MA 214, Introduction to Numerical Analysis, Spring 2018
Tutorial 1: Extra Problem

1. Let $p_n(x)$ be a polynomial of degree $n \geq 1$.

   (a) If c is a simple root of p_n, then show that there exists a polynomial $q_{n-1}(x)$ of degree $n - 1$ such that
   $$p_n(x) = (x - c)q_{n-1}(x) \quad \text{and} \quad q_{n-1}(c) \neq 0.$$

   (b) If c is a double root of p_n, then show that there exists a polynomial $q_{n-2}(x)$ of degree $n - 2$ such that
   $$p_n(x) = (x - c)^2 q_{n-2}(x) \quad \text{and} \quad q_{n-2}(c) \neq 0.$$

   (c) If c is a root of p_n of multiplicity m, then show that there exists a polynomial $q_{n-m}(x)$ of degree $n - m$ such that
   $$p_n(x) = (x - c)^m q_{n-m}(x) \quad \text{and} \quad q_{n-m}(c) \neq 0.$$