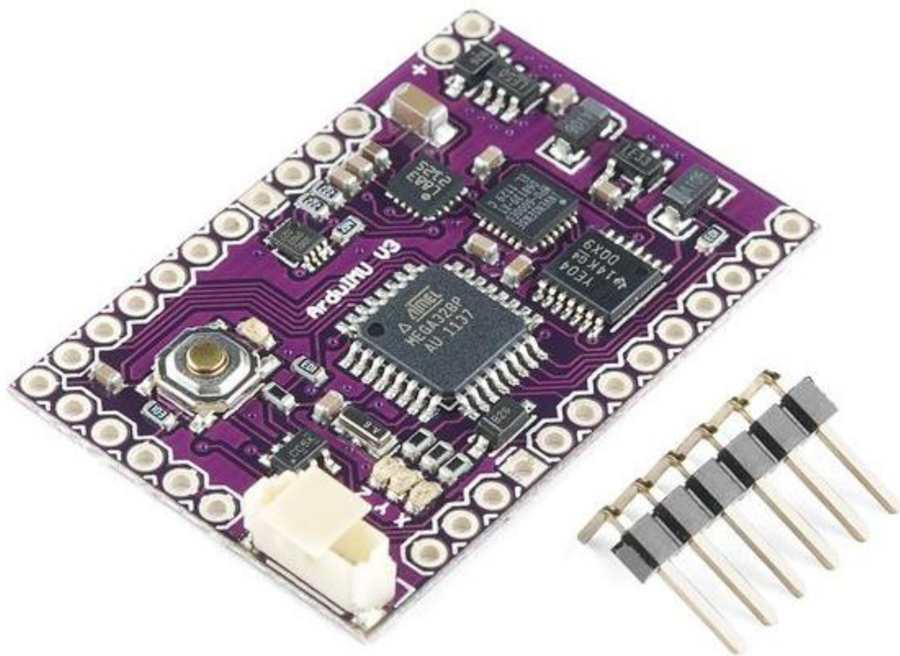
AE 242
Aerospace Measurements Laboratory

Displacement and velocity estimation using Inertial Measurement Unit

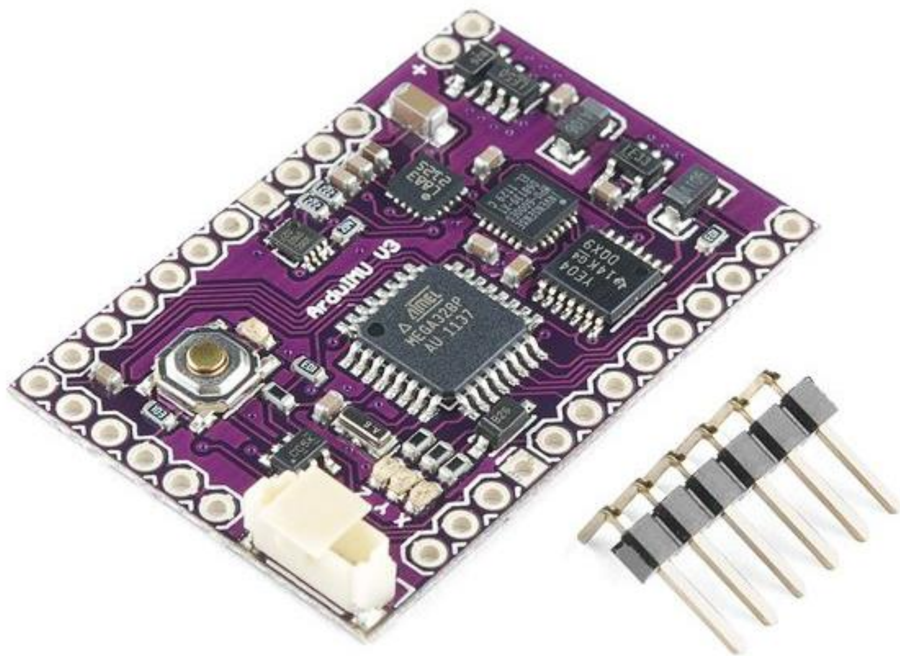


3 axis – accelerometer

3 axis – gyroscope

3 axis - Magnetometer

Displacement and velocity estimation using Inertial Measurement Unit



3 axis – accelerometer: Calibration

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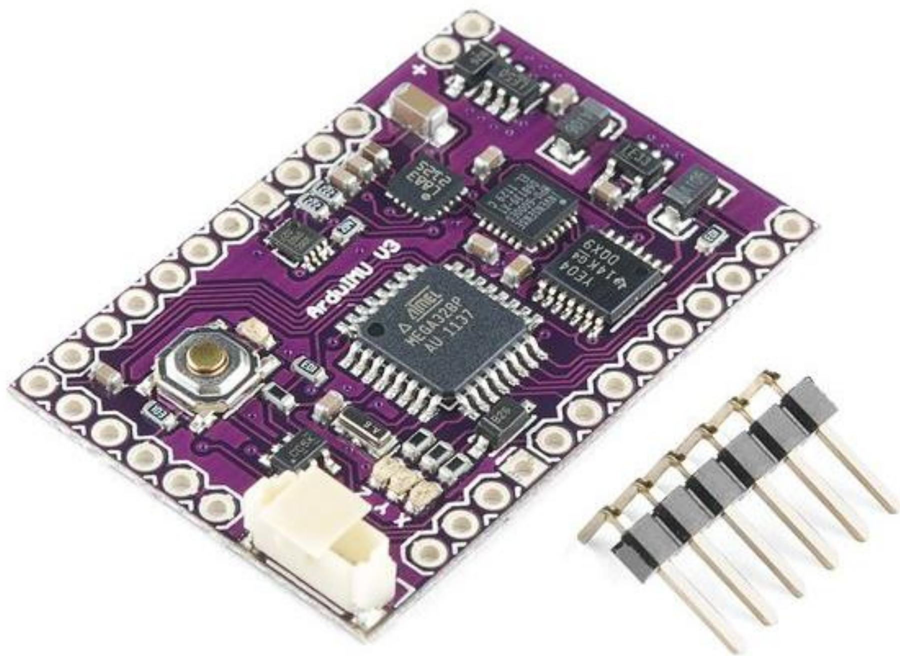
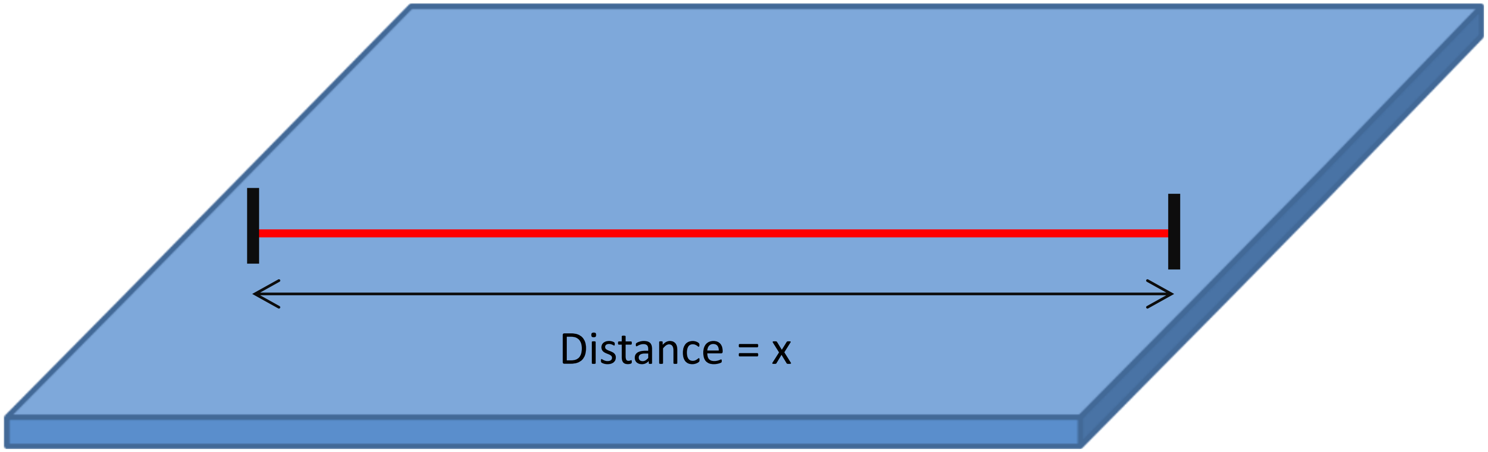


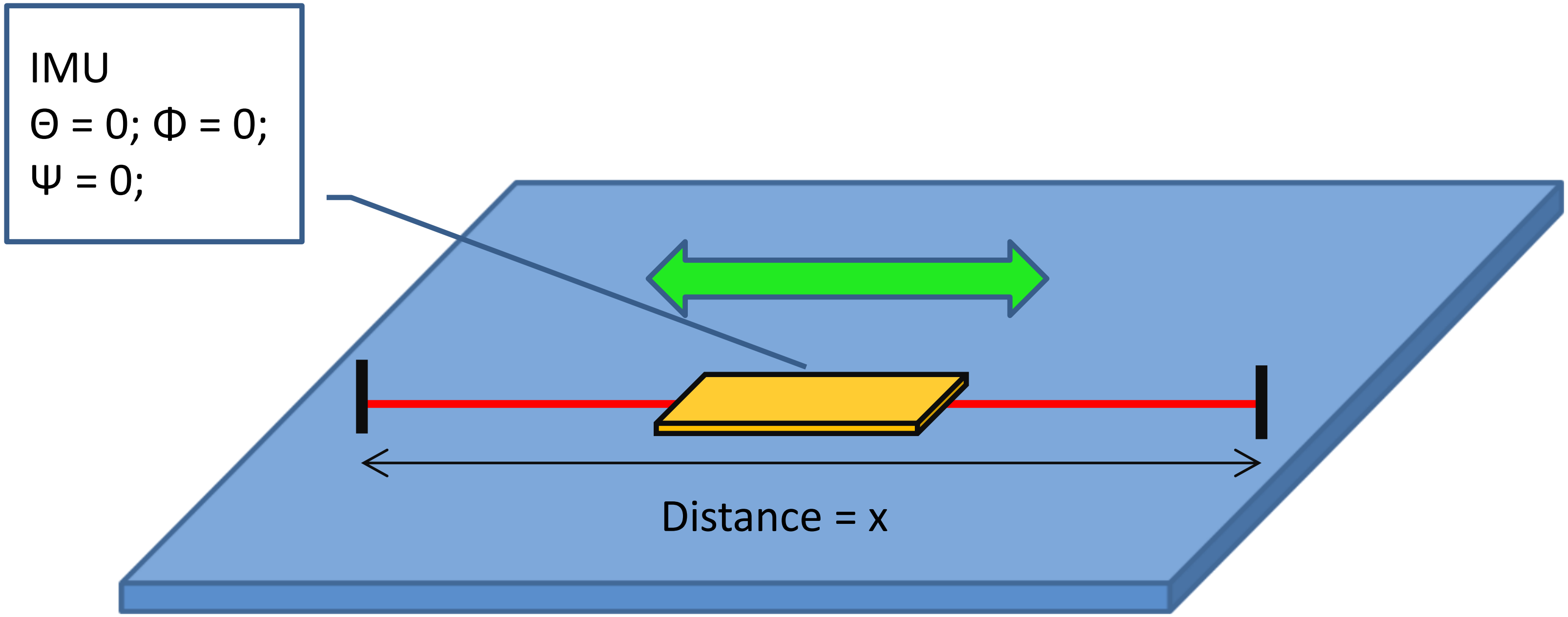
3 axis – accelerometer: Calibration

Calibration – static
By observing gravity

Displacement and velocity estimation using Inertial Measurement Unit

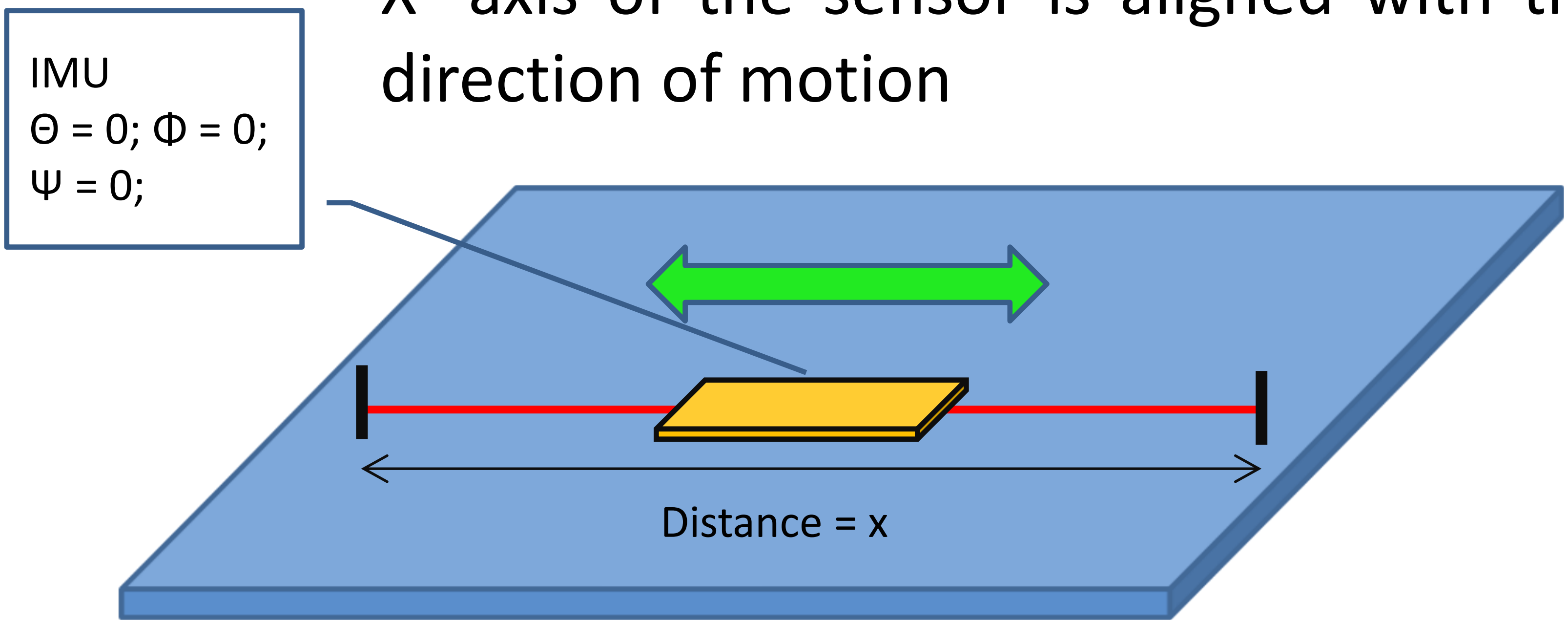


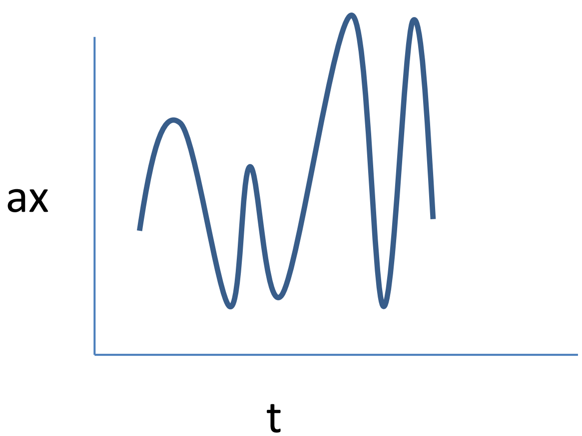
Displacement and velocity estimation using IMU



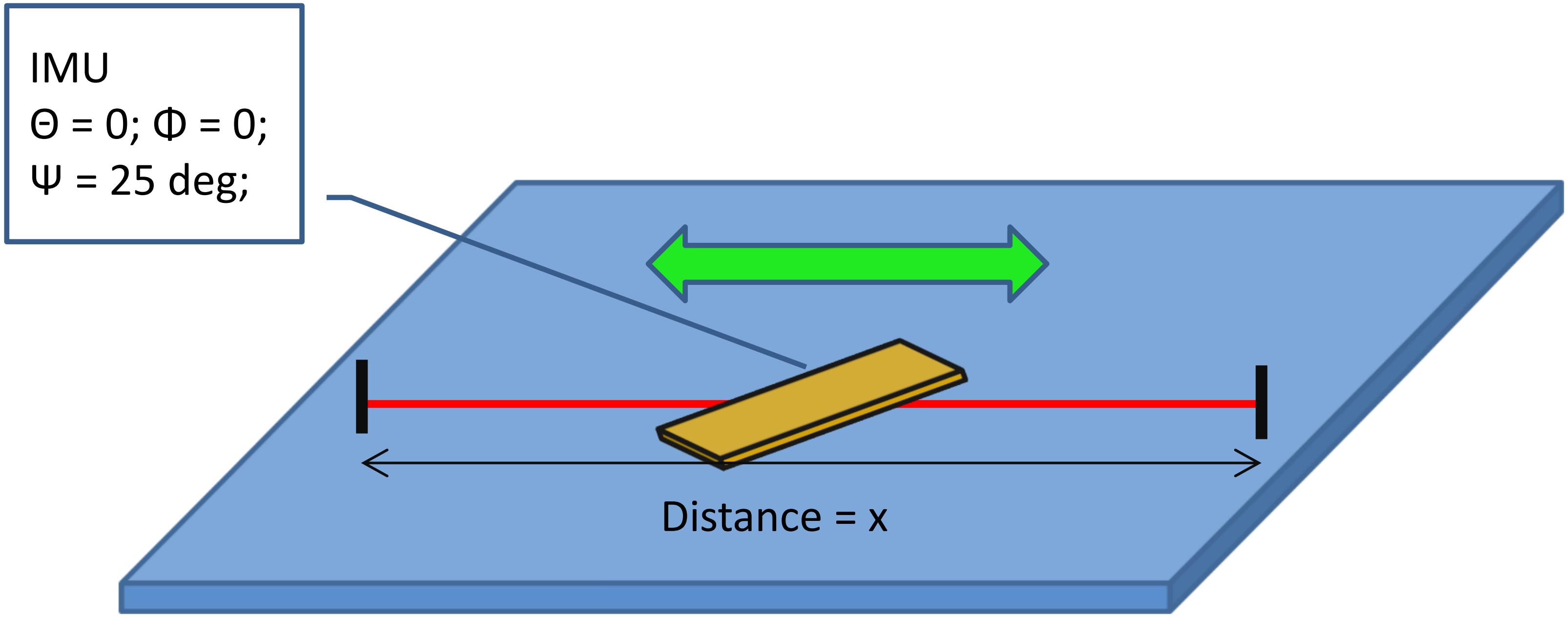
Displacement and velocity estimation using IMU

X- axis of the sensor is aligned with the direction of motion



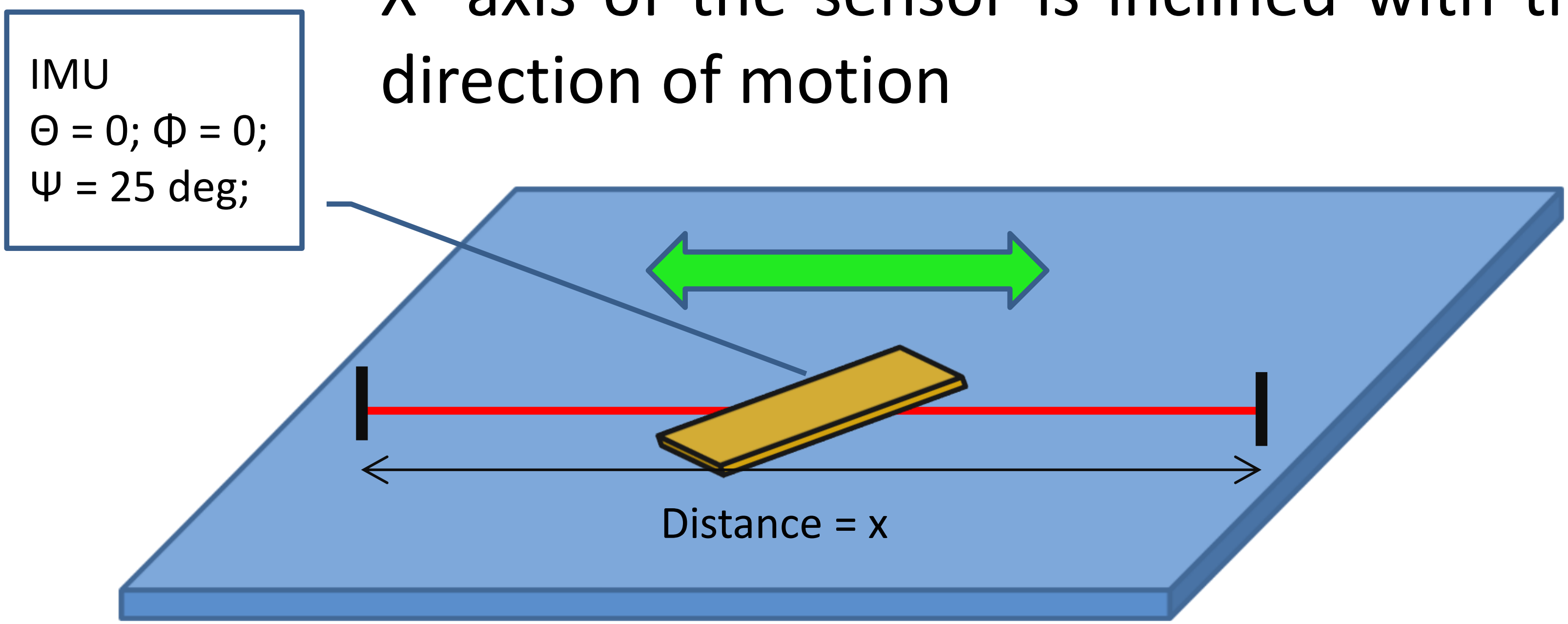


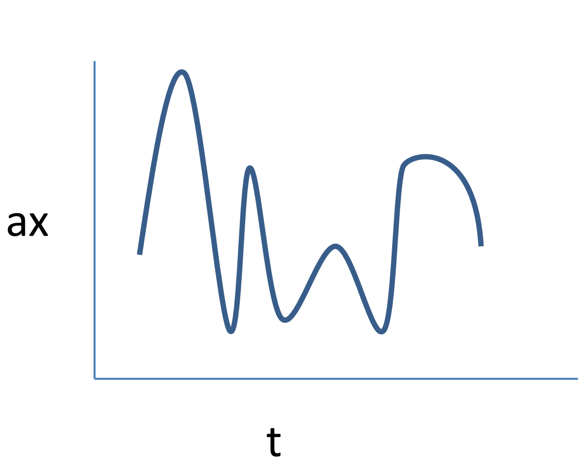
Displacement and velocity estimation using IMU

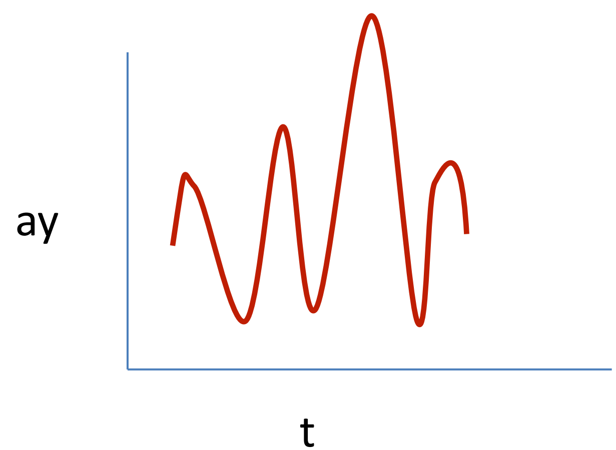


Displacement and velocity estimation using IMU

X- axis of the sensor is inclined with the direction of motion



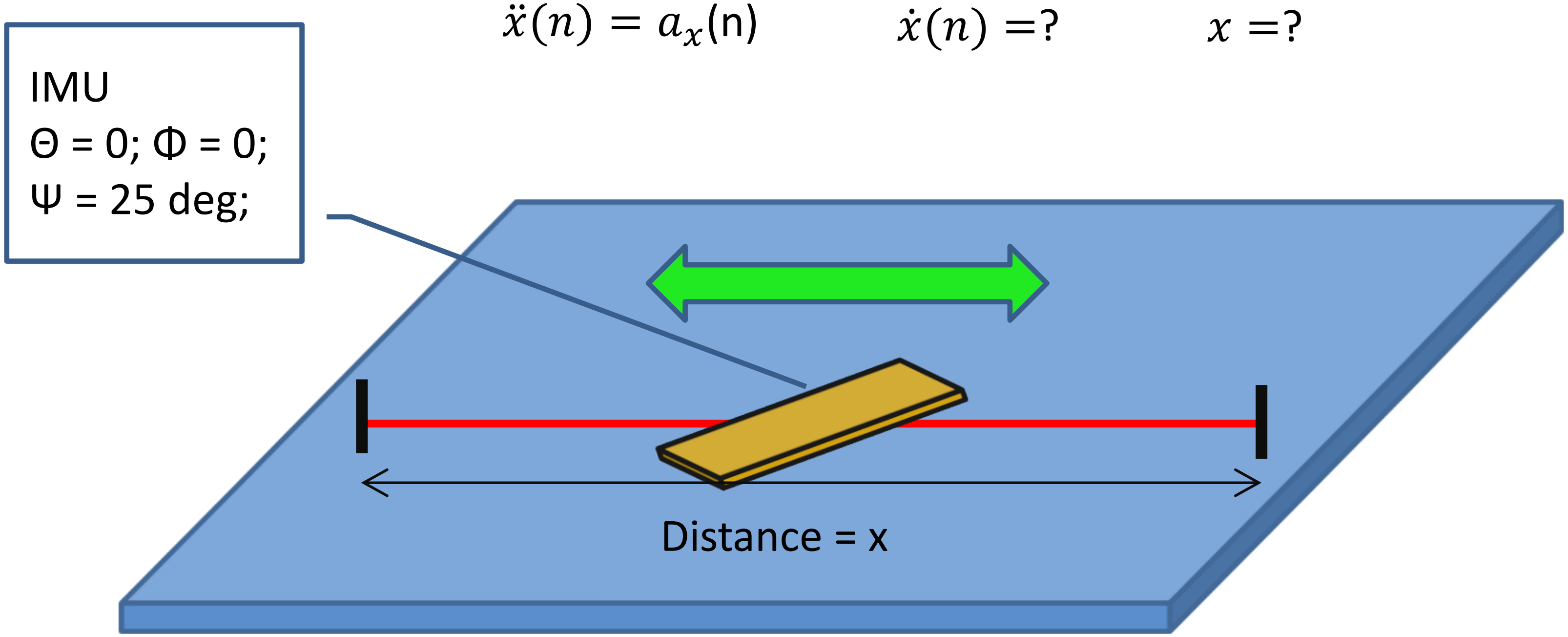


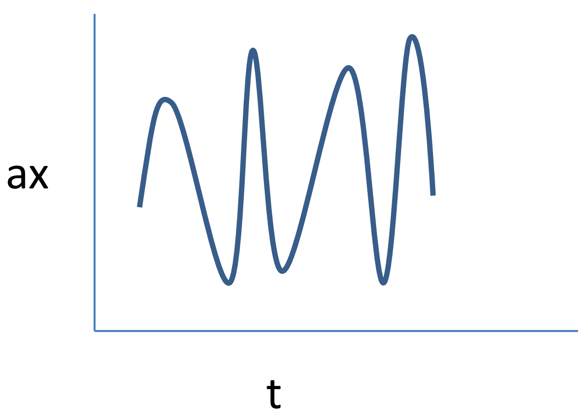


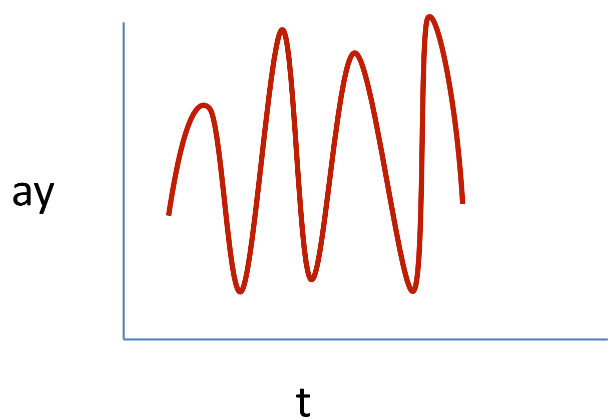
Displacement and velocity estimation using IMU

$\ddot{x} = a_x$(t)

$\ddot{x}(n) = a_x$(n) $\quad\quad \dot{x}(n) = ?$ $\quad\quad x = ?$







Solution methods for ODEs

Set of simultaneous differential equations – linear or non-linear

Solution methods : a) Analytically

b) Analog computation (old method)

c) Digital simulation

Analytical solutions are limited to linear equations and very few non-linear problems. These solutions are used for verification of the models.

Analog computation was popular when digital computers were not available. Linear problems are easy to solve. Simulation setup is complex. Limited by saturation voltages.

Solution methods for ODEs

$$\frac{\mathbf{dx}}{\mathbf{dt}} = \mathbf{f}(\mathbf{t}, \mathbf{x}) \qquad \mathbf{x}(\mathbf{t}_0) = \mathbf{x}_0$$

First order initial value problem

where $\mathbf{x}(\mathbf{t}) = [\mathbf{x}_1(\mathbf{t}), \mathbf{x}_2(\mathbf{t}), \ldots\ldots, \mathbf{x}_\mathbf{n}(\mathbf{t})]$ states of the system

Initial conditions $\mathbf{x}(0) = [\mathbf{x}_1(0), \mathbf{x}_2(0), \ldots\ldots, \mathbf{x}_\mathbf{n}(0)]$

In general n^{th} order differential equation can be converted into n first order differential equation

Solution methods for ODEs – Euler Method

In limit $\dfrac{\mathbf{dx}}{\mathbf{dt}} = \mathbf{f}(\mathbf{t}, \mathbf{x})$ Equals to $\lim\limits_{\mathbf{h} \to 0} \dfrac{\mathbf{x}(\mathbf{t} + \mathbf{h}) - \mathbf{x}(\mathbf{t})}{\mathbf{h}} = \mathbf{f}(\mathbf{t}, \mathbf{x})$

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If h (time step) is small, above function can be approximated as

$$\mathbf{x}(\mathbf{t}+\mathbf{h}) \approx \mathbf{x}(\mathbf{t}) + \mathbf{hf}(\mathbf{t}, \mathbf{x})$$

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Let $\mathbf{t} = \mathbf{t}_{\mathbf{k}} = \mathbf{hk} + \mathbf{t}_0$ where $\mathbf{t}_0 \le \mathbf{t} \le \mathbf{t}_{\mathbf{n}}$ $\mathbf{k} = 0,1,2,\ldots,\mathbf{n}$

Above equation reduces to

$$x\big(h(k+1)+t_0\big) \approx x\big(hk+t_0\big) + hf\left[hk+t_0, x\big(hk+t_0\big)\right]$$

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Introducing new variable x(k)

$$\boxed{x(k+1) = x(k) + hf\left[t(k), x(k)\right]}$$

Value at k+1 is value at k + step x slope at k

Solution methods for ODEs

Analytical solution not possible. Numerical methods are used and solution is obtained at discrete steps. Intermediate information is not available.