AE-410 GPS (Assignment-1 Fundamentals of GPS)

Rollno. 20D170022 Name - Lyric Khare

```
In [ ]:  # Shri Ganeshaya Namah
         # @author.lyrickhare
```

Some important Relations / Properties

- $C_i(\eta - \tau)$ is the PRN code (similar to a vector/array with 1023 dimensions/elements) for $i - th$ satellite, delayed by τ chips

- $C_i(\eta) = C_i(\eta + 1023)$ i.e. PRN codes are periodic with period = 1023 chips = 1 millisecond

- $\frac{C_i(\eta) \oplus C_j(\eta - \tau)}{1023} = 1$ {iff $i = j$ and $\tau = 0$, = 0 otherwise}

- $\oplus$ in the above point denotes correlation, i.e. element wise multiplication:

    - $C_i(\eta) \oplus C_j(\eta - \tau) = \sum_{r=0}^{r=1022} C_i(\eta)[r] \cdot C_j(\eta - \tau)[r]$
    - $C_i(\eta)[r]$ is a scalar value having 0 or 1 value

References / Resources

- Github repository having 32 GPS PRN codes https://github.com/danipascual/GNSS-matlab.git

```
In [ ]:  # importing necessary libraries
         import numpy as np
         import pandas as pd
         import matplotlib.pyplot as plt
         import scipy
```

```
In [ ]:  # GPS PRN Extraction
         codesFile = scipy.io.loadmat("randomData/codes_L1CA.mat")
         codes = np.array(codesFile['codes_L1CA'])
```

```
In [ ]:  # shape of codes array
         print(codes.shape)
```

(1023, 32)

Proof of $\frac{C_i(\eta) \oplus C_j(\eta - \tau)}{1023} = 1$ {iff $i = j$ and $\tau = 0$, = 0 otherwise}

```
In [ ]:  corrArr = np.zeros((2,2,1023))
         for i in range(0,2):
             for k in range(0,2):
                 for j in range(0,len(corrArr)):
                     corrArr[i][k][j]=  np.correlate(codes[:,i],np.roll(codes[:,k],-j),'valid')
```

```
C:\Users\lyric\AppData\Local\Temp\ipykernel_22732\3946328126.py:5: DeprecationWarning: Conversion of an array with ndim >
0 to a scalar is deprecated, and will error in future. Ensure you extract a single element from your array before performi
ng this operation. (Deprecated NumPy 1.25.)
  corrArr[i][k][j]=  np.correlate(codes[:,i],np.roll(codes[:,k],-j),'valid')
```

```
In [ ]:  %matplotlib inline
         plt.subplot(2,2,1)
         plt.plot(corrArr[0,0,:])
         plt.ylabel('correlation')
         plt.xlabel('chip delay')
         plt.title('SAT-0 with SAT-0')
         plt.subplot(2,2,2)
         plt.plot(corrArr[0,1,:])
         plt.ylabel('correlation')
         plt.xlabel('chip delay')
         plt.yticks(np.linspace(-500,500,5))
         plt.title('SAT-0 with SAT-1')
         plt.subplot(2,2,3)
         plt.plot(corrArr[1,0,:])
         plt.ylabel('correlation')
         plt.xlabel('chip delay')
         plt.yticks(np.linspace(-500,500,5))
         plt.title('SAT-1 with SAT-0')
         plt.subplot(2,2,4)
         plt.plot(corrArr[1,1,:])
```

```
plt.ylabel('correlation')
plt.xlabel('chip delay')
plt.title('SAT-1 with SAT-1')

plt.tight_layout()
```
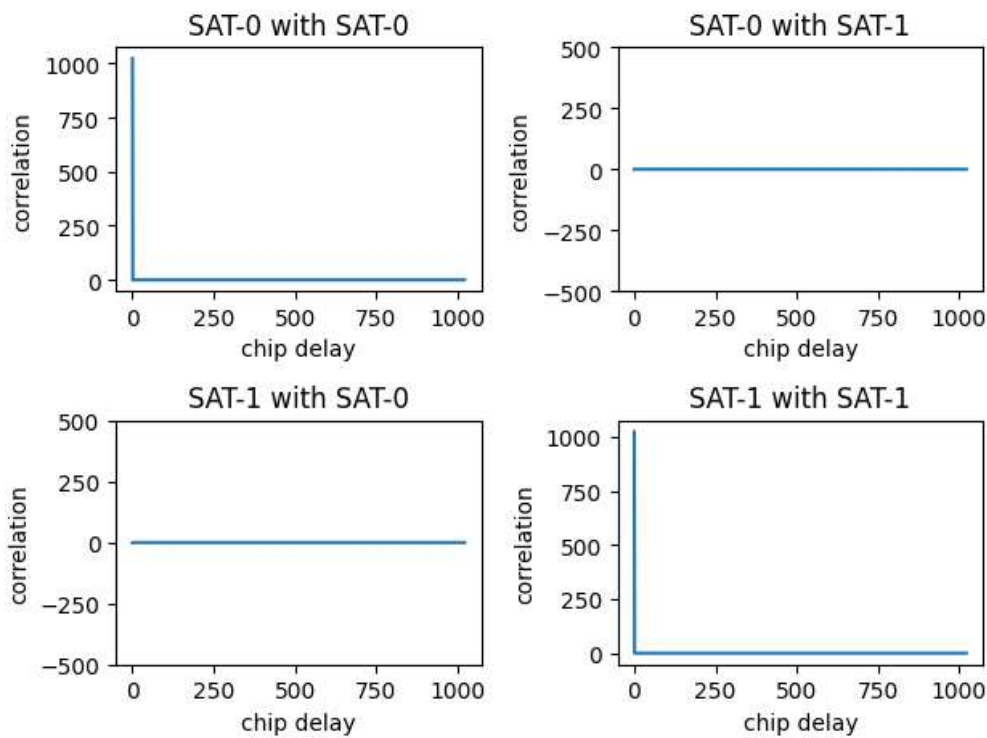

It is clear that the correlation is high only when there is 0 delay and same satellite PRN

Q1

Write a MATLAB/Python/C/C++ program to compute circular autocorrelation of PRN 8 with a delayed PRN code by 200 chips and plot the results
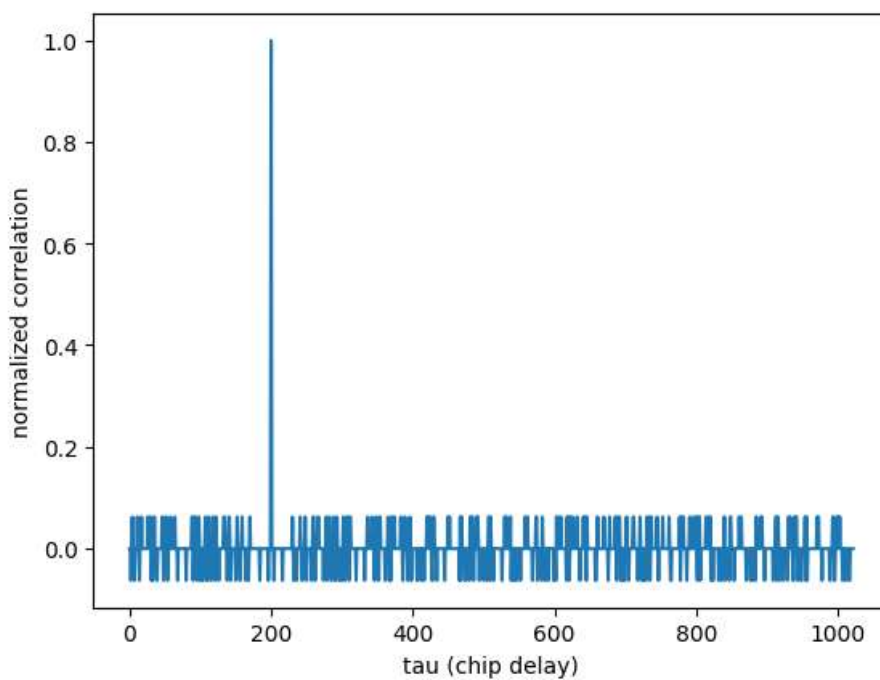
Soln:

$$\frac{C_8(\eta) \oplus C_8(\eta-200)}{1023}$$

In [ ]: `np.correlate(codes[:,8],np.roll(codes[:,8],-200),'valid')/1023`

Out[ ]: `array([-0.00097752])`

In [ ]:
```
# plot for different delays of main PRN
tempval = [np.correlate(np.roll(codes[:,8],-i),np.roll(codes[:,8],-200),'valid')/1023 for i in range(0,1023)]
plt.plot(tempval)
plt.xlabel("tau (chip delay)")
plt.ylabel("normalized correlation")
print("maximum value comes when two overlap fully, which is at " + str(np.argmax(tempval)))
```

maximum value comes when two overlap fully, which is at 200

Q2

Write a MATLAB/ Python/C/C++ program to compute circular cross correlation of PRN 8 with a delayed copy of a PRN 16 by 900 chips and plot the results.
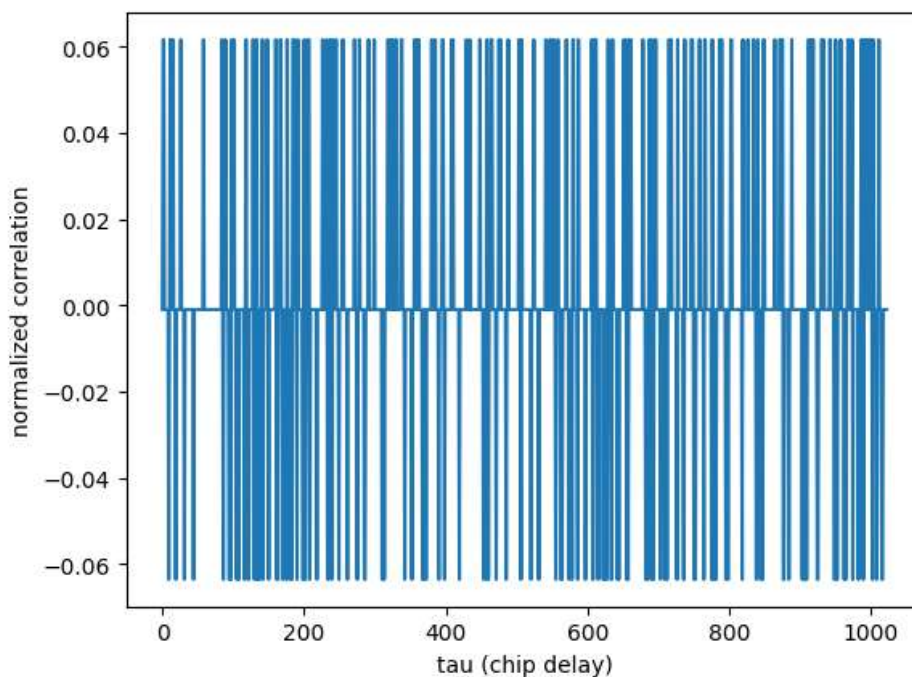
Soln:

$$\frac{C_8(\eta) \oplus C_{16}(\eta - 900)}{1023}$$

```
In [ ]: np.correlate(codes[:,8],np.roll(codes[:,16],-900),'valid')/1023
```

```
Out[ ]: array([-0.00097752])
```

```
In [ ]: # plot for different delays in main PRN
        tempval = [np.correlate(np.roll(codes[:,8],-i),np.roll(codes[:,16],-900),'valid')/1023 for i in range(0,1023)]
        plt.plot(tempval)
        plt.xlabel("tau (chip delay)")
        plt.ylabel("normalized correlation")
        print("maximum value never comes as both PRN are different, can be clearly seen as max(abs(correlation)) = "+str(max(np.a
```

maximum value never comes as both PRN are different, can be clearly seen as max(abs(correlation)) = [0.06353861]


Q3

Write a MATLAB/Python/C/C++ program to compute autocorrelation of PRN 8 with a delayed PRN code by -1, 0, and 1 chip, respectively and plot the correlation with the delay chip.

Soln:

- $\dfrac{C_8(\eta)\oplus C_8(\eta+1)}{1023}$
- $\dfrac{C_8(\eta)\oplus C_8(\eta-0)}{1023}$
- $\dfrac{C_8(\eta)\oplus C_8(\eta-1)}{1023}$

```
In [ ]:  cor = np.zeros((3,1))
         cor[0] = np.correlate(codes[:,8],np.roll(codes[:,8],1),'valid')/1023
         cor[1] = np.correlate(codes[:,8],np.roll(codes[:,8],0),'valid')/1023
         cor[2] = np.correlate(codes[:,8],np.roll(codes[:,8],-1),'valid')/1023
         plt.plot(np.array([-1,0,1]),cor)
         plt.xlabel("tau (chip delay)")
         plt.ylabel("normalized correlation")
```

```
Out[ ]:  Text(0, 0.5, 'normalized correlation')
```
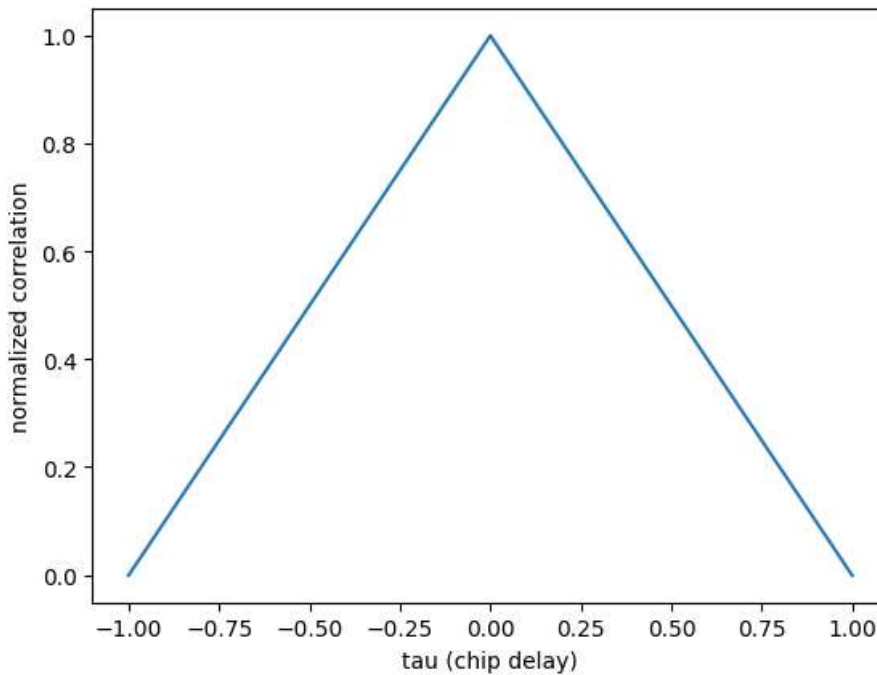

Q4

Write a MATLAB/ Python/C/C++ program to compute circular autocorrelation of PRN 8 with a noisy PRN code delayed by 200 chips and plot the results. Assume the noise is white Gaussian additive and generated with mean zero and standard deviation of 4.

Soln:

- $PRN_{noisy} = (C_8(\eta - 200) + N(0,4))$
- $C_8(\eta) \oplus PRN_{noisy}$

```
In [ ]:  noisy = (np.array(codes[:,8])+ (np.random.normal(0,4,1023)))
         np.correlate(codes[:,8],np.roll(noisy,-200),'valid')/1023
```

```
Out[ ]:  array([0.08707181])
```

```
In [ ]:  # plot for different delays of main PRN
         tempval = [np.correlate(np.roll(codes[:,8],-i),np.roll(noisy,-200),'valid')/1023 for i in range(0,1023)]
         plt.plot(tempval)
         plt.xlabel("tau (chip delay)")
         plt.ylabel("normalized correlation")
         print("here we can see that due to noise the non-peak values have a higher range")
```

here we can see that due to noise the non-peak values have a higher range
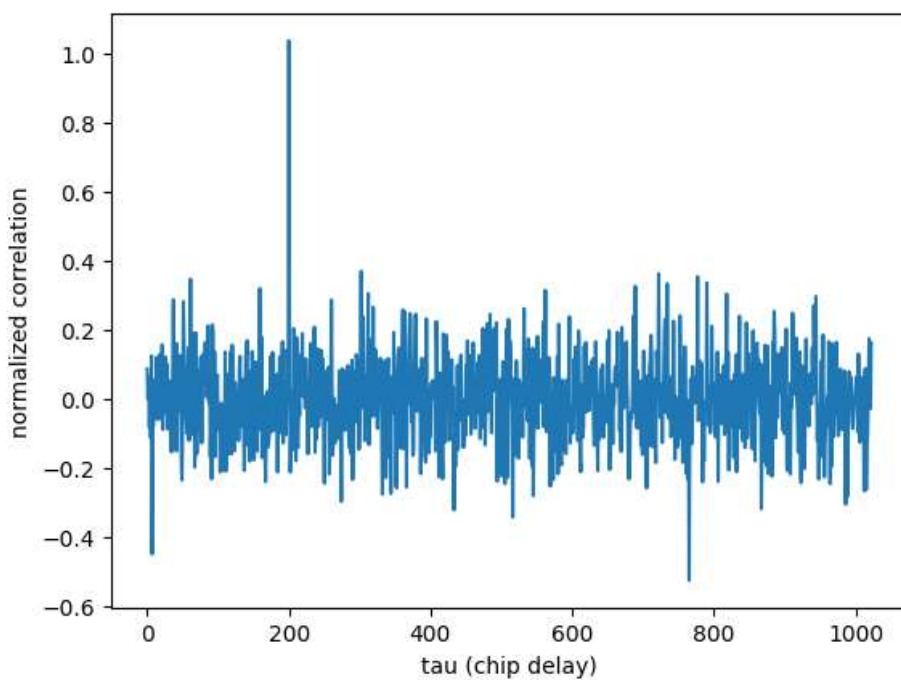
Q5

Write a MATLAB/ Python/C/C++ program to implement serial search/parallel code phase search acquisition algorithm. Identify the satellites (PRN IDs), carrier frequency, and code phase using the acquisition algorithm in the data file provided in the Google drive

Sol.

- we know the signal received at the antenna is:

$s(t) = \sqrt{2P}D(t-\tau)C_j(t-\tau)cos((w_L + w_D)t + \theta)$ where : - P is power - τ is delay ($\tau = \tau_{int} + \tau_{frac}$) - w_L is L-5 band frequency and w_D is doppler frequency

- After RF front-end processing we get:

$I = \sqrt{2P_1}D(t-\tau)C_j(t-\tau)cos((w_D)t + \theta)$

$Q = \sqrt{2P_1}D(t-\tau)C_j(t-\tau)sin((w_D)t + \theta)$

where: - I is Inphase samples - Q is Quadrature samples

Note : slight abuse of notation is used as we defined C for discrete values while here we are considering it as continous, but this discrepancy will be resolved in program as we have input data as discrete samples

Now,

for serial search we need to get 3 parameters

- i the satellite number
- $\tau_{frac} \in [0, 1022]$
- $w_D \in [-10kHz : 500Hz : 10kHz]$

Steps

- Firstly, we will rewrite signal as $S = I + Qj = \sqrt{2P_1}D(t-\tau)C(t-\tau)e^{jw_dt}$

- Then, we will multiply S by $C(t-\hat{\tau})e^{-j\hat{w}_dt}$

- We will get:

$Corr = \sqrt{2P_1}D(t-\tau)C_j(t-\tau) \oplus C_i(t-\hat{\tau})e^{j(w_d - \hat{w}_d)t}$

it will attain it's maximum when :

$w_d - \hat{w}_d = 0$

$\tau = \hat{\tau}$

$i = j =$ satellite number of incoming signal

```
In [ ]:   # Assumptions:
          #   - Sampling frequency is Fs which is a multiple of 1.023 MHz
          #   - Fs = n*1.023 MHz

          def modPRN(PRN,n,tau):
              """
              function to get modified PRN for some sampling frequency & delay
              Inputs:
                  - PRN is the PRN code for single satellite with 1023 length
                  - n = SamplingFrequency/1.023e6
              Output:
                  - PRN code for 1 millisecond with length = 1023*n
                  - This PRN code is delayed by tau chips
              """

              # return np.roll(np.repeat(PRN,n),-tau)
              return np.repeat(np.roll(PRN,-tau),n)
```

```
In [ ]:   # Checking modPRN
          someArrForModPRN = np.array([1,2,3,4,5])
          modPRN(someArrForModPRN,3,4)
```

```
Out[ ]:   array([5, 5, 5, 1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4])
```

```
In [ ]:   wds = np.linspace(-10000,10000,41)*2*np.pi
          def serialSearch(I, Q, knownPRNs, Fs, wds=wds):
              rcvdSignal = np.array(I+Q*1j)
              n = int(Fs/1.023e6)
              t = np.linspace(0,1,n*1023)*1e-3
              totalSatellites = knownPRNs.shape[1]
              corr = np.zeros((totalSatellites,len(wds),1023))
              for x in range(0,totalSatellites):
                  for y in range(0,len(wds)):
                      for z in range(0,1023):
                          generatedSignal = modPRN(knownPRNs[:,x],n,z)*np.array(np.exp(-1j*wds[y]*t))
                          corr[x,y,z] = np.abs(np.sum(generatedSignal*rcvdSignal))
              return corr
```

```
In [ ]:   t = np.linspace(0,1,1023)*1e-3
```

```
In [ ]:   # sirPRNI = modPRN(codes[:,20],1,82)*np.cos(2*np.pi*675*t)
          # sirPRNQ = modPRN(codes[:,20],1,82)*np.sin(2*np.pi*675*t)
```

```
In [ ]:   # dataSet1 = scipy.io.loadmat("Dataset1.mat")
          # sirPRNI = np.array(dataSet1['sim_doppler_I'])
          # sirPRNQ = np.array(dataSet1['sim_doppler_Q'])
          dataSet2 = scipy.io.loadmat("Dataset2.mat")
          sirPRNI = np.array(dataSet2['sim_doppler_I'])
          sirPRNQ = np.array(dataSet2['sim_doppler_Q'])
```

```
In [ ]:   # corr = serialSearch(sirPRNI,sirPRNQ,codes,16*1.023e6)
```

```
In [ ]:   corr2 = serialSearch(sirPRNI,sirPRNQ,codes,16*1.023e6)
```

```
In [ ]:   # satNum,wdstar,taustar = np.unravel_index(corr.argmax(), corr.shape)
```

```
In [ ]:   satNum,wdstar,taustar = np.unravel_index(corr2.argmax(), corr2.shape)
```

```
In [ ]:   # satNum
```

```
Out[ ]:   7
```

```
In [ ]:   satNum #for2
```

```
In [ ]:   # wds[wdstar]/(2*np.pi)
```

```
Out[ ]:   2500.0
```

```
In [ ]:   wds[wdstar]/(2*np.pi) #for2
```

```
In [ ]:   # taustar
```
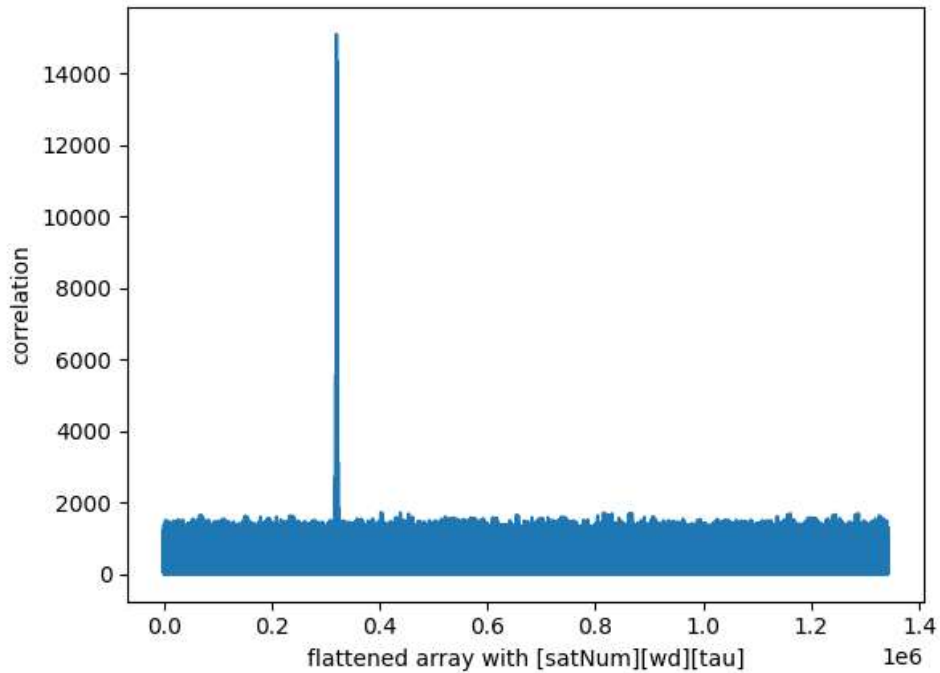
```
Out[ ]:   823
```

```
In [ ]:   taustar #for2
```

We can clearly see that we got the desired result

```
In [ ]:  # plt.plot(corr.flatten())
         # plt.ylabel("correlation")
         # plt.xlabel("flattened array with [satNum][wd][tau]")
         # plt.title("""Recieved signal has satNum=20, delay=82chips, Fs = 1.023e6
         #           with normal noise (0,4)""")
```

Out[ ]: Text(0.5, 1.0, 'Recieved signal has satNum=20, delay=82chips, Fs = 1.023e6\n   with normal noise (0,4)')


```
In [ ]:  plt.plot(corr2.flatten())
         plt.ylabel("correlation")
         plt.xlabel("flattened array with [satNum][wd][tau]")
         plt.title("""Recieved signal has satNum=20, delay=82chips, Fs = 1.023e6
                   with normal noise (0,4)""")
```

In [ ]: