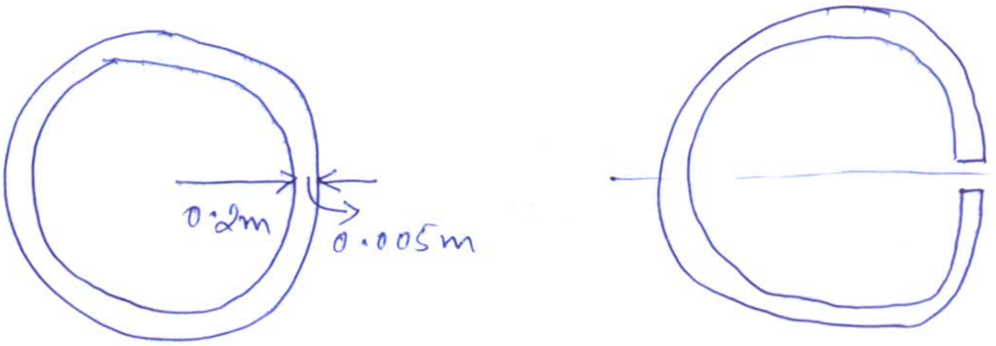
Torsion : Closed cell thin-walled section problems

(1) Consider a thin-walled tube with the cross-section shown below. The wall thickness is $t = 0.005$ m and the average radius is 0.2025 m.

closed section



slit-section

$$\bar{A} = \pi (0.2025)^2 = 0.129 \text{ m}^2 \rightarrow \text{(for closed-section)}$$

$$\oint \frac{ds}{t} = \frac{\pi \times 0.405}{0.005} = 254$$

$\therefore$ Torsion constant is obtained as,

$$J_1 = \frac{4\bar{A}^2}{\oint \frac{ds}{t}} = \frac{4 \times (0.129)^2}{254} = 2.62 \times 10^{-4} \text{ m}^4$$

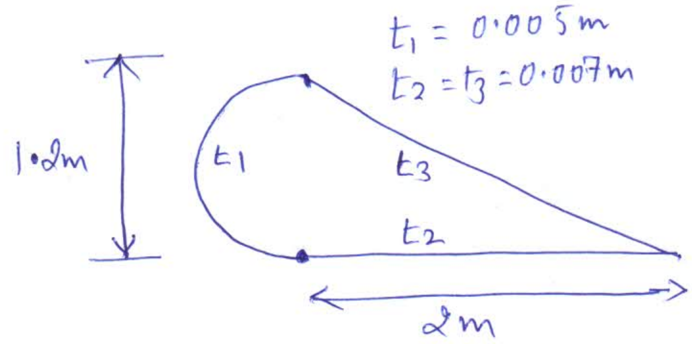
For the slit-section

$$J_2 = \frac{bt^3}{3} = \frac{\pi \times 0.04 \times (0.005)^3}{3} = 5.24 \times 10^{-8} \text{ m}^4$$

$$\therefore \frac{J_1}{J_2} = 5000 \Rightarrow$$

closed section has a much higher torsional rigidity than slit-section

(2)



Consider a three-stringer thin-walled beam with the cross-section as shown in figure. The contribution of individual stringers to the overall torsional rigidity of the thin-walled structure is small and can be neglected. Hence, this structure can be considered as a single-cell closed section with a non-uniform wall thickness and shear flow is constant along the wall.

If torque T (N·m) is given, then the shear flow is obtained from the relation:

$$T = 2\bar{A}q$$

Where $\bar{A} = \frac{1}{2}\pi(0.6)^2 + \frac{1}{2}(2\times 1.2) = 1.765\ m^2$

$$\therefore q = \frac{T}{2\bar{A}} = \frac{T}{3.53}\ N/m$$

Twist angle is obtained by,

$$\theta = \frac{1}{2\bar{A}G} q \oint \frac{ds}{t}$$

$$= \frac{q}{2\times 1.765\,G} \cdot \left\{ \frac{1.2\pi}{t_1} + \frac{2}{t_2} + \frac{2.33}{t_3} \right\}$$

$$\theta = 79\frac{T}{G}\ rad/m$$

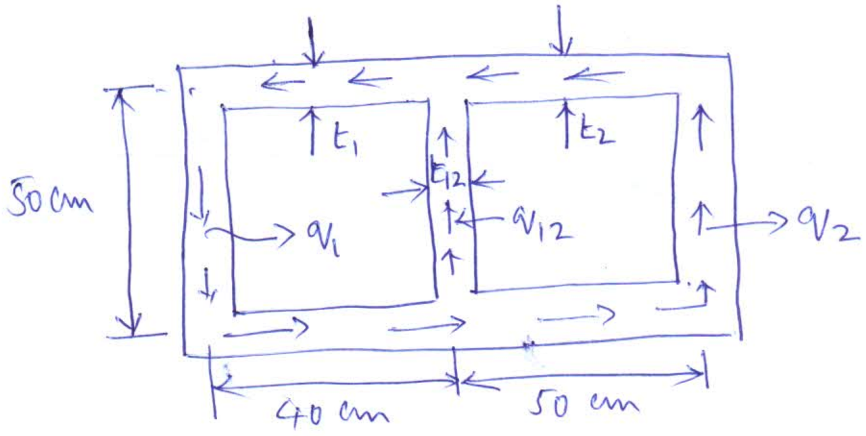
Because of its smaller thickness, the shear stress in the curved wall is higher than that in straight walls.

$$\therefore \tau_{max} = \frac{q}{t_1} = \frac{T}{0.005\times 3.53} = 56.66\,T = \tau_{allowable}$$

If allowable shear stress is 200MPa then max torque $= T_{max} = \frac{\tau_{allow}}{56.66}$

$$\Rightarrow T_{max} = \underline{3.53\times 10^6\ N\cdot m}$$

(3) A two-cell thin-walled box beam is subjected to a torque T that causes a twist angle $\theta = 5°/m$ (0.0872 rad/m). Assume $G = 27$ GPa.



$t_1 = 0.2$ cm

$t_2 = 0.4$ cm

$t_{12} = 0.3$ cm

$$\therefore \theta_1 = \frac{1}{2G\bar{A}_1} \oint_{cell_1} \frac{q\,ds}{t}$$

$$= \frac{1}{2G\bar{A}_1}\left[\frac{q_1(0.4+0.5+0.4)}{t_1} + \frac{\overset{(q_1-q_2)}{q_{12}}(0.5)}{t_{12}}\right] \rightarrow (1)$$

$$\Rightarrow \theta_1 = \theta$$

$$\Rightarrow 0.087 = 7.56\times10^{-8} q_1 - 1.55\times10^{-8} q_2$$

$$\theta_2 = \theta = \frac{1}{2G\bar{A}_2}\left[\frac{q_2(1.5)}{t_2} - \frac{\overbrace{(q_1-q_2)}^{q_{12}}(0.5)}{t_{12}}\right]$$

$$0.087 = -1.24\times10^{-8} q_1 + 4.01\times10^{-8} q_2 \rightarrow (2)$$

$\therefore$ from (1) & (2) we have,

$q_1 = 1.7\times10^{6}$ N/m

$q_2 = 2.7\times10^{6}$ N/m

Torque that produces the given twist angle is

$$T = 2\bar{A}_1 q_1 + 2\bar{A}_2 q_2 = 2.03 \times 10^6 \ N \cdot m$$

Torsion constant

$$J = \frac{T}{G\theta} = \frac{2.03 \times 10^6}{(27 \times 10^9)(0.087)} = \underline{\underline{0.86 \times 10^{-3} \ m^4}}$$