1 or 0

(a) (1 Mark) Then a value of $k \in \mathbb{N}$ such that $\|e^{(k)}\|_\infty \leq 10^{-3}$ is

(a) ≥ 10

(b) (1 Mark) Justification for your answer in (a) is

$\|e^{(k)}\|_\infty \leq \mu^k \|e^{(0)}\|_\infty$.

$\mu_i = \dfrac{\sum_{\substack{j=1 \\ i\neq j}}^{n} |a_{ij}|}{|a_{ii}|}$ $\quad \mu = \max_{1\leq i\leq n} \mu_i$

1/2

compute μ 1/2

11. (2 Marks) Assuming that every diagonally dominant matrix is invertible, show that each eigenvalue of a matrix belongs to at least one of the Gerschgorin disks associated to it.

Spring 2017, IIT Bombay

MA 214: Numerical Analysis

Midsemester Examination, Code AA

Roll no.: Tut.Batch T

1. Duration of the exam is 2 hours. This exam has **11** questions, for a total of **30** marks in **4** pages.
2. Answer all the questions, in the space provided at the end of each question. **ONLY FINAL Answer** must be written, except for Questions 10(b) and 11.

1. Let A denote the matrix

$$A = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 8 & 0 \\ 1 & 0 & \alpha \end{pmatrix}$$

(a) (3 Marks) Gerschgorin theorem was used to conclude that the matrix A satisfies **Hypothesis (H1)** of Power method. The set of all such values of α is

$\alpha \in (-\infty, -11) \cup (-5, 5) \cup (11, \infty)$

1 + 1 + 1

(b) (2 Marks) Gerschgorin theorem was used to conclude that the matrix A has **distinct eigenvalues**. The set of all such values of α is

$\alpha \in (-\infty, -4) \cup (4, 5) \cup (11, \infty)$

Atleast 2 to get 1 mark

2. (2 Marks) The set of all $\alpha \in \mathbb{R}$ such that the vector $(\alpha-2, \alpha, \alpha+2)^T$ satisfies **Hypothesis (H3)** of Power method for the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -3 \end{pmatrix}$ is

$\alpha \neq -2$

2 or 0

3. (3 Marks) Consider the linear system

$$\begin{aligned} 4x_1 - x_2 - x_3 &= 1 \\ 3x_1 + 5x_2 + x_3 &= 2 \\ x_1 - 3x_2 + 6x_3 &= 3 \end{aligned}$$

(a) Let $\boldsymbol{x}^{(0)} = (0,0,0)^T$. Then first two members $\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}$ of the Jacobi iterative sequence for solving the given linear system are (show as many digits after decimal as your calculator allows)

1/2

1/2

$x^{(1)} = (.25 ; .4 , .5)$

$x^{(2)} = (.475 , .15 , .6583333)$

(b) Let $\boldsymbol{x}^{(0)} = (0,0,0)^T$. Then first two members $\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}$ of the Gauss-Seidel iterative sequence for solving the given linear system are (show as many digits after decimal as your calculator allows)

1

1

$x^{(1)} = (.25 , .25 , .5833333)$

$x^{(2)} = (.45833325 , .00833345 , .42777785)$

1 or 0

4. (a) (1 Mark) The set of all values of $\beta \in \mathbb{R}$ such that the matrix $\begin{pmatrix} 2 & \beta \\ \beta & 3 \end{pmatrix}$ is **diagonally dominant but NOT positive definite** is

no such β exists

1 or 0

(b) (1 Mark) The set of all values of $\alpha \in \mathbb{R}$ such that the matrix $A = \begin{pmatrix} \alpha & 2 \\ 2 & \alpha \end{pmatrix}$ is **positive definite but NOT diagonally dominant** is

no such α exists

2 or 0

(c) (2 Marks) Let $A = (a_{ij})$ be a positive definite matrix with 5 rows and 5 columns. If $a_{11} = 1, a_{22} = 4, a_{33} = 9, a_{44} = 16, a_{55} = 25$, then the value of a_{23} **must be greater than or equal to**

(c) $-4.8.5$

2 or 0

5. (2 Marks) The set of all values of $\alpha, \beta \in \mathbb{R}$ such that the matrix $\begin{pmatrix} -1 & \beta & \alpha \\ \beta & 2 & \beta \\ 0 & 0 & \alpha \end{pmatrix}$ has a **doolittle decomposition** is

$\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}$

3 or 0

6. (3 Marks) Let L be a lower triangular matrix having positive real numbers on its diagonal. Further L satisfies

$$LL^T = \begin{pmatrix} 4 & -2 & -4 \\ -2 & 5 & -4 \\ -4 & -4 & 29 \end{pmatrix}.$$

Then L is equal to

6. $\begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ -2 & -3 & 4 \end{bmatrix}$

1 for each bound

1 + 1

7. (2 Marks) For solving a linear system of equations $A\boldsymbol{x} = \boldsymbol{b}$, an iterative sequence given by $\boldsymbol{x}^{(k+1)} = B\boldsymbol{x}^{(k)} + \boldsymbol{c}$ (for $k \geq 0$) is proposed, where $B = \begin{pmatrix} 3 & \alpha \\ \beta & -3 \end{pmatrix}$. If the sequence $\left(\boldsymbol{x}^{(k)}\right)_{k \geq 0}$ converges to the exact solution (assumed to be unique) of the linear system $A\boldsymbol{x} = \boldsymbol{b}$ for every choice of initial guess $\boldsymbol{x}^{(0)}$, then $\alpha, \beta \in \mathbb{R}$ must be such that

$-9 < \alpha\beta < -8$ $\mathbb{R}$

$-10 < \alpha\beta < -8$ $\mathbb{C}$

1 for each bound

1 + 1

8. (2 Marks) Let λ denote any eigenvalue of the matrix $\begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \\ 3 & -4 & -5 \end{pmatrix}$. The optimal bounds $\alpha, \beta \in \mathbb{R}$ such that $\alpha \leq |\lambda| \leq \beta$, obtained using Gerschgorin theorem are given by

$-4 \leq |\alpha| \leq 12$

9. This question is concerned with power method iterations for the matrix $\begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \\ 3 & -4 & -5 \end{pmatrix}$, with initial guess $\boldsymbol{x}^{(0)} = (1,2,3)^T$.

1

(a) (1 Mark) μ_1 is equal to

(a) -20

1

(b) (1 Mark) μ_2 is equal to

(b) 4.6

1

(c) (1 Mark) $\boldsymbol{x}^{(2)}$ is equal to

(c) $(.76086 , 1 , -.93478)^t$

10. For the linear system in **Question no. 3**, Jacobi method is applied with an initial guess vector $\boldsymbol{x}^{(0)}$ such that the maximum norm of the error $\boldsymbol{e}^{(0)}$ is 1.