ {iff $i = j$ and $\tau = 0$, = 0 otherwise}
- $\oplus$ in the above point denotes correlation, i.e. element wise multiplication:
  - $C_i(\eta) \oplus C_j(\eta - \tau) = \sum_{r=0}^{r=1023} C_i(\eta)[r] \cdot C_j(\eta - \tau)[r]$
  - $C_i(\eta)[r]$ is a scalar value having 0 or 1 value

References / Resources

- Github repository having 32 GPS PRN codes https://github.com/danipascual/GNSS-matlab.git

In [ ]:
```
# importing necessary libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy
```

In [ ]:
```
# GPS PRN Extraction
codesFile = scipy.io.loadmat("randomData/codes_L1CA.mat")
codes = np.array(codesFile['codes_L1CA'])
```

In [ ]:
```
# shape of codes array
print(codes.shape)
```

```
(1023, 32)
```

Proof of $\frac{C_i(\eta) \oplus C_j(\eta-\tau)}{1023} = 1$ {iff $i = j$ and $\tau = 0$, = 0 otherwise}

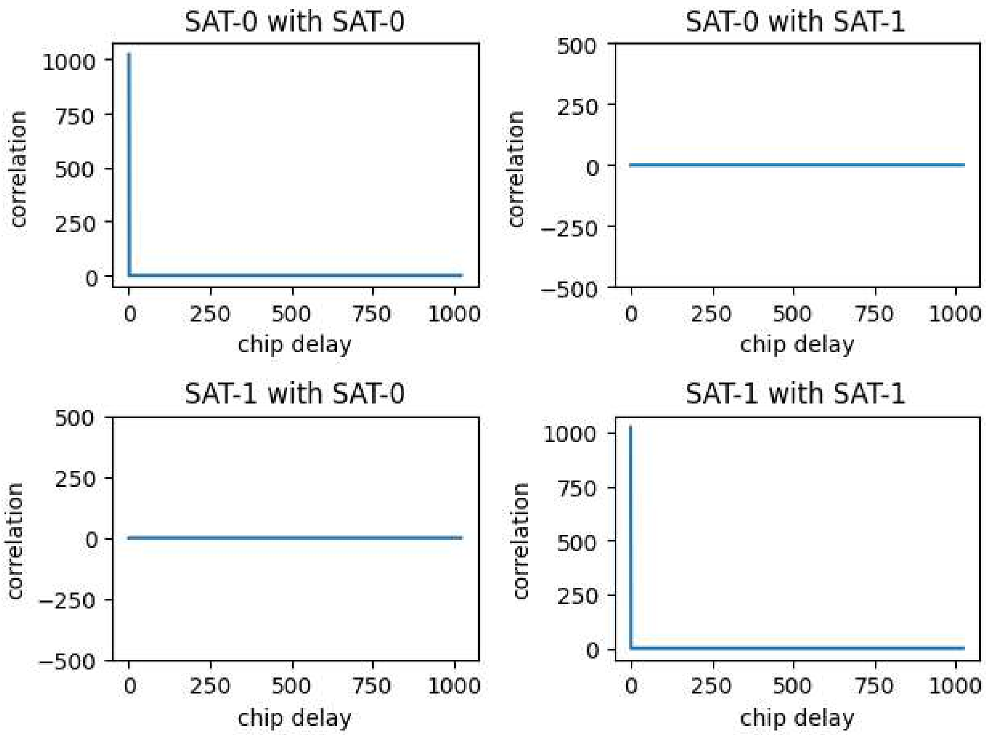
In [ ]:
```
corrArr = np.zeros((2,2,1023))
for i in range(0,2):
    for k in range(0,2):
        for j in range(0,len(corrArr)):
            corrArr[i][k][j]=  np.correlate(codes[:,i],np.roll(codes[:,k],-j),'valid')
```

```
C:\Users\lyric\AppData\Local\Temp\ipykernel_15676\3946328126.py:5: DeprecationWarning: Conversion of an array with ndim >
0 to a scalar is deprecated, and will error in future. Ensure you extract a single element from your array before performi
ng this operation. (Deprecated NumPy 1.25.)
  corrArr[i][k][j]=  np.correlate(codes[:,i],np.roll(codes[:,k],-j),'valid')
```

In [ ]:
```
%matplotlib inline
plt.subplot(2,2,1)
plt.plot(corrArr[0,0,:])
plt.ylabel('correlation')
plt.xlabel('chip delay')
plt.title('SAT-0 with SAT-0')
plt.subplot(2,2,2)
plt.plot(corrArr[0,1,:])
plt.ylabel('correlation')
plt.xlabel('chip delay')
plt.yticks(np.linspace(-500,500,5))
plt.title('SAT-0 with SAT-1')
plt.subplot(2,2,3)
plt.plot(corrArr[1,0,:])
plt.ylabel('correlation')
plt.xlabel('chip delay')
plt.yticks(np.linspace(-500,500,5))
plt.title('SAT-1 with SAT-0')
plt.subplot(2,2,4)
plt.plot(corrArr[1,1,:])
```

```
plt.ylabel('correlation')
plt.xlabel('chip delay')
plt.title('SAT-1 with SAT-1')

plt.tight_layout()
```



It is clear that the correlation is high only when there is 0 delay and same satellite PRN

Q1

Write a MATLAB/Python/C/C++ program to compute circular autocorrelation of PRN 8 with a delayed PRN code by 200 chips and plot the results

Soln:

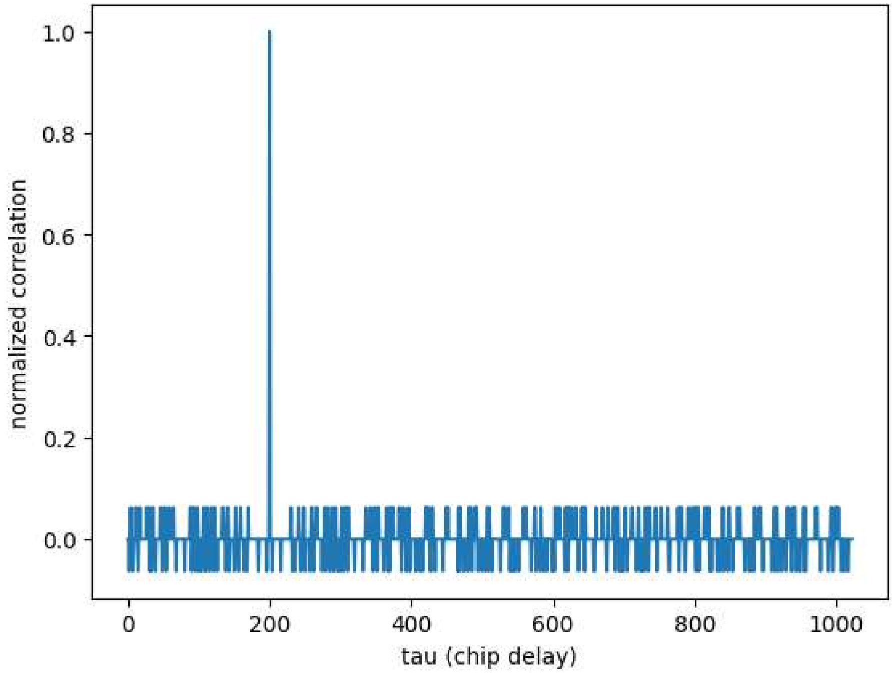
$$\frac{C_8(\eta)\oplus C_8(\eta-200)}{1023}$$

In [ ]:
```
np.correlate(codes[:,8],np.roll(codes[:,8],-200),'valid')/1023
```

Out[ ]:
```
array([-0.00097752])
```

In [ ]:
```
# plot for different delays of main PRN
tempval = [np.correlate(np.roll(codes[:,8],-i),np.roll(codes[:,8],-200),'valid')/1023 for i in range(0,1023)]
plt.plot(tempval)
plt.xlabel("tau (chip delay)")
plt.ylabel("normalized correlation")
print("maximum value comes when two overlap fully, which is at " + str(np.argmax(tempval)))
```

```
maximum value comes when two overlap fully, which is at 200
```



Q2

Write a MATLAB/ Python/C/C++ program to compute circular cross correlation of PRN 8 with a delayed copy of a PRN 16 by 900 chips and plot the results.

Soln:

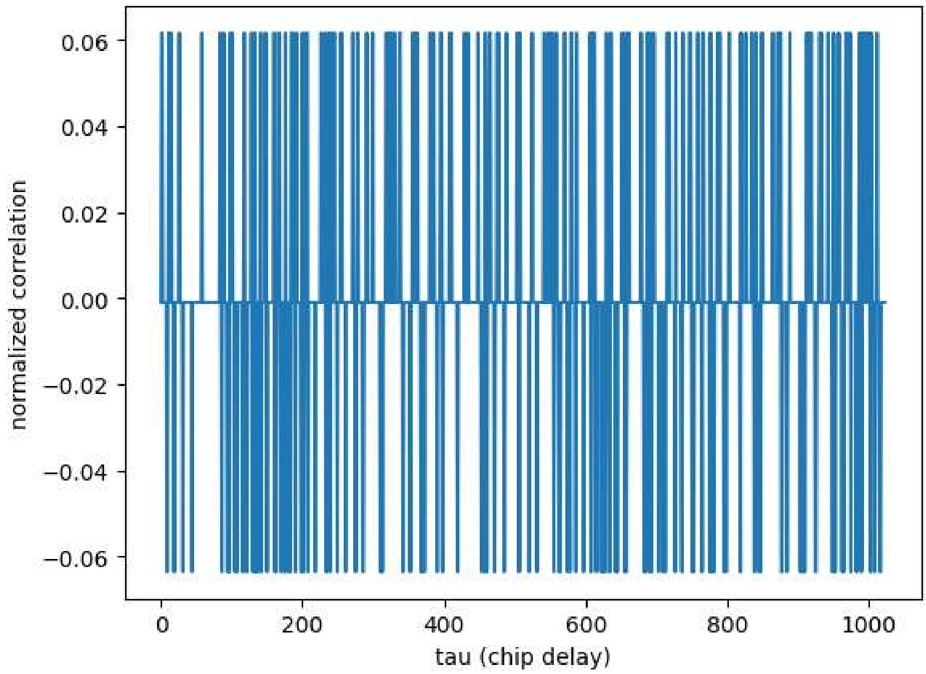
$\frac{C_8(\eta) \oplus C_{16}(\eta - 900)}{1023}$

In [ ]:
```
np.correlate(codes[:,8],np.roll(codes[:,16],-900),'valid')/1023
```

Out[ ]:
```
array([-0.00097752])
```

In [ ]:
```
# plot for different delays in main PRN
tempval = [np.correlate(np.roll(codes[:,8],-i),np.roll(codes[:,16],-900),'valid')/1023 for i in range(0,1023)]
plt.plot(tempval)
plt.xlabel("tau (chip delay)")
plt.ylabel("normalized correlation")
print("maximum value never comes as both PRN are different, can be clearly seen as max(abs(correlation)) = "+str(max(np.a
```

```
maximum value never comes as both PRN are different, can be clearly seen as max(abs(correlation)) = [0.06353861]
```



Q3

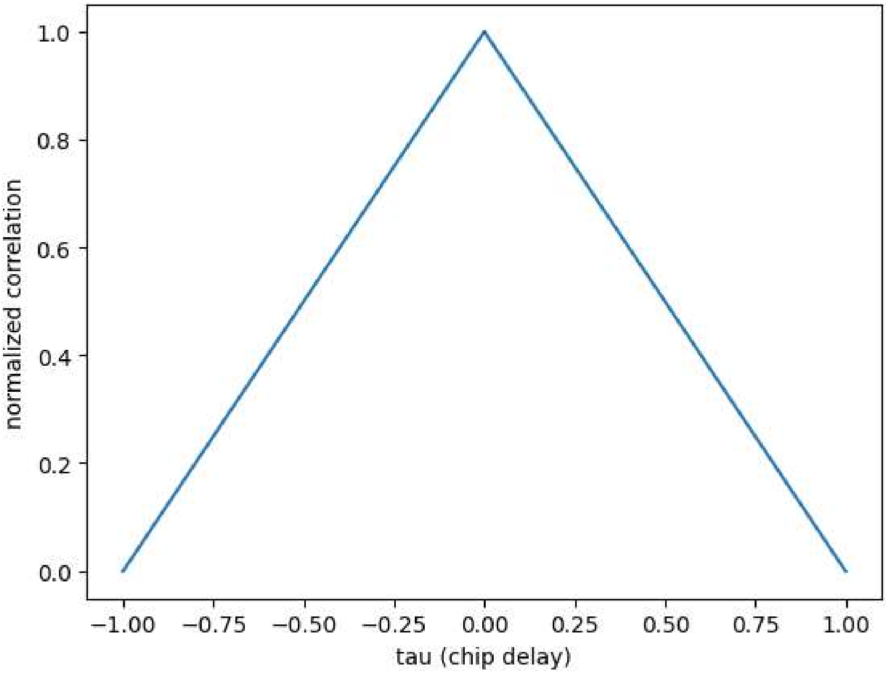
Write a MATLAB/Python/C/C++ program to compute autocorrelation of PRN 8 with a delayed PRN code by -1, 0, and 1 chip, respectively and plot the correlation with the delay chip.

Soln:

- $\frac{C_8(\eta) \oplus C_8(\eta+1)}{1023}$
- $\frac{C_8(\eta) \oplus C_8(\eta-0)}{1023}$
- $\frac{C_8(\eta) \oplus C_8(\eta-1)}{1023}$

In [ ]:
```
cor = np.zeros((3,1))
cor[0] = np.correlate(codes[:,8],np.roll(codes[:,8],1),'valid')/1023
cor[1] = np.correlate(codes[:,8],np.roll(codes[:,8],0),'valid')/1023
cor[2] = np.correlate(codes[:,8],np.roll(codes[:,8],-1),'valid')/1023
plt.plot(np.array([-1,0,1]),cor)
plt.xlabel("tau (chip delay)")
plt.ylabel("normalized correlation")
```

Out[ ]:
```
Text(0, 0.5, 'normalized correlation')
```



Q4

Write a MATLAB/ Python/C/C++ program to compute circular autocorrelation of PRN 8 with a noisy PRN code delayed by 200 chips and plot the results. Assume the noise is white Gaussian additive and generated with mean zero and standard deviation of 4.

Soln:

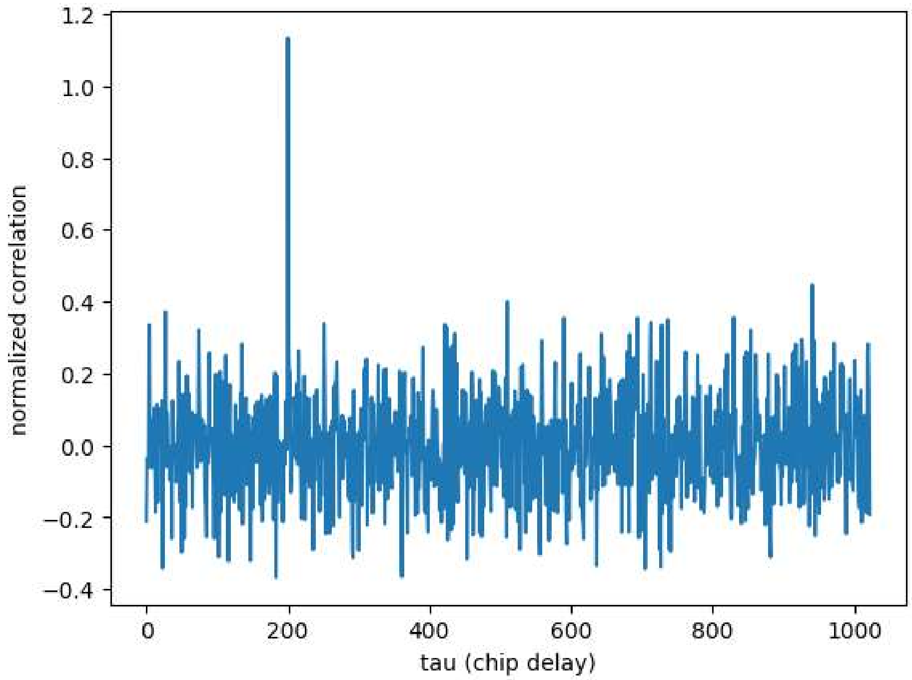
- $PRN_{noisy} = (C_8(\eta - 200) + N(0,4))$
- $C_8(\eta) \oplus PRN_{noisy}$

In [ ]:
```
noisy = (np.array(codes[:,8])+ (np.random.normal(0,4,1023)))
np.correlate(codes[:,8],np.roll(noisy,-200),'valid')/1023
```

Out[ ]:
```
array([-0.21093067])
```

In [ ]:
```
# plot for different delays of main PRN
tempval = [np.correlate(np.roll(codes[:,8],-i),np.roll(noisy,-200),'valid')/1023 for i in range(0,1023)]
plt.plot(tempval)
plt.xlabel("tau (chip delay)")
plt.ylabel("normalized correlation")
print("here we can see that due to noise the non-peak values have a higher range")
```

```
here we can see that due to noise the non-peak values have a higher range
```



Q5

Write a MATLAB/ Python/C/C++ program to implement serial search/parallel code phase search acquisition algorithm. Identify the satellites (PRN IDs), carrier frequency, and code phase using the acquisition algorithm in the data file provided in the Google drive

Sol.

- we know the signal received at the antenna is:

$s(t) = \sqrt{2P}D(t-\tau)C(t-\tau)cos((w_L + w_D)t + \theta)$ where : - P is power - τ is delay ($\tau = \tau_{int} + \tau_{frac}$) - w_L is L-5 band frequency and w_D is doppler frequency

- After RF front-end processing we get:

$I = \sqrt{2P_1}D(t-\tau)C(t-\tau)cos((w_D)t + \theta)$

$Q = \sqrt{2P_1}D(t-\tau)C(t-\tau)sin((w_D)t + \theta)$

where: - I is Inphase samples - Q is Quadrature samples

Note : slight abuse of notation is used as we defined C for discrete values while here we are considering it as continous, but this discrepancy will be resolved in program as we have input data as discrete samples

Now,

for serial search we need to get 3 parameters

- i the satellite number
- $\tau_{frac} \in [0, 1022]$
- $w_D \in [-20kHz : 500Hz : 20kHz]$

Steps

- Firstly, we will rewrite signal as $S = I + Qj = \sqrt{2P_1}D(t-\tau)C(t-\tau)e^{jw_d t}$

- Then, we will multiply S by $C(t-\hat{\tau})e^{-j\hat{w}_d t}$

- We will get:

$Corr = \sqrt{2P_1}D(t-\tau)C(t-\tau) \oplus C_i(t-\hat{\tau})e^{j(w_d - \hat{w}_d)t}$

it will attain it's maximum when :

$w_d - \hat{w}_d = 0$

$\tau = \hat{\tau}$

$i =$ satellite number of incoming signal

In [ ]:
```python
# Assumptions:
#   - Sampling frequency is Fs which is a multiple of 1.023 MHz
#   - Fs = n*1.023 MHz

def modPRN(PRN,n,tau):
    """
    function to get modified PRN for some sampling frequency & delay
    Inputs:
        - PRN is the PRN code for single satellite with 1023 length
        - n = SamplingFrequency/1.023e6
    Output:
        - PRN code for 1 millisecond with length = 1023*n
        - This PRN code is delayed by tau chips
    """

    # return np.roll(np.repeat(PRN,n),-tau)
    return np.repeat(np.roll(PRN,-tau),n)
```

In [ ]:
```python
# Checking modPRN
someArrForModPRN = np.array([1,2,3,4,5])
modPRN(someArrForModPRN,3,4)
```

Out[ ]:
```
array([5, 5, 5, 1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4])
```

In [ ]:
```python
wds = np.linspace(-20000,20000,41)*2*np.pi
def serialSearch(I, Q, knownPRNs, Fs, wds=wds):
    rcvdSignal = np.array(I+Q*1j)
    n = int(Fs/1.023e6)
    t = np.linspace(0,1,n*1023)*1e-3
    totalSatellites = knownPRNs.shape[1]
    corr = np.zeros((totalSatellites,len(wds),1023*n))
    for x in range(0,totalSatellites):
        for y in range(0,len(wds)):
            for z in range(0,1023*n):
                generatedSignal = modPRN(knownPRNs[:,x],n,z)*np.array(np.exp(-1j*wds[y]*t))
                corr[x,y,z] = np.abs(np.sum(generatedSignal*rcvdSignal))
    return corr
```

In [ ]:
```python
t = np.linspace(0,1,1023)*1e-3
```

In [ ]:
```python
sirPRNI = modPRN(codes[:,20],1,82)*np.cos(2*np.pi*675*t)+np.random.normal(0,4,1023)
sirPRNQ = modPRN(codes[:,20],1,82)*np.sin(2*np.pi*675*t)+np.random.normal(0,4,1023)
```

In [ ]:
```python
corr = serialSearch(sirPRNI,sirPRNQ,codes,1.023e6)
```

In [ ]:
```python
satNum,wdstar,taustar = np.unravel_index(corr.argmax(), corr.shape)
```

In [ ]:
```python
satNum
```

Out[ ]:
```
20
```

In [ ]:
```python
wds[wdstar]/(2*np.pi)
```

Out[ ]:
```
999.9999999999999
```

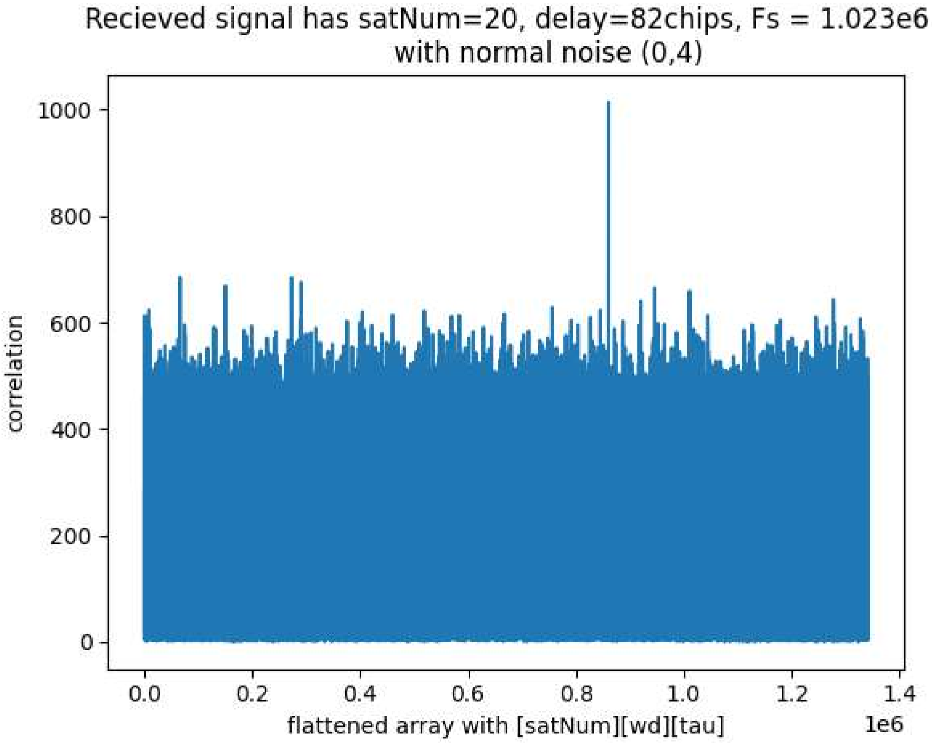
In [ ]:
```python
taustar
```

Out[ ]:
```
82
```

We can clearly see that we got the desired result

In [ ]:
```python
plt.plot(corr.flatten())
plt.ylabel("correlation")
plt.xlabel("flattened array with [satNum][wd][tau]")
plt.title("""Recieved signal has satNum=20, delay=82chips, Fs = 1.023e6
          with normal noise (0,4)""")
```

Out[ ]:
```
Text(0.5, 1.0, 'Recieved signal has satNum=20, delay=82chips, Fs = 1.023e6\n          with normal noise (0,4)')
```


Recieved signal has satNum=20, delay=82chips, Fs = 1.023e6
with normal noise (0,4)
1000
800
600
400
200
0
correlation
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1e6
flattened array with [satNum][wd][tau]

In [ ]:

In [ ]: