AE 234 Aircraft Propulsion
Quiz 1

- This exam is for 90 minutes, and counts for 20 points.
- You should scan the answer sheet and upload it on moodle within the given time.
- This is an open book exam.

Airbus A380 has four turbofan engines of the Rolls-Royce Trent 900 series. These engines have a bypass ratio of 8.5 and handle around 1,200 kg/s of air mass flow rate at sea level take-off conditions, producing a peak thrust of 340 kN. The bypass ratio drops to 7.1 under cruise conditions. The aircraft cruises at Mach 0.85 at an altitude of 35,000 ft. The aircraft has a lift-to-drag ratio of 19 and a range of 14,800 km. Let the bypass exhaust jet speed is 80% of the core exhaust jet speed, the fuel-air ratio in the core engine be 0.02.
The take-off mass of A380 is 575 tons. The airplane has an empty mass of 277 tons. For this study, consider a payload mass of 44 tons. Let the fuel consumption for take-off and climb be 2% of the take-off weight and that another 0.2% of the take-off weight for descent and landing. Fuel reserves of 4% of take-off weight are retained after landing. The Trent 900 engine is said to generate a thrust of 62,275 N during A380 cruise.

Question 1 . . . . *6 points*

Draw the schematic of a turbofan engine similar to the description of the Trent 900 engine. Clearly label the components and state the processes happening in each of the component using a sentence (or two) each. Provide a qualitative plot of the variation of the flow velocity, temperature and pressure along the engine.

Question 2 . . . . *4 points*

Let $\beta = V_b/V_c$ be the ratio of the speeds of the bypass exhaust jet and core exhaust jet. Obtain the expressions for thrust and the three efficiencies of a turbofan engine in terms of the bypass ratio (α), β and any other appropriate non-dimensional parameter(s).

Question 3 . . . . *6 points*

Obtain the following from the data given above: core and bypass exhaust jet speeds, efficiencies and TSFC.

Question 4 . . . . *1 points*

The engine thrust under cruise conditions is given as 62,275 N. Is this a realistic condition for A380?

Question 5 . . . . *3 points*

For a take-off speed of 110 m/s, calculate the TSFC for the engine at take-off for Mumbai in March. Assume the engine thermal efficiency to the same at cruise and take-off conditions.

Question 2

We have $\beta = V_b/V_c$ and $r = V_a/V_c$. Assuming the exhaust streams to be perfectly expanded, thrust can be written as

$$\begin{aligned}\mathcal{T} &= \dot{m}_c\left([1+f]\,V_c - V_a\right) + \dot{m}_b\left(V_b - V_a\right)\\ &= \dot{m}_c\left\{\left([1+f]\,V_c - V_a\right) + \alpha\left(\beta V_c - V_a\right)\right\} = \dot{m}_a\left\{\left(\frac{1+f+\alpha\beta}{1+\alpha}\right)V_c - V_a\right\}\end{aligned}$$

So, we can write $\mathcal{T} = \dot{m}_a\left(V_e - V_a\right)$, and the effective exhaust velocity is

$$V_e = \frac{1+f+\alpha\beta}{1+\alpha}V_c$$

We can neglect the fuel fraction wherever possible. Power to vehicle is

$$\begin{aligned}\mathcal{P}_v = \mathcal{T}V_a &= \dot{m}_a V_a\left\{\left(\frac{1+\alpha\beta}{1+\alpha}\right)V_c - V_a\right\}\\ &= \dot{m}_a V_c^2 r\left(\frac{1+\alpha\beta}{1+\alpha} - r\right)\end{aligned}$$

Jet power is

$$\begin{aligned}\mathcal{P}_j &= \frac{1}{2}\dot{m}_c\left(V_c^2 - V_a^2\right) + \frac{1}{2}\dot{m}_b\left(V_b^2 - V_a^2\right)\\ &= \frac{1}{2}\dot{m}_c V_c^2\left\{\left(1-r^2\right) + \alpha\left(\beta^2 - r^2\right)\right\}\\ &= \frac{1}{2}\dot{m}_a V_c^2\left\{\frac{1+\alpha\beta^2}{1+\alpha} - r^2\right\}\end{aligned}$$

Fuel energy is

$$\mathcal{P}_f = \dot{m}_f Q_R \equiv f\dot{m}_c Q_R = \frac{\dot{m}_a}{1+\alpha}fQ_R$$

So, the efficiencies are

$$\begin{aligned}\text{Propulsive: } \eta_p &= \frac{\mathcal{P}_v}{\mathcal{P}_j} = \frac{\dot{m}_a V_c^2 r\left(\frac{1+\alpha\beta}{1+\alpha} - r\right)}{\frac{1}{2}\dot{m}_a V_c^2\left\{\frac{1+\alpha\beta^2}{1+\alpha} - r^2\right\}} = 2r\frac{(1+\alpha\beta) - r\,(1+\alpha)}{(1+\alpha\beta^2) - r^2\,(1+\alpha)}\\ &= 2r\frac{(1-r) + \alpha\,(\beta - r)}{(1-r^2) + \alpha\left(\beta^2 - r^2\right)}\\ \text{Thermal: } \eta_{th} &= \frac{\mathcal{P}_j}{\mathcal{P}_f} = \frac{\frac{1}{2}\dot{m}_a V_c^2\left\{\frac{1+\alpha\beta^2}{1+\alpha} - r^2\right\}}{\frac{\dot{m}_a}{1+\alpha}fQ_R} = \frac{V_c^2}{2fQ_R}\left\{\left(1-r^2\right) + \alpha\left(\beta^2 - r^2\right)\right\}\\ \eta_{th} &= \frac{\left(1-r^2\right)}{E}\left(1 + \alpha\frac{\beta^2 - r^2}{1-r^2}\right)\\ \text{Overall: } \eta_{ov} &= \frac{\mathcal{P}_v}{\mathcal{P}_f} = \frac{\dot{m}_a V_c^2 r\left(\frac{1+\alpha\beta}{1+\alpha} - r\right)}{\frac{\dot{m}_a}{1+\alpha}fQ_R} = \frac{2r\,(1-r)}{E}\left\{1 + \alpha\left(\frac{\beta - r}{1-r}\right)\right\}\end{aligned}$$

where $E = V_c^2/(2f\mathcal{Q}_R)$ is the ratio of jet absolute kinetic energy to the chemical energy added from to the stream.

For $\alpha = 0$, the above expressions reduce to those derived in the lectures.

Question 3

Range is given by the below expression:

$$R = \frac{V_a}{g_0 \mathsf{TSFC}} \frac{\mathcal{L}}{\mathcal{D}} \ln \frac{\mathcal{M}_{ini}}{\mathcal{M}_{fin}} \implies \mathsf{TSFC} = \frac{V_a}{g_0 R} \frac{\mathcal{L}}{\mathcal{D}} \ln \frac{\mathcal{M}_{ini}}{\mathcal{M}_{fin}}$$

The temperature at 35,000 ft is 218.80 K as per the ISA model, giving a vehicle speed of $V_a = 252 m/s$. Knowing that the temperature stays constant at around 216 K beyond 11 km, a reasonable range to guess would be between 200-220 K. An alternative approach would be to directly guess the aircraft speed to be 250 m/s.
Using the given information, we obtain TSFC = 16.2 mg/N-s.

$$\eta_{ov} = \frac{\mathcal{T} V_a}{\dot{m}_f \mathcal{Q}_R} \equiv \frac{V_a}{\mathsf{TSFC}\mathcal{Q}_R} \implies \mathsf{TSFC} = \frac{V_a}{\eta_{ov} \mathcal{Q}_R}$$

$$\mathsf{TSFC} = \frac{\dot{m}_f}{\mathcal{T}} = \frac{f\dot{m}_c}{\dot{m}_c \{(1 + f + \alpha\beta) V_c - (1+\alpha) V_a\}}$$
$$\implies V_c = \left(\frac{1}{1 + f + \alpha\beta}\right) \left\{\frac{f}{\mathsf{TSFC}} + (1+\alpha) V_a\right\}$$

For $\alpha = 7.1$, $\beta = 0.8$ and $f = 0.02$, we obtain, $V_c = 452\ m/s$, $V_b = 362\ m/s$ and $V_e = 374\ m/s$. Using the formulas from Question 2, we get $\eta_p = 0.80$, $\eta_{th} = 0.46$ and $\eta_{ov} = 0.37$.

Question 4

For level flight, we have

$$\mathcal{T}_{engine} = \frac{1}{4}\mathcal{T}_{total} = \frac{1}{4}\mathcal{M}_{a/c} g_0 \frac{\mathcal{D}}{\mathcal{L}}$$
$$\implies \mathcal{M}_{a/c} = 4 \frac{\mathcal{T}_{engine}}{g_0} \frac{\mathcal{L}}{\mathcal{D}}$$

For $\mathcal{T}_{engine} = 62,250\ N$, we get $\mathcal{M}_{a/c} = 483$ tons, which falls within the range of variation of aircraft mass during cruise.

Question 5

At sealevel, we are given the flight speed, mass flowrate, thrust and α values. These values differ from those of Question 3.

Mumbai in February/March is dry. So, we can neglect humidity in air. Modifying the thrust expressions, we get:

$$V_c = \left(\frac{1}{1+f+\alpha\beta}\right)\left\{\frac{(1+\alpha)\,\mathcal{T}}{\dot{m}_a} + (1+\alpha)\,V_a\right\}$$

From the exhaust speeds, we calculate efficiencies, and obtain find that TSFC $= 13mg/N-s$ from the overall efficiency.

The ambient air temperature at sea level can be taken as 300 K (around 30^oC). For accuracy one can try scaling the mass flowrate inversely with temperature. If we consider this reduces mass flowrate, for the same thrust, we get TSFC $= 13.3mg/N-s$.