4 *Handbook of Aeronautics*, *No. 1, Structural Principles and Data*, 4th edition. Published under the authority of the Royal Aeronautical Society, The New Era Publishing Co. Ltd., London, 1952.
5 Timoshenko, S. and Goodier, J. N., *Theory of Elasticity*, 2nd edition, McGraw-Hill Book Company, New York, 1951.

Problems

P.3.1 Show that the stress function $\phi = k(r^2 - a^2)$ is applicable to the solution of a solid circular section bar of radius a. Determine the stress distribution in the bar in terms of the applied torque, the rate of twist and the warping of the cross-section.

Is it possible to use this stress function in the solution for a circular bar of hollow section?

Ans. $\tau = Tr/I_p$ where $I_p = \pi a^4/2$,
$\mathrm{d}\theta/\mathrm{d}z = 2T/G\pi a^4$, $w = 0$ everywhere.

P.3.2 Deduce a suitable warping function for the circular section bar of P.3.1 and hence derive the expressions for stress distribution and rate of twist.

Ans. $\psi = 0$, $\tau_{zx} = -\dfrac{Ty}{I_p}$, $\tau_{zy} = \dfrac{Tx}{I_p}$, $\tau_{zs} = \dfrac{Tr}{I_p}$, $\dfrac{\mathrm{d}\theta}{\mathrm{d}z} = \dfrac{T}{GI_P}$

P.3.3 Show that the warping function $\psi = kxy$, in which k is an unknown constant, may be used to solve the torsion problem for the elliptical section of Example 3.2.

P.3.4 Show that the stress function

$$\phi = -G\frac{\mathrm{d}\theta}{\mathrm{d}z}\left[\frac{1}{2}(x^2 + y^2) - \frac{1}{2a}(x^3 - 3xy^2) - \frac{2}{27}a^2\right]$$

is the correct solution for a bar having a cross-section in the form of the equilateral triangle shown in Fig. P.3.4. Determine the shear stress distribution, the rate of twist and the warping of the cross-section. Find the position and magnitude of the maximum shear stress.

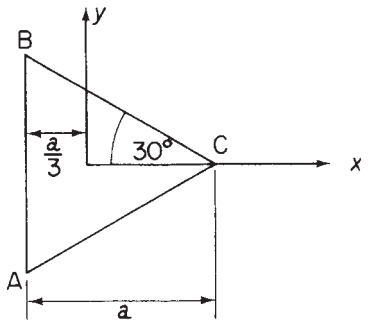

Fig. P.3.4

Ans.

$$\tau_{zy} = G\frac{\mathrm{d}\theta}{\mathrm{d}z}\left(x - \frac{3x^2}{2a} + \frac{3y^2}{2a}\right)$$

$$\tau_{zx} = -G\frac{\mathrm{d}\theta}{\mathrm{d}z}\left(y + \frac{3xy}{a}\right)$$

$$\tau_{\max}\ (\text{at centre of each side}) = -\frac{a}{2}G\frac{\mathrm{d}\theta}{\mathrm{d}z}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{15\sqrt{3}T}{Ga^4}$$

$$w = \frac{1}{2a}\frac{\mathrm{d}\theta}{\mathrm{d}z}(y^3 - 3x^2y).$$

P.3.5 Determine the maximum shear stress and the rate of twist in terms of the applied torque T for the section comprising narrow rectangular strips shown in Fig. P.3.5.

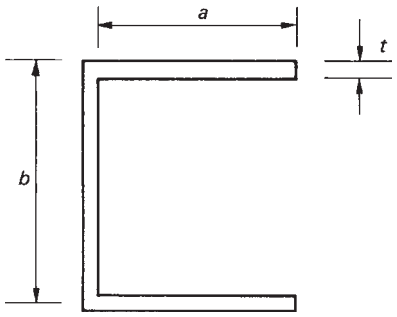

Fig. P.3.5

Ans. $\tau_{\max} = 3T/(2a+b)t^2$, $\mathrm{d}\theta/\mathrm{d}z = 3T/G(2a+b)t^3$.