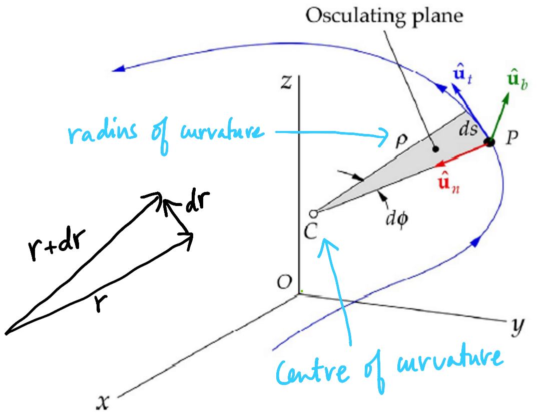
- Suppose a particle P is moving on a curved path with velocity v



- $$\hat{u}_t = \frac{v}{\|v\|} = \frac{v_x}{\|v\|}\hat{i} + \frac{v_y}{\|v\|}\hat{j} + \frac{v_z}{\|v\|}\hat{k}$$

- $$a = a_t \hat{u}_t + a_n \hat{u}_n$$
$$a_t = \ddot{s} = \dot{\widehat{\|v\|}}$$
$$a_n = \frac{\|v\|^2}{\rho}$$

- $$r_{c/p} = \rho \hat{u}_n$$

- Binormal $\hat{u}_b = \hat{u}_t \times \hat{u}_n$
- Now, $v \times a = \|v\| \hat{u}_t \times \left(\dot{\widehat{\|v\|}} \hat{u}_t + \frac{\|v\|^2}{\rho} \hat{u}_n \right)$
$$= \frac{\|v\|^3}{\rho} \hat{u}_t \times \hat{u}_n$$
$$= \|v \times a\| \hat{u}_b$$

- Alternatively, $\hat{u}_b = \frac{v \times a}{\|v \times a\|}$

- Note that:
$\hat{u}_t \times \hat{u}_n = \hat{u}_b$, $\hat{u}_n \times \hat{u}_b = \hat{u}_t$, $\hat{u}_b \times \hat{u}_t = \hat{u}_n$

- Note that $ds = \rho\, d\phi \Rightarrow \dot{s} = \rho \dot{\phi}$, or
$$\dot{\phi} = \frac{\dot{s}}{\rho} = \frac{\|v\|}{\rho}$$

Example

Relative to a Cartesian coordinate system, the position, velocity, and acceleration of a particle P at a given instant are

$$\mathbf{r} = 250\hat{\mathbf{i}} + 630\hat{\mathbf{j}} + 430\hat{\mathbf{k}}\,(\mathrm{m}) \qquad \text{(a)}$$

$$\mathbf{v} = 90\hat{\mathbf{i}} + 125\hat{\mathbf{j}} + 170\hat{\mathbf{k}}\,(\mathrm{m/s}) \qquad \text{(b)}$$

$$\mathbf{a} = 16\hat{\mathbf{i}} + 125\hat{\mathbf{j}} + 30\hat{\mathbf{k}}\,(\mathrm{m/s^2}) \qquad \text{(c)}$$

Find the coordinates of the center of curvature at that instant.

Details

$$r_c = r + \rho\,\hat{u}_n$$

ρ and $\hat{u}_n$ are unknowns

- Finding $\hat{u}_n$

$$\hat{u}_t = \frac{v}{\|v\|}$$

$$\hat{u}_b = \frac{v \times a}{\|v \times a\|}$$

$$\hat{u}_n = \hat{u}_b \times \hat{u}_t$$

- Finding ρ

$$\rho = \frac{\|v\|^2}{a_n}$$

$$a_n = a \cdot \hat{u}_n$$

like length and time

- Mass is a primitive physical concept, i.e., it cannot be defined in terms of any other physical concept.

- More practically, mass is the measure of the inertia of a body.

- Force is the action of one physical body on another, either through direct contact

- on another, either through direct contact or through a distance

$$F_g = \frac{G m_1 m_2}{r^2}$$

$$G = 6.6742 \times 10^{-11} \ m^3/kg \cdot s^2$$

(universal gravitational constant)

- The force of a large mass (such as the earth) on a mass many orders of magnitude smaller (such as a person) is called weight

$$W = \frac{GMm}{r^2}$$

$$= m\underbrace{\left(\frac{GM}{r^2}\right)}$$

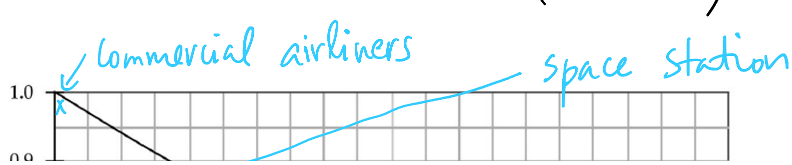
$$= m\underbrace{g}$$

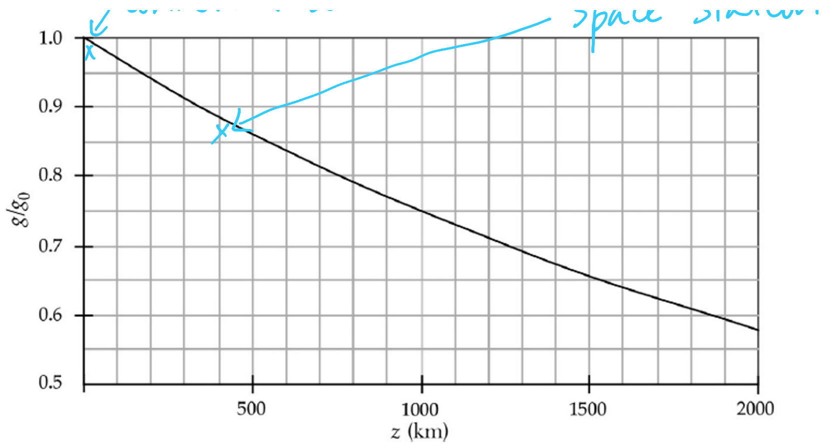
acceleration due to gravity

- Standard sea-level value of g:

$$g_0 = \frac{GM}{R_E^2} = 9.807 \ m/s^2$$

- $$g = g_0 \frac{R_E^2}{(R_E+z)^2} = \frac{g_0}{(1+z/R_E)^2}$$





- Force is not a primitive concept.
- Newton's second law: $F_{net} = ma$.
- Impulse of a force F over a time interval:

$$I = \int_{t_1}^{t_2} F\,dt$$

- Net impulse on a body:

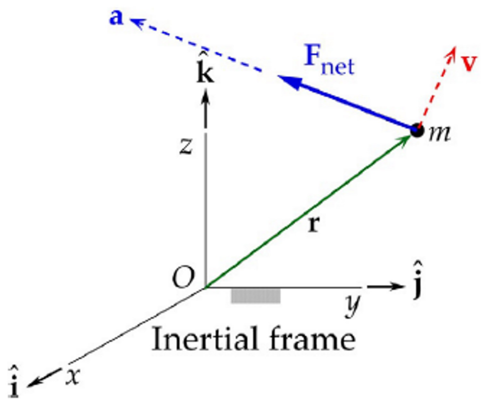
$$I_{net} = \int_{t_1}^{t_2} \underbrace{m\frac{dv}{dt}}\,dt = mv_2 - mv_1$$

$$\Rightarrow \Delta v = v_2 - v_1 = \frac{I_{net}}{m}$$

- If F_{net} is a constant, then $I_{net} = F_{net}\Delta t$ and so,

$$\Delta v = F_{net}\,\Delta t$$

$$\Delta v = \frac{F_{net}}{m} \Delta t$$



- The moment of the net force about O is

$$M_o)_{net} = r \times F_{net}$$
$$= r \times m \frac{dv}{dt}$$
$$= \frac{d}{dt}(r \times mv) - (v \times mv)$$
$$= \frac{d}{dt}(r \times mv)$$
$$= \frac{d}{dt} \underbrace{H_0}_{\uparrow},$$

angular momentum about O

$$\frac{d}{dt}(r \times mv) = \frac{dr}{dt} \times mv + r \times m \frac{dv}{dt}$$
$$= \underbrace{v \times mv}_{0} + r \times m \frac{dv}{dt}$$
$$= r \times m \frac{dv}{dt}$$

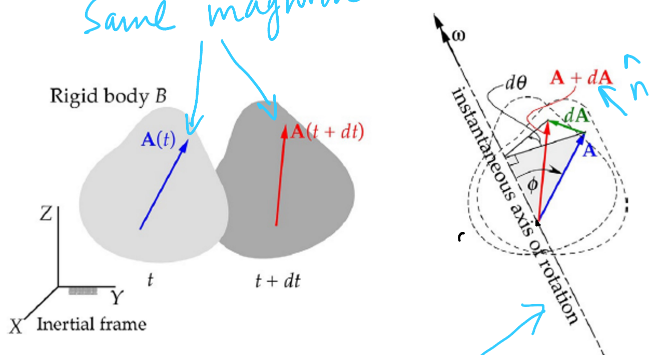
- Net angular impulse:

$$\int^{t_2} M_0)_{net} = H_0)_2 - H_0)_1$$

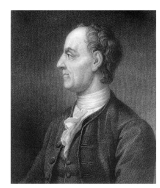
$$\int_{t_1} M_0)_{net} = H_0)_2 - H_0)_1$$

- Just as the net force on a particle changes its linear momentum mv, the moment of that force about a fixed point changes the moment of its linear momentum about that point.

Same magnitude



(a) (b)



Leonhard Euler
1707-1783

- Rigid body motion
- Fluid mechanics
- Solid mechanics
- Number theory
- Real and complex analysis
- Calculus of variations
- Differential geometry and topology
- Differential equations
- Mathematical notation

$\frac{dA}{dt}$?

unique (follows from one of Euler's theorem)

- We have

$$dA = \underbrace{\left[\left(\|A\|\sin\phi\right)d\theta\right]}_{\text{magnitude of } dA}\hat{n}$$

- By definition, $d\theta = \|\omega\|\,dt$

- So, we have

$$dA = \left[(\|A\| \sin\phi) \|\omega\| \right] \hat{n}\, dt$$

$$\Rightarrow \frac{dA}{dt} = \omega \times A \quad \left(\text{if } \frac{d}{dt}\|A\| = 0 \right)$$

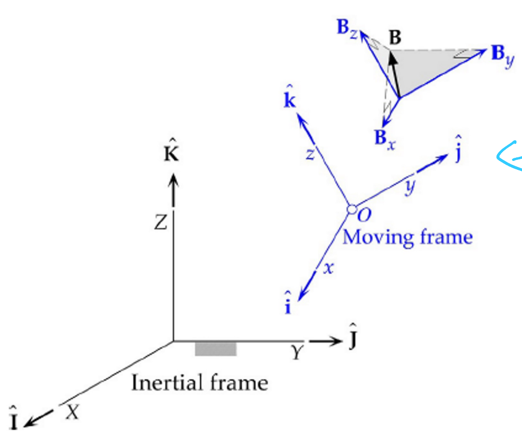
Example

Calculate the second time derivative of a vector **A** of constant magnitude, expressing the result in terms of **ω** and its derivatives and **A**.

Details

$$\frac{d^2A}{dt^2} = \frac{d}{dt}\left(\frac{dA}{dt}\right) = \frac{d}{dt}(\omega \times A)$$

$$= \frac{d\omega}{dt} \times A + \omega \times \frac{dA}{dt} = \frac{d\omega}{dt} \times A + \omega \times (\omega \times A)$$



can translate and also rotate freely

- Absolute vs. Relative measurements!

- Let B be any time-dependent vector and assume that the absolute angular velocity of the moving frame is Ω.

$$B = B_X \hat{I} + B_Y \hat{J} + B_Z \hat{K}$$

$$\frac{dB}{dt} = \frac{dB_X}{dt}\hat{I} + \frac{dB_Y}{dt}\hat{J} + \frac{dB_Z}{dt}\hat{K}$$

$$B = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\frac{dB}{dt} = \frac{dB_x}{dt}\hat{i} + \frac{dB_y}{dt}\hat{j} + \frac{dB_z}{dt}\hat{k} + B_x \underbrace{\frac{d\hat{i}}{dt}}_{\Omega \times \hat{i}} + B_y \underbrace{\frac{d\hat{j}}{dt}}_{\Omega \times \hat{j}} + B_z \underbrace{\frac{d\hat{k}}{dt}}_{\Omega \times \hat{k}}$$

$$= \frac{dB_x}{dt}\hat{i} + \frac{dB_y}{dt}\hat{j} + \frac{dB_z}{dt}\hat{k} + \underbrace{\Omega \times B_x\hat{i} + \Omega \times B_y\hat{j} + \Omega \times B_z\hat{k}}_{\Omega \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})}$$

$$\frac{dB_x}{dt}\hat{i} + \frac{dB_y}{dt}\hat{i} + \frac{dB_z}{dt}\hat{k} + \Omega \times B$$

$$= \frac{dB_x}{dt}\hat{i} + \frac{dB_y}{dt}\hat{j} + \frac{dB_z}{dt}\hat{k} + \Omega \times B$$

$$\frac{dB}{dt} = \left.\frac{dB}{dt}\right)_{rel} + \Omega \times B$$