AE 242 Aerospace Measurements Laboratory

Attitude Determination

Angular displacement between two frames.

AHRS – Attitude heading reference system





Attitude Determination

By using kinematic equations: involves integration of differential equations. Prone to errors during long time estimations.

Using algebraic methods: Uses measurement of a vector fixed in inertial space in body frame and obtains body attitude with respect to the vector. To find body's attitude with respect to inertial frame, components of the vector fixed in inertial frame must be known. Such combination will give only two attitude angles, it is an under determined problem. Two vector measurements can be used for finding the attitude. When two vectors are used it is an over determined problem.

Can we use accelerometers to determine attitude?

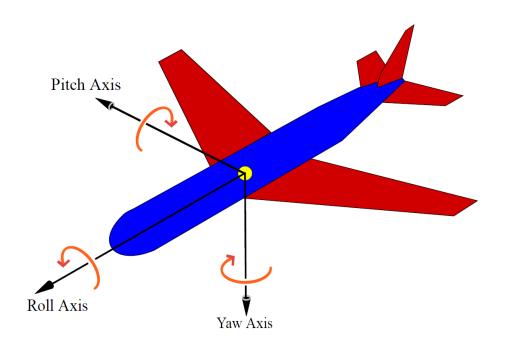
What are its limitations?

Roll - Pitch - Yaw

Roll – Rotation about x-axis

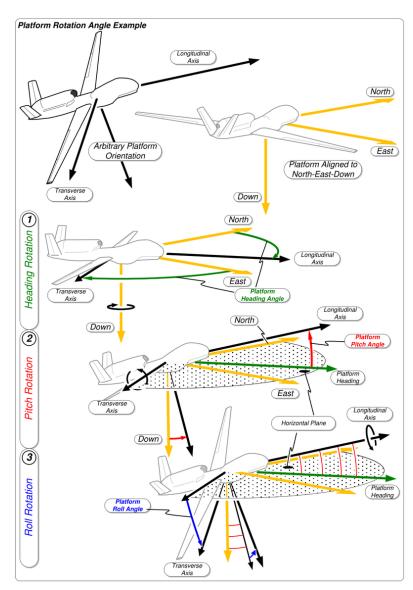
Pitch – Rotation about y-axis

Yaw – Rotation about z axis

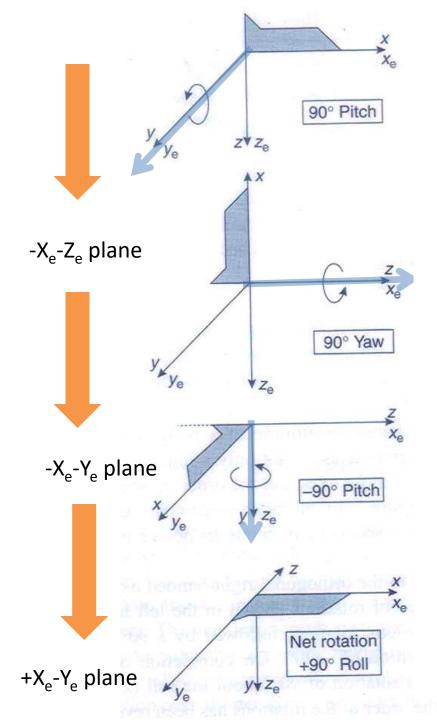


By Yaw_Axis.svg: Auawisederivative work: Jrvz (talk) - Yaw_Axis.svg, CC BY-SA 3.0,

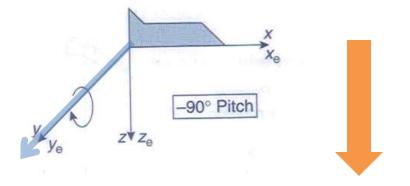
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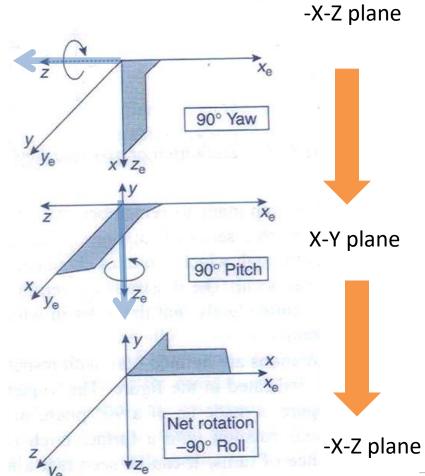
https://en.wikipedia.org/wiki/Aircraft_principal_axes

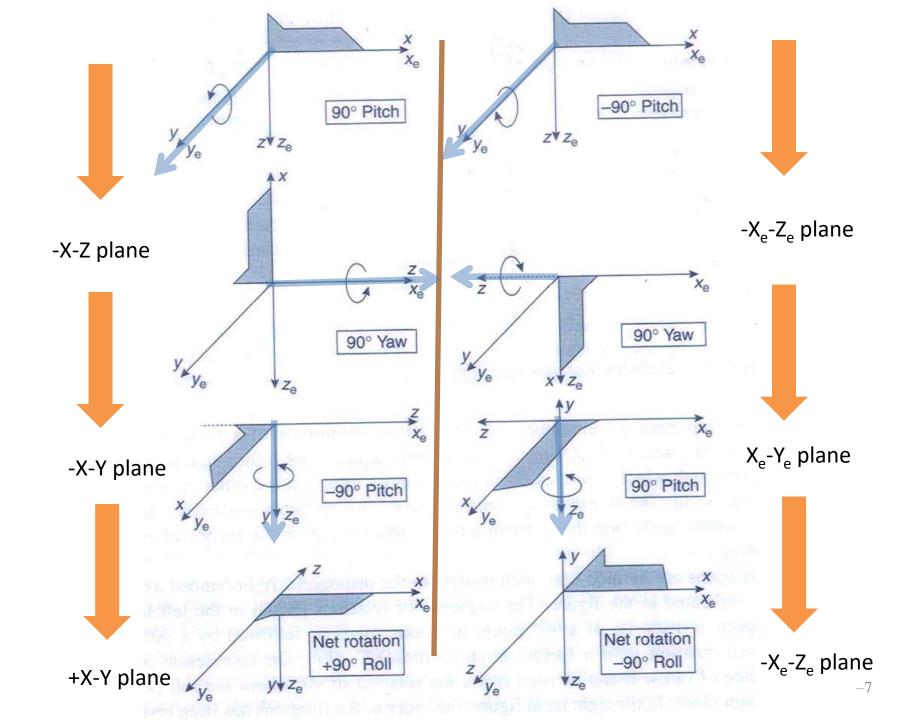


Pitch = 90 degree Yaw = 90 degree Pitch = -90 degree



Pitch = -90 degree Yaw = 90 degree Pitch = 90 degree

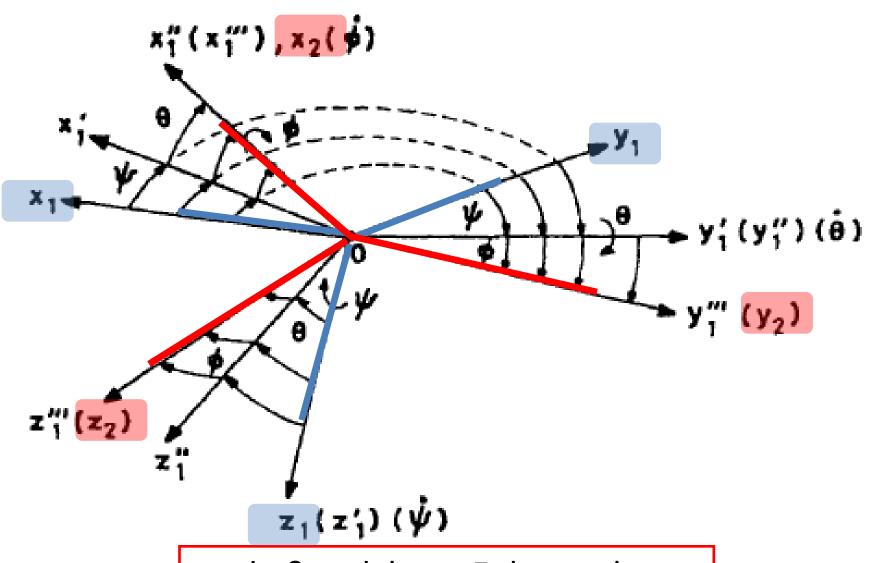




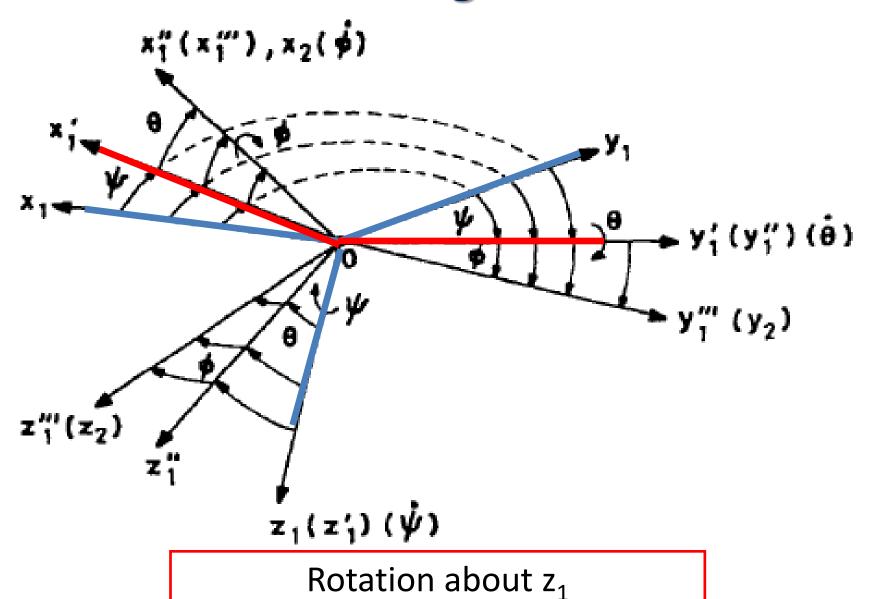
Euler angles are also used to describe the orientation of a frame of reference relative to another. They are typically denoted as ϕ , θ , ψ .

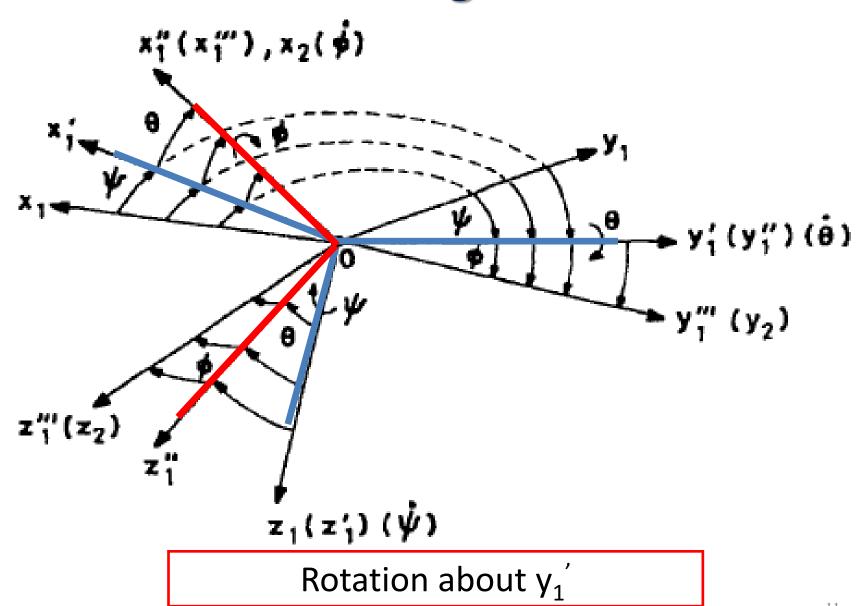
-Three parameterization.

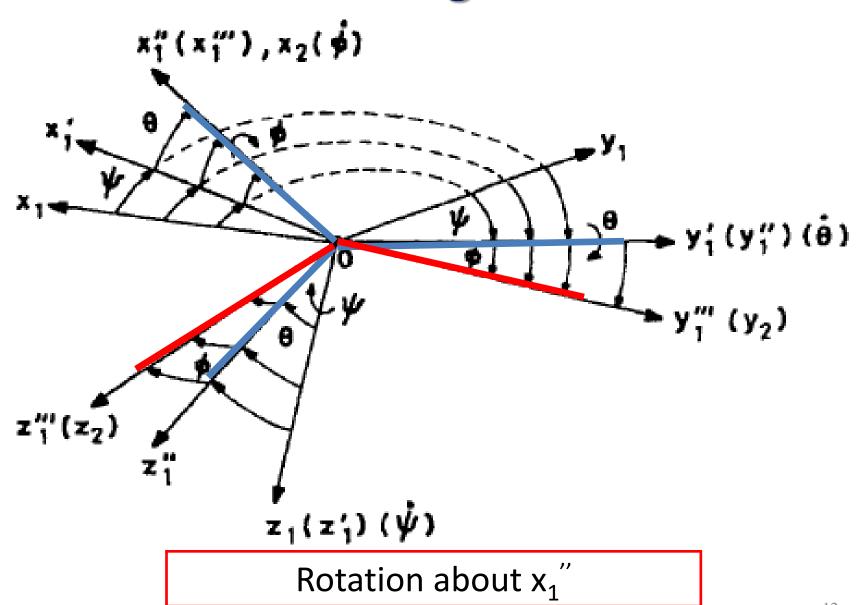
Euler angles represent a sequence of three elemental rotations, i.e. rotations about the axes of a coordinate system



 Φ , θ and ψ are Euler angles







Euler angle rates are not directly measured. Angular velocity components p, q, r, which are the body axes components of angular velocity of vehicle with respect to inertial frame are available. These are available from onboard gyros.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 1 & \tan \theta \sin \phi & \cos \phi \tan \theta \end{bmatrix} \begin{bmatrix} p \\ q \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & \cos \phi & -\sin \phi & q \\ 0 & \sec \theta \sin \phi & \sec \theta \cos \phi \end{bmatrix} \begin{bmatrix} r \end{bmatrix}$$

Euler angle rates expressed as function of body angular rate

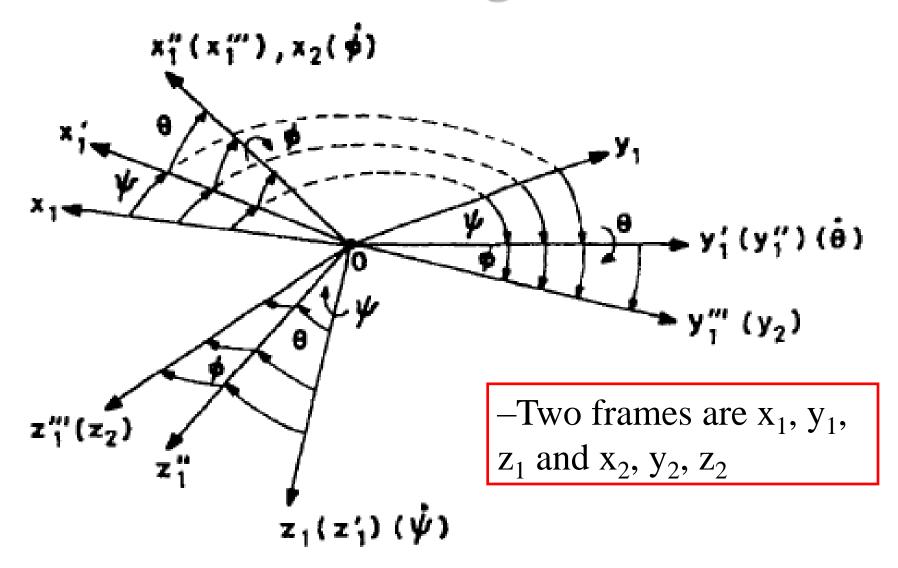


Fig. 4.3 Euler angles.

Axes Transformation

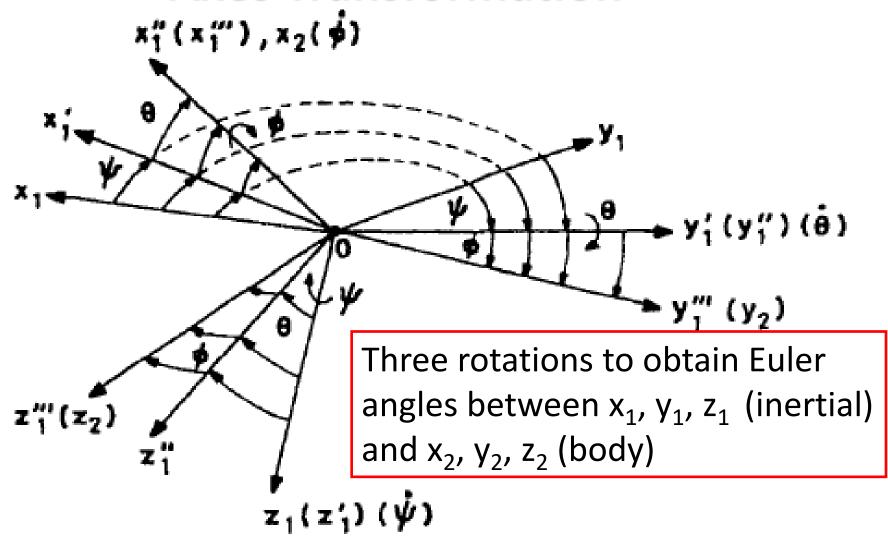
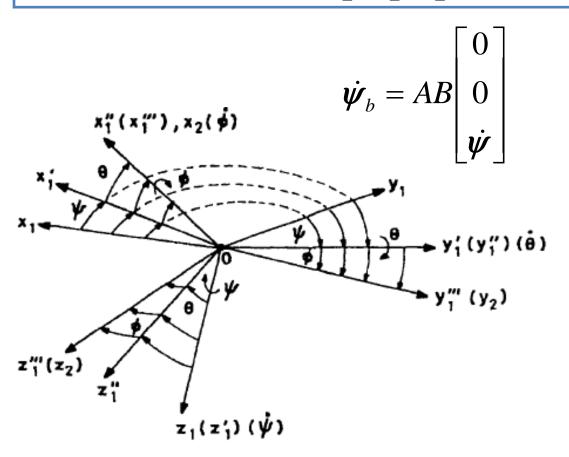


Fig. 4.3 Euler angles.

Consider $\dot{\psi}$ vector. It has to be transformed from $Ox_i y_i z_i$ system to $Ox_b y_b z_b$ (x₂, y₂, z₂)



$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix}$$

$$B = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

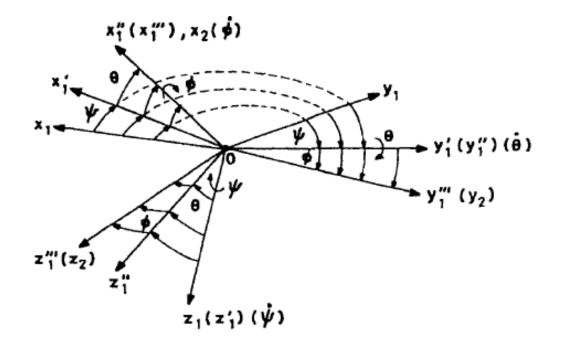


Fig. 4.3 Euler angles.

Consider $\dot{\theta}$ vector. It has to be transformed from $Ox_i^{\alpha}y_i^{\alpha}z_i^{\alpha}$ system to $Ox_b y_b z_b$ (x₂, y₂, z₂)

$$\dot{m{ heta}}_b = A egin{bmatrix} 0 \ \dot{m{ heta}} \ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix}$$

Consider ϕ vector. It has to be transformed from $Ox_b y_b z_b$ system to $Ox_b y_b z_b$ (x₂, y₂, z₂)

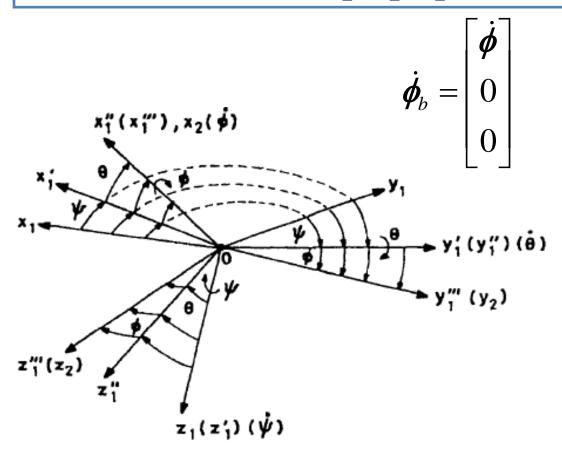


Fig. 4.3 Euler angles.

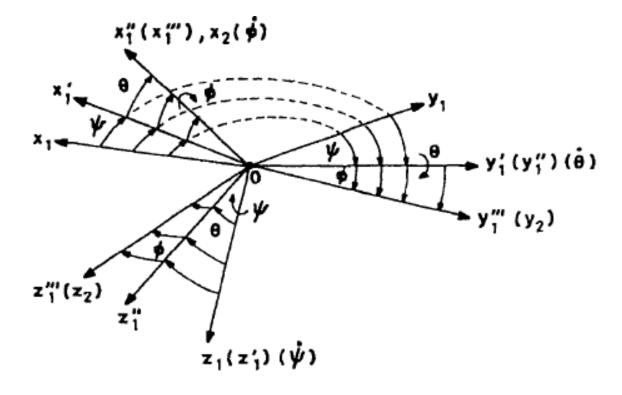


Fig. 4.3 Euler angles.

Angular velocity vector in the body axes system is given as

$$\boldsymbol{\omega}_{i,b}^b = \begin{bmatrix} p \\ q \end{bmatrix} = \dot{\boldsymbol{\psi}}_b + \dot{\boldsymbol{\theta}}_b + \dot{\boldsymbol{\phi}}_b$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = AB \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + A \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix}$$

$$B = \begin{bmatrix} \cos \boldsymbol{\theta} & 0 & -\sin \boldsymbol{\theta} \\ 0 & 1 & 0 \\ \sin \boldsymbol{\theta} & 0 & \cos \boldsymbol{\theta} \end{bmatrix}$$

$$\dot{\boldsymbol{\psi}}_b = AB \begin{bmatrix} 0 \\ 0 \\ \dot{\boldsymbol{\psi}} \end{bmatrix} \quad \dot{\boldsymbol{\theta}}_b = A \begin{bmatrix} 0 \\ \dot{\boldsymbol{\theta}} \\ 0 \end{bmatrix} \qquad \dot{\boldsymbol{\phi}}_b = \begin{bmatrix} \dot{\boldsymbol{\phi}} \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{m{ heta}}_b = A egin{bmatrix} 0 \ \dot{m{ heta}} \ 0 \end{bmatrix}$$

$$\dot{\phi}_b = \begin{vmatrix} \phi \\ 0 \\ 0 \end{vmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = AB \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + A \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix}$$

$$B = \begin{bmatrix} \cos \boldsymbol{\theta} & 0 & -\sin \boldsymbol{\theta} \\ 0 & 1 & 0 \\ \sin \boldsymbol{\theta} & 0 & \cos \boldsymbol{\theta} \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = AB \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + A \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} B = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix}$$

$$B = \begin{bmatrix} \cos \boldsymbol{\theta} & 0 & -\sin \boldsymbol{\theta} \\ 0 & 1 & 0 \\ \sin \boldsymbol{\theta} & 0 & \cos \boldsymbol{\theta} \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ \sin \phi \sin \theta & \cos \phi & \sin \phi \cos \theta \\ \cos \phi \sin \theta & -\sin \phi & \cos \phi \cos \theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \psi \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = AB \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + A \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix}$$

$$B = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ \sin \phi \sin \theta & \cos \phi & \sin \phi \cos \theta \\ \cos \phi \sin \theta & -\sin \phi & \cos \phi \cos \theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \psi \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = L_{w} \begin{bmatrix} \dot{\boldsymbol{\phi}} \\ \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{\psi}} \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = L_w \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \qquad L_w = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix}$$

$$p = \dot{\phi} - \dot{\psi} \sin \theta$$

$$q = \dot{\theta} \cos \phi + \dot{\psi} \sin \phi \cos \theta$$

$$r = \dot{\psi} \cos \phi \cos \theta - \dot{\theta} \sin \phi$$

Euler angle rates can be obtained by inverting above matrix (It is not an orthogonal matrix)

$$\begin{bmatrix} \dot{\boldsymbol{\phi}} \\ \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{\psi}} \end{bmatrix} = L_w^{-1} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad L_w^{-1} = \left(\frac{1}{\Delta(L)}\right) adj(L_w)$$

$$L_{w}^{-1} = \left(\frac{1}{\Delta(L)}\right) adj(L_{w})$$

$$\Delta(L) = \cos^2 \phi \cos \theta + \sin^2 \phi \cos \theta = \cos \theta$$

$$adj(L_w) = \begin{bmatrix} \cos \theta & \sin \theta \sin \phi & \cos \phi \sin \theta \\ 0 & \cos \phi \cos \theta & -\sin \phi \cos \theta \\ 0 & \sin \phi & \cos \phi \end{bmatrix}$$

$$L_{w}^{-1} = \left(\frac{1}{\Delta(L)}\right) adj(L_{w})$$

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$$L_{w}^{-1} = \left(\frac{1}{\Delta(L)}\right) adj(L_{w}) \qquad L_{w}^{-1} = \begin{bmatrix} 1 & \tan\theta\sin\phi & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sec\theta\sin\phi & \sec\theta\cos\phi \end{bmatrix}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan \theta \sin \phi & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sec \theta \sin \phi & \sec \theta \cos \phi \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad \begin{aligned} \dot{\phi} &= p + \tan \theta (q \sin \phi + r \cos \phi) \\ \dot{\theta} &= q \cos \phi - r \sin \phi \\ \dot{\psi} &= \sec \theta (q \sin \phi + r \cos \phi) \end{aligned}$$

Euler angle rates expressed as function of body angular rate

$$\dot{\phi} = p + \tan \theta (q \sin \phi + r \cos \phi)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\psi} = \sec \theta (q \sin \phi + r \cos \phi)$$

Singularity when pitch angle is 90 degree. It is for 3-2-1 rotation

When body angular rate and Euler angle rates will be equal?

$$\dot{\phi} = p + \tan \theta (q \sin \phi + r \cos \phi)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\psi} = \sec \theta (q \sin \phi + r \cos \phi)$$

When Euler angles are zero?

$$\dot{\phi} = p$$

$$\dot{\theta} = q$$

$$\dot{\psi} = r$$

$$\dot{\phi} = p$$

$$\dot{\theta} = q$$

$$\dot{\psi} = r$$

$$\dot{\phi} = p + \tan \theta (q \sin \phi + r \cos \phi)$$

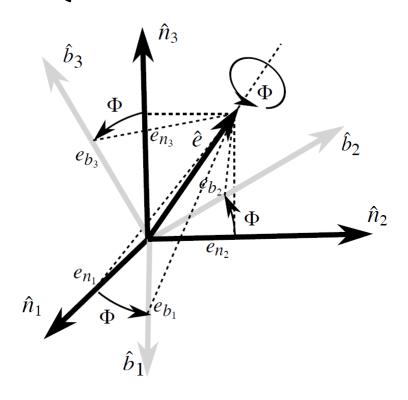
$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\psi} = \sec \theta (q \sin \phi + r \cos \phi)$$

Singularity when pitch angle is 90 degree. It is for 3-2-1 rotation

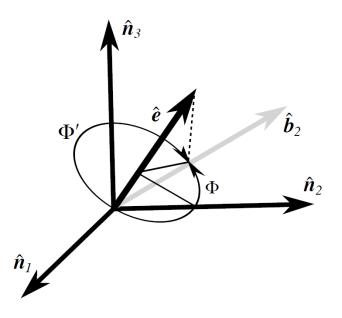
How to overcome singularity? In the framework of Euler Angles.

Quarternions



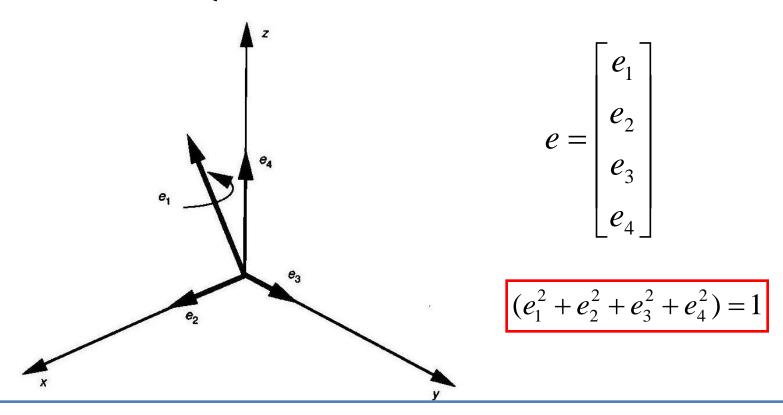
A rigid body or coordinate reference frame can be brought from an arbitrary initial orientation to an arbitrary final orientation by a single rigid rotation through a principal angle "about the principal axis e^; the principal axis being a judicious axis fixed in both the initial and final orientation.31

Quarternions



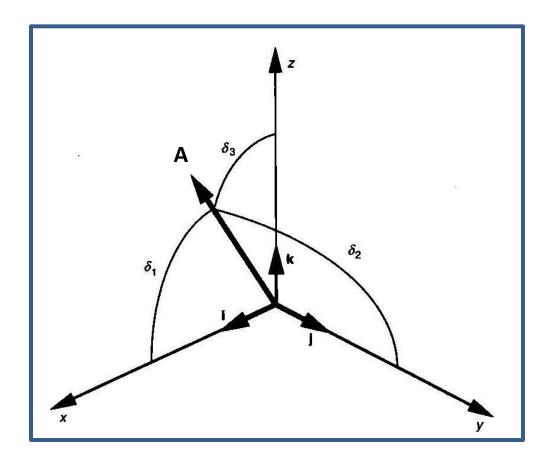
Rotation of the judiciously selected axis is not unique. Rotation can be achieved in clockwise or anticlockwise i.e. one rotation can be more than the other. It becomes tricky in some situations.

Quarternions



A solid body rotation from one attitude to another, by a single rotation about some axis in reference frame. Four parameters are, three direction cosines of a unit vector aligned with rotational axis and fourth parameter is rotation angle

Direction Cosine Matrix



A vector A makes angle δ_1 , δ_2 and δ_3 with the axis x, y and z

Axes Transformation

Consider a vector

$$\vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$$

$$\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \boldsymbol{\delta}_3$$

 $\delta_1, \delta_2, \delta_3$ angles with the x, y and x axis

$$\cos \boldsymbol{\delta}_1 = \frac{A_x}{|A|}$$

$$\cos \boldsymbol{\delta}_2 = \frac{A_y}{|A|}$$

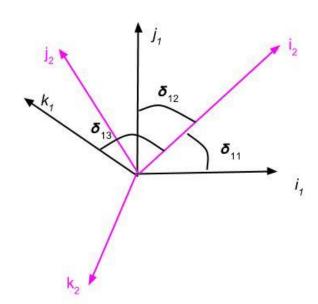
$$\cos \boldsymbol{\delta}_3 = \frac{A_z}{|A|}$$

$$\cos^2 \boldsymbol{\delta}_1 + \cos^2 \boldsymbol{\delta}_2 + \cos^2 \boldsymbol{\delta}_3 = 1$$

 $\cos \delta_1, \cos \delta_2, \cos \delta_3$

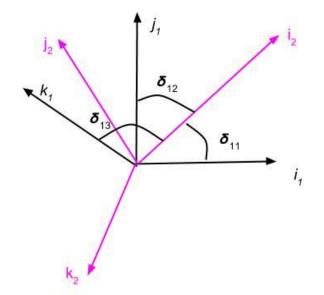
direction cosines of vector A respect to x, y and z axis 35

Consider the transformation of a vector from one reference frame to another reference frame. Transformation from x_1 , y_1 , z_1 to x_2 , y_2 , z_2



Consider the transformation of a vector from one reference frame to another reference frame. Transformation from x_1 , y_1 , z_1 to x_2 , y_2 , z_2

$$\hat{i}_2 = C_{11}\hat{i}_1 + C_{12}\hat{j}_1 + C_{13}\hat{k}_1$$



$$C_{11} = \cos(\delta_{11})$$

$$C_{12} = \cos(\delta_{12})$$

$$C_{13} = \cos(\delta_{13})$$

Consider the transformation of a vector from one reference frame to another reference frame. Transformation from x_1 , y_1 , z_1 to x_2 , y_2 , z_2

$$\hat{i}_{2} = C_{11}\hat{i}_{1} + C_{12}\hat{j}_{1} + C_{13}\hat{k}_{1}$$

$$\hat{j}_{2} = C_{21}\hat{i}_{1} + C_{22}\hat{j}_{1} + C_{23}\hat{k}_{1}$$

$$\hat{k}_{2} = C_{31}\hat{i}_{1} + C_{32}\hat{j}_{1} + C_{33}\hat{k}_{1}$$

 C_{11}, C_{12}, C_{13} are the direction cosines of the i_2 unit vector with respect to the x_1 , y_1 , z_1 axis system

$$\begin{split} \hat{i}_2 &= C_{11}\hat{i}_1 + C_{12}\hat{j}_1 + C_{13}\hat{k}_1 \\ \hat{j}_2 &= C_{21}\hat{i}_1 + C_{22}\hat{j}_1 + C_{23}\hat{k}_1 \\ \hat{k}_2 &= C_{31}\hat{i}_1 + C_{32}\hat{j}_1 + C_{33}\hat{k}_1 \end{split}$$

In matrix form

$$\begin{bmatrix} \hat{i}_2 \\ \hat{j}_2 \\ \hat{k}_2 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \hat{i}_1 \\ \hat{j}_1 \\ \hat{k}_1 \end{bmatrix}$$

$$C_{1}^{2} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

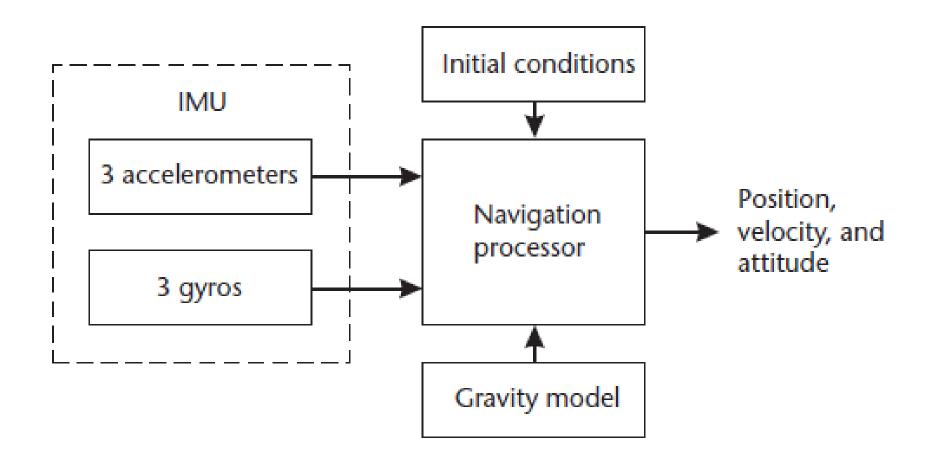
$$C_2^1 = \left[C_1^2\right] = \left[C_1^2\right]^T$$

$$C_2^1 C_1^2 = I$$

Nine equations can be obtained by matrix multiplication. DCM is an orthogonal matrix

$$\begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

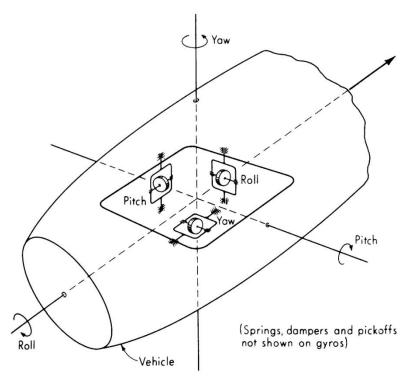
Out of nine equations three equations are repeated. We will have six equations relating nine parameters. There are six constraint equations and nine parameters. Only three of them are independent.



Schematic of Inertial Navigation System

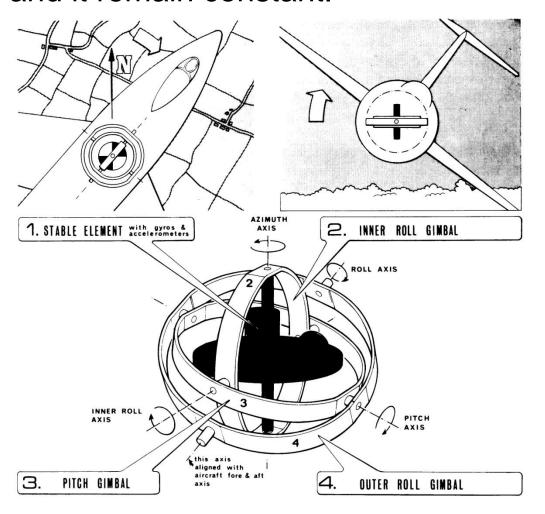
Gyroscope

Gyroscope measures body angular rates in their respective axis. Three single axis gyroscopes can be used to find three angular rates or rotations. Angular rates can be used to obtain the attitude of the aircraft.



Attitude/heading reference systems

Stable platform: Rate gyros and accelerometers are suspended on a set of gimbals. Angular rotation can be obtained by the position of gimbal. The gimbals maintain the orientation and it remain constant.



Attitude/heading reference systems

Strap down system: Rate gyros, accelerometers are strapped to the body of aircraft or all the sensor are fixed with respect to the body of aircraft. All the quantities are measured with respect to the body axis. Computation required are extensive.

