Impact of Gravity on Design

Impact of Gravity on Trajectory

An **important** feature of the solution **obtained** under the gravity is that **we** get lower burnout **velocity**, resulting in reduction in kinetic **energy**.

However, as **rocket** now does work against **gravity**, this work (or energy) appears as **potential** energy, which is reflected in the **altitude** achieved at burnout.

Energy Conservation Concept

As **spherically** symmetric gravity is **conservative**, total energy must be **constant.**

In view of **this**, we note that the sum of **potential** and kinetic energies **must** be equal to ideal **burnout** energy.

This **aspect** is examined next.

Impact of Gravity on Trajectory

The **total** energy in these two **cases** is as given below.

Ideal Burnout Case: $E_{ideal} = 5.327\times10^6$

Burnout under **gravity**: $E_{gravity} = 3.393\times10^6$

We see that total energy under **gravity** is significantly less in **comparison** to the total energy under **ideal** burnout.

Impact of Gravity on Ascent Mission

Thus, **we** note that the total **energy** is not conserved even if the **force** field is conservative, which **needs** to be understood in the **present** context.

In this regard, we note that **energy** conservation holds **good** only if mass is also **conserved.**

Impact of Gravity on Ascent Mission

In the **present** case, we find that, though in an **overall** sense, mass is conserved, **burnt** mass is useless and therefore, **energy** associated with this is a **loss.**

In addition, as **mass** is burnt and lost in a **sequential** manner, the energy **imparted** to the unburnt **propellant** by the burning **propellant** is also lost in the next **instant**.

Therefore, these '**losses**' to the final burnout **mass**, which is typically the **mission** payload, results in the **non-conservative** nature of gravity in the **ascent** missions.

Gravity Loss Mitigation

As **we** have seen from the **example**, the loss of energy due to **gravity** is quite significant at ~**36**% and needs to be **reduced** in order to make the **mission** more efficient.

In **this** regard, we know that **both** velocity and altitude are **functions** of burnout time and that, **larger** the time, lower are the **values** of altitude and velocity.

Therefore, **one** way to reduce the **loss** is to reduce the burnout **time**, which can be done by **increasing** the burn rate for a given **propellant** mass.

Burn Rate Vs. Gravity Loss Example

Let us **consider** the previous example and **generate** the terminal conditions for **two** burn rates of 600 and 1200 kg/s **respectively**, as obtained below.

$V_{ideal} = 3.264$ km/s

$\beta = 600$ kg/s: $V_b = 2.283$ km/s; $h_b = 77.6$ km

$B = 1200$ kg/s: $V_b = 2.773$ km/s; $h_b = 51.1$ km

Let **us** now compare the three **energies**.

Energy Loss Vs. Burn Rate

The **total** energy in these three **cases** is as given below.

Ideal Burnout Case: $E_{ideal} = 5.327\times10^6$

Burnout for $\boldsymbol{\beta = 600}$: $E_{\beta = 600} = 3.393\times10^6$

Burnout for $\boldsymbol{\beta = 1200}$: $E_{\beta = 1200} = 4.346\times10^6$

We see that gravity energy **loss** is significantly lower at **18**% for $\beta = 1200$ kg/s, and hence, is a **viable** option for improving the **efficiency** of the ascent mission.

In **fact** we see that we get '**zero**' gravity loss for $\beta = \infty$ (or **impulsive** launch), but altitude is **zero**.

Drawback of Higher Burn Rate

Therefore, **higher** burn rate, though **beneficial** from total energy point of **view**, has the a few **drawbacks**.

Firstly, we see that the **altitude** is lower which may have an **impact** on the desired terminal **performance**.

Secondly, a larger velocity occurs in **lower** (and denser) atmosphere, which can have **both** control and aerodynamic related **implications**.

Gravity Loss Expression

We can arrive at closed form **expression** for the gravity loss as a **function** of the burn rate, as **shown** below.

$$V_\beta = V_{ideal} - g_0\left(\Lambda\frac{m_0}{\beta}\right); \quad h_\beta = \frac{m_0 g_0 I_{sp}}{\beta}\left[(1-\Lambda)\ln(1-\Lambda)+\Lambda\right] - \frac{1}{2}g_0\left(\Lambda\frac{m_0}{\beta}\right)^2$$

$$\Delta E = \frac{1}{2}V_{ideal}^2 - \frac{1}{2}V_\beta^2 - g_0 h_\beta = V_{ideal}\, g_0\left(\Lambda\frac{m_0}{\beta}\right) - \frac{1}{2}g_0^2\left(\Lambda\frac{m_0}{\beta}\right)^2 - g_0 h_\beta$$

$$= V_{ideal}\, g_0\left(\Lambda\frac{m_0}{\beta}\right) - \cancel{\frac{1}{2}g_0^2\left(\Lambda\frac{m_0}{\beta}\right)^2} - \frac{m_0 g_0^2 I_{sp}}{\beta}\left[(1-\Lambda)\ln(1-\Lambda)+\Lambda\right] + \cancel{\frac{1}{2}g_0^2\left(\Lambda\frac{m_0}{\beta}\right)^2}$$

$$= -\frac{m_0 g_0^2 I_{sp}}{\beta}\Lambda\ln(1-\Lambda) - \frac{m_0 g_0^2 I_{sp}}{\beta}\left[(1-\Lambda)\ln(1-\Lambda)+\Lambda\right] = -\frac{m_0 g_0^2 I_{sp}}{\beta}\left[\ln(1-\Lambda)+\Lambda\right]$$

Summary

Thus, to **summarize**, impact of gravity is to **reduce** the terminal total **energy**, in comparison to ideal **burnout**.

We have also noted that the **above** loss is inversely proportional to the **burn** rate.

Lastly, we have also established that while **we** can reduce the loss by **increasing** burn rate, there is an **impact** on the trajectory and aerodynamic **effects** that needs study.