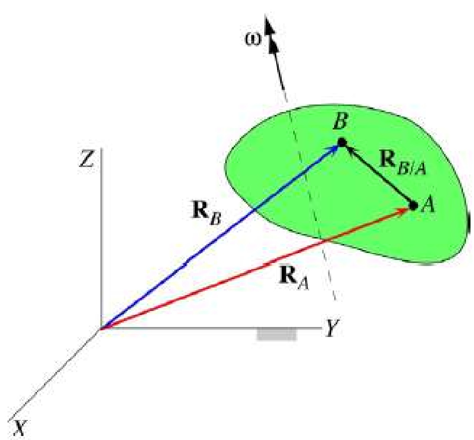
Rigid Body Dynamics

- A rigid body is a collection of points, each of which may or may not have mass, such that the distance between any two points on the body is a constant.

- Chasles' theorem states that the motion of a rigid body can be described by the displacement of any point of the body (the base point) plus a rotation about a unique axis through that point.

Kinematics



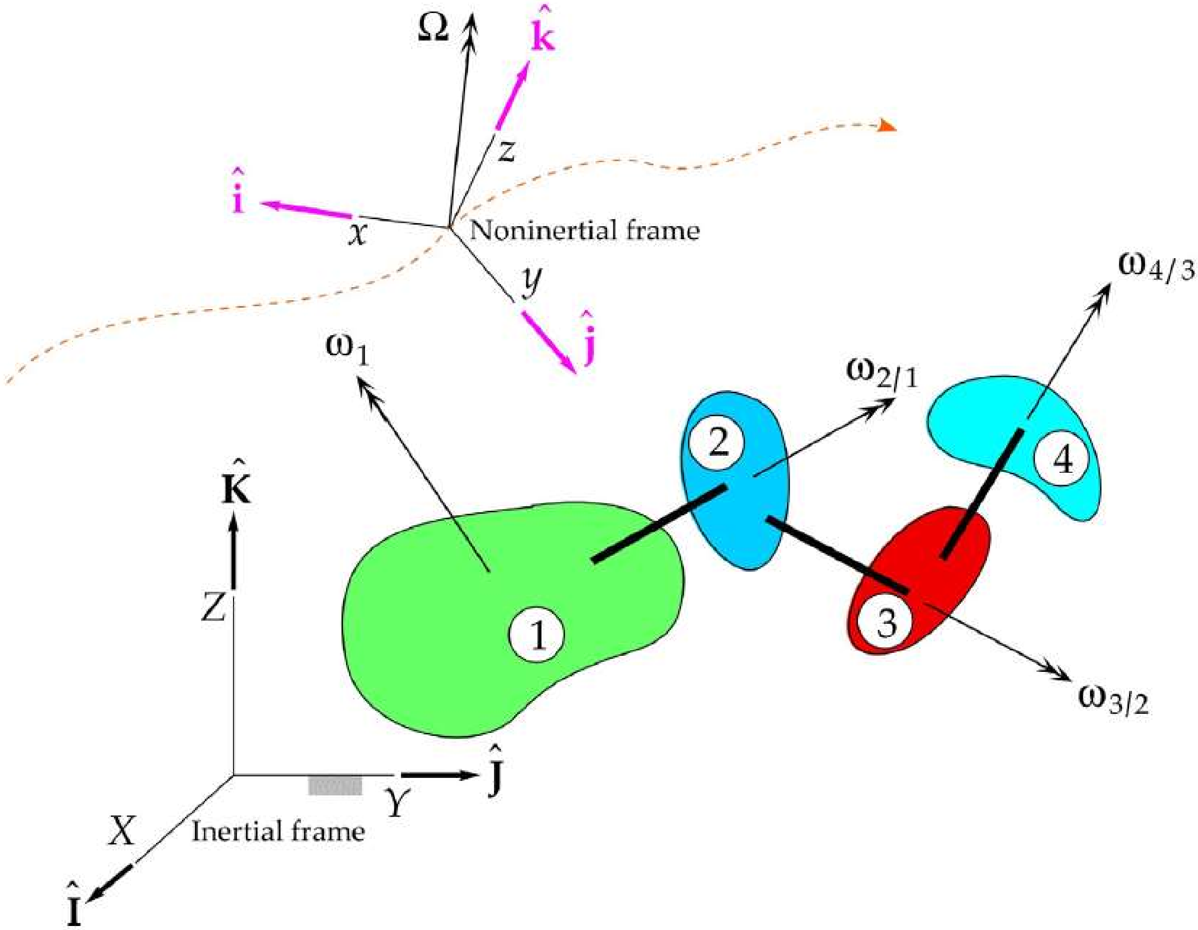
- $R_B = R_A + R_{B/A}$

- $\dot{R}_B = \dot{R}_A + \frac{d}{dt} R_{B/A}$

  $= \dot{R}_A + \omega \times R_{B/A}$

- $V_B = V_A + \omega \times R_{B/A}$

$$- a_B = a_A + \alpha \times R_{B/A} + \omega \times (\omega \times R_{B/A})$$



$$- \omega = \omega_1 + \omega_{2/1} + \omega_{3/2} + \omega_{4/3}$$

$$- \omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

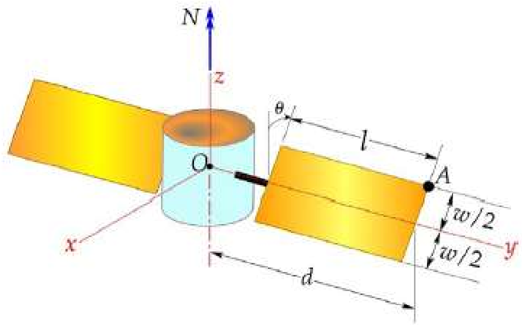
$$- \alpha = \left.\frac{d\omega}{dt}\right)_{rel} + \Omega \times \omega$$

$$- \left.\frac{d\omega}{dt}\right)_{rel} = \dot{\omega}_x \hat{i} + \dot{\omega}_y \hat{j} + \dot{\omega}_z \hat{k}$$

<u>Example</u>

The satellite in Fig. 11.4 is rotating about the z axis at a constant rate N. The xyz axes are attached to the spacecraft, and the z axis has a fixed orientation in inertial space. The solar panels rotate at a constant rate $\dot{\theta}$ in the direction shown. Relative to point O, which lies at the center of the spacecraft and on the centerline of the panels, calculate for point A on the panel

(a) its absolute velocity and
(b) its absolute acceleration.



Details

(a) $$\omega_{panel} = \omega_{satellite} + \omega_{panel/satellite} = N\hat{k} - \dot{\theta}\hat{j}$$

$$r_{A/O} = -\frac{w}{2}\sin\theta\,\hat{i} + d\hat{j} + \frac{w}{2}\cos\theta\,\hat{k}$$

$$V_{A/O} = V_A - V_O = \omega_{panel} \times r_{A/O}$$

(b) $$\alpha_{panel} = \left.\frac{d\omega_{panel}}{dt}\right)_{rel} + \underbrace{\Omega}_{N\hat{k}} \times \omega_{panel}$$

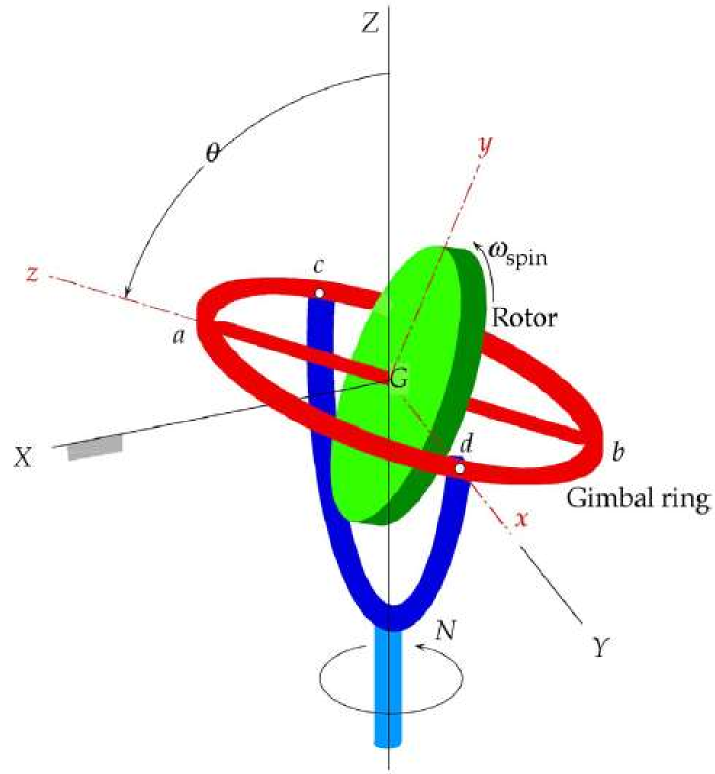
$$a_{A/O} = a_A - a_O = \alpha_{panel} \times r_{A/O} + \omega_{panel} \times (\omega_{panel} \times r_{A/O})$$

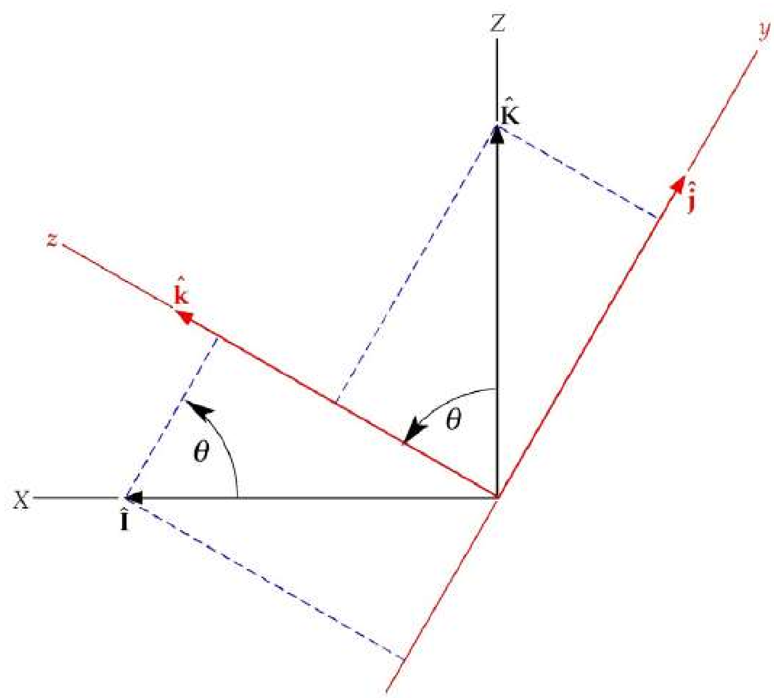
Example

The gyro rotor illustrated in Fig. 11.5 has a constant spin rate ω_{spin} around axis b–a in the direction shown. The XYZ axes are fixed. The xyz axes are attached to the gimbal ring, whose angle θ with the vertical is increasing at a constant rate $\dot{\theta}$ in the direction shown. The assembly is forced to precess at a constant rate N around the vertical. For the rotor in the position shown, calculate

(a) the absolute angular velocity and
(b) the absolute angular acceleration.

Express the results in both the fixed XYZ frame and the moving xyz frame.





Details

$$\hat{I} = -\cos\theta\,\hat{j} + \sin\theta\,\hat{k}$$
$$\hat{J} = \hat{i}$$

$$\hat{K} = \sin\theta\,\hat{j} + \cos\theta\,\hat{k}$$

$$[Q]_{xX} = \begin{bmatrix} 0 & -\cos\theta & \sin\theta \\ 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \end{bmatrix}$$

(a) $$\omega_{gimbal} = \omega_{base} + \omega_{gimbal/base}$$
$$= N\hat{K} + \dot{\theta}\hat{i}$$
$$= \dot{\theta}\hat{i} + N\sin\theta\,\hat{j} + N\cos\theta\,\hat{k}$$

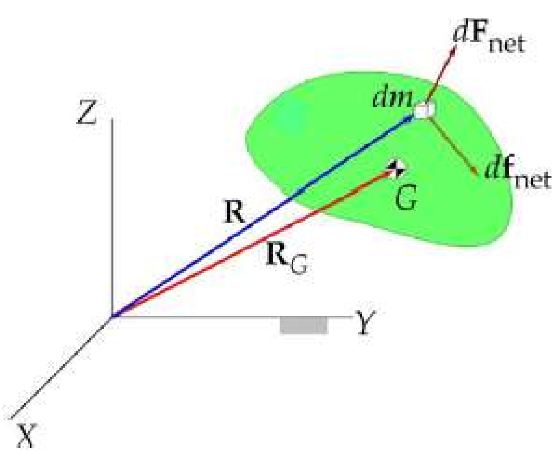
$$\omega_{rotor} = \omega_{gimbal} + \omega_{spin}\hat{k}$$

$$\omega_{rotor}|_{XYZ} = [Q]_{xX}\,\omega_{rotor}|_{xyz}$$

(b) $$\alpha_{rotor} = \left.\frac{d\omega_{rotor}}{dt}\right)_{rel} + \underbrace{\Omega}_{\downarrow\ \omega_{gimbal}} \times \omega_{rotor}$$

$$\alpha_{rotor}|_{XYZ} = [Q]_{xX}\,\alpha_{rotor}|_{xyz}$$

Equations of Translational Motion



- $m R_G = \int_m R \, dm$

- $dF_{net} + df_{net} = dm \, \ddot{R}$

- $\int_m dF_{net} + \underbrace{\int_m df_{net}}_{0} = \int_m \ddot{R} \, dm$

- $F_{net} = \int_m \ddot{R} \, dm$

$$= m \ddot{R}_G$$