AE234 Endsem Soham S. Phanse 19D170030.

→ Given Data :

1. $M_{cruise} = 0.85$ 2. $h_{cruise} = 41000$ ft

3. Max Takeoff weight = 170600 kg

4. empty weight = 71700 kg.

5. Max Fuel carried at Takeoff = 75750 kg.

6. Fuel required for Takeoff = 0 kg

7. Fuel required for Landing = 0 kg

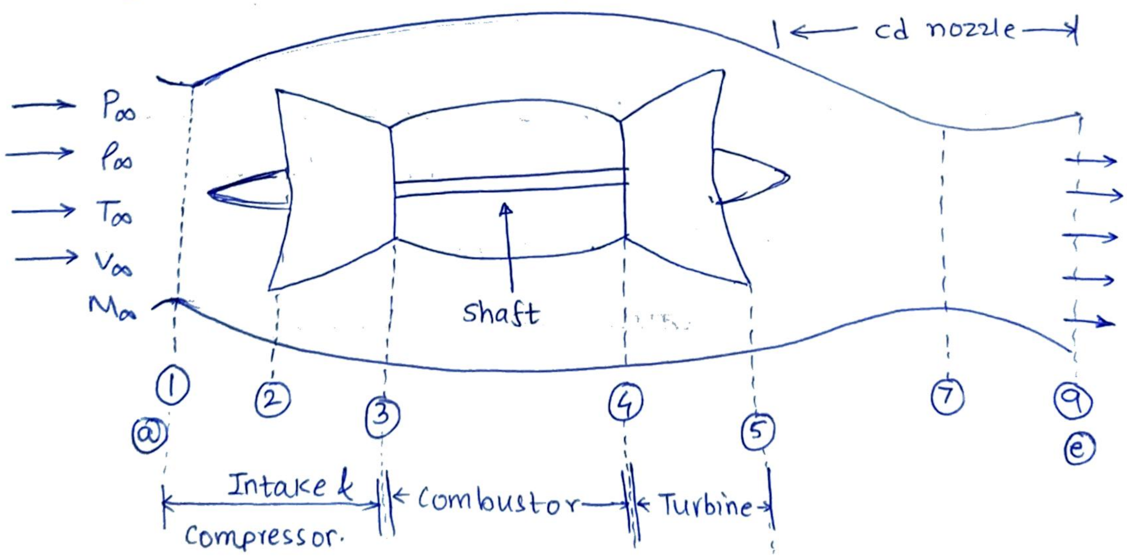
8. Weight of 1 F118 engine = 1450 kg.

9. Overall compression ratio = $\dfrac{P_{stg,\ after\ compressor}}{P_{ambient}} = 35.1$

10. Maximum Thrust produced = 85 kN.

11. $\eta_{compressor} = \eta_t = 0.90$ and $Q_R = 45$ MJ/kg

Q1). Engine schematic. (Turbojet)



① ⓐ Intake ② Compressor entry ③ Compressor exit & combustor entry

④ combustor exit ⑤ Turbine ~~entry~~ exit ⑦ @ throat of cd nozzle.

⑨ nozzle exit ($P_e, T_e, \rho_e, M_e, V_e$) etc.

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here Pamb ≠ Tamb, Pamb will be calculated from the International Standard Atmosphere tables.

@ 41,000 ft ≈ 40,000 ft [entry]

$T_{amb} = -56.5°C = 216.5\ K$

$P_{amb} = 18.8\ kPa$

$\sigma = \frac{\rho_{amb}}{\rho_{SL}} = 0.2462$

$\therefore \rho_{amb} = 0.2462 \times 1.225$
$= 0.301595\ kg/m^3$

Now as we don't have losses in the diffuser we have

$P_{stg,2} = P_{stg,amb} = 30.1523\ kPa = P_{t2}$ (1)

$T_{stg,2} = T_{stg,amb} = 247.796\ K = T_{t2}$ (2)

Now we use Isentropic gas tables. @ $M = 0.85$

$\frac{T}{T_0} = 0.8737 \Rightarrow T_{0,amb} = 247.796\ K$

$\frac{P}{P_0} = 0.6235 \Rightarrow P_{0,amb} = 30.1523\ kPa.$

Next we have non-isentropic compression with $\pi_c = 35.1$ and $\eta_{compressor} = 0.9$ But $\pi_c = \frac{P_{t3}}{P_{amb}} = 35.1 \Rightarrow \pi_{c,actual} = \frac{P_{t3}}{P_{t2}} = 21.88$

$\therefore \pi_c = \frac{V_{amb}}{V_3} = 35.1$ hence we use the relation

$$\frac{T_{t3}}{T_{t2}} = 1 + \frac{1}{\eta_c}\left[\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right] = 1 + \frac{1}{0.9}\left[\frac{\cancel{35.1}}{21.88}^{\frac{1.4-1}{1.4}} - 1\right]$$

$$= \cancel{2.9598}\ 2.5719 \Rightarrow \boxed{T_{t3} = \cancel{733.4381}\ K} \quad (3) \quad \boxed{T_{t3} = 637.315}$$

Now we do apply ideal gas equation.

$$\cancel{\frac{P_{t2} V_2}{T_{t2}} = \frac{P_{t3} V_3}{T_{t3}} \quad \text{But } \pi_c = 35.1 = \frac{V_2}{V_3}} = \frac{V_{amb}}{V_3}$$

$$\therefore \cancel{P_{t3} = \frac{P_{t2} \cdot T_{t3}}{T_{t2}} \cdot 35.1 = [30.1523\ kPa] \times \left[\frac{733.4381}{247.796}\right] \times 35.1}$$

$$\therefore \boxed{P_{t3} = \cancel{3132.5408\ kPa}} \quad (4) \quad \boxed{P_{t3} = P_{amb} \times 35.1 = 659.88\ kPa}$$

Next, we have the the combustor stage where we have,

$$\dot{q}_{burner} = \dot{m}_a C_p (T_{t4} - T_{t3}) = \dot{m}_a \times \frac{\gamma R}{\gamma - 1}(T_{t4} - T_{t3}) = \dot{m}_f Q_R$$

at maximum conditions we have $Thrust_{max} = 85\ kN.$ >>>

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Thrust $= \dot{m}_a((1+f)V_e - V_a) = 85\ kN$ $\quad \because f = \frac{\dot{m}_f}{\dot{m}_a}$

also we have $\dot{m}_f Q_R = \dot{m}_a C_p(T_{t4} - T_{t3})$

$\Rightarrow f = \frac{C_p(T_{t4} - T_{t3})}{Q_R}$ also as it is not specified we can take perfect expansion of nozzle $\therefore [\pi_t = \pi_c]$

$\therefore$ Thrust $= \dot{m}_a((1+f) M_e\sqrt{\gamma R T_e} - M_a\sqrt{\gamma R T_a})$

$\because$ We know, that $\boxed{T_{t4} = \text{maximum allowable temperature} = 1400K}$ (5)

$\therefore$ Now we have. $f = \frac{\gamma R}{\gamma - 1} \times \frac{(1400 - \cancel{733.4381}\ 637.315)}{45 \times 10^6}$

$\therefore \boxed{f = \cancel{0.01488}\ 0.01703 \approx 0.017}$ (6) $\quad f = 0.017$.

~~As~~ Now, after that, we have non isentropic expansion, & $\pi_t = \pi_c$

$\therefore \frac{T_{t5}}{T_{t4}} = 1 - \eta_t\left[1 - \pi_t^{\frac{1-\gamma}{\gamma}}\right] = \cancel{0.4256}\ 0.4727$ $\quad \therefore \boxed{T_{t5} = \cancel{595.88}\ 661.794\ K}$ (7)

We must remember that we have isobaric combustion in the combustor. Hence $\boxed{P_{t4} = P_{t3} = \cancel{3132.5408}\ 659.88\ kPa}$ (8)

~~Now, applying ideal gas law in stages ④ & ⑤~~

$\cancel{\frac{P_{t4}V_4}{T_{t4}} = \frac{P_{t5}V_5}{T_{t5}}}$ But $\pi_t = \cancel{35.1}\ 21.88 = \cancel{\frac{V_5}{V_4}} = \frac{P_{t4}}{P_{t5}}$

$\therefore \boxed{P_{t5} = \cancel{\frac{P_{t4}}{T_{t4}}} \times \cancel{\frac{1}{35.1}} \times \cancel{T_{t5}} = \cancel{37.985}\ 30.159\ kPa}$ (9)

Now as as we don't have any losses in the nozzle and we have perfect expansion, we have isentropic process from stage ⑤ Turbine exit to stage ⑨ nozzle exit.

$\therefore \boxed{T_{t9} = T_{t5} = \cancel{595.88}\ 661.794\ K}$ (10) and $\boxed{M_e = 0.85}$ (12)

$\boxed{P_{t9} = P_{t5} = \cancel{37.985}\ 30.159\ kPa}$ (11) $\therefore \boxed{T_e = \cancel{520.646}\ 578.238\ K}$ (13)

$\therefore \boxed{V_e = M_e\sqrt{\gamma R T_e} = \cancel{388.84}\ 409.782\ m/s}$ (14)

$V_a = M_a\sqrt{\gamma R T_a} = 250.74\ m/s$ (15)

PTO

∴ we had expression as

$$\text{Thrust}_{max} = \dot{m}_a[(1+f)V_e - V_a]$$

$$85\times10^3 = \dot{m}_a((1+\cancel{0.015}\,\overset{0.017}{})V_e - V_a)$$

$$85\times10^3 = \dot{m}_a((\cancel{1.015}\,\overset{1.017}{} \times \cancel{388.84}\,\overset{409.782}{}) - 250.74)$$

$$85\times10^3 = \dot{m}_a \times \cancel{143.9226}\,\overset{166.008}{} \Rightarrow \boxed{\dot{m}_a = \cancel{590.55}\,\underset{512.048}{} \text{ kg/s}} \quad (16)$$

also $\boxed{\dot{m}_f = f.\dot{m}_a = \cancel{0.015}\,\overset{0.017}{} \times \cancel{590.55}\,\overset{512.048}{} \text{ kg/s} = \cancel{8.858}\,\overset{8.7048}{} \text{ kg/s.}}$ (17)

$$\boxed{\dot{q}_{burner} = \dot{m}_f Q_R = \cancel{398.62}\,\underset{391.716}{} \times 10^6 \ \cancel{M}\frac{J}{s}} \quad (18)$$

Now, just for extra, we had $\text{Mass}_{fuel} = 75750$ kg

& $\dot{m}_f = 8.588$ kg/s ∴ General Endurance (for 1 engine) $= \dfrac{75750}{\cancel{8.588}\ 8.7048} = \cancel{8820.44}\ 8702.095$ sec

$= \cancel{147.007}\ 145.034$ minutes

Q2 We basically have the following processes:

① - ② i.e ambient to compressor entry : isentropic

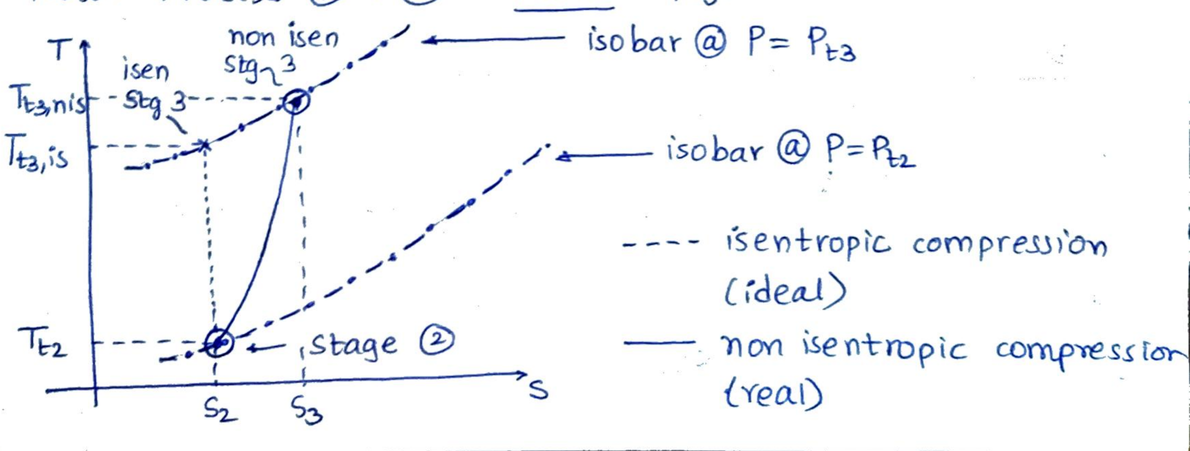
② - ③ i.e compressor entry to compressor exit
: non-isentropic compression ($\because \eta_c \neq 1$)

③ - ④ i.e combustor entry to combustor exit
: isobaric combustion (Brayton's cycle)

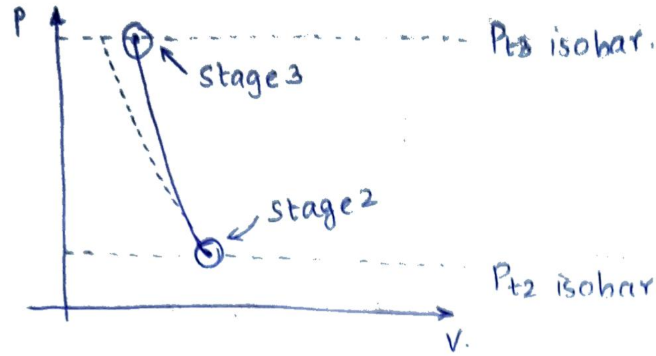
④ - ⑤ i.e combustor exit or turbine entry to turbine exit
: non isentropic expansion ($\because \eta_t \neq 1$)

⑤ - ⑨ isentropic flow (perfectly expanded nozzle).

Now. Process ② → ③. PLOTS. T-s.



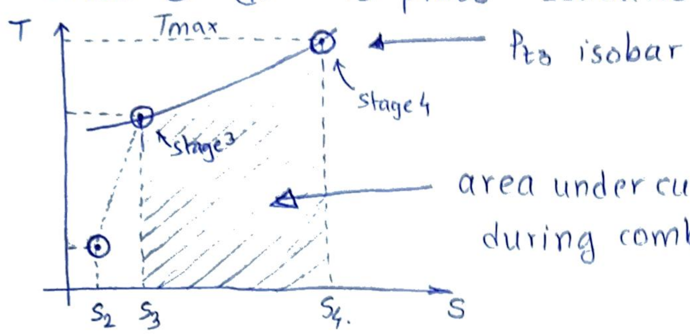
P-V plots - for 2 - 3



--- isentropic compression

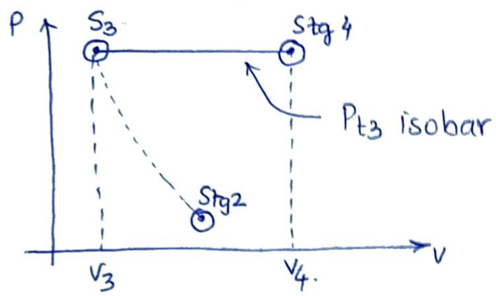
— non isentropic compression.

Process 3 - 4 Ts plots. Isobaric combustion



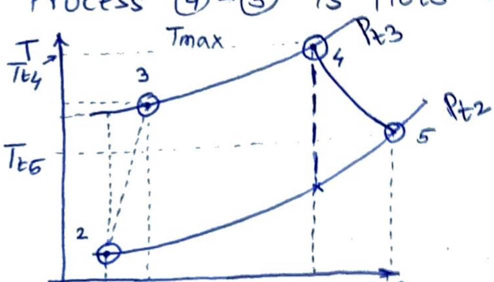
area under curve, heat supplied during combustion

Process. 3 - 4 PV plots.



isobaric combustion @ isobar of P_{t3}

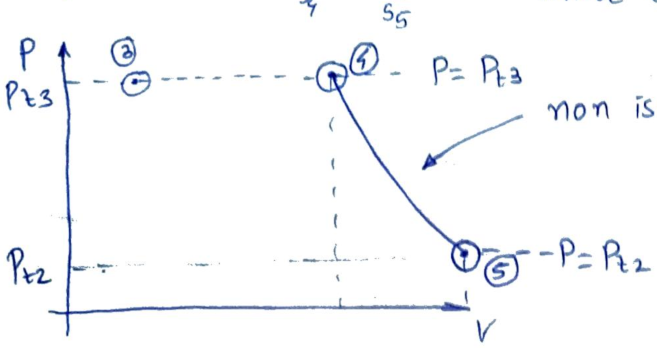
Process 4 - 5 TS Plots Non-isentropic expansion in turbine



--- isentropic ~~compression~~ expansion (ideal).

— 4 to 5 non isentropic ~~compression~~ expansion (real)

finite entropy change $S_5 - S_4 \neq 0$



non isentropic expansion

expressions for these are same as stated above

for process 2 → 3

$$\frac{T_{t3}}{T_{t2}} = 1 + \frac{1}{\eta_c}\left[\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right]$$ and units are 'K'

$$\frac{P_{t3}}{P_{t2}} = \left(\frac{\cancel{T_{t3}}}{\cancel{T_{t2}}}\right)\pi_c$$ .... units kPa ... (just ideal gas eqn)

for process 3 → 4

$$P_{t3} = P_{t4}$$ --- units kPa (isobaric).

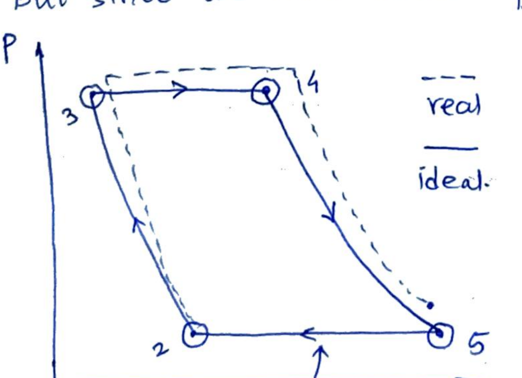
$$\dot{q}_{burner} = \dot{m}_a C_p (T_{t4} - T_{t3})$$ (isobaric combustion) units MJ/s.

for process 4 → 5

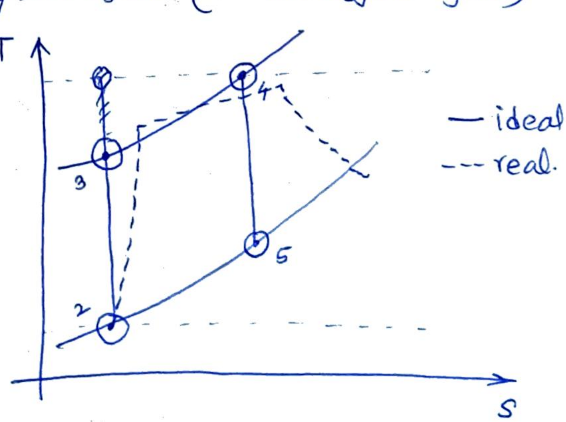
$$\frac{T_{t5}}{T_{t4}} = 1 - \eta_t\left[1 - \pi_t^{\frac{1-\gamma}{\gamma}}\right]$$ --- units 'K'

$$\frac{P_{t5}}{P_{t4}} = \frac{\cancel{T_{t5}}}{\cancel{T_{t4}}} \cdot \frac{1}{\pi_t}$$ --- units kPa (ideal gas law).

In general we see that for $\eta_c, \eta_t \neq 1$ to achieve the same compressor ratio in compressor, T_{t3} & P_{t3} are higher than their normal isentropic values. In nozzle expansion the resulting P_t & T_t are again higher, if it were isentropic expansion then the T_t amb P_o amb & P_{t5} must be same as the ideal Brayton cycle proposes that But since we have losses. Brayton cycle (~turbojet cycle)



This branch doesn't exist real as gas is exhausted to amtosphere



Q3. TSFC

Now we had $f = \frac{\dot{m}_f}{\dot{m}_a} = \cancel{0.015}\ 0.017$

also we have that $TSFC = \frac{\dot{m}_f}{\tau} = \frac{\dot{m}_f}{\dot{m}_a((1+f)V_e - V_a)}$

$$\therefore TSFC = \frac{f}{(1+f)V_e - V_a} = \frac{\cancel{0.015}\ 0.017}{(\underset{1.017}{\cancel{1.015}} \times \underset{409.782}{\cancel{388.84}}) - (250.74)}$$

$$= \frac{0.0157}{\underset{166.008}{\cancel{143.9326}}} = \frac{\cancel{1.04215 \times 10^{-4}}}{1.024 \times 10^{-4}} \frac{s}{m} = \frac{\cancel{1.04215} \times 10^{-4}}{1.024} \frac{kg}{NS}$$

$$= \frac{\cancel{10}\ \cancel{1.04215}}{1.024} \times \frac{10^3 \times 10^{-4}}{10^{-3}} \frac{gm}{KNS} = \cancel{104.215}\ \boxed{102.4} \frac{gm}{KN\text{-}s}.$$

$$\eta_{thermal} = \eta_{Brayton\ cycle} = \frac{Work_{net}}{heat_{added}}$$

$$= 1 - \left(\frac{T_{t5} - T_{t2}}{T_{t4} - T_{t3}}\right) = 1 - \left(\frac{\overset{661.794}{\cancel{595.88}} - 247.796}{1400 - \underset{637.315}{\cancel{733.4381}}}\right)$$

$$= 1 - \left(\frac{348.084}{666.5619}\right) = \underset{0.45718}{\cancel{0.47779}} \approx \boxed{\cancel{0.48} = \eta_{Th}}$$

$$\eta_{propulsive} = \frac{2V_a}{V_a + V_e} = \frac{2 \times 250.74}{250.74 + \underset{409.782}{\cancel{388.84}}} = \frac{501.485}{\underset{660.522}{\cancel{639.58}}}$$

$$\boxed{\eta_{propulsive} = \cancel{0.784}}\ 0.75922$$

$$\eta_{overall} = \eta_{propulsive} \times \eta_{Thermal} = \cancel{0.784} \times \cancel{0.48} = \cancel{0.376}$$
$$0.759 \times 0.457 = 0.3468$$

$$\therefore \boxed{\eta_{overall} = 0.3468}$$

Q4. Intake $d = 1.2\ m \Rightarrow A_{intake} = 1.13097\ m^2$

$$Thrust_{@\ cruise} = \dot{m}_a((1+f)V_e - V_a)$$

But $\dot{m}_a = \rho_\infty A_\infty V_\infty = 0.301595\ \frac{kg}{m^3} \times 1.13097\ m^2 \times 250.74$

$$= 85.526\ kg/s.$$

$\therefore Thrust_{@\ cruise} = 85.526\ kg/s \times ((1.017 \times 409.782) - 250.74)$

1 engine $= 14.198\ kN$. all four engines $= 56.792\ kN$

Q5. Range for constant altitude and speed

$$R = \frac{V_a}{g_o \, TSFC} \frac{L}{D} \log \frac{W_{ini}}{W_{fin}}$$

We have $W_{ini} = 170600$ kg

$W_{fuel} = 75750$ kg assuming $M_{fuel, TO} = 0$

$M_{fuel, land} = 0$

and all of the fuel is consumed during the flight we have, $W_{final} = 170600 - 75750$

$= 94850$ kg

$$\therefore \text{Range} = \frac{250.74}{9.81 \times 1.024 \times 10^{-4}} \times 20 \times \log\left(\frac{170600}{94850}\right)$$

$$= \boxed{2930.496 \text{ km.}}$$

Q6. Now after the combustor we had properties,

$P_{t2} = 30.1523$ kPa $\qquad$ $P_{t3} = \pi_c \cdot P_{amb} = 659.88$

$T_{t2} = 247.796$ K $\qquad$ $T_{t3} = 637.315$ K

$P_{t4} = 659.88$ kPa $\qquad$ $P_{t5} = 30.159$ kPa

$T_{t4} = 1400$ K $\qquad$ $T_{t5} = 661.794$ K

after combustor we have stage 4 & 5

Now we have overall compression $= 35.1 = \frac{P_{t3}}{P_{amb}}$

but $\pi_c = \frac{P_{t3}}{P_{t2}} = \frac{P_{amb} \times 35.1}{P_{o,amb}} = \frac{659.88}{30.1523} \approx 21.88$

but we have perfect expansion hence $\pi_t = 21.88$

now if we have $\gamma = 1.33$ we have,

$$\frac{T_{t5}}{T_{t4}} = 1 - \eta_t\left[1 - \pi_t^{\frac{1-\gamma}{\gamma}}\right] = 0.5185 \Rightarrow \boxed{T_{t5} = 725.972} \text{ K}$$

also we have $\pi_t = 21.88 = \frac{P_{t4}}{P_{t5}} \Rightarrow \boxed{P_{t5} = 30.159 \text{ K}}$

But as we have no losses in the nozzle we have,

$T_{t9} = T_{t5} = 725.972 \qquad P_{t9} = P_{t5} = 30.159 \text{ kPa}$

no loss in stagnation pressure as there is no heat addition, isentropic flow and no losses.

$\therefore M_{exit} = 0.85 \Rightarrow T_{exit} = 648.645 \text{ K}$

$P_{exit} = 18.80 \text{ kPa}.$

$\therefore a_{exit} = \sqrt{\gamma R T_{exit}} = 497.675$

$V_{exit} = M_e a_{exit} = 423.023 \text{ m/s}$

$V_a = 250.74 \text{ m/s}.$

we have $f = \dfrac{c_p(T_{t4} - T_{t3})}{Q_R} = 0.017$

$$TSFC = \frac{f}{(1+f)V_e - V_a} = \frac{0.017}{(1.017 \times 423.023) - 250.74}$$

$$= \frac{0.017}{179.474} = 9.472 \times 10^{-5} = \cancel{0.947 \times 10^{-5}}$$

$= 94.72 \times 10^{-4}$ (TSFC decreased)

(earlier it was around 102.4×10^{-4})

$$\eta_{thermal} = 1 - \left(\frac{T_{t5} - T_{t2}}{T_{t4} - T_{t3}}\right) = 1 - \left(\frac{725.972 - 247.796}{1400 - 637.315}\right)$$

$= 0.373$ (decreased) (earlier $\eta_{th} = 0.457$)

$$\eta_{propulsive} = \frac{2V_a}{V_e + V_a} = \frac{2 \times 250.74}{423.023 + 250.74} = 0.7442$$

(decreased) (earlier $\eta_{propulsive} = 0.75922$)

$\eta_{overall} = 0.7442 \times 0.373 = 0.2775$

(decreased) (@ $\gamma = 1.4$ $\eta_{overall} = 0.3468$)

Q7. Engine Improvement.

Overall compression ratio $= \dfrac{P_{t3}}{P_{amb}} = 45$

$P_{t3} = P_{amb} \times 45 = 18.8 \text{ kPa} \times 45 = 846 \text{ kPa}$

actual compression ratio $= \frac{P_{t3}}{P_{t2}} = \pi_c = \frac{846 \text{ kPa}}{30.1523 \text{ kPa}}$

$\therefore \pi_c = 28.057$

Hence what we have now is,

$T_{t2} = 247.\cancel{2}96 \text{ K} \quad P_{t2} = 30.1523 \text{ kPa}$

$\boxed{P_{t3} = 846 \text{ kPa}}$ (1)

$\therefore \frac{T_{t3}}{T_{t2}} = 1 + \frac{1}{\eta_c}\left[\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right]$ [here $\gamma = 1.33$ after combustor hence use $\gamma = 1.4$ here]

$\therefore \frac{T_{t3}}{T_{t2}} \Rightarrow \boxed{T_{t3} = 686.269 \text{ K}}$ (2)

Now, $\boxed{T_{t4} = 1400 \text{ K}}$ $\therefore f = \frac{c_p(T_{t4} - T_{t3})}{Q_R} = 0.01593 \approx 0.016.$

(here too, as process before combustor using $\gamma = 1.4$)

Now as perfect expansion, $\pi_t = \pi_c = 28.057$

$\therefore \pi_t = \frac{P_{t4}}{P_{t5}} = 28.057$ But combustion is isobaric hence $\boxed{P_{t4} = 846 \text{ kPa}}$

$\therefore \boxed{P_{t5} = 30.1529 \text{ kPa}}$ and as we have that, in turbine

$\frac{T_{t5}}{T_{t4}} = 1 - \eta_t\left(1 - \pi_t^{\frac{1-\gamma}{\gamma}}\right)$ now use $\gamma = 1.33$ as after the combustor process.

$\therefore \frac{T_{t5}}{T_{t4}} = 0.4935 \quad \boxed{T_{t5} = 690.9118 \text{ K}}$

$\left.\begin{aligned} \therefore T_{t9} &= T_{t5} = \cancel{681}.\ 690.9118 \\ P_{t9} &= P_{t5} = 30.1529 \text{ kPa} \end{aligned}\right\}$ (no losses in the nozzle)

and for perfect expansion $M_e = 0.85$

$\therefore$ Texit or $T_9 = \frac{T_{stg,9}}{\left(1 + \frac{\gamma-1}{2}M^2\right)} = \frac{690.9118 \text{ K}}{\left(1 + \left(\frac{1.33-1}{2}\right)(0.85)^2\right)}$

(here $\gamma = 1.33$)

$\therefore \boxed{T_{exit} = 617.319 \text{ K}}$

$V_{exit} = M_e\sqrt{\gamma R T_e} = 412.682 \text{ m/s}.$

PTO

We have Thrust $= \dot{m}_a((1+f)V_e - V_a)$

$$\text{Thrust} = \dot{m}_a((1+0.017)\,412.682 - 250.74)$$

$$= \dot{m}_a(168.957)$$

Hence Thrust per unit mass flow rate $= 168.957 \ \frac{N}{kg/s}$

also $TSFC = \frac{\dot{m}_f}{\tau} = \frac{f}{(1+f)V_e - V_a} = \frac{0.017}{168.957}$

$\therefore$ updated $TSFC = 1.00617 \times 10^{-4} \ \frac{kg}{Ns}$