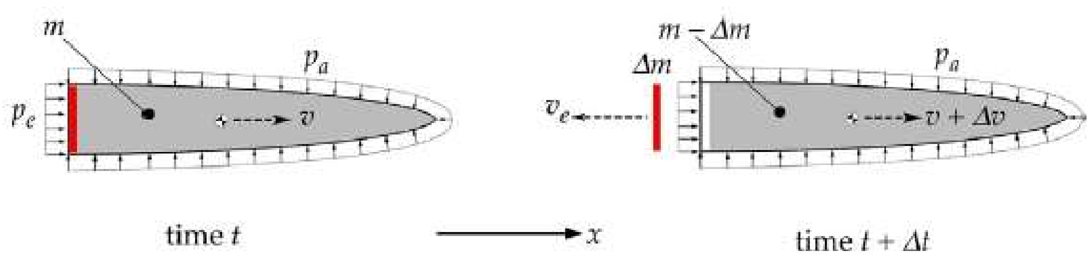
Orbital Manoeuvres

- Orbital manoeuvres transfer a spacecraft from one orbit to another.

- Changing of orbits requires the firing of on-board rocket engines.

- Impulsive manoeuvres are those in which brief firings of on-board rocket motors change the magnitude and the direction of the velocity vector instantaneously.

The Thrust Equation



- According to Newton's second law of motion:

  (Momentum of system at $t+\Delta t$) − (momentum of system at t) = Net external impulse

  $$\left[(m-\Delta m)(v+\Delta v)\hat{i} + \Delta m(-v_e\hat{i})\right] - mv\hat{i} = (p_e - p_a)A_e\Delta t\,\hat{i}$$

- $\dfrac{dm}{dt} = -\dot{m}_e$

- $m(t) = m_0 - \dot{m}_e t$ (assuming $\dot{m}_e$ is a constant)

- $\Delta m = \dot{m}_e \Delta t$

- $\left[(m - \dot{m}_e \Delta t)(v + \Delta v)\hat{i} + \dot{m}_e \Delta t(-v_e \hat{i})\right] - mv\hat{i} = (p_e - p_a)A_e \Delta t\,\hat{i}$

- $m\Delta v\hat{i} - \dot{m}_e \Delta t(v_e + v)\hat{i} - \dot{m}_e \Delta t \Delta v\hat{i} = (p_e - p_a)A_e \Delta t\,\hat{i}$

- $m\dfrac{dv}{dt} - \dot{m}_e \underbrace{c_a}_{v_e + v} = (p_e - p_a)A_e$

- $m\dfrac{dv}{dt} = \dot{m}_e c_a + (p_e - p_a)A_e$

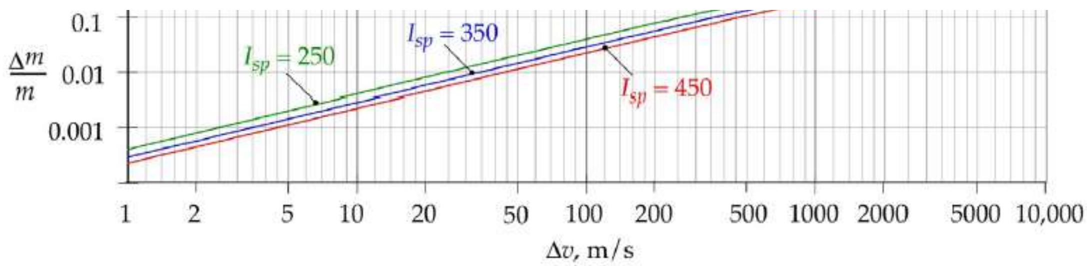
- $T = \dot{m}_e\left[c_a + \dfrac{(p_e - p_a)A_e}{\dot{m}_e}\right] = \dot{m}_e c$

- Specific impulse:

$$I_{sp} = \frac{T}{\dot{m}_e g_0} \quad \left(\frac{\text{Thrust}}{\text{Sea-level weight-rate of consumption}}\right)$$

Table 6.1 Some typical specific impulses

Propellant	I_{sp} (s)
Cold gas	50
Monopropellant hydrazine	230
Solid propellant	290
Nitric acid/monomethylhydrazine	310
Liquid oxygen/liquid hydrogen	455
Ion propulsion	>3000



- $c = I_{sp} g_0$

Rocket Performance

- $T = -I_{sp} g_0 \frac{dm}{dt}$

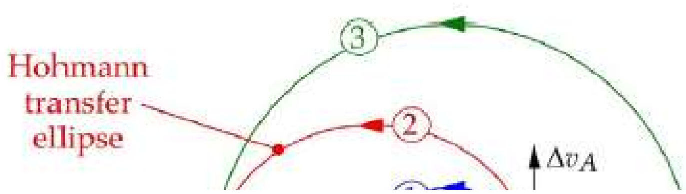
- $m \frac{dv}{dt} = -I_{sp} g_0 \frac{dm}{dt}$

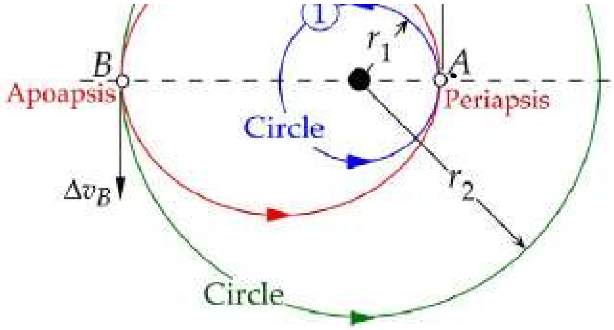
- $\Delta v = I_{sp} g_0 \ln\left(\frac{m_0}{m_f}\right)$

- $\frac{\Delta m}{m} = \frac{m_0 - m_f}{m_0} = 1 - \frac{m_f}{m_0} = 1 - e^{\frac{-\Delta v}{I_{sp} g_0}}$

Hohmann Transfer [Walter Hohmann (1880 - 1945)]

- The most energy-efficient two-impulse manoeuvre for transferring between two co-planar circular orbits sharing a common focus.





- $\Delta V_{total} = \Delta V_A + \Delta V_B$

- $\|e\| = \dfrac{r_a - r_p}{r_a + r_p}$

- $r_p = \dfrac{\|h\|^2}{\mu} \dfrac{1}{1+\|e\|}$

- $r_p = \dfrac{\|h\|^2}{\mu} \dfrac{r_a + r_p}{2r_a}$

- $\|h\| = \sqrt{2\mu} \sqrt{\dfrac{r_a r_p}{r_a + r_p}}$ $(\|h\| = \|r\| v_\perp)$

- $\|h\| = \sqrt{\mu \|r\|}$ (circular orbit)

- $\|v\| = \sqrt{2\mu} \sqrt{\dfrac{1}{\|r\|} - \dfrac{1}{2a}}$

Example

A 2000-kg spacecraft is in a 480 km by 800 km earth orbit (orbit 1 in Fig. 6.3). Find
(a) The Δv required at perigee A to place the spacecraft in a 480 km by 16,000 km transfer ellipse (orbit 2).
(b) The Δv (apogee kick) required at B of the transfer orbit to establish a circular orbit of 16,000 km altitude (orbit 3).
(c) The total required propellant if the specific impulse is 300 s.

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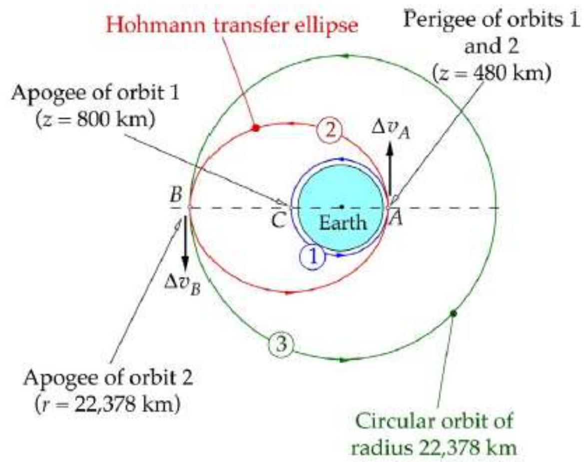


Details

Orbit 1: $r_p = 6378 + 480$ Km, $r_a = 6378 + 800$ Km

Orbit 2: $r_p = 6378 + 480$ Km, $r_a = 6378 + 16000$ Km

Orbit 3: $r_p = r_a = 6378 + 16000$ Km

(a) $V_A)_1 = \frac{\|h_1\|}{r_A}$, $V_A)_2 = \frac{\|h_2\|}{r_A}$

$$\Delta V_A = V_A)_2 - V_A)_1$$

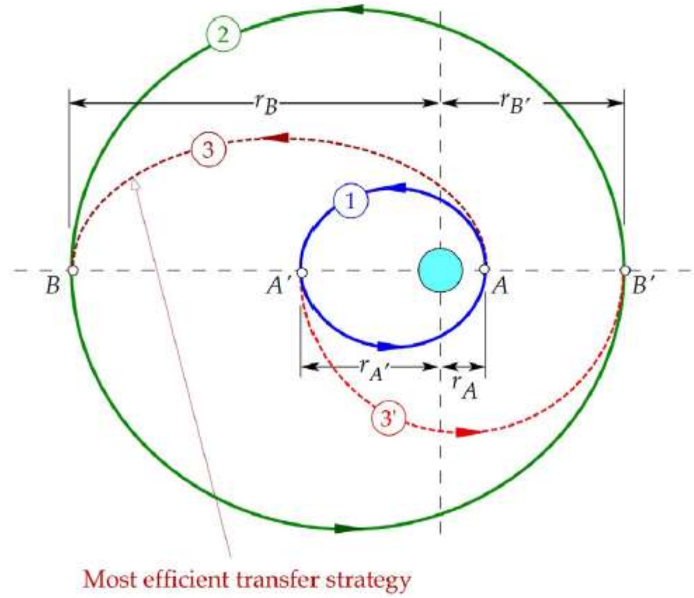
(b) $V_B)_2 = \frac{\|h_2\|}{r_B}$, $V_B)_3 = \frac{\|h_3\|}{r_B}$

$$\Delta V_B = V_B)_3 - V_B)_2$$

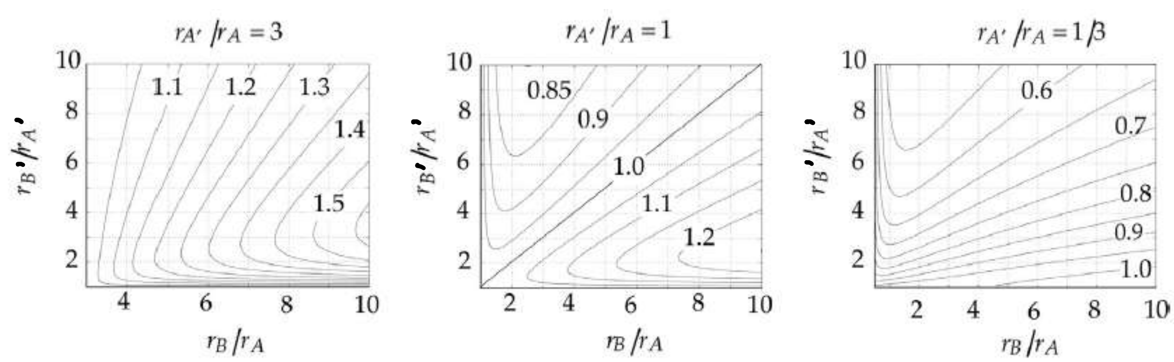
(c) $\Delta V_{total} = |\Delta V_A| + |\Delta V_B|$

$$\underline{\Delta m} = 1 - e^{\frac{-\Delta V_{total}}{I_{sp} g_0}}$$

m



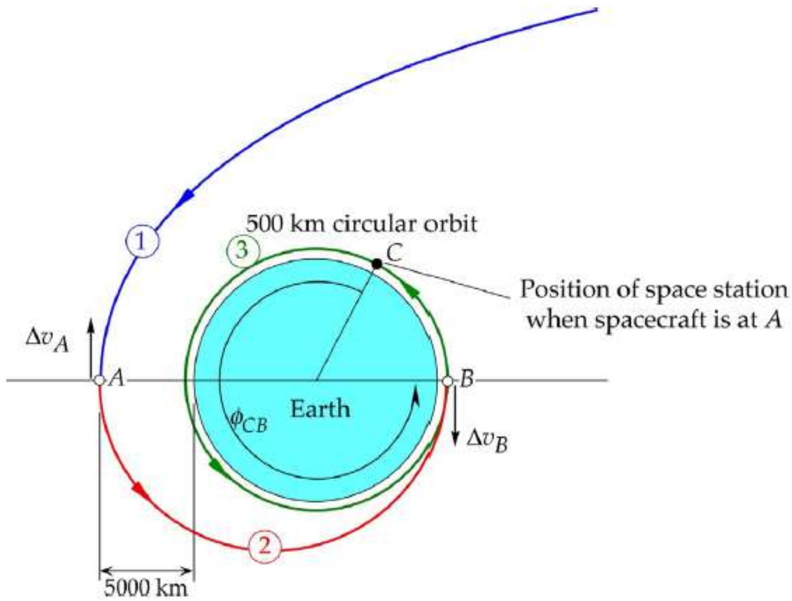
- $\Delta V_{total})_3 = \Delta V_A + \Delta V_B$, $\Delta V_{total})_{3'} = \Delta V_{A'} + \Delta V_{B'}$
- $\Delta V_A = |V_A)_3 - V_A)_1|$, $\Delta V_B = |V_B)_2 - V_B)_3|$
- $\Delta V_{A'} = |V_{A'})_{3'} - V_{A'})_1|$, $\Delta V_{B'} = |V_{B'})_2 - V_{B'})_{3'}|$



(a) A is periapsis of orbit 1 (b) Orbit 1 is circular (c) A is apoapsis of orbit 1

Example

A spacecraft returning from a lunar mission approaches earth on a hyperbolic trajectory. At its closest approach A it is at an altitude of 5000 km, traveling at 10 km/s. At A retrorockets are fired to lower the spacecraft into a 500-km-altitude circular orbit, where it is to rendezvous with a space station. Find the location of the space station at retrofire so that rendezvous will occur at B (Fig. 6.6).



Details

$$r_A = 5000 + 6378 \text{ Km}, \quad r_B = 500 + 6378 \text{ km}$$

$$a = \frac{1}{2}(r_A + r_B)$$

$$T_2 = \frac{2\pi}{\sqrt{\mu}} a^{3/2}$$

$$T_3 = \frac{2\pi}{\sqrt{\mu}} r_B^{3/2}$$

$$\Delta t_{CB} = \frac{T_2}{2}$$

$$\frac{\phi_{CB}}{\Delta_{CB}} = \frac{360^\circ}{T_3}$$