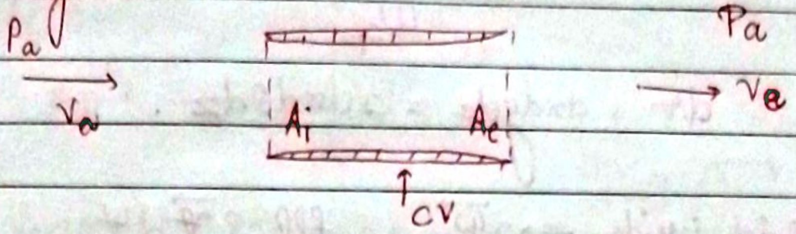
AE 234

Thrust :-

Exert force on fluid $\rightarrow$ Reac^n $\rightarrow$ Generate a force
$\uparrow$ [Air pushes back.]
Push the air back

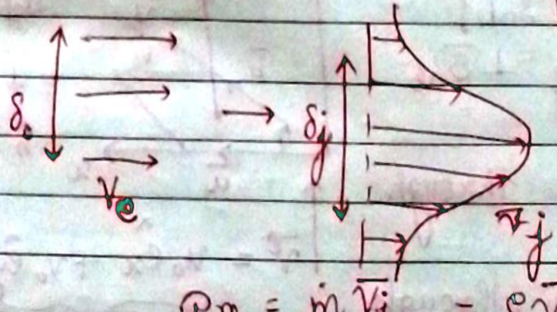
Simple engine-



$$\dot{m} = \rho_a A_i V_a$$

$$F_{\text{air by engine}} = \dot{m}(V_e - V_a) = T$$

But, - pressure at exit need not equal to P_a
- V_e & V_a are usually not uniform/ finite.



momentum

$$P_m = \rho V_e^2 \delta_o = \rho \bar{V}_j^2 \delta_j$$

$$\dot{m} = \rho V_e \delta_o = \rho \bar{V}_j \delta_j$$

$$P_m = \dot{m}\bar{V}_j = \rho \bar{V}_j^2 \delta_j \Rightarrow \bar{V}_j = \sqrt{\frac{P_m}{\rho \delta_j}}$$

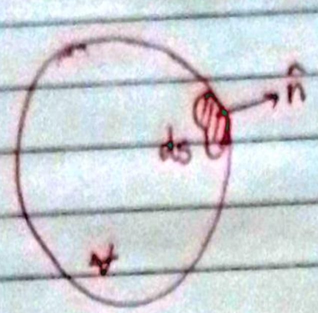
$\delta_j \uparrow \rightarrow$ jet widens $\Rightarrow \bar{V}_j \downarrow$

$\dot{m} = \sqrt{P_m \rho \delta_j} \uparrow$ - (entrainment)

- Due to wing effect, vel. & pressure different from $V_a/P_a \rightarrow$ errors in estimation

- Energy is added through combustion → fuel is consumed
  $\dot{m}_e = \dot{m}_a + \dot{m}_f$

Newton's laws for CV:-



Inside the vol., we've fluid, ρ, $\vec{v}$

$$m = \iiint_{\forall} \rho \, d\forall$$

$$d\forall = dx\,dy\,dz = r\,dr\,d\theta\,dz.$$

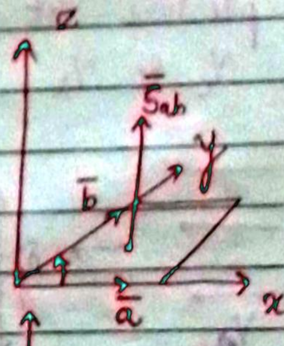
Momentum of fluid inside the vol. $\longrightarrow \vec{P}_m = \iiint_{\forall} \rho \vec{v} \, d\forall$

Avg. vel. of fluid in $\forall$ $\rightarrow \vec{u} = \frac{\vec{P}_m}{m} = \frac{\iiint_{\forall} \rho \vec{v} \, d\forall}{\iiint_{\forall} \rho \, d\forall}$

$$d\vec{S} = dS\,\hat{n}$$

How much flow goes through $\bar{S}_{ab}$?

$\vec{V}_z \parallel \bar{S}_{ab}$, $\vec{V}_{xy} \perp \bar{S}_{ab}$



$\bar{S}_{ab} = \bar{c} = \bar{a} \times \bar{b}$

$\hat{n} = \frac{\bar{c}}{|\bar{c}|}$

$\vec{V} \cdot \bar{S}_{ab}$ ← volumetric flow through $\bar{S}_{ab}$

$$\vec{v} = \underbrace{u_0 \hat{e}_x + v_0 \hat{e}_y}_{\vec{V}_{xy}} + \underbrace{w_0 \hat{e}_z}_{\vec{V}_z}$$

$\rho \vec{V} \cdot \bar{S}_{ab}$ ← mass flow rate through $\bar{S}_{ab}$.

flow rate → well-defined surface area * normal velocity

mass flow rate across a surface $\longrightarrow \dot{m} = \iint \rho \vec{v} \cdot d\vec{S}$ → >0 outflow, <0 inflow

$$m(t+\Delta t) - m(t) = \oiint \rho \vec{V} \cdot d\vec{S} \, \Delta t$$

$$\lim_{\Delta t \to 0} \frac{m(t+\Delta t) - m(t)}{\Delta t} = - \oiint \rho \vec{V} \cdot d\vec{S}$$

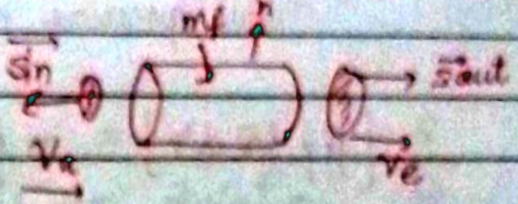
$$\frac{dm}{dt} = -\oiint \rho \vec{V} \cdot d\vec{S}$$

$$\frac{d}{dt}\left(\iiint \rho \, d\forall\right) = -\oiint \rho \vec{V} \cdot d\vec{S}$$

For a fixed c.v., (boundaries do not move)

$$\iiint \frac{\partial}{\partial t}(\rho)\, d\forall + \oiint \rho \vec{V} \cdot d\vec{S} = 0 \quad \text{--- (1)}$$

Going back to engine.



$$\oiint \rho \vec{V} \cdot d\vec{S} = -\rho V_a S_{in} + \rho V_e S_{out}$$

$$\iiint \frac{\partial \rho}{\partial t}\, d\forall = 0 \quad \text{--- (steady state)}$$

$\therefore$ By cont. eq^n (1), $\rho V_a S_{in} = \rho V_e S_{out}$

For fuel coming inside the engine,

$$\oiint \rho \vec{V} \cdot d\vec{S} = -\rho V_a S_{in} + \rho V_e S_{out} - \dot{m}_f \quad \{\text{in steady state}\}$$

$$\therefore \rho V_e S_{out} = \rho V_a S_{in} + \dot{m}_f$$

$$\dot{m} = \dot{m}_a + \dot{m}_f$$

$$f = \frac{\dot{m}_f}{\dot{m}_a} << 1$$

$$\dot{m} = \dot{m}_a(1+f)$$

Momentum consv.

$$P_m = \iiint \rho \vec{V}\, d\forall$$

$$\partial \dot{m} = \rho \vec{V} \cdot d\vec{S}$$

$$\partial \dot{p}_m = \partial \dot{m}\, \vec{V} \equiv \rho \vec{V}\, \vec{V} \cdot d\vec{S}$$

Over Δt,

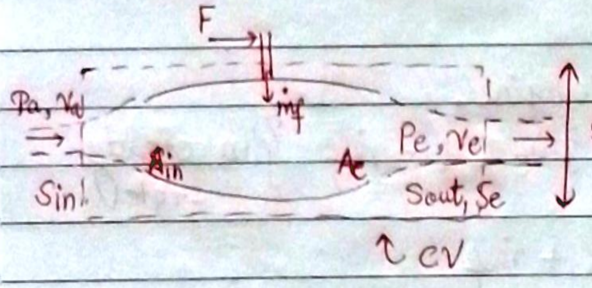
$$P_m = \Delta t \iint \rho \bar{V}^2 \cdot d\bar{S} \leftarrow$$ loss of momentum of fluid in CV due to advection.

Momentum to be affected by → pressure, shear stress, body forces, interaction with other fluids

Pressure force at ds part $= -p\,d\bar{S}$

Total pressure force $= -\oiint p\,d\bar{S}$

$$\left.\begin{aligned} &\frac{\partial}{\partial t}\left[\iiint \rho \bar{V}\, d\forall\right] = -\oiint \rho \bar{V}\bar{V}\cdot d\bar{S} - \oiint p\,d\bar{S} + \bar{F}_{other} \\ &\text{For steady state,} \\ &F_{other} = \oiint \rho \bar{V}\bar{V}\cdot d\bar{S} + \oiint p\,d\bar{S} \end{aligned}\right\}$$ Momentum Consv.



$$\oiint \rho \bar{V}\bar{V}\cdot d\bar{S} = -\rho_i V_a^2 S_{in} + \rho_e V_e^2 S_e$$

$$\oiint p\,d\bar{S} = -P_a S + P_e S_e + P_a[S - S_e]$$
$$= [P_e - P_a] S_e$$

$$F = \oiint \rho \bar{V}\bar{V}\cdot d\bar{S} + \oiint p\,d\bar{S} = [\rho_e V_e^2 S_e - \rho_{in} V_a^2 S_{in}] + (p_e - p_a) S_e$$

$$F = \rho_e V_e^2 S_e - \rho_{in} V_a^2 S_{in} + (p_e - p_a) S_e$$

$$\dot{m}_a = \rho_{in} V_a S_{in} \Rightarrow \rho_{in} V_a^2 S_{in} = \dot{m}_a V_a$$

$$\dot{m}_e = \rho_e V_e S_e \Rightarrow \rho_e V_e^2 S_e = \dot{m} V_e$$

$$T = F = \dot{m} V_e - \dot{m}_a V_a + (p_e - p_a) S_e$$
$$= \dot{m}_a \{(1+f) V_e - V_a\} + (p_e - p_a) S_e$$

Considering the CV only till Ae, i.e. no effect of the aircraft on the engine flows

$\therefore$ uninstalled thrust $\rightarrow$ $T = \dot{m}V_e - \dot{m}_a V_a + (p_e - p_a)A_e$

Installed thrust $\rightarrow$ $T_{inst} = T - T_{losses}$

Ram Drag $\rightarrow D_0 = \dot{m}_a V_a \equiv \rho_a V_a^2 S_{in} \approx \rho_a V_a^2 A_{in}$

Gross thrust $\rightarrow$ $T_g = \dot{m}V_e + (p_e - p_a)A_e$

$$T = T_g - D_0$$

Fuels :- i) Piston engines $\rightarrow$ Avgas $\rightarrow$ Aviation gasoline

$\longrightarrow$ added (tetraethyl lead) < 0.56 gm/l

$Q_R = 44$ MJ/kg | $\rho = 0.71$ kg/l

$= 31.6$ MJ/l

ii) Jet engines $\rightarrow$ kerosene $\rightarrow$ Jet A, Jet A-1 (JP-1A)

Jet B (cold climate), TS-1 (russian)

JP-4 - 50 gasoline + 50 kerosene

JP-8 - for military purpose

$Q_R = 43$ MJ/kg | $\rho = 0.775 - 0.84$ kg/l

$= 38.6$ MJ/l

iii) Diesel $\rightarrow Q_R = 45.6$ MJ/kg

$H_2 \rightarrow Q_R = 120-140$ MJ/kg

(gas) $\rightarrow Q_R = 0.01$ MJ/l

(liq.) $\rightarrow Q_R = 8.5-10$ MJ/l $\leftarrow$ storage wt. requirements

Fuel requirements :-

How much air do we need to burn 1kg of fuel (kerosene) ? $\left[\frac{\dot{m}_a}{\dot{m}_f}\right]$

$C_{17n}C_{36n} \approx 85\% C, 15\% H$

26 x 3.72

$$C_{17n}H_{36n} + 26n(O_2 + 3.72N_2) \longrightarrow 17nCO_2 + 18nH_2O + nN_2$$

$$f_{st} = \frac{\dot{m}_f}{\dot{m}_a} = \frac{1}{14.75} = 0.068$$ (for stiochiometric comb^n)

$f < f_{st} \longrightarrow$ fuel lean — used for RAMjets, afterburning Turbojets

$f > f_{st} \longrightarrow$ fuel reach

$f = 0.005 - 0.02 \leftarrow$ for most gas-Turbines

$f << 1$

For subsonic engines, $p_e = p_a$

$$\therefore T = \dot{m}_a\left[V_e - V_a + fV_e\right]$$

$$T \approx \dot{m}_a\left[V_e - V_a\right] \quad , \quad fV_e << V_e - V_a$$

$$r = \frac{V_a}{V_e} \quad , \quad 1 - r >> f$$

near take-off r is very small $\Rightarrow 1 - r \approx 1$

$$T = \dot{m}_a V_e\left[(1-r) + f\right]$$

$$T \approx \dot{m}_a V_e (1-r)$$

r shd not be too large

$$r << 1 - f$$

$$\therefore r << 0.98$$

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$V_a \sim 250\,m/s$

$r \approx 1/2$

$V_e \sim 500\,m/s$

Efficiency :- $\dot{m}_f Q_R \rightarrow T \rightarrow$ pushing the vehicle at speed V_a

Power gained $\rightarrow T V_a$

~~wake~~ work $=$ F × dist. , over Δt , dist. $= V_a \Delta T$, $F = T$,

$W = F\,\text{dist.} = T V_a \Delta T$

$P = T V_a$

Total power from fuel $\rightarrow \dot{m}_f Q_R$,

fuel also had KE $\rightarrow \frac{1}{2} \dot{m}_f V_a^2$

$$\frac{\frac{1}{2}\dot{m}_f V_a^2}{\dot{m}_f Q_R} = \frac{V_a^2}{2Q_R} \approx \frac{250 \times 250}{2 \times 40 \times 10^6} = 8 \times 10^{-5} << 1$$

overall efficiency :-

$$\text{Efficiency} - \frac{\text{useful work}}{\text{Total energy input}}$$

$$\eta_T = \frac{T V_a}{\dot{m}_f Q_R} = \frac{\dot{m}_a [V_e - V_a] V_a}{\dot{m}_f Q_R}$$

$$= \frac{\dot{m}_a V_e^2 (1 - V_a/V_e) V_a/V_e}{f \dot{m}_a Q_R}$$

$$\eta_T = \frac{(1-r) r}{f(Q_R/V_e^2)}$$

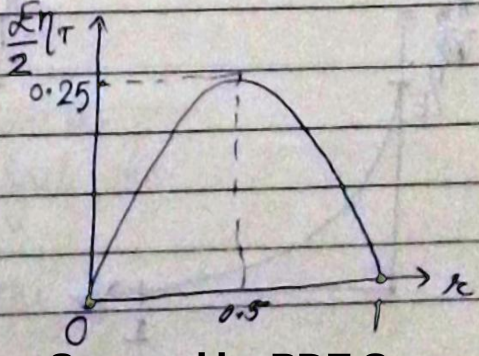
$$E = \frac{f Q_R}{\frac{1}{2} V_e^2} = \frac{2 f Q_R}{V_e^2} = \frac{2 \times 10^{-2} \times 40 \times 10^6}{500 \times 500}$$

$$= 3.2$$

$$\therefore \eta_T = \frac{2r(1-r)}{E}$$

$$E \rightarrow 4 - 10$$

$$\eta_{T/opt.} = \frac{1}{2E} \rightarrow 0.05 - 0.125$$



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Losses →

fuel energy (A) $\xrightarrow{\text{burns}}$ Thermal energy (B) → lost to surrounding

Thermal energy → air flow → air jet KE (C)

air jet KE (C) → vehicle power (E)

air jet KE (C) → residual KE of exhaust jet (F)

air flow → air thermal energy (D) (Internal energy of exhaust jet)

$$u_e = V_e - V_a$$

Propulsive efficiency → $\eta_p = \dfrac{\text{Power to vehicle}}{\text{vehicle power} + \text{Residual jet KE}}$

$$\eta_p = \frac{TV_a}{TV_a + \frac{1}{2}\dot{m}u_e^2} = \frac{TV_a}{\frac{1}{2}\dot{m}(V_e^2 - V_a^2)}$$

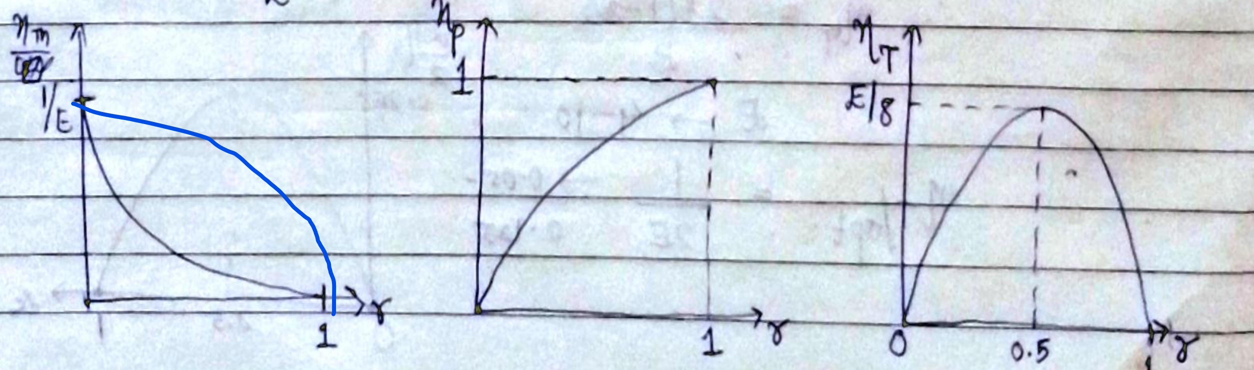
$$= \frac{\dot{m}(V_e - V_a)V_a}{\frac{1}{2}\dot{m}(V_e^2 - V_a^2)} = \frac{2V_a}{V_e + V_a} = \frac{2r}{1+r}$$

overall efficiency ⇒ (A) → (E) Jet KE ⇒ (A) → (C)

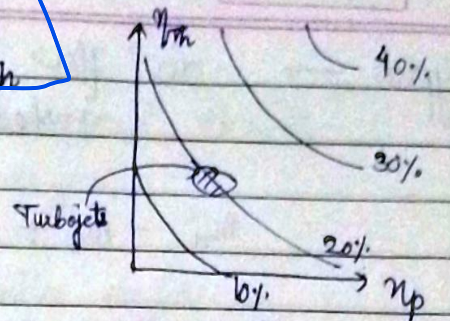
Propulsive efficiency → (C) → (E)

$$\eta_{Th} = \frac{\text{Jet KE}}{\text{fuel energy}}, \quad \eta_{Th} = \frac{\frac{1}{2}\dot{m}(V_e^2 - V_a^2)}{\dot{m}_f Q_R} = \frac{\dot{m}_a V_e^2(1-r^2)}{2f\dot{m}_a Q_R}$$

$$\eta_{Th} = \frac{1-r^2}{E}$$



$$\therefore \boxed{\eta_T = \eta_p \eta_{Th}}$$



Limiting factor -
$V_e = 2V_a$,

Most subsonic jets $\rightarrow M = 0.85$. , $h \sim 35,000$ ft., $T_a = 216K$
$V_a \sim 250$ m/s

$f \sim 0.005 - 0.02 \leftarrow$ cannot reduce f further while sustaining combustion.

$$\eta_T = \frac{V_a^2}{f}$$

$$\dot{m}_f Q_R = \frac{1}{2}[\dot{m} V_e^2] + \dot{m} C T_e - \left(\frac{1}{2}\dot{m} V_a^2 + \dot{m} C T_a\right)$$

$$= \frac{1}{2}\dot{m}[V_e^2 - V_a^2] + \dot{m} C (T_e - T_a)$$

$$1 = \underbrace{\frac{\frac{1}{2}\dot{m}(V_e^2 - V_a^2)}{\dot{m}_f Q_R}}_{\eta_{Th}} + \frac{\dot{m} C (T_e - T_a)}{\dot{m}_f Q_R}$$

$$\eta_{Th} = 1 - \underbrace{\left(\frac{C(T_e - T_a)}{f Q_R}\right)}_{\text{The term reducing } \eta_{Th}}$$

Can $(T_e - T_a)$ be reduced or energy $C(T_e - T_a)$ be utilised in thrust?

$$T = \underset{\substack{\uparrow \\ \text{can we } \uparrow ?}}{\dot{m}} \underbrace{(V_e - V_a)}_{}, \text{ fixed (restricted by } \eta_p, \tau)$$

Core engine $\rightarrow$ air flow rate $\rightarrow \dot{m}_c$
reacts with fuel $\dot{m}_f$
$f_c = \frac{\dot{m}_f}{\dot{m}_c}$, $\leftarrow$
gains energy
$\hookrightarrow$ runs the fan
$f = 0.005 - 0.02$
$\hookrightarrow$ jet

Bypass $\longrightarrow$ air flow rate $\rightarrow \dot{m}_b = \alpha \dot{m}_c$
pushed by fan.

$$\dot{m}_a = \dot{m}_b + \dot{m}_c = \dot{m}_c(1+\alpha)$$

Core thrust, $T_c = \dot{m}_c\left[(1+f_c)V_e - V_a\right]$

Bypass thrust, $T_b = \dot{m}_b\left[V_{e_2} - V_a\right]$

If $V_{e_2} = V_e \Rightarrow T_b = \alpha \dot{m}_c\left[V_e - V_a\right]$

$$T = T_c + T_b = \dot{m}_c\left[(1+\alpha+f_c)V_e - (1+\alpha)V_a\right]$$

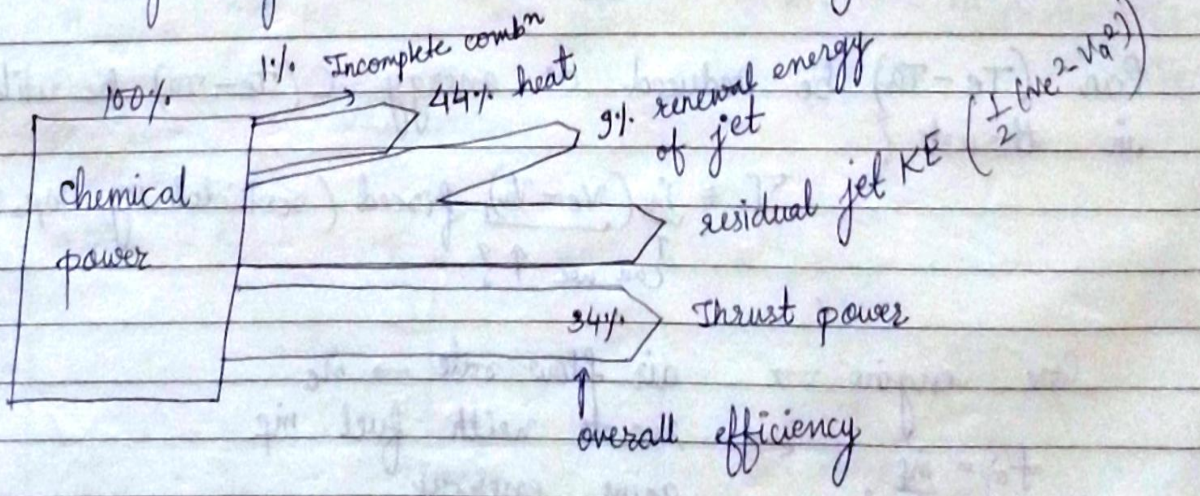
$$= \dot{m}_c(1+\alpha)\left\{\left[1+\frac{f_c}{1+\alpha}\right]V_e - V_a\right\}$$

$$= \dot{m}_a\left[(1+f)V_e - V_a\right] \quad \text{where } f = \frac{f_c}{1+\alpha}$$

$$\eta \propto \frac{V_a^2}{f} \propto \frac{V_a^2(1+\alpha)}{f_c}$$

$$\therefore \alpha\uparrow \Rightarrow \eta\uparrow$$

very large $\alpha \Rightarrow \dot{m}_b >> \dot{m}_c \Rightarrow$ drag $\uparrow$ (vel. of fan tip $\uparrow$)



Specific fuel consumption :-

↳ fuel needed for unit thrust

$$\underset{\text{Thrust}}{T}SFC = \frac{\dot{m}_f}{T} \leftarrow \text{Jet engines}$$

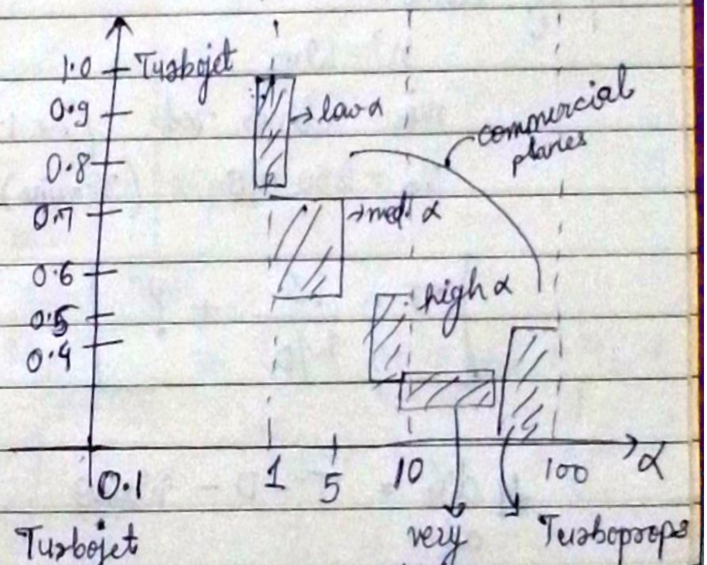
$$\underset{\text{Brake}}{B}SFC = \frac{\dot{m}_f}{P} \longleftarrow \text{Propeller engines.}$$

$$\frac{\dot{m}_f}{T} = \frac{\dot{m}_f}{\dot{m}_a(V_e - V_a)} = \frac{f}{V_e - V_a} = \frac{10^{-2}}{200} \sim 5 \times 10^{-5}\ kg/N\text{-}s$$

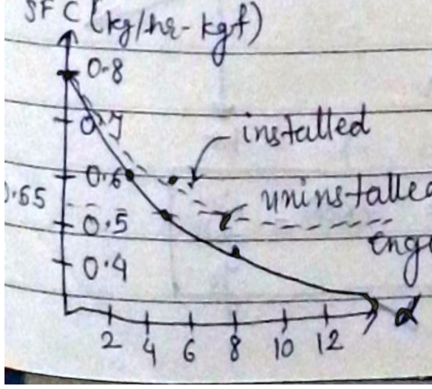
SFC shd be as small as possible.

$$\eta_T = \frac{TV_a}{\dot{m}_f Q_R} = \frac{1}{SFC}\frac{V_a}{Q_R}, \quad SFC \propto \frac{1}{\eta_T} \propto V_a$$

Improve η_T by $\alpha \uparrow$ (bypass ratio)



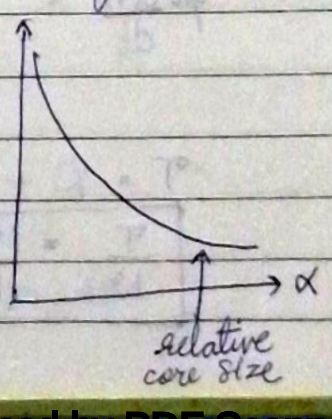
Turbofans :- Relative core size. $= \frac{\dot{m}_c}{\dot{m}_a} = \frac{1}{1+\alpha}$

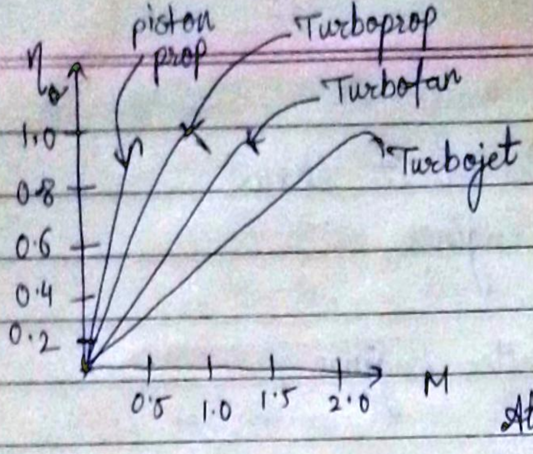


[1kgf = 9.8 N]

α 0 ↔ ∞

↑ Turbojet ↑ Turboprop





Turbojets:-

RR/Snecma Olympus 593 – Concorde

		SFC (mg/N-s)	T (kN)
M = 2	cruise –	33.8	139
At sea level (wet)		39	169

T/W

–

5.4

PW J-58 (JP-7) — SR-71

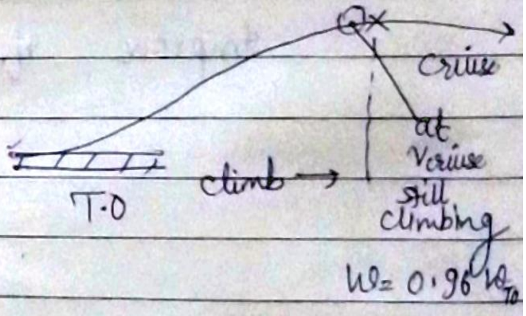
	SFC	T (kN)	T/W
M = 3.2 (cruise wet cond")	54	151 (bare)	5.2
		113	

Top of climb:-

$W = W_{TO}$

min. climb rate ($\sim 1.5\ m/s$)

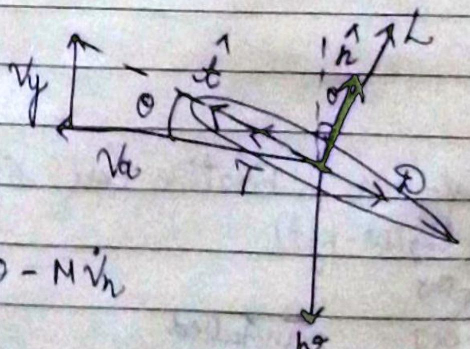
$V_a = 250\ m/s$ (V_{cruise})



$$\boxed{T = \frac{W}{L/D} + ?}$$

$$M\frac{dV_t}{dt} = T - D - W\sin\theta$$

$$M\frac{dV_n}{dt} = L - W\cos\theta$$



$$L = W\cos\theta - M\dot{V}_n$$

$$T = D + W\sin\theta + M\dot{V}_t$$

$$\boxed{\frac{T}{W} = \frac{D}{W} + \sin\theta + \frac{\dot{V}_t}{g_o}, \quad \frac{L}{W} = \cos\theta - \frac{\dot{V}_n}{g_o}}$$

$$\tan\theta = \frac{V_y}{V_a} = \frac{1.5}{250}$$

$$\theta \approx \tan\theta = 0.006 \text{ radian} \approx 0.34^\circ$$

$$L \approx W \quad , \quad \frac{T}{W} = \frac{D}{W} + \theta + \underbrace{\frac{\dot{V}_t}{g_0}}_{\text{assume negligible}}$$

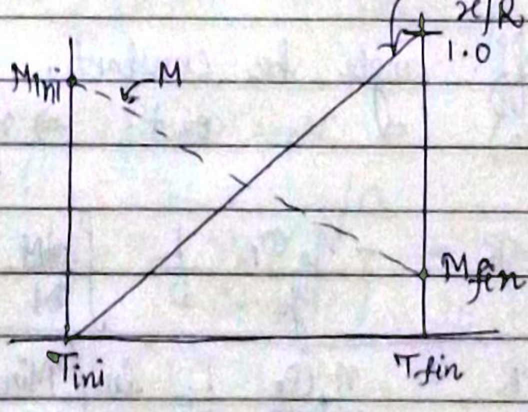
$$\frac{T}{W} = \frac{1}{L/D} + \theta$$

For specific 747, $T\Big|_{\substack{\cancel{\text{Take off}} \\ \text{Top of climb}}} = 1.11\, T\Big|_{\text{cruise}}$

$W_{TO}, T \rightarrow$ Range, Endurance

$$R = \int dx \equiv \int V dt \quad , \quad T = \int dt$$

During cruise,



$$M_{\text{Take-off}(TO)} = \underset{\substack{\uparrow \\ \cancel{4\%}\text{(say)}}}{M_{f_{cruise}}} + \underset{\substack{\uparrow \\ 10\% \text{ (say)}}}{M_{f_{reserved}}} + M_{\text{aircraft structure}} + \underset{\substack{\uparrow \\ \text{payload}}}{M_L} + \underset{\substack{\uparrow \\ (4\% \text{ of } TO)}}{M_{f_{climb}}}$$

$$M_{ini} = M_{TO} - M_{f_{climb}}$$

$$M_{fin} = M_{TO} - M_{f_{climb}} - M_{f_{cruise}}$$

$$R = \int V_a \, dt$$

over $dt \rightarrow$ distance covered $\rightarrow V dt$

fuel consumed $\rightarrow -\dot{m}_f dt$

$$dM = -\dot{m}_f\,dt$$

$$dx = V\,dt.$$

$$\therefore \frac{dM}{dx} = -\frac{\dot{m}_f}{V}$$

$$dx = -V\frac{dM}{\dot{m}_f} \quad , \quad \dot{m}_f = \frac{V_a}{\eta_o Q_R}\cdot\frac{W}{L/D} = \frac{V_a \cdot Mg}{\eta_o Q_R\, L/D}$$

$$\cancel{dx} \quad \therefore \frac{V}{\dot{m}_f} = \frac{\eta_o Q_R}{M g_o}\cdot\frac{L}{D}$$

$$\int_{T_{in}}^{T_{fin}} dx = \int_{T_{in}}^{T_{fin}} -\frac{\eta_o Q_R}{g_o}\frac{L}{D}\frac{dM}{M}$$

$$R = -\int_{t_{in}}^{t_{fin}} \frac{\eta_o Q_R}{g_o}\left(\frac{L}{D}\right)\frac{dM}{M}$$

Let 'α' – eff. angle be constant. $\Rightarrow \frac{L}{D}$ = const, η_{ov} = const.

$\eta_{ov} = \eta_{ov}(V_a) \Rightarrow V_a$ = const. $\Rightarrow \eta_o$ = const.

$$\therefore R = -\frac{\eta_o Q_R}{g_o}\frac{L}{D}\int\frac{dM}{M}$$

$$\boxed{R = \frac{\eta_o Q_R}{g_o}\frac{L}{D}\ln\left(\frac{M_{in}}{M_{fin}}\right)}$$

$$T = \frac{\eta_o Q_R}{g_o V}\frac{L}{D}\ln\left[\frac{M_{in}}{M_{fin}}\right]$$

$$TSFC = \frac{\dot{m}_f}{T} = \frac{V_a}{\eta_o Q_R}$$

$$\therefore R = \frac{1}{TSFC}\frac{V_a}{g_o}\frac{L}{D}\ln\left[\frac{M_{in}}{M_{fin}}\right]$$

$$R \propto \eta_o \propto \frac{1}{TSFC}$$

fuel fraction $\longrightarrow$ $\dfrac{M_{fer}}{M_{TO}} \approx \dfrac{M_{ini} - M_{fin}}{M_{in}} = 1 - \dfrac{M_{fin}}{M_{in}}$

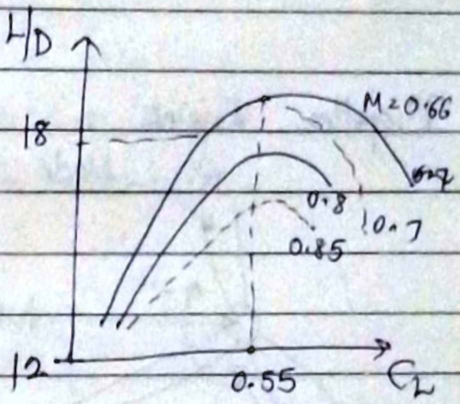
$$\frac{M_{ini}}{M_{fin}} = \exp\left[\frac{TSFC}{L/D}\,\frac{g_0 R}{V_a}\right] \qquad = 1 - \exp\left[\frac{-TSFC}{L/D}\,\frac{g_0 R}{V_a}\right]$$

Range - $dx = -V_a \dfrac{dM}{\dot{m}_f} \equiv -V_a \dfrac{dM}{T(SFC)}$

$$T = D = \frac{W}{L/D} = \frac{M g_0}{L/D}$$

$$R = \int dx = \int_{t_{in}}^{t_{fin}} \frac{-V_a}{SFC}\,\frac{L/D}{g_0}\,\frac{dM}{M}$$

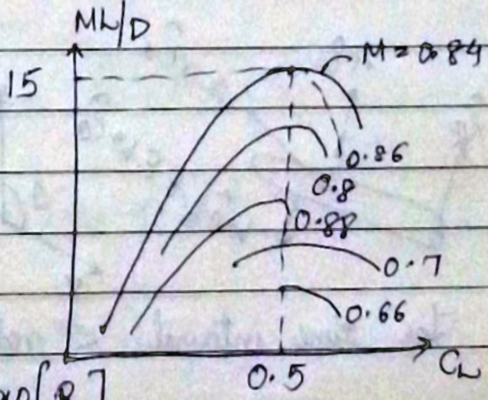
$$R = V_a\left(\frac{L}{D}\right)\frac{1}{g_0\,SFC}\,\ln\left[\frac{M_{in}}{M_{fin}}\right]$$



Considering velocity effect too,

$$M = V_a/C_s \quad , \quad C_s = \sqrt{\gamma R T_a}$$

$$\frac{ML}{D} = \frac{V_a}{C_s}\left(\frac{L}{D}\right)$$



Length scale, $R_0 = V_a\left(\dfrac{L}{D}\right)\dfrac{1}{g_0\,SFC}$

$$R = R_0 \ln\left[\frac{M_{in}}{M_{fin}}\right]$$

$$\ln\left[\frac{M_{in}}{M_{fin}}\right] = \frac{R}{R_0} \Rightarrow \frac{W_{in}}{W_{fin}} = \exp\left[\frac{R}{R_0}\right]$$

$$\frac{W_{in}}{W_{fin}} = 1 + \frac{W_f}{W_{engine\,(empty)} + W_L}$$

$$\frac{W_f}{W_e + W_L} = \exp\left[\frac{R}{R_0}\right] - 1$$

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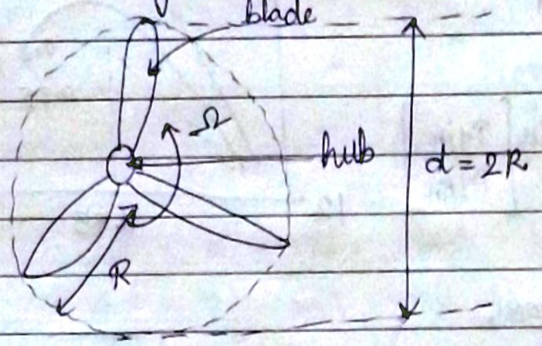
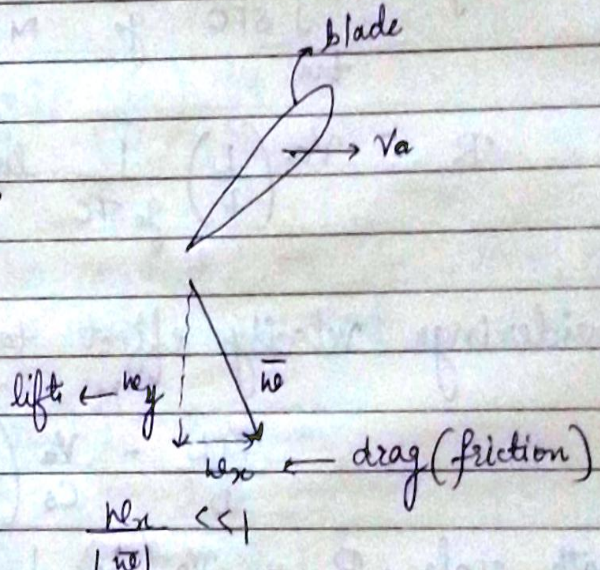
$$\frac{w_f}{w_e} = \left[1+\frac{w_e}{w_L}\right]\left[\exp\left(\frac{R}{R_0}\right)-1\right]$$

$$\frac{w_f}{R w_e} = \left[1+\frac{w_e}{w_L}\right]\frac{1}{R}\left\{\exp\left(\frac{R}{R_0}\right)-1\right\}$$

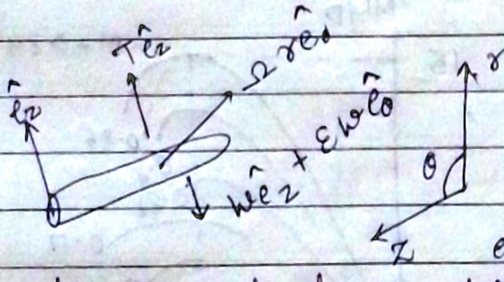
For the same range,
small fuel consump^n $\Rightarrow$ large R_0 & small $\ln[w_{in}/w_{fin}]$
$\Rightarrow$ small R_0 & large $\ln[w_{in}/w_{fin}]$

Propeller Engines :-



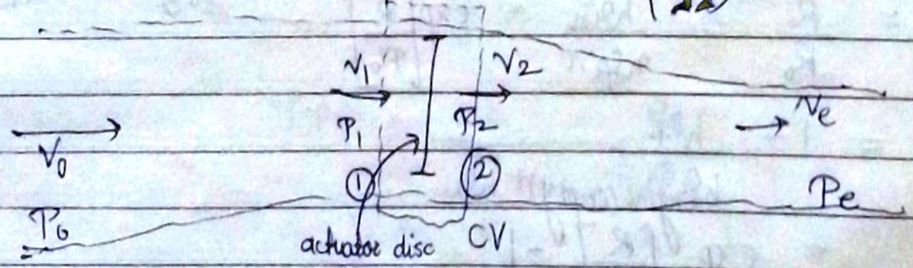


$$\frac{w_x}{|\bar{w}|} << 1$$



over one rotation, blades are smeared everywhere. imagine an actuator disk.

For time interval < rotation time $\left(\frac{2\pi}{\Omega}\right)$ $\longrightarrow$ blades are like unsteady flow



, $P_e = P_0 = P_a$

Streamtube passing through the actuator disk.

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$P_{useful} = T v_0$ & $P_{induced} = T w$

$T_{SSL} = P_{to}^{2/3} (2\rho A)^{1/3}$ ← induced power , $w_0 (v_0 = 0) = \sqrt{\frac{T}{2\rho A}}$

$\dot{m} = \rho_0 A_0 v_0 = \rho_1 A_1 v_1 = \rho_2 A_2 v_2 = \rho_e A_e v_e$.

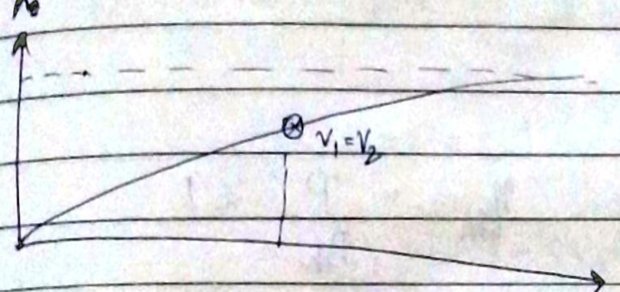
$A_1 = A_2 = \frac{\pi d^2}{4} = A_d$

Assump^n :- Incompressible flow

$\rho_0 = \rho_1 = \rho_2 = \rho_e = \rho$.

$A_0 v_0 = A_1 v_1 = A_2 v_2 = A_e v_e$

$T = \dot{m} (v_e - v_0)$



$T = \dot{m} v_e - \dot{m} v_0$

$= \rho A_e v_e v_e - \rho A_0 v_0 v_0$

$= \rho (A_e v_e^2 - A_0 v_0^2)$

Push force → $P_2 > P_1$

Inviscid, irrotational, steady Bernoulli eq^n -

$P_0 + \frac{1}{2} \rho v_0^2 = P_1 + \frac{1}{2} \rho v_1^2$ ← upstream

$P_2 + \frac{1}{2} \rho v_2^2 = P_e + \frac{1}{2} \rho v_e^2$ ← downstream

$T = \dot{m} v_2 - \dot{m} v_1 + (P_2 - P_1) A_d$ ← for CV.

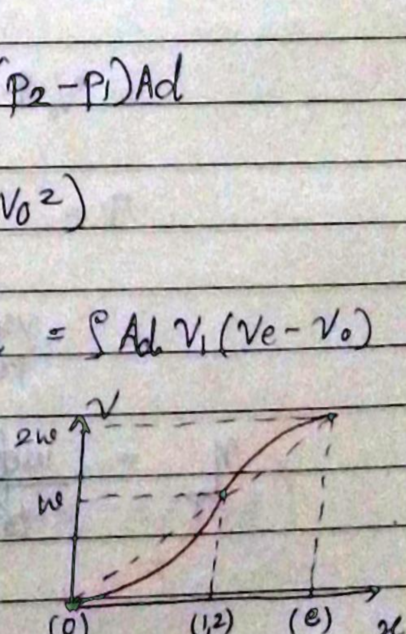
$\dot{m} = \rho v_1 A_d = \rho v_2 A_d$

$\dot{m}(v_2 - v_1) = 0 \implies T = (P_2 - P_1) A_d$

$\therefore P_2 - P_1 = \frac{1}{2} \rho (v_e^2 - v_0^2)$

$\therefore T = \frac{1}{2} \rho (v_e^2 - v_0^2) A_d = \rho A_d v_1 (v_e - v_0)$

$\therefore v_1 = \frac{v_e + v_0}{2} = v_2$



If $v_1 = v_0 + w \Rightarrow v_e = v_0 + 2w$

$$w = \frac{1}{2}\left[-V_0 + \sqrt{V_0^2 + \left(\frac{2T}{\rho A}\right)}\right]$$

$$\therefore T = 2\rho A_d w (V_0 + w)$$

$$\text{Power} = \frac{1}{2}\dot{m}(V_e^2 - V_0^2) = \frac{1}{2}\dot{m}(V_e - V_0)(V_e + V_0)$$

$$= \frac{1}{2}\dot{m} \times 2w \times 2(V_0 + w) = 2\dot{m}w(V_0 + w)$$

$$= 2\rho A_d V_1 w (V_1) = 2\rho A_d V_1^2 w = 2\rho A_d (V_0 + w)^2 w$$

$$P, \text{Power} = T(V_0 + w)$$

$$P_{shaft} > P \qquad \eta_{sh} = \frac{P}{P_{sh}} < 1$$

$$\eta_{power} = \frac{TV_0}{P_{shaft}} = \eta_{sh}\frac{TV_0}{P} = \eta_{sh}\frac{TV_0}{TV_1}$$

$$V_1 = \frac{V_0 + V_e}{2} \Rightarrow \eta_p = \eta_{sh}\left[\frac{2V_0}{V_0 + V_e}\right]$$

Ideal case $\Rightarrow \eta_{sh} = 1 \Rightarrow \eta_{power} = \frac{2V_0}{V_0 + V_e}$

$$T = \frac{P}{V_0 + w} = \frac{\eta_{sh} P_{sh}}{V_0 + w}$$ engine output

$$V_0 \uparrow \Rightarrow T \downarrow$$

$$P = \underset{\substack{\uparrow \\ \text{useful} \\ \text{power}}}{TV_0} + \underset{\substack{\uparrow \\ \text{induced} \\ \text{power}}}{Tw}$$

$$\eta_p = \frac{\text{useful power}}{\text{total power}}$$

$$P_1 = Pa + \frac{1}{2}\rho Va^2 - \frac{1}{2}\rho V_1^2$$

$$= Pa + \frac{1}{2}\rho\left[Va^2 - (Va+w)^2\right] = Pa - \frac{1}{2}\rho w Va\left[\frac{w}{Va} + 2\right]$$

$$= Pa - \rho w Va\left[\frac{w}{2Va} + 1\right]$$

$w = \varepsilon Va \quad \Big| \quad r = \frac{Va}{Ve} = \frac{Va}{Va+2w} = \frac{1}{1+2\varepsilon}$

$$\therefore P_1 = Pa - \rho Va^2 \varepsilon\left(1 + \frac{\varepsilon}{2}\right)$$

$$\therefore \frac{P_1}{Pa} = 1 - \frac{\rho Va^2}{Pa}\varepsilon\left(1 + \frac{\varepsilon}{2}\right)$$

$$= 1 - \gamma Ma^2 \varepsilon\left(1 + \frac{\varepsilon}{2}\right)$$

$$P_2 = Pa + \frac{1}{2}\rho Ve^2 - \frac{1}{2}\rho V_1^2$$

$$= Pa + \rho w Va\left[1 + \frac{3}{2}\varepsilon\right]$$

$$\therefore \frac{P_2}{Pa} = 1 + \gamma Ma^2 \varepsilon\left(1 + \frac{3}{2}\varepsilon\right)$$

$$\therefore \frac{P_2}{P_1} = \frac{1 + \gamma Ma^2 \varepsilon\left[1 + \frac{3}{2}\varepsilon\right]}{1 - \gamma Ma^2 \varepsilon\left[1 + \frac{\varepsilon}{2}\right]}$$

$$\varepsilon << 1$$

$$\therefore \frac{P_2}{P_1} \approx 1 + \gamma Ma^2 \varepsilon\left[1 + \frac{3}{2}\varepsilon\right] + \gamma Ma^2 \varepsilon\left(1 + \frac{\varepsilon}{2}\right)$$

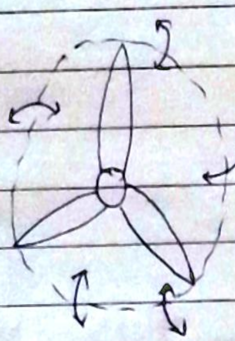
$$\therefore \frac{P_2}{P_1} \approx 1 + 2\gamma Ma^2 \varepsilon(1+\varepsilon) \leftarrow \text{pressure ratio}$$

The pressure ↑ from $P_a \rightarrow P_1$, then ↓ from $P_2 \rightarrow P_a$,

$$P_2 - P_1 = \frac{T}{A_d} \leftarrow \text{disk loading}$$

$$P_{0_a} = P_{0_1}, \quad P_{0_e} = P_{0_2}$$

Assump^n:- - net effect is pushing air back.
- neglect drag friction losses. (or swirl)



pushes air flow radially out $\Longrightarrow$ blade-tip vortex shedding

- neglect blade-tip shedding vortices.

Efficiency:-

$$P_{sh} \longrightarrow \text{propeller} \longrightarrow \text{fluid jet} \rightarrow P_{jet} = \frac{1}{2}\dot{m}(V_e^2 - V_a^2)$$

$$\underbrace{P_{sh} \rightarrow P_{jet}}_{\eta_{prop}}, \qquad P_{jet} \xrightarrow{\eta_p} TV_a$$

$$\eta_p = \frac{TV_a}{P_{jet}} = \frac{2V_a}{V_a + V_e}$$

$$\eta_{prop} = \frac{P_j}{P_{sh}}$$

$$\eta_T = \eta_{prop}\,\eta_p = \frac{TV_a}{P_{sh}}$$

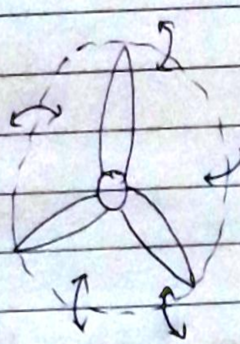
$$\therefore \frac{P_2}{P_1} \approx 1 + 2\gamma Ma^2 \varepsilon (1+\varepsilon) \leftarrow \text{pressure ratio}$$

The pressure ↑ from $P_a \rightarrow P_1$, then from $P_2 \rightarrow P_a$,

$$P_2 - P_1 = \frac{T}{A_d} \leftarrow \text{disk loading}$$

$$P_{0a} = P_{01}, \quad P_{0e} = P_{02}$$

Assump^n :- - net effect is pushing air back.
- neglect drag friction losses. (or swirl)



pushes air flow radially out $\Longrightarrow$ blade-tip vortex shedding

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Efficiency :-

$$P_{sh} \rightarrow \text{propeller} \rightarrow \text{fluid jet} \rightarrow P_{jet} = \frac{1}{2}\dot{m}(V_e^2 - V_a^2)$$

$P_{sh} \xrightarrow{\eta_{prop}} P_{jet} \xrightarrow{\eta_p} TV_a$

$$\eta_p = \frac{TV_a}{P_{jet}} = \frac{2V_a}{V_a + V_e}$$

$$\eta_{prop} = \frac{P_j}{P_{sh}}$$

$$\eta_T = \eta_{prop}\,\eta_p = \frac{TV_a}{P_{sh}}$$

$$T = P_{sh}\left(A_d \frac{\rho}{\rho_{sl}}\right) \frac{\eta_{propeller}}{V_0}$$

$$P_{jet} = \frac{T(V_e + V_a)}{2} = T(V_a + w) = \underset{\substack{\uparrow \\ \text{vehicle power}}}{T V_a} + \underset{\substack{\uparrow \\ \text{Thrust induced power}}}{T w}$$

$$\eta_p = \frac{T V_a}{P_{jet}} = \frac{2 V_a}{V_e + V_a} = \frac{1}{1 + w/V_a}$$

$$P_{jet} \Rightarrow T \alpha w$$

$$\frac{T}{4 q A_d} = \frac{w}{V_a}\left(1 + \frac{w}{V_a}\right)$$

$$\therefore \left(\frac{w}{V_a}\right)^2 + \left(\frac{w}{V_a}\right) - \frac{T}{4 q A_d} = 0$$

$$\therefore \frac{w}{V_a} = \frac{1}{2}\left[-1 \pm \sqrt{1 + \frac{T}{q A_d}}\right]$$

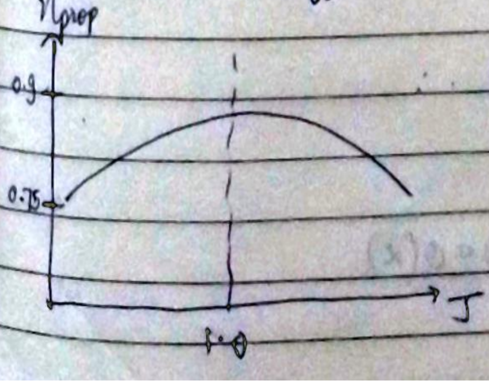
$$w = \frac{V_a}{2}\left[\sqrt{1 + \frac{T}{q A_d}} - 1\right]$$

T/qA_d must be larger (at least not small due to larger wt & touching to grnd)

Advance ratio $\rightarrow$ $J = \dfrac{V_a \leftarrow \text{vehicle speed}}{n d \leftarrow \text{diameter}} = \dfrac{\pi V_a}{\omega r}$

n ↑ rpm $\qquad$ ωr ↑ Tip speed

Power co-eff., $c_p = \dfrac{P_{sh}}{\rho n^3 d^5}$

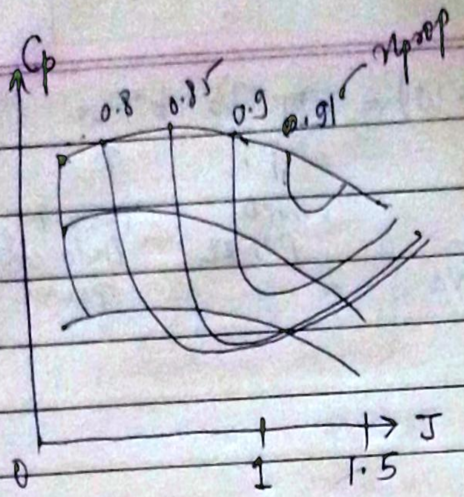


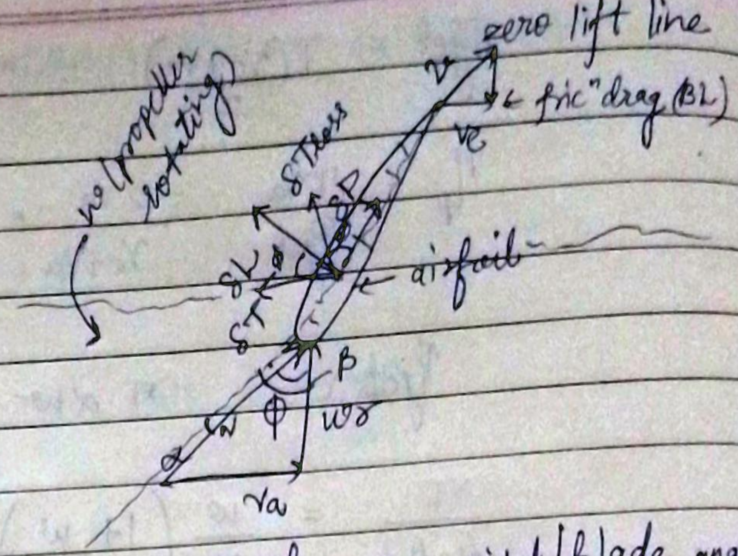
$$\eta_{prop} = \frac{\pi}{2} \frac{J^3}{C_p} \frac{\eta_p^{\,3}}{(1 - \eta_p)}$$

McCauley propellers, rpm ~ 2700

$n = 45$

$$\tan\phi = \frac{V_a}{\omega r}$$





$\tan\phi = \frac{V_a}{\omega r}$, α - angle of attack , β - pitch/blade angle

$$\alpha = \beta - \phi$$

$$\delta L = \frac{1}{2}\rho V^2 \delta S C_L \equiv q C_L \delta S$$

$$\delta D = q C_D \delta S$$

$$\delta T = \delta L\cos\phi - \delta D\sin\phi$$

$$\delta T_{loss} = \delta L\sin\phi + \delta D\cos\phi$$

Torque to drive propeller $= \int \delta T_{loss}\, \omega r$

$$= \int (C_L\sin\phi - C_D\cos\phi)\, q \cancel{\cos}\, c\,\omega r\, dr$$

$$\eta_{prop})_r = \frac{\delta T\, V_a}{\delta T_{loss}\,\omega r} = \frac{q\, ds\, [C_L\cos\phi - C_D\sin\phi]\, V_a}{q\, ds\, [C_L\sin\phi + C_D\cos\phi]\,\omega r}$$

$$\eta_{prop} \approx \left[\frac{\frac{C_L}{C_D} - \tan\phi}{1 + \frac{C_L}{C_D}\tan\phi}\right]\frac{V_a}{\omega r}$$

$$\frac{C_L}{C_D} \propto f(\alpha)$$

Let $\frac{C_L}{C_D} = \tan\theta \Rightarrow \theta = \theta(\alpha)$

Propeller Analysis - Blade Element Theory

$$\therefore \eta_{prop} = \tan\phi\,(\tan(\theta-\phi))$$

$$\eta_{prop|opt} \rightarrow \frac{d\eta_{prop}}{d\phi} = 0 \Rightarrow \phi_{opt} = \frac{\theta}{2}$$

$$\left(\frac{V_a}{\omega r}\right)_{opt} = -\frac{C_D}{C_L} + \sqrt{\left(\frac{C_D}{C_L}\right)^2 + 1}$$

$$\eta_{prop|opt} = \tan^2\left(\frac{\theta}{2}\right) = \left(\frac{V_a}{\omega r}\right)^2_{opt}.$$

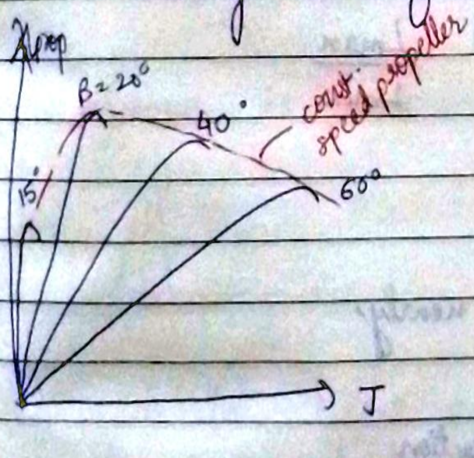
$$J = \frac{\pi^2}{\pi}\left(\frac{V_a}{\omega r_{max}}\right)$$

$$\therefore J_{opt} = \pi\left[1 - \frac{C_D}{C_L} + \frac{1}{2}\left(\frac{C_D}{C_L}\right)^2\right]$$

For opt. performance of propeller, η_{prop} is high. can be achieved,

$\Rightarrow$ change $\beta \rightarrow \beta_{hub} + \beta_{tip}$

change ~~st~~ airfoil shape $\rightarrow$ hubs & tips have different airfoil shapes.



$$V_{tip} = \sqrt{V_a^2 + \left(\frac{\omega d}{2}\right)^2}$$

$$M_t = \sqrt{M_a^2 + M_{\omega}^2}$$

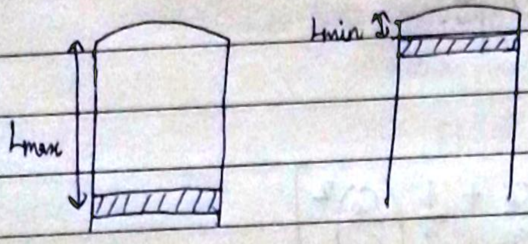
$$M_a = \frac{V_a}{a}, \quad M_{\omega} = \frac{\omega d}{2a}$$

Piston Engines :-

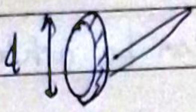
Piston movement $\rightarrow$ rotates a shaft $\rightarrow$ rotates the propeller

IC / Reciprocating engine :-

Combustion $\rightarrow$ Piston $\rightarrow$ Shaft $\xrightarrow{\eta_{sh}}$ Propeller $\xrightarrow{\eta_{prop}}$ Thrust $\rightarrow$ vehicle power



$l = L_{max} - L_{min}$ $\leftarrow$ piston movement distance



$A_d = \frac{\pi d^2}{4}$

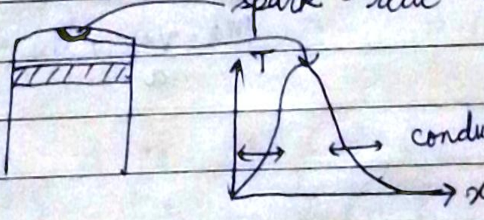
$\underset{(P_0)}{\forall_0} = \frac{\pi d^2}{4} L_{max} \longrightarrow \underset{(P_c)}{\forall_c} = \frac{\pi d^2}{4} L_{min}$ (volume)

Compression ratio - $r = \frac{\forall_o}{\forall_c} \approx \frac{L_{max}}{L_{min}}$

$\frac{P_c}{P_0} = f(r)$

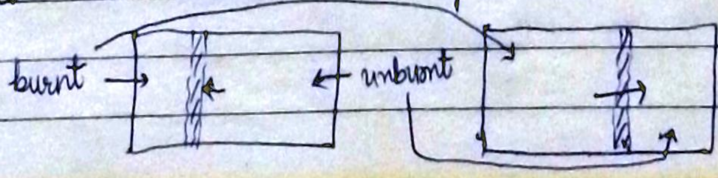
$r \sim 14$

For $r > 14$, spark - reac^n nearly



If conduction dominates $\Rightarrow$ surrounding gas heats up & starts reacting

fuel - air mixture $\Rightarrow$ premixed comb^n.



flame speed - S_f

combus^n - fuel-air mixture should get heated up to overcome the activation energy.

Reac^n spreads everywhere quickly.

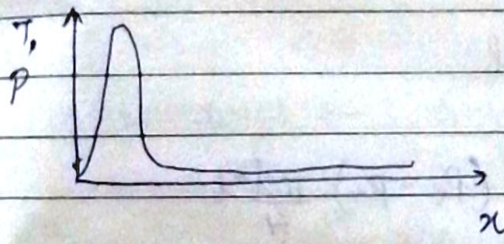
If τ_f - Time for reac^n to spread all over the box
τ_r - time for the reac^n to complete $\propto \frac{1}{w_r}$

$w_r = f(P)$

high P (moderately) ← τ_r is small, but large enough to spread flame everywhere

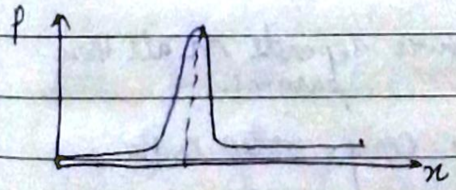
low P ← reac^n is too slow for proper combustion or to generate sufficient energy to heat up the remaining gas.

very high P - reac^n is too fast before surrounding gas heats upto react, this region has reacted.



very high P diff. → shock wave
As the shock passes, ~~gat~~ gas heats up & reacts.

This wave is known as Detonation wave.
↕
knocking



Damages material

Measured in terms of octane no. Higher octane rating can be compressed to higher pressures.

Aviation Gases - AvGas > 100

Why compress?
We can Intake ⇒ burn ⇒ T ↑ P ↑ ⇒ drive shaft
But why don't we do this & compress?

Reason - $p\uparrow \rightarrow$ reac^n rates $\uparrow$

to push the piston $\rightarrow p_r > p_{amb}$.

$p_r \rightarrow \rho \Rightarrow$ How much air can be packed into cylinder
↓ fixes

How much fuel can be burned.
↓

1 kg fuel $\leftrightarrow$ 14.7 kg of air   Energy add^n to gas
↓

vehicle $\leftarrow$ propeller $\leftarrow$ Power to shaft

Let P_e - post comb^n pressure, $A_d = \frac{\pi d^2}{4}$

$F = (P_e - P_\infty)\frac{\pi d^2}{4}$, disp. $\rightarrow l$

$\therefore$ Work $= (P_e - P_\infty)\frac{\pi d^2}{4} l$

1 cycle - 2 revolutions of shaft - $(P_e - P_\infty)\frac{\pi d^2}{4} l$

N (rpm) $\sim$ 2500 - 6000

For N/2 cycles

Power $= \frac{N}{2}(P_e - P_\infty) A_d l$ ← Power depends on all these params.

$(P_e - P_\infty)$ depends on heat release $\rightarrow$ comp. ratio & P_∞

Mainly power $\propto$ air intake $(P_\infty, \rho_\infty, T_\infty)$
rpm, cylinder size, 'l'

Gagg & Ferrar :-

$\overset{\text{power}}{P} = P_{sl}[1.132\sigma - 0.132]$

$\sigma = \rho/\rho_{sl}$

Simplest model $\rightarrow$ atmp. $\rightarrow$ $\overset{\text{power}}{P} = P_{sl}\,\sigma$

Power → rpm — higher rpm ⇒ higher power
→ [S/P in the cylinder] [super charger / turbo charger]
→ volume $\left[\frac{\pi d^2}{4} l\right]$

Quantative analysis - Thermodynamics:-

System ← identifiable / well-defined collection of particles.
← does not change unless disturbed from outside (eqbm)

what if the system is disturbed? → stays the same (stable eqbm)
→ changes (unstable eqbm)

surroundings ← everything else (except the system)

Interactions betn the system & surroundings happen through the boundary.
- boundary can move
- system cannot gain/lose mass.

State - internal configuration of a system that's at eqbm.

Properties - measurable params.
↳ extensive - depends on size
↳ intensive - independent of size

Quasi - steady process - changes are gradual
- constraint changes a little, system adjusts instantaneously.

changes are slower than the relaxation processes

– gradual change in constraints, such that all the intermediate states are well-defined

Thermd. can only tell us about end states. Since most real processes are transients.

I Law – For any process (involving no net effects external to the system except disp. of a mass in a gravitational field betn specified levels), the magnitude of the mass is fixed by the end states of the system & it is independent of the details of the process. *

II law – Among all allowed states of a system with given values of energy (E), no. of particles (N) & constraints, one & only one is a stable eqbm state. Such a state can be reached from any of the said allowable states without any net effects on the envmt.

Postulate II – ∃ a func^n (called entropy S) of the extensive parameters of any composite system, defined for all eqbm states & having the following property: "values assumed by the extensive params in the absence of an internal constraint are those that maximize the entropy over the manifold of constrained eqbm states."

Postulate III – The entropy of a composite system is additive over the constitute subsystems. The entropy is continuous & differentiable & is a monotonically increasing func^n of energy.

$$Tds = du + pdv$$
$$Tds = dh - vdP$$

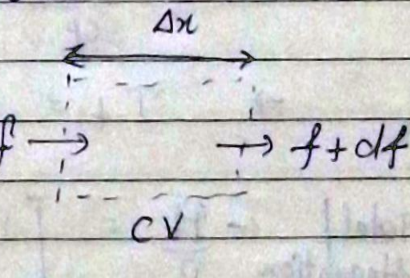
$$ds = \frac{1}{T}de + \frac{P}{T}dv - \sum_i \frac{\mu_i}{T} dN_i$$

<u>Basic eq^n</u>:-
$$u_2 - u_1 = C_v(T_2 - T_1)$$
$$h_2 - h_1 = C_p(T_2 - T_1)$$
$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)$$
$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$
$$C_p - C_v = R$$

* <u>Slow down gas flow</u>

$$\frac{\partial}{\partial t}\left[\rho\left(e + \frac{1}{2}u^2\right)\right] + \frac{\partial}{\partial x}\left[\rho u\left[h + \frac{1}{2}u^2\right]\right] = \frac{\partial q_{cond}}{\partial x} + \dot{q}''' - w_{sh}'''$$

$$u = 0 \Rightarrow \frac{\partial}{\partial t}(\rho e) = \frac{\partial q_c}{\partial x} + \dot{q}''' - w_{sh}'''$$
$$de = \delta q - \delta w_{sh} \equiv \delta q - \delta w$$



$u \neq 0$ - steady

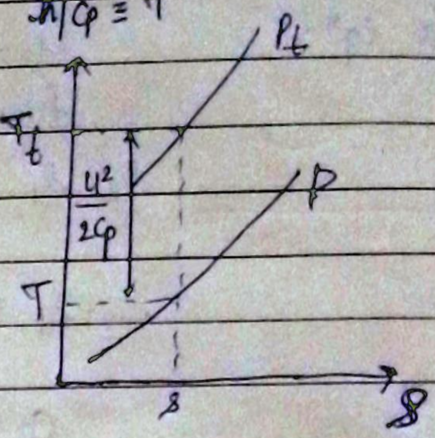
$$d\left[\rho u\left(h + \frac{1}{2}u^2\right)\right] = \delta\dot{q} - \delta w_{sh}$$

$$h_t = h + \frac{1}{2}u^2$$ - [enthalpy when flow stops - total / stagnation enthalpy]

$$d[\rho u h_t] = \delta\dot{q} - \delta w_{sh}$$
$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0 \rightarrow \text{steady} \Rightarrow d(\rho u) = 0$$
$$f = \frac{\dot{m}}{A} = \rho u = \text{const.}$$

$$C_p dT_t = dh_t = \frac{\delta q}{\dot{m}/A} - \frac{\delta w_{sh}}{\dot{m}/A} \equiv T ds - \delta w_{sh}$$

$\uparrow$ δq $\qquad$ $\uparrow$ δw_{sh}



$$T_t - T = \frac{2u^2}{c_p}$$

$$P_t = P + \frac{\rho u^2}{2} \, ? \leftarrow \text{(NO)}$$

$$dh = T ds + v dp$$

$$c_p dT = T ds + \frac{1}{\rho} dp \qquad (de = T ds - p dv)$$

$$c_p dT = T ds + RT \frac{dp}{p}$$

$$\frac{\gamma}{\gamma-1} \frac{dT}{T} = \frac{ds}{R} + \frac{dp}{p} \rightarrow \frac{P_f}{P_i} = \left(\frac{T_f}{T_i}\right)^{\frac{\gamma}{\gamma-1}} \exp\left(\frac{-\Delta s}{R}\right)$$

Isentropic $\rightarrow \Delta s = 0 \Rightarrow \frac{P_t}{P} = \left(\frac{T_t}{T}\right)^{\gamma/\gamma-1}$

$$T_t = \frac{h_t}{c_p} = \frac{h + \frac{1}{2}u^2}{c_p} = \frac{h}{c_p} + \frac{\gamma-1}{2\gamma RT} u^2 T$$

$$\Rightarrow T_t = T\left[1 + \frac{\gamma-1}{2} M^2\right]$$

Total/stagnation pressure $\leftarrow \frac{P_t}{P} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}$

$dh_t = T ds - \delta w_{sh} \leftarrow$ flow eqⁿ

$dh = T ds + v dp \leftarrow$ not valid when there's flow

$dh_t = T_t ds + v_t dp_t$

$$T ds - \delta w_{sh} = T_t ds + \frac{1}{\rho_t} dp_t$$

W.D by the flow $\Rightarrow$ $\dot{s}_{\text{sh}} > 0$ $\Rightarrow$ $dp_t < 0$

$$= T_t\, ds + RT_t \frac{dp_t}{p_t}$$

$$\boxed{\frac{dp_t}{p_t} = -\left[1 - \frac{T}{T_t}\right]\frac{ds}{R} - \frac{\dot{s}_{\text{sh}}}{RT_t}}$$

W.D. on the flow $\Rightarrow$ $\dot{s}_{\text{sh}} < 0$ $\Rightarrow$ $dp_t > 0$

heat add^n $\Rightarrow ds > 0 \Rightarrow dp_t < 0 \Rightarrow u_e \downarrow$

$$T_t = T + \frac{u^2}{2c_p}, \quad 1 - \frac{T_t}{T} = \frac{u^2/2c_p}{T + u^2/c_p} = \frac{u^2}{2c_pT_t} > 0$$

$$\left(1 - \frac{T_t}{T}\right) \propto \frac{u^2}{2c_pT_t} = \frac{(\gamma-1)\, M^2/2}{1 + \frac{\gamma-1}{2}M^2} \propto M^2$$

$M\downarrow \Rightarrow$ low p_t loss $(M_b \sim 0.3)$ $(M_a \sim 0.85)$

Flow shd be slowed down before burner $\xleftarrow{\text{reason 1}}$ combustion

reason 2 p_t

How to slow down the flow,

$$\dot{m} = \rho A u = \frac{p}{RT} AM\sqrt{\gamma RT} = \frac{p}{\sqrt{T}}\sqrt{\frac{\gamma}{R}}\, MA$$

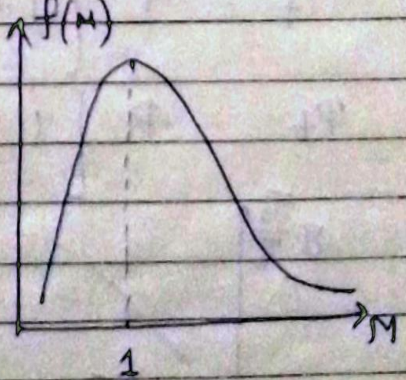
$$= \frac{P_0}{\sqrt{T_0}}\left[1 + \frac{\gamma-1}{2}M^2\right]^{-\frac{\gamma+1}{2(\gamma-1)}} M\sqrt{\frac{\gamma}{R}}\, A$$

$f(M, \gamma, R)$

energy

$$\dot{m} = A\frac{P_0}{\sqrt{T_0}} f(M, \gamma, R)$$

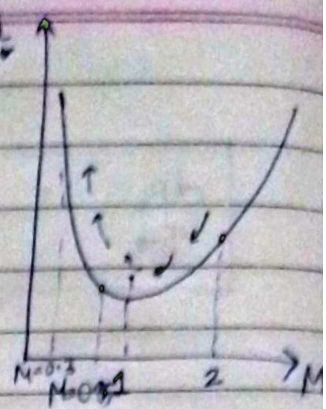
nozzle/diffuser prpty. gas prpty.

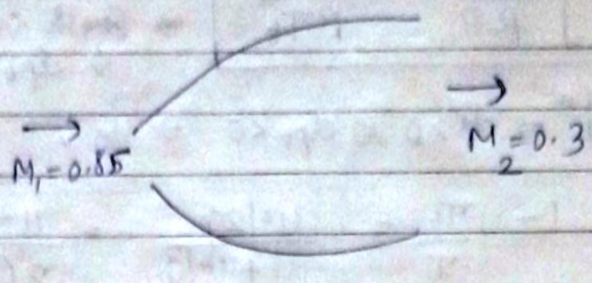


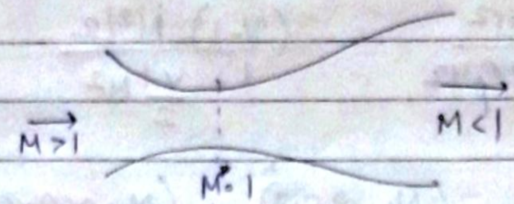
$$f_* = f(1, \gamma, R) = \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}\sqrt{\frac{\gamma}{R}}$$

$$\rightarrow \dot{m} = A_* \frac{P_t}{\sqrt{T_t}} f_*$$

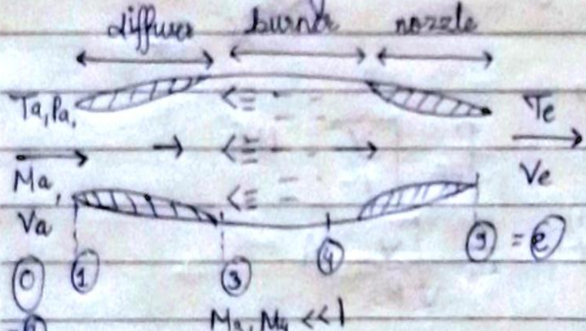
$$A = \frac{\dot{m}}{f(M)} \frac{\sqrt{T_t}}{P_t} \quad , \quad \frac{A}{A^*} = \frac{f_*}{f(M)}$$







RAMJET Engines :-



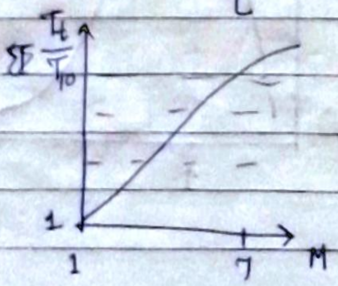
Stagnation states while analyzing the burner

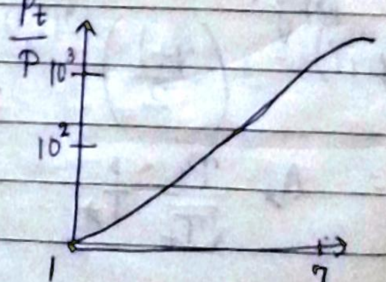
① - ③ → static - stagnation

T_0, P_0 $\begin{pmatrix} T_a, P_a \\ M_a \end{pmatrix}$ M_0 $\quad T_{t0}, P_{t0} = T_{t3}, P_{t3}$

$$T_t = T\left[1 + \frac{\gamma-1}{2} M^2\right] = T\left[1 + 0.2M^2\right]$$

$$P_t = P\left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}} = P\left[1 + 0.2M^2\right]^{3.5}$$

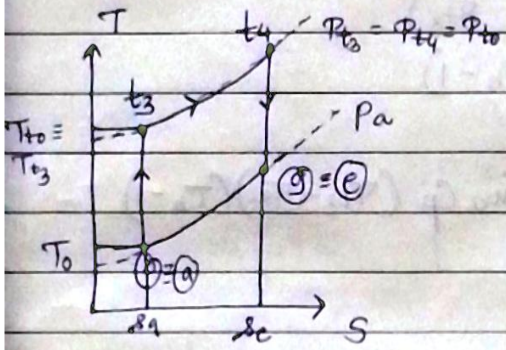




0 → 3 — Isentropic compression
0 → t_3 $(T_0, P_0) \longrightarrow (T_{t3}, P_{t3})$

3 - 4 — Isobaric combustion
$t_3 - t_4$ $(T_{t3}, P_{t3}) \longrightarrow (T_{t4}, P_{t4})$

4 - 9 — Isentropic expansion
$(T_{t4}, P_{t4}) \longrightarrow (T_9, P_9) \equiv (T_e, P_a)$



$$\frac{P_{t3}}{P_a} = \left[1 + \frac{\gamma-1}{2} M_a^2\right]^{\gamma/\gamma-1}$$

$$\frac{P_{t4}}{P_e} = \left[1 + \frac{\gamma-1}{2} M_e^2\right]^{\gamma/\gamma-1}$$

$$M_e = M_a$$

$$\frac{P_{t3}}{P_0} = \frac{P_{t4}}{P_e} \Rightarrow \underbrace{\frac{T_{t3}}{T_0}}^{\text{determ. by } M_a} = \frac{T_{t4}}{T_e}$$

$\underbrace{T_0 \simeq 220\,K}$ (fixed by altitude) | T_{t4} ← $\underbrace{\text{max. / peak temperature}}$ limited by material safety (fixed by tech.)

$$T_e = \frac{T_{t4}}{T_{t3}} \cdot T_a \quad \Big| \quad \tau_b = \frac{T_{t4}}{T_{t3}} \Rightarrow T_e = \tau_b T_a$$

Burner:- $\dot{m}_f Q_R = \dot{m} C_p (T_{t4} - T_{t3})$

- assume $f = \frac{\dot{m}_f}{\dot{m}_a} \ll 1$ & $c_p(\gamma, R)$ are not changing due to combustion.

$$\boxed{\frac{f Q_R}{C_p T_{t3}} = \tau_b - 1}$$

$$\tau_b = 1 + \frac{f Q_R}{c_p T_3} \leftarrow \text{flow enthalpy}$$

Surplus
Thermal energy of jet -

$$q_{thj} = \dot{m}_a C_p (T_e - T_a)$$
$$q_{loss} = \dot{m}_a C_p T_a (\tau_b - 1)$$

Combustion heat release

$$q_b = \dot{m}_a C_p (T_{t4} - T_{t3})$$
$$= \dot{m}_a C_p T_{t3} (\tau_b - 1)$$

WD $\leftrightarrow$ Energy that was utilized.

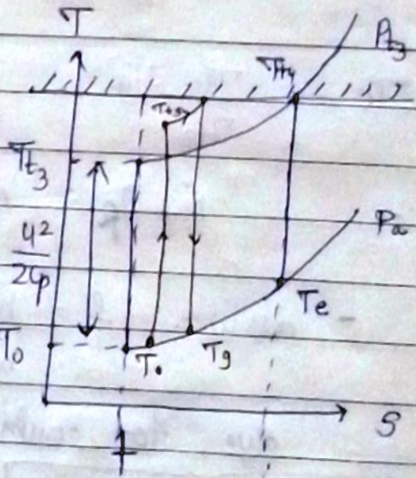
$$w = q_b - q_{loss} = \dot{m}_a C_p (T_{t3} - T_a)(\tau_b - 1)$$

Efficiency $\rightarrow$ $\boxed{\eta = \frac{w}{q_b} = 1 - \frac{T_a}{T_{t3}}}$

$$\eta = 1 - \frac{T_a}{T_{t3}} = 1 - \left(\frac{P_a}{P_{t3}}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{1}{1 + \frac{\gamma-1}{2} Ma^2}$$
$$= \frac{(\gamma-1) Ma^2/2}{1 + \frac{\gamma-1}{2} Ma^2}$$

$$\eta \uparrow \leftarrow Ma \uparrow$$

$$Ma \uparrow \Rightarrow \frac{u_a^2}{2c_p} = \frac{(\gamma-1) u_a^2}{2R\gamma} = \frac{\gamma-1}{2} Ma^2 T_a$$



$$q_b = \dot{m} C_p T_{t3} (\tau_b - 1)$$

If Ma is too large, not much heat add^n is possible $\rightarrow$ τ_b is small.

η is high but not much work output.

How high can M_a be?

When is work highest?

$$w = q_b - q_{loss} = \dot m_a c_p T_{t_3}(\tau_b - 1)(T_{t_3} - T_a)$$

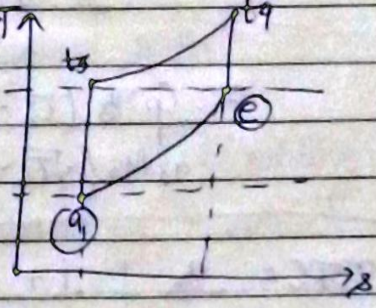
$$\frac{w}{\dot m c_p T_a} = \frac{T_{t_3}}{T_a}(\tau_b - 1) - (\tau_b - 1) \equiv \frac{T_{t_4}}{T_a} - \frac{T_{t_3}}{T_a} - \frac{T_e}{T_a} + 1$$

$$\frac{P_{t_4}}{P_e} = \frac{P_{t_3}}{P_a} \Rightarrow \frac{T_{t_4}}{T_e} = \frac{T_{t_3}}{T_a} \Rightarrow$$

$$\frac{w}{\dot m c_p T_a} = \left[\frac{T_{t_4}}{T_a} + 1\right] - \frac{T_{t_3}}{T_a} - \frac{T_{t_4}}{T_{t_3}} \equiv f(T_{t_3})$$

$$\frac{d}{dT_{t_3}}\left[\frac{w}{\dot m c_p T_a}\right] = 0 \Rightarrow -\frac{1}{T_a} + \frac{T_{t_4}}{T_{t_3}^2} = 0$$

$$\Rightarrow \boxed{T_{t_3} = \sqrt{T_a T_{t_4}}} \longrightarrow \boxed{T_e = T_{t_3}}$$



$$\eta = 1 - \frac{T_a}{T_{t_3}}$$

$$\eta_{opt} = 1 - \frac{T_a}{\sqrt{T_a T_{t_4}}} = 1 - \sqrt{\frac{T_a}{T_{t_4}}}$$

To improve efficiency $\Rightarrow$ increase T_{t_4}.

Optimum M_a?

$$\frac{T_{t_3}}{T_a} = 1 + \frac{\gamma - 1}{2} M_a^2 = \sqrt{\frac{T_{t_4}}{T_a}}$$

$$\Rightarrow \boxed{M_a = \sqrt{\frac{2}{\gamma - 1}\left[\sqrt{\frac{T_{t_4}}{T_a}} - 1\right]}}$$

Thrust analysis -

$$T = \dot{m}_a (V_e - V_a) = \dot{m}_a V_a \left[\frac{V_e}{V_a} - 1 \right]$$

If $M_e = M_a$,

$$T = \dot{m}_a M_a a_a \left[\sqrt{\frac{T_e}{T_a}} - 1 \right]$$

normalised thrust $\rightarrow$ $\frac{T}{\dot{m}_a a_0} = M_0 \left[\sqrt{\tau_b} - 1 \right]$

$\frac{T}{\dot{m}_a} \leftarrow$ Thrust per unit mass $\rightarrow$ m/s

↑ Normalised with ambient speed of sound.

$$\tau_b = \frac{T_{t4}}{T_{t3}} = \frac{T_{t4}}{T_0} \left[1 + \frac{\gamma - 1}{2} M_0^2 \right]^{-1}$$

$$TSFC = \frac{\dot{m}_f}{T} = \frac{f \dot{m}_a}{\dot{m}_a (V_e - V_a)} = \frac{f}{V_e - V_a}$$

$$= \frac{f}{V_a (\sqrt{\tau_b} - 1)}$$

$$= \frac{C_p T_{t3} (\tau_b - 1)}{a_0 M_0 (\sqrt{\tau_b} - 1)}$$

$$f Q_R = C_p T_{t3} (\tau_b - 1)$$

$$C_p T_3 = \frac{\gamma R T_0}{\gamma - 1} \left(1 + \frac{\gamma - 1}{2} M_0^2 \right)$$

$$= \frac{a_0^2}{\gamma - 1} \left[1 + \frac{\gamma - 1}{2} M_0^2 \right]$$

$$\boxed{TSFC = \frac{a_0}{\gamma - 1} \frac{1}{Q_R} \frac{1 + \frac{\gamma - 1}{2} M_0^2}{M_0} (\sqrt{\tau_b} + 1)}$$

TSFC increases with T_{t4}.

Super charger in piston engine is used to increase 'p'.

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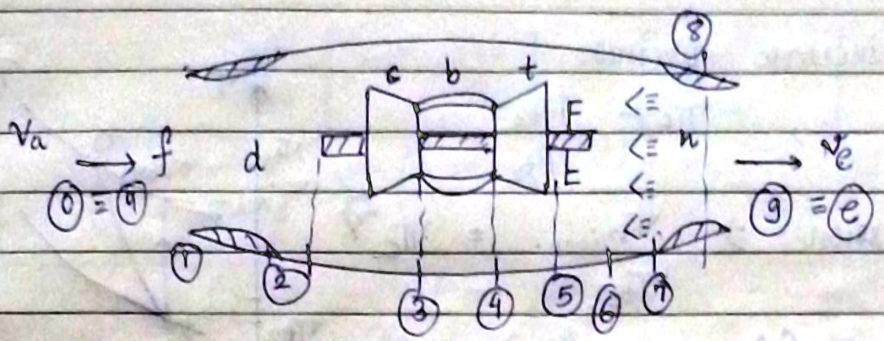
Thrust when $v=0 \Rightarrow P_{t3} = P_{t0}$:-

Use compressor to increase P_{t3} ($P_{t3} > P_{t0}$).

Turbine extracts energy after the burner to drive the compressor

$$\frac{dp_t}{P_t} = -\left[1 - \frac{T}{T_t}\right]\frac{ds}{R} - \frac{\delta w_{sh}}{RT_t}$$



Gasturbine → Turbojet engine · with afterburner

Gas Turbine Cycle :-

RAMJET engine → $M \sim 2-4$

Compression was mostly from flow K.E.

In above diagram, 2 − 3 → compressor − adding work
4 − 5 → Turbine − extract work

Stagnation states → $t2 \rightarrow t3$ in compressor
$t4 \rightarrow t5$ in turbine

$$\dot{w}_c = \dot{m}_a C_p (T_{t3} - T_{t2}) \equiv \dot{m}_a C_p T_{t2}\left[\frac{T_{t3}}{T_{t2}} - 1\right]$$

const.

$$\dot{w}_t = \dot{m}_a C_p (T_{t4} - T_{t5}) \equiv \dot{m}_a C_p T_{t4}\left[1 - \frac{T_{t5}}{T_{t4}}\right]$$

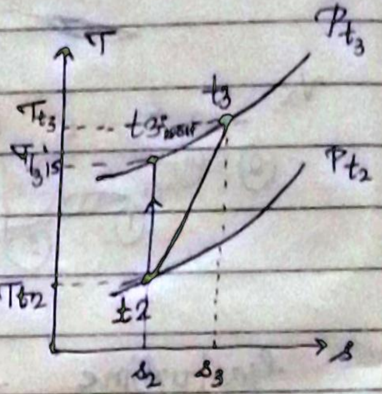
$$\dot{W}_{net} = \dot{W}_t - \dot{W}_c = \dot{m}_a C_p T_{t_2} \left\{ \frac{T_{t4}}{T_{t_2}}\left[1 - \frac{T_{t5}}{T_{t4}}\right] - \left[\frac{T_{t3}}{T_{t2}} - 1\right] \right\}$$

$$T_{t_2} = T_2\left[1 + \frac{\gamma-1}{2} M_2^2\right] \approx T_{t0} = T_0\left[1 + \frac{\gamma-1}{2} M_0^2\right]$$

As altitude increases in the flight, T_{t_2} reduces, inspite of M_a increase.

Compressor $\rightarrow$ increase pressure

$P_{t_2} \rightarrow P_{t_3}$, $\pi_c = \frac{P_{t3}}{P_{t2}}$



Isentropic compression $\Rightarrow$ $\frac{T_{t3 isen.}}{T_{t2}} = \pi_c^{\frac{\gamma-1}{\gamma}}$

$$h_{t_3} - h_{t_2} = (h_{t_3 isen.} - h_{t_2}) + \Delta h_{diss.}$$

$$h_{t_3} = h_{t3 isen.} + \Delta h_{diss.}$$

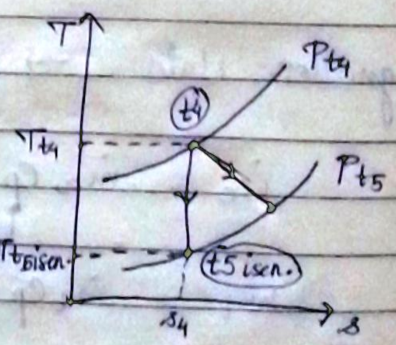
Compressor Efficiency :- $\eta_c = \frac{\text{ideal work input}}{\text{actual work input}} = \frac{h_{t3 isen.} - h_{t2}}{h_{t_3} - h_{t_2}}$

Expts. $\rightarrow$ $\eta_c = \frac{T_{t_3 isen.} - T_{t2}}{T_{t_3} - T_{t_2}} = \frac{T_{t3 isen.}/T_{t2} - 1}{T_{t3}/T_{t2} - 1}$ Isentropic relations

$\frac{T_{t3 isen.}}{T_{t_2}} = \pi_c^{\gamma-1/\gamma}$ $\Rightarrow$ $\boxed{\frac{T_{t3}}{T_{t2}} = 1 + \frac{1}{\eta_c}\left[\pi_c^{\gamma-1/\gamma} - 1\right]}$

As π_c increases, η_c decreases.

Turbine - $\pi_t = \frac{P_{t4}}{P_{t5}}$



$$\frac{T_{t5 isen.}}{T_{t4}} = \frac{1}{\pi_t^{\frac{\gamma-1}{\gamma}}}$$

$$h_{t5} - h_{t4} = (h_{t5 isen.} - h_{t4}) + \Delta h_{diss.}$$

$$h_{t5} = h_{t4} - (h_{t4} - h_{t5\,isen.}) + \Delta h_{diss.}$$

Efficiency - $\eta_t = \dfrac{\text{actual work extracted}}{\text{ideal work extracted}} = \dfrac{h_{t4} - h_{t5}}{h_{t4} - h_{t5\,isen.}}$

$$\eta_t = \frac{T_{t4} - T_{t5}}{T_{t4} - T_{t5\,isen.}} = \frac{1 - T_{t5}/T_{t4}}{1 - \pi_t^{-\left(\frac{\gamma-1}{\gamma}\right)}}$$

$$\frac{T_{t5}}{T_{t4}} = 1 - \eta_t \left[1 - \pi_t^{-\left(\frac{\gamma-1}{\gamma}\right)}\right]$$

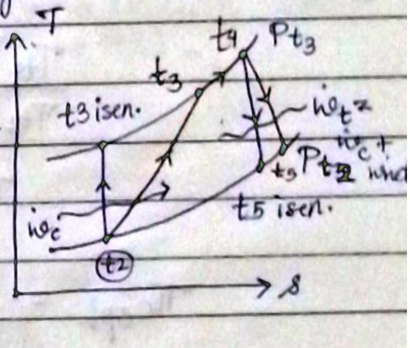
$$\frac{\dot{w}_{net}}{\dot{m}_a c_p T_{t2}} = \frac{T_{t4}}{T_{t2}}\left[1 - \frac{T_{t5}}{T_{t4}}\right] - \left[\frac{T_{t3}}{T_{t2}} - 1\right]$$

$$\boxed{\frac{\dot{w}_o}{\dot{m}_a c_p T_{t2}} = \frac{T_{t4}}{T_{t2}} \cdot \eta_t \left[1 - \pi_t^{-\left(\frac{\gamma-1}{\gamma}\right)}\right] - \frac{1}{\eta_c}\left[\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

What is the gasturbine usage?

- → shaft work
- → jet
- → combination (fan + jet) → $P_{t5} > P_{t2}$

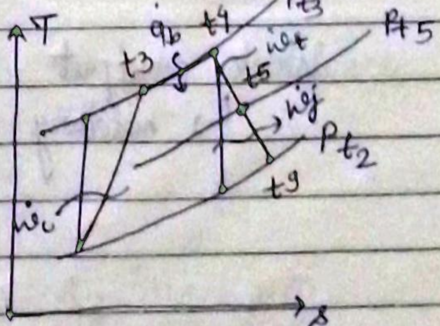
Shaft power only.



$$\dot{q}_b = \dot{m}_a c_p (T_{t4} - T_{t3}) = \dot{m}_f q_R = \dot{m}_a q_R f$$

$$f = \frac{c_p (T_{t4} - T_{t3})}{q_R}.$$

Including jet



$$\dot{w}_j = \frac{v_{jet}^2}{2}$$

Fan Turbine $\dot{w}_t = \underset{\text{compressor}}{\dot{w}_c} + \underset{\text{fan}}{\dot{w}_f}$

Core engine → compressor - burner - turbine

$$\Omega = \frac{\dot{W}_{net}}{\dot{m}_a C_p T_{t2}} = \frac{T_{t4}}{T_{t2}} \eta_t \left[1 - \pi_t^{-\frac{\gamma-1}{\gamma}}\right] - \frac{1}{\eta_c}\left[\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\Omega = \Omega\left(\frac{T_{t4}}{T_{t2}}, \eta_c, \eta_t, \pi_c, \pi_t, \gamma\right)$$

$\Omega = \Omega(\pi_c, \pi_t)$ if others are fixed

Simple Gas-Turbine $\rightarrow$ shaft power only

$$\Rightarrow \pi_t = \pi_c$$

$$\Omega = \Omega(\pi_c) \qquad g = \frac{\gamma-1}{\gamma}$$

$$\Omega = \frac{T_{t4}}{T_{t2}} \eta_t (1 - \pi_c^{-g}) - \frac{1}{\eta_c}(\pi_c^{g} - 1)$$

When is work output maximized?

$$\frac{d\Omega}{d\pi_c} = 0 \qquad \frac{T_{t4}}{T_{t2}} \eta_t g \pi_c^{-g-1} - \frac{1}{\eta_c} g \pi_c^{g-1} = 0$$

$$\frac{T_{t4}}{T_{t2}} \eta_t g \pi_c^{-g-1} = \frac{1}{\eta_c} g \pi_c^{g-1}$$

$$\pi_c|_{opt.}^{2g} = \frac{T_{t4}}{T_{t2}} \eta_c \eta_t \rightarrow \pi_c|_{opt.} = \left[\frac{T_{t4}}{T_{t2}} \eta_c \eta_t\right]^{1/2g}$$

$$\pi_{c\,opt.} = \left[\frac{T_{t4}}{T_{t2}} \eta_c \eta_t\right]^{\frac{\gamma}{2(\gamma-1)}}$$

Work output ~~to~~ be maximised $\rightarrow$ $\pi_c \approx 13$.

$\hookrightarrow$ military engines $\rightarrow$ $\pi_c \sim 12-14$

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