Constant Pitch Rate Problems

Problem No. 01

Consider the following rocket **configuration**.

m_0 = 74 Tons, m_p = 54 Tons, I_{sp} = 240 s, g_0 = 9.81m/s^2, V_0 = 85.4 m/s, $\theta_0 = 5^o$, h_0 = 415 m.

Determine all the **terminal** parameters in case **rocket** executes the above manoeuvre for another **90s**, assuming sea-level **gravity** and vacuum condition.

Solution No. 01

The V, θ **solutions** are as follows.

$$q_0 = \frac{\tilde{g}\sin\theta_0}{V_0} = \frac{9.81\times\sin 5^o}{85.4} = 0.01 rad/s = 0.573^o/s$$

$$\theta_{i=90} = \theta_0 + q_0\Delta t = 5 + 0.573\times 90 = 56.6^o$$

$$V_{i=90} = \frac{\tilde{g}\sin\theta_{90}}{q_0} = \frac{9.81\times 0.835}{0.01} = 819 m/s$$

Solution No. 01

The mass, x and y **solutions** are as follows.

$$\ln\frac{m_0}{m_{t=90}} = \frac{2\tilde{g}}{q_0 g_0 I_{sp}}(\sin\theta_{t=90} - \sin\theta_0) = 0.62 \rightarrow m_{90} = 39.7T$$

$$h_{t=90} = 415 + \frac{9.81}{4\times 0.01^2}(0.9848 + 0.3939) = 33813 \text{ m}$$ +415

$$x_{t=90} = \frac{9.81}{2\times 0.01^2}\left[0.900 - \frac{(\sin 2\theta_{t=90} - \sin 2\theta_0)}{2}\right] = 25862m$$

Problem No. 02

Can all the **propellant** can be burnt to **reach** 90^o**?** If yes**, give** final burnout **parameters.** If no**,** give the **trajectory** parameters when all the **propellant** is consumed.

Solution No. 02

No. All the propellant **cannot** be consumed while reaching 90^o. The burnout **trajectory** parameters as are follows.

$$t_{\theta=90} = \frac{90-5}{0.573} = 148.3s; \quad V_{90} = \frac{\tilde{g}\sin 90}{q_0} = \frac{9.81}{0.01} = 981 m/s$$

$$\ln\frac{m_0}{m_{\theta=90}} = \frac{2\tilde{g}}{q_0 g_0 I_{sp}}(\sin 90 - \sin\theta_0) = 0.7607 \rightarrow m_{\theta=90} = 34.6T$$

$$h_{\theta=90} = 415 + \frac{9.81}{4\times 0.01^2}(0.9848 + 1.000) = 49092 \text{ m}$$

$$x_{\theta=90} = \frac{9.81}{2\times 0.01^2}\left[1.4837 - \frac{(\sin 180 - \sin 2\theta_0)}{2}\right] = 77034m$$

Problem No. 03

What should be θ_0 and q_0, for the same V_0, if all **fuel** is to be burnt for θ_b = **90°**? Also, determine the **mission** terminal parameters in this case?

Solution No. 03

Here, **we** know mass **fraction** and final angle so we can **obtain** initial angle through **solution** of two nonlinear equations, as **follows**.

$$V_0 = 85.4 = \frac{9.81 \times \sin\theta_0}{q_0}; \quad \ln\frac{m_{t-5}}{m_{\theta-90}} = 1.308 = \frac{2}{q_0 \times 240}(1.000 - \sin\theta_0)$$

$$\sin\theta_0 = 8.705 q_0; \quad \frac{2}{q_0 \times 240}(1.000 - 8.705 q_0) = 1.308; \quad q_0 = 0.00604$$

$$\sin\theta_0 = 8.705 \times 0.00604 = 0.00526 \rightarrow \theta_0 = 3.012^o; \quad t_b = 5 + \frac{90 - 3.012}{0.00604 \times 57.29} = 251.4s$$

$$V_b = \frac{9.81 \times \sin 90}{0.00604} = 1624.2 m/s; \quad h_b = 415 + \frac{9.81}{4 \times 0.00604^2}(0.9945 + 1.000) = 134080 \text{ m}$$

$$x_b = \frac{9.81}{2 \times 0.00604^2}\left[1.518 - \frac{(\sin 180 - \sin 2\theta_0)}{2}\right] = 211147m$$