AE 242
Aerospace Measurements Laboratory

Inertial sensors

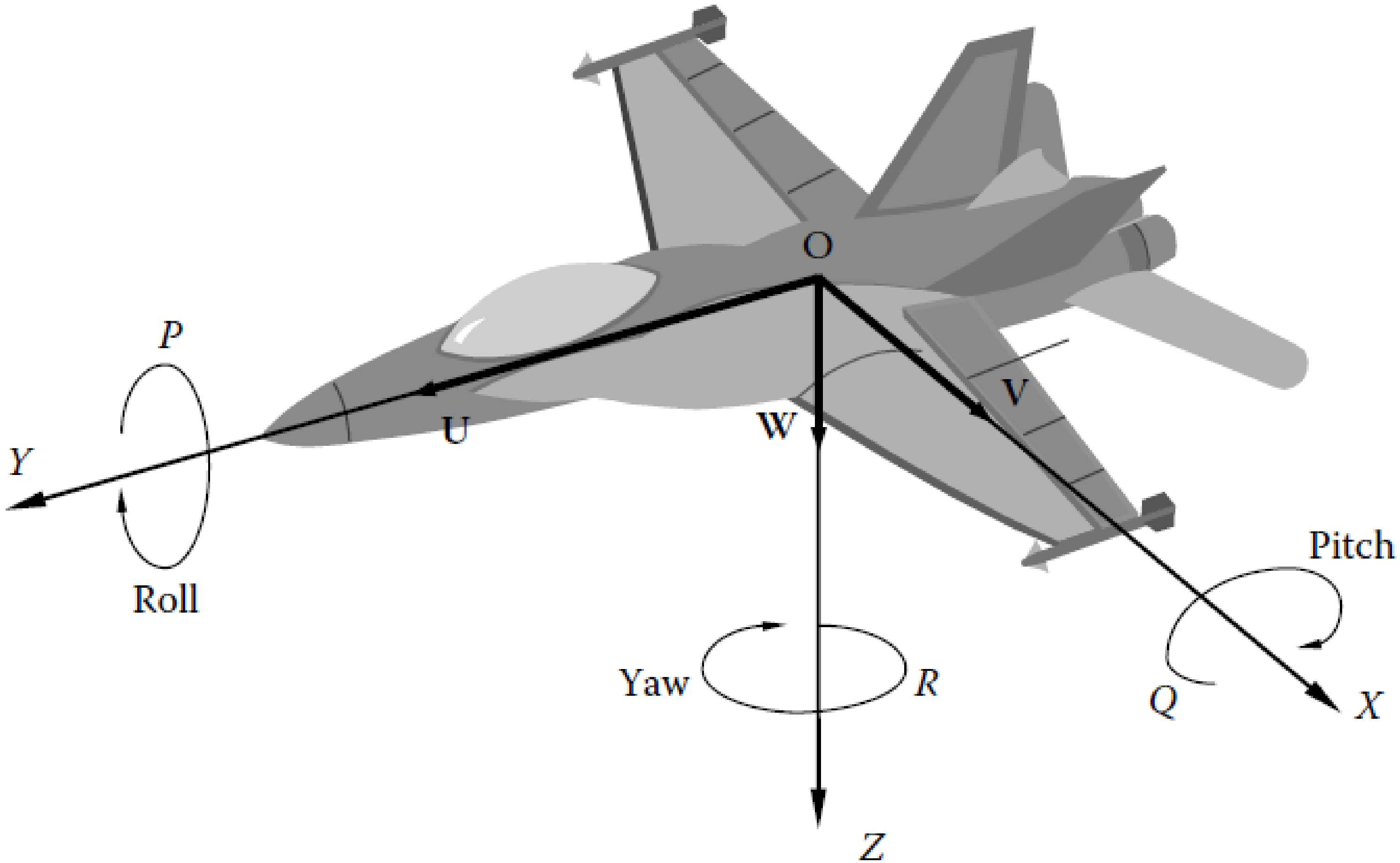
Sensed quantity is related to inertia (mass) of the sensor: Newton’s law

Accelerometers : Measures acceleration of an object. Change in velocity per unit time, commonly expressed in ‘g’. Can be used for estimating velocity and displacement, with respect to initial condition.

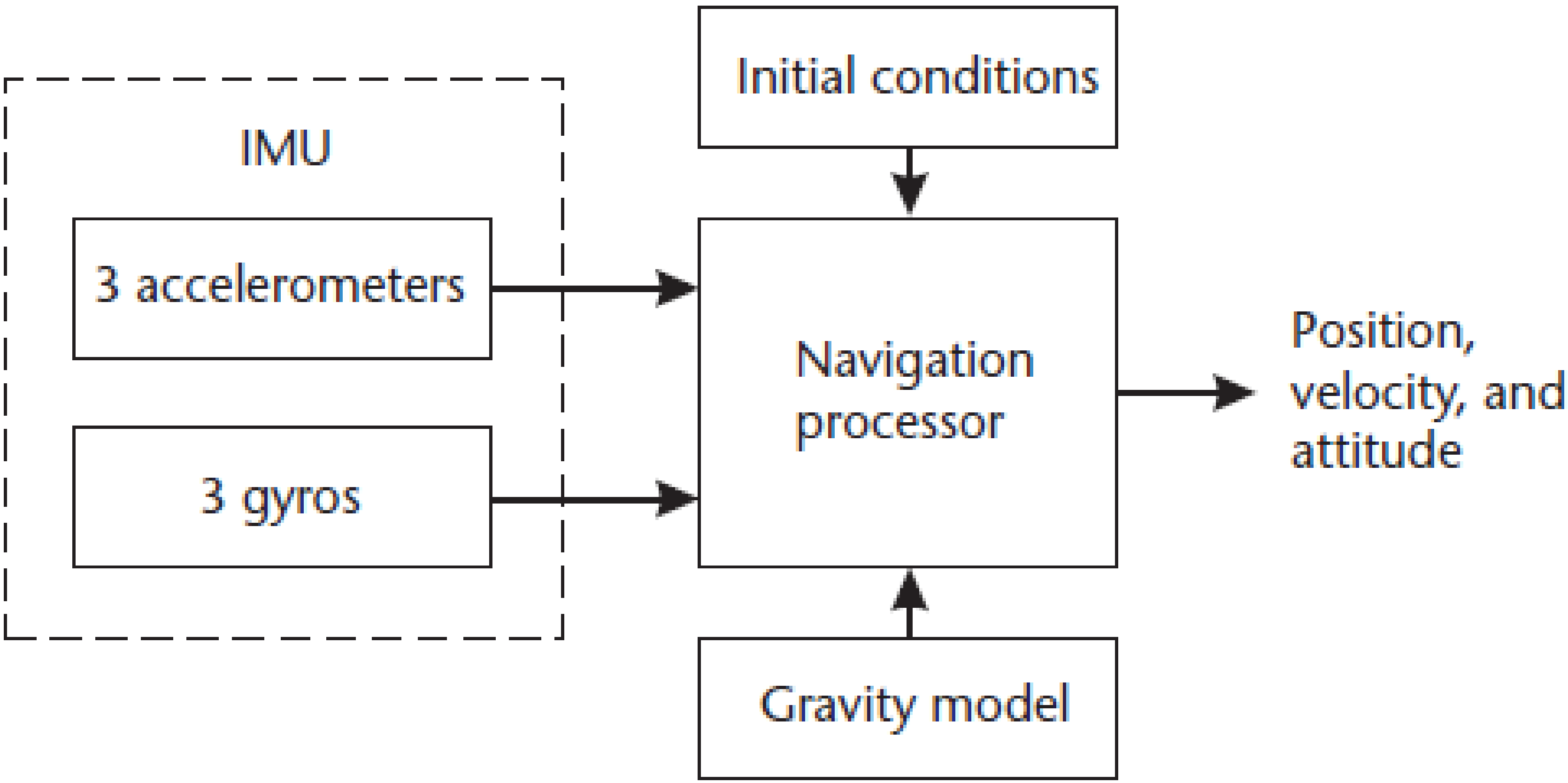
Gyroscope : To measure angular motion about an axis. Can be used to estimate the attitude of an object.

Accelerometers and gyroscopes are key sensors for Inertial Navigation System (INS). INS is used in missiles, aircraft, submarine, ship, satellite launch vehicle, satellite etc. It is a independent system and no outside signal is required for its operation, or it is a self contained system.

Inertial sensors



Inertial sensors are used to measure acceleration, angular rates of aircraft body. Process these quantities to obtain velocity, position and attitude.



Schematic of Inertial Navigation System

Accelerometer

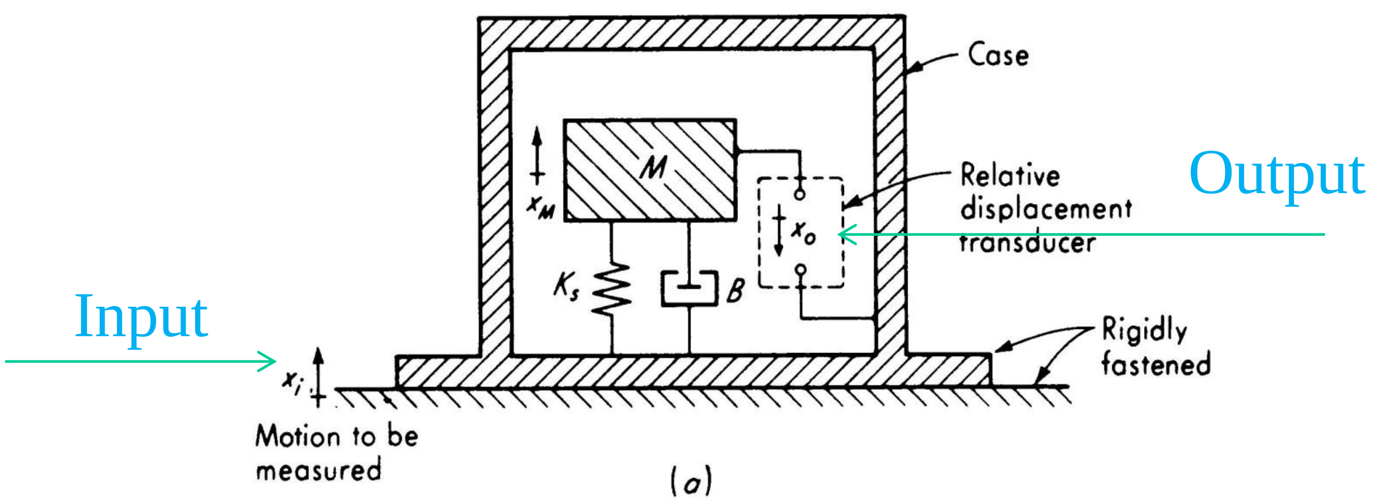
Popularity of accelerometers:

1) Frequency response is from zero to high limiting value. Piezo electric type are generally for dynamic measurement.

2) Displacement and velocity can be obtained by integration. A preferred way compared to differentiation.

3) Destructive forces are more close to acceleration as compared to velocity or displacement.

4) Measurement of transients (shock) motion is more readily achieved than with displacement or velocity pickup.



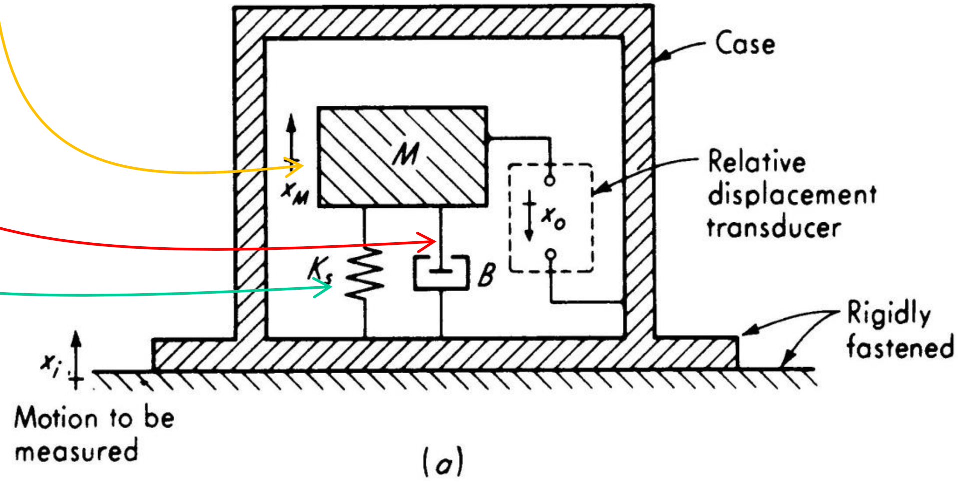
Accelerometer

Governing equation for such a system:

$$\mathbf{K_s x_o} + \mathbf{B\dot{x}_o} = \mathbf{M\ddot{x}_M} = \mathbf{M(\ddot{x}_i - \ddot{x}_o)}$$

$$\omega_n = \sqrt{\frac{\mathbf{K_s}}{\mathbf{M}}}$$

$$\zeta = \frac{\mathbf{B}}{2\sqrt{\mathbf{K_s M}}}$$



Accelerometer

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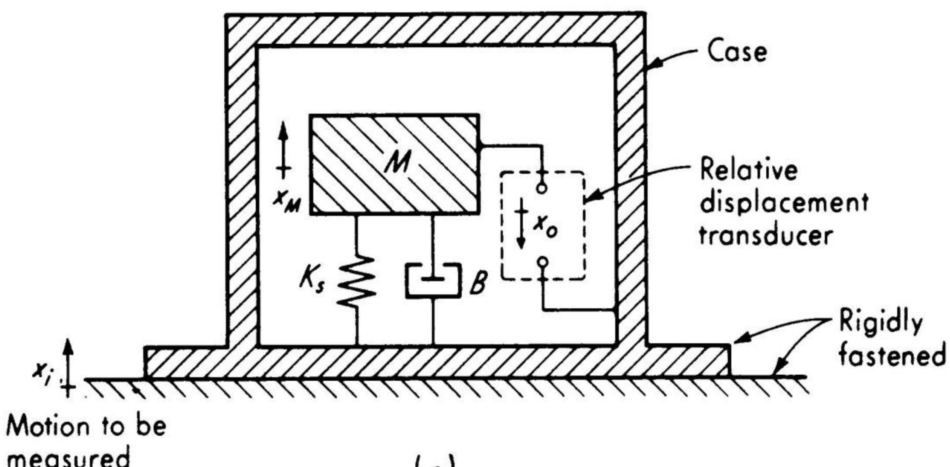
$$\omega_{\mathbf{n}} = \sqrt{\frac{\mathbf{K_s}}{\mathbf{M}}}$$

$$\zeta = \frac{\mathbf{B}}{2\sqrt{\mathbf{K_s M}}}$$

$$\boxed{M\ddot{x}_o + B\dot{x}_o + K_s x_o = M\ddot{x}_i}$$

$$\frac{\mathbf{x_0}}{\mathbf{x_i}}(\mathbf{D}) = \frac{\mathbf{D}^2/\omega_{\mathbf{n}}^2}{\mathbf{D}^2/\omega_{\mathbf{n}}^2 + 2\zeta\mathbf{D}/\omega_{\mathbf{n}} + 1}$$



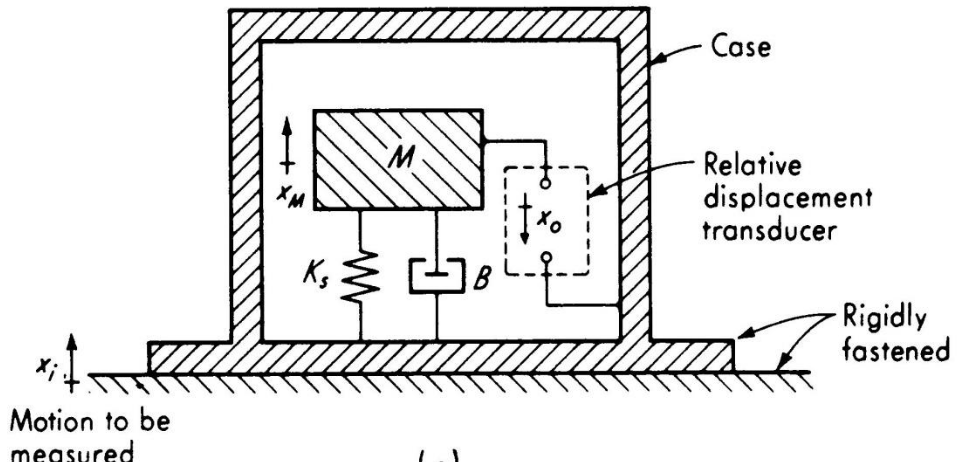
(a)

Accelerometer

Output is proportional to acceleration and it is a second order system. All the properties of the second order system are applicable. High natural frequency for a flat response. Frequency response of output will be from zero to some fraction of natural frequency.

$$\boxed{M\ddot{x}_o + B\dot{x}_o + K_s x_o = M\ddot{x}_i}$$

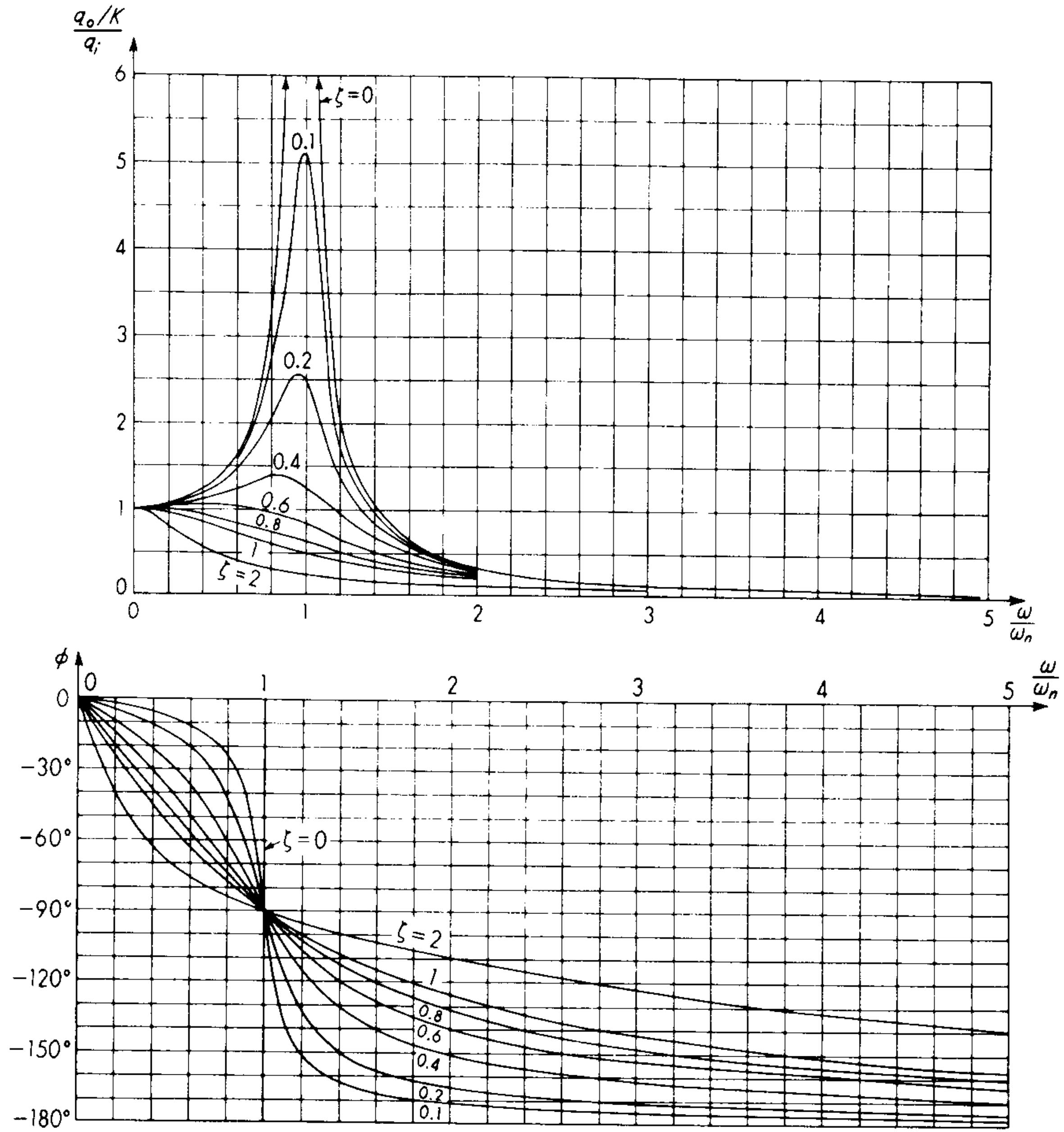
$$\frac{\mathbf{x}_0}{\mathbf{D}^2\mathbf{x}_\mathbf{i}}(\mathbf{D}) = \frac{\mathbf{x}_0}{\ddot{\mathbf{x}}_\mathbf{i}}(\mathbf{D}) = \frac{\mathbf{K}}{\mathbf{D}^2/\boldsymbol{\omega}_\mathbf{n}^2 + 2\zeta\mathbf{D}/\boldsymbol{\omega}_\mathbf{n} + 1} \qquad \mathbf{K} = \frac{1}{\boldsymbol{\omega}_\mathbf{n}^2}$$



(a)

Accelerometer

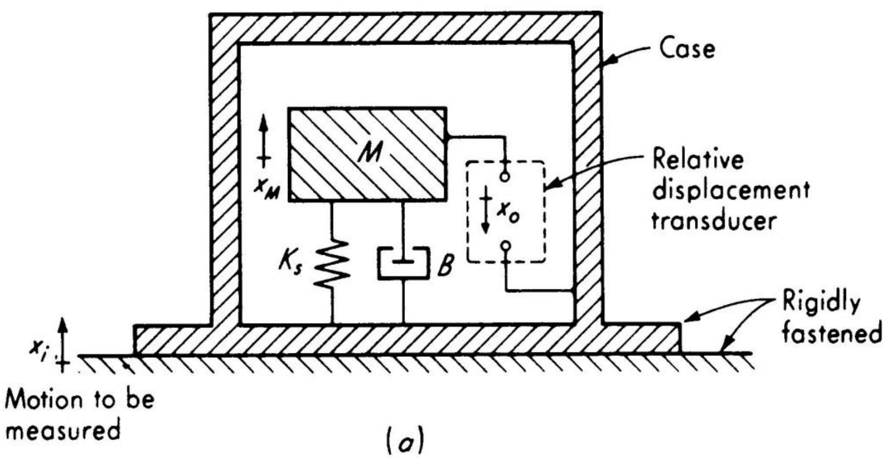
Damping ratio of 0.6 – 0.8 gives a flat response upto some fraction of natural frequency. Sensitivity is inversely proportional to square of natural frequency. For large flat response, low sensitivity. Phase angle is linearly dependent of frequency.

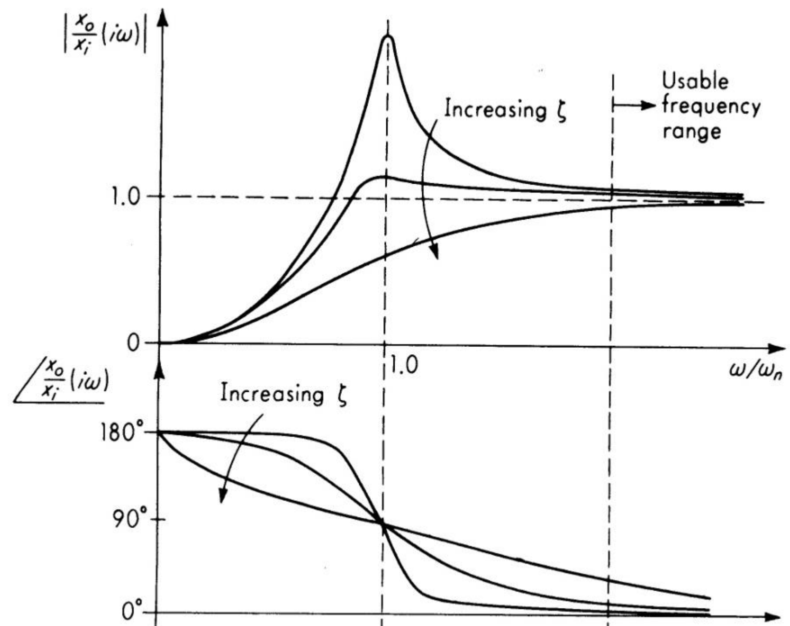


Accelerometer as Displacement Pickup

Desired output is displacement. For frequencies much above natural frequency the output is perfect measurement. For accurate displacement natural frequency of the sensor must be much small compared to lowest vibration frequency.

$$\frac{\mathbf{x}_0}{\mathbf{x}_\mathbf{i}}(\mathbf{D}) = \frac{\mathbf{D}^2/\omega_\mathbf{n}^2}{\mathbf{D}^2/\omega_\mathbf{n}^2 + 2\zeta\mathbf{D}/\omega_\mathbf{n} + 1} \qquad \omega_\mathbf{n} = \sqrt{\frac{\mathbf{K}_\mathbf{s}}{\mathbf{M}}} \qquad \zeta = \mathbf{B}/2\sqrt{\mathbf{K}_\mathbf{s}\mathbf{M}}$$



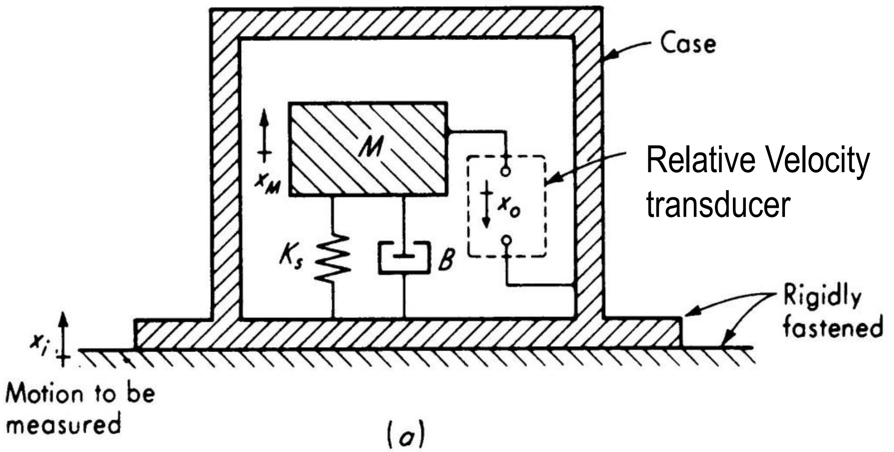


Accelerometer as Velocity Pickup

If the sensing element is dependent on velocity, then the relative displacement sensor described earlier can be used. Response will be same as the displacement sensor i.e. output will be flat for $\omega >> \omega_n$
Required displacement can be obtained by integrating.

$$\boxed{M\ddot{x}_o + B\dot{x}_o + K_s x_o = M\ddot{x}_i}$$

$$\frac{\dot{\mathbf{x}}_0}{\dot{\mathbf{x}}_\mathbf{i}}(\mathbf{D}) = \frac{\mathbf{D}^2/\boldsymbol{\omega}_\mathbf{n}^2}{\mathbf{D}^2/\boldsymbol{\omega}_\mathbf{n}^2 + 2\zeta\mathbf{D}/\boldsymbol{\omega}_\mathbf{n} + 1}$$



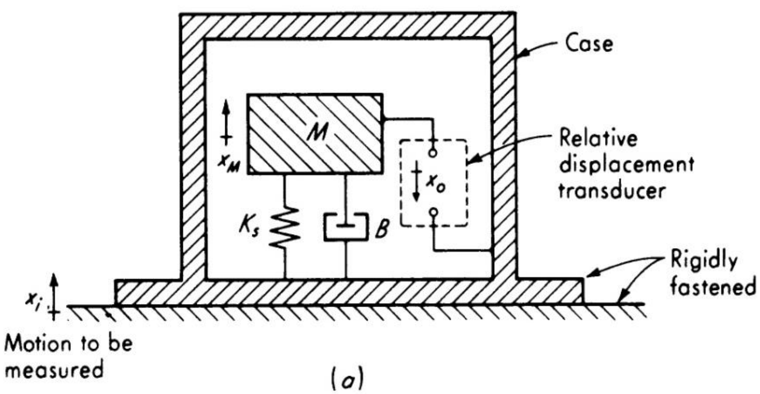
(a)

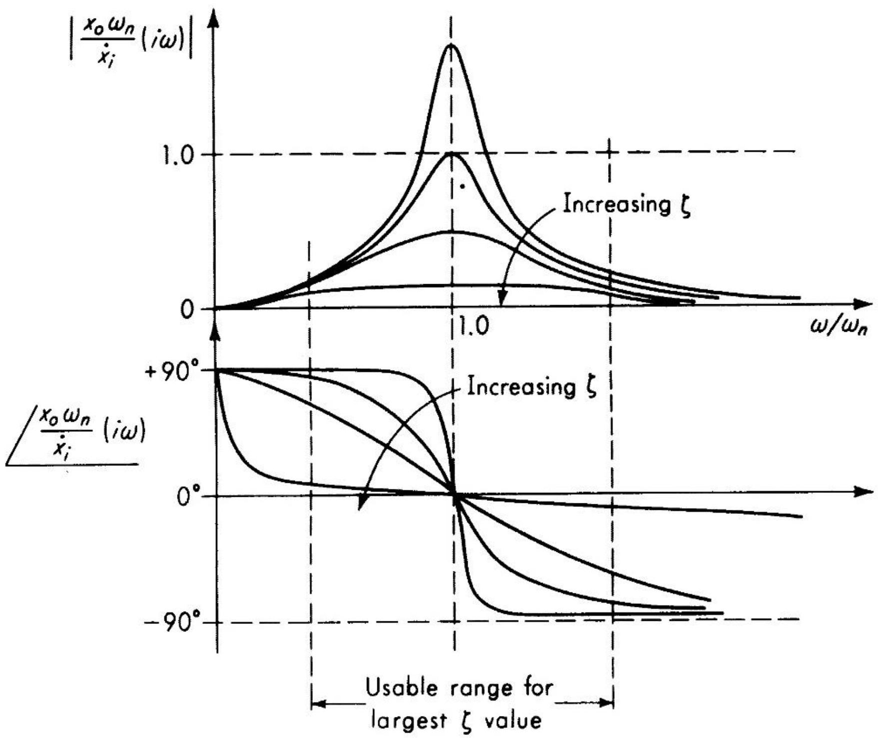
Accelerometer as Velocity Pickup

Velocity can be obtained by differentiation of displacement or integration of acceleration, both have problems in some form. If the damping is made large, output remains constant for large frequency variation. This also reduces sensitivity. It is meant for vibratory velocities.

$$\boxed{M\ddot{x}_o + B\dot{x}_o + K_s x_o = M\ddot{x}_i}$$

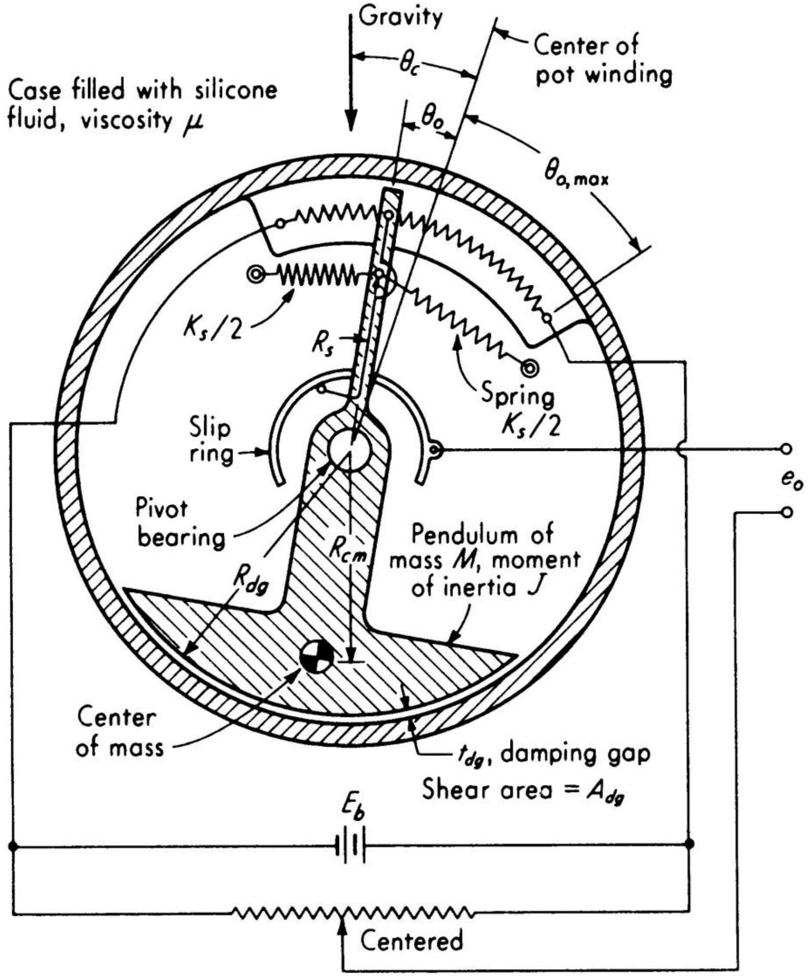
$$\frac{\mathbf{x}_0}{\mathbf{D}\mathbf{x}_\mathbf{i}}(\mathbf{D}) = \frac{\mathbf{x}_0}{\dot{\mathbf{x}}_\mathbf{i}}(\mathbf{D}) = \frac{\mathbf{D}}{\mathbf{D}^2 + 2\zeta\omega_\mathbf{n}\mathbf{D} + \omega_\mathbf{n}^2}$$



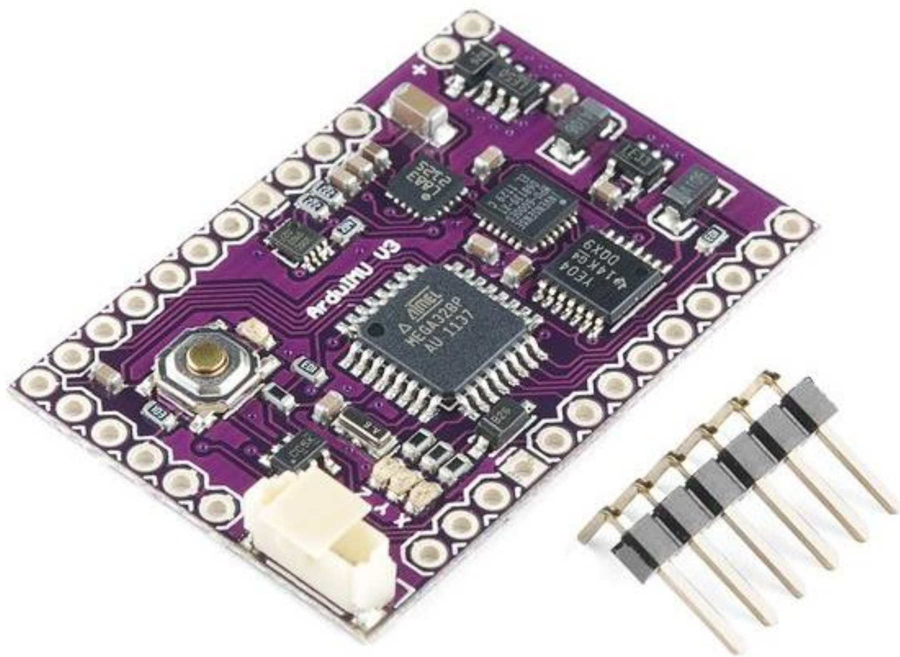


Pendulous Angular Displacement Sensor

Angular displacement relative to local vertical can be found. Good for non rotating and accelerating motion. Springs create a notch filter characteristics for dynamic motion.

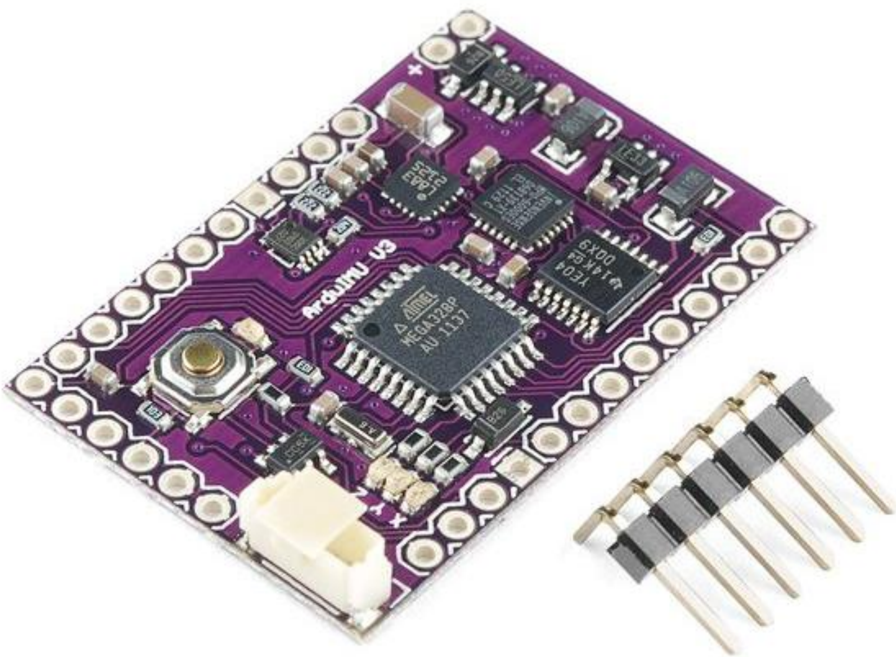


Displacement and velocity estimation using Inertial Measurement Unit



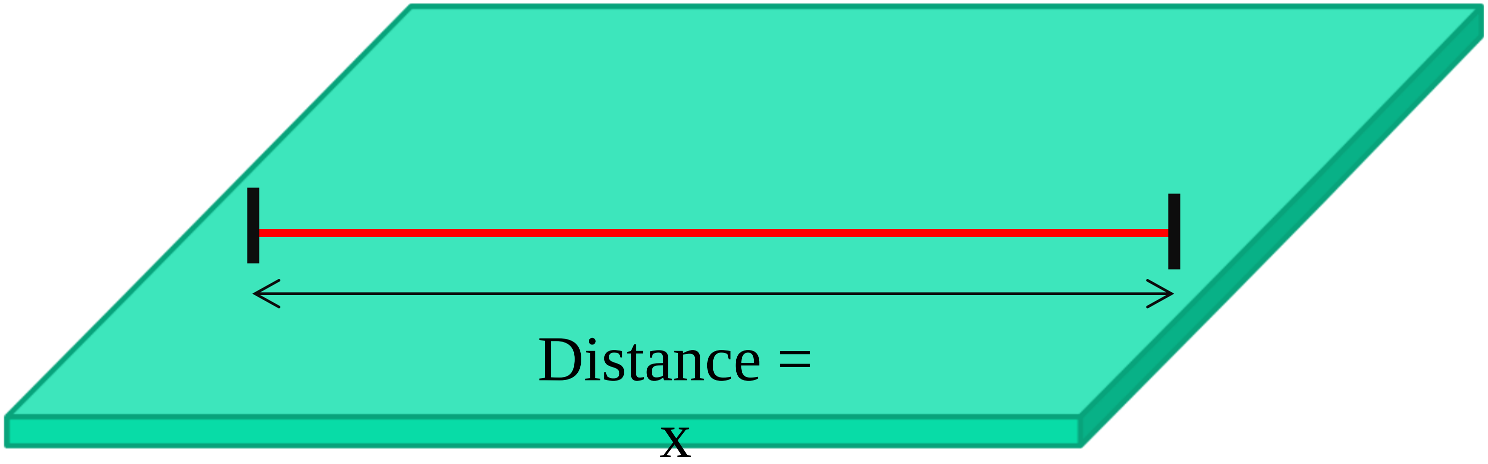
3 axis – accelerometer
3 axis – gyroscope
3 axis - Magnetometer

Displacement and velocity estimation using Inertial Measurement Unit

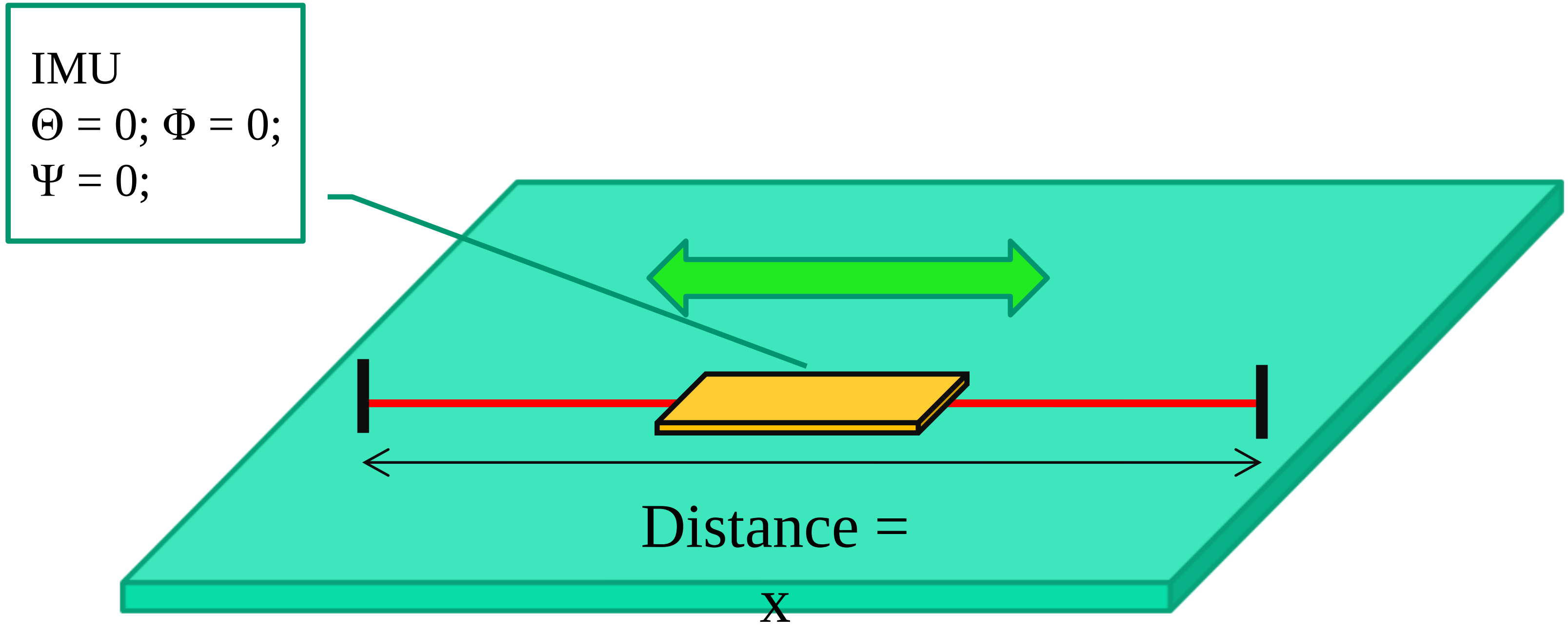


3 axis – accelerometer: Calibration

Displacement and velocity estimation using Inertial Measurement Unit



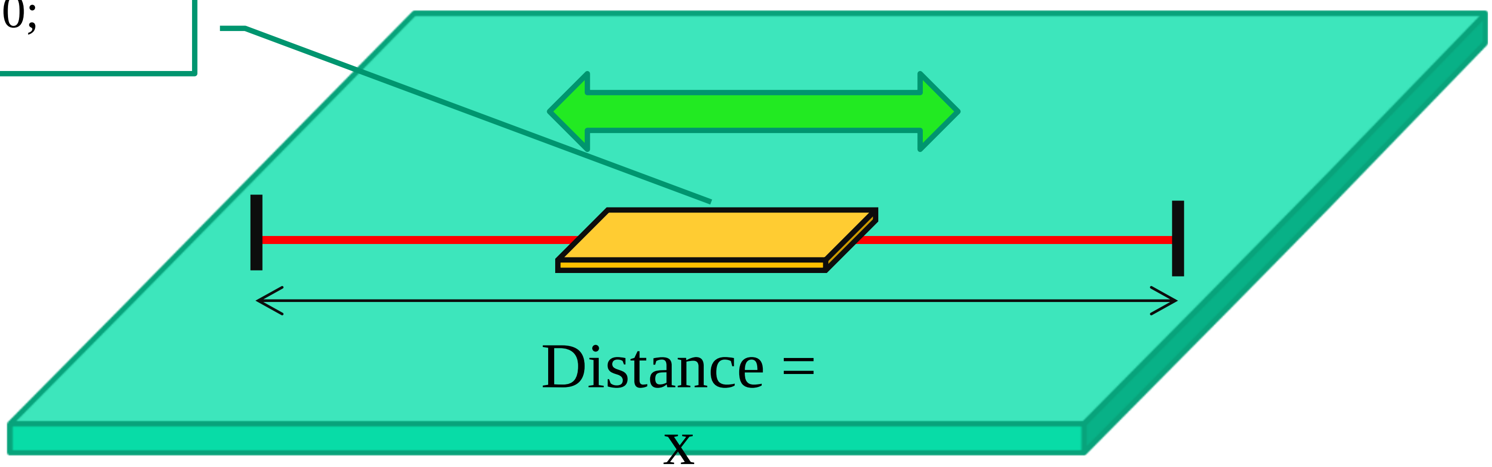
Displacement and velocity estimation using IMU

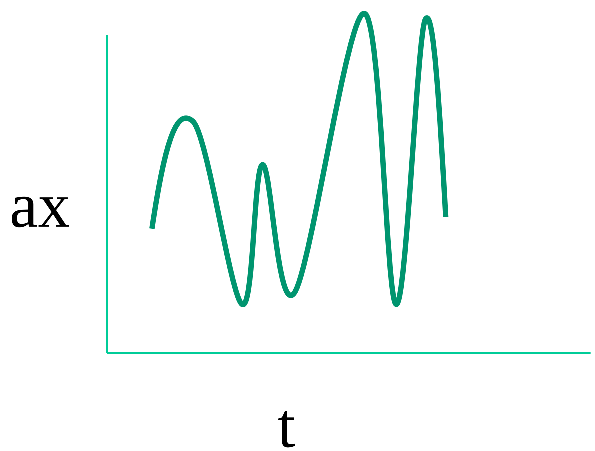


Displacement and velocity estimation using IMU

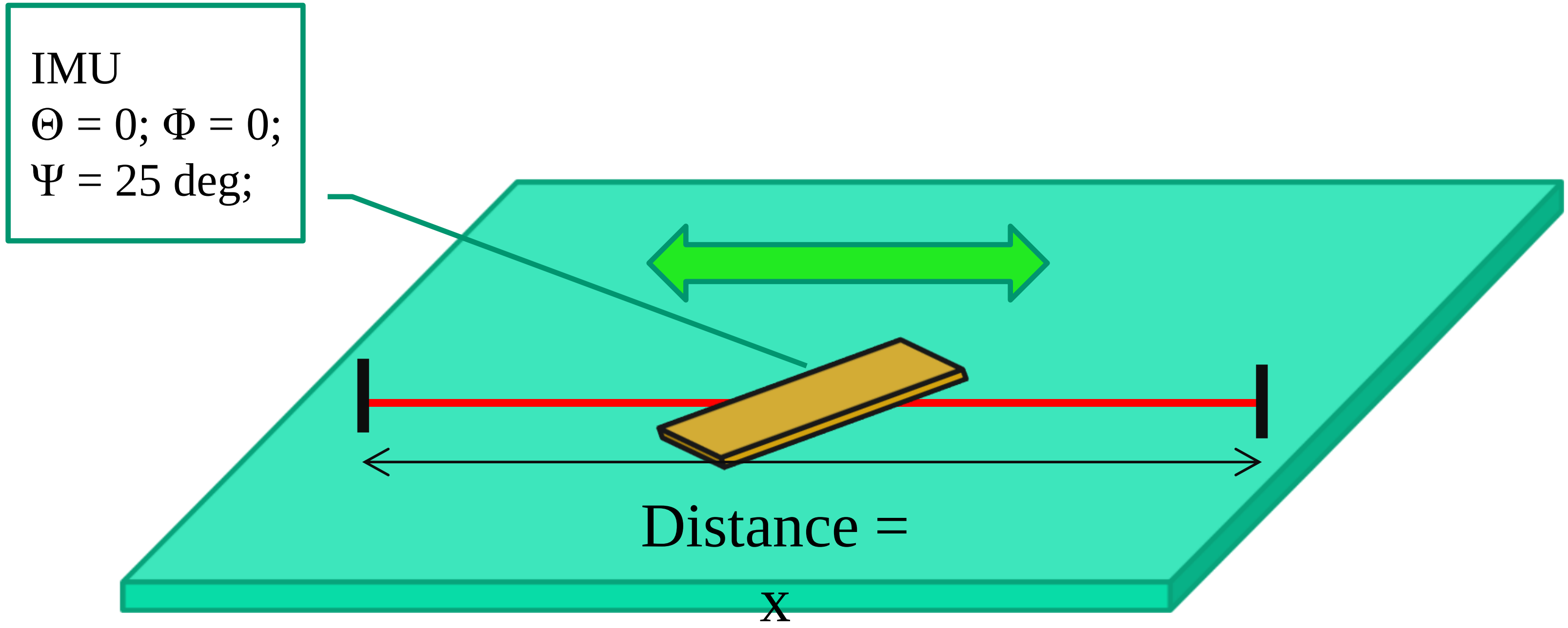
X- axis of the sensor is aligned with the direction of motion

IMU
$\Theta = 0; \Phi = 0;$
$\Psi = 0;$



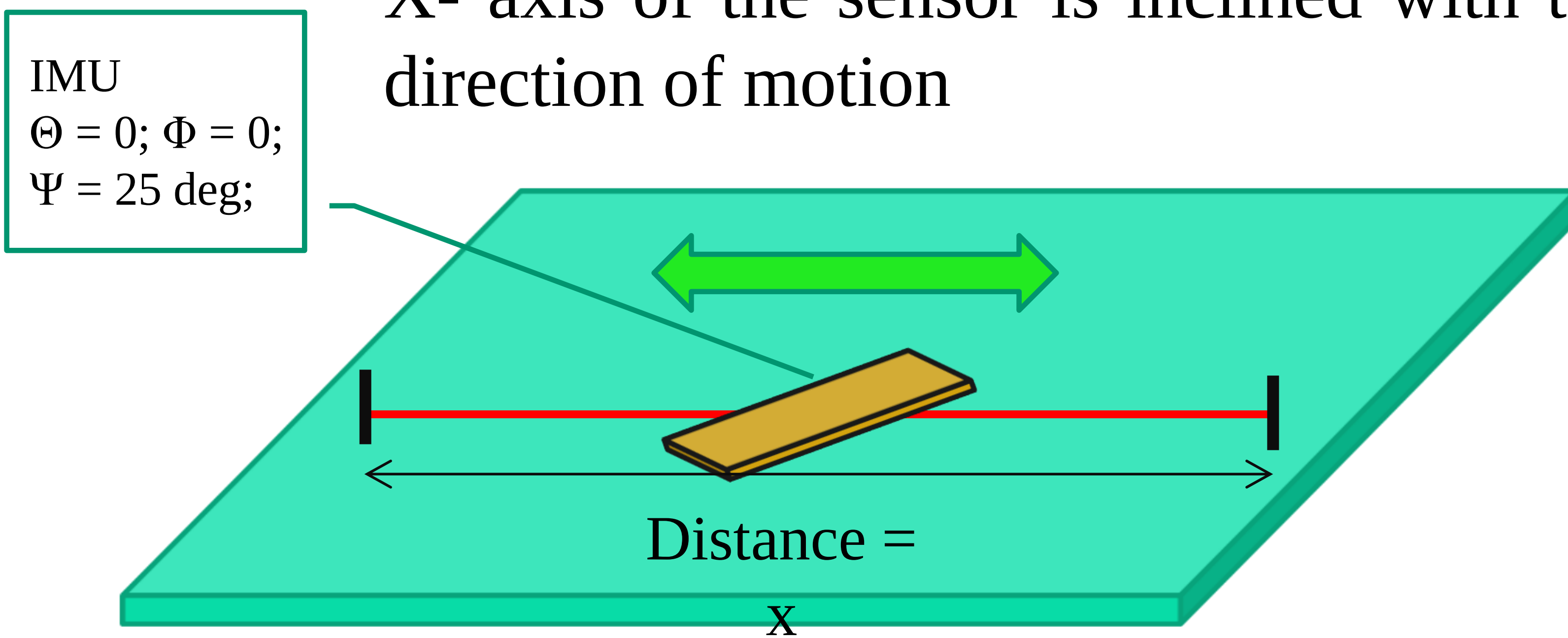


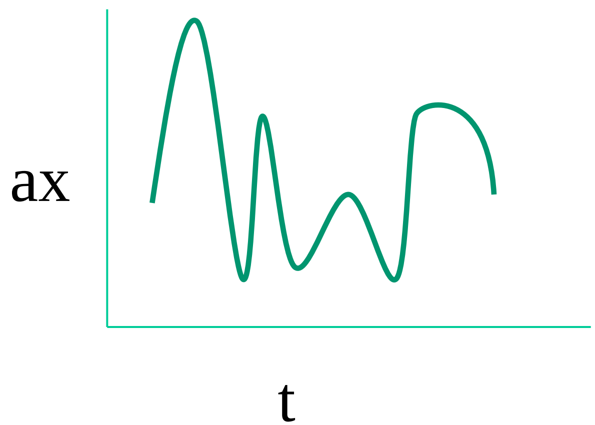
Displacement and velocity estimation using IMU

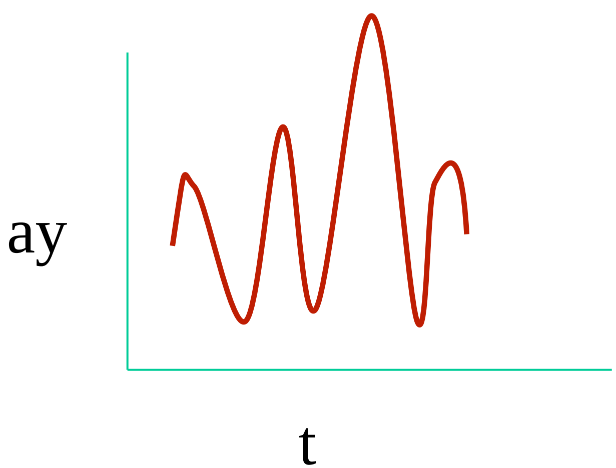


Displacement and velocity estimation using IMU

X- axis of the sensor is inclined with the direction of motion





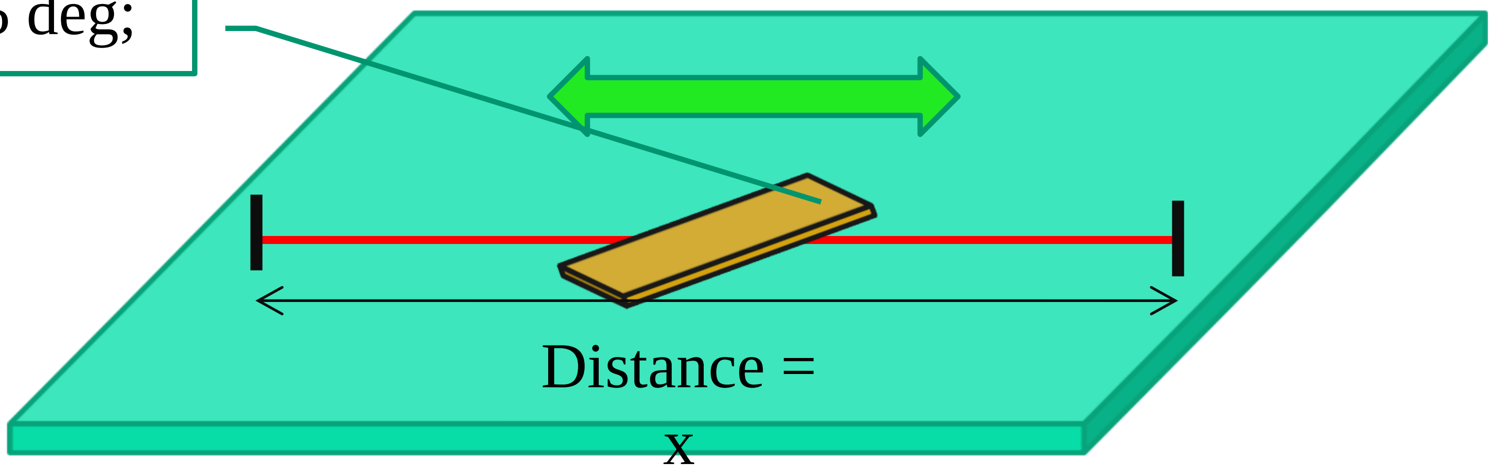


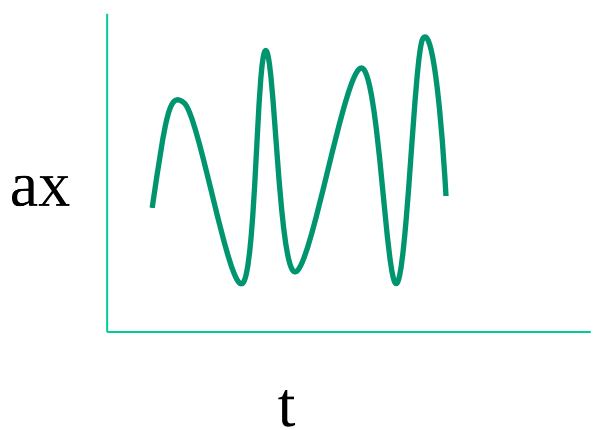
Displacement and velocity estimation using IMU

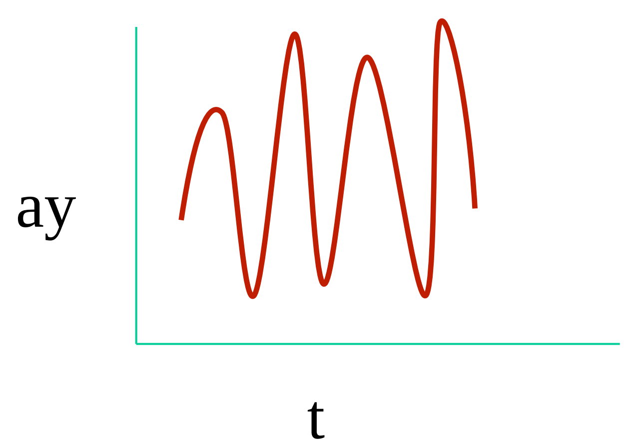
$$\ddot{x}(t) = a(t) \qquad\qquad \dot{x}(N) = ?$$

$$\ddot{x}(N) = a(N) \quad \text{Discrete Sampling time } \Delta t \qquad x(N) = ?$$

IMU
$\Theta = 0$; $\Phi = 0$;
$\Psi = 25$ deg;







Solution methods for ODEs

Set of simultaneous differential equations – linear or non-linear

Solution methods : a) Analytically

b) Analog computation (old method)

c) Digital simulation

Analytical solutions are limited to linear equations and very few non-linear problems. These solutions are used for verification of the models.

Analog computation was popular when digital computers were not available. Linear problems are easy to solve. Simulation setup is complex. Limited by saturation voltages.

Solution methods for ODEs

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{t}, \mathbf{x}) \qquad \mathbf{x}(\mathbf{t}_0) = \mathbf{x}_0$$

First order initial value problem

where $\mathbf{x}(\mathbf{t}) = [\mathbf{x}_1(\mathbf{t}), \mathbf{x}_2(\mathbf{t}), \ldots\ldots, \mathbf{x}_\mathbf{n}(\mathbf{t})]$ states of the system

Initial conditions $\mathbf{x}(0) = [\mathbf{x}_1(0), \mathbf{x}_2(0), \ldots\ldots, \mathbf{x}_\mathbf{n}(0)]$

In general n^{th} order differential equation can be converted into n first order differential equation

Solution methods for ODEs – Euler Method

In limit $\dfrac{\mathbf{dx}}{\mathbf{dt}} = \mathbf{f}(\mathbf{t}, \mathbf{x})$ Equals to $\lim\limits_{\mathbf{h}\to 0} \dfrac{\mathbf{x}(\mathbf{t}+\mathbf{h}) - \mathbf{x}(\mathbf{t})}{\mathbf{h}} = \mathbf{f}(\mathbf{t}, \mathbf{x})$

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If h (time step) is small, above function can be approximated as

$$\mathbf{x}(\mathbf{t}+\mathbf{h}) \approx \mathbf{x}(\mathbf{t}) + \mathbf{h}\mathbf{f}(\mathbf{t}, \mathbf{x})$$

Solution methods for ODEs – Euler Method

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If h (time step) is small, above function can be approximated as

$$\mathbf{x}(\mathbf{t}+\mathbf{h}) \approx \mathbf{x}(\mathbf{t}) + \mathbf{hf}(\mathbf{t},\mathbf{x})$$

Let $\mathbf{t} = \mathbf{t}_\mathbf{k} = \mathbf{hk} + \mathbf{t}_0$ where $\mathbf{t}_0 \le \mathbf{t} \le \mathbf{t}_\mathbf{n}$ $\mathbf{k} = 0,1,2,\ldots,\mathbf{n}$

Above equation reduces to

$$x(h(k+1)+t_0) \approx x(hk+t_0) + hf\left[hk+t_0, x(hk+t_0)\right]$$

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Above equation reduces to

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Introducing new variable x(k)

$$\boxed{x(k+1) = x(k) + hf\left[t(k), x(k)\right]}$$

Value at k+1 is value at k **+** step x slope at k