AE234 Aircraft Propulsion

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June 1, 2021

1 Question

Consider a 3-blade propeller, with a diameter of 3 m and an input power of 746 kW. Assume that the propeller is limited to a tip Mach number of 0.8.

1. Estimate the thrust, speed of the exit stream (downstream from the propeller), and rpm for static sea-level operation.

2. Estimate how the following change as the vehicle flies at a Mach number of 0.2 near sea-level conditions:

   - Permitted rpm
   - Thrust
   - Range of values taken by the parameter ϕ if the hub radius is 20% of the tip radius

2 Solution

Since we have SSL[1], we have $\rho = 1.225\frac{kg}{m^3}$ and T = 288.15 K. Also we have the following conditions:

$$\frac{V_{tip}}{a_{SSL}} = M_{tip} \leq 0.8$$

$$a_{SSL} = \sqrt{\gamma RT} = \sqrt{1.4 \times 287.1 \times 288.15} = 340.321\,\frac{m}{s}$$

Hence we get the following,

$$V_{tip} \leq 0.8 \times \sqrt{1.4 \times 287.1 \times 288.15} = 272.257\,\frac{m}{s}$$

$$\Omega R_{tip} = V_{tip} \leq 272.257\,\frac{m}{s}$$

$$\Omega \leq 181.50\,\frac{rad}{s}$$

[1]Standard Sea Level Conditions

Now, we have Input Power to the Propeller = Thrust to Vehicle + Induced Power to Flow. Let the velocity of the incoming flow far upstream be v_0, upstream near the propeller be v_1 and velocity of outgoing flow just downstream of the propeller be v_2 and far downstream be v_e. Hence,

$$\tau = 2\rho A_d w(v_0 + w) \longrightarrow P_{input} = \tau v_1 = (v_0 + w) = 2\rho A_d w(v_0 + w)^2$$

where τ is the thrust produced and w is the downwash velocity imparted to the flow by the propeller. As we have static sea level operation, we have $v_0 = 0$. Hence we have,

$$746\,kW = P_{input} = \tau v_1 = 2\rho A_d w^3$$

Solve for w from the above expression. Here A_d is the area of the propeller. Hence $A_d = \pi r^2$. Now, $v_1 = v_2 = v_0 + w$ and $v_e = v_0 + 2w$.

$$v_e = v_0 + 2w = 2w$$

$$\tau = \frac{P_{input}}{v_1} = \frac{P_{input}}{v_0 + w} = \frac{P_{input}}{v_0}$$

Now, we get to the next part of the question. The vehicle flies at M = 0.2 at sea-level conditions. Hence, $a_{SSL} = \sqrt{\gamma RT}$, hence $v_0 = M \times a_{SSL}$. Again we use the earlier expressions,

$$P_{input} = \tau(v_0 + w) = 2\rho A_d w(v_0 + w)^2$$

You get a cubic equation in w. Solve that to find, $v_1 = v_0 + w = v_2$ and $v_e = v_0 + 2w$.

$$\tau = \frac{P_{input}}{v_0 + w}$$

Now, the propeller tip will experience 2 velocities, as given in the diagram, hence the relative velocity is as follows,

$$M_{tip} \times a_{SSL} = v_{res,tip} = \sqrt{v_0^2 + (\Omega R)^2}$$

Apply the constrain that $M_{tip} \leq 0.8$. And get the following,

$$\frac{\sqrt{v_0^2 + (\Omega R)^2}}{a_{SSL}} = M_{tip} \leq 0.8$$

Solve this to get the upper limit on the angular velocity on the propeller. Now, we have been given, $r_{hub} = 20$ % of $r_{tip} = 0.3$m. We also have,

$$tan\phi = \frac{v_a}{\Omega R}$$

Here take the Ω take the upper limit of Ω to find out the limiting case. Hence, we get

$$\phi = tan^{-1}\frac{v_a}{\Omega r}$$

The propeller blade profile will be as follows – straight from the above expression.

% of R_{tip}	length (in m)	ϕ
10%	0.15m	$\phi = 68.823^0$
20%	0.30m	$\phi = 52.231^0$
30%	0.45m	$\phi = 40.71^0$
40%	0.60m	$\phi = 32.835^0$
50%	0.75m	$\phi = 27.305^0$
60%	0.90m	$\phi = 23.728^0$
70%	1.05m	$\phi = 20.242^0$
80%	1.20m	$\phi = 17.883^0$
90%	1.35m	$\phi = 16.003^0$
100%	1.50m	$\phi = 14.473^0$

Table 1: Propeller Blade Profile