

(a) Rectilinear Trajectories

Q.1 A rocket having a payload of 450 kg needs to achieve an ideal burnout velocity of 450 m/s. If a fuel of I_{sp} of 400s is to be used and it is known that rocket motor empty shell has a mass of 600 kg, determine the propellant and lift-off masses of the rocket, along with the effective payload fraction.

Q.2 A rocket is launched vertically from the surface of the Earth and expels burnt gases at a uniform rate of 0.003 times the initial mass of rocket per second. For an effective exhaust speed of 4500 m/s, and assuming a uniform sea level gravity model ($g_0 = 9.805$), find the speed and altitude of the rocket 15s after the lift-off. Further, determine the velocity and altitude after 80s from lift-off and the energy loss due to gravity for 80s mission.

Q.3 A rocket has the following lift – off parameters.

$m_0 = 80$ Tons, $m_p = 60$ Tons, $I_{sp} = 300$ s, $g_0 = 9.81 \text{ m/s}^2$, $\beta = 1000$ kg/s. Assuming rectilinear vertical motion, determine the trajectory parameters at the end of propellant burnout, while considering the effects of gravity, altitude and atmospheric drag. $C_{D0} = 1.0$, $S_{ref} = \pi \text{ m}^2$. Assume peak drag acceleration time to be 30s. What is the extent of correction due to drag? Also, at what time does the actual peak acceleration occur? Does it cause any error?

(b) Constant 'q' Gravity Turn Ascent Trajectory

Q.1 A rocket with a lift-off mass of 100 tons and propellant mass of 90 tons ($I_{sp} = 250$ s) is launched directly into a gravity turn manoeuvre, with a pitch down rate of 0.3 deg./sec. It is desired that the trajectory becomes 45° to horizontal axis (neglect Earth's curvature) after some time from the launch. Find initial speed for 0.1° initial angle, which will make this manoeuvre possible. Also, determine speed, altitude and horizontal distance at the end of the manoeuvre and the time taken for it. Is the propellant sufficient to perform the above mission? If yes, determine how much is consumed. If no, how much more is needed to perform the mission? Is it necessary to take into account the effect of earth's curvature?

(c) Constant Velocity Gravity Turn Trajectories

Q.1 A Launch vehicle with a mass of 5000 kg and flying at an angle of 70° with local vertical travels at the speed of 4000 m/s for a horizontal distance of 30 km. Determine the terminal angle reached at the end of the manoeuvre, along with the time taken and the amount of propellant ($I_{sp} = 300$ s) burnt. Assume earth to be flat and constant sea-level gravity.

(d) Constant Specific Thrust Gravity Turn Trajectories

Q.1 A rocket with a lift-off mass of 80 tons and propellant mass of 65 tons ($I_{sp} = 350$ s) is launched directly near vertically into a gravity turn trajectory, during which the specific thrust (T/m) is kept constant at 1.1g. It is desired that the trajectory becomes parallel to the local horizon (neglect Earth's curvature) at the end of burnout while consuming all the propellant. Find the speed at the 'pitch-over' angle command of 0.5° and the time taken for the manoeuvre. Also, determine the speed, altitude and horizontal distance at the end of the manoeuvre. Is it necessary to take into account the effect of earth's curvature?