

AE 234/711 Aircraft Propulsion

Tutorial 6

1. **Compression** - Ideal vs Real (with dissipation) The fuel-air mixture inside a cylinder is initially occupying a volume \mathcal{V}_i . It has a mass m_i , pressure p_i , specific internal energy e_i and a temperature T_i . This mixture needs to be energised by adding energy of magnitude \mathcal{E} to the system. Compare the various ways of adding this energy, through isentropic, isobaric, isochoric and isothermal processes. Calculate the final state for each of these processes. Represent these processes on $T - s$ and $p - v$ plots. Comment on the relative effectiveness of these processes.

2. A ramjet is to propel an aircraft at Mach 3 at high altitude where the ambient pressure is 8.5 kPa and the ambient temperature T_a is 220 K. The temperature of the gases leaving the combustor is 2540 K.

If all components of the engine are ideal - that is, frictionless - determine the thrust, thermal, propulsion and overall efficiencies of the engine.

Repeat the calculations for $\eta_d = 0.7$, $\eta_b = 0.95$ and $\eta_n = 0.98$.

Let the specific heat ratio be $\gamma = 1.4$ and make the approximations appropriate to $f \ll 1$.

Verify your results with the given plots.

Ideal: $\mathcal{T}/\dot{m}_a = 1000 \text{ m/s}$, TSFC = $0.045 \text{ kg/kN} - s$, $\eta_p = 0.68$, $\eta_{th} = 0.64$, $\eta_{ov} = 0.44$

Real: $\mathcal{T}/\dot{m}_a = 931.9 \text{ m/s}$, TSFC = $0.049 \text{ kg/kN} - s$, $\eta_p = 0.70$, $\eta_{th} = 0.58$, $\eta_{ov} = 0.41$

These values are close to the values for $M = 3$ in the plots on page 2.

3. Given in the table on the next page are the JANAF coefficients that describe the thermodynamic properties of N_2 . Plot the variation of c_p vs temperature for $T \in [200, 6000]$. Estimate the error in assuming a constant value for c_p . **Compare your results with the tabulated values at <https://janaf.nist.gov/tables/N-023.html>**

We typically assume that the thermodynamic properties (h , s etc.) and transport properties (μ , κ etc.) of gases vary in a simple manner, easy for our calculations. However, they vary nonlinearly with temperature.

US armed forces had put in a lot of effort to compile these properties for individual species. Tabulated data, called the JANAF (Joint Army Navy and AirForce) tables, were first available for different temperature levels; later curvefits were generated for these properties. The curvefits are of this form

$$\frac{1}{R}c_p(T) = a_1T^{-2} + a_2T^{-1} + a_3 + a_4T + a_5T^2 + a_6T^3 + a_7T^4$$

The following expressions for specific enthalpy and entropy follow from the above express for c_p

$$\begin{aligned} \frac{1}{RT}h(T) &= -a_1T^{-2} + a_2\frac{1}{T}\ln T + a_3 + \frac{1}{2}a_4T + \frac{1}{3}a_5T^2 + \frac{1}{4}a_6T^3 + \frac{1}{5}a_7T^4 + b_1T^{-1} \\ \frac{1}{R}s(T) &= -\frac{1}{2}a_1T^{-2} - a_2T^{-1} + a_3\ln T + a_4T + \frac{1}{2}a_5T^2 + \frac{1}{3}a_6T^3 + \frac{1}{4}a_7T^4 + b_2 \end{aligned}$$

200 < T < 1000	a_1	a_2	a_3	a_4
	$2.210371497 \cdot 10^4$	$-3.818461820 \cdot 10^2$	6.082738360	$-8.530914410 \cdot 10^{-3}$
a_5	a_6	a_7	b_1	b_2
$1.384646189 \cdot 10^{-5}$	$-9.625793620 \cdot 10^{-9}$	$2.519705809 \cdot 10^{-12}$	$7.108460860 \cdot 10^2$	-10.76003744
1000 < T < 6000	a_1	a_2	a_3	a_4
	$5.877124060 \cdot 10^5$	$-2.239249073 \cdot 10^3$	6.066949220	$-6.139685500 \cdot 10^{-4}$
a_5	a_6	a_7	b_1	b_2
$1.491806679 \cdot 10^{-7}$	$-1.923105485 \cdot 10^{-11}$	$1.061954386 \cdot 10^{-15}$	$1.283210415 \cdot 10^4$	-15.86640027

Values of the coefficients for most chemicals at temperatures upto 20,000 K can be found in the report NASA/TP-2002-211556.

Specific enthalpy at 298.15 K for N_2 is 8670.104 J/mol . where $\mathcal{R} = 8.314510 \text{ J}/(\text{mol}\cdot\text{K})$ and h is in J/mol. Molecular weight for N_2 is 28.0134 gm/mol.

Data for various other species can also be found at <https://janaf.nist.gov/>

