AE 236: Compressible Fluid Mechanics

(Module I : Isentropic Flow)

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What is a 'compressible flow'?

Density changes w.r.t. pressure are significant

$\rho = \rho(p, T)$ $\qquad \frac{1}{\rho}\frac{d\rho}{dp}$ compressibility

Temperature changes appreciably ⇒ we need to worry about thermodynamics in addition to the flow dynamics

Primarily gases (gas dynamics)

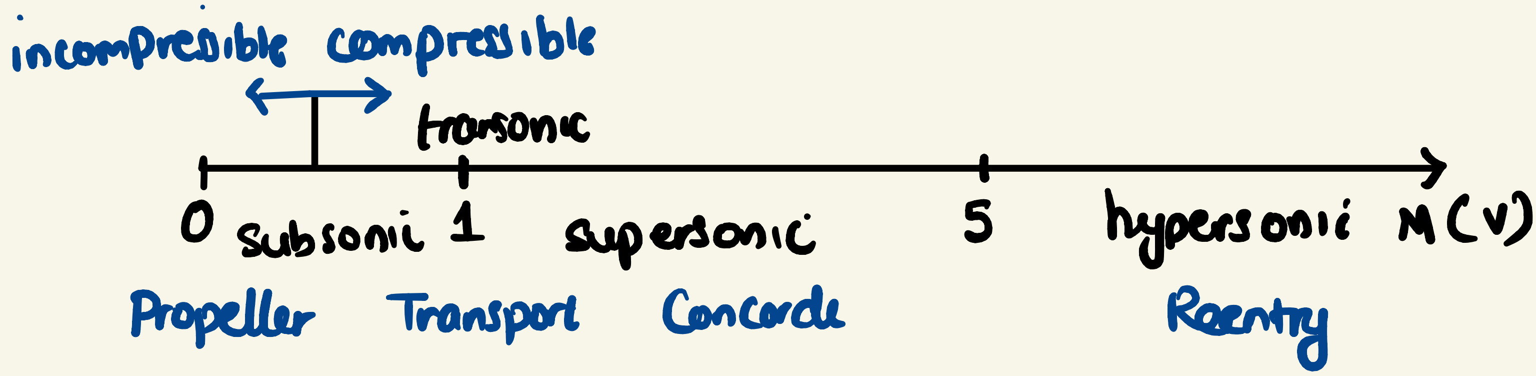
Practical scenarios

Gas turbines - flow through blading and nozzles
Steam turbines
Reciprocating engines
Natural gas transmission lines
'Combustion chambers'

High vs low speeds

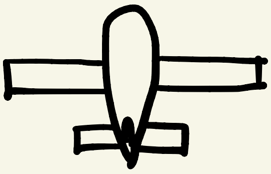
Speed of sound is the reference chosen to compare velocities (a)

$$\text{Mach number } (M) = \frac{\text{gas velocity}}{\text{speed of sound}} = \frac{V}{a}$$

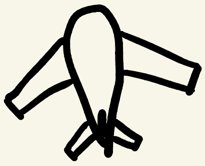


Aircrafts

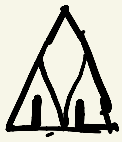
Propeller (low speed)
rounded nose
unswept wings

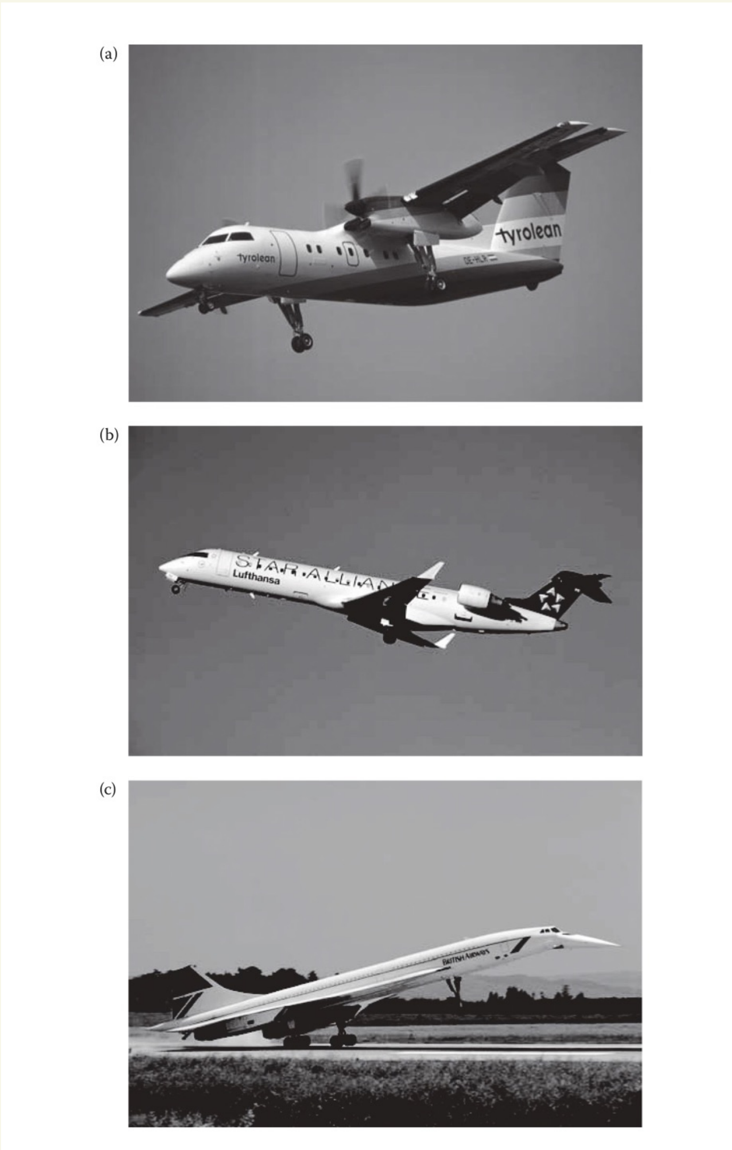


Transport (higher speeds)
rounded nose
swept wings



Concorde (even higher)
sharp nose
more sweep





low subsonic

transonic

M

supersonic

Compressible flow effects play a major role in aircraft design

Fundamental assumptions

1. Gas is continuous — Mean free path $\lambda <$ flow dimensions

2. No chemical changes — No reactions, ionizations, dissociations

3. Gas is perfect

$$p = \rho R T$$ (perfect / ideal gas equation)

$$R = \frac{R_u}{M}$$

R_u = universal gas constant
M = molecular weight

$$= \frac{8314.3}{28.966} = 287.04 \text{ J/kgK (for air)}$$

Calorically perfect — c_p, c_v are constants

$$\gamma = c_p / c_v, \quad c_p - c_v = R$$

Thermally perfect — c_p, c_v are fn's of T

4. Gravitational effects are negligible
No gravitational potential

5. Magnetic & electric effects not considered

6. Effect of viscosity negligible
except close to bodies and shocks

$$F = \mu A \frac{dv}{dy}$$

μ small; this is also small

Under these assumptions, the flow is completely described in terms of following variables

(a) V (b) p (c) ρ (d) T

Conservation laws

Additional assumptions:

1. Flow is steady
2. Not considering cases where gas does work

$\boxed{\text{Mass}}$

$$\cancelto{0}{\text{Rate of change of mass in c.v.}} = \text{Rate of mass entry} - \text{Rate of mass exit}$$

$\boxed{\text{Momentum}}$

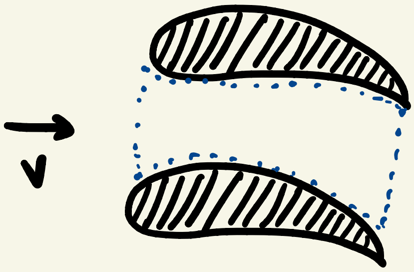
$$\text{Net force acting on the gas in c.v.} = \cancelto{0}{\text{Rate of increase of momentum in the c.v.}} + \text{Rate of momentum exit} - \text{Rate of momentum entry.}$$

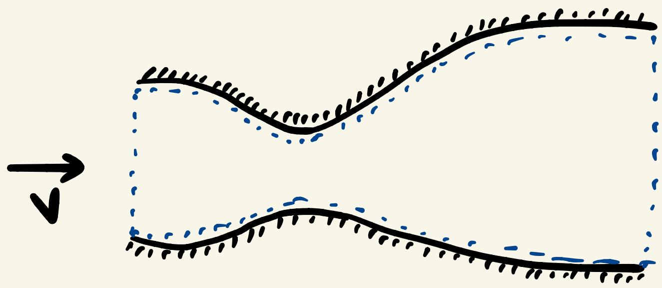
$\boxed{\text{Energy}}$

$$\cancelto{0}{\text{Rate of increase of internal energy}} + \text{Rate of exit of enthalpy \& k.e.} - \text{Rate of entry of enthalpy \& k.e.}$$

$$= \text{Rate heat is transferred to c.v.} - \cancelto{0}{\text{Rate of work done by the gas in c.v}}$$

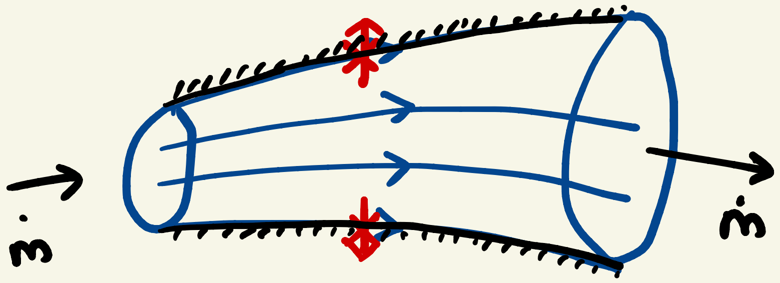
Quasi 1D steady equations





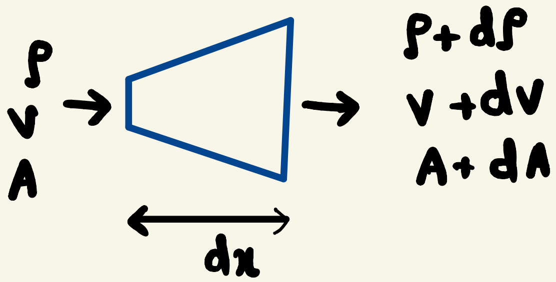
Area and curvature vary slowly with x that you can assume flow is locally 1D $A = A(x)$

Streamtube



Any duct can be treated as a streamtube

Differential control volume



We neglect $dV \cdot d\rho$ and other second and higher order terms

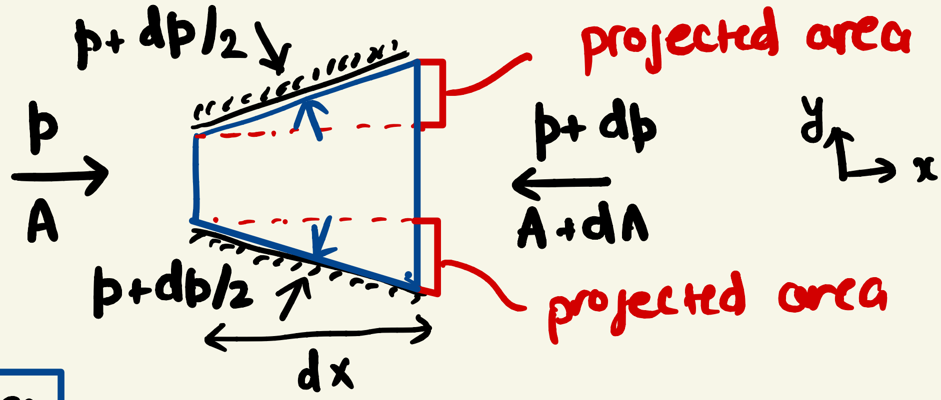
Continuity equation

$$\rho \cancel{V} A = (\rho + d\rho)(V + dV)(A + dA)$$

$$= \rho V dA + A V d\rho + \rho A dV + \rho \cancel{V} A$$

$$\boxed{\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0}$$

Momentum equation (Euler's equation)



Net forces

$$pA - (p+dp)(A+dA) + \left(p+\frac{dp}{2}\right)dA$$

$$= \cancel{pA} - \cancel{pA} - \cancel{pdA} - Adp + \cancel{pdA} = -Adp$$

Momentum flux

$$\rho v A[(v+dv)-v] = \rho v A dv$$

we have

$$\rho v \cancel{A} dv = -\cancel{A} dp$$

$$\boxed{-\frac{dp}{\rho} = v dv}$$

Euler's equation

If velocity increases pressure decreases and vice versa.

For incompressible flow, $\rho =$ const.

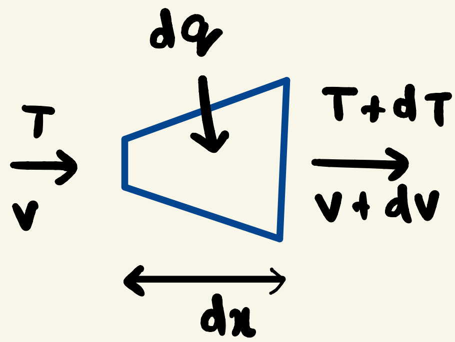
$$\frac{p}{\rho} + \frac{v^2}{2} = \text{const.}$$ Bernoulli's equation

For general flows, we need to know $\rho = \rho(p)$ → Barotropic flows

$$\frac{v^2}{2} + \int \frac{dp}{\rho} = \text{const.}$$

Energy equation

$$h + dh + \frac{(V+dV)^2}{2} = h + \frac{V^2}{2} + dq$$



Now, $h = C_p T$

$$C_p(T+dT) + \frac{(V+dV)^2}{2} = C_p T + \frac{V^2}{2} + dq$$

$$\cancel{C_p T} + C_p dT + \cancel{\frac{V^2}{2}} + V dV = \cancel{C_p T} + \cancel{\frac{V^2}{2}} + dq$$

$$\boxed{C_p dT + V dV = dq}$$

Adiabatic flow : $dq = 0$

$$C_p dT + V dV = 0$$

$$\underbrace{C_p T}_{h} + \frac{V^2}{2} = \text{const}$$

If velocity increases temperature decreases and vice versa.

State equation

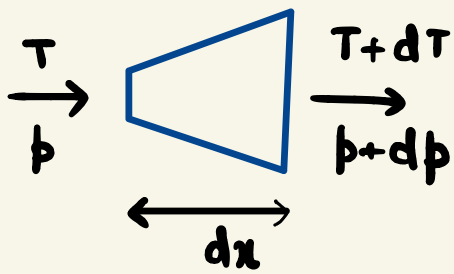
$$\frac{p}{\rho T} = R = \text{const.}$$

$$\cancel{\frac{p}{\rho T}} = \frac{p + dp}{(\rho + d\rho)(T + dT)}$$

$$= \cancel{\frac{p}{\rho T}} \left[\left(1 + \frac{dp}{p}\right)\left(1 - \frac{d\rho}{\rho}\right)\left(1 - \frac{dT}{T}\right) \right]$$

$$\boxed{\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0}$$

Entropy considerations



$$C_p - C_v = R$$

$$C_p / C_v = \gamma$$

$$C_p = \frac{R\gamma}{\gamma - 1}$$

$$C_v = \frac{R}{\gamma - 1}$$

$$T\,ds = dh - dp/\rho$$

$$= C_p\,dT - dp/\rho$$

$$ds = C_p \frac{dT}{T} - R\frac{dp}{p}$$

$$s_2 - s_1 = C_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

$$\frac{s_2 - s_1}{C_p} = \ln\frac{T_2}{T_1} - \frac{\gamma-1}{\gamma}\ln\frac{p_2}{p_1}$$

$$= \ln\left[\left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Isentropic flow : $ds = s_2 - s_1 = 0$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}}$$

Relation b/n p & T

From the state relation, we have

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\cdot\frac{\rho_1}{\rho_2} \Rightarrow \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{p_2}{p_1}\cdot\frac{\rho_1}{\rho_2}$$

$$\boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}}$$

Relation b/n p & ρ

Equivalence of momentum and energy equations

$$ds = c_p \ln\left(\frac{T+dT}{T}\right) - R\ln\left(\frac{p+dp}{p}\right)$$

$$= c_p \frac{dT}{T} - R\frac{dp}{p}$$

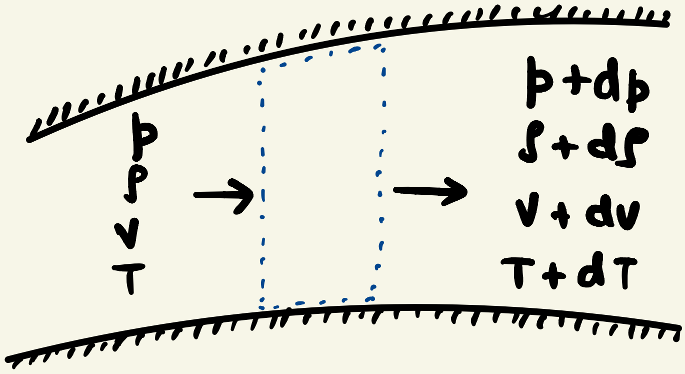
Isentropic flow: $ds = 0$

$$c_p\, dT = \frac{RT}{p}\, dp = \frac{dp}{\rho} = -V dV$$ from energy equation

$\therefore$ we have $\frac{dp}{\rho} = -V dV$ which is Euler's equation

Momentum and energy equations give the same information for isentropic flows.

Isentropic flow in a stream tube



$\boxed{\text{Pressure}}$

$$\frac{dp}{\rho} = -V dV \Rightarrow \frac{dp}{p} = -\underbrace{\frac{\gamma\rho V^2}{\gamma p}}_{\frac{1}{a^2}}\frac{dV}{V}$$

$$\frac{dp}{p} = -\gamma\frac{V^2}{a^2}\frac{dV}{V} = -\gamma M^2\frac{dV}{V}$$

$$\boxed{\frac{dp}{p} = -\gamma M^2 \frac{dV}{V}}$$

Fractional change in p depends on M^2

Temperature

$$C_p\, dT = -v\, dv$$

$$\frac{dT}{T} = -\frac{v}{C_p T}\, dv = -\frac{(\gamma-1)}{\underbrace{T R \gamma}_{a^2}} v^2 \frac{dv}{v}$$

$$\boxed{\frac{dT}{T} = -(\gamma-1) M^2 \frac{dv}{v}}$$

Fractional change in T depends on M^2

Density

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

$$\boxed{\frac{d\rho}{\rho} = -M^2 \frac{dv}{v}}$$

Fractional change in ρ depends on M^2

M is the parameter that determines the importance of compressibility

$$\frac{d\rho/\rho}{dv/v} = -M^2$$

$M = 0.1$ $d\rho = 1\%\, dv$

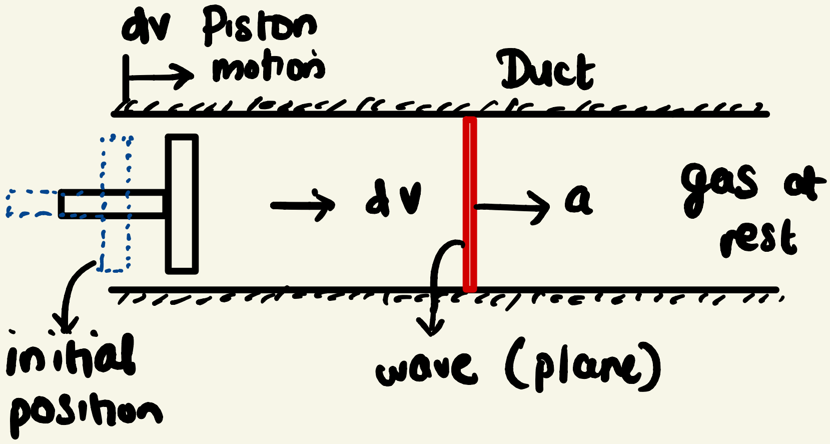
$M = 0.33$ $d\rho = 10\%\, dv$

$M = 0.4$ $d\rho = 16\%\, dv$

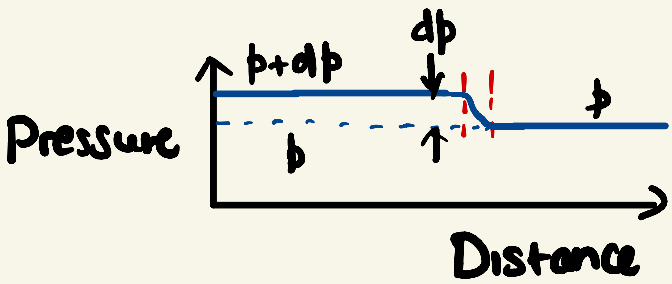
The Mach number at which compressibility becomes important depends on the flow situation and required accuracy.

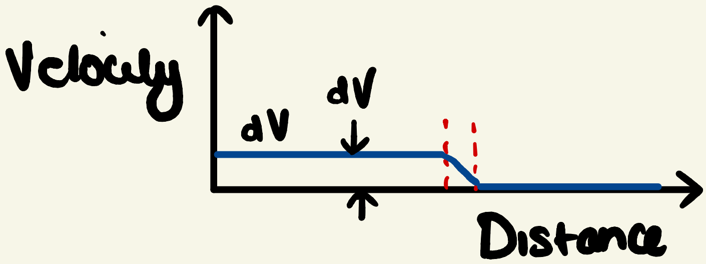
Speed of sound

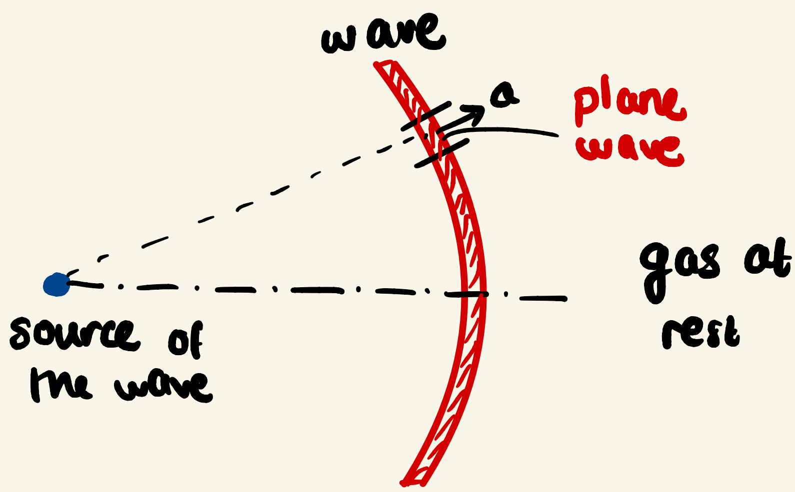
Speed at which very weak pressure waves are transmitted through the gas.



We have a plane wave propagating through the duct.

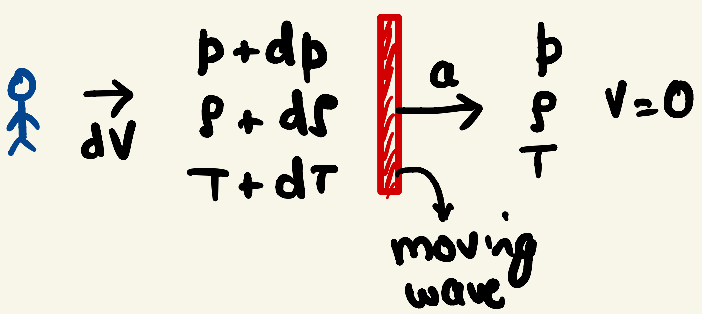




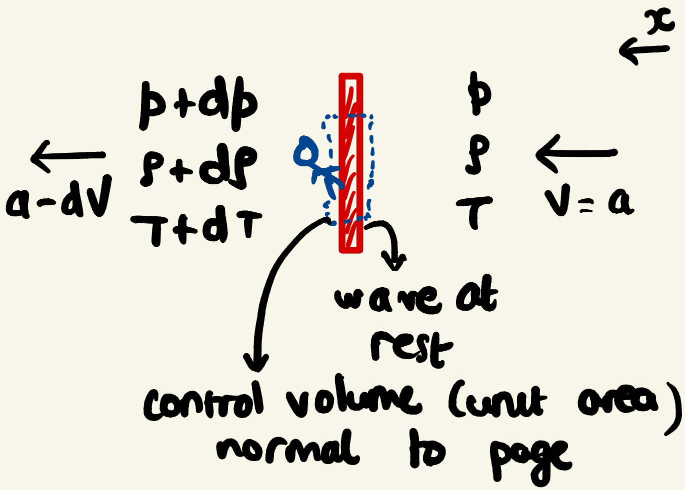


Plane wave is a small, essentially plane portion of a spherical wave moving outward through the gas from a point of disturbance.

Lab frame



Wave frame



Continuity

$$\frac{\dot{m}}{A} = \rho a = (\rho + d\rho)(a - dv)$$

$$d\rho\, a - \rho\, dv = 0$$

$$d\rho = \frac{\rho}{a} dv \quad - (1)$$

Momentum

$$pA - (p + dp)A = \dot{m}[(a - dv) - a]$$

$$-dp = \frac{\dot{m}}{A}(-dv)$$

$$dp = \rho a\, dv \quad - (2)$$

$$(2)/(1) \Rightarrow \frac{dp}{d\rho} = \frac{\rho a}{\rho / a} = a^2$$

$$\boxed{a = \sqrt{\frac{dp}{d\rho}}}$$

To evaluate this expression. we need to know the thermodynamic process the gas undergoes in passing through the wave

Wave is weak $\Rightarrow$ dv, dT are small

i.e. gradients of velocity and temperature are negligible $\Rightarrow$ heat transfer and viscous effects on the flow through the wave are negligible.

Gas undergoes an isentropic process.

For isentropic sound propogation,

$$\frac{p}{\rho^{\gamma}} = \text{const.} = c$$

$$\left.\frac{dp}{d\rho}\right)_s = \gamma c \rho^{\gamma-1} = \frac{\gamma p}{\rho} = \gamma R T$$

$$\boxed{a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma R T}}$$

Speed of sound depends only on the absolute temperature for a given gas.

γ does not vary greatly between gases.

$$a = \sqrt{\gamma \frac{R_u}{m} T}$$

Speed of sound at a given T is approximately inversely proportional to $\sqrt{m}$

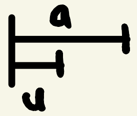
Mach waves

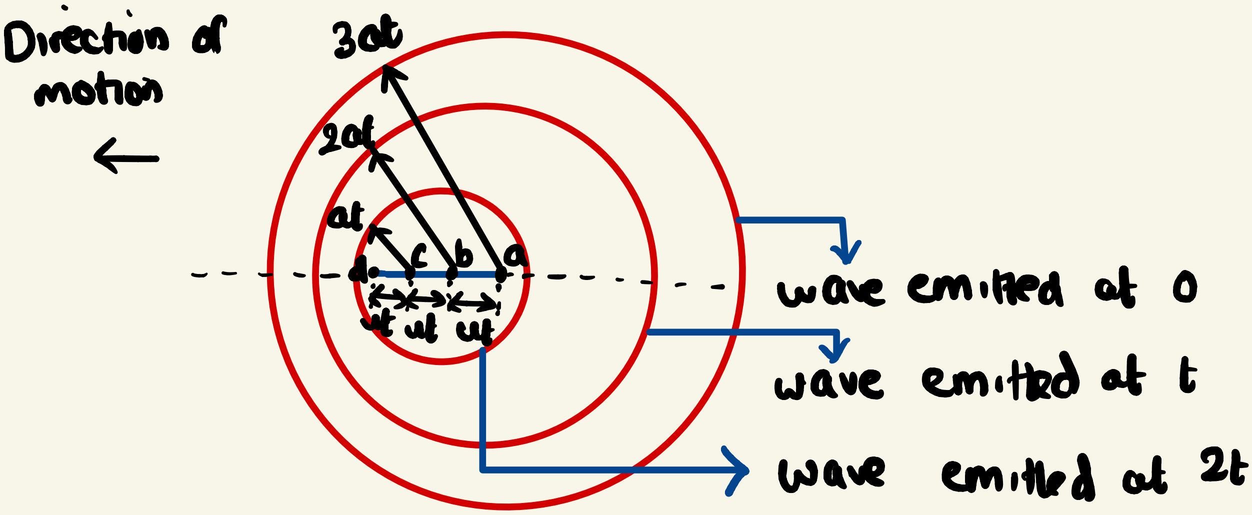
For the gas to move smoothly over a body, disturbances propagate ahead of the body to 'warn' the gas of the approach of the body.

Pressure at the surface of the body > Pressure of the gas nearby

$\Rightarrow$ The disturbances (pressure waves) propagate at the speed of sound.

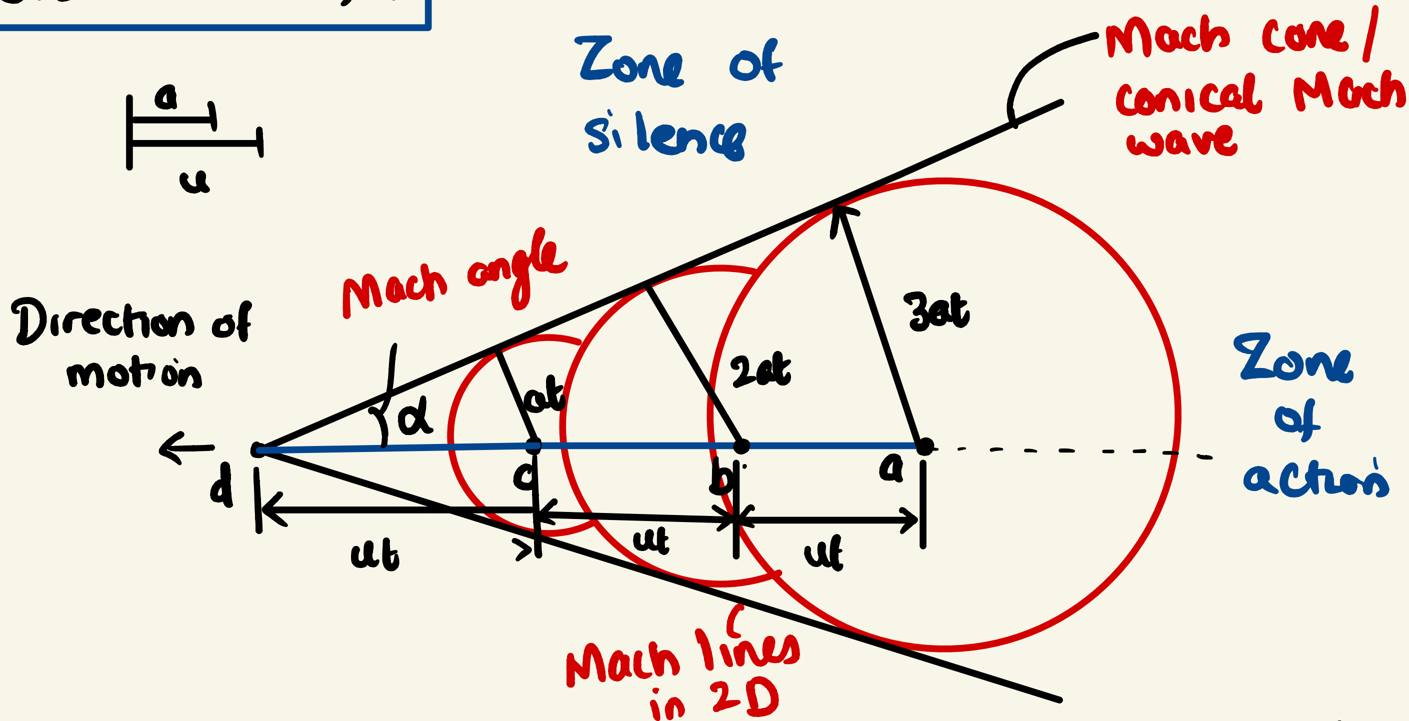
$\boxed{\text{Case 1 : } M < 1}$





waves emitted at times 0, t, 2t as observed at time 3t when the body has moved from a-d.

$\boxed{\text{Case 2 : } M > 1}$



waves emitted at times 0, t, 2t as observed at time 3t when the body has moved from a-d

$$\sin\alpha = \frac{at}{ut} = \frac{1}{M}$$

$$\boxed{\alpha = \sin^{-1}\frac{1}{M}}$$ Mach angle

Used in the measurement of Mach numbers of a gas flow.

Problems

① $M_{max} = 0.91$ @ sea level. Find V_{max} at

(a) $T = 5°C$, (b) $45°C$

$$V_{max} = M_{max} \cdot a$$

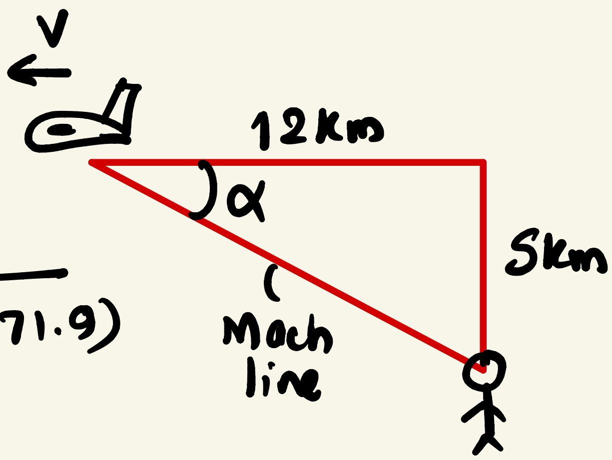
(a) $V_{max}\big|_{5°C} = 0.91 \cdot \sqrt{(1.4)\cdot(287.04)\cdot(278)}$

$= 304 \, m/s$

(b) $V_{max}\big|_{45°C} = 325 \, m/s$

Better to set land speed records on a hot summer day!

② Find v if T @ 5km $= 271.9\,K$



$a = \sqrt{(1.4)\cdot(287.04)\cdot(271.9)}$

$= 330.6 \, m/s$

$\tan\alpha = \frac{5}{12} = 0.417 = \frac{1}{\sqrt{M^2-1}}$

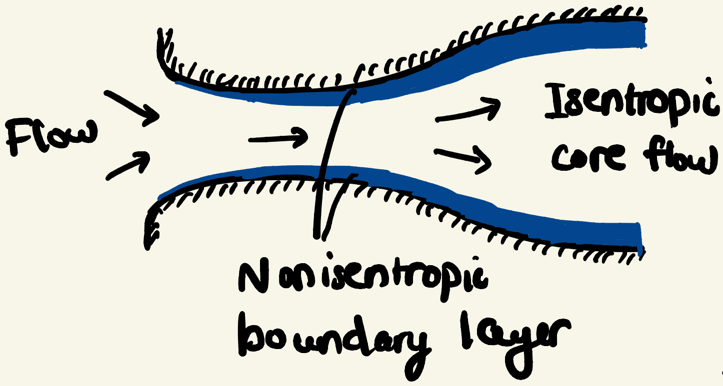
$M = 2.6$

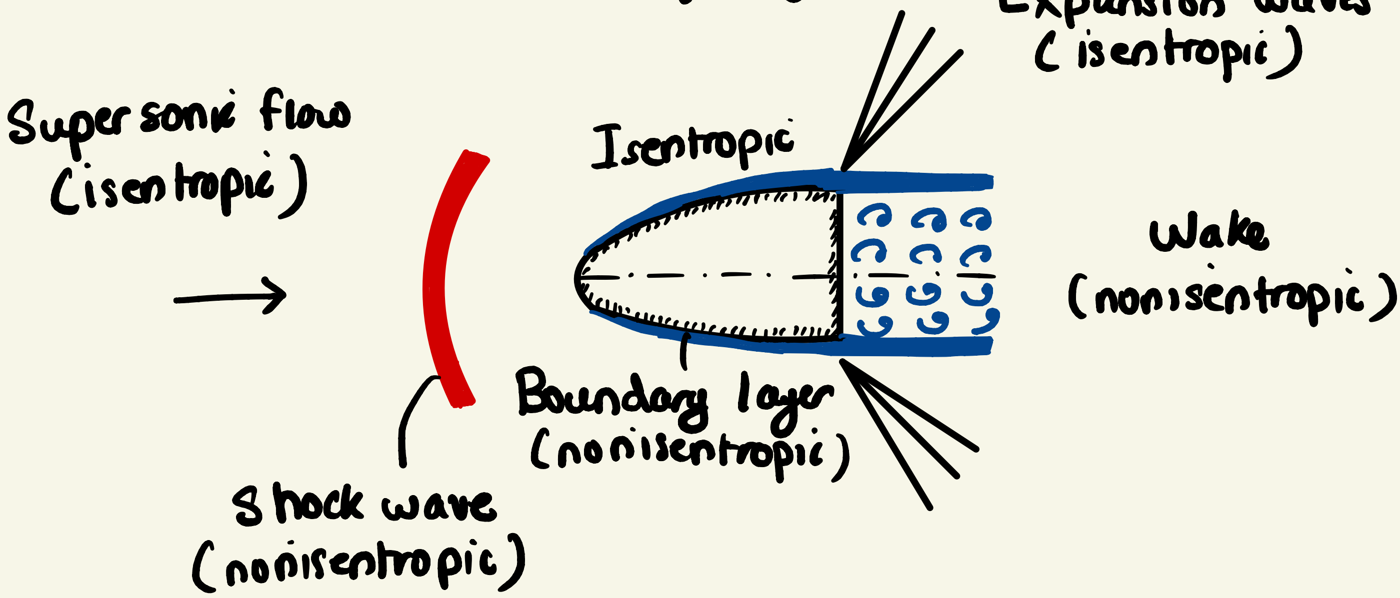
$v = M \cdot a = 859.6 \, m/s$

1D isentropic flow

Effects of viscosity and heat transfer are restricted to the boundary layers, wakes and 'shock waves'.

Internal flow





Governing equations

State equation

$$\frac{p}{\rho^\gamma} = \text{const.} \qquad \frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma$$

$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2} \Rightarrow \frac{T_2}{T_1} = \frac{p_2}{p_1}\cdot\frac{\rho_1}{\rho_2} \; ; \quad a = \sqrt{\gamma R T}$$

$$\boxed{\frac{a_2}{a_1} = \left(\frac{T_2}{T_1}\right)^{1/2} = \left(\frac{\rho_2}{\rho_1}\right)^{\frac{\gamma-1}{2}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{2\gamma}}}$$

Energy equation

$$C_p T_1 + \frac{V_1^2}{2} = C_p T_2 + \frac{V_2^2}{2}$$

$$T_1\left(1 + \frac{V_1^2}{2C_p T_1}\right) = T_2\left(1 + \frac{V_2^2}{2C_p T_2}\right)$$

$$\frac{T_2}{T_1} = \frac{1 + \frac{V_1^2(\gamma-1)}{2\gamma R T_1}}{1 + \frac{V_2^2(\gamma-1)}{2\gamma R T_2}}$$

$$\boxed{\frac{T_2}{T_1} = \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}} \quad \text{— (1)}$$

This equation applies in adiabatic flow

$$\boxed{\frac{p_2}{p_1} = \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}\right]^{\frac{\gamma}{\gamma-1}}} \quad \text{— (2)}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}\right]^{\frac{1}{\gamma-1}}} \quad \text{— (3)}$$

Only valid for isentropic flows

Continuity equation

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

$$\boxed{\left(\frac{\rho_2}{\rho_1}\right)\cdot\left(\frac{V_2}{V_1}\right) = \frac{A_1}{A_2}} \quad \text{— (4)}$$

Eqns. ① - ④ sufficient to determine all features of 1D isentropic flow

Why did we not use momentum equation?

$$\frac{V_2^2}{2} - \frac{V_1^2}{2} + \int_1^2 \frac{dp}{\rho} = 0$$

$$\boxed{\begin{aligned} \frac{p}{\rho^\gamma} &= \frac{p_1}{\rho_1^\gamma} \\ \rho &= \left(\frac{p}{p_1}\right)^{\frac{1}{\gamma}} \rho_1 \end{aligned}}$$

$$\int_1^2 \frac{dp}{\rho} = \int_1^2 \frac{dp}{(p/p_1)^{\frac{1}{\gamma}} \rho_1}$$

$$= \frac{\gamma}{\gamma-1} \frac{p_1}{\rho_1} \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$V_2^2 - V_1^2 + \frac{2}{\gamma-1} a_1^2 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right] = 0$$

$$\left(p_2/p_1\right)^{\frac{\gamma-1}{\gamma}}$$

$$\underbrace{\frac{V_2^2}{a_2^2}}_{M_2^2} \cdot \frac{a_2^2}{a_1^2} - \underbrace{\frac{V_1^2}{a_1^2}}_{M_1^2} + \frac{2}{\gamma-1} \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right] = 0$$

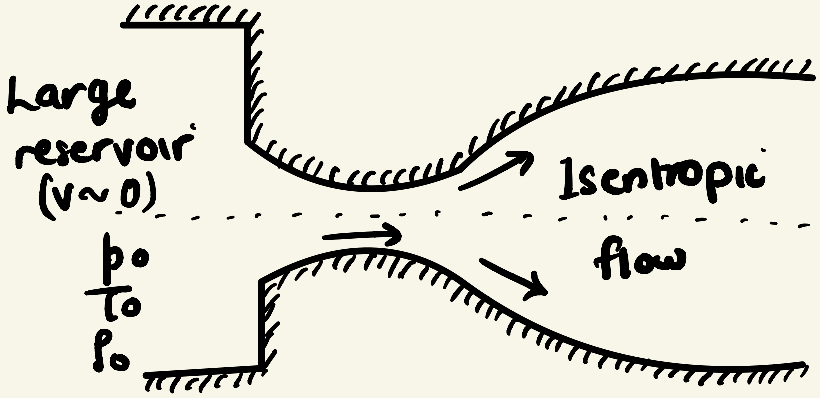
$$1 + \frac{\gamma-1}{2} M_1^2 = \left[1 + \frac{\gamma-1}{2} M_2^2\right] \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\frac{p_2}{p_1} = \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{\frac{\gamma}{\gamma-1}}}$$

which is the expression we derived without momentum equation

Energy equation is simpler to apply than the momentum equation.

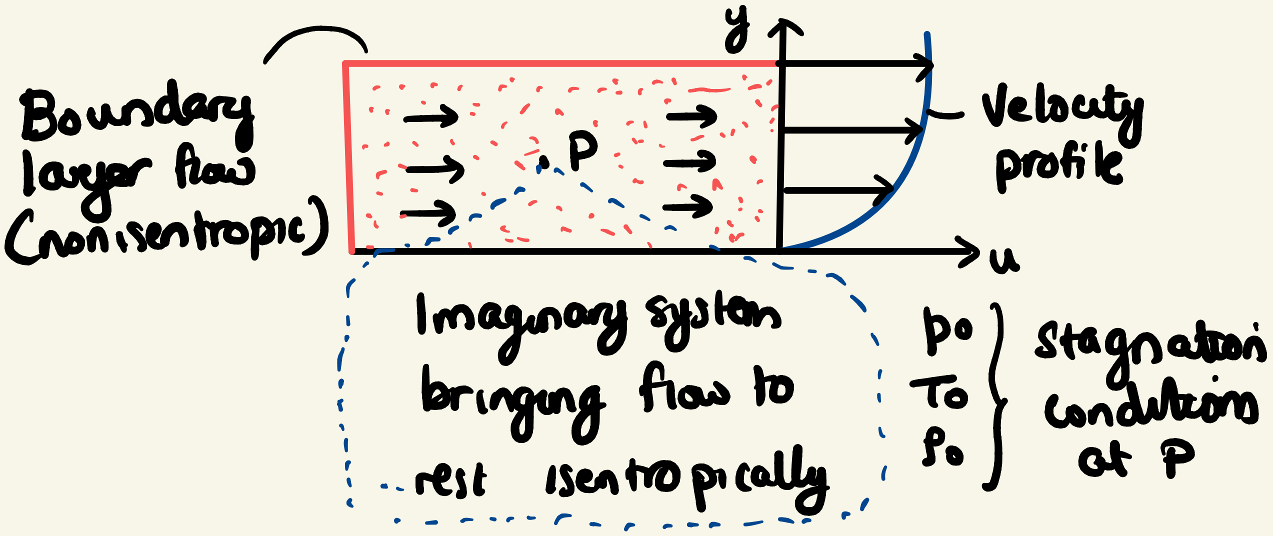
Stagnation conditions



Stagnation condition at all points is p_0, ρ_0, T_0

Stagnation conditions are those that would exist if the flow at any point is brought to rest isentropically

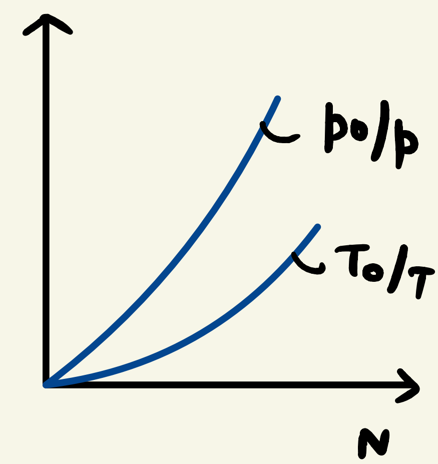
For an isentropic flow, the stagnation conditions at all points in the flow will be those existing at the zero velocity point



Even for non isentropic flows, we can imagine stagnation conditions at any local point

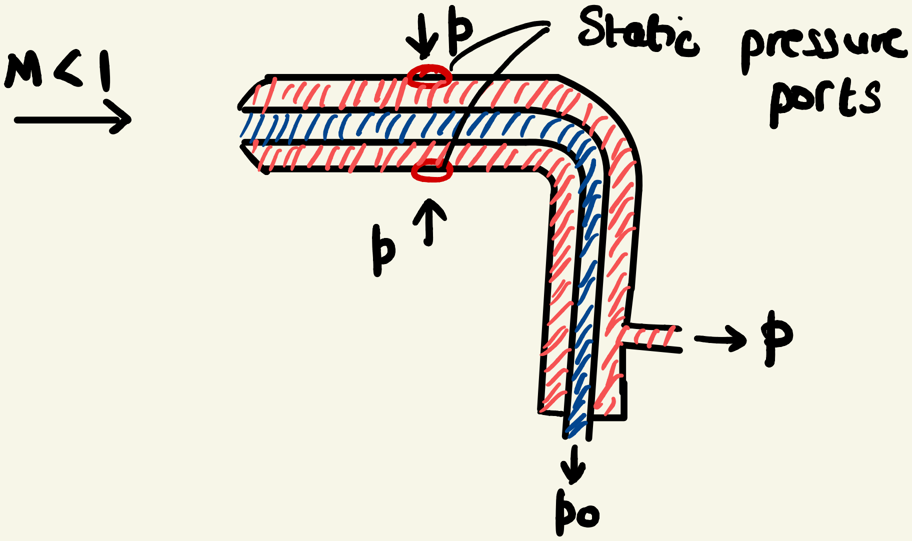
$$\boxed{\begin{aligned}\frac{T_0}{T} &= 1 + \frac{\gamma-1}{2}M^2\\ \frac{p_0}{p} &= \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}\\ \frac{\rho_0}{\rho} &= \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{1}{\gamma-1}}\end{aligned}}$$

For air, $\gamma = 1.4$



Pitot - static tube

Pitot-static can be used to measure Mach number in subsonic flow.



For incompressible flows.

$$p_0 = p + \frac{1}{2}\rho V_{inc}^2$$

$$V_{inc} = \sqrt{\frac{2(p_0 - p)}{\rho}}$$

For compressible flows,

$$\frac{p_0}{p} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow M = \sqrt{\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

$$V_{com} = M a = M\sqrt{\gamma R T}$$

What is the error in using incompressible formula?

$$p_0 - p = p\left[\frac{p_0}{p} - 1\right]$$

$$= \left(\frac{1}{2}\rho V^2\right)\left(\frac{2\gamma p}{\gamma \rho V^2}\right)\left\{\left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}} - 1\right\}$$

$$= \left(\frac{1}{2}\rho V^2\right)\cdot\left(\frac{2}{\gamma M^2}\right)\left\{ \quad '' \quad \right\}$$

$$V_{com} = \sqrt{\frac{2(p_0 - p)}{\rho}}\left\{\frac{2}{\gamma M^2}\left[\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]\right\}^{-\frac{1}{2}}$$

Error using incompressible formula.

$$\epsilon = \left|\frac{V_{com} - V_{inc}}{V_{com}}\right| = \left|1 - \frac{V_{inc}}{V_{com}}\right|$$

$$\boxed{\epsilon = \left|1 - \left\{\left(\frac{2}{\gamma M^2}\right)\left[\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]\right\}^{\frac{1}{2}}\right|}$$

For $M \sim 0.3$, $\epsilon < 1\%$

For $M \sim 0.6$, $\epsilon \sim 5\%$

Problems

③ Subsonic: $p = 96$ kPa, $T = 300$ K, $p_0 - p = 32$ kPa

Find v using (a) incompressible and (b) compressible formula.

$$\rho = \frac{p}{RT} = 1.115 \text{ kg/m}^3$$

(a) $$V_{inc} = \sqrt{\frac{2.(32.10^3)}{1.115}} = 239.6 \text{ m/s}$$

(b) $$\frac{p_0 - p}{p} = \frac{p_0}{p} - 1 = \frac{32}{96}$$

$$\Rightarrow \frac{p_0}{p} = 1.33 = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow M = 0.654$$

$$V_{com} = Ma = M\sqrt{\gamma RT}$$

$$= 225.7 \text{ m/s}$$

To determine velocity in incompressible flow, only Δp $(p_0 - p)$ has to be measured

For compressible flows, to find the Mach number, p_0 & p have to be separately measured and for velocity, temperature also has to be measured (for the speed of sound)

Critical conditions

Conditions that would exist if M is isentropically changed to 1 (v^*, p^*, T^*, ρ^*)

$$\frac{T^*}{T} = \frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2 = \left(\frac{a^*}{a}\right)^2$$

$$\frac{p^*}{p} = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho^*}{\rho} = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{1}{\gamma-1}}$$

Relation b/n stagnation and critical points

$$\frac{T^*}{T_0} = \frac{2}{\gamma+1} = \left(\frac{a^*}{a_0}\right)^2 = 0.833$$

$$\frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.528$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} = 0.634$$

Remember these.

Maximum discharge velocity ($\hat{v}$)

It is the velocity that would be generated if the gas was adiabatically expanded until its temperature had dropped to absolute zero.

Also called maximum escape velocity

$$\frac{\hat{V}^2}{2} = \frac{V^2}{2} + C_p T = C_p T_0$$

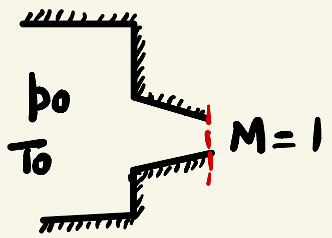
$$\boxed{\hat{V} = \sqrt{2 C_p T_0} = \sqrt{V^2 + 2 C_p T} = \sqrt{\left(V^2 + \frac{2a^2}{\gamma - 1}\right)} = \sqrt{\frac{2a_0^2}{\gamma - 1}}}$$

Maximum velocity that can be generated in a gas for a given T_0 ($M \to \infty$ as $a \to 0$)

Cannot be achieved in reality because at very low T, the underlying assumptions cease to apply (eg. liquefaction)

Problems

④



$p_0 = 300\,kPa$, $T_0 = 50°C$

Find p, T, V @ exit plane for air ($\gamma = 1.4$)

$$p^* = p_0 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 158.5\,kPa$$

$$T^* = T_0 \left(\frac{2}{\gamma+1}\right)$$

$$= 269.2\,K = -3.8°C$$

$$v^* = Ma = a$$

$$= \sqrt{1.4 \cdot 287.04 \cdot 269.2}$$

$$= 328.9 \text{ m/s}$$

What if we had He instead of air?

$$p^*_{He} = 146.1 \text{ kPa}$$

$$T^*_{He} = 242.2 \text{ K} = -30.8°C$$

$$V^*_{He} = 916.1 \text{ m/s}$$

Using gas tables (isentropic)

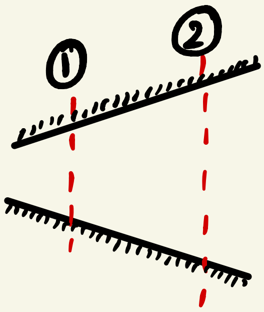
⑤ At some point, $V_1 = 600$ m/s, $p_1 = 70$ kPa, $T_1 = 5°C$ (steady flow of air)

At some other point $p_2 = 30$ kPa

Find M_2, T_2, V_2 assuming 1D isentropic flow.

$$M_1 = 600 / \sqrt{1.4 \cdot 287.04 \cdot 278}$$

$$= 1.8$$



From gas tables @ $M = 1.8$

$$\frac{T_0}{T_1} = 1.65 \qquad \frac{p_0}{p_1} = 5.75$$

given

$$\frac{p_2}{p_1} = \frac{p_0/p_1}{p_0/p_2} \Rightarrow p_0/p_2 = 13.42$$

given — from gas table

From gas tables for this p_0/p_2

$M_2 = 2.345$ $\quad T_0/T_2 = 2.10$

$$\frac{T_2}{T_1} = \frac{T_0/T_1}{T_0/T_2} = \frac{1.65}{2.1} \Rightarrow T_2 = 218.4\,K$$

$$V_2 = 2.345 \cdot \sqrt{1.4 \cdot 287.04 \cdot 218.4} = 694.7\,m/s$$