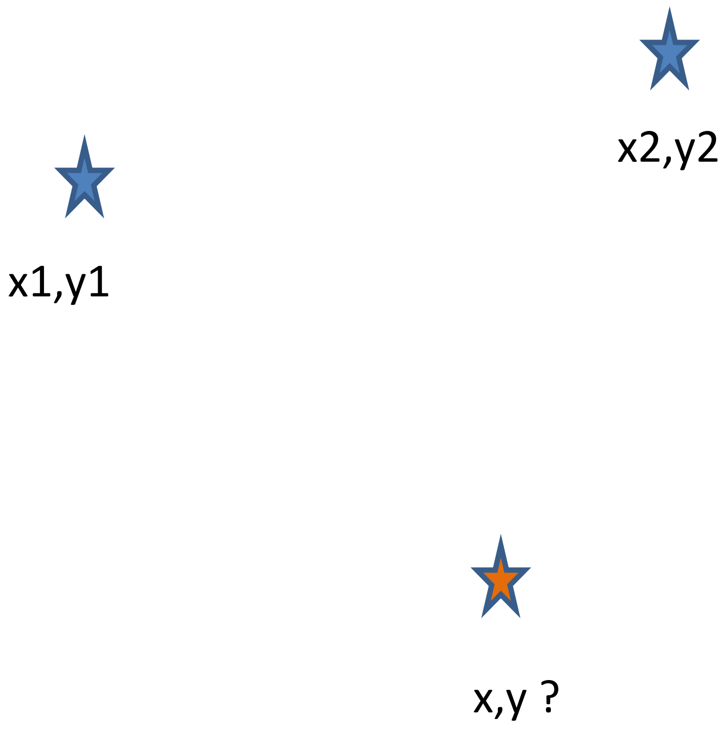
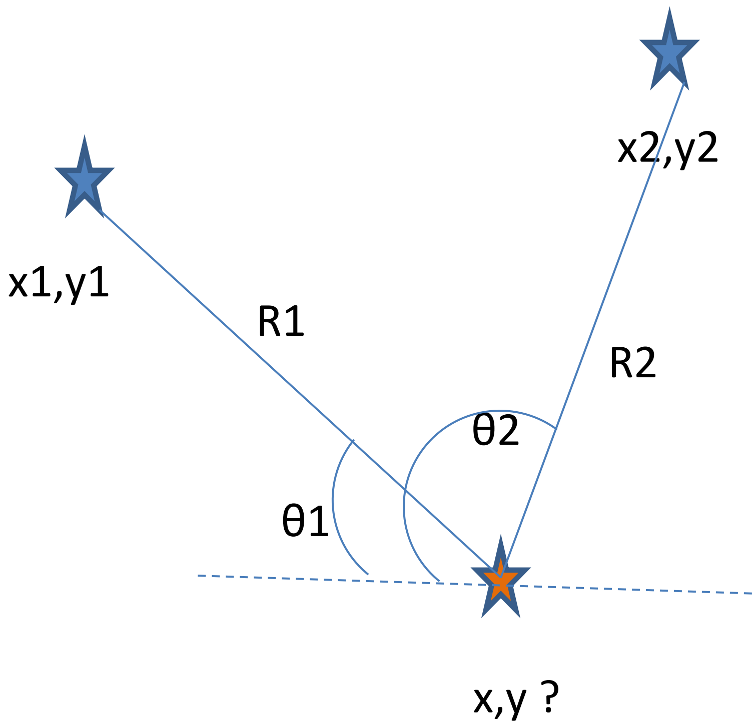
AE 242
Aerospace Measurements Laboratory

Position determination



Position determination



Presentation material from book “Understanding GPS principles and applications”
by
Elliot D. Kaplan and Christopher J. Hegarty (editors)

Use of GPS

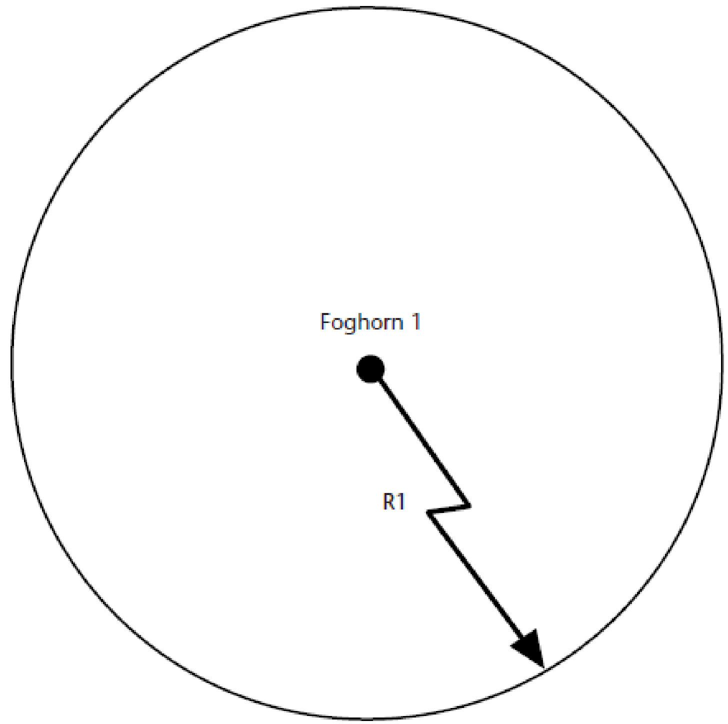
Introduction

- GPS – Global Positioning system
- Owned and operate by DoD, USA
- Similar to radio service
- Free to use service
- Designed in such a way that can be used over the whole globe
- Consist of three segment: Space, Control & Monitor and user

Introduction

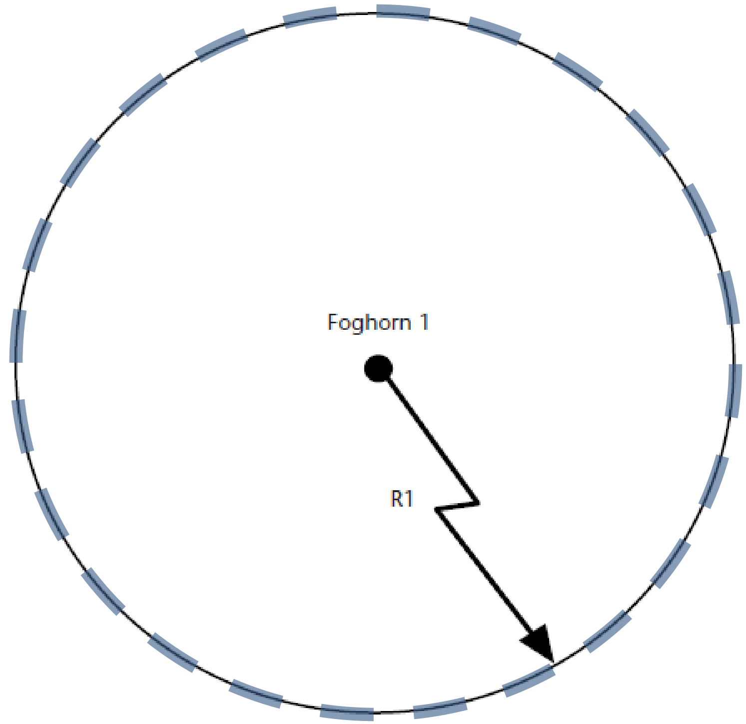
- Space- Space vehicle (satellites)
- Control & Monitor – Few stations on the ground, which monitor satellite motion, clock etc. sends correction terms to satellite for rebroadcast
- **User – Radio receiver for processing information received from GPS satellites**

Ranging using TOA Measurements



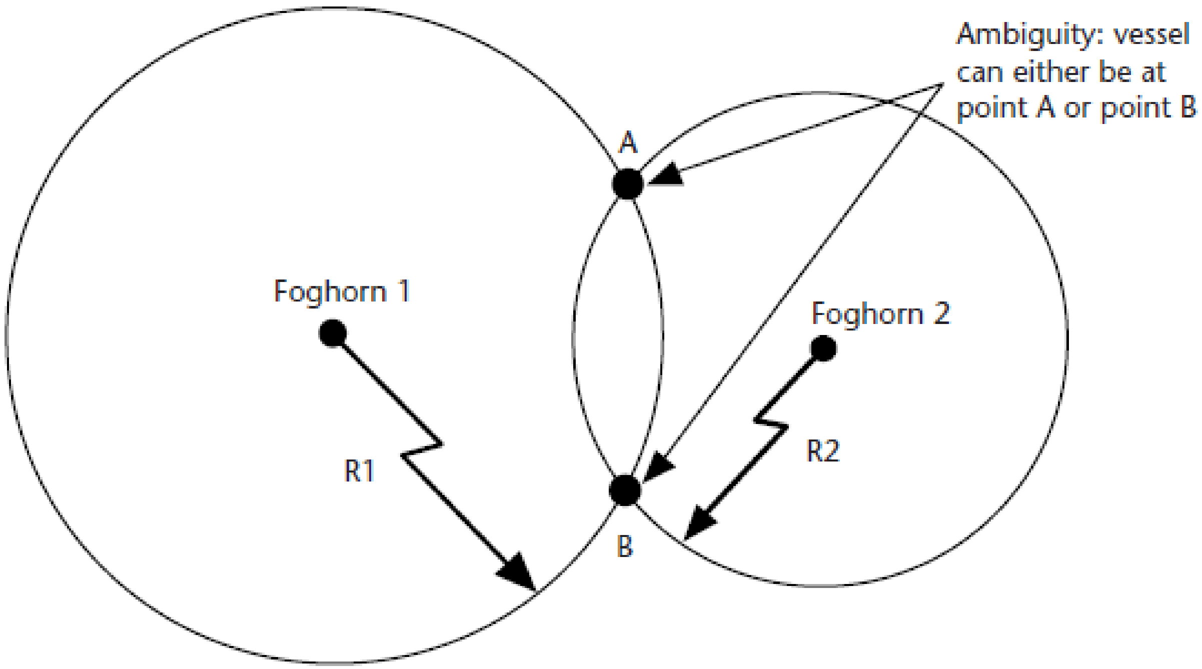
- Transmitter (Foghorn) – Transmitting signal at regular interval
- *How to find distance from foghorn?*

Ranging using TOA Measurements



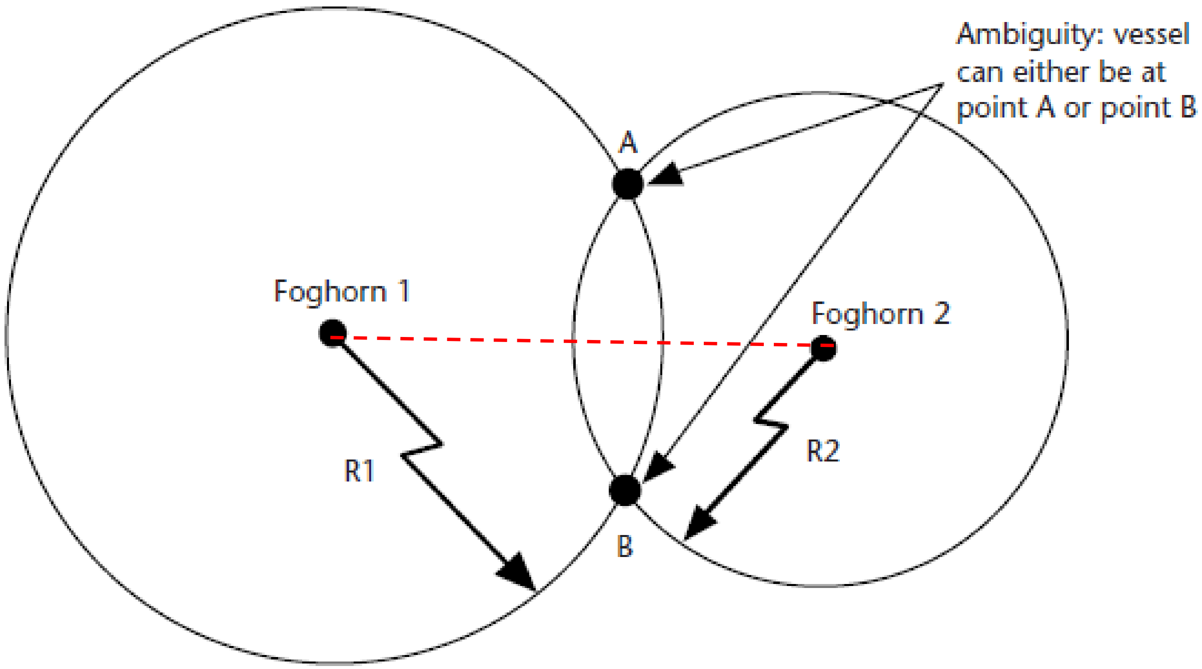
- A receiver having precise sycronised clock with transmitter clock
- By measuring time of signal travel, distance from transmitter can be estimated
- Receiver can be somewhere on the circle

Ranging using TOA Measurements



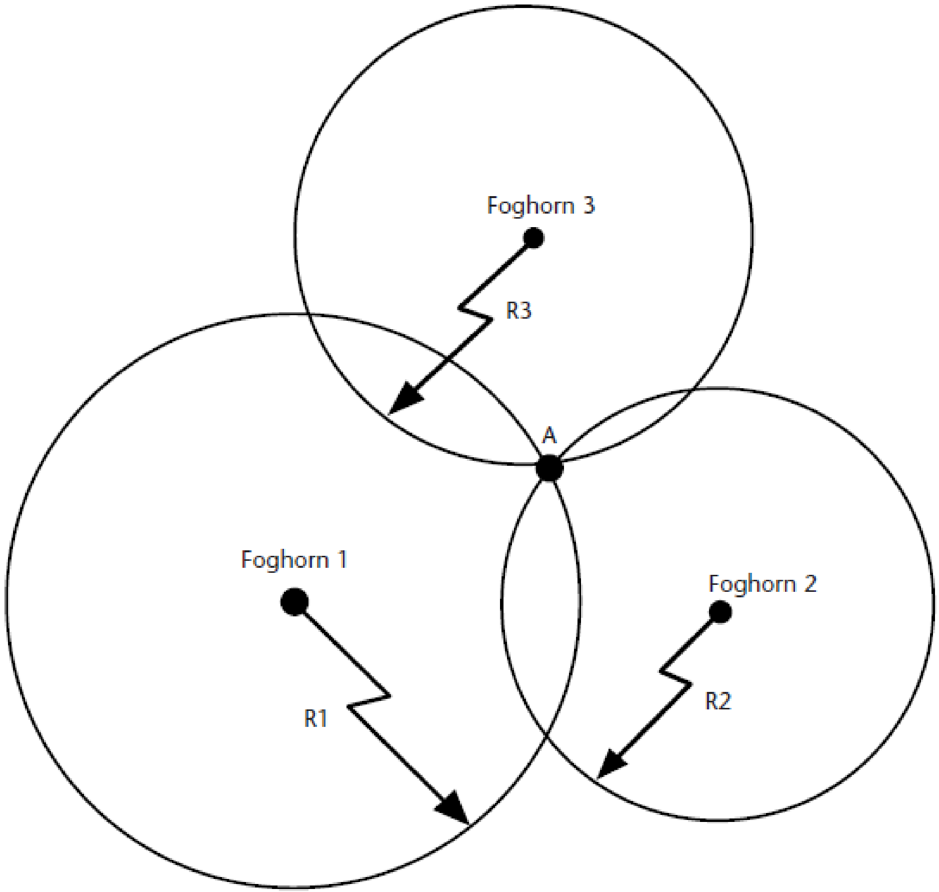
- By measuring time of signal travel from two transmitter
- Receiver can be at the intersection of two circles

Ranging using TOA Measurements



- By measuring time of signal travel from two transmitter
- Receiver can be at the intersection of two circles
- Correct location can be obtained if we know receiver is on which side of line joining transmitters

Ranging using TOA Measurements



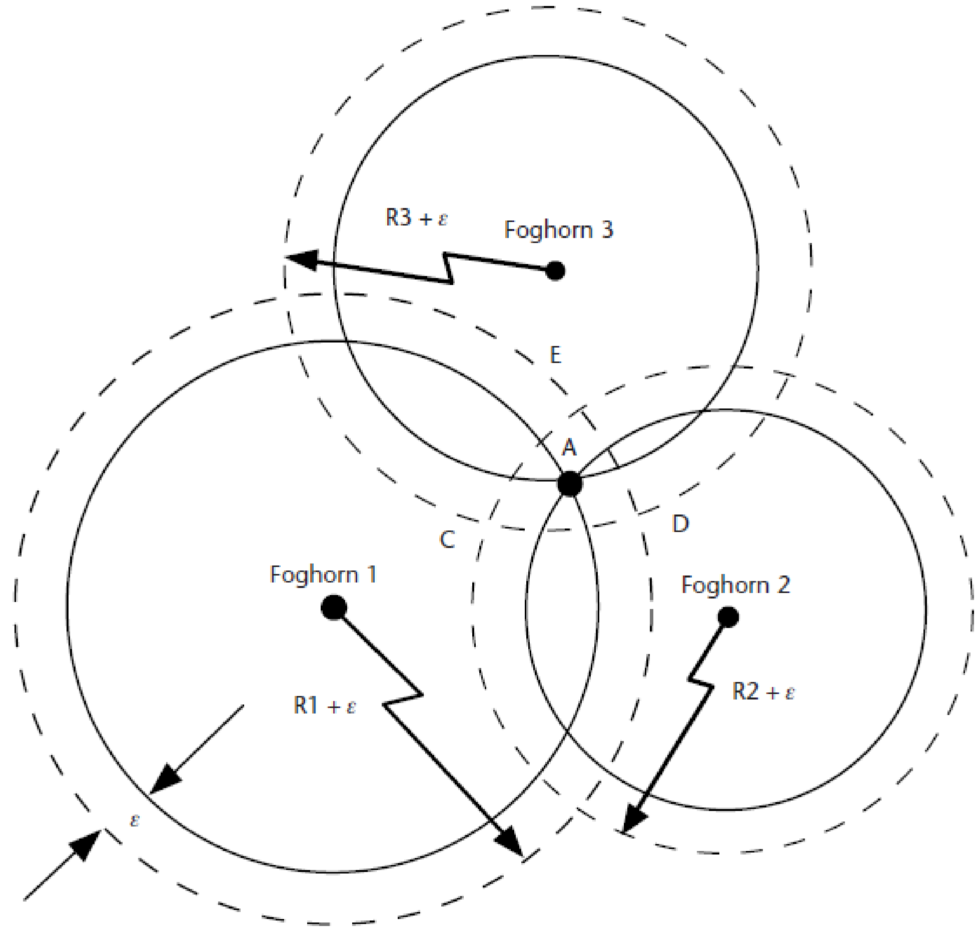
- By measuring time of signal travel from three transmitters
- Receiver will be at the intersection of three circles, ambiguity in position is resolved

Ranging using TOA Measurements

Assumptions

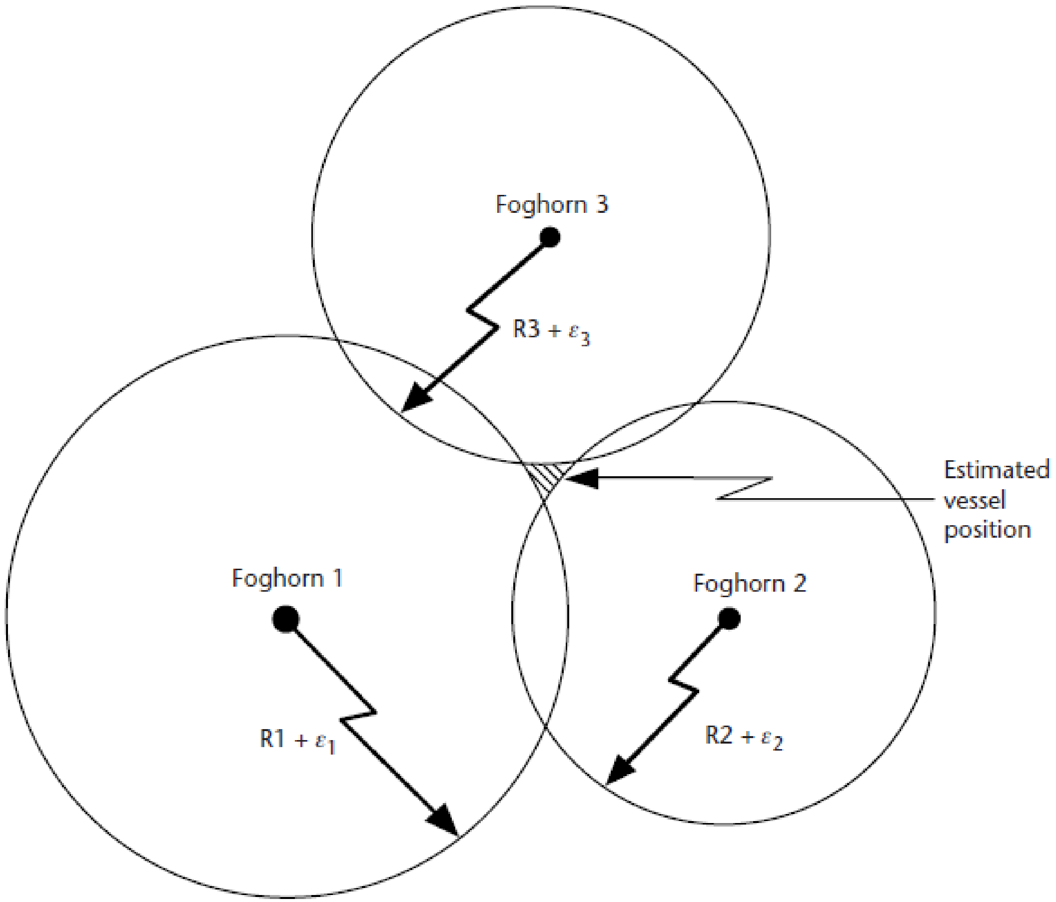
- Transmitter and receiver clocks are synchronised
- No errors in measurement of time

Ranging using TOA Measurements



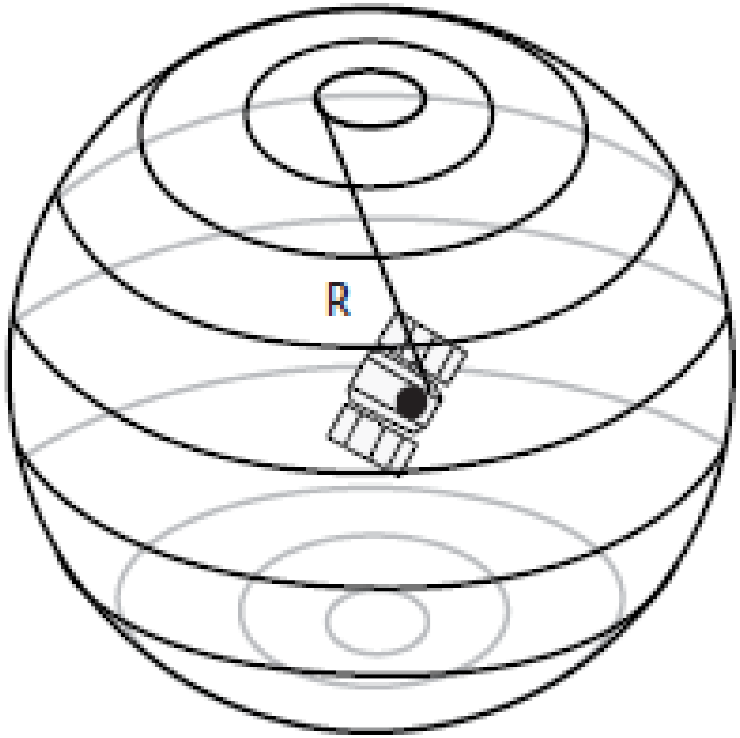
- Position estimate will have error
- Bounds on error will depend on error band of individual receiver

Ranging using TOA Measurements



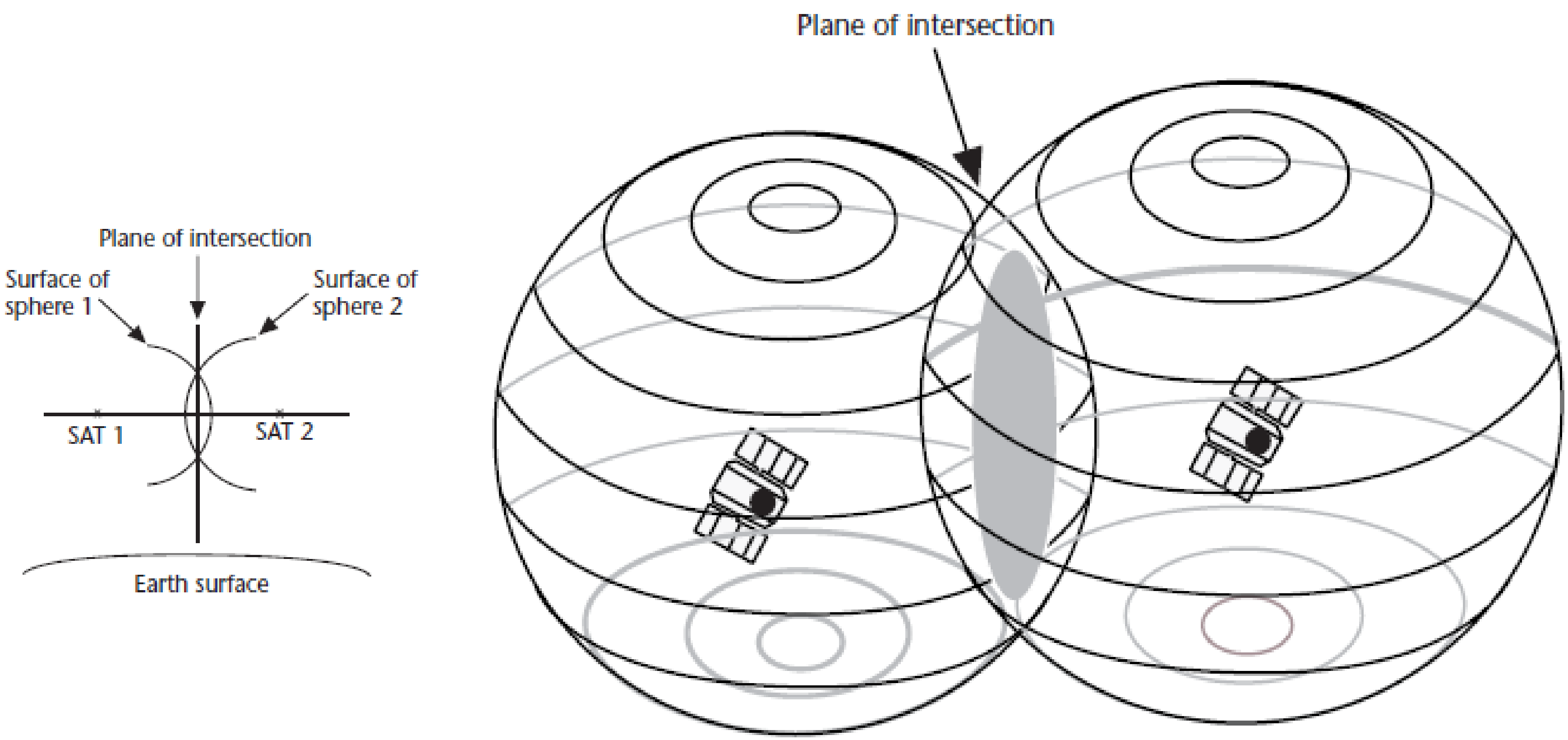
- Errors may not be constant in all the receivers
- Is the error dependent on the placement of transmitters?
- Can this concept extended to 3D positioning?

Three dimensional Position Location



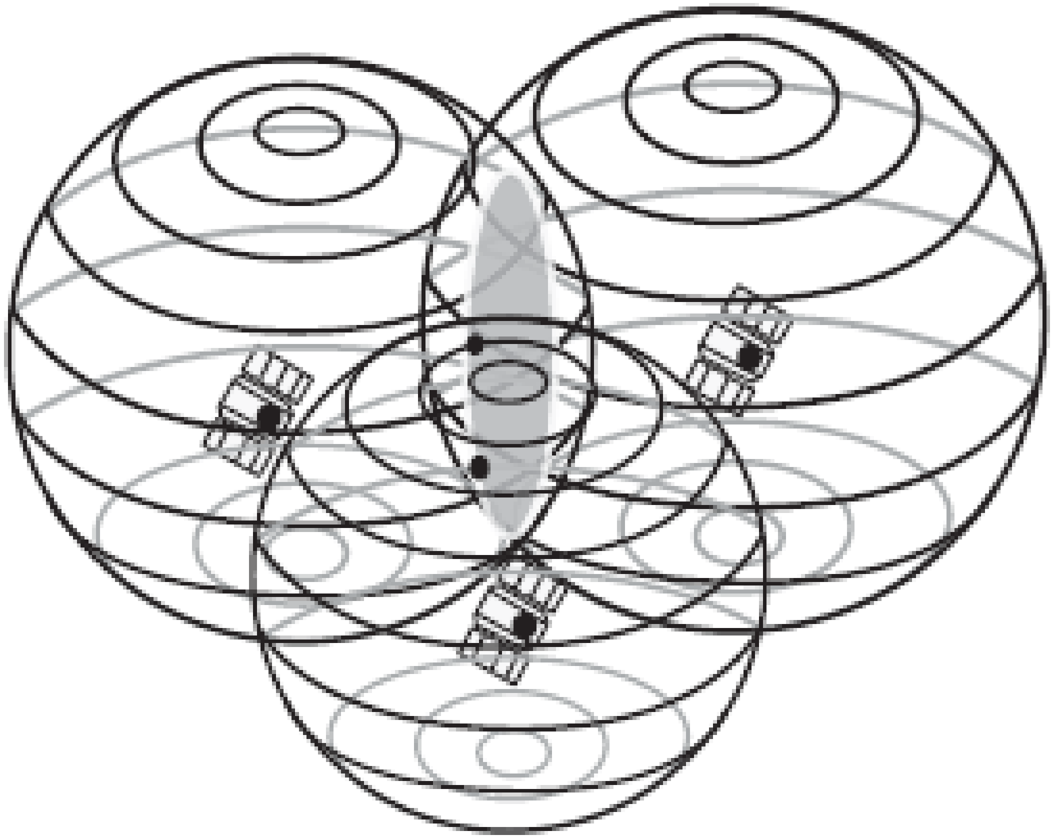
- Assume ranging measurement possible using signal from single satellite
- User will be somewhere on the sphere

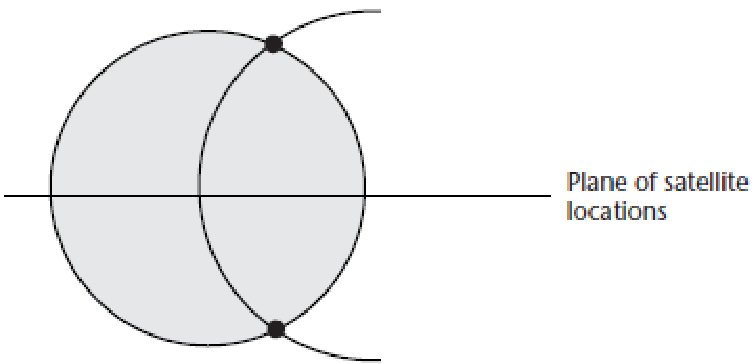
Three dimensional Position Location



- Assume ranging measurement possible using signal from two satellites
- User will be somewhere on the intersection of two spheres (circle)

Three dimensional Position Location





- Assume ranging measurement possible using signal from three satellites
- User will be somewhere on the intersection of third sphere with the circle (two points)

Three dimensional Position Location

- Range measurement
  - Position of transmitter (satellite) is required
  - Clock synchronisation (between satelliite and user) is required

- **How many satellites for three dimensional position estimation?**

Reference coordinate systems

1) ECI – Earth Centered Inertial coordinate system
2) ECEF – Earth Centered Earth Fixed coordinate system
3) WGS – World Geodetic System

ECI – Earth Centered Inertial coordinate system

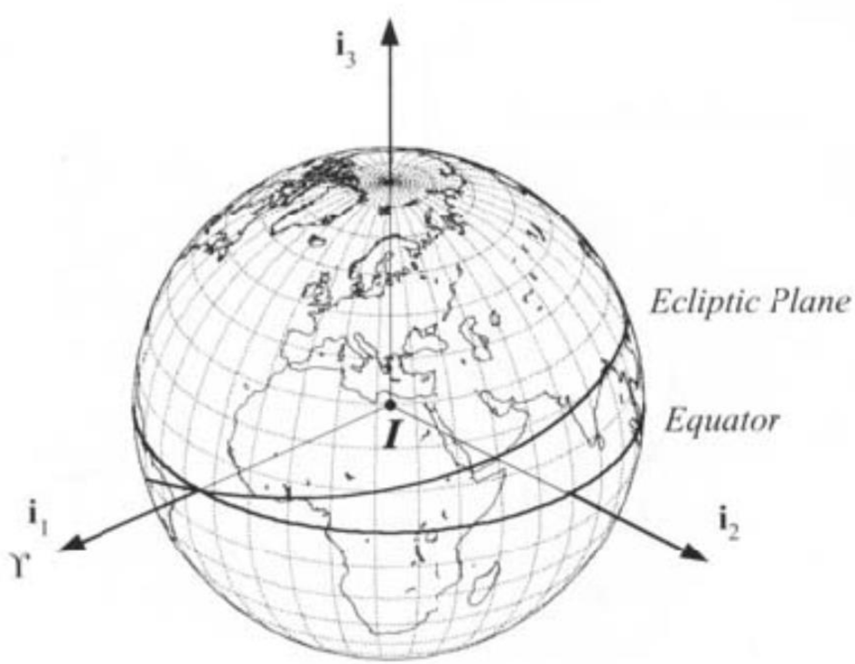


Fig. 3.5 Inertial reference frame.

Geocentric Inertial frame: Collocated with center of earth. Its orientation remain fixed in ecliptic. x –axis is the direction of vernal equinox; z-axis is axis of earth rotation and y-axis completes the triad. Used for obtaining orbit of satellites around earth. ECI Earth centered inertial frame

ECEF – Earth Centered Earth Fixed coordinate system

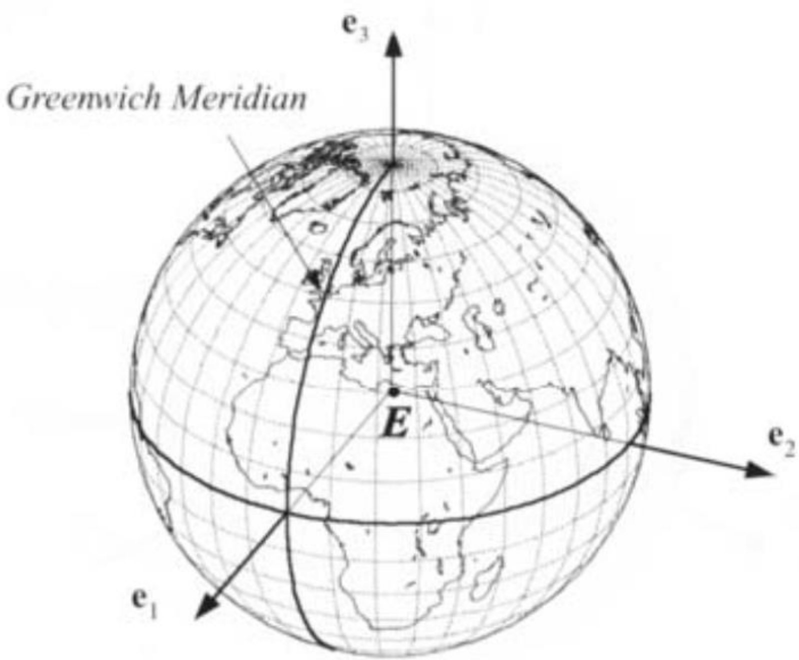


Fig. 3.6 Earth frame.

Earth frame: Base point is center of earth. Prime meridian that traces through the Royal observatory at Greenwich, is intersection with equator is x-axis. Earth axis of rotation is z-axis and y-axis completes the triad. (Meridian is a circle of constant longitude). Used for travel over earth surface. ECEF Earth centered earth fixed frame

Latitude

Longitude

Non-Spherical Earth

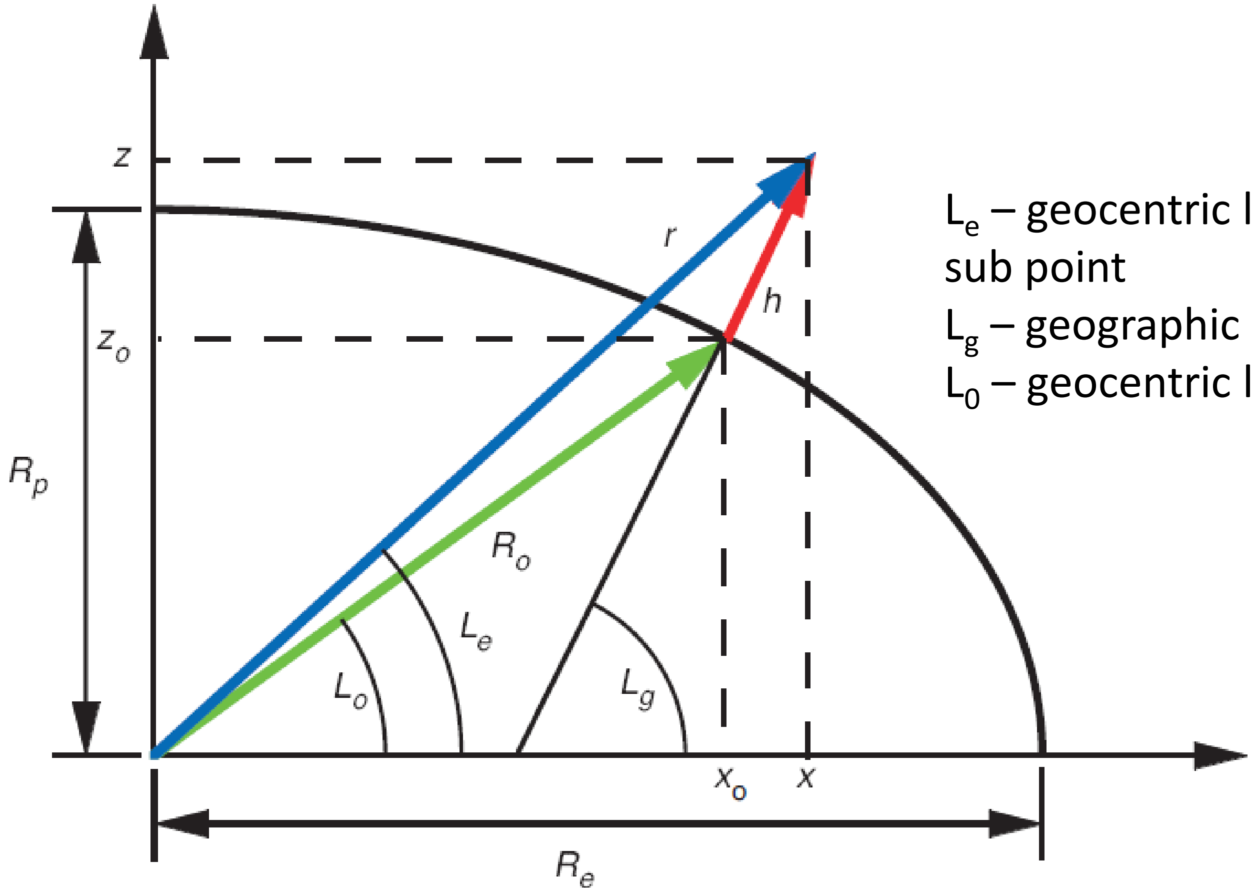


Fig. 3.3-2. Coordinates of the ellipsoidal reference frame.

L_e – geocentric latitude of sub point
L_g – geographic latitude
L_0 – geocentric latitude

Flight dynamics- Robert Stengel

WGS-84 – standard for defining earth surface. Each cross section is an ellipse. Polar radius is short by 22 km

WGS – World Geodetic System

$$\frac{x_o^2}{R_e^2} + \frac{z_o^2}{R_p^2} = 1$$

Equation satisfying cross section of earth which is ellipse
R_e and R_p are equatorial and polar earth radius and e is Ellipticity and k is Eccentricity

$$x_o^2 = R_o^2 \cos^2 L_o \qquad z_o^2 = R_o^2 \sin^2 L_o$$

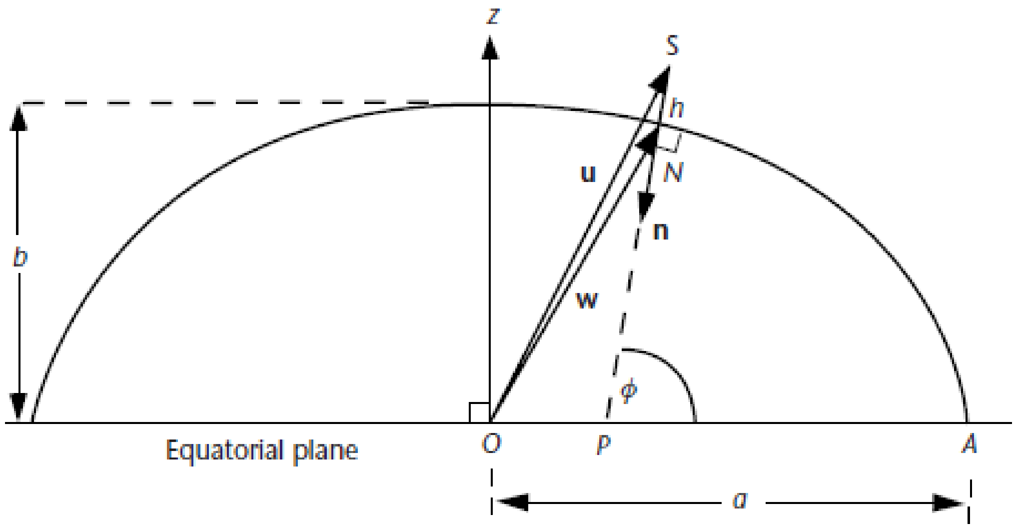
$$R_o^2 = \frac{R_p^2}{1 - \left[1 - \left(R_p / R_e\right)^2\right] \cos^2 L_o} = \frac{R_p^2}{1 - k^2 \cos^2 L_o}$$

$$e = \frac{R_e - R_p}{R_e} \qquad k^2 = 2e\left(1 - \frac{e}{2}\right)$$

TABLE 3.3-1 WORLD GEODETIC SYSTEM ELLIPSOID [A-7]

Equatorial Radius R_e, m	6,378,137
Polar Radius R_p, m	6,356,752
Mean Radius R_m, m	6,367,435
Ellipticity (or Flattening, f) e	1/298.257223563
Eccentricity k	0.08181919085

WGS – World Geodetic System



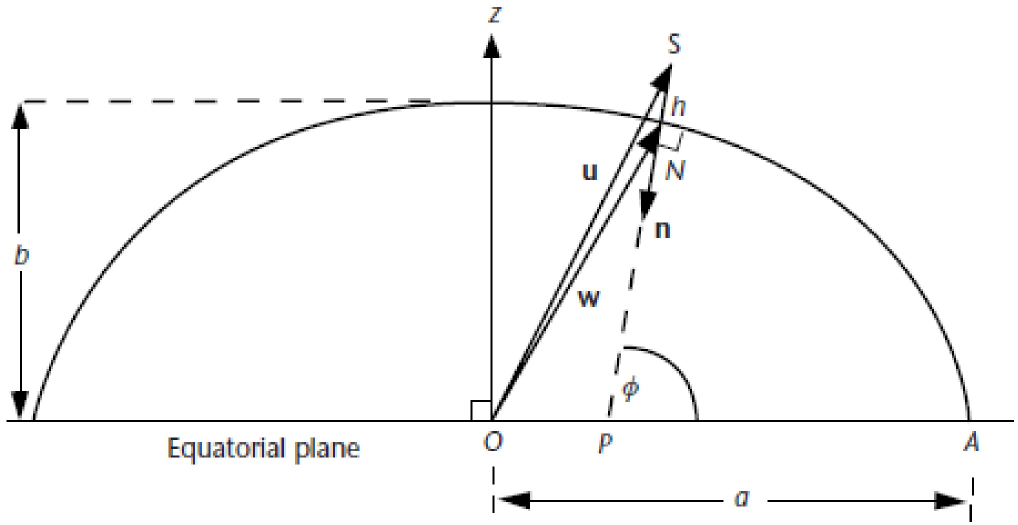
$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e' = \sqrt{\frac{a^2}{b^2} - 1} = \frac{a}{b} e$$

User is concerned about latitude and longitude on earth surface. Earth is modeled as ellipsoidal shape. Cross-section of the earth parallel to equatorial plane are circular.

Equatorial cross-section of earth a – 6378.137 km
Polar radius of the earth b – 6356.752 km

WGS – World Geodetic System



ECEF is affixed to WGS84 reference ellipsoid. Latitude, longitude and altitude wrt WGS84

Longitude

$$\lambda = \begin{cases} \arctan\left(\dfrac{y_u}{x_u}\right), & x_u \geq 0 \\ 180^0 + \arctan\left(\dfrac{y_u}{x_u}\right), & x_u < 0 \;\; and \;\; y_u \geq 0 \\ -180^0 + \arctan\left(\dfrac{y_u}{x_u}\right), & x_u < 0 \;\; and \;\; y_u < 0 \end{cases}$$

WGS – World Geodetic System

$$p = \sqrt{x^2 + y^2}$$

$$\tan u = \left(\frac{z}{p}\right)\left(\frac{a}{b}\right)$$

Iteration Loop

$$\cos^2 u = \frac{1}{1 + \tan^2 u}$$

$$\sin^2 u = 1 - \cos^2 u$$

$$\tan \varphi = \frac{z + e'^2 b \sin^3 u}{p - e^2 a \cos^3 u}$$

$$\tan u = \left(\frac{b}{a}\right) \tan \varphi$$

until $\tan u$ *converges, then*

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$$

$$h = \frac{p}{\cos \phi} - N \qquad \varphi \neq \pm 90^0$$

otherwise

$$h = \frac{z}{\sin \phi} - N + e^2 N \qquad \varphi \neq 0$$

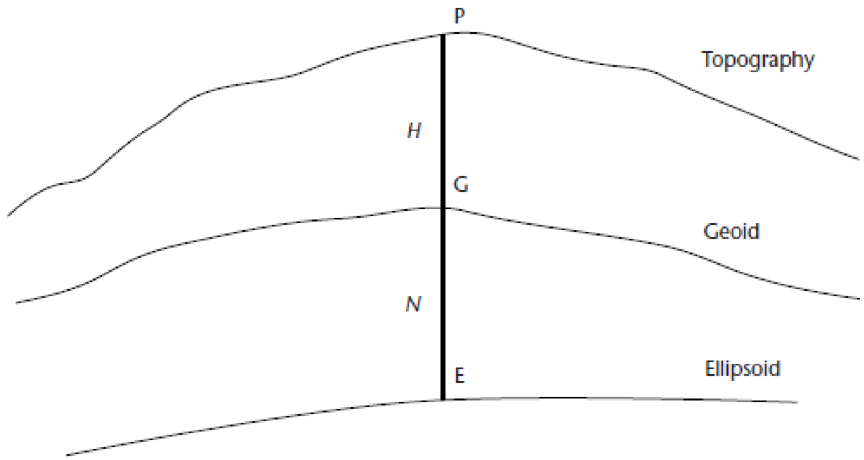
Computation of geodetic latitude and height

WGS – World Geodetic System

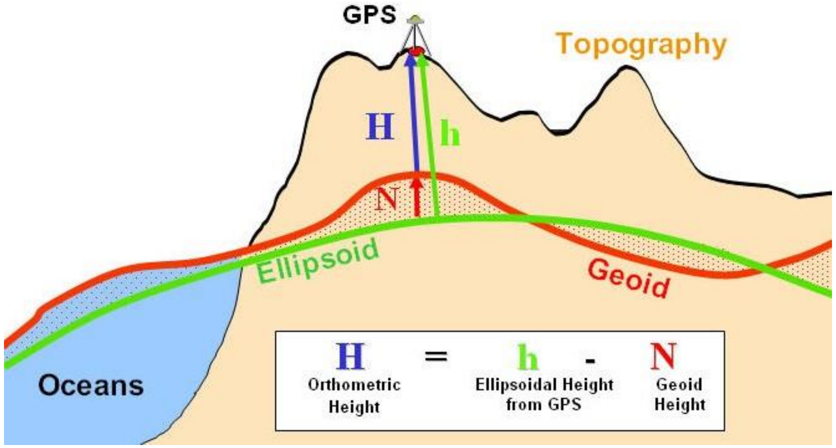
$$\mathbf{u} = \begin{bmatrix} \dfrac{a\cos\lambda}{\sqrt{1+\left(1-e^2\right)\tan^2\phi}} + h\cos\lambda\cos\phi \\ \dfrac{a\sin\lambda}{\sqrt{1+\left(1-e^2\right)\tan^2\phi}} + h\sin\lambda\cos\phi \\ \dfrac{a\left(1-e^2\right)\sin\phi}{\sqrt{1-e^2\sin^2\phi}} + h\sin\phi \end{bmatrix}$$

Conversion from geodetic coordinates to cartesian coordinates

WGS – World Geodetic System



$$h = H + N$$



Historically, height is measured relative to geoid, a surface of constant geopotential; global mean sea level in mean square sense.
Ellipsoid height h is height of point above the surface of the ellipsoid. Point E can be above or below G

Time

How time is defined?

Time

How time is defined?

Atomic clock: The second is the duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133atom.

For info http://www.bipm.org/en/si/

Solar year: the length of time that the Sun takes to return to the same position in the cycle of seasons, as seen from Earth; for example, the time from vernal equinox to vernal equinox, or from summer solstice to summer solstice

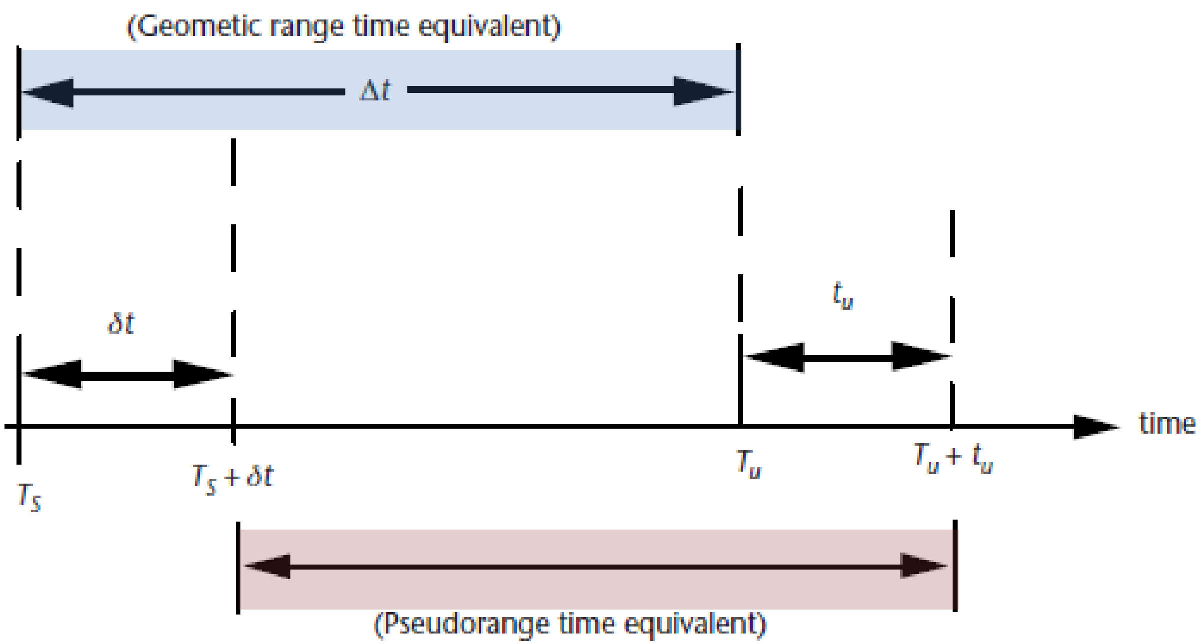
Source: http://en.wikipedia.org/wiki/Tropical_year

Satellite Position

Satellite motion is governed by different forces acting on it i.e. earth, moon, sun gravitational force, solar radiation pressure. Most dominant is earth gravitational force. To model satellite motion accurately all the forces must be modeled. Satellite position with respect to earth, moon and sun must be known and for solar pressure attitude with respect to sun must be known.

Satellite motion can be expressed using Kaplerian orbital elements. These are transmitted by the satellite and have good life (few hours).

Range Measurement Time



$T_s = System\ time\ at\ which\ the\ signal\ left\ the\ satellite$

$T_u = System\ time\ at\ which\ the\ signal\ reached\ the\ user\ receiver$

$\delta t = Offset\ of\ the\ satellite\ clock\ from\ systsem\ time\left[advance\ is\ positive; retardation\left(delay\right) is\ negative\right]$

$t_u = Offset\ of\ the\ receiver\ clock\ from\ system\ time$

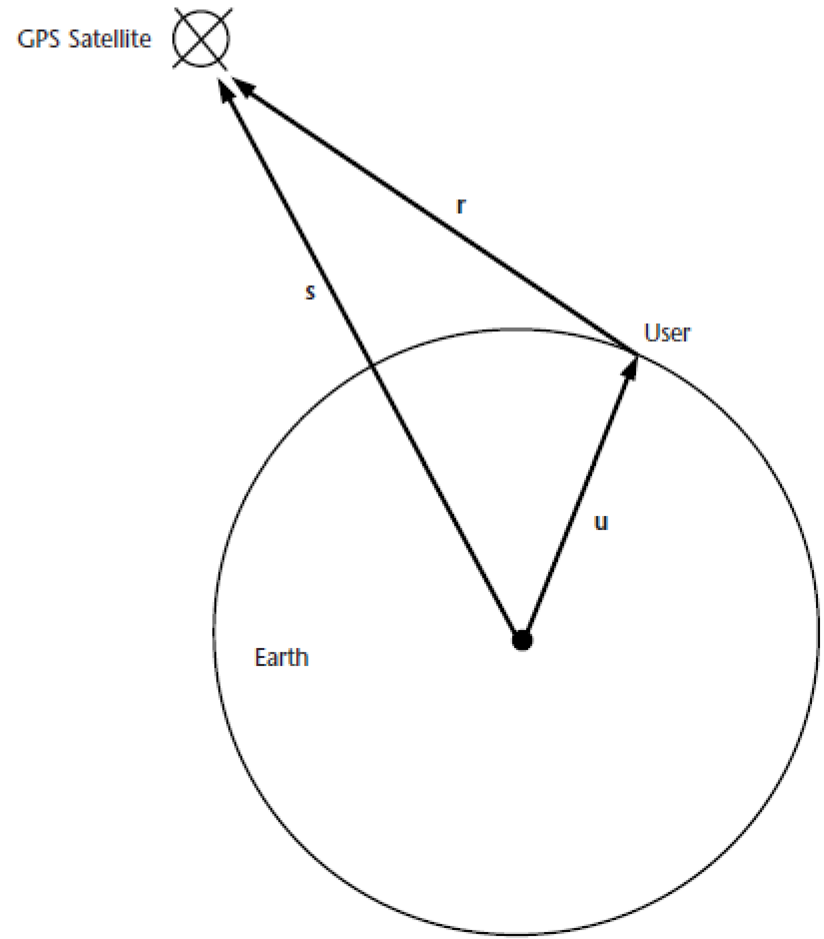
$T_s + \delta t = Satellite\ clock\ reading\ at\ the\ time\ that\ the\ signal\ left\ the\ satellite$

$T_u + t_u = User\ receiver\ clock\ reading\ at\ the\ time\ the\ signal\ reached\ the\ user\ receiver$

$c = speed\ of\ light$

Pseudorange

Pseudorange is range determined by signal propagation velocity and time difference between two non-synchronised clocks (receiver clock and satellite clock)



$$Geometric\ range, r = c(T_u - T_s) = c\Delta t$$

$$Pseudorange, \rho = c[(T_u + t_u) - (T_s + \delta t)] = c(T_u - T_s) + c(t_u - \delta t) = r + c(t_u - \delta t)$$

$$\rho - c(t_u - \delta t) = \|\mathbf{s} - \mathbf{u}\|$$

$$\rho - ct_u = \|\mathbf{s} - \mathbf{u}\|$$

Calculation of Receiver Position

$$\rho_j = \left\| \mathbf{s}_j - \mathbf{u} \right\| + ct_u$$

$$\rho_1 = \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} + ct_u$$

$$\rho_2 = \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} + ct_u$$

$$\rho_3 = \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} + ct_u$$

$$\rho_4 = \sqrt{(x_4 - x_u)^2 + (y_4 - y_u)^2 + (z_4 - z_u)^2} + ct_u$$

$$\rho_j = \sqrt{(x_j - x_u)^2 + (y_j - y_u)^2 + (z_j - z_u)^2} + ct_u$$

$$= f(x_u, y_u, z_u, t_u)$$

One receiver and four satellite. Receiver clock bias will be same for all the measurements.

Data format

NMEA

Propriety sentences - Sirf

GPS applications

NMEA – National Marine Electronics Association

Propriety sentences – Manufacturer specific: SiRF

GPS data

NMEA – National Marine Electronics Association

1. GGA (GPS Fix Data, fixed data for the Global Positioning System)

2. GLL (Geographic Position – Latitude/Longitude)

3. GSA (GNSS DOP and Active Satellites, degradation of accuracy and the number of active satellites in the Global Satellite Navigation System)

4. GSV (GNSS Satellites in View, satellites in view in the Global Satellite Navigation System)

5. RMC (Recommended Minimum Specific GNSS Data)

6. VTG (Course over Ground and Ground Speed, horizontal course and horizontal velocity)

7. ZDA(Time&Date)

GPS data

$GPGGA,130305.0,4717.115,N,00833.912,E,1,08,0.94,00499,M,047,M,,*58<CR><LF>

Field	Description
$	Start of the data set
GP	Information originating from a GPS appliance
GGA	Data set identifier
130305.0	UTC positional time: 13h 03min 05.0sec
4717.115	Latitude: 47° 17.115 min
N	Northerly latitude (N=north, S= south)
00833.912	Latitude: 8° 33.912min
E	Easterly longitude (E= east, W=west)
1	GPS quality details (0= no GPS, 1= GPS, 2=DGPS)
08	Number of satellites used in the calculation
0.94	Horizontal Dilution of Precision (HDOP)
00499	Antenna height data (geoid height)
M	Unit of height (M= meter)
047	Height differential between an ellipsoid and geoid
M	Unit of differential height (M= meter)
,,	Age of the DGPS data (in this case no DGPS is used)
0000	Identification of the DGPS reference station
*	Separator for the checksum
58	Checksum for verifying the entire data set
<CR><LF>	End of the data set

GPS data

Propriety sentences – Manufacturer specific: SiRF

SiRF- Data set No.	Name	Description
2	Measured Navigation Data	Position, speed and time
4	Measured Tracking Data	Signal-to-noise ratio, elevation and azimuth
5	Raw Track Data	Raw distance measurement data
6	SW Version	Receiver software
7	Clock Status	Time measurement status
8	50 BPS Subframe Data	Receiver information (ICD format)
9	Throughput	CPU throughput
11	Command Acknowledgment	Reception confirmation
12	Command NAcknowledgment	Failed inquiry
13	Visible List	Number of visible satellites
14	Almanac Data	Almanac data
15	Ephemeris Data	Ephemeris data
18	OkToSend	CPU On/Off status (trickle power)
19	Navigation Parameters	Reply to the POLL command
255	Development Data	Various internal items of information

GPS data

Propriety sentences – Manufacturer specific: SiRF

Received binary data (Hex. code) with a repetition rate of 1Hz

A0A2002902FFD6F78CFFBE536E003AC00400030104A00036B039780E30612190E160F04000000000000
09BBB0B3

- Start sequence:

  A0A2

- Length of the information in bytes

  0029

- Information:

  02FFD6F78CFFBE536E003AC00400030104A00036B039780E30612190E160F04000000000000

- Checksum:

  09BB

- End sequence

  B0B3

The 41 bytes of information are divided up as follows:

Name	Bytes	Scaling	Value (Hex)	Unit	Scaling	Value (Decimal)
Message ID	1		02			2
X-position	4		FFD6F78C	m		-2689140
Y-position	4		FFBE536E	M		-4304018
Z-position	4		003AC004	m		3850244
X-velocity	2	*8	0000	m/s	Vx/8	0
Y-velocity	2	*8	0003	m/s	Vy/8	0.375
Z-velocity	2	*8	0001	m/s	Vz/8	0.125
Mode 1	1		04		Bitmap	4
DOP	1	*5	A		/5	2.0
Mode 2	1		00	Bitmap		0
GPS Week	2		036B			875
GPS TOW	4	*100	039780E3	S	/100	602605.79
SVs in Fix	1		06			6
CH 1	1		12			18
CH 2	1		19			25
CH 3	1		0E			14
CH 4	1		16			22
CH 5	1		0F			15
CH 6	1		04			4
CH 7	1		00			0
CH 8	1		00			0
CH 9	1		00			0
CH 11	1		00			0
CH 11	1		00			0
CH 12	1		00			0

GPS applications

- Seismology (geophysics), Glaciology (geophysics), Geology (mapping)
- Surveying deposits (mineralogy,geology)
- Physics (time standardisation measurement)
- Scientific expeditions
- General construction industry
- Geo-information technology
- Forestry and agricultural sciences
- Geodesy
- Aerospace sciences
- Precision delivery
- Communication
- Power generation
- Marine navigation
- Hiking