MA 214: Introduction to Numerical Analysis

Indian Institute of Technology Bombay Quiz 3

Marks: 15 Date: 27-03-2019

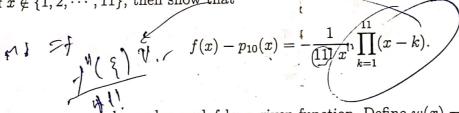
Time: 1 hour Instructors: S. Baskar and S. Sivaji Ganesh

- (1) Write your Name, Roll Number, and Tutorial Batch clearly on your answer book as well as every supplement you may use. A penalty of -1 mark will be awarded for failing to do so.
- (2) Number the pages of your answer book and make a question-page index on the front page.
- (3) The answer to each question should start on a new page. If the answer for a question is split into two parts and written in two different places, the first part alone will be corrected.
- (4) Only scientific calculators are allowed. Any kind of programing device is not allowed.
- (5) Formulas used need not be proved but needs to be stated clearly.
- (6) The question paper contains 4 questions. Answer all the questions.
- (1) (a) Draw the graph of a function for which the Newton-Raphson iterates satisfy $x_0=x_3=0$ (in the usual notation of Newton-Raphson method). Justify your answer graphically.
 - (b) Draw the graph of a function for which secant method iterates satisfy $x_0 = 0$, $x_1 = 3$, and $x_2 = 1$, $x_3 = 2$ (in the usual notation of secant method). Justify your answer graphically.

[3 Marks]

[Note: You need not define the functions explicitly]

(2) Let $p_{10}(x)$ be the interpolating polynomial for the function $f(x) = \frac{1}{x}$ at the nodes $\{1, 2, \dots, 11\}$. + . (?) If $x \notin \{1, 2, \dots, 11\}$, then show that



[4 Marks]

(3) Let x_0, x_1, \dots, x_n be nodes, and f be a given function. Define $w(x) = \prod_{i=0}^n (x - x_i)$. Prove that

$$f[x_0, x_1, \cdots, x_n] = \sum_{i=0}^n \frac{f(x_i)}{w'(x_i)}$$

[4 Marks]

Let $g:[a,b] \to [a,b]$ be a continuously differentiable function with $\lambda:=\max_{x\in [a,b]}|g'(x)|<1$. Let $\{x_n\}$ be a sequence generated by the fixed point iteration method with iteration function g. Assume that the sequence $\{x_n\}$ converges to the fixed point x^* .

(a) Show that

$$|x_{n+1} - x^*| \le \frac{\lambda}{1-\lambda} |x_{n+1} - x_n|$$

(b) Assuming that $x^* \neq x_n$ for all $n = 0, 1, 2, \cdots$, show that

$$\lim_{n\to\infty} \frac{x^* - x_{n+1}}{x^* - x_n} = g'(x^*).$$

[4 Marks]