AE-410 GPS (Assignment-1 Fundamentals of GPS)

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In [ ]:
```
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```

Some important Relations / Properties

- $C_i(\eta - \tau)$ is the PRN code (similar to a vector/array with 1023 dimensions/elements) for $i - th$ satellite, delayed by τ chips
- $C_i(\eta) = C_i(\eta + 1023)$ i.e. PRN codes are periodic with period = 1023 chips = 1 millisecond
- $\frac{C_i(\eta) \oplus C_j(\eta - \tau)}{1023} = 1$ {iff $i = j$ and $\tau = 0$, = 0 otherwise}
- $\oplus$ in the above point denotes correlation, i.e. element wise multiplication:
  - $C_i(\eta) \oplus C_j(\eta - \tau) = \sum_{r=0}^{r=1023} C_i(\eta)[r] \cdot C_j(\eta - \tau)[r]$
  - $C_i(\eta)[r]$ is a scalar value having 0 or 1 value

References / Resources

- Github repository having 32 GPS PRN codes https://github.com/danipascual/GNSS-matlab.git

In [ ]:
```
# importing necessary libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy
```

In [ ]:
```
# GPS PRN Extraction
codesFile = scipy.io.loadmat("randomData/codes_L1CA.mat")
codes = np.array(codesFile['codes_L1CA'])
```

In [ ]:
```
# shape of codes array
print(codes.shape)
```

```
(1023, 32)
```

Proof of $\frac{C_i(\eta) \oplus C_j(\eta - \tau)}{1023} = 1$ {iff $i = j$ and $\tau = 0$, = 0 otherwise}

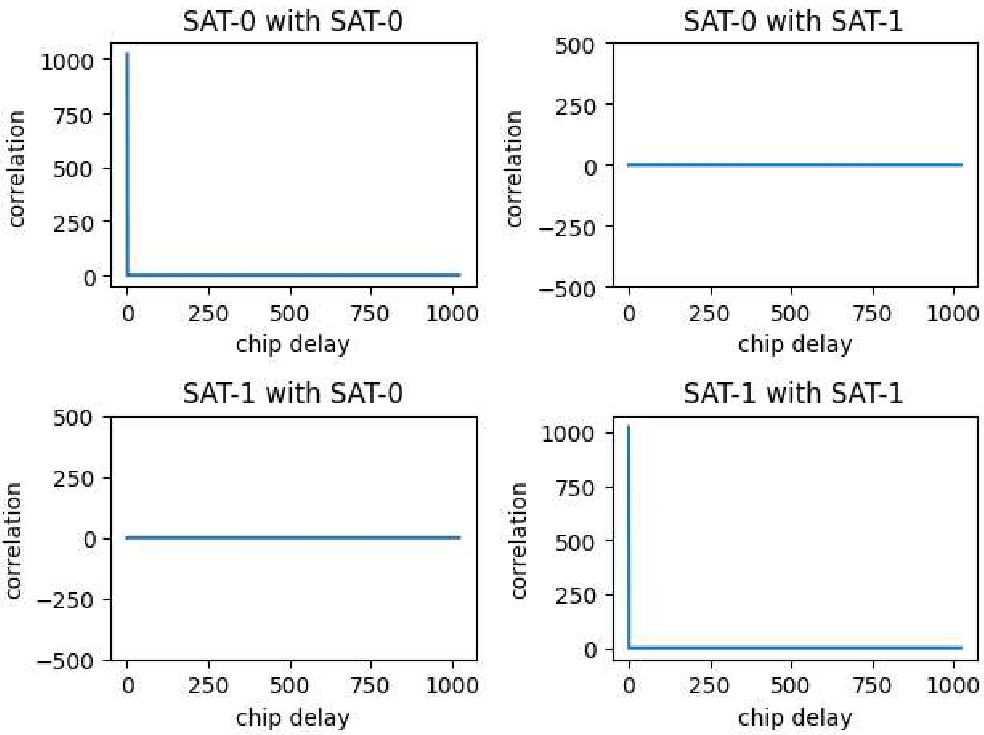
In [ ]:
```
corrArr = np.zeros((2,2,1023))
for i in range(0,2):
    for k in range(0,2):
        for j in range(0,len(corrArr)):
            corrArr[i][k][j]=  np.correlate(codes[:,i],np.roll(codes[:,k],-j),'valid')
```

```
C:\Users\lyric\AppData\Local\Temp\ipykernel_10380\3946328126.py:5: DeprecationWarning: Conversion of an array with ndim >
0 to a scalar is deprecated, and will error in future. Ensure you extract a single element from your array before performi
ng this operation. (Deprecated NumPy 1.25.)
  corrArr[i][k][j]=  np.correlate(codes[:,i],np.roll(codes[:,k],-j),'valid')
```

In [ ]:
```
%matplotlib inline
plt.subplot(2,2,1)
plt.plot(corrArr[0,0,:])
plt.ylabel('correlation')
plt.xlabel('chip delay')
plt.title('SAT-0 with SAT-0')
plt.subplot(2,2,2)
plt.plot(corrArr[0,1,:])
plt.ylabel('correlation')
plt.xlabel('chip delay')
plt.yticks(np.linspace(-500,500,5))
plt.title('SAT-0 with SAT-1')
plt.subplot(2,2,3)
plt.plot(corrArr[1,0,:])
plt.ylabel('correlation')
plt.xlabel('chip delay')
plt.yticks(np.linspace(-500,500,5))
plt.title('SAT-1 with SAT-0')
plt.subplot(2,2,4)
plt.plot(corrArr[1,1,:])
```

```python
plt.ylabel('correlation')
plt.xlabel('chip delay')
plt.title('SAT-1 with SAT-1')

plt.tight_layout()
```



It is clear that the correlation is high only when there is 0 delay and same satellite PRN

Q1

Write a MATLAB/Python/C/C++ program to compute circular autocorrelation of PRN 8 with a delayed PRN code by 200 chips and plot the results

Soln:

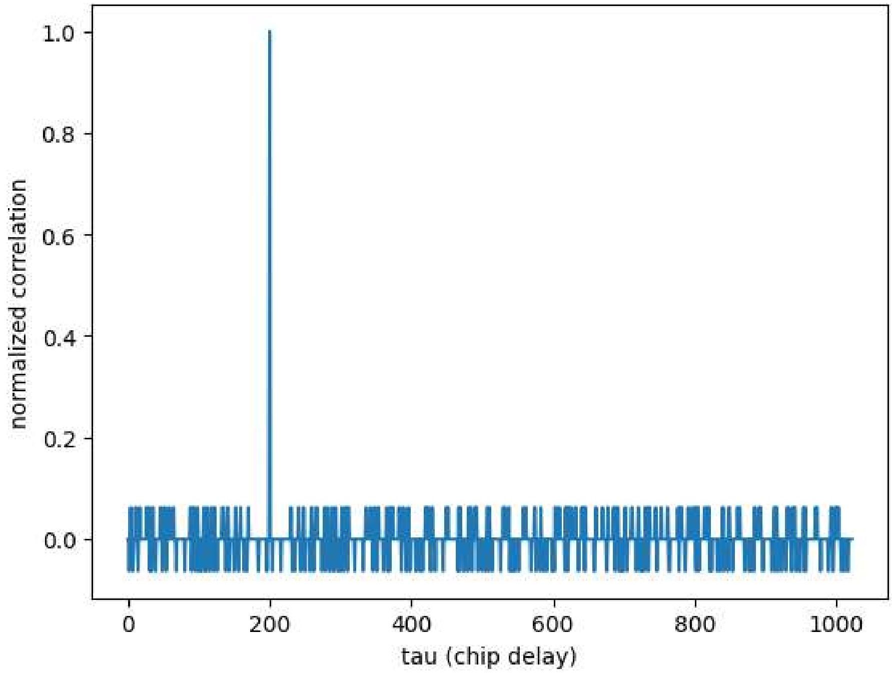
$\frac{C_8(\eta) \oplus C_8(\eta-200)}{1023}$

In [ ]:
```python
np.correlate(codes[:,8],np.roll(codes[:,8],-200),'valid')/1023
```

Out[ ]:
```
array([-0.00097752])
```

In [ ]:
```python
# plot for different delays of main PRN
tempval = [np.correlate(np.roll(codes[:,8],-i),np.roll(codes[:,8],-200),'valid')/1023 for i in range(0,1023)]
plt.plot(tempval)
plt.xlabel("tau (chip delay)")
plt.ylabel("normalized correlation")
print("maximum value comes when two overlap fully, which is at " + str(np.argmax(tempval)))
```

```
maximum value comes when two overlap fully, which is at 200
```



Q2

Write a MATLAB/ Python/C/C++ program to compute circular cross correlation of PRN 8 with a delayed copy of a PRN 16 by 900 chips and plot the results.

Soln:

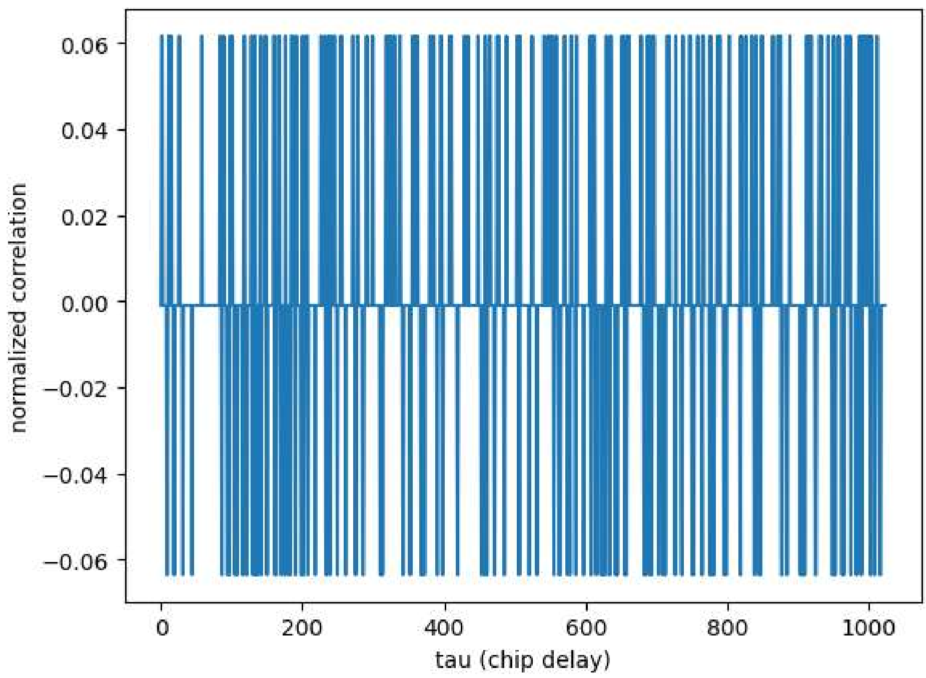
$$\frac{C_8(\eta)\oplus C_{16}(\eta-900)}{1023}$$

In [ ]:
```
np.correlate(codes[:,8],np.roll(codes[:,16],-900),'valid')/1023
```

Out[ ]:
```
array([-0.00097752])
```

In [ ]:
```
# plot for different delays in main PRN
tempval = [np.correlate(np.roll(codes[:,8],-i),np.roll(codes[:,16],-900),'valid')/1023 for i in range(0,1023)]
plt.plot(tempval)
plt.xlabel("tau (chip delay)")
plt.ylabel("normalized correlation")
print("maximum value never comes as both PRN are different, can be clearly seen as max(abs(correlation)) = "+str(max(np.a
```

```
maximum value never comes as both PRN are different, can be clearly seen as max(abs(correlation)) = [0.06353861]
```



Q3

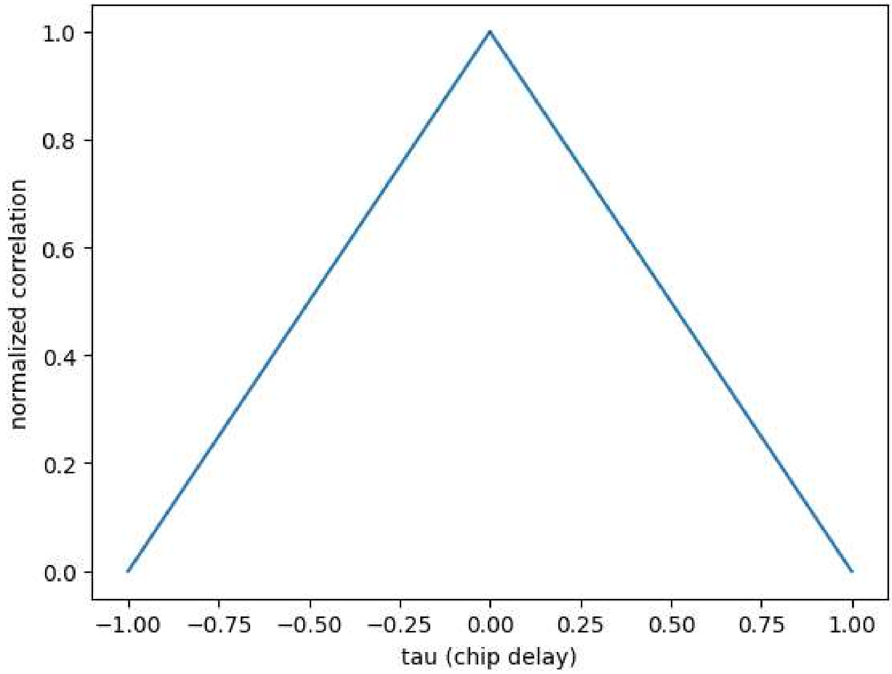
Write a MATLAB/Python/C/C++ program to compute autocorrelation of PRN 8 with a delayed PRN code by -1, 0, and 1 chip, respectively and plot the correlation with the delay chip.

Soln:

- $\frac{C_8(\eta) \oplus C_8(\eta+1)}{1023}$
- $\frac{C_8(\eta) \oplus C_8(\eta-0)}{1023}$
- $\frac{C_8(\eta) \oplus C_8(\eta-1)}{1023}$

In [ ]:
```
cor = np.zeros((3,1))
cor[0] = np.correlate(codes[:,8],np.roll(codes[:,8],1),'valid')/1023
cor[1] = np.correlate(codes[:,8],np.roll(codes[:,8],0),'valid')/1023
cor[2] = np.correlate(codes[:,8],np.roll(codes[:,8],-1),'valid')/1023
plt.plot(np.array([-1,0,1]),cor)
plt.xlabel("tau (chip delay)")
plt.ylabel("normalized correlation")
```

Out[ ]:
```
Text(0, 0.5, 'normalized correlation')
```



Q4

Write a MATLAB/ Python/C/C++ program to compute circular autocorrelation of PRN 8 with a noisy PRN code delayed by 200 chips and plot the results. Assume the noise is white Gaussian additive and generated with mean zero and standard deviation of 4.

Soln:

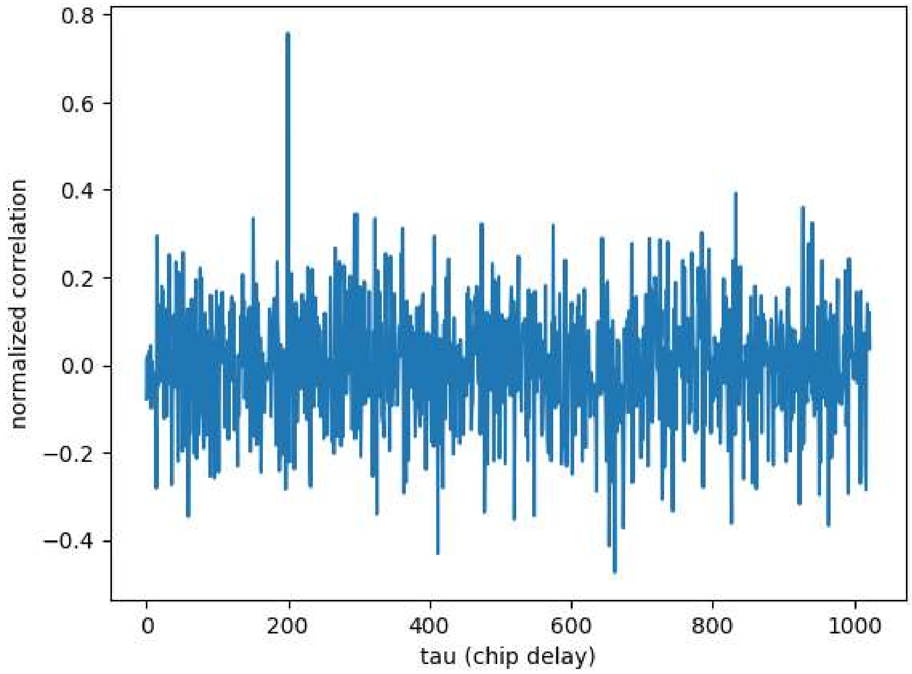
- $PRN_{noisy} = (C_8(\eta - 200) + N(0,4))$
- $C_8(\eta) \oplus PRN_{noisy}$

In [ ]:
```
noisy = (np.array(codes[:,8])+ (np.random.normal(0,4,1023)))
np.correlate(codes[:,8],np.roll(noisy,-200),'valid')/1023
```

Out[ ]:
```
array([-0.07845267])
```

In [ ]:
```
# plot for different delays of main PRN
tempval = [np.correlate(np.roll(codes[:,8],-i),np.roll(noisy,-200),'valid')/1023 for i in range(0,1023)]
plt.plot(tempval)
plt.xlabel("tau (chip delay)")
plt.ylabel("normalized correlation")
print("here we can see that due to noise the non-peak values have a higher range")
```

```
here we can see that due to noise the non-peak values have a higher range
```



Q5

Write a MATLAB/ Python/C/C++ program to implement serial search/parallel code phase search acquisition algorithm. Identify the satellites (PRN IDs), carrier frequency, and code phase using the acquisition algorithm in the data file provided in the Google drive

Sol.

- we know the signal received at the antenna is:

$s(t) = \sqrt{2P}D(t-\tau)C(t-\tau)cos((w_L + w_D)t + \theta)$ where : - P is power - τ is delay ($\tau = \tau_{int} + \tau_{frac}$) - w_L is L-5 band frequency and w_D is doppler frequency

- After RF front-end processing we get:

$I = \sqrt{2P_1}D(t-\tau)C(t-\tau)cos((w_D)t + \theta)$

$Q = \sqrt{2P_1}D(t-\tau)C(t-\tau)sin((w_D)t + \theta)$

where: - I is Inphase samples - Q is Quadrature samples

Note : slight abuse of notation is used as we defined C for discrete values while here we are considering it as continous, but this discrepancy will be resolved in program as we have input data as discrete samples

Now,

for serial search we need to get 3 parameters

- i the satellite number
- $\tau_{frac} \in [0, 1022]$
- $w_D \in [-20kHz : 500Hz : 20kHz]$

Steps

- Firstly, we will multiply $cos(\hat{w}_D t)$ and $sin(\hat{w}_D t)$ to I and Q respectively, and then pass it through a low pass filter

- Then, we will multiply $C_i(t-\hat{\tau})$ to both I and Q

- We will get:

$\hat{I} = \sqrt{2P_1}D(t-\tau)C(t-\tau)cos((w_D - \hat{w}_D)t + \theta) \oplus C_i(t-\hat{\tau})$

$\hat{Q} = \sqrt{2P_1}D(t-\tau)C(t-\tau)sin((w_D - \hat{w}_D)t + \theta) \oplus C_i(t-\hat{\tau})$

when $w_D = \hat{w}_D$:

$\hat{I}_{w_D = \hat{w}_D} = \sqrt{2P_1}D(t-\tau)C(t-\tau)cos(\theta) \oplus C_i(t-\hat{\tau})$

$\hat{Q}_{w_D = \hat{w}_D} = \sqrt{2P_1}D(t-\tau)C(t-\tau)sin(\theta) \oplus C_i(t-\hat{\tau})$

Finally, we can take norm of $[\hat{I}, \hat{Q}]$ to get final correlation, that we wish to get maximised

$$corr = \sqrt{\hat{I}^2 + \hat{Q}^2}$$

In [ ]:
```
# Assumptions:
#   - Sampling frequency is Fs which is a multiple of 1.023 MHz
#   - Fs = n*1.023 MHz

def modPRN(PRN,n,tau):
    """
    function to get modified PRN for some sampling frequency & delay
    Inputs:
        - PRN is the PRN code for single satellite with 1023 length
        - n = SamplingFrequency/1.023e6
    Output:
        - PRN code for 1 millisecond with length = 1023*n
        - This PRN code is delayed by tau chips
    """

    return np.roll(np.repeat(PRN,n),-tau)
```

In [ ]:
```
# Serial Search Function
wd = np.linspace(-20000,20000,41)
def serialSearch(I,Q,Fs,IRNSS_PRN_Codes,wD=wd):
    """
    Assumption for this serial search:
        - we are getting data from only 1 satellite
    Note:
        - Can improve this function by:
            - setting a threshold based on std dev of the correlationn
            - using clustering to differentiate high values from low value
    Inputs:
        - I = Inphase component for 1 millisecond i.e. length = 1023*Fs/1.023e6
        - Q = Quadrature component for 1 millisecond i.e. length = Fs*1e-3
        - Fs = sampling frequency in Hz
        - IRNSS_PRN_Codes = 7 PRN codes each of 1023 length; i.e (1023 x 7)
        - wD =  array of possible doppler frequencies
    Outputs:
        - satNum = satellite number = {0,1,2,3,4,5,6}
        - taustar = codePhase (fractional part)
        - wdstar = doppler frequency
    """
    n = Fs/1.023e6
    totalSatNum = IRNSS_PRN_Codes.shape[1] # 7 for IRNSS
    tau = np.linspace(0,1023*n,1023*n)
    correlation = np.zeros((totalSatNum,len(wD),len(tau)))
    t = np.linspace(0,1,1023*n)*1e-3

    for i in range(0,totalSatNum):
        for j in range(0,len(wD)):
            for k in range(0,len(tau)):
                I_hat = I*np.cos(wD[j]*t)
                Q_hat = Q*np.sin(wD[j]*t)
                I_hat= np.correlate(I_hat,modPRN(IRNSS_PRN_Codes[:,i],n,tau),'valid')/1023
                Q_hat= np.correlate(Q_hat,modPRN(IRNSS_PRN_Codes[:,i],n,tau),'valid')/1023
                correlation[i,j,k] = np.sqrt(I_hat**2+Q_hat**2)
    satNum,wdstar,taustar = np.unravel_index(correlation.argmax(), correlation.shape)
    return satNum,wdstar,taustar
```

Question can't be solved further

There are some issues in the data .BIN files:

- 2 seconds file is for L1 band but IRNSS satellites operate on L5 and S band
- The data format is not clear because if we assume that each byte i.e. 8 bits contain 1 data point for each I and Q then the sampling frequency and data points are giving contradiction

In [ ]:
```
with open('L1_DHRV.BIN', 'rb') as f:
    # Read the data into a NumPy array
    array = np.fromfile(f, dtype=np.uint8)
```

In [ ]:
```
Fs = 23e6 # sampling frequency
t = 2 # time for which signal is received
```

In [ ]:
```
totalSamples = Fs*t
```

In [ ]:
```
nbytes = len(array) # no. of bytes
```

In [ ]:
```
totalSamples/nbytes # this value should have been 1
```

Out[ ]:
```
4.010881696428571
```

In [ ]:
```
```