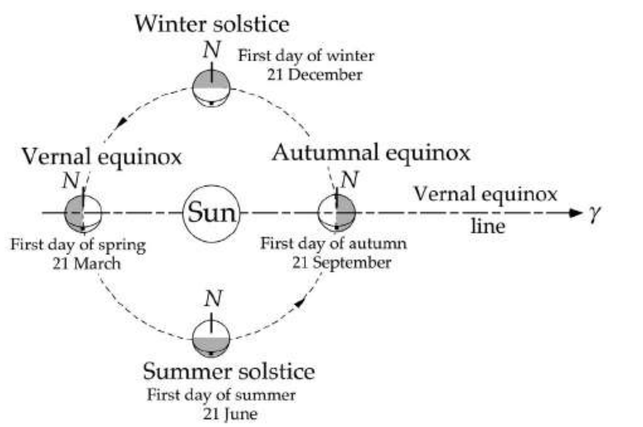
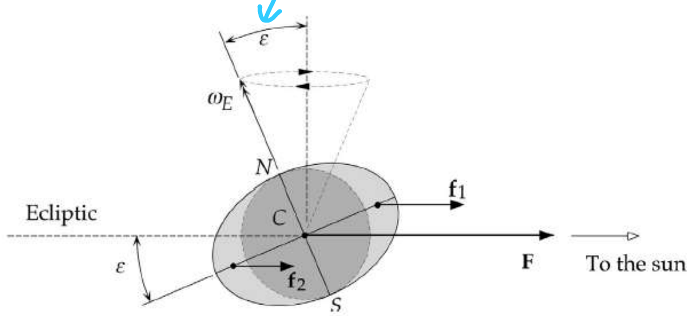
- How to describe orbits in three-dimensional space (which, of course, is the setting for real missions and orbital manoeuvres)?

Geocentric Right Ascension - Declination Frame

- Cooridinate system used to describe earth orbits in three dimensions: earth's equitorial plane, the ecliptic and the earth's axis of rotation.



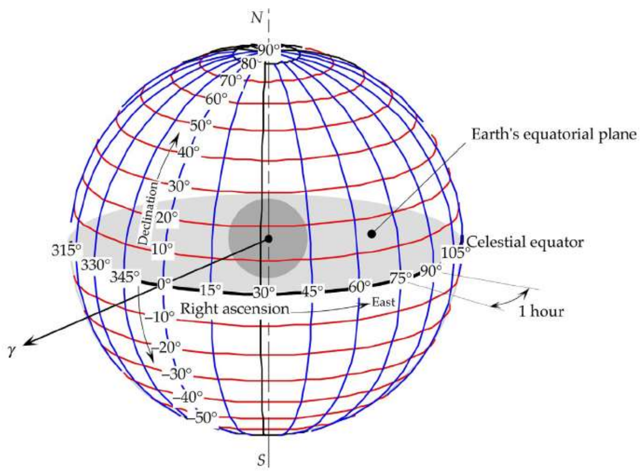
obliquity of the ecliptic



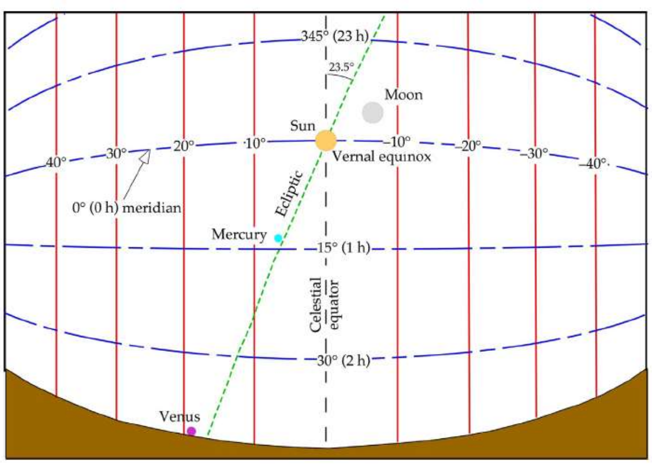
- Earth's equitorial plane and the ecliptic intersect along a line, which is known as the "vernal equinox" line.

- For many practical purposes, the vernal equinox line may be considered fixed in space. However, it actually rotates slowly because the earth's tilted spin axis precesses westward around the normal to the ecliptic at the rate of about 1.4° per century.

- To the human eye, objects in the night sky appear as points on a celestial sphere surrounding the earth.



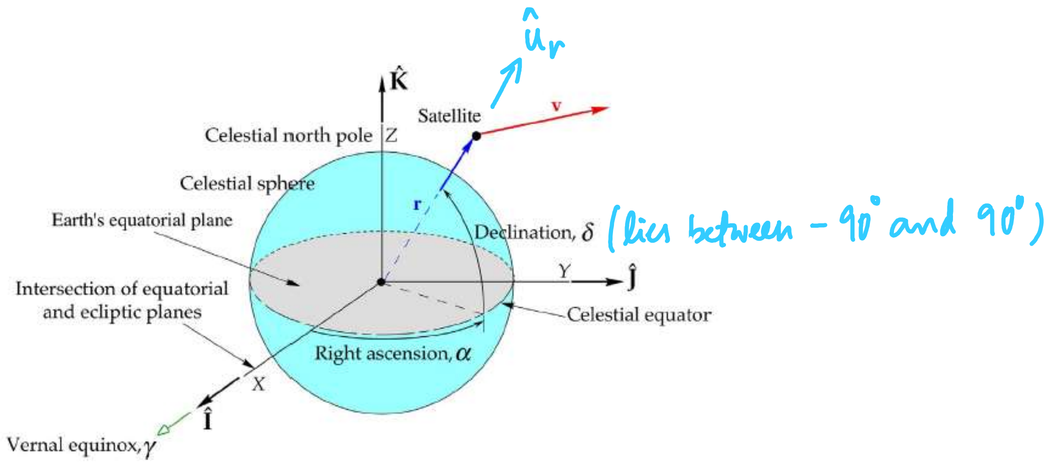
- Coordinates of latitude and longitude are used to locate points on the celestial sphere in much the same way as the surface of the earth.



State Vector and the Geocentric Equitorial Frame

- State vector of a satellite: $[r \ v]$

- $\ddot{r} = -\dfrac{\mu}{\|r\|^3} r$



- $\dot{r} = v$ and $\ddot{r} = a$ must be measured in a non-rotating frame attached to the earth.

- The non-rotating geocentric equitorial frame serves as an inertial frame for the two-body earth-satellite problem. However, it is not truly an inertial frame, since the centre of the earth is always accelerating towards a third body, the sun (to say nothing of the moon), a fact that is ignored in the two-body formulation).

- $r = X\hat{I} + Y\hat{J} + Z\hat{K}$
  $v = V_x\hat{I} + V_y\hat{J} + V_z\hat{K}$

- $r = \|r\|\,\hat{u}_r$

- $\hat{u}_r = \underbrace{l\hat{I} + m\hat{J} + n\hat{K}}_{\text{direction cosines}}$
  $= \cos\delta\cos\alpha\,\hat{I} + \cos\delta\sin\alpha\,\hat{J} + \sin\delta\,\hat{K}$

- $\delta = \sin^{-1}(n)$

- $\cos\delta$ cannot be negative.

- $\alpha = \cos^{-1}\left(\dfrac{l}{\cos\delta}\right)$ (lies between $0°$ and $360°$)

$\cos\delta$)

- To determine the correct quadrant for α, check the sign:

$$m = \cos\delta \sin\alpha$$

- If $\sin\alpha > 0$, then α lies in the range $0°$ to $180°$. On the other hand, if $\sin\alpha < 0$, then α lies in the range $180°$ to $360°$.

1. Calculate the magnitude of **r**:	$r = \sqrt{X^2 + Y^2 + Z^2}$.
2. Calculate the direction cosines of **r**:	$l = X/r \quad m = Y/r \quad n = Z/r$
3. Calculate the declination:	$\delta = \sin^{-1} n$
4. Calculate the right ascension:	$\alpha = \begin{cases} \cos^{-1}(l/\cos\delta) & (m > 0) \\ 360° - \cos^{-1}(l/\cos\delta) & (m \le 0) \end{cases}$

Example

If the position vector of the International Space Station in the geocentric equatorial frame is $\mathbf{r} = -5368\hat{\mathbf{I}} - 1784\hat{\mathbf{J}} + 3691\hat{\mathbf{K}}$ (km), what are its right ascension and declination?

Details

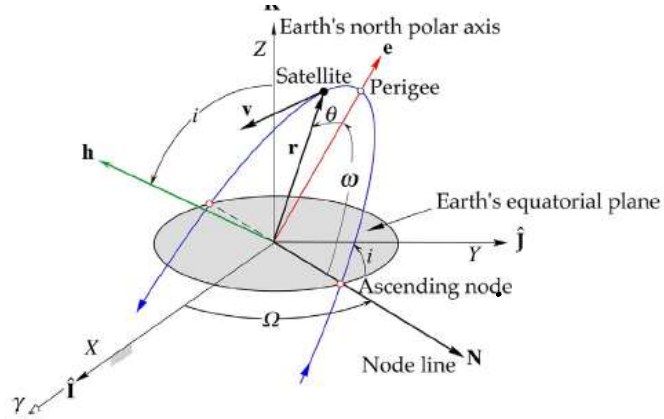
Follow the above algorithm.

Orbital Elements and the State Vector

- To define an orbit in a plane requires two parameters: eccentricity and angular momentum. To locate a point on the orbit requires a third parameter, the true anomaly.

- Describing an orbit in three dimensions requires three additional parameters, called the Euler angles.

$\hat{\mathbf{K}}$



- Six orbital elements :
  - $\|h\|$ - specific angular momentum
  - i - inclination (lies between 0° and 180°)
  - Ω - right ascension of the ascending node (lies between 0° and 360°)
  - $\|e\|$ - eccentricity
  - ω - argument of perigee (lies between 0° and 360°)
  - θ - true anomaly