

AE 236 : Compressible Fluid Mechanics

(Module IV : Variable Area Flow)

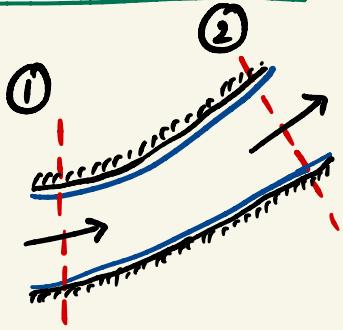
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Quasi - 1 D flow



$\frac{dA}{dx}$ is small

Assumptions

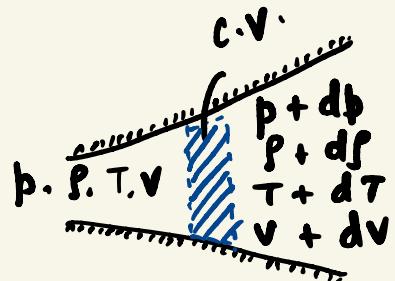
1. Flow is isentropic (unless otherwise mentioned)
2. Neglect effects of friction and heat transfer
3. Steady flow

Effect of dA on dV (or dM)

mass $pAV = \text{mass flow rate} = \text{constant} ;$

$$\frac{dp}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \quad \textcircled{1}$$

Energy $C_p T + \frac{V^2}{2} = \text{constant}$



$$C_p dT + V dV = 0 \quad \textcircled{2}$$

State $p = \rho RT$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T} \quad \textcircled{3}$$

Isentropic $p/p^r = \text{constant}$

$$\frac{dp}{p} = r \frac{ds}{s} \quad \textcircled{4}$$

Eqn. ② can be written as

$$\frac{dT}{T} + \frac{V^2}{C_p T} \frac{dV}{V} = 0 \quad - \textcircled{5}$$

Substitute ④ and ⑤ in ③

$$(r-1) \frac{df}{P} + \frac{V^2}{C_p T} \frac{dV}{V} = 0$$

$$\frac{V^2}{C_p T} = (r-1) M^2 \Rightarrow \frac{dT}{T} + (r-1) M^2 = 0 \quad - \textcircled{6}$$

$$\Rightarrow \cancel{(r-1)} \frac{df}{P} + \cancel{(r-1)} M^2 \frac{dV}{V} = 0$$

$$\frac{df}{P} = - M^2 \frac{dV}{V} \quad - \textcircled{7}$$

Substitute ⑦ in ①

$$-M^2 \frac{dV}{V} + \frac{dA}{A} + \frac{dV}{V} = 0$$

$$\boxed{\frac{dA}{A} = (M^2 - 1) \frac{dV}{V}}$$

or

$$\boxed{\frac{dA}{dV} = (M^2 - 1) \frac{A}{V}}$$

Cases

1. $M < 1$, dA and dV have opposite signs

$$dA > 0 \Leftrightarrow dV < 0$$

$$dA < 0 \Leftrightarrow dV > 0$$

2. $M > 1$, dA & dV have the same sign

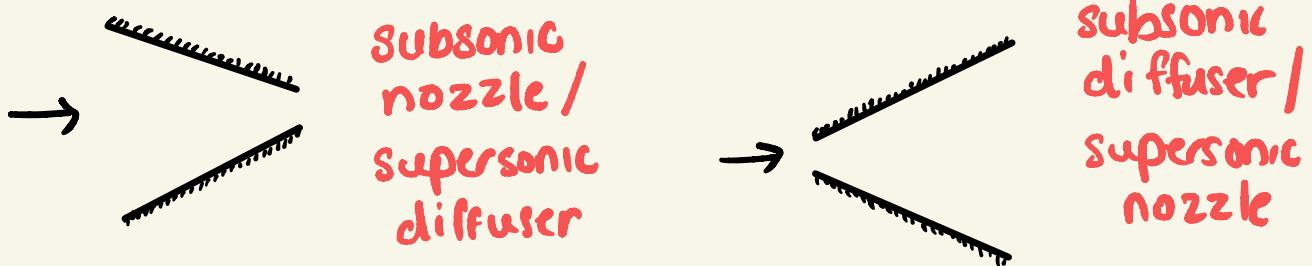
$$dA < 0 \Leftrightarrow dV < 0$$

$$dA > 0 \Leftrightarrow dV > 0$$

3. $M = 1$, sonic flow, $\frac{dA}{dV} = 0$

$\Rightarrow A$ reaches an extremum (minimum)

$M = 1$ can only exist at A_{\min}



$$V = Ma$$

$$\frac{dV}{V} = \frac{dM}{M} + \frac{da}{a} = \frac{dM}{M} + \frac{1}{2} \frac{dT}{T}$$

$$\frac{dT}{T} = -(\gamma - 1) M^2 \frac{dV}{V}$$

From eqn. ⑥

$$\frac{dM}{M} = \left[1 + \frac{(\gamma - 1)}{2} M^2 \right] \frac{dV}{V}$$

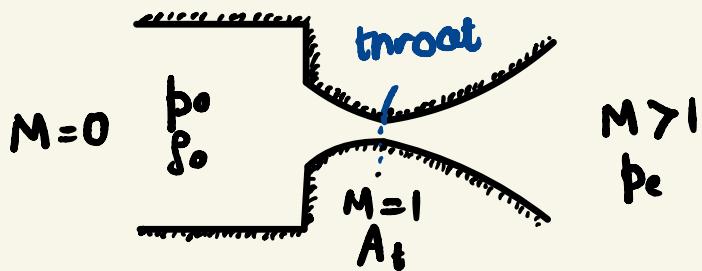


$$\frac{dA}{A} = \frac{(M^2 - 1)}{\left[1 + \frac{\gamma - 1}{2} M^2 \right]} \frac{dM}{M}$$

Cases

1. $M < 1$, dA and dM have opposite signs
2. $M > 1$, dA and dM have the same sign
3. $M = 1$, $dA = 0$, A is a minimum (A_t)

To accelerate a subsonic flow to a supersonic flow, we need a **convergent-divergent nozzle (c-d nozzle)** or a **de Laval nozzle**

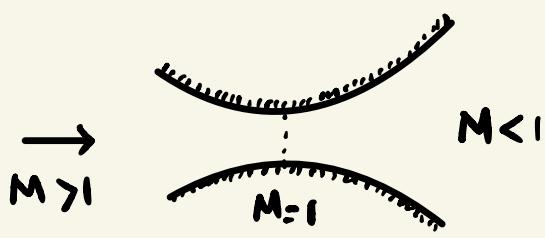


- (a) Pressure decreases across the nozzle
(b) $M=1$ at the throat

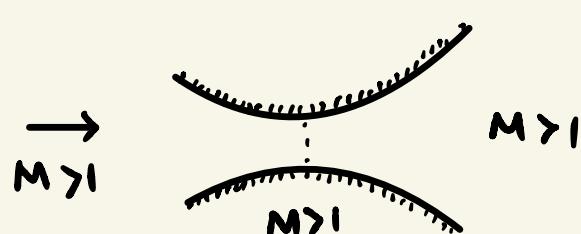
Supersonic flow is achieved only if p_0/p_e is 'large enough'. If p_0/p_e is not large, v increases till flow reaches throat ($M=1$) and then decreases like a venturi; ie, flow is subsonic throughout.

If initial flow is supersonic, two things happen

Case 1



Case 2

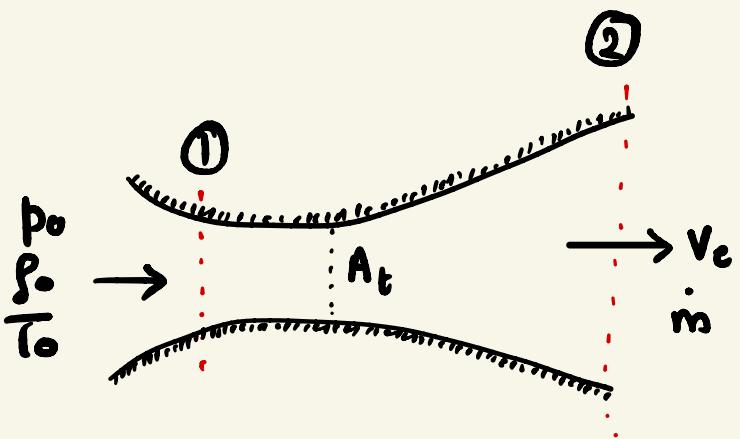


Finding V_e , m , A_t

$$\dot{m} = \rho_0 V_1 A_1 = \text{constant}$$

Energy Eqⁿ

$$V_1^2 + \frac{2 \alpha_0^2}{(\gamma - 1)} = \frac{2 \alpha_0^2}{\gamma - 1} - \textcircled{1}$$



Flow isentropic

$$\frac{\alpha_1}{\alpha_0} = \left(\frac{T_1}{T_0} \right)^{\gamma/2} = \left(\frac{p_1}{p_0} \right)^{\frac{\gamma-1}{2\gamma}} = \left(\frac{p_1}{p_0} \right)^{\frac{\gamma-1}{2}}$$

$$V_1 = \left[\left(\frac{2}{\gamma-1} \right) \alpha_0^2 \left[1 - \left(\frac{\alpha_1}{\alpha_0} \right)^2 \right] \right]^{\gamma/2}$$

$$V_1 = \left\{ \left(\frac{2\gamma}{\gamma-1} \right) \frac{p_0}{\rho_0} \left[1 - \left(\frac{p_1}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{\gamma/2} - \textcircled{2}$$

$$\dot{m} = \rho_0 V_1 A_1 \frac{p_1}{\rho_0}$$

$$\dot{m} = \rho_0 A_1 \left(\frac{p_1}{p_0} \right)^{\frac{1}{\gamma}} \left\{ \left(\frac{2\gamma}{\gamma-1} \right) \left(\frac{p_0}{\rho_0} \right) \left[1 - \left(\frac{p_1}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{\gamma/2}$$

At station 2

$$\dot{m} = \rho_0 A_2 \left(\frac{p_2}{p_0} \right)^{\frac{1}{\gamma}} \left\{ \left(\frac{2\gamma}{\gamma-1} \right) \left(\frac{p_0}{\rho_0} \right) \left[1 - \left(\frac{p_2}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{\gamma/2}$$

$$\frac{A_2}{A_1} = \left(\frac{p_1}{p_2} \right)^{\frac{1}{\gamma}} \left\{ \frac{1 - \left(\frac{p_1}{p_0} \right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{p_2}{p_0} \right)^{\frac{\gamma-1}{\gamma}}} \right\}^{\gamma/2}$$

- $\star\star\star$
Connects area
and pressure
ratio

Critical conditions (p^* , T^* , s^*)

A convenient reference point in presenting the flow equations in a variable area duct.

Point in the flow at which $M=1$ (may not actually exist)

Energy equation at critical condition gives ($V^* = a^*$)

$$V^{*2} = \frac{2}{\gamma+1} a_0^2$$

$$\text{Also, } \left(\frac{a^*}{a_0}\right)^2 = \frac{T^*}{T_0} = \frac{2}{\gamma+1}$$

$$\Rightarrow \frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}, \quad \frac{P^*}{P_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$

For air ($\gamma=1.4$) Remember these!!!

$$\frac{T^*}{T_0} = 0.833$$

$$\frac{p^*}{p_0} = 0.528$$

$$\frac{P^*}{P_0} = 0.634$$

Now \star at critical condition gives

$$\begin{aligned} \dot{m} &= P_0 A^* \left(\frac{p^*}{p_0}\right)^{\frac{1}{\gamma}} \left\{ \left(\frac{2\gamma}{\gamma-1}\right) \left(\frac{p_0}{P_0}\right) \left[1 - \left(\frac{p^*}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{1/2} \\ &= P_0 A^* \left(\frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}} \left\{ \left(\frac{2\gamma}{\gamma-1}\right) \left(\frac{p_0}{P_0}\right) \left[\frac{\frac{\gamma-1}{\gamma}}{\gamma+1} \right] \right\}^{1/2} \end{aligned}$$

$$\Rightarrow A^* = \frac{\dot{m}}{\sqrt{\gamma p_0}} \left(\frac{2}{\gamma+1} \right)^{-\frac{\gamma+1}{2(\gamma-1)}} - \star \star \star$$

The area at any section of the duct, in terms of the critical area A^* is

$$\frac{A}{A^*} = \left(\frac{P_e}{P_0}\right)^{\frac{1}{r}} \left\{ \frac{1 - \left(\frac{P_e}{P_0}\right)^{\frac{r-1}{r}}}{1 - \left(\frac{P}{P_0}\right)^{\frac{r-1}{r}}} \right\}^{\frac{1}{2}}$$

$$\Rightarrow \boxed{\frac{A}{A^*} = \frac{\left(\frac{2}{r+1}\right)^{\frac{r+1}{2(r-1)}} \left(\frac{r-1}{2}\right)^{\frac{1}{2}}}{\left\{ \left(\frac{P_e}{P_0}\right)^{\frac{2}{r}} - \left(\frac{P_e}{P_0}\right)^{\frac{r+1}{r}} \right\}^{\frac{1}{2}}} - \text{---} \star \star \star}$$

Let us say, a nozzle is designed for a given m , with an overall pressure ratio of P_e/P_0

Discharge velocity is obtained as

$$V_c = \left\{ \left(\frac{2r}{r-1}\right) \left(\frac{P_0}{P_e}\right) \left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{r-1}{r}} \right] \right\}^{\frac{1}{2}} - ①$$

Exit area A_c is found from

$$m = P_0 A_c \left(\frac{P_e}{P_0} \right)^{\frac{1}{r}} \left\{ \left(\frac{2r}{r-1}\right) \left(\frac{P_0}{P_e}\right) \left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{r-1}{r}} \right] \right\}^{\frac{1}{2}} - ②$$

Throat area A^* is found from

$$\frac{A_c}{A^*} = \frac{\left(\frac{2}{r+1}\right)^{\frac{r+1}{2(r-1)}} \left(\frac{r-1}{2}\right)^{\frac{1}{2}}}{\left\{ \left(\frac{P_e}{P_0}\right)^{\frac{2}{r}} - \left(\frac{P_e}{P_0}\right)^{\frac{r+1}{r}} \right\}^{\frac{1}{2}}} - ③$$

A/A* in terms of M

Energy equation gives

$$v = Ma_0 \left[1 + \frac{\gamma-1}{2} M^2 \right]^{-\frac{1}{2}}$$

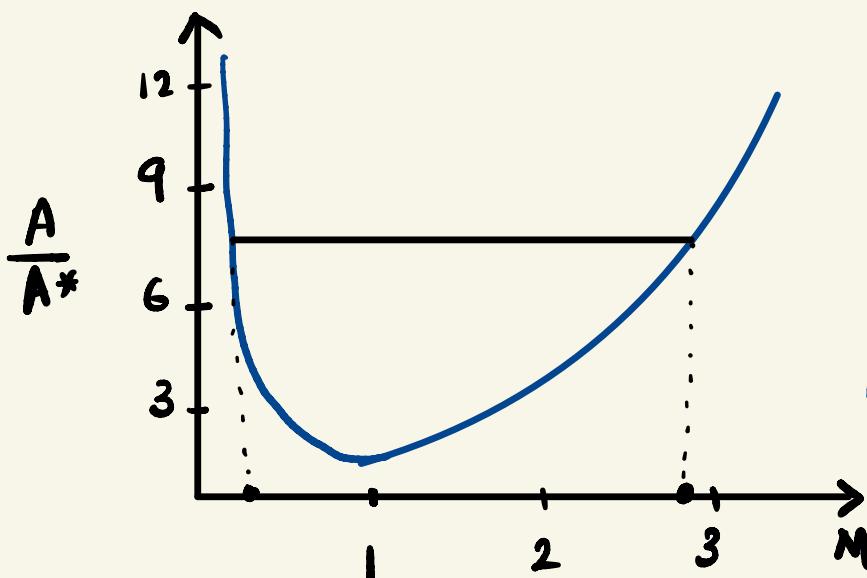
$$\frac{P}{P_0} = \left[1 + \frac{\gamma-1}{2} M^2 \right]^{\frac{-1}{\gamma-1}}$$

$$\begin{aligned} \frac{\dot{m}}{A} &= \rho v = P_0 v \cdot \frac{P}{P_0} \\ &= \frac{P_0 a_0 M}{\left[1 + \frac{\gamma-1}{2} M^2 \right]^{\frac{\gamma+1}{2(\gamma-1)}}} \end{aligned}$$

$$\Rightarrow \boxed{\frac{A_2}{A_1} = \left(\frac{M_1}{M_2} \right) \left\{ \frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2} \right\}^{\frac{\gamma+1}{2(\gamma-1)}}} \quad \text{is constant}$$

For $A_1 = A^*$, $A_2 = A$, $M_2 = M$, $M_1 = 1$, we have

$$\boxed{\frac{A}{A^*} = \frac{1}{M} \left\{ \left(\frac{2}{\gamma+1} \right) \left[1 + \frac{\gamma-1}{2} M^2 \right] \right\}^{\frac{\gamma+1}{2(\gamma-1)}}}$$



(a) $\frac{A}{A^*}$ reaches a minimum at $M=1$

(b) For a given A/A^* there are two M

Problems

① H_2 Find \dot{m} , A_e ?

$$p_0 = 600 \text{ kPa}$$

$$T_0 = 40^\circ\text{C}$$

$$A_t = 0.0001 \text{ m}^2 = A^*$$

$$p_e = 130 \text{ kPa}$$

$$\frac{p^*}{p_0} = \left(\frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}} = 0.528$$

$$\frac{p_e}{p_0} = \frac{130}{600} = 0.2167 < \frac{p^*}{p_0}$$

$\Rightarrow M = 1$ at throat and flow is supersonic through the nozzle diverging section

$$\dot{m} = p_0 A^* \left(\frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}} \left\{ \left(\frac{2\gamma}{\gamma-1} \right) \frac{p_0}{p_e} \left(\frac{\gamma-1}{\gamma+1} \right) \right\}^{\frac{1}{2}}$$

$$p_0 = \frac{p_e}{R T}, \quad R = \frac{8314 \cdot 3}{2.016} = 4124.2$$

$$\Rightarrow p_0 = 0.4648 \text{ kg/m}^3$$

$$\dot{m} = 0.03622 \text{ kg/s}$$

$$\frac{A_e}{A^*} = \frac{\left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(\frac{\gamma-1}{2} \right)^{\frac{1}{2}}}{\left\{ \left(\frac{p_e}{p_0} \right)^{\frac{2}{\gamma}} - \left(\frac{p_e}{p_0} \right)^{\frac{\gamma+1}{\gamma}} \right\}^{\frac{1}{2}}}$$

$$A_e = 0.0006416 \text{ m}^2$$

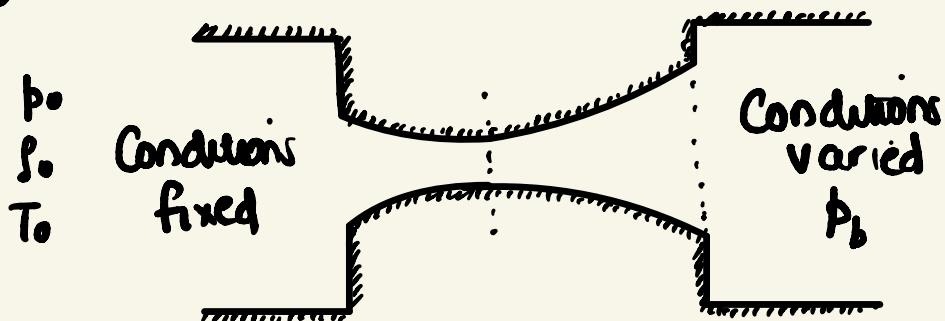
Operating characteristics of nozzles

Nozzle - a shape that accelerates gas flow

Diffuser - a shape that decelerates gas flow

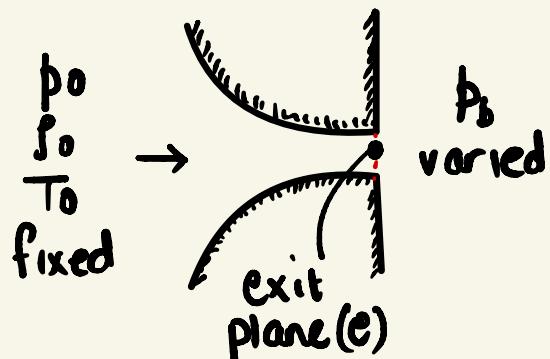
Assumptions

- (1) Upstream conditions are stagnation conditions
- (2) Only conditions downstream of nozzle varied



Downstream pressure is called back pressure (p_b)

Case 1: convergent nozzle



$p_b = p_0 \Rightarrow$ no flow ($M_e = 0$)

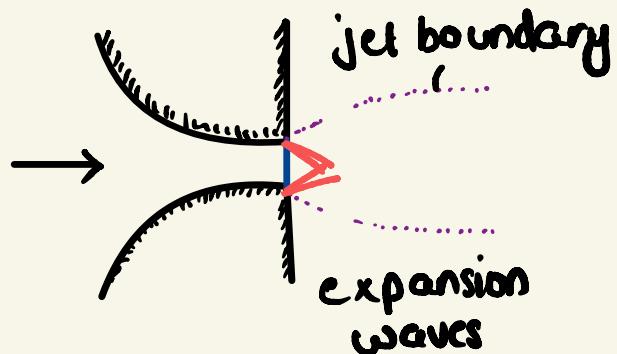
$p_b < p_0 \Rightarrow$ flow ($M_e \neq 0$)

$$\text{When } p_b = p^* = p_0 \left(\frac{2}{r+1} \right)^{\frac{r}{r-1}} \quad M_e = 1$$

For $p_b > p^*$, reduction in p_b increases the mass flow

For $p_b \leq p^*$, reduction in p_b have no effect on the flow in the nozzle, mass flow rate remains constant and $M_e = 1 \Rightarrow$ Nozzle is choked

When $p_b < p^*$, expansion from p_c to p_b takes place outside the nozzle through a series of expansion waves



$$p_b > p^*$$

$$p_c = p_b$$

discharge velocity $V_c = \left\{ \left(\frac{2r}{r-1} \right) \left(\frac{p_0}{p_c} \right) \left[1 - \left(\frac{p_b}{p_c} \right)^{\frac{r-1}{r}} \right] \right\}^{\frac{1}{2}}$

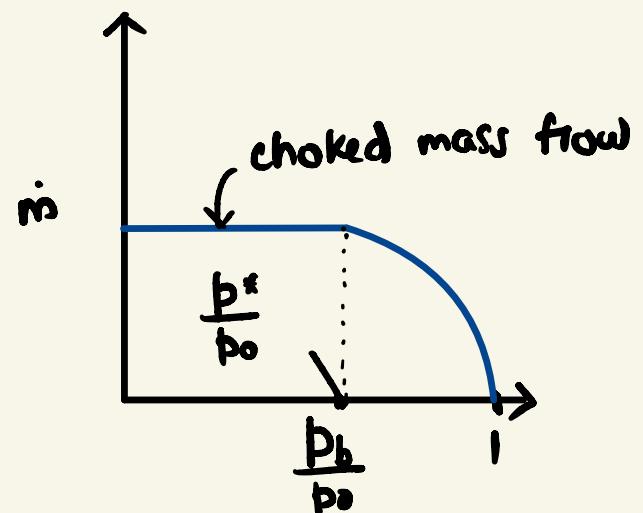
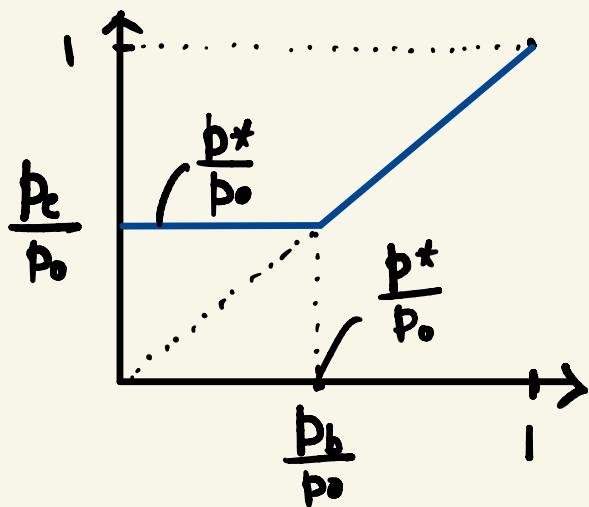
$$\dot{m} = \rho_0 A_c \left(\frac{p_b}{p_0} \right) \left\{ \left(\frac{2r}{r-1} \right) \left(\frac{p_0}{p_b} \right) \left[1 - \left(\frac{p_b}{p_0} \right)^{\frac{r-1}{r}} \right] \right\}^{\frac{1}{2}}$$

$$p_b \leq p^*$$

$$p_c = p_0 \left(\frac{2}{r+1} \right)^{\frac{r}{r-1}}$$

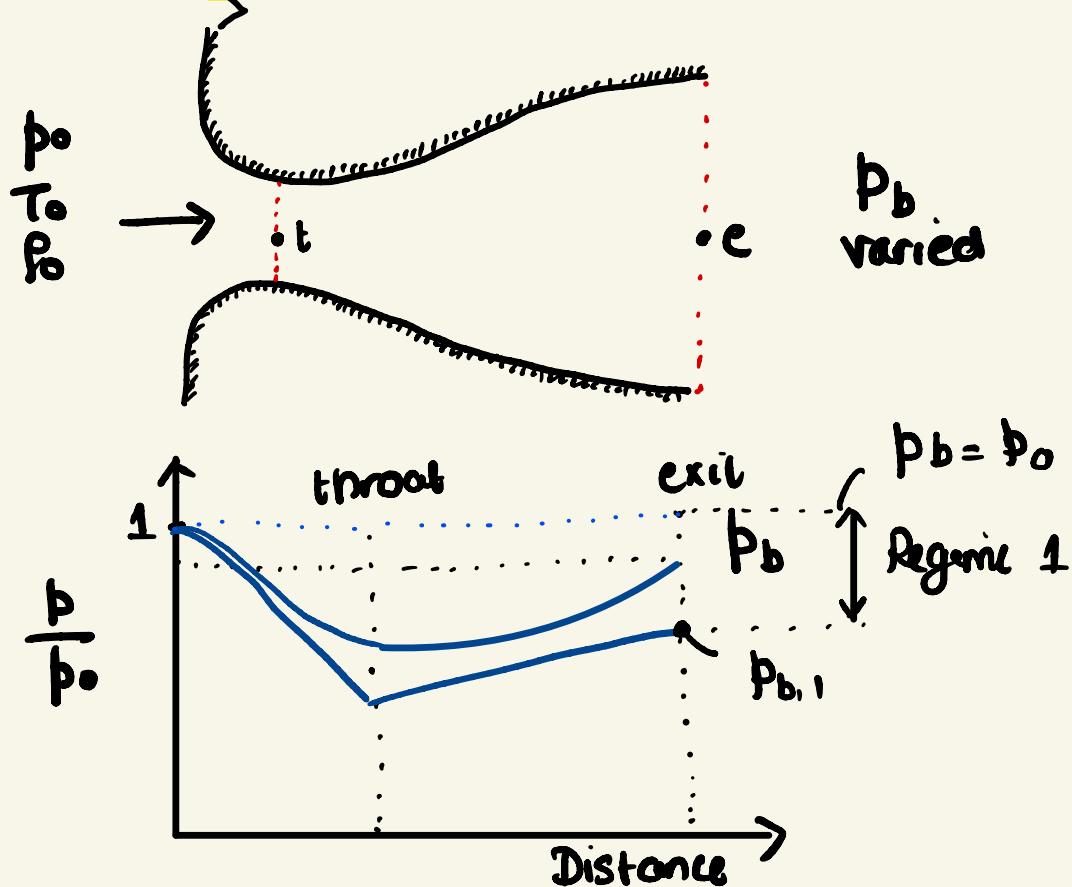
$$V_c = \sqrt{\left(\frac{2r}{r+1} \right) \frac{p_0}{p_c}}$$

$$\dot{m} = \sqrt{r \rho_0 A_c} \left(\frac{2}{r+1} \right)^{\frac{r+1}{2(r-1)}} \quad \left. \begin{array}{l} \text{independent of } p_b \\ \text{and invariant for} \\ p_b \leq p^* \end{array} \right\}$$



Case 2: Convergent-divergent nozzle

Regime 1 : $p_t < p^*$, Flow subsonic throughout



Nozzle operates like a venturi until p_b decreases enough to make $M_t = 1$

$$p_e = p_b$$

$$\frac{A_e}{A^*} = \left(\frac{p_t}{p_b}\right)^{\frac{1}{r}} \left[\frac{1 - \left(\frac{p_t}{p_0}\right)^{\frac{r-1}{r}}}{1 - \left(\frac{p_b}{p_0}\right)^{\frac{r-1}{r}}} \right]^{\frac{1}{2}}$$

$$\left(\frac{p_t}{p_0}\right)^{\frac{1}{r}} \left\{ 1 - \left(\frac{p_t}{p_0}\right)^{\frac{r-1}{r}} \right\}^{\frac{1}{2}} =$$

$$\left(\frac{A_e}{A^*}\right) \left(\frac{p_b}{p_0}\right)^{\frac{1}{r}} \left\{ 1 - \left(\frac{p_b}{p_0}\right)^{\frac{r-1}{r}} \right\}$$

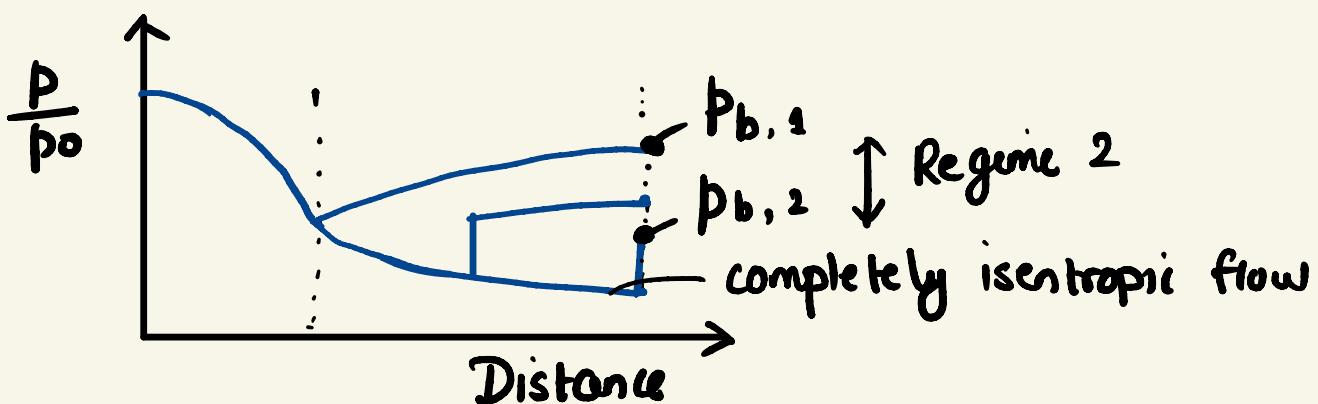
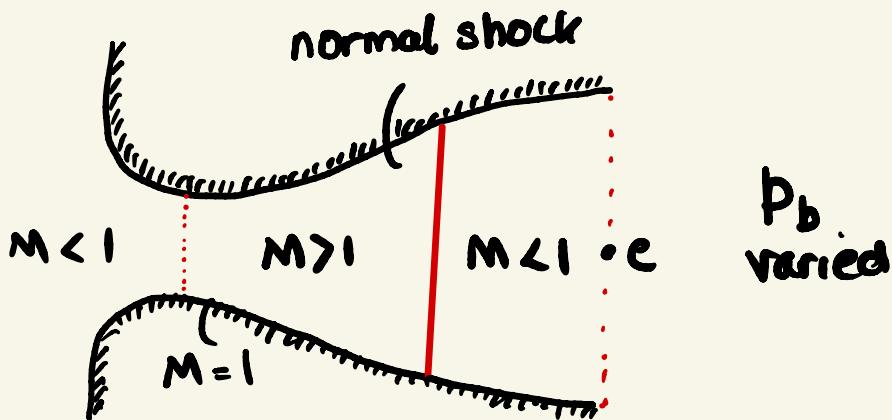
When $M_t = 1$

$$P_t = P^* = P_0 \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}}$$

$$\left(\frac{A_e}{A^*} \right) \left(\frac{P_{b,1}}{P_0} \right)^{\frac{1}{\gamma}} \left\{ 1 - \left(\frac{P_{b,1}}{P_0} \right)^{\frac{\gamma-1}{\gamma}} \right\}^{\frac{1}{2}} = \left(\frac{\gamma-1}{\gamma+1} \right) \left(\frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}}$$

$$P_{b,2} = P_b \text{ at which } P_t = P^*$$

Regime 2: From the end of regime 1 until there is a normal shock at the exit ($P_b = P_{b,2}$)



For $P_b < P_{b,1}$, there will be a normal shock in the divergent portion and this shock will move downstream as P_b is decreased until it reaches the exit at $P_b = P_{b,2}$.

As p_b decreases in this regime, we have stronger normal shocks as M upstream is larger.

When the shock reaches the exit plane, flow in the nozzle is isentropic throughout and p_e/p_0 is the design pressure ratio and M_e is the design Mach number

In this limit, we have

$$\left(\frac{p_e}{p_0}\right)^{\frac{2}{r}} - \left(\frac{p_e}{p_0}\right)^{\frac{r+1}{r}} = \left(\frac{2}{r+1}\right)^{\frac{r+1}{r-1}} \left(\frac{r-1}{2}\right) \left(\frac{A^*}{A_e}\right)^2$$

$$\frac{1}{M_e} \left\{ \left(\frac{2}{r+1}\right) \left[1 + \left(\frac{r-1}{2}\right) M_e^2 \right]^{\frac{r+1}{2(r-1)}} \right\} = \frac{A_e}{A^*}$$

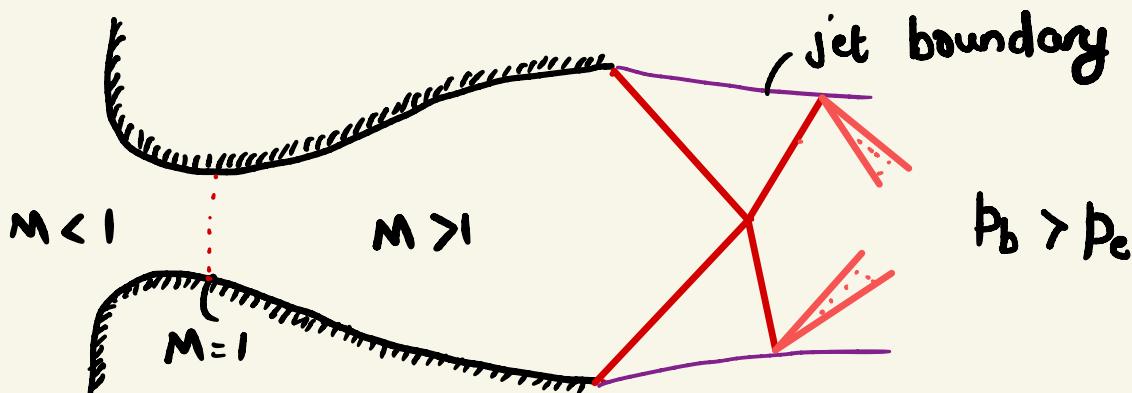
Also, from normal shock relationships

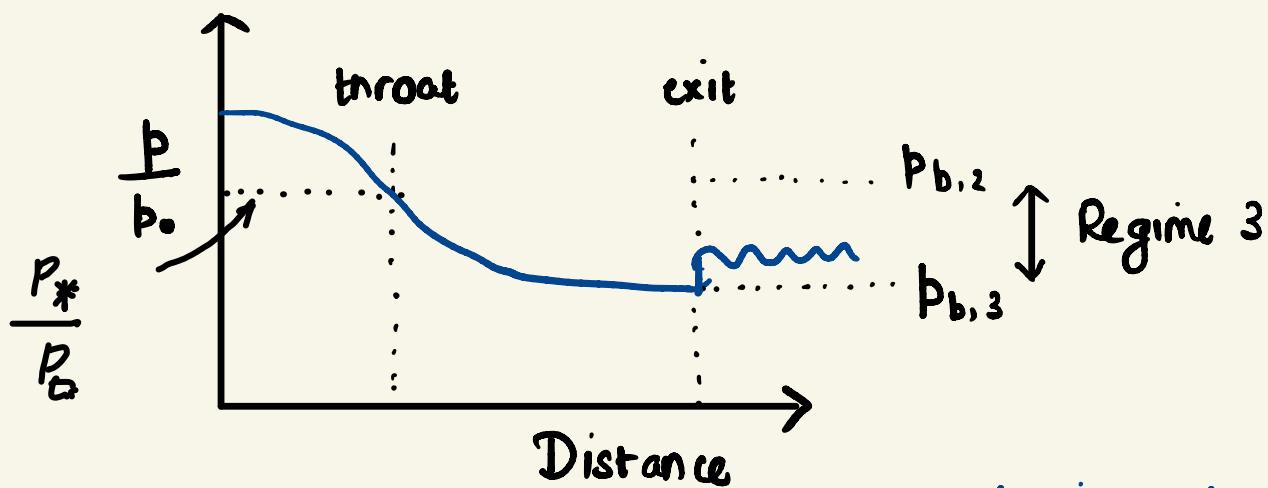
$$\frac{p_{b,2}}{p_e} = \frac{2rM_e^2 - (r-1)}{r+1}$$

$$\frac{p_{b,2}}{p_0} = \frac{p_{b,2}}{p_e} \cdot \frac{p_e}{p_0}$$

Once shock reaches the exit, further reductions in p_b will not affect the nozzle

Regime 3 (overexpanded regime): p_b at which oblique shocks are present at the nozzle exit

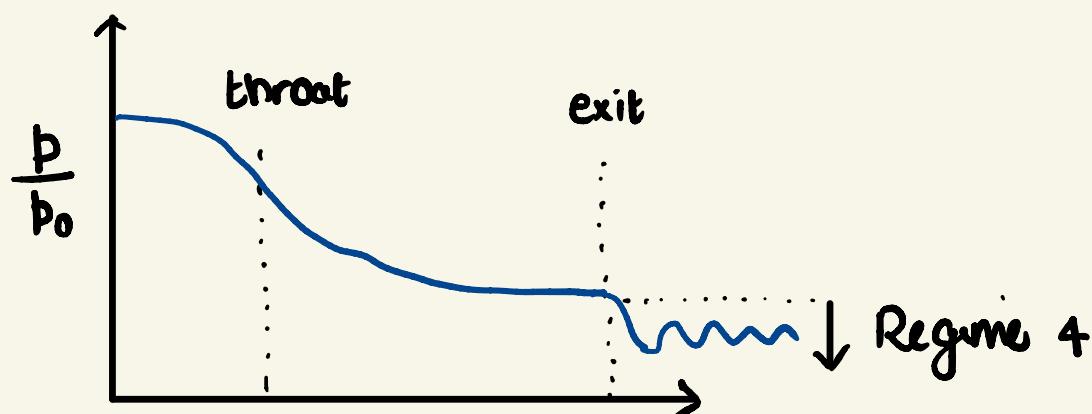
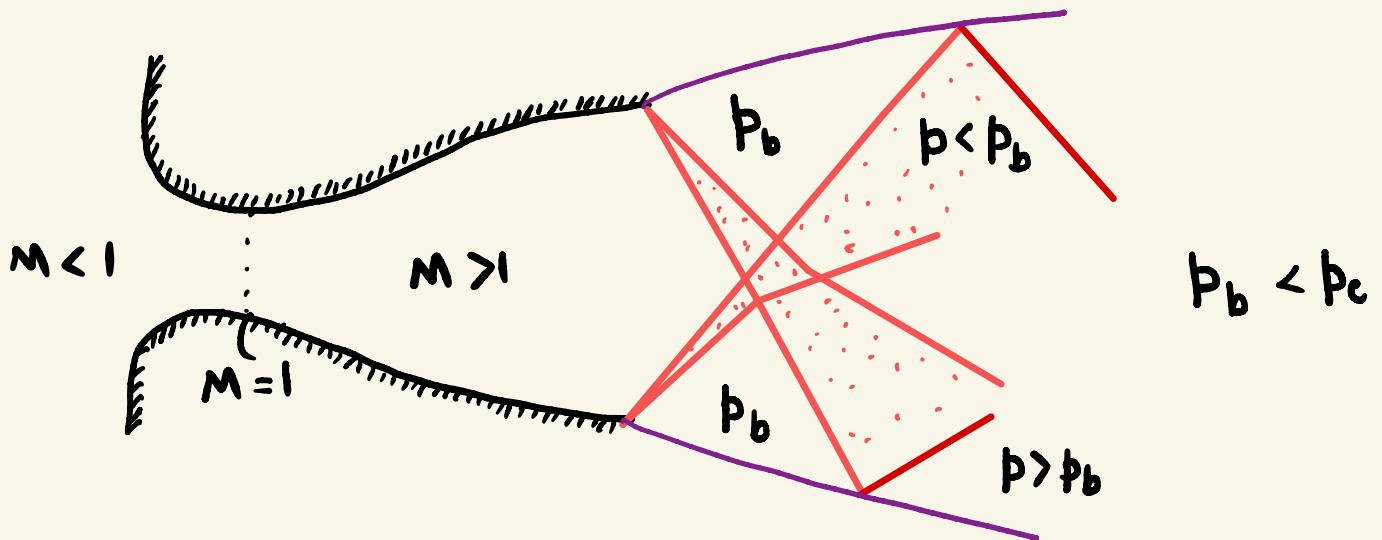




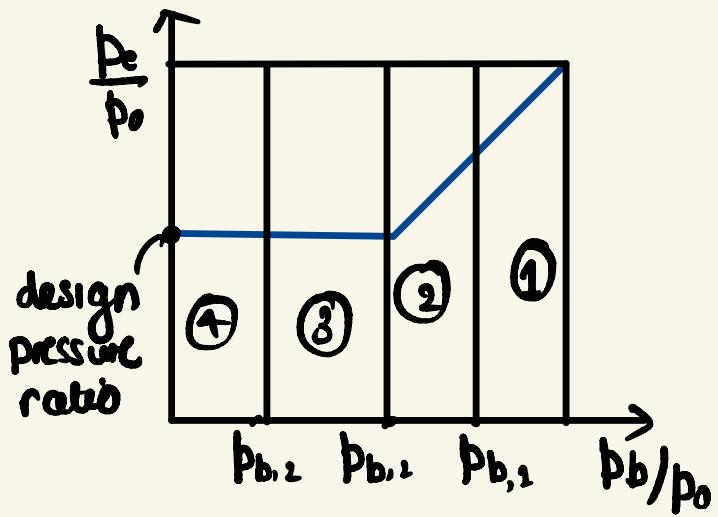
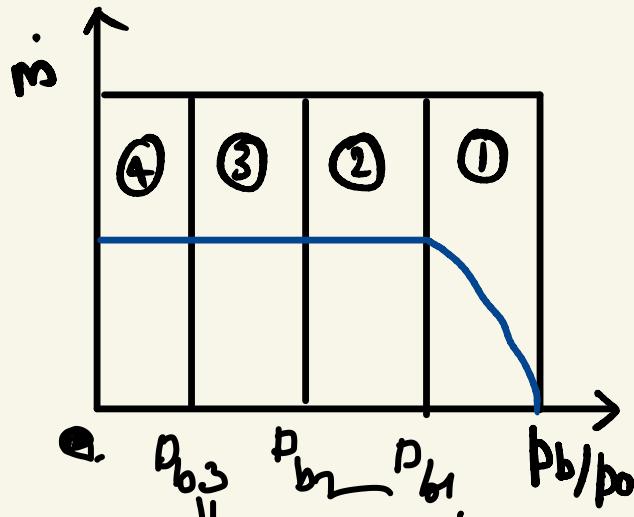
As P_b decreases and approaches P_c , oblique shock becomes weaker until it vanishes at $P_b = P_c$. The nozzle is now operating at **design conditions** and there are no waves inside or outside the nozzle ($P_b = P_{b,3}$)

Shock

Regime 4 (underexpanded regime): P_b at which expansion waves are present at the nozzle exit



$$\left(\frac{P_b}{P_0}\right)^{\frac{2}{r}} - \left(\frac{P_b}{P_0}\right)^{\frac{r+1}{r}} = \left(\frac{2}{r+1}\right)^{\frac{r+1}{r-1}} \left(\frac{r-1}{2}\right) \left(\frac{A^*}{A_e}\right)^2$$



Problems $P_e/\text{design} \rightarrow p^t \approx p^*$

② Air flow in a C-d nozzle. Reservoir: $p_0 = 600 \text{ kPa}$,

$T = 40^\circ\text{C}$. $P_{b,\text{design}} = 100 \text{ kPa}$

Find (a) A_e/A_t (b) V_e (c) P_b at which normal shock is present in the exit plane

$$(a) \frac{P_e}{P_0} = \frac{100}{600} = 0.1667$$

From isentropic tables, $M_e = 1.83$

At $M=1.83$, isentropic tables $\frac{A_e}{A^*} = 1.472$

$$(b) \frac{a_e}{a_0} = 0.7739 \quad (\text{from tables})$$

$$a_0 = \sqrt{\gamma R T_0} = 345 \text{ m/s}$$

$$V_e = \frac{V_e}{a_e} \cdot \frac{a_e}{a_0} \cdot a_0 = 502.1 \text{ m/s}$$

(c) From normal shock tables at $M_e = 1.63$

$$\frac{P_2}{P_1} = 3.74$$

$$\Rightarrow P_b = 3.74 \cdot 100 = 374 \text{ kPa}$$

③ c-d nozzle: $P_0 = 800 \text{ kPa}$, $T_0 = 40^\circ\text{C}$, $M = 2.7$

$$A^* = 0.08 \text{ m}^2$$

1. A_e ?

Isentropic tables at $M_c = 2.7$ $\frac{A}{A^*} = 3.183$

$$A_e = 3.183 \times 0.08 = 0.255 \text{ m}^2$$

2. \dot{m}_{design} ?

$$\dot{m} = \rho^* V^* A^*$$

$$\rho_0 = \frac{P_0}{RT_0} = 8.91 \text{ kg/m}^3$$

$$\frac{\rho^*}{\rho_0} = 0.63394 \Rightarrow \rho^* = 5.65 \text{ kg/m}^3$$

$$V^* = Q^* = \sqrt{\gamma R T^*}$$

$$\frac{T^*}{T_0} = 0.83055 \Rightarrow T^* = 260 \text{ K}$$

$$V^* = 323.2 \text{ m/s}$$

$$\begin{aligned}\dot{m} &= 5.65 \times 323.2 \times 0.08 \\ &= 146 \text{ kg/s}\end{aligned}$$

3. Design back pressure

$$\left. \frac{p_0}{p_b} \right|_{M=2.7} = 23.283 \Rightarrow p_{b, \text{design}} = 34.36 \text{ kPa}$$

4. Lowest p_b for which only subsonic flow

$$\frac{A}{A^*} = 3.183 \Rightarrow \left. \frac{p_0}{p_b} \right|_{\text{subsonic}} = 1.025 \text{ for the subsonic case}$$

$$\left. p_b \right|_{\text{subsonic}} = \frac{800}{1.025} = 780.5 \text{ kPa}$$

5. p_b at which normal shock in exit plane

From normal shock tables at $M=2.7$

$$\frac{p_b}{p_{b, \text{design}}} = 8.33832$$

$$\Rightarrow p_b = 8.33832 \times 34.36 = 286.5 \text{ kPa}$$

6. p_b below which no shock waves exist inside the nozzle.

$$p_b < 286.5 \text{ kPa}$$

7. Range of p_b over which oblique shock waves exist

$$34.36 \text{ kPa} < p_b < 286.5 \text{ kPa}$$

8. Range of p_b over which expansion waves exist

$$p_b < 34.36 \text{ kPa}$$

Convergent-Divergent Supersonic Diffusers

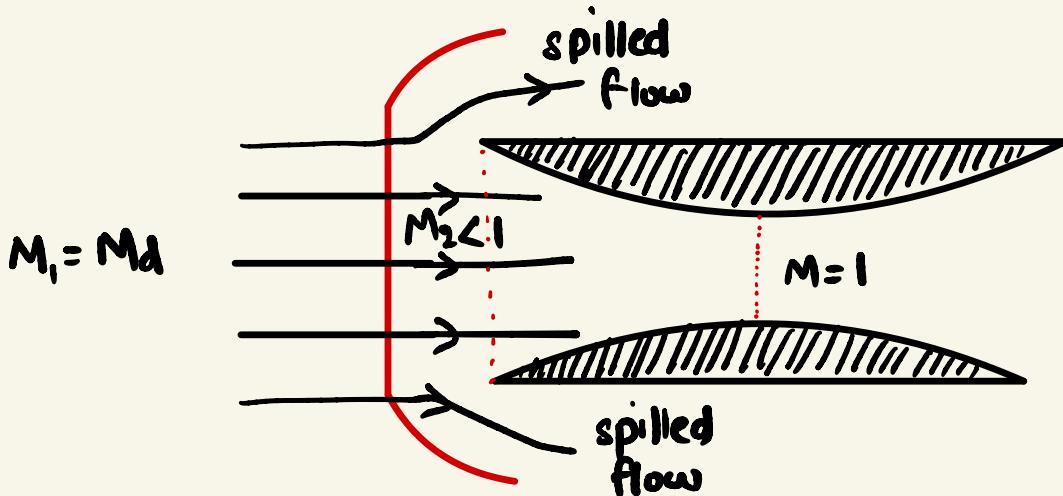


Ideally, we want shockless diffusion to $M < 1$ before flow reaches engine

Q. What happens at off-design conditions?

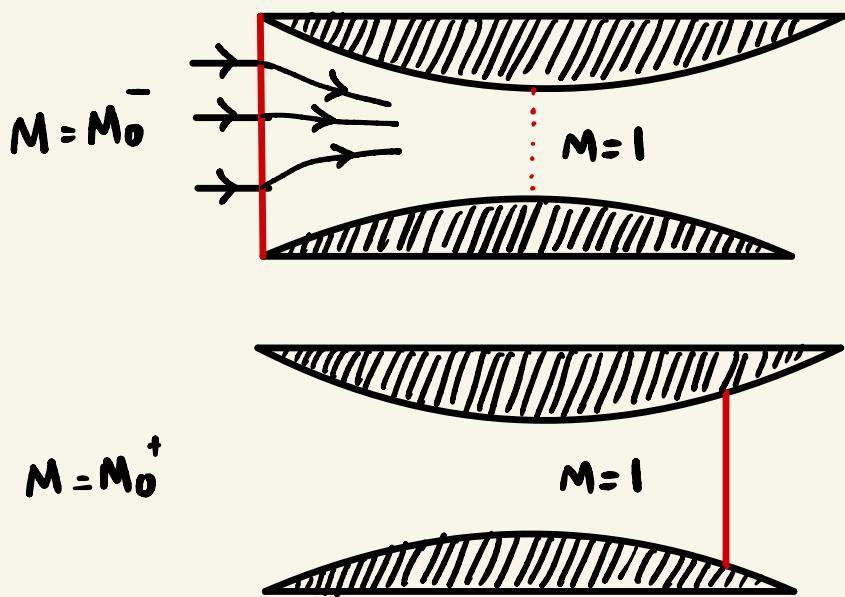
Fixed diffusers: A_i, A_t are fixed $\Rightarrow \frac{A^*}{A}$ fixed.

$\frac{A^*}{A}$ is larger at $M < M_d$ which for a fixed diffuser results in a larger mass flow rate trying to enter it. As a result, normal shock is present before the inlet which persists even at $M = M_d$ although the shock is now closer to the intake



Q. How to get rid of the shock in a fixed diffuser?

Overspeeding: Increase freestream M until shock reaches the inlet and is swallowed. Shock swallowing happens when M_2 has the design A/A^* value. ($M_i = M_o$). All of the mass flow is ingested

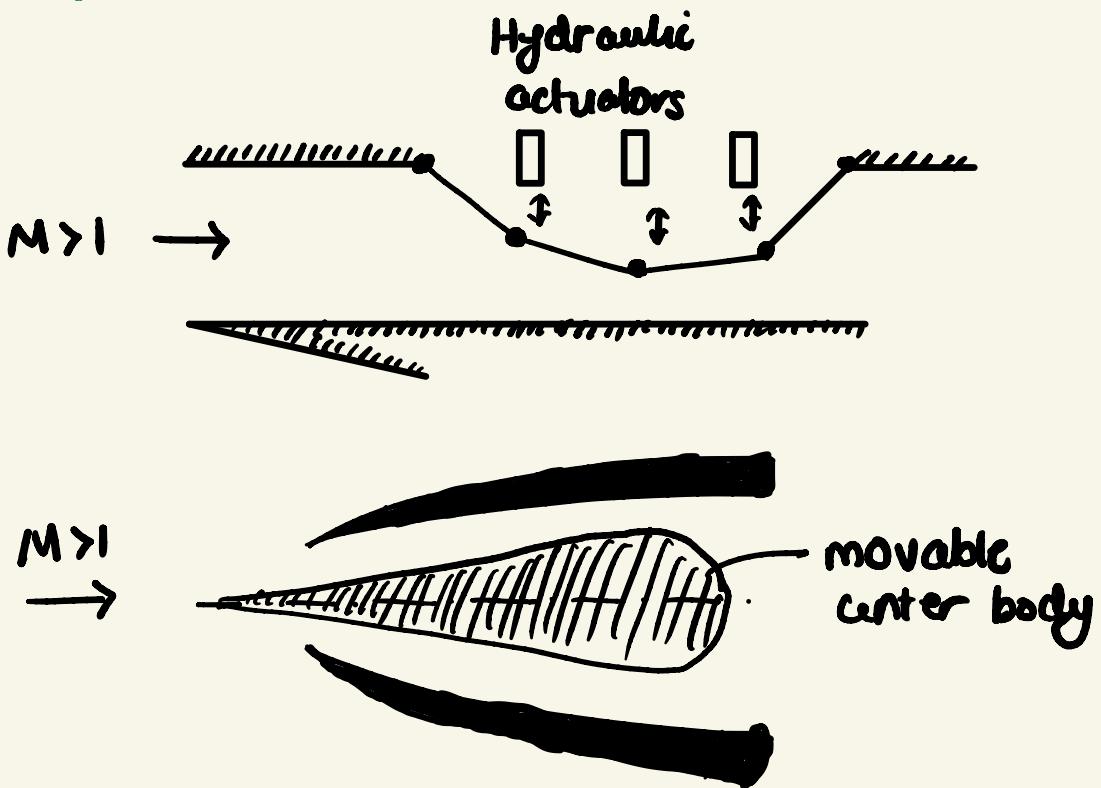


Once the shock is swallowed, the flight Mach number can be decreased which moves the shock upstream towards the throat until at $M = M_d$, the shock reaches throat and vanishes.

In reality, it would not be practical to operate under these design conditions because the slightest decrease in M would cause the shock to be **disengaged**, i.e., move out of the diffuser

Because of poor performance at off-design conditions and requirement of overspeeding, fixed C-D diffusers are seldom used.

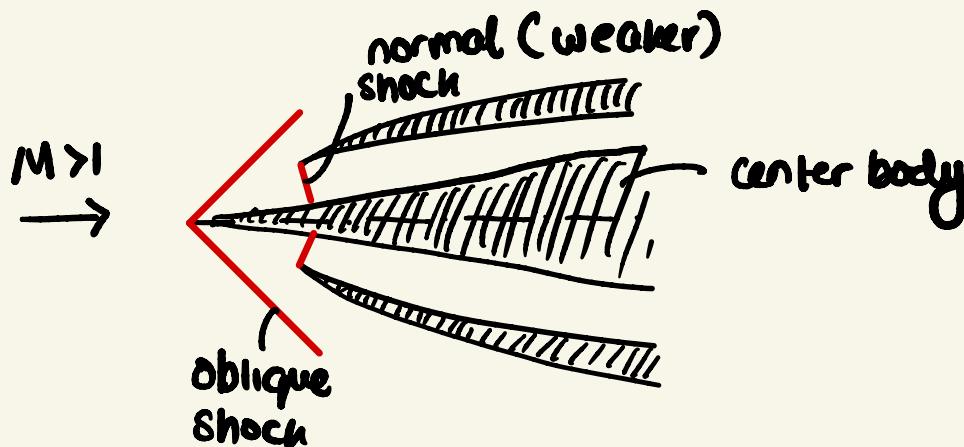
Solution: Diffuser with a variable area throat



With a variable A/A^* , the throat can be opened to allow the shock to be swallowed and then the throat area can be decreased until the shock disappears at all Mach numbers \Rightarrow no overspeeding required.

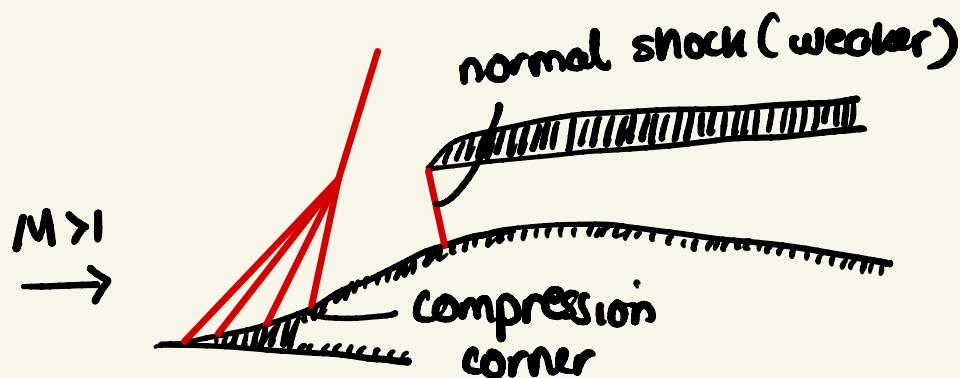
Other improvements:

1. Oblique shock diffusers



Much of the deceleration done through oblique shocks
less stagnation pressure loss

2. Curved compressive corner



Measure of diffuser performance

$$\text{Stagnation pressure recovery ratio} = \frac{p_0 \text{ in diffuser}}{p_0 \text{ in freestream}}$$

Problems

④ Jet aircraft : $M = 1.7$, fixed A/A^* c-d diffuser.

Find ideal A/A^* and M to which aircraft must be taken to swallow the shock.

$$\text{At } M = 1.7 \Rightarrow \frac{A}{A^*} = 1.338$$

Subsonic M which has $\frac{A}{A^*} = 1.338$ is $M_2 = 0.5$

\Rightarrow Mach number before the shock, $M_1 = 2.64$

⑤ Jet aircraft : $M_{\text{cruise}} = 2.5$, variable area diffuser

What is $\frac{A^*_{\text{cruise}}}{A^*_{M=1.3}}$ assuming fixed A_e ?

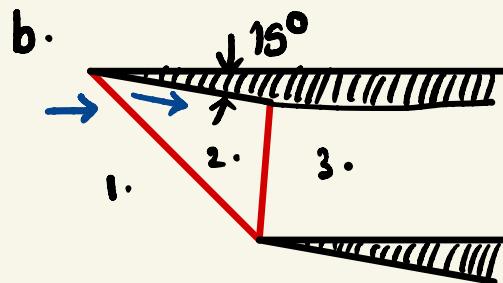
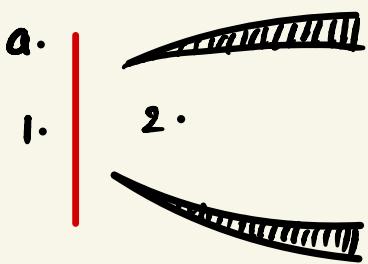
$$A^*_{M=1.3}$$

$$\text{At } M = 2.5 \quad A/A^* = 2.6367$$

$$M = 1.3 \quad A/A^* = 1.0663$$

$$\frac{A^*_{\text{cruise}}}{A^*_{M=1.3}} = \frac{1.0663}{2.6367} = 0.404$$

⑥ Aircraft cruising at $M = 3$. $P_0 = 400 \text{ kPa}$



Find P_0 at exit of these diffusers.

a. At $M = 3$ $\frac{p_{02}}{p_{01}} = 0.3283$

$$\Rightarrow p_{02} = 0.3283 \times 400 = 131.3 \text{ kPa}$$

b. $\delta = 15^\circ$, $M = 3$, $M_{n1} = 1.6$, $M_2 = 2.255$

$$\frac{p_{02}}{p_{01}} = 0.8952$$

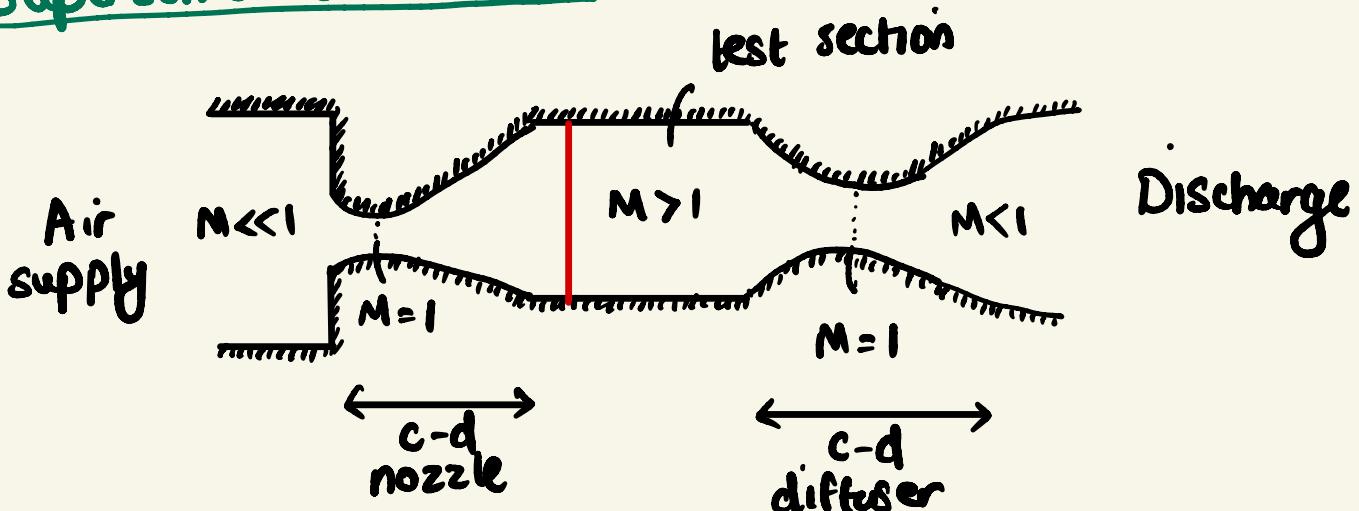
At $M_2 = 2.255$, normal shock tables give

$$\frac{p_{03}}{p_{02}} = 0.6168$$

$$\Rightarrow p_{03} = 0.8952 \times 0.6168 \times 400 = 220.9 \text{ kPa}$$

c. p_0 fixed = 400 kPa.

Supersonic Wind tunnel



The tunnel is started by either increasing the pressure ahead of the nozzle or decreasing the pressure behind the diffuser.

A normal shock wave exists in the nozzle during startup and moves downstream to the test section. Because of p_0 loss across the shock, A_t of diffuser $>$ A_t of nozzle to swallow the starting shock.

Problems

⑦ Wind tunnel: $M_d = 3$, variable area diffuser

Find $\frac{A^*_{\text{diffuser/designed}}}{A^*_{\text{diffuser/shock}}}$

At design condition, $\frac{A^*}{A} = 0.236$

Using normal shock tables at $M_d = 3$

$$M_2 = 0.475$$

Using isentropic tables at $M = 0.475$

$$\frac{A^*}{A} = 0.72$$

$$\text{Required ratio} = \frac{0.236}{0.72} = 0.328$$