AE 242
Aerospace Measurements Laboratory

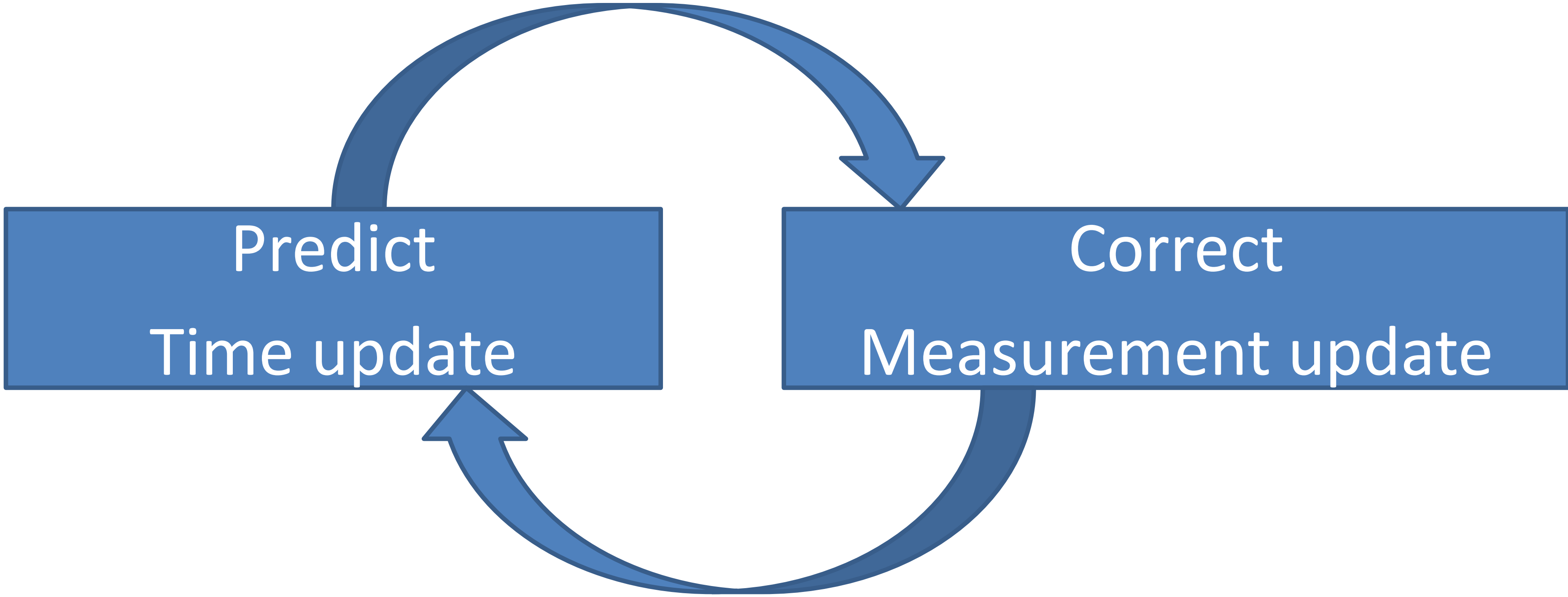
Kalman Filter

Kalman filter is an state estimator. It fuses states from dynamical model and measurement to give best estimates. It is applicable to linear system.

First major application was in Apollo space navigation. It is used in almost all the systems e.g. aerospace, powerplant, electronics, automobile, marine etc.

Other popular extension of Kalman filter for nonlinear systems are Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF) etc,

Kalman Filter



Using system model states can be predicted. It will have uncertainty due to modeling errors.

Measurement can be used to compare with estimated state and weighted difference is added to update the predicted state

Kalman Filter

Predict

- Propagate state
- Propagate error covariance

Measurement update

- Compute Kalman gain
- Update state with measurement
- Update error covariance

Kalman Filter

Let us take a linear discrete system

$$x_{k+1} = Ax_k + Bu_k + w_k$$

Using above equation state x_{k+1} can be obtained at $k+1$ instant from the state and input at k instant. w_k is process noise. A is transition matrix and B is control matrix.

$$y_k = Cx_k + z_k$$

y_k is measurement at k instant. C is output matrix and z_k is measurement noise.

$$P_k = AP_{k-1}A^T + S_w$$

Error covariance prediction (difference of estimated and updated states)

Kalman Filter

Based on the error covariance, Kalman gain is calulated

$$K_k = AP_kC^T\left(CP_kC^T + S_z\right)^{-1}$$

Error between measurement and predicted state also called as innovation

$$\boxed{innov = y_{k+1} - C\hat{x}_k}$$

Update state $$\boxed{\hat{x}_{k+1} = \left(A\hat{x}_k + Bu_k\right) + K_k\left(y_{k+1} - C\hat{x}_k\right)}$$

Update error covariance

$$P_{k+1} = AP_kA^T + S_w - AP_kC^TS_z^{-1}CP_kA^T$$

Kalman Filter

A vehicle moving in a straight line. Input is commanded acceleration. We can obtain velocity and position by integration. *T* is integration time step. Velocity:

$$v_{k+1} = v_k + Tu_k$$ Ideal – no noise

$$v_{k+1} = v_k + Tu_k + v_k^{\sim}$$ Real - noise

$v_k^{\sim}$ noise in velocity

Position :

$$p_{k+1} = p_k + Tv_k + \frac{1}{2}T^2 u_k + p_k^{\sim}$$

$p_k^{\sim}$ noise in position

Kalman Filter

Let the state vector be position and velocity

$$x_k = \begin{bmatrix} p_k \\ v_k \end{bmatrix}$$

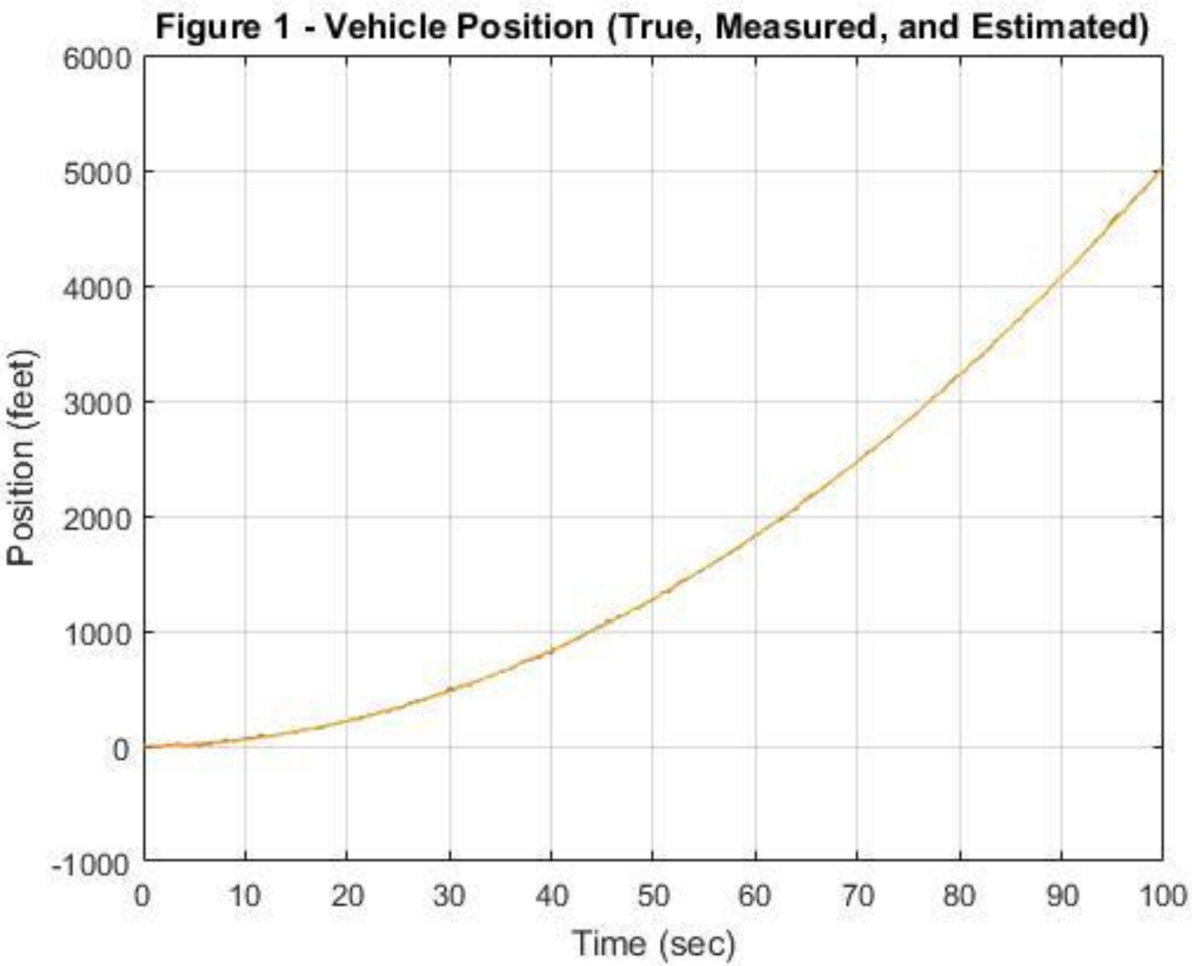
System equation can be written as

$$x_{k+1} = \begin{bmatrix} p_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} u_k + w_k$$

Measurement equation

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + z_k$$ Position measurement with noise

Kalman Filter



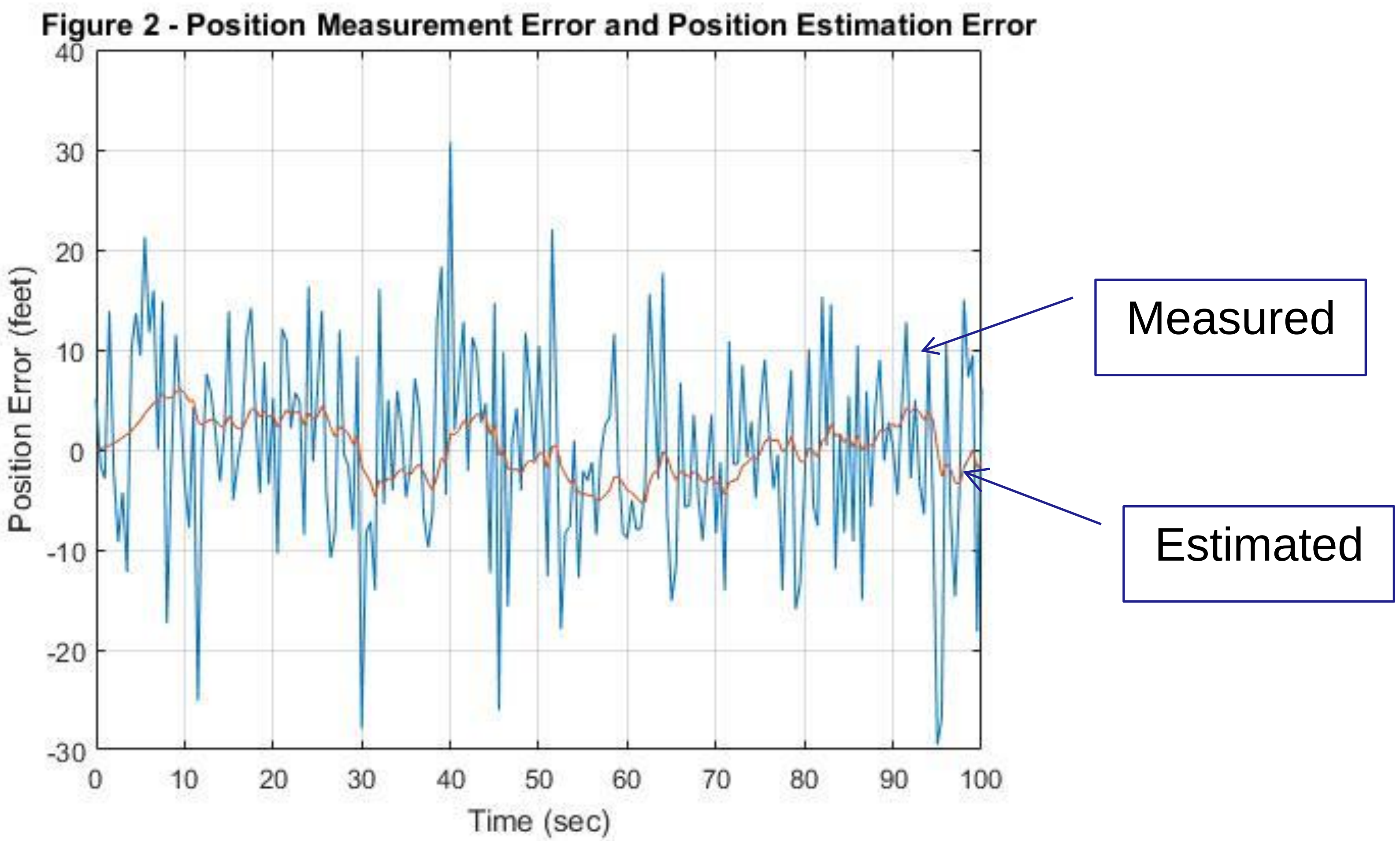
Don Simon, Kalman Filtering, June 2001, Embedded Systems Programming

U= 1 ft / square (sec)

Process model error: 0.2 feet/square(sec)

Position error model: 10 feet

Kalman Filter



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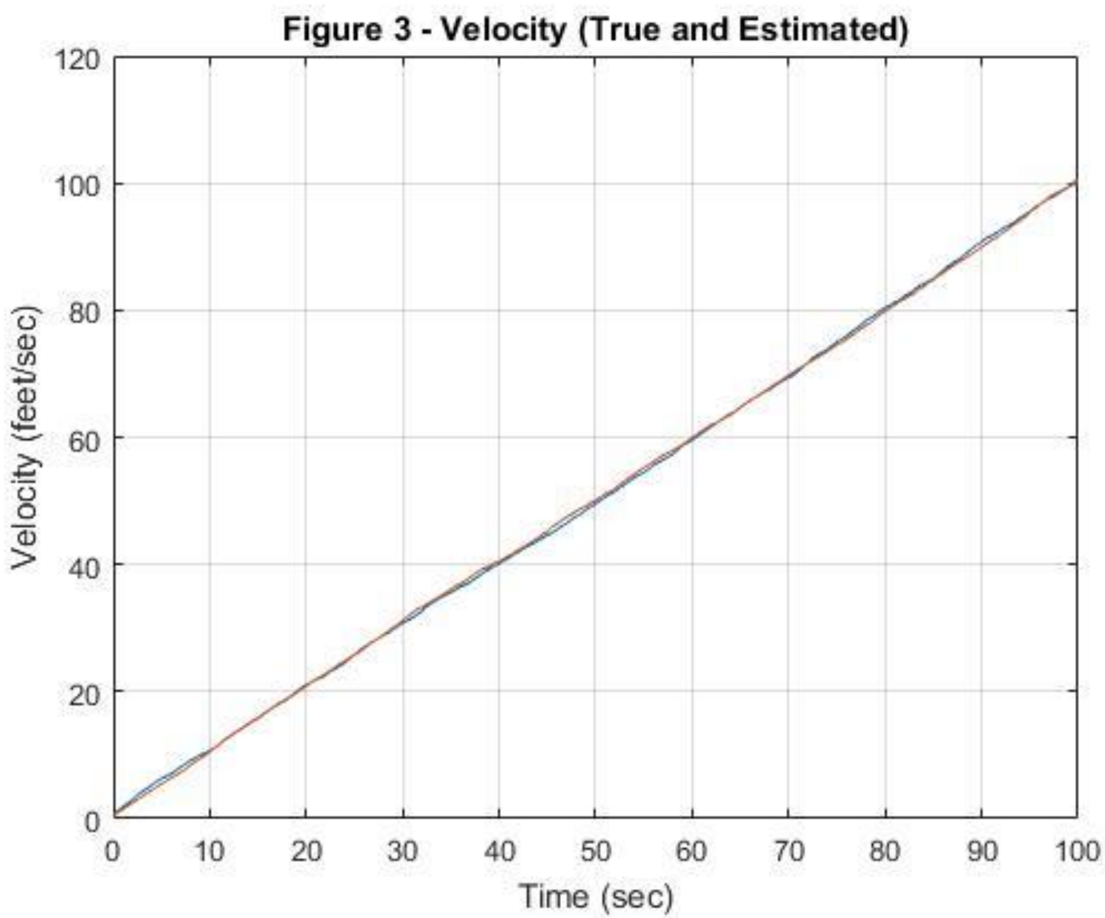
Kalman Filter



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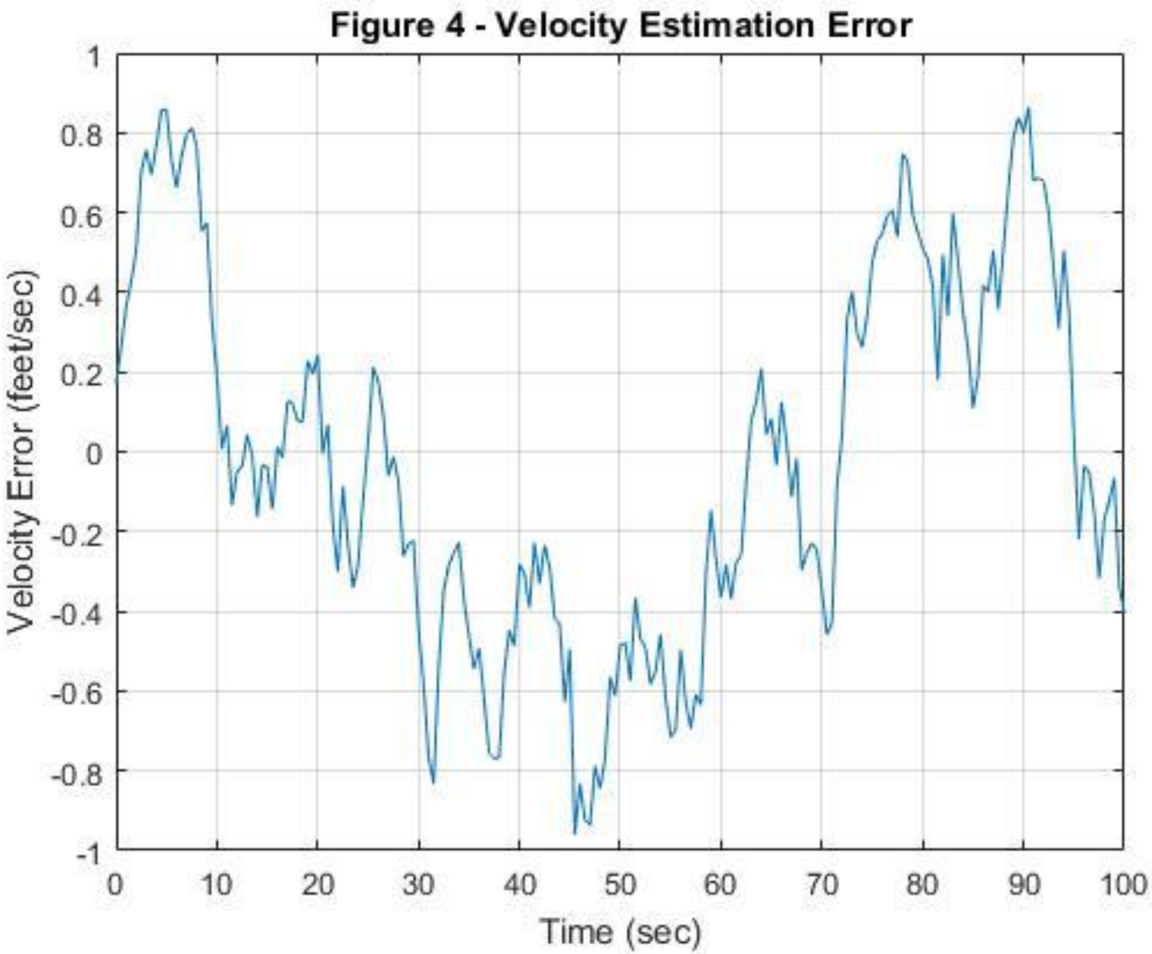
Kalman Filter



Process model error: 0.2 feet/square(sec)

Position error model: 10 feet

The Equations of Motion for INS

$$\dot{u} = a_x - qw + rv - g\sin\theta$$

$$\dot{v} = a_y - ru + pw + g\cos\theta\sin\phi$$

$$\dot{w} = a_z - pv + qu + g\cos\theta\cos\phi$$

$$\dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

$$\dot{\psi} = \sec\theta(q\sin\phi + r\cos\phi)$$

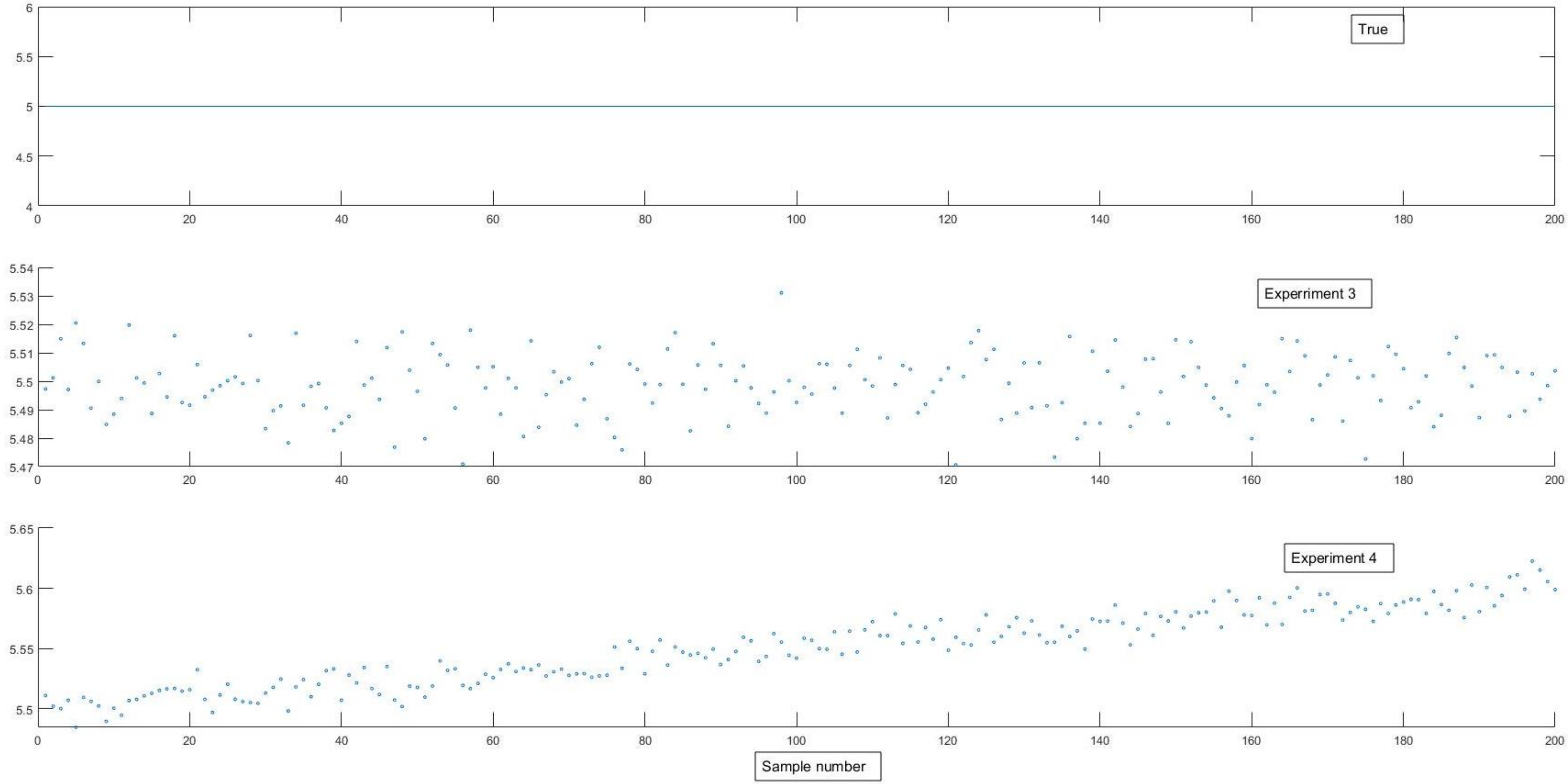
The Navigation equations

$$\left[v\right]_I = R_B^I \left[v\right]_B$$

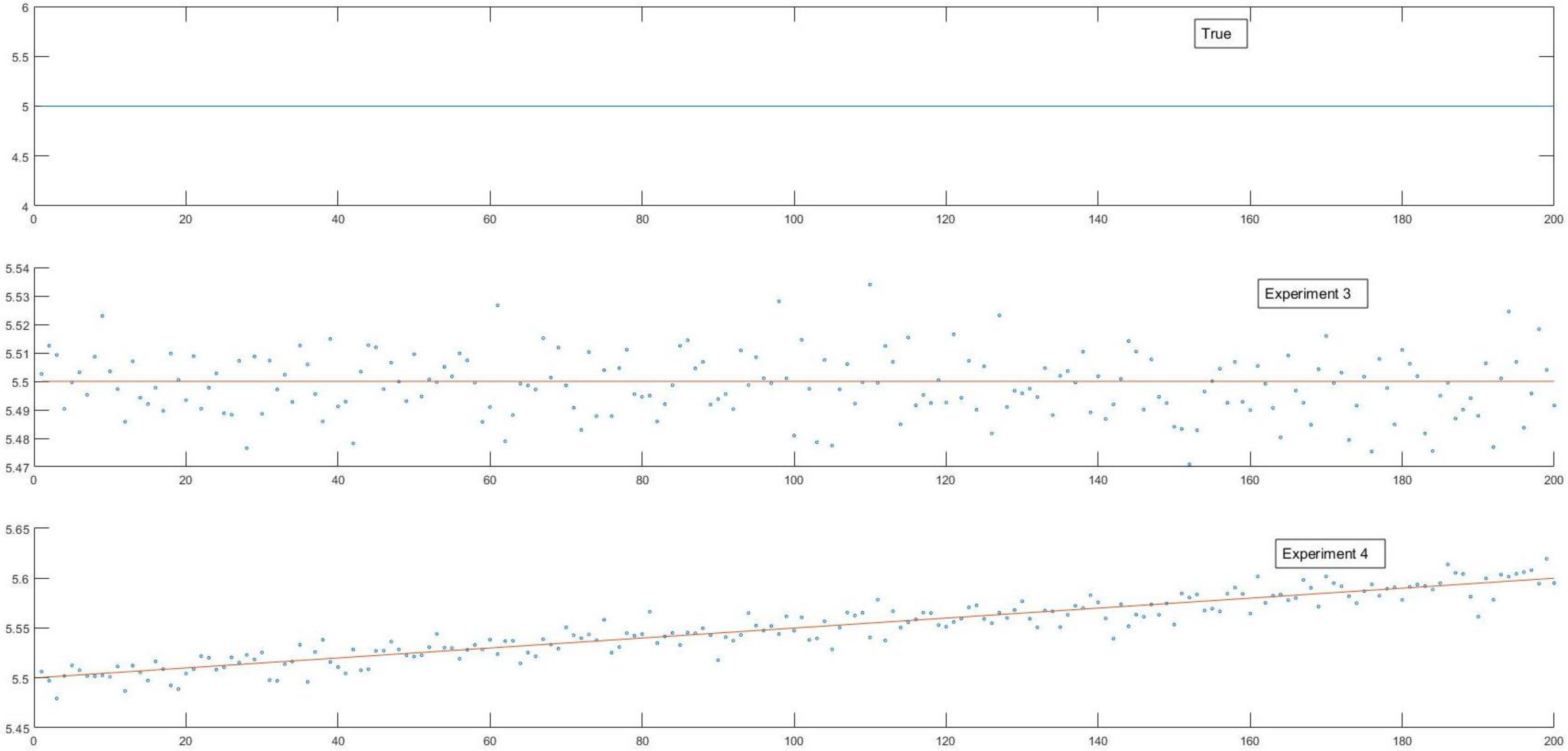
$$\dot{x} = u\cos\theta\cos\psi + v(\sin\varphi\sin\theta\cos\psi - \cos\phi\sin\psi) +$$
$$w(\cos\phi\sin\theta\cos\psi + \sin\varphi\sin\psi)$$

$$\dot{y} = u\cos\theta\sin\psi + v(\sin\theta\sin\phi\sin\psi + \cos\phi\cos\psi)$$
$$+ w(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)$$

$$\dot{z} = -u\sin\theta + v\sin\phi\cos\theta + w\cos\phi\cos\theta$$



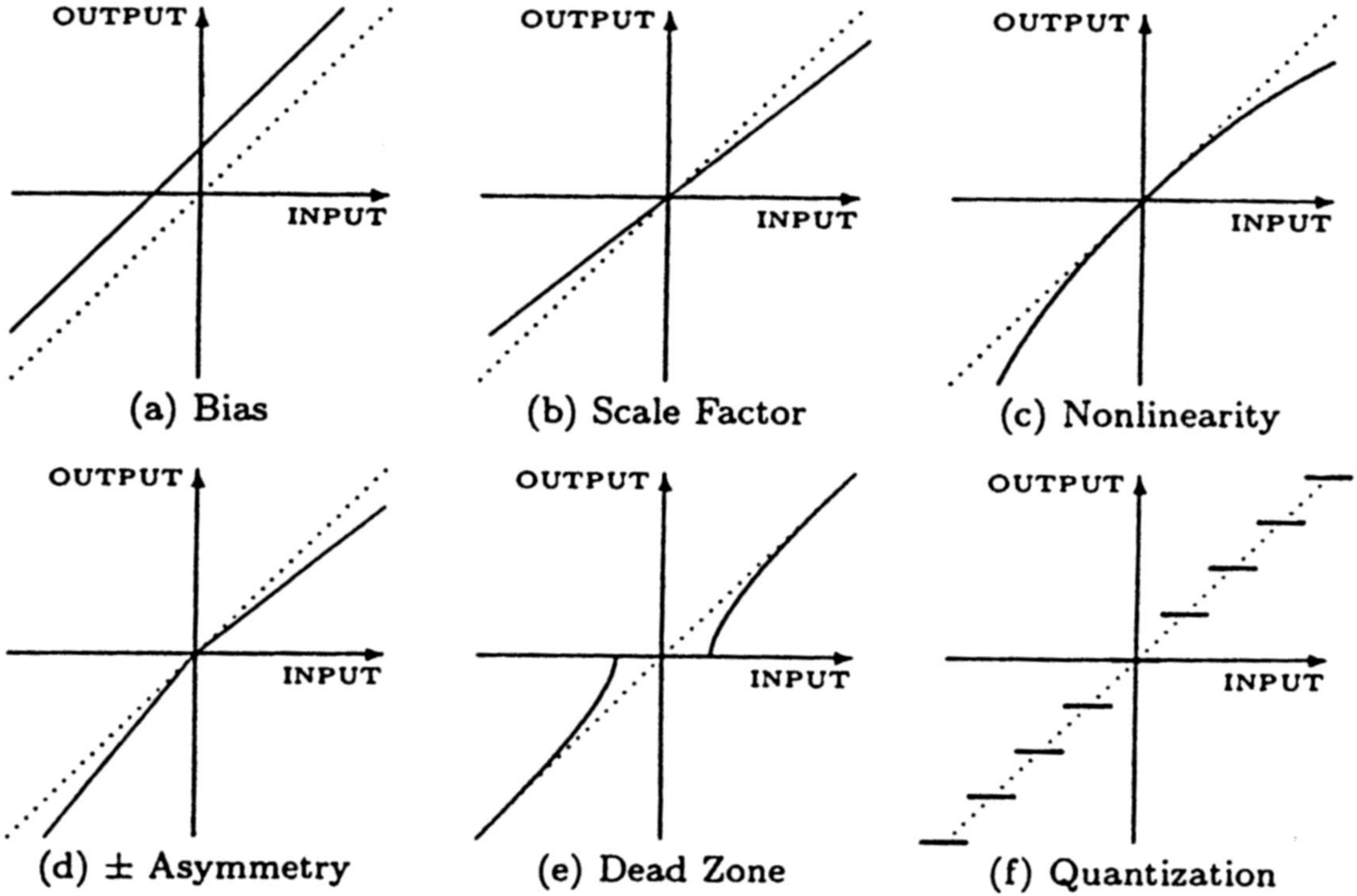
Two thermometers are used for measuring temperature. 200 samples are recorded when temperature input is 5 units. From the above two observations, what can be deduced about thermometers?



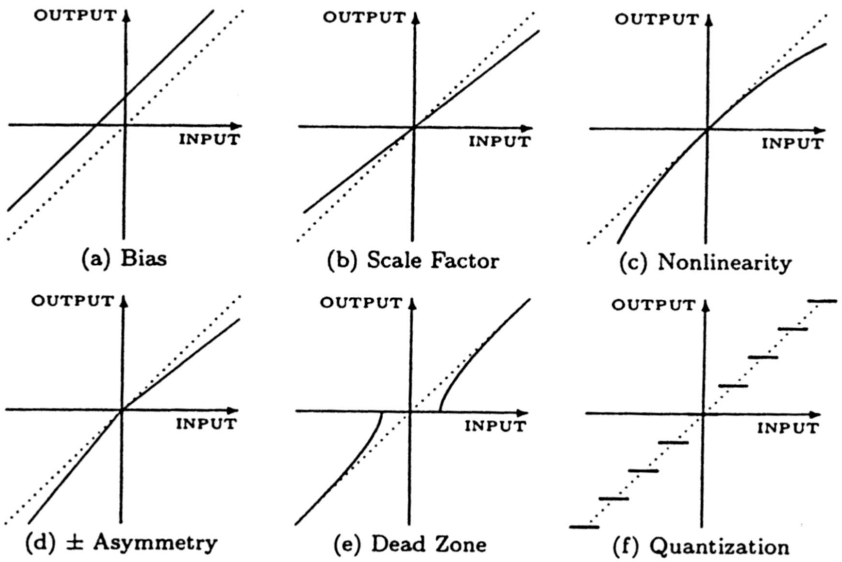
Bias and drift can be estimated by suitable curve fitting (static case).

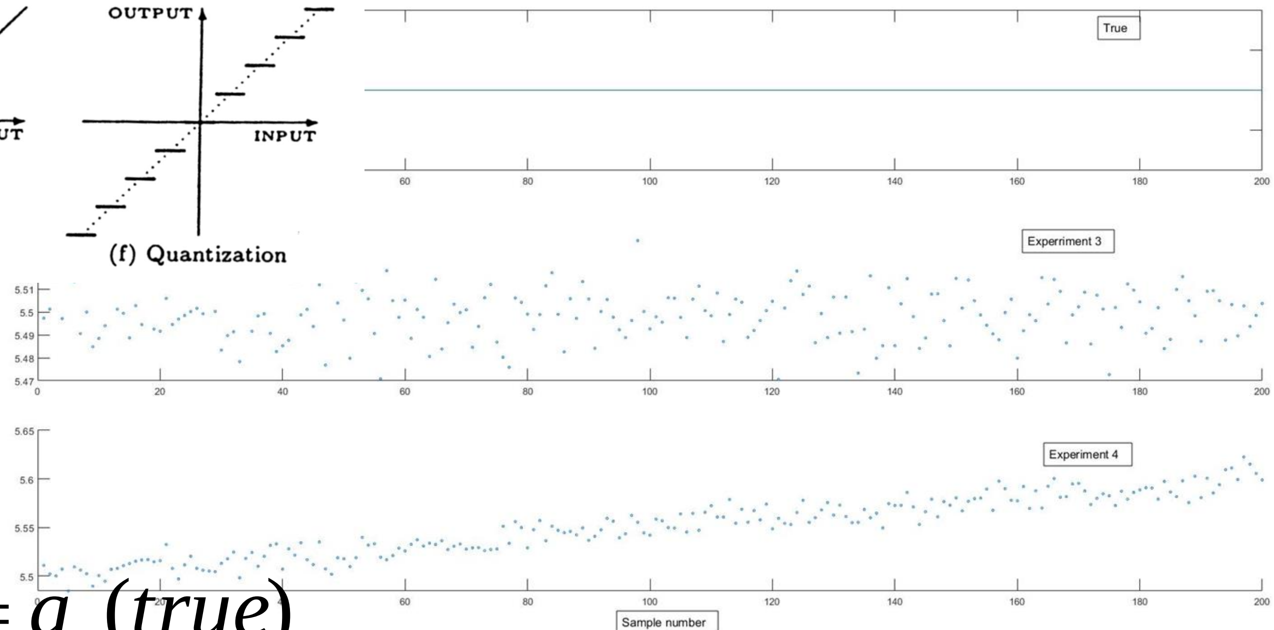
Bias and drift (time varying) can be estimated by using estimation methods like Kalman filter (Dynamic case).

Common error models - deterministic



Common error models





$$a_x(measured) = a_x(true)$$

$$+ \text{deterministic errors}$$

$$+ \text{stocahstic errors}$$

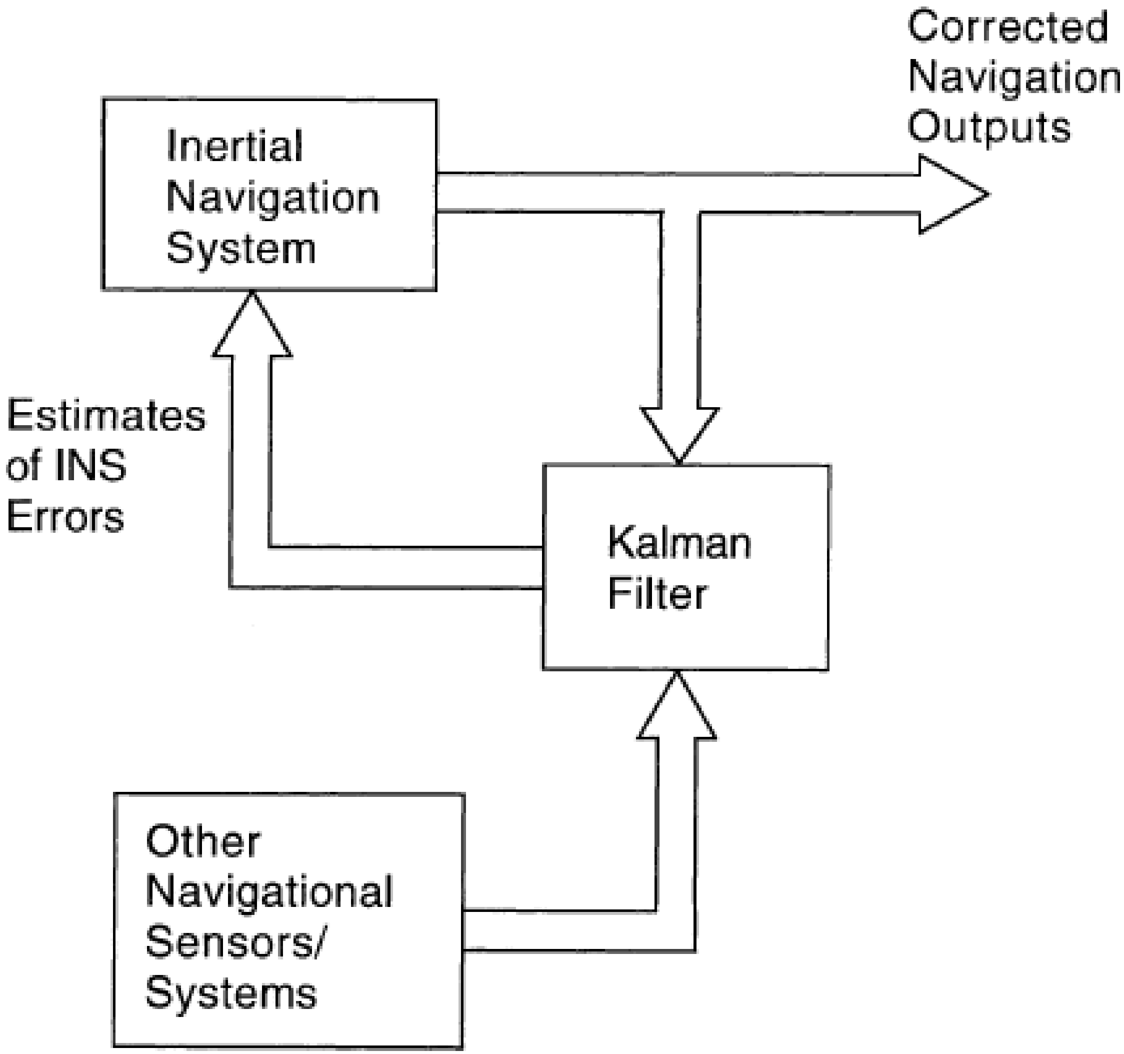


Fig. 6.21 Block diagram of aided IN system with Kalman filter.

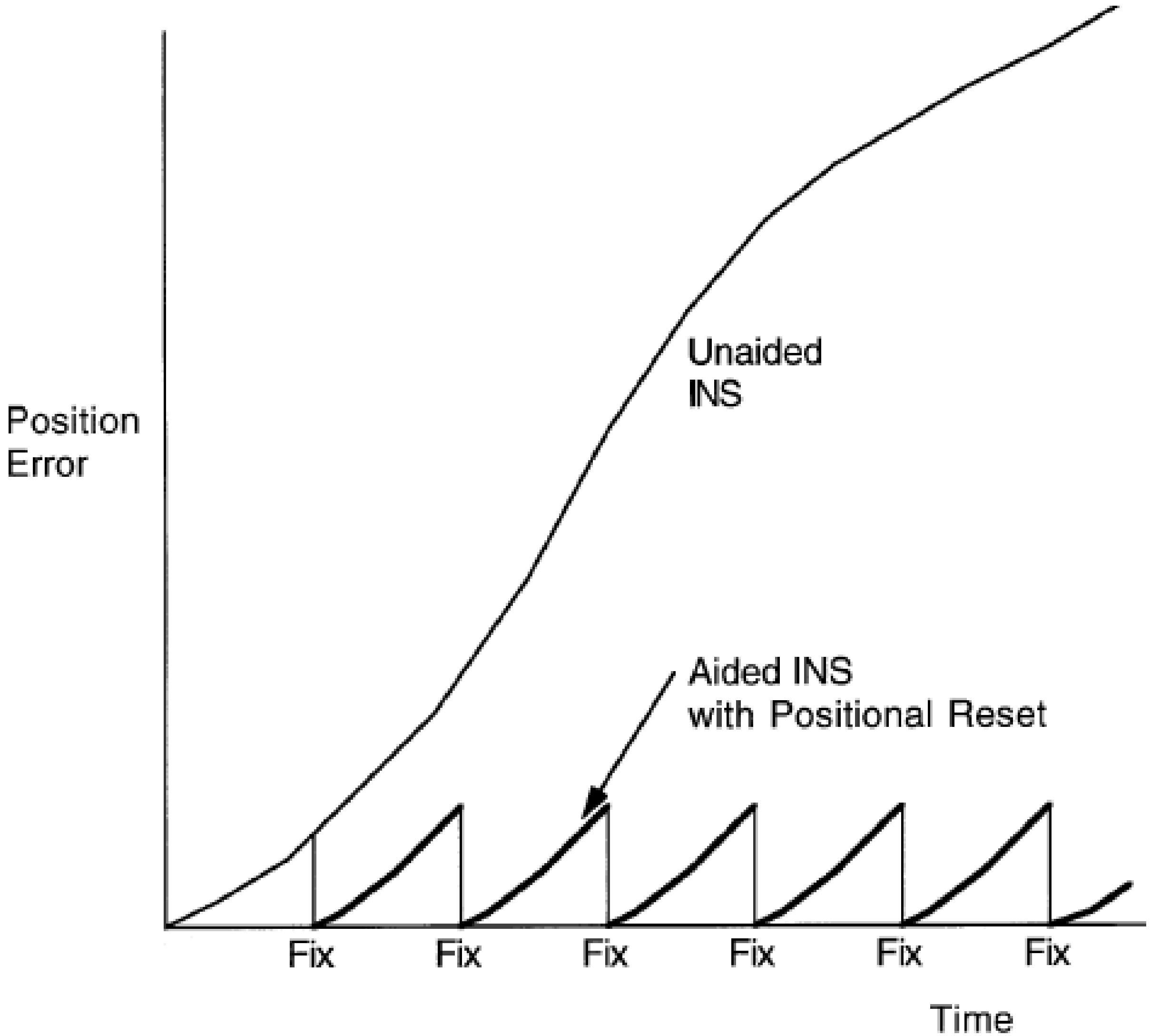


Fig. 6.20 Aided INS with simple positional reset.

Aiding

<u>Position based:</u>

GPS / NaVIC etc (Outdoor): Limited to satellite visibility; prone to spoofing.

Range measurement (indoor): UWB sensors; TOF; ultrasonic;

Vision: Processing camera pose and image of known object (position)

<u>Velocity:</u>

Odometry – Independent measurement of speed / velocity. Wheel speed; Optic flow;

Summary of the lectures

- What is measurement, instrumentation?
- Functional elements of a measurement system
- Standards
- Measurement of resistance
- Wheatstone bridge : Bending, tension, temperature compensation
- Dynamics characteristics
- Analog to digital conversion
- Operational amplifier

Summary of the lectures

- Digital electronics: Combinational and sequential
- GPS and its applications
- Inertial sensors
- Attitude measurement
- Air Data system
- Temperature measurement
- Kalman filter