MA 214: Introduction to Numerical Analysis

Indian Institute of Technology Bombay

Quiz 3

Marks: 15

Date: 27-03-2019

Time: 1 hour

Instructors: S. Baskar and S. Sivaji Ganesh

Instructions:

(1) Write your **Name, Roll Number, and Tutorial Batch** clearly on your answer book as well as every supplement you may use. A **penalty of -1 mark will be awarded** for failing to do so.

(2) Number the pages of your answer book and make a question-page index on the front page.

(3) The answer to each question should start on a new page. If the answer for a question is split into two parts and written in two different places, the first part alone will be corrected.

(4) Only scientific calculators are allowed. Any kind of programing device is not allowed.

(5) Formulas used need not be proved but **needs to be stated clearly**.

(6) The question paper contains 4 questions. **Answer all the questions.**

(1) (a) Draw the graph of a function for which the Newton-Raphson iterates satisfy $x_0 = x_3 = 0$ (in the usual notation of Newton-Raphson method). Justify your answer graphically.

(b) Draw the graph of a function for which secant method iterates satisfy $x_0 = 0$, $x_1 = 3$, and $x_2 = 1$, $x_3 = 2$ (in the usual notation of secant method). Justify your answer graphically.

[3 Marks]

[**Note:** You need not define the functions explicitly]

(2) Let $p_{10}(x)$ be the interpolating polynomial for the function $f(x) = \frac{1}{x}$ at the nodes $\{1, 2, \cdots, 11\}$. If $x \notin \{1, 2, \cdots, 11\}$, then show that

$$f(x) - p_{10}(x) = \frac{1}{11!} f^{11}(\xi) \prod_{k=1}^{11} (x - k).$$

[4 Marks]

(3) Let $x_0, x_1, \cdots, x_n$ be nodes, and f be a given function. Define $w(x) = \prod_{i=0}^{n}(x - x_i)$. Prove that

$$f[x_0, x_1, \cdots, x_n] = \sum_{i=0}^{n} \frac{f(x_i)}{w'(x_i)}.$$

[4 Marks]

(4) Let $g : [a, b] \to [a, b]$ be a continuously differentiable function with $\lambda := \max_{x \in [a,b]} |g'(x)| < 1$. Let $\{x_n\}$ be a sequence generated by the fixed point iteration method with iteration function g. Assume that the sequence $\{x_n\}$ converges to the fixed point x^*.

(a) Show that

$$|x_{n+1} - x^*| \leq \frac{\lambda}{1 - \lambda} |x_{n+1} - x_n|.$$

(b) Assuming that $x^* \neq x_n$ for all $n = 0, 1, 2, \cdots$, show that

$$\lim_{n \to \infty} \frac{x^* - x_{n+1}}{x^* - x_n} = g'(x^*).$$

[4 Marks]