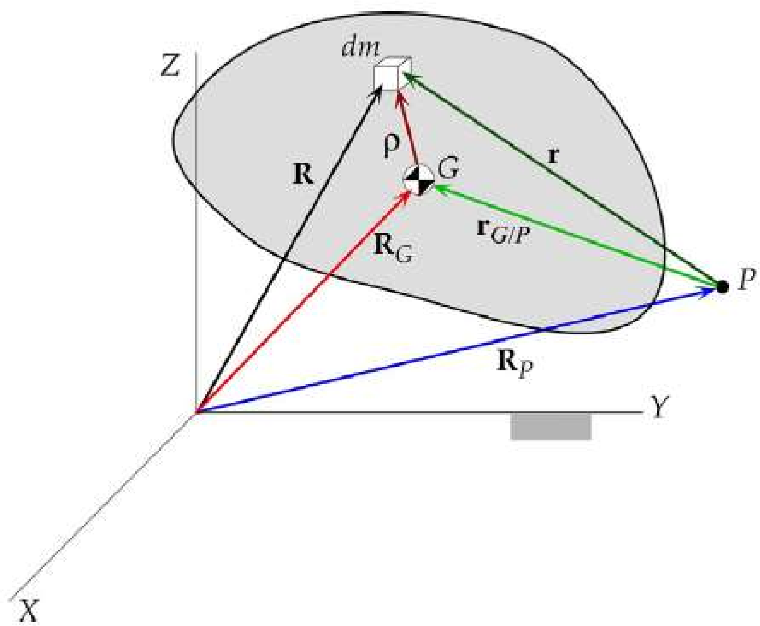
Lecture 25

Equations of Rotational Motion



$- dM_P = r \times dF_{net} + r \times df_{net}$

$- M_P)_{net} = \int_m r \times (dF_{net} + df_{net})$

$$= \int_m r \times \ddot{R}\, dm$$

$- r \times \ddot{R} = \frac{d}{dt}(r \times \dot{R}) - \dot{r} \times \dot{R}$

$- \dot{r} \times \dot{R} = (\dot{R} - \dot{R}_P) \times \dot{R}$

$$= - \dot{R}_P \times \dot{R}$$

$$- M_P)_{net} = \frac{d}{dt} \int_m r \times \dot{R}\, dm + \dot{R}_P \times \int_m \dot{R}\, dm$$

$$- H_P = \int_m r \times \dot{R}\, dm$$

$$= r_{G/P} \times \underbrace{\int_m \dot{R}\, dm}_{mV_G} + \underbrace{\int_m \rho \times \dot{R}\, dm}_{H_G}$$

$$- H_G = \int_m \rho \times \dot{R}_G\, dm + \int_m \rho \times \dot{\rho}\, dm$$

$$= \underbrace{\left(\int_m \rho\, dm\right)}_{0} \times \dot{R}_G + \int_m \rho \times \dot{\rho}\, dm$$

$$- H_G = H_G)_{rel}$$

$$- H_P = \underbrace{\left(\int_m r\, dm\right)}_{m r_{G/P}} \times \dot{R}_P + \underbrace{\int_m r \times \dot{r}\, dm}_{H_P)_{rel}}$$

$$- H_P)_{rel} = H_G + r_{G/P} \times m V_{G/P}$$

$$- M_P)_{net} = \dot{H}_P + \dot{R}_P \times m \dot{R}_G$$

$$= \dot{H}_P + V_P \times m V_G$$

$= H_P + r_P \times m v_G$

- $M_G)_{net} = \dot{H}_G$

- $\int_{t_1}^{t_2} M_G)_{net}\, dt = H_G)_2 - H_G)_1$

- $\dot{H}_P)_{rel} = \dot{H}_G + r_{G/P} \times m a_{G/P}$

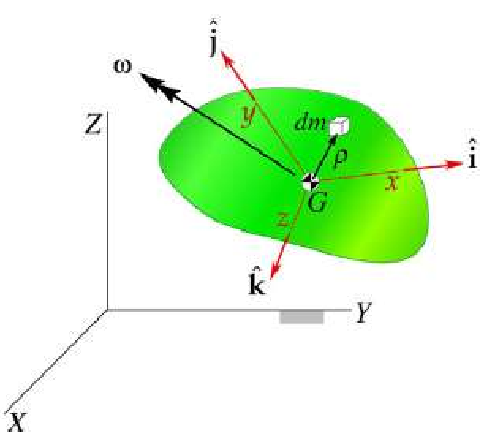
- $M_P)_{net} = \dot{H}_G + r_{G/P} \times m a_G$

- $M_G)_{net} = \dot{H}_P)_{rel} + r_{G/P} \times m a_{P/G}$

- $H_G = \int_m \rho \times (\omega \times \rho)\, dm$ (rigid body)

- $H_P = \int_m r \times (\omega \times r)\, dm$ (rigid body rotating about a fixed point P)

Moments of Inertia



- $\rho \times (\omega \times \rho) = \omega \|\rho\|^2 - \rho(\omega \cdot \rho)$

- $\rho = x\hat{i} + y\hat{j} + z\hat{k}, \quad \omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$

- $$\rho \times (\omega \times \rho) = \left[(y^2+z^2)\omega_x - xy\,\omega_y - xz\,\omega_z\right]\hat{i} + \left[-yx\,\omega_x + (x^2+z^2)\omega_y - yz\,\omega_z\right]\hat{j} + \left[-zx\,\omega_x - zy\,\omega_y + (x^2+y^2)\omega_z\right]\hat{k}$$

- $H_G = H_x \hat{i} + H_y \hat{j} + H_z \hat{k}$

- $$\underbrace{\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}}_{H} = \underbrace{\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}}_{I} \underbrace{\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}}_{\omega}$$

$$\begin{aligned}
I_x &= \int_m (y^2+z^2)\mathrm{d}m & I_{xy} &= -\int_m xy\mathrm{d}m & I_{xz} &= -\int_m xz\mathrm{d}m \\
I_{yx} &= -\int_m yx\mathrm{d}m & I_y &= \int_m (x^2+z^2)\mathrm{d}m & I_{yz} &= -\int_m yz\mathrm{d}m \\
I_{zx} &= -\int_m zx\mathrm{d}m & I_{zy} &= -\int_m zy\mathrm{d}m & I_z &= \int_m (x^2+y^2)\mathrm{d}m
\end{aligned}$$