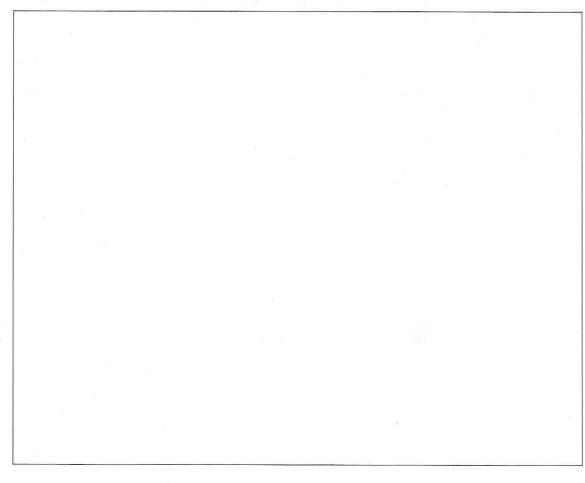




(a) (1 Mark) Then a value of $k \in \mathbb{N}$ such that $\|e^{(k)}\|_{\infty} \le 10^{-3}$ is

(b) (1 Mark) Justification for your answer in (a) is

11. (2 Marks) Assuming that every diagonally dominant matrix is invertible, show that each eigenvalue of a matrix belongs to at least one of the Gerschgorin disks associated to it.





Spring 2017, IIT Bombay

MA 214: Numerical Analysis

Midsemester Examination, Code AA

Roll no .:

 $\mathsf{Tut}.\mathsf{Batch}\ T$

- 1. Duration of the exam is 2 hours. This exam has 11 questions, for a total of 30 marks in 4
- 2. Answer all the questions, in the space provided at the end of each question. ONLY FINAL Answer must be written, except for Questions 10(b) and 11.
- 1. Let A denote the matrix

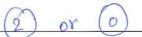
$$A = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 8 & 0 \\ 1 & 0 & \alpha \end{pmatrix}$$

(a) (3 Marks) Gerschgorin theorem was used to conclude that the matrix A satisfies Hypothesis (H1) of Power method. The set of all such values of α is

(b) (2 Marks) Gerschgorin theorem was used to conclude that the matrix A has **distinct eigenvalues**. The set of all such values of α is

2. (2 Marks) The set of all $\alpha \in \mathbb{R}$ such that the vector $(\alpha - 2, \alpha, \alpha + 2)^T$ satisfies **Hypothesis** (H3) of Power method for the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -3 \end{pmatrix}$

$$\alpha \neq -2$$



3. (3 Marks) Consider the linear system

$$4x_1 - x_2 - x_3 = 1$$

$$3x_1 + 5x_2 + x_3 = 2$$

$$x_1 - 3x_2 + 6x_3 = 3$$

(a) Let $x^{(0)} = (0,0,0)^T$. Then first two members $x^{(1)}, x^{(2)}$ of the Jacobi iterative sequence for solving the given linear system are (show as many digits after decimal as your calculator allows)





$$n^{(2)} = (.475, .15, .6583333)$$

(b) Let $x^{(0)} = (0,0,0)^T$. Then first two members $x^{(1)}, x^{(2)}$ of the Gauss-Seidel iterative sequence for solving the given linear system are (show as many digits after decimal as your calculator allows)

$$\chi^{(2)} = (.45.833325, 100833345, .42777785)$$

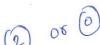
4. (a) (1 Mark) The set of all values of $\beta \in \mathbb{R}$ such that the matrix $\begin{pmatrix} 2 & \beta \\ \beta & 3 \end{pmatrix}$ is **diagonally dominant but NOT** positive definite is

no such B exist



(b) (1 Mark) The set of all values of $\alpha \in \mathbb{R}$ such that the matrix $A = \begin{pmatrix} \alpha & 2 \\ 2 & \alpha \end{pmatrix}$ is **positive definite but NOT diagonally dominant** is

no such a exists



(c) (2 Marks) Let $A = (a_{ij})$ be a positive definite matrix with 5 rows and 5 columns. If $a_{11} = 1$, $a_{22} = 4$, $a_{33} = 9$, $a_{44} = 16$, $a_{55} = 25$, then the value of a_{23} must be greater than or equal to

(c) -48.5

(5) or 65.

5. (2 Marks) The set of all values of $\alpha, \beta \in \mathbb{R}$ such that the matrix $\begin{pmatrix} -1 & \beta & \alpha \\ \beta & 2 & \beta \\ 0 & 0 & \alpha \end{pmatrix}$ has a **doolittle decomposition**

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6. (3 Marks) Let L be a lower triangular matrix having positive real numbers on its diagonal. Further L satisfies

$$LL^T = \begin{pmatrix} 4 & -2 & -4 \\ -2 & 5 & -4 \\ -4 & -4 & 29 \end{pmatrix}.$$

Then L is equal to

$$\begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ -2 & -3 & 4 \end{bmatrix}$$

7. (2 Marks) For solving a linear system of equations Ax = b, an iterative sequence given by $x^{(k+1)} = Bx^{(k)} + c$ (for $k \ge 0$) is proposed, where $B = \begin{pmatrix} 3 & \alpha \\ \beta & -3 \end{pmatrix}$. If the sequence $(x^{(k)})_{k \ge 0}$ converges to the exact solution (assumed to be unique) of the linear system Ax = b for every choice of initial guess $x^{(0)}$, then $\alpha, \beta \in \mathbb{R}$ must be such that

(1) + (1)

-9 < x \beta < -8 R -10 < a \beta < -8 C

8. (2 Marks) Let λ denote any eigenvalue of the matrix $\begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \\ 3 & -4 & -5 \end{pmatrix}$. The optimal bounds $\alpha, \beta \in \mathbb{R}$ such that $\alpha \leq |\lambda| \leq \beta$, obtained using Gerschgorin theorem are given by

1) for each

9. This question is concerned with power method iterations for the matrix $\begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \\ 3 & -4 & -5 \end{pmatrix}$, with initial guess $\boldsymbol{x}^{(0)} = (1,2,3)^T$.

(a) (1 Mark) μ_1 is equal to

(a) <u>- 20</u>

(b) (1 Mark) μ_2 is equal to

(b) 4.6

(c) (1 Mark) $x^{(2)}$ is equal to

(d. 76086, 1, -193478

10. For the linear system in **Question no.** 3, Jacobi method is applied with an initial guess vector $x^{(0)}$ such that the maximum norm of the error $e^{(0)}$ is 1.