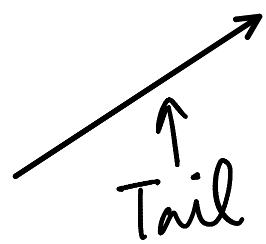
What is a vector?

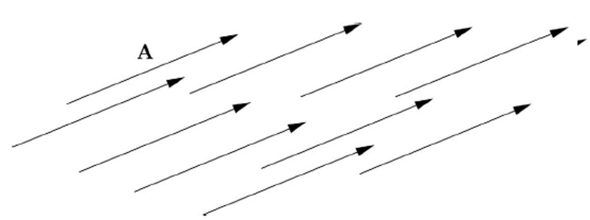
- A vector is an object, which is Specified by both magnitude and a direction.



- Length of the vector is proportional to the magnitude of the vector.

E.g. A car is traveling eastwards at 80 Km/hr.

direction = east
magnitude = 80 Km/hr



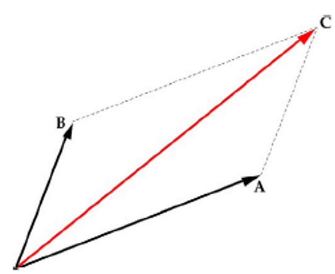
- The magnitude of a vector A is denoted

- The magnitude of a vector is denoted by $\|A\|$.

- A vector divided by its magnitude produces a vector, which is dimensionless and has magnitude 1. Such vectors are called as unit vectors.

E.g.: A, $\hat{u}_A = \dfrac{A}{\|A\|}$

- The sum or "resultant" of two vectors is defined by the parallelogram rule



$$B + A = C = A + B$$

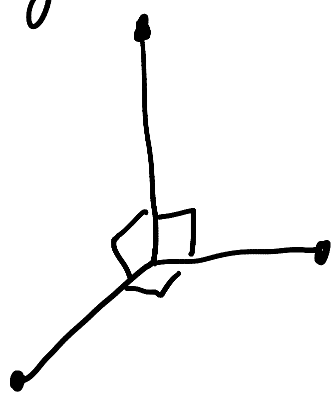
$$A + B = B + A \text{ (Commutative)}$$

What is a reference frame?

- Qualitatively, a reference frame is a ... from which observations

- Qualitatively, a reference [top of line cut off] perspective from which observations are made regarding the motion of a system.

- A reference frame is a collection of at least three noncolinear points in the three-dimensional Euclidean space such that the distance between any points in the collection does not change with time.



What are inertial and non-inertial frames?

- An inertial or Newtonian reference frame is one whose points are either absolutely fixed in space or atmost translate relative ...lutely fixed set of or atmost

to an absolutely fixed set of points with the same constant velocity.

- A non-inertial or non-Newtonian reference frame is one whose points accelerate with time.

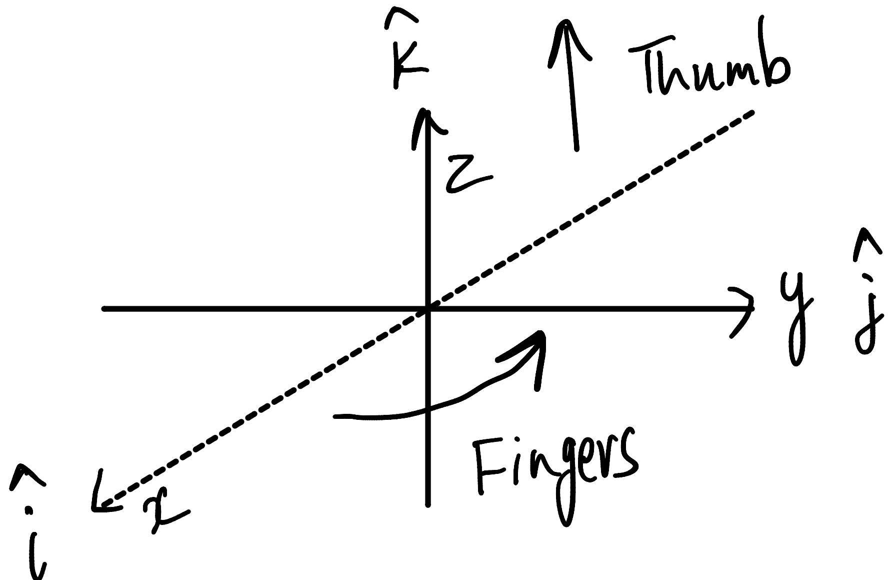
- Note carefully, that in Newton's second law, the acceleration must be calculated with respect to an inertial frame of reference.

What is a coordinate system?

- A coordinate system provides a means of measuring the observations that may be made of the motion by an observer fixed in a reference frame.

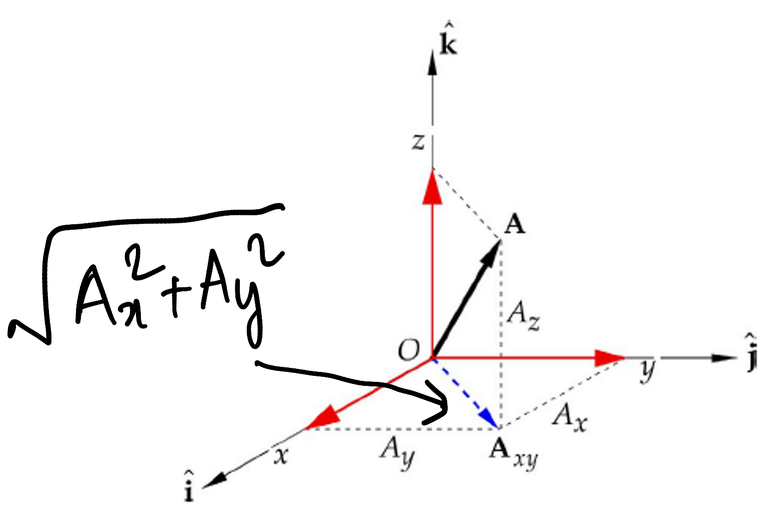
Cartesian

- *E.g.* Right-handed Cartesian coordinate system



Given a vector A, we can write it in the above coordinate system as follows

$$A = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$



$$\|A\| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\hat{u}_A = \frac{A}{\|A\|} = \frac{A_x}{\|A\|}\hat{i} + \frac{A_y}{\|A\|}\hat{j} + \frac{A_z}{\|A\|}\hat{k}$$

$$= \cos\theta_x \hat{i} + \cos\theta_y \hat{j} + \cos\theta_z \hat{k}$$

(in terms of the direction angles)

- For two vectors A and B, the operations AB and A/B are undefined. However, there are two well-known operations on vectors:

- The dot product of two vectors is a scalar defined as follows:

(Geometric) $\boxed{A \cdot B = \|A\| \|B\| \cos\theta}$

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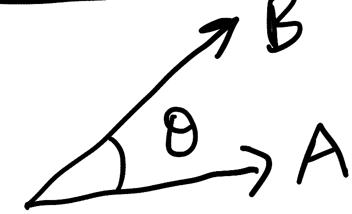


$$A \cdot B = B \cdot A \text{ (commutative)}$$

- If two vectors are perpendicular to each other, i.e., $\theta = 90^\circ$, then

$$A \cdot B = 0$$

- $\hat{i}, \hat{j}, \hat{k}$

$$\hat{i} \cdot \hat{j} = 0, \quad \hat{i} \cdot \hat{k} = 0,$$
$$\hat{j} \cdot \hat{k} = 0$$
$$\hat{i} \cdot \hat{i} = 1, \quad \hat{j} \cdot \hat{j} = 1, \quad \hat{k} \cdot \hat{k} = 1$$

- Given two vectors A and B,

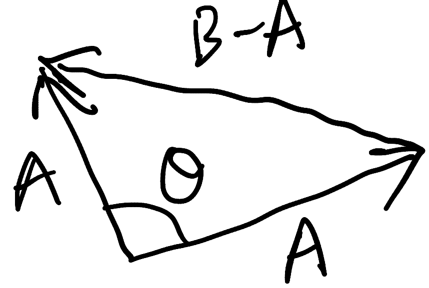
$$A = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$
$$B = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$= (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$A \cdot B = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

(Algebraic) $\boxed{A \cdot B = A_x B_x + A_y B_y + A_z B_z}$

<u>Why are the two equivalent?</u>



Law of cosines $\|B-A\|^2 = \|A\|^2 + \|B\|^2 - 2\|A\|\|B\|\cos\theta$

$A = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$A_x^2 + A_y^2 + A_z^2 = A \cdot A = \|A\|^2 = A_x^2 + A_y^2 + A_z^2$

$$(B-A)\cdot(B-A) = \|B-A\|^2$$

$$\|$$

$$A \cdot A + B \cdot B - 2A \cdot B$$

$$= \|A\|^2 + \|B\|^2 - 2A \cdot B$$

$$A \cdot B = \|A\| \|B\| \cos\theta$$