Mid-semester Examination *AE 240/713* *Monday February 26, 2018*
Weightage: 20% *Time: 1100 – 1300 Hrs.* *Venue: LH101*
Examination is closed Notes/Books. ***Only Calculator is permitted.***

Q.1 (a) Derive the expressions for m_b, t_b & x_b, starting from $t_0 = 0$ & $x_0 = 0$, in respect of constant speed ($V = V_0$, zero drag) gravity turn trajectory, using the following equations of motion. (Assume gravity to be constant sea level value of g_0) (3)

$$\dot{V} = -\frac{\dot{m} g_0 I_{sp}}{m} - g_0 \cos\theta; \quad \dot{\theta} = \frac{g_0 \sin\theta}{V}; \quad \frac{dx}{dt} = V \sin\theta$$

(b) A rocket having $m_0 = 30$ T is impulsively launched into a constant velocity gravity turn trajectory from earth's surface by burning instantaneously, 15 T of propellant having I_{sp} of 250s. If it is found that after burning another 10T of propellant of same I_{sp} during the above gravity turn manoeuvre, it attains $\theta_b = 60^o$, determine (1) the extent of initial 'pitch kick' (θ_0) required, (2) time taken to complete the gravity turn manoeuvre and (3) horizontal (x) distance travelled. ($g_0 = 9.81$ m/s^2). (3)

Q.2 (a) Formulate the augmented objective function for optimizing the Mission payload fraction under the ideal burnout velocity constraint and give the corresponding solution for π_i, using the Lagrange multiplier approach. (Hint: Use following relations as required). (1)

$$\pi_* = \frac{m_*}{m_0} = \prod_{i=1}^{n} \pi_i; \quad \pi_i = \frac{m_{0i+1}}{m_{0i}}; \quad \varepsilon_i = \frac{m_{si}}{m_{si} + m_{pi}}; \quad m_{0i+1} = m_{0i} - m_{si} - m_{pi}; \quad m_{fi} = m_{0i} - m_{pi}$$

$$V_* = \sum_{i=1}^{n} \Delta V_i = -\sum_{i=1}^{n} g_0 I_{spi} \ln \frac{m_{0i} - m_{pi}}{m_{0i}} = -g_0 \sum_{i=0}^{n} I_{spi} \ln\left(\varepsilon_i + \pi_i \times (1 - \varepsilon_i)\right)$$

(b) Further, also obtain the expression for π_* for an N-stage rocket, based on the results obtained in (a) above, if all the stages have same ε and I_{sp}. (2)

(c) Using the result of (b), determine π_i and π_* of a 2-stage rocket with same structural ratio, ε of 0.1 and same specific impulse, I_{sp} of 300s for both the stages, so that it achieves the ideal burnout velocity of 8000 m/s. (2)

Q.3 (a) Show that, starting from first principles, the propellant trade-off ratios for a N-stage rocket are given by the following expression. (2)

$$\frac{\delta m_*}{\delta m_{pi}}\Big|_{dV_*=0} = -\frac{\sum_{j=1}^{i-1} I_{spj}\left(\frac{1}{m_{0j}} - \frac{1}{m_{fj}}\right) + \frac{I_{spi}}{m_{0i}}}{\sum_{k=1}^{N} I_{spk}\left(\frac{1}{m_{0k}} - \frac{1}{m_{fk}}\right)}$$

You may use the following relations, as applicable.

$$m_{0i} = m_* + m_{pi} + m_{si} + \sum_{j=i+1}^{N}\left(m_{sj} + m_{pj}\right); \quad m_{fi} = m_* + m_{si} + \sum_{j=i+1}^{N}\left(m_{sj} + m_{pj}\right); \quad V_b = g_0 \sum_{i=1}^{N} I_{spi} \ln\left(\frac{m_{0i}}{m_{fi}}\right)$$

(b) Obtain $\frac{\partial m_*}{\partial m_{p1}}$ & $\frac{\partial m_*}{\partial m_{p2}}$ for a rocket with $m_{p1} = 21000kg$; $m_{s1} = 1300kg$; $m_{p2} = 3900kg$; $m_{s2} = 350kg$; $m_* = 650kg$; $I_{sp1} = 250s$; $I_{sp2} = 300s$. (2)

Q.4 (a) Derive the relations between ellipse parameters 'a', 'e' & initial momentum and energy parameters 'h', 'ε'. (Hint: Use following expressions). (1)

$$h = r_p v_p = r_a v_a = \sqrt{\mu a\left(1-e^2\right)}; \quad \varepsilon = \frac{1}{2}v^2 - \frac{\mu}{r}; \quad r_p = a(1-e); \quad r_a = a(1+e);$$

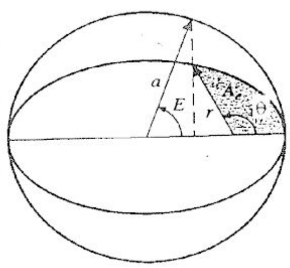
(b) A spacecraft is expected to form an orbit around earth, with perigee altitude of 200 km and apogee altitude of 20,000 km respectively, above earth's surface. Determine the velocity with which it should be injected parallel to local horizon. (2)

(c) In case the propellant over-performs with a 1% higher velocity, is it possible to still achieve the same orbit by changing the velocity vector direction with respect to local horizon? If yes, give the solution. If no, give reasons. (1)

(Hint: Use results of (a), along with following data. $R_E = 6{,}378$ km, $\mu = 3.986\times10^{14}$).

Q.5 Derive the expressions for mean angular velocity, 'n', and mean angle, 'M', for an elliptic orbit, based on Kepler's 3rd law and circle-ellipse equivalence. (Hint: Use following expression and figure as necessary). (1)

Orbital Time Period: $T = 2\pi\sqrt{a^3}/\sqrt{\mu}$



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