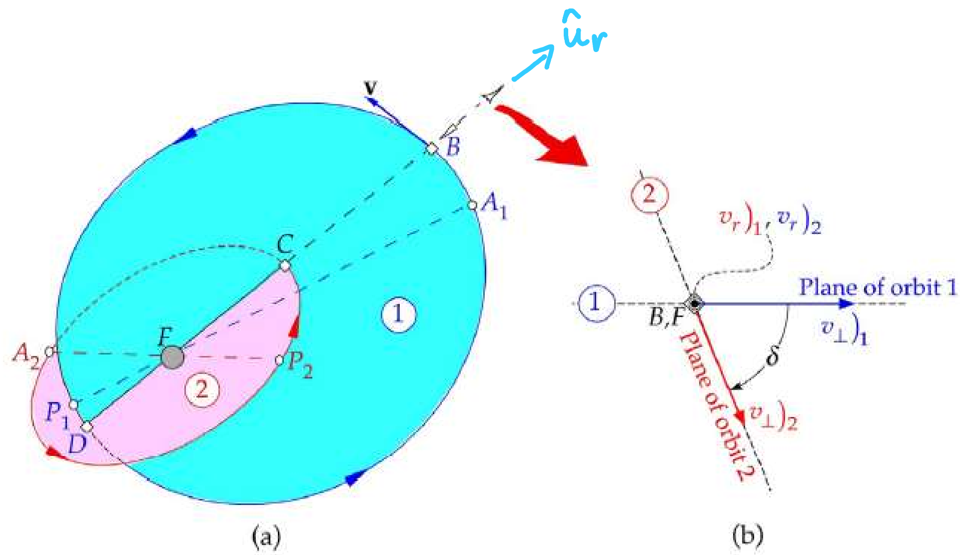
Plane Change Manoeuvres



- $\|V_1\| = V_{r_1}\hat{u}_r + V_{\perp_1}\hat{u}_{\perp_1}$
  $\|V_2\| = V_{r_2}\hat{u}_r + V_{\perp_2}\hat{u}_{\perp_2}$

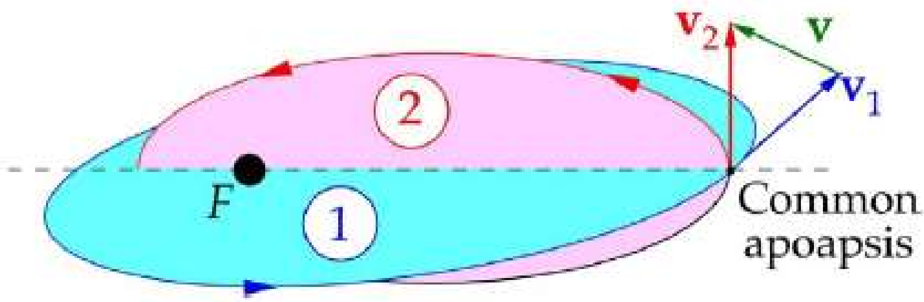
- $\Delta V = V_2 - V_1 = (V_{r_1} - V_{r_2})\hat{u}_r + V_{\perp_2}\hat{u}_{\perp_2} - V_{\perp_1}\hat{u}_{\perp_1}$

- $\|\Delta V\| = \sqrt{\Delta V \cdot \Delta V} = \sqrt{(V_{r_2} - V_{r_1})^2 + V_{\perp_1}^2 + V_{\perp_2}^2 - 2V_{\perp_1}V_{\perp_2}\cos\delta}$

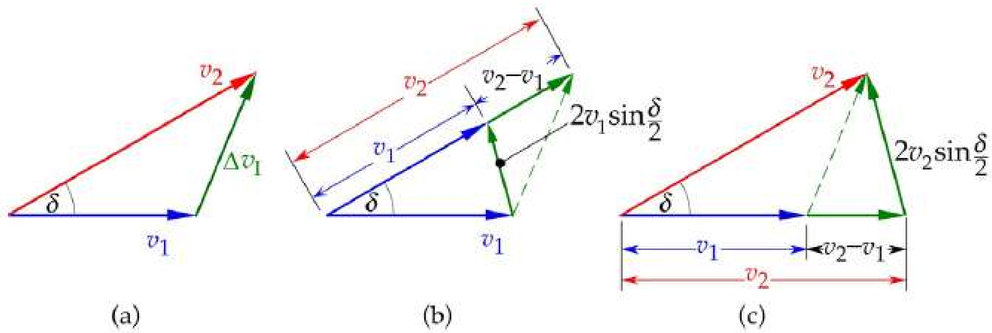
- $V_{r_1} = \|V_1\|\sin\gamma_1 \quad , \quad V_{\perp_1} = \|V_1\|\cos\gamma_1$
  $V_{r_2} = \|V_2\|\sin\gamma_2 \quad , \quad V_{\perp_2} = \|V_2\|\cos\gamma_2$

- $\|\Delta V\| = \sqrt{\|V_1\|^2 + \|V_2\|^2 - 2\|V_1\|\|V_2\|[\cos\Delta\gamma - \cos\gamma_2\cos\gamma_1(1-\cos\delta)]}$

- $\|\Delta V\| = \sqrt{\|V_1\|^2 + \|V_2\|^2 - 2\|V_1\|\|V_2\|\cos\Delta\gamma}$ $(\delta = 0°)$



- $\|\Delta V\| = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\delta}$ $(V_{r_1} = V_{r_2} = 0,\ V_{\perp_1} = V_1,\ V_{\perp_2} = V_2)$

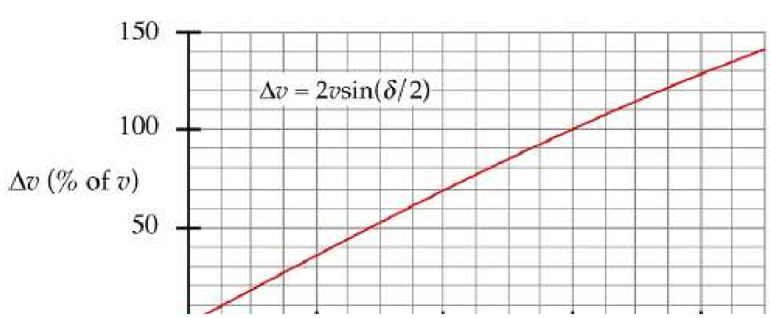


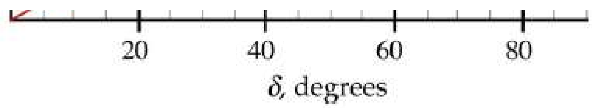
- $\Delta V_{\mathrm{I}} = \sqrt{(V_2 - V_1)^2 + 4V_1V_2\sin^2\delta/2}$

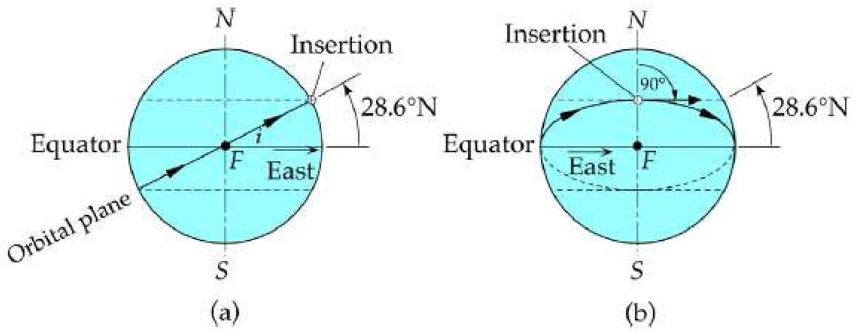
- $\Delta V_{\mathrm{II}} = 2V_1\sin\delta/2 + |V_2 - V_1|$

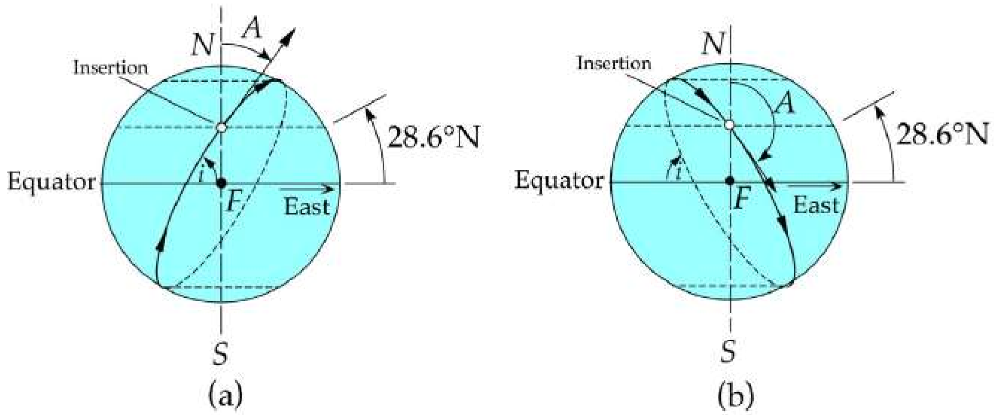
- $\Delta V_{\mathrm{III}} = |V_2 - V_1| + 2V_2\sin\delta/2$

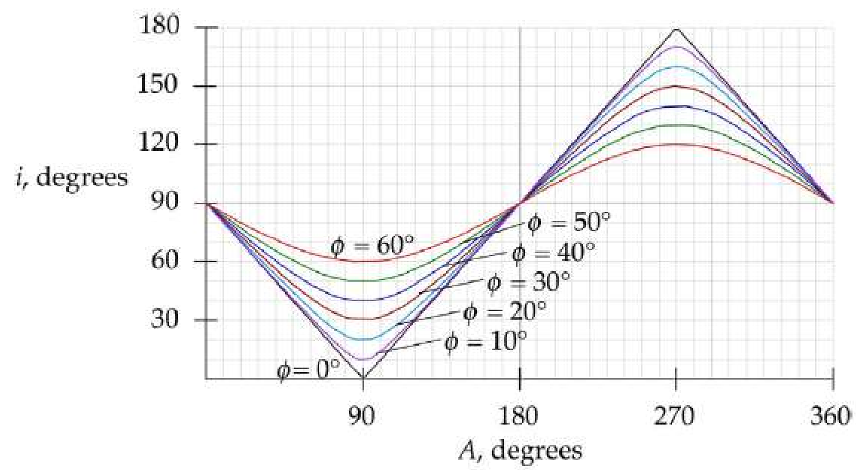
- $\Delta V_{\mathrm{II}} > \Delta V_{\mathrm{I}}$, $\Delta V_{\mathrm{III}} > \Delta V_{\mathrm{I}}$



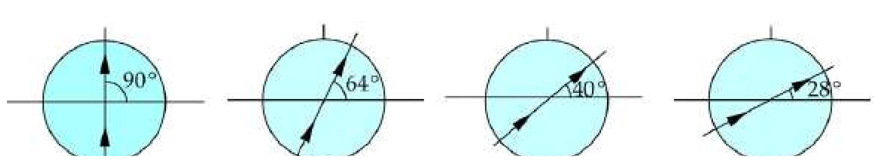


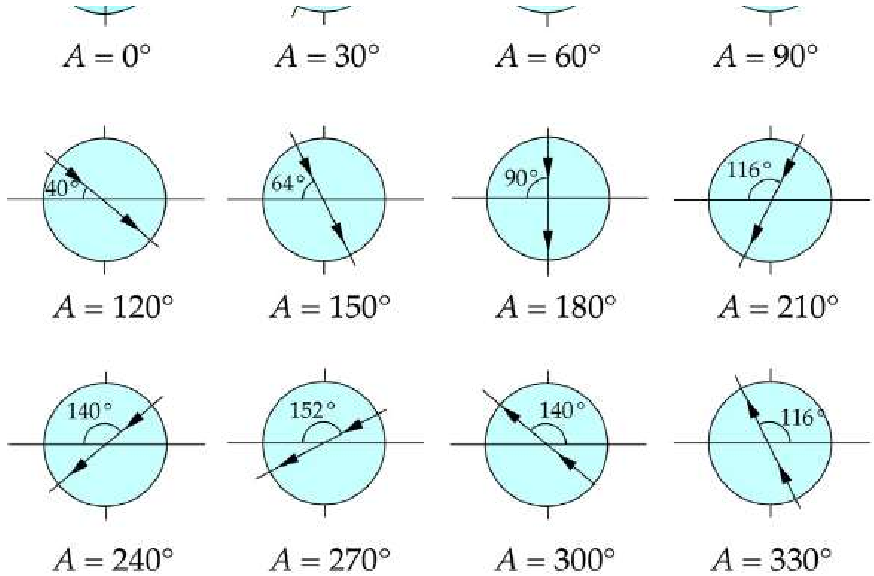






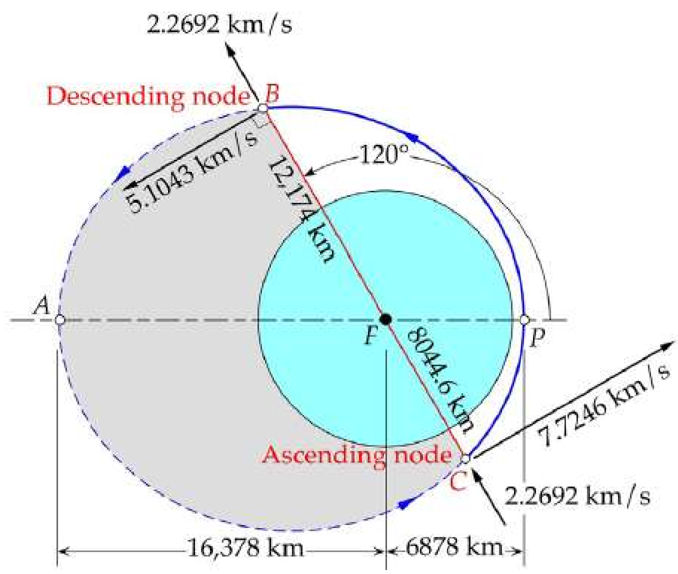
$\cos i = \cos\phi \sin A$





Example

A spacecraft is in a 500 km by 10,000 km altitude geocentric orbit that intersects the equatorial plane at a true anomaly of 120° (see Fig. 6.33). If the orbit's inclination to the equatorial plane is 15°, what is the minimum velocity increment required to make this an equatorial orbit?



Details

$$\|e\| = \frac{r_A - r_p}{r_A + r_p} \quad , \quad \|h\| = \sqrt{2\mu}\sqrt{\frac{r_A r_p}{r_A + r_p}}$$

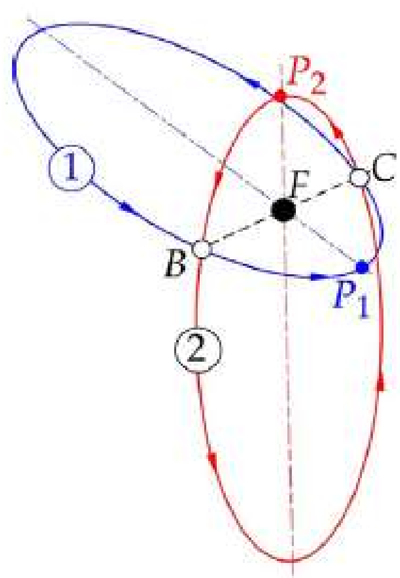
$$r_B = \frac{\|h\|^2}{\mu}\frac{1}{1+\|e\|\cos\theta_B}, \quad V_{\perp B} = \frac{\|h\|}{r_B}, \quad V_{r_B} = \frac{\mu}{\|h\|}\sin\theta_B$$

$$r_C = \frac{\|h\|^2}{\mu}\frac{1}{1+\|e\|\cos\theta_C}, \quad V_{\perp} = \frac{\|h\|}{r_C}, \quad V_{r_C} = \frac{\mu}{\|h\|}\sin\theta_C$$

$$\Delta V = 2V_{\perp}\sin\delta/2$$

Example

Orbit 1 in Fig. 6.34 has angular momentum h and eccentricity e. The direction of motion is shown. Calculate the Δv required to rotate the orbit 90° about its latus rectum BC without changing h and e. The required direction of motion in orbit 2 is shown.



Details

$$\theta_B)_1 = -90^\circ, \quad \theta_B)_2 = 90^\circ$$

$$r_B = \frac{h^2}{\mu}$$

$$V_{\perp B})_1 = \frac{h}{r_B} = \frac{\mu}{h}$$

$$V_{r_B})_1 = \frac{\mu}{h} e \sin \theta_B)_1 = -\frac{\mu e}{h}$$

$$V_{r_B})_2 = \frac{\mu}{h} e \sin \theta_B)_2 = \frac{\mu e}{h}$$

$$\Delta V_B = \ldots$$