CHAPTER 4 THE GAS TURBINE CYCLE[1]

4.0 INTRODUCTION

The gas turbine has many important applications but it is most widely used as the jet engine. In the last few years, since the regulations changed to permit natural gas to be burned for electricity generation, gas turbines have become important prime movers for this too. Many of the gas turbines used in land-based and ship-based applications are derived directly from aircraft engines; other gas turbines are designed specifically for land or marine use but based on technology derived for aircraft propulsion.

The attraction of the gas turbine for aircraft propulsion is the large power output in relation to the engine weight and size – it was this which led the pre-Second World War pioneers to work on the gas turbine. Most of the pioneers then had in mind a gas turbine driving a propeller, but Whittle and later von Ohain realised that the exhaust from the turbine could be accelerated to form the propulsive jet.

This chapter looks at the operation of simple gas turbines and outlines the method of calculating the power output and efficiency. The treatment is simplified by treating the working fluid as a perfect gas with the properties of air, but later some examples are discussed to assess the effect of adopting more realistic assumptions. It is assumed throughout that there is a working familiarity with thermodynamics – this is not the place to give a thorough treatment of the first and second laws (something covered very fully in many excellent text books, for example Van Wylen and Sonntag, 1985). Nevertheless, in the appendix to this chapter a brief account is given to remind those whose knowledge of engineering thermodynamics is rusty or to familiarise those who have learned thermodynamics in connection with a different field.

4.1 GAS TURBINE PRINCIPLES

The essential parts of a gas turbine are shown schematically in Fig. 4.1. Air is compressed in the compressor and in the combustor (combustion chamber) fuel is burned in this high-pressure air. The hot, high-pressure gas leaving the combustor enters the turbine. In most cases there are several turbine stages, one or more to drive the compressor, the others to drive the load. The turbine driving the load may be on the same shaft as the compressor or it may be on a separate

[1] Purists will object to this description of a **cycle**. Strictly speaking a cycle uses a fixed parcel of fluid which in a gas turbine would be compressed, heated in a heat exchanger, expanded in a turbine and then cooled in a heat exchanger; the ideal gas turbine is sometimes called a Joule or Brayton cycle. The gas turbine 'cycle' we consider here takes in fresh air, burns fuel in it and then discharges it after the turbine: in other words it does not cycle the air. Here we are adopting the standard terminology of the industry.

shaft. The load may be an electric alternator, a ship's propeller or the fan on the front of a high bypass ratio jet engine.

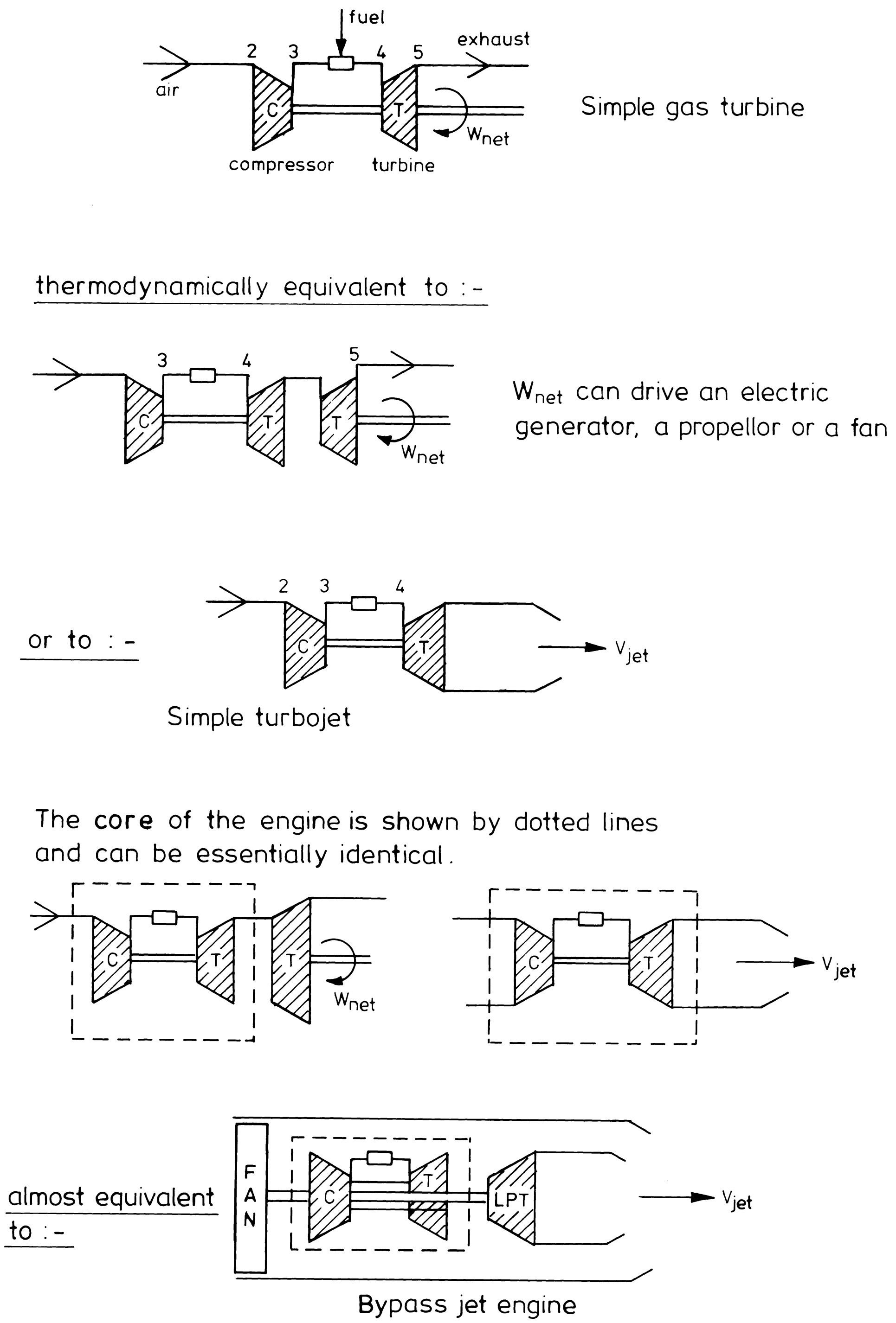

Figure 4.1. Gas turbines – variations on a core theme

The central part of a gas turbine, the compressor, combustor and turbine driving the compressor, is often referred to as the **core** and the same core can be put to many different applications. In Fig. 4.1 the turbine power $\dot{W}_t$ would partly be used to supply the compressor power $\dot{W}_c$ and partly to supply the useful or net power $\dot{W}_{net}$ which is equal to $\dot{W}_t - \dot{W}_c$. At this stage we do not need to consider how $\dot{W}_{net}$ is taken from the core, but it is worth remembering that there is the special case of the pure jet engine when all the power is used to accelerate the core stream and produce a jet at exit. This was the basis of the engine envisaged and then constructed by Whittle and is still used for propulsion at supersonic speeds; Concorde, for example, is propelled by a pure jet engine. Pure jets are also used at subsonic speeds when fuel economy is unimportant but first cost and weight do matter, for example to propel missiles or target drones.

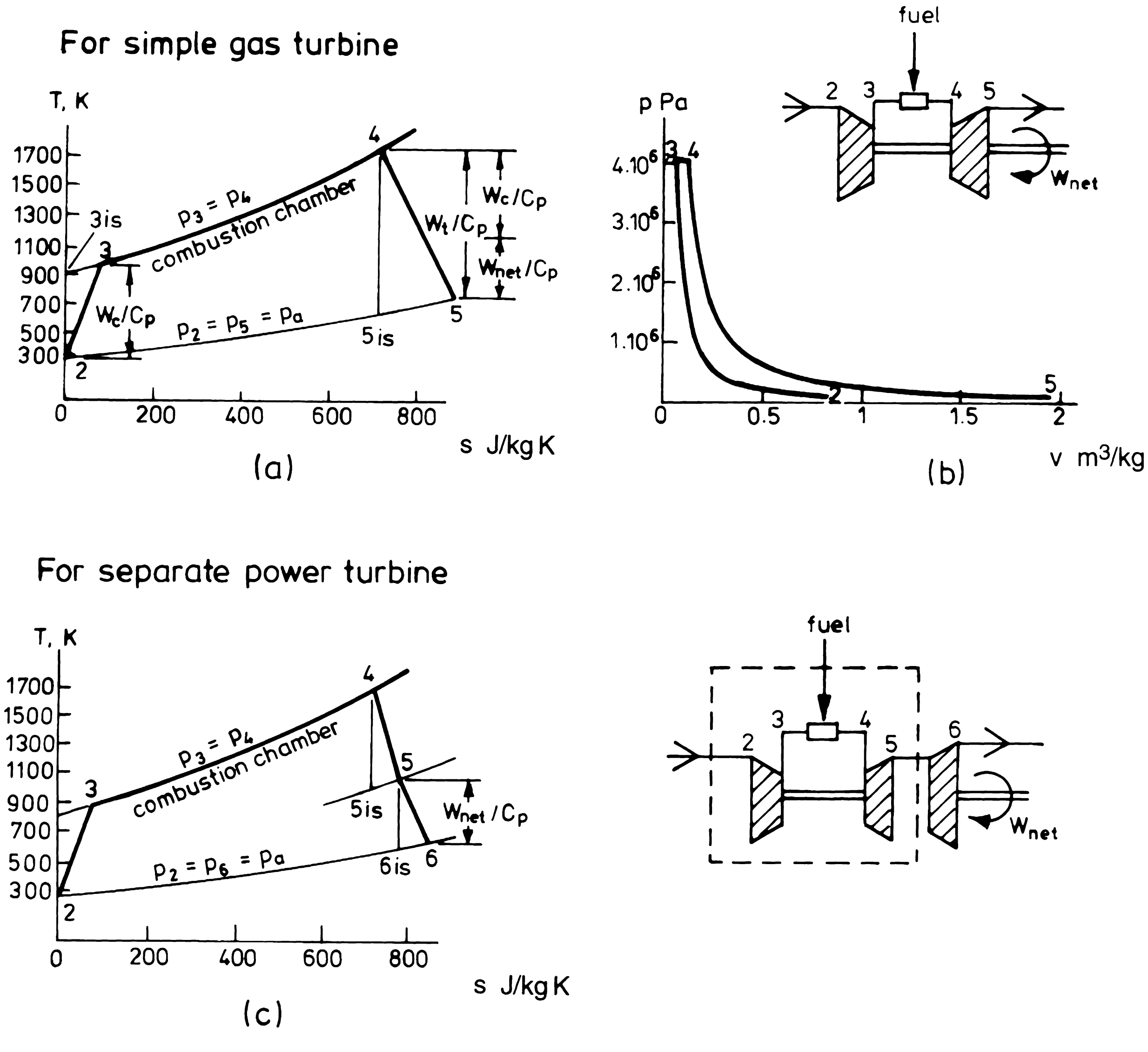

Figure 4.2. Scale diagrams of temperature–entropy and pressure–volume for gas turbine cycles. Pressure ratio 40, $T_2 = 288$, $T_4 = 1700$K, $\eta_c = \eta_t = 0.90$; $s = 0$ for $p = p_a$ and $T = 288$ K

The first law of thermodynamics can be applied to a steady process through the engine, where air enters the engine at temperature T_2 and exhaust products leave at temperature[2] T_5. (It may seem odd to make the entry condition station 2, but this is chosen here to be compatible with the normal recommended practice for aircraft engines.) If the effect of combustion is represented by an equivalent net heat transfer to the gas $\dot{Q}_{net}$, the first law may be written

$$\dot{Q}_{net} - \dot{W}_{net} = \dot{m}_{air} \Delta h \tag{4.1}$$

where Δh is the enthalpy difference between the inlet air and the exhaust based on stagnation conditions. The mass flow of fuel is neglected in this equation as a small quantity. If the exhaust gas can be modelled as a perfect gas with the same properties as air, equation 4.1 reduces to

$$\dot{Q}_{net} - \dot{W}_{net} = \dot{m}_{air} c_p (T_5 – T_2).$$

The combustion process, which is represented as an equivalent heat transfer

$$\dot{Q}_{net} = \dot{m}_{air} c_p (T_4 - T_3)$$

can in turn be written in terms of the lower calorific value of the fuel

$$\dot{m}_f LCV = \dot{m}_{air} c_p (T_4 - T_3). \tag{4.2}$$

For kerosene, or similar fuels used in aircraft engines, $LCV = 43$ MJ / kg. This magnitude is so large in relation to the specific heat of air (taken here to be $c_p = 1.005$ kJ/kgK) that a small flow rate of fuel is sufficient to produce a substantial temperature rise in a much greater mass flow rate of air.

Important processes in a gas turbine which burns fuel can be represented by an equivalent closed-cycle gas turbine, and by doing this it is easy to represent the processes graphically. Figure 4.2 shows the temperature–entropy (T–s) and pressure–volume (p–v) diagrams for a closed-cycle gas turbine. At entry to the compressor the temperature is T_2 and the ambient pressure p_a. The upper pressure $p_3 = p_4$ is that at which the heat transfer (equivalent to the combustion) takes place; for the present simple example it is assumed that there is no pressure drop in the combustor. In place of a heat exchanger to remove heat at the lower pressure the turbine exhausts to atmosphere in open-cycle gas turbine and the compressor draws in new air at the same pressure but at ambient temperature. After the combustion process the temperature entering the turbine is T_4. The work[3] exchanges per unit mass of air are shown on the T–s diagrams in Fig. 4.2 (though what is actually shown is work divided by specific heat, W/c_p).

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[2] To relate this to what is discussed later in these notes it is appropriate to mention that the temperatures used in connection with the gas turbine cycle are the **stagnation** temperatures. Similarly it is **stagnation** pressures which are used here too. Stagnation and static properties are explained in Chapter 6.

[3] Work W is produced per unit mass of gas through the engine (units: Joule per kg) whereas power $\dot{W}$ is produced per unit mass flow rate (units: Watt per kg/s). The units J/kg can be shown to be identical with W/(kg/s).

In Figure 4.2 (and throughout Part 1: Design of the engines for a new 600-seat aircraft) the properties of the combustion products will be treated as pure air with the properties of a perfect gas: $c_p = 1.005$ kJ/kg K, $\gamma = 1.40$, $R = 0.287$ kJ/kg K. This is an approximation which can easily be removed (and is considered in more detail in section 4.4 below, and then in Chapter 11), but for the present purpose it is sufficiently accurate and is a substantial convenience.

4.2 Isentropic Efficiency and the Exchange of Work

In the diagrams of Fig. 4.2 process 2–3 is the compression, and 4–5 is the expansion through the turbine. In practice the compression and expansion occur virtually without heat transfer to anything outside them, that is to say they may be taken to be adiabatic. Also shown in Fig. 4.2 are the hypothetical process 2–3is, which is an adiabatic and reversible (i.e. isentropic) compression, and the hypothetical process 4–5is, which is the isentropic expansion through the turbine. These isentropic processes are those which ideal compressors and turbines would perform. As can be seen, the actual compression process involves a greater temperature rise than that of the isentropic compressor for the same pressure rise,

$$T_3 - T_2 > T_{3is} - T_2;$$

in other words the work input to the actual compressor for each unit mass of air is greater than the work input for the ideal one. Similarly the actual turbine produces a smaller temperature drop than that in the ideal turbine, that is

$$T_4 - T_5 < T_4 - T_{5is},$$

and therefore for the same pressure ratio the actual turbine produces less work than the reversible adiabatic one.

For compressors and turbines it is normal to define efficiencies which relate actual work per unit mass flow to that of an ideal (i.e. loss-free) machine with equivalent pressure change;

$$\eta_{comp} = \frac{\text{Ideal work}}{\text{Actual work}} \quad \text{and} \quad \eta_{turb} = \frac{\text{Actual work}}{\text{Ideal work}}. \tag{4.3}$$

Note that the efficiency definitions are different for a compressor or turbine so that their values are always less than unity. For an adiabatic machine the ideal equivalent process is reversible and the corresponding efficiencies are referred to as **isentropic efficiencies.**

Treating the fluid as a perfect gas, for which $h = c_p T$,

$$\eta_{comp} = \frac{T_{3is} - T_2}{T_3 - T_2} \quad \text{and} \quad \eta_{turb} = \frac{T_4 - T_5}{T_4 - T_{5is}}. \tag{4.4}$$

Nowadays the isentropic efficiencies in a high quality aircraft engine for use on a civil aircraft are likely to be around 90% for compressors and turbines and this round number will normally be used in this book when a numerical value is needed. For the simple gas turbine of Fig. 4.2 the pressure rise across the compressor is equal to the pressure fall across the turbine and the corresponding pressure ratios are equal. For a jet engine, however, the pressure ratio across the turbine must be less than the pressure ratio across the compressor because some of the expansion is used to accelerate the jet. The pressure out of the turbine is p_5 and downstream of the propelling nozzle the pressure is the atmospheric static pressure, p_a.

The isentropic temperature change can very easily be found once the pressure ratio is specified. It may be recalled that for an adiabatic and reversible process

$$p/T^{\gamma/(\gamma-1)} = \text{constant}$$

which, in the present case, means

$$T_{3\text{is}}/T_2 = (p_3/p_a)^{(\gamma-1)/\gamma} \qquad \text{and} \qquad T_4/T_{5\text{is}} = (p_4/p_a)^{(\gamma-1)/\gamma}.$$

Neglecting any pressure drop in the combustor gives $p_3 = p_4$ and writing $p_3/p_a = p_4/p_a = r$ gives

$$T_{3\text{is}}/T_2 = T_4/T_{5\text{is}} = r^{(\gamma-1)/\gamma}. \tag{4.5}$$

The power which must be supplied to the compressor is given by

$$\dot{W}_c = \dot{m}_{air}\, c_p\, (T_3 - T_2) \tag{4.6}$$

and expressing this in terms of the isentropic temperature rise gives

$$\dot{W}_c = \frac{\dot{m}_{air}\; c_p\; (T_{3\text{is}} - T_2)}{\eta_{\text{comp}}}$$

$$= \frac{\dot{m}_{air}\; c_p\; T_2(T_{3\text{is}}/T_2 - 1)}{\eta_{\text{comp}}} = \frac{\dot{m}_{air}\; c_p\; T_2(r^{(\gamma-1)/\gamma} - 1)}{\eta_{\text{comp}}}\;. \tag{4.6}$$

Similarly, the power available from the turbine, when the mass flow of fuel in the gas stream is neglected, is given by

$$\dot{W}_t = \dot{m}_{air}\, c_p\, ((T_4 - T_5)$$

or

$$\dot{W}_t = \eta_{\text{turb}} \dot{m}_{air}\, c_p\, (T_4 - T_{5\text{is}}) = \eta_{\text{turb}} \dot{m}_{air}\, c_p T_4\; (1 - r^{-(\gamma-1)/\gamma}). \tag{4.7}$$

The turbine power must be greater than the power required to drive the compressor and the difference W_{net} , which is available to drive the load or accelerate the jet, is

$$\dot{W}_{net} = \dot{m}_{air}\, c_p\, T_2 \left(\eta_{\text{turb}} \frac{T_4}{T_2} (1 - 1/r^{(\gamma-1)/\gamma}) - \frac{(r^{(\gamma-1)/\gamma} - 1)}{\eta_{\text{comp}}} \right). \tag{4.8}$$

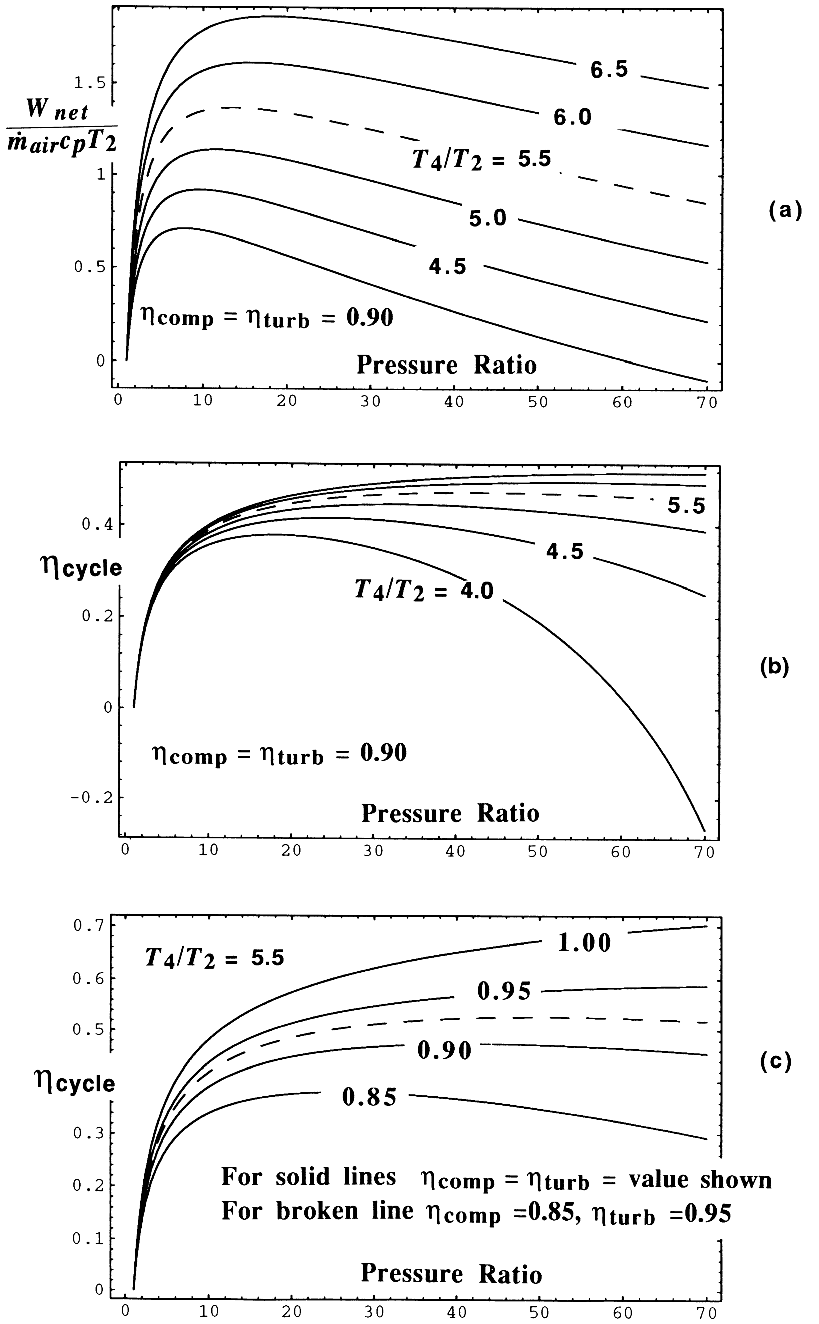

Figure 4.3. Non-dimensional power and cycle efficiency for an idealised gas turbine

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Exercises

4.1 Air enters a compressor at a temperature of 288 K and pressure of 1 bar. If the exit pressure is 45 bar (a plausible value for take off in a new design engine) find the temperature at compressor outlet for isentropic efficiencies of 100% and 90%. What is the work input per unit mass flow for the irreversible compressor? (**Ans:** 854.6 K; 917.5 K. W_C = 633 kJ/kg)

4.2 For the engine of Exercise 4.1 find the work per kg which could be extracted from a turbine for a pressure ratio of 45 when the turbine inlet temperature is 1700 K (a plausible value at take off) and η_{turb} = 0.90. Compare the work per kg in compressing the air in Exercise 4.1. (**Ans:** 1019 kJ/kg)

Certain features can be determined directly from the equation for the net power per unit mass flow rate. The pressure ratio is crucial and if this tends to unity the net power goes to zero. The ratio of turbine inlet temperature to compressor inlet temperature T_4/T_2 is important and, for a given pressure ratio, increasing the temperature ratio brings a rapid rise in net power. This is effectively how the engine is controlled, because increasing the fuel flow increases T_4 and thence the power. Note, however, that it is the *ratio* T_4/T_2 which is involved, so that at high altitude, when the inlet temperature T_2 is low, a high value of temperature ratio can be obtained with a *comparatively* low value of T_4. In fact the highest values of T_4/T_2 are normally achieved at top-of-climb, the condition when the aircraft is just climbing to its cruising altitude. The dependence of power per unit mass flow rate of air, $\dot{W}_{net}$, on T_4/T_2 and pressure ratio is shown by the curves in Fig. 4.3(a). At small values of pressure ratio, increasing the pressure ratio brings a rapid increase in $\dot{W}_{net}$ but the rate of increase falls and there is a pressure ratio at which $\dot{W}_{net}$ peaks; the value of the pressure ratio for peak net power increases as T_4/T_2 rises but never exceeds about 20 for all practical values of temperature ratio.

4.3 THE GAS TURBINE THERMAL AND CYCLE EFFICIENCY

The efficiency of a closed-cycle gas turbine, such as are displayed in the T–s and p–v diagrams in Fig. 4.2, can be written as the ratio of net power out to the heat transfer rate to the air in the process replacing combustion,

$$\eta_{cycle} = \frac{\dot{W}_{net}}{\dot{m}_{air}\ c_p (T_4 - T_3)}$$

$$= \frac{\dot{W}_{net}}{\dot{m}_{air}\ c_p T_2 (T_4/T_2 - T_3/T_2)} . \tag{4.9}$$

The actual temperature ratio across the compressor in equation 4.9 is given by

$$T_3/T_2 = 1 + \frac{r^{(\gamma-1)/\gamma} - 1}{\eta_{comp}} .$$

The thermal efficiency of the open-cycle gas turbine is the ratio of the net power to the energy input by the combustion of fuel and is

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{m}_f\ LCV} . \tag{4.10}$$

but using equation 4.2

$$\dot{m}_f LCV = \dot{m}_{air}\, c_p (T_4 - T_3)$$

it is clear that within the approximations we are adopting the thermal efficiency of the open-cycle gas turbine and the cycle efficiency of the closed-cycle gas turbine are equal, $\eta_{th} = \eta_{cycle}$. When the thermal efficiency was introduced in section 3.3 it was specific to the jet engine and all the power was transferred into the increase in kinetic energy of the flow through the engine, assuming that the jet nozzle was loss free.

In calculating the thrust and propulsive efficiency it was assumed that the mass flow of fuel is negligible compared to that of air. According to the approximations we have adopted the temperature rise in the combustion chamber is related to the fuel flow by equation 4.2. Suppose the turbine inlet temperature $T_4 = 1700$ K and the compressor produces a pressure ratio of 45 at an efficiency of 90% with air entering it at $T_2 = 288$ K. The compressor outlet temperature is therefore 912.5 K and the temperature rise in the combustor is 787.5 K, requiring an enthalpy rise of 786 kJ/kg. For a fuel with $LCV = 43$ MJ/kg, the fuel flow per unit mass flow of air is therefore 0.018. This is the mass flow of air through the core; were the bypass ratio to be 6, for example, the ratio of fuel to air would be down to 3 parts in 1000 for the whole engine.

Exercise

4.3* **a**) For the example of Exercises 4.1 and 4.2, take off on a standard day, calculate the thermal efficiency and the net work per kg of air flowing. Assume isentropic efficiencies of 90% for the compressor and turbine. (**Ans:** $\eta_{th} = 0.492$, $W_{net} = 387$ kJ/kg)

The calculation **could** be repeated for the following:

b) $T_2 = 308$ K, $T_4 = 1700$K (as for a take off on a hot day), $PR = 45$,
(**Ans:** $\eta_{th} = 0.475$, $W_{net} = 342$ kJ/kg)

c) $T_2 = 259.5$K, $T_4 = 1575$K (representative of top of climb at 31000ft, $M = 0.85$), $PR = 45$,
(**Ans:** $\eta_{th} = 0.498$, $W_{net} = 374$ kJ/kg)

d) $T_2 = 259.5$K, $T_4 = 1450$K (representative of initial cruise conditions), $PR = 40$,
(**Ans:** $\eta_{th} = 0.478$, $W_{net} = 313$ kJ/kg)

e) $T_2 = 259.5$K, $T_4 = 1450$K (representative of initial cruise conditions), $PR = 35$,
(**Ans:** $\eta_{th} = 0.475$, $W_{net} = 326$ kJ/kg)

and **f**) $T_2 = 259.5$K, $T_4 = 1450$K (representative of initial cruise conditions), $PR = 40$ when for the compressor and turbine $\eta_c = \eta_t = 0.85$. (**Ans:** $\eta_{th} = 0.375$, $W_{net} = 234$kJ/kg)

Discuss these results of a–f, commenting on the effect of inlet temperature, turbine inlet temperature, pressure ratio and component isentropic efficiency.

Notes

1) The answers here show the highest work output for conditions at take off on a standard day – note that this is actually work per unit mass flow. In fact the power from any gas turbine falls rapidly with altitude because as the density of the air drops the mass flow rate of air through the engine falls.

2) These efficiencies are really quite high. For comparison, a high quality diesel engine in a large truck has an overall efficiency only just about 40%.

3) The gas turbine is not alone in suffering a reduction in power output and efficiency as the inlet air temperature rises. In road vehicles we normally pay no attention to the maximum temperature in the engine (assuming that the engine has been developed to stand the worst conditions to which we are likely to expose it) but the peak temperature will rise with inlet temperature, assuming the same fuel input. Gasoline engines and diesel engines experience a marked reduction in power output as the inlet air density drops, which is most obvious at high altitudes; one of the Interstate freeways in the USA exceeds 12000 feet and there the effect is pronounced.

Figure 4.3(b) shows the cycle efficiency for a gas turbine as a function of pressure ratio, with the curves denoting different values of the temperature ratio T_4/T_2. It is again assumed that $\eta_{comp} = \eta_{turb} = 0.90$. It is clear how important both the temperature ratio and pressure ratio are in determining the efficiency of the cycle. (For the ideal gas turbine, in which $\eta_{comp} = \eta_{turb} = 1.00$, the efficiency is independent of T_4/T_2.) Put simply, for realistic cycles (where the components are not assumed to be isentropic) the cycle efficiency continues to rise as the mean temperature of heat input rises relative to T_2, the ambient temperature, and in practical gas turbines it is essential for T_4, the temperature of the gas entering the turbine, to be high[4].

The cycle efficiency depends on pressure ratio and for a given value of T_4/T_2 there is a pressure ratio corresponding to the peak value of η_{cy}. The pressure ratio at which the peak cycle efficiency occurs depends on the isentropic efficiencies of the compressor and turbine. For the higher values of T_4/T_2 the variation of η_{cy} with pressure ratio is very slight (i.e. the curves are fairly flat) to a considerable distance either side of the maximum. A pressure ratio for maximum efficiency occurs because increasing pressure ratio raises the mean temperature of heat addition, but the extra power into the compressor has to be supplied by the turbine and irreversibilities in these take a larger share of the available power. As will be discussed in Chapter 5, the value of T_4/T_2 appropriate for cruise is approximately 5.5.

By comparing Fig. 4.3(a) and 4.3(b) it may be noted that peak efficiency occurs at a substantially higher pressure ratio than that for the maximum net power output per unit mass flow rate of air: for $T_4/T_2 = 5.5$, for example, the pressure ratios for maximum power and maximum efficiency are around 12 and 32 respectively. Thus for engines for which maximum thrust is the goal (the normal requirement for military engines) the pressure ratio might be around 12 for $T_4/T_2 = 5.5$, whilst for engines for which efficiency is most important (most civil engines) the pressure ratio would be nearer 40. The reason for the maximum power occurring at lower pressure ratio than maximum efficiency is as follows. At low pressure ratios the power output increases with pressure ratio because the efficiency rises so rapidly that more of the heat is converted to work. As the pressure ratio increases, the temperature at compressor outlet also increases and if the turbine inlet temperature is held constant the permissible heat input (equivalent to amount of fuel burned) reduces. At higher pressure ratios the heat input decreases more rapidly with pressure than the increase in efficiency (the conversion of heat into work) and where the two effects have equal magnitude defines the pressure ratio for maximum power output per unit mass flow rate of air.

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[4] Because here we are not dealing with a real closed cycle, but with an open system in which the heat input is replaced by a combustion process, the effect of temperature at the end of the combustion process should really be handled more carefully. Combustion produces flame temperatures typically in excess of 2300 K, which are higher than the temperature allowed into the turbine (typically no more than 1500 K for prolonged operation). The dilution of the combustion products to lower the temperature is associated with a rise in entropy and a loss in the capability of turning thermal energy into work.

The inlet temperature T_2 is determined by the altitude and the forward speed and at 31000 feet and M = 0.85 this is T_2 =259.5 K. As discussed later in section 5.2, the turbine inlet temperature is fixed by metallurgical considerations (i.e. what temperature can the metal stand at a given level of stress), by cooling technology and by considerations of longevity – lower temperatures lead to longer life. With T_4/T_2 =5.5 the peak cycle efficiency is about 0.474 and occurs at a pressure ratio of about 40; the variation in efficiency to either side of this pressure ratio is small. As the pressure ratio increases it becomes harder to design a satisfactory compressor and the isentropic efficiency tends to fall, so there is some advantage in staying to the lower side of the peak in η_{cy}. Furthermore, by putting the cruise at a pressure ratio of, say, 40 allows the maximum climb condition to occur in an acceptable range of pressure ratios up to, say, 45.

There is an additional issue concerning pressure ratio, which will be discussed more fully in Chapter 6. For an aircraft cruising at a Mach number of 0.85 the pressure at inlet to the compressor is raised by a factor of 1.60, compared to the surrounding atmosphere, by the forward motion. At outlet from the engine (i.e. at nozzle outlet) the pressure is not raised in this way, but remains at the atmospheric pressure. The effect of forward speed is therefore to raise the compression ratio of the whole engine by 1.60; raising the effective pressure ratio from 40 to about 64 has only a small effect on the cycle efficiency, which we can neglect.

Figure 4.3(c) also shows η_{cycle} versus pressure ratio but this time for a fixed temperature ratio T_4/T_2 =5.5 and various values of compressor and turbine efficiency. It can be seen that the cycle efficiency is very sensitive to the component efficiencies: at a pressure ratio of 40 a reduction in compressor and turbine efficiencies from 90% to 85% would lower the cycle efficiency from 47% to 37%, corresponding to about 21% less power for the same rate of energy input in the form of fuel.

Taking equal turbine and compressor efficiencies is an oversimplification, and for the pressure ratios now being considered this becomes significant. As the pressure ratio increases the efficiency of the turbine tends to rise, whilst that of the compressor falls by a similar amount. The broken curve in Fig. 4.3(c) explores this, with equal magnitude changes of component isentropic efficiency of opposite sign in turbine and compressor. Although the alteration in η_{cy} is significant, the trends found with equal values of isentropic efficiency are not altered.

4.4 THE EFFECT OF WORKING GAS PROPERTIES

The analysis up to now, and in most of what follows up to Chapter 10, treats the working fluid in the gas turbine as a perfect gas with the same properties as air at standard conditions. This is done to make the treatment as simple as possible, and it does yield the correct trends. In a serious design study, however, the gas would be treated as semi-perfect and the products of combustion in the stream through the turbine would be included. In the semi-perfect gas approximation c_p, R, and $\gamma = c_p/c_v$ are functions of temperature and composition but *not* of pressure. With this approximation, which is sufficiently accurate for all gas turbine applications, the useful relation between gas properties $p/\rho = RT$ is retained. As is discussed in more detail in Chapter 11, R is virtually independent of temperature and composition for the gases occurring throughout a gas turbine, and it may be taken to be constant at 0.287 kJ/kgK. The specific heat capacity may be obtained from $c_p = \gamma R/(\gamma-1)$.

Table 4.1 compares the results of Exercise 4.3 (based on a gas with $\gamma = 1.40$, giving c_p = 1.005 kJ/kgK) with results for the same pressure ratio and compressor and turbine entry temperatures but using accurate values for γ and c_p based on local temperature and composition. In these more accurate calculations (under the heading 'Variable γ') the mass flow rate of fuel is added to the exhaust gas. The columns show the overall cycle efficiency and the net work produced per unit mass of air through the engine.

Table 4.1 Comparison of cycle efficiency and net work based on constant gas properties and those related to local conditions

Exercise number	T_2 K	T_4 K	$\eta_c=\eta_t$	p_3/p_2	p_2 bar	$\gamma = 1.40$ η_{cy}	$\gamma = 1.40$ W_{net} kJ/kg	Variable γ η_{cy}	Variable γ W_{net} kJ/kg
4.3a	288	1700	0.90	45	1.00	0.492	387	0.477	496
4.3b	308	1700	0.90	45	1.00	0.475	342	0.466	454
4.3c	259.5	1575	0.90	45	0.46	0.498	374	0.485	466
4.3d	259.5	1450	0.90	40	0.46	0.478	313	0.469	387
4.3e	259.5	1450	0.90	35	0.46	0.475	326	0.464	397
4.3f	259.5	1450	0.85	40	0.46	0.375	234	0.385	305

The table shows that treating the gas with equal properties throughout gives quite good estimates for the efficiency, reflecting the variation with operating condition and, most significantly, with component efficiency. With the simple gas treatment, however, the work output is underestimated by around 20%, though the trends are similar to those calculated with the more realistic assumptions for the gas properties. It would be possible to improve the constant-property gas model by having different values of c_p for the compressor and turbine (a much better approximation for the flow in the turbine would be to take c_p =1.244 kJ/kg and γ=1.3). The mass flow through the turbine should be about 2% larger than through the compressor because of the fuel, but the effect of cooling air and combustor pressure loss need to be included. These complications are, however, unwarranted for the present purpose but are revisited in Chapter 20.

The underestimate of power predicted assuming a perfect gas with equal properties for the flow through the turbine and compressor is likely to lead to a core being designed which is about 20% bigger (in terms of the mass of air which passes through it) than that which would result if the correct gas properties were used. In fact the perfect gas design is not likely to give such an oversized core for a number of reasons. The model has neglected the cooling flows (which may take 20% of the air passing through the compressor). For an aircraft, air must be bled off the compressor to pressurise the cabin and to bring about de-icing of the wing and nacelle, whilst electrical and hydraulic power is taken from the engine. There is also drop in pressure in the combustor (perhaps 5% of the local pressure) is also neglected. Lastly the 90% isentropic efficiencies for compressor and turbine used in the design calculations are quite ambitious and these might not be achieved; the net power has been shown to be strongly affected by compressor and turbine isentropic efficiency and this too means that the actual core might be larger than a design shown here using real gas properties and $\eta_c=\eta_t=0.90$.

4.5 THE GAS TURBINE AND THE JET ENGINE

High cycle efficiency therefore depends on having a high temperature ratio and a pressure ratio appropriate to this. The net power generated by the core could all be used to accelerate the flow through it, in which case we would have produced a simple turbojet, as in Fig. 5.1. The problem that this creates is that with the high turbine inlet temperatures necessary to give high efficiency there is such a large amount of net work available that if all of this were put into the kinetic energy of the jet, the jet velocity would be very high. As a result the propulsive efficiency η_p would be low and therefore so too would be the overall efficiency. The way around this is to use the available energy of the flow out of the core to drive a turbine which is used to move a much larger mass flow of air, either by a propeller (to form a turboprop) or a fan in a duct (as in the high bypass ratio engine used now on most large aircraft).

The noise power generated by jets is approximately proportional to the eighth power of jet velocity, so there is an incentive, in addition to propulsive efficiency, to reduce jet velocity.

Exercise

4.4 **a**) A simple (no bypass) turbojet engine flies at 256.5 m/s. The compressor and turbine are as described in Exercises 4.1,4.2 and 4.3. Find the jet velocity which would be produced with the temperatures given as appropriate for initial cruise (31000 feet, M =0.85) in part d of Exercise 4.3. Assume that all the net work is used to increase the kinetic energy of the flow.

Calculate the propulsive efficiency η_p and the overall efficiency $\eta_o=\eta_p\eta_{cycle}$ at this flight speed. Explain why the propulsive efficiency and overall efficiency is low. Indicate ways in which η_o could be raised at this flight speed. (**Ans:** V_j= 832 m/s, η_p =0.471, η_o= 0.225)

b) Recalculate the efficiencies if the flight speed were 600 m/s (M =1.99 at 31000 feet).
(**Ans:** V_j= 993 m/s, η_p =0.753, η_o= 0.360)

SUMMARY CHAPTER 4

The gas turbine may be envisaged as a core with various loads fitted to it; one possible load is a nozzle to create a simple turbojet, another is a turbine driving a bypass fan. In a closed cycle gas turbine a fixed amount of air is compressed, heated, expanded in a turbine and cooled; the efficiency of such a cycle is the cycle efficiency η_{cy}. Most gas turbines, and all jet engines, are open cycle, replacing the heating process by combustion of fuel and eliminating the cooling process by instead taking in fresh air to the compressor. For the open-cycle gas turbine the measure of performance is thermal efficiency η_{th}. Both η_{cy} and η_{th} vary in a similar way with pressure ratio and the ratio of turbine inlet temperature to compressor inlet temperature, T_4/T_2.

Isentropic efficiencies are the conventional method of relating actual compressor and turbine performance to the ideal; nowadays isentropic efficiencies of around 90% are expected, perhaps a few per cent lower for the compressor and a similar amount higher for the turbine.

The cycle efficiency, the ratio of the net power to the equivalent heat input rate, is strongly dependent on the isentropic efficiencies. It is also a strong function of T_4/T_2. For any given value of T_4/T_2 the pressure ratio to give maximum net power would be lower than the pressure ratio for maximum cycle efficiency.

To obtain high cycle efficiency it is essential to operate at high values of T_4/T_2. This invariably means that there is a high level of power output, and if this were used to accelerate only the core flow it would lead to jet velocities unacceptably high for subsonic aircraft. The solution is to choose a combination of high temperature ratio and pressure ratio for the core, but to use the available power output via a turbine to drive a fan, supplying a relatively small increment in kinetic energy to a large bypass stream.

The elementary description of the gas turbine has made it possible to show what is needed to obtain large power output and high cycle efficiency and, equivalently, high thermal efficiency. Some important limitations of the present treatment are:

- no cooling flows (air taken from the compressor to cool or shield the turbine blades);
- a perfect gas with properties of ambient air (γ=1.40) has been assumed throughout;
- the mass of fuel in calculating power from the turbine has been neglected;
- pressure drop in the combustor, which may be 5% of pressure, is omitted;
- simple assumptions have been made for compressor and turbine efficiency;
- electrical power off-takes and air for cabin pressurisation have been neglected.

Despite these shortcomings the trends predicted are correct and that the magnitudes for efficiency and available power are plausible. The manner in which this can be used effectively is explored in the next chapter.

APPENDIX

A brief summary of thermodynamics from an engineering perspective

This is not the place to give a detailed account of thermodynamic theory and principles, but a brief summary in the nature of revision may be helpful. For those who find the treatment here insufficient it is recommended that one of the very many texts be consulted. (There are so many books on engineering thermodynamics available that it is invidious to select only one, but some guidance does seem in order. The text by Van Wylen and Sonntag, 1985, gives an excellent treatment.) In the interest of brevity and simplicity details will be omitted wherever possible and no attempt is made to include every restriction and caveat. A feature of engineering thermodynamics is the common use of a closed control surface to enclose the process or device under consideration; when using a control surface attention is directed at what crosses the surface, both material flows of gas and liquid as well as work and heat being transferred across it.

The **first law** is a formal statement of the conservation of energy. For a *fixed* mass m of a substance this can be written in differential form as

$$\mathrm{d}Q - \mathrm{d}W = m\,\mathrm{d}e$$

where Q is heat transferred to the substance, W is work extracted from the substance and e is the energy per unit mass. We will neglect changes in energy associated with chemical, electrical and magnetic changes. If potential and kinetic mechanical energy are also neglected for the moment, energy e is restricted to the internal energy denoted by u, the specific internal energy. The work here can be done by, for example, gas expanding and pushing back the atmosphere, or it could be work driving a shaft or electrical work.

The application we have for the first law in the gas turbine and jet engine is for steady flow of gas through a device (either a whole engine or a component like a compressor or turbine). To facilitate this we draw an imaginary control surface around the device or the process we are considering. The control surface is closed, so any matter entering or leaving the device must cross the control surface; properties entering are given subscript 1 whilst those leaving have subscript 2. We specify here that there is a steady mass flow $\dot{m}$ into and out of the control surface. We also have heat transfer rate *into* the control surface $\dot{Q}$ and work transfer rate *out from* the control surface $\dot{W}_s$. The first law is now written for the closed control surface as

$$\dot{Q} - \dot{W}_s = \dot{m}\{(h_2 + V_2^2/2) - (h_1 + V_1^2/2)\}. \tag{A1}$$

The terms on the right hand side are the energy transported across the control surface by the flow. The kinetic energy is recognisable as $V^2/2$ and h is the enthalpy. For a flow process it is appropriate to use the specific enthalpy $h = u + p/\rho$, where u is the internal energy per unit mass,

p is the pressure and ρ is the density. Enthalpy allows for the displacement work done by flow entering and leaving the control surface. The work $\dot{W}_s$ in equation A1 is often referred to as shaft work (to distinguish it from the so-called displacement work done by flow entering and leaving the control surface) even though the work may be removed without a shaft, for example electrically.

For an ideal gas of fixed composition the enthalpy is a function of only the temperature. The formal expression is

$$\mathrm{d}h = c_p \, \mathrm{d}T$$

where c_p is the specific heat capacity at constant pressure. If c_p is constant, as for a perfect gas, equation (A1) can be written

$$\dot{Q} - \dot{W} = \dot{m}\{(c_p T_2 + V_2^2/2) - (c_p T_1 + V_1^2/2)\}. \tag{A.2}$$

It is frequently useful to refer to the stagnation enthalpy h_0, defined by

$$h_0 = h + V^2/2.$$

and the corresponding stagnation temperature

$$T_0 = T + V^2/2c_p.$$

The **second law** leads to the property entropy. Entropy is useful because in an ideal process for changing energy from one form to another the net change of entropy of the system and the environment is zero. Since zero entropy increase gives the ideal limit, the magnitude of entropy rise gives a measure of how far short of ideal the real process is.

Entropy, as a property, can be expressed in terms of other properties, for example $s = s(T,p)$. It is defined for an ideal process (normally referred to as a reversible process) by

$$\mathrm{d}s = \mathrm{d}Q/T \tag{A3}$$

where s denotes the specific entropy (entropy per unit mass). By using the first law in differential form the change in entropy in equation (A3) can be rewritten as

$$\mathrm{d}s = \mathrm{d}h/T - \mathrm{d}p/\rho T$$

and this can be integrated for a perfect gas to give

$$s - s_{ref} = c_p \ln(T/T_{ref}) - R \ln(p/p_{ref}) \tag{A4}$$

where s_{ref} is the entropy at the reference pressure p_{ref} and temperature T_{ref}.

As the energy of a gas is increased the temperature must also increase; if pressure is held constant equation (A4) shows that entropy also rises. But if temperature is held constant and the pressure is reduced, entropy also rises. The implications of equation (A4) can be understood a little better by considering the adiabatic (no heat transfer) flow of gas along a pipe or through a throttle; in both cases there is a drop in pressure but, because there is no work or heat transfer, no change in energy and therefore no change in temperature. The fall in pressure therefore leads to a rise in entropy. In fact the magnitude of the entropy rise is a measure of the pressure loss. An entropy rise in the absence of heat transfer is evidence of a dissipative process; the effect is irreversible, since reversing the process would not reduce the entropy. Ideal processes are therefore often referred to as reversible, whereas processes which lead to loss (that is, processes for which the entropy rises by more than follows from equation (A3)) are often referred to as irreversible.

Compressors and turbines have a small external area in relation to the mass flow passing through them. As a result the heat transfer from them is small and a good approximation is normally to treat both compressors and turbines as adiabatic. When ideal (hypothetical) machines are used as a standard to compare with real machines, it is normal to specify them to be adiabatic. The ideal compressor or turbine will have no dissipative processes – the flow is loss free and is normally described as reversible. A reversible adiabatic flow will experience no rise in entropy and such ideal processes are normally referred to as isentropic.

An ideal compressor or turbine, producing a reversible and adiabatic change in pressure, produces no change in the entropy of the flow passing through. With no change in entropy equation (A4) leads immediately to

$$0 = \ln(T_2/T_1) - \frac{R}{c_p} \ln(p_2/p_1),$$

and since $c_p = \dfrac{\gamma R}{\gamma - 1}$, the familiar expression relating temperature and pressure changes results

$$T_2/T_1 = (p_2/p_1)^{(\gamma-1)/\gamma}.$$