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AE 234
Endsem

$10+2+3+1+1+2+2.5$
$= 21.5$

Values used throughout:

$M = 0.85$

$h = 41,000$ ft

$W_{max}|_{TO} = 170,600$ kg

$W_{empty} = 71,700$ kg

$W_{max\,fuel}|_{TO} = 75750$ kg

$W_{engine} = 1450$ kg/engine (4 engines)

$\pi_c|_{overall} = 35.1$

(Thrust) $T_{max} = 85$ kN / engine.

$\eta_c = \eta_t = 0.9$

$Q_R = 45$ MJ / kg

(Temp). $T_{max} = T_{t4} = 1400K$

$g_0 = 9.81\ m/s^2$

$R = 287\ J/kg\,K$

$\frac{L}{D} = 20$ (for range)

$c_p = 1005\ J/kg\,K$

Turbojet engine

From STD. Atm. data, at 41,000 ft,

$T_a = 216.65K$

$P_a = 17.9$ kPa

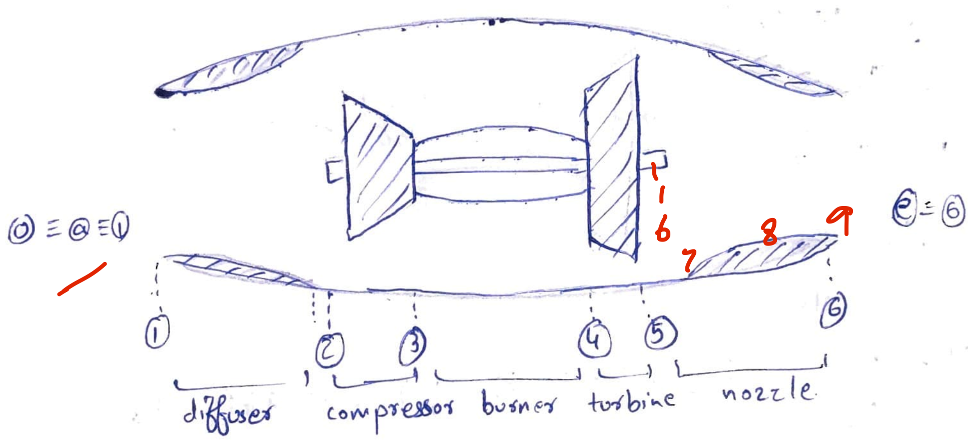
$\rho_a = 0.287\ kg/m^3$

$C_a = \sqrt{\gamma R T_a} = \sqrt{1.4 \times 287 \times 216.65}$
$= 295.04\ m/s$

$\therefore V_a = 250.79\ m/s$

$T_{ta} = 216.65(1 + 0.2(0.85)^2)$
$= 247.96$ K

Question 1



Assuming no losses in the diffuser & nozzle, no pressure loss in combustor, nozzle is expanded perfectly.

$\therefore$ Properties at $1 \approx 2$ & ~~[illegible]~~ $5 \approx 6$

Need not be true

$T_2 = 216.65K$; $T_{t_2} = 247.96$ K

$P_{a2} = 17.9$ kPa; $P_{t_2} = 28.71$ kPa

$\pi_c|_{overall} = 35.1 = \frac{P_{t3}}{P_2} \Rightarrow P_{t3} = 628.29$ kPa

~~$\frac{T_{t3}}{T_{t2}}$ is $= \pi_c^{\frac{\gamma-1}{\gamma}} = 2.764$~~ $\pi_c = \frac{P_{t3}}{P_{t2}} = 21.89 = \pi_t$

$\pi_c \neq \pi_t$

η_c $\frac{T_{t3}}{T_{t2}} = 1 + \frac{1}{\eta_c}\left[\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right]$

$\therefore T_{t3} = 247.96\left(1 + \frac{1}{0.9}\left(21.89^{\frac{0.4}{1.4}} - 1\right)\right)$

$T_{t3} = 637.82\,K$

Now, ~~$T = \dot{m}_a(v_e - v_a)$~~
~~85×10^3~~

~~$W_{TO} = W_{empty} + W_{fuel} + W_{payload}$~~

$P_{t4} = P_{t3} = 628.29\,kPa$ (Assuming no loss of pressure in combustor)

$T_{t4} = 1400\,K$ (Given T_{max})

$\therefore$ ~~T For perf~~

$W_{turbine} = W_{compressor}$

~~$\dot{m}_a C_p(T_{t4} - T_{t5}) = \dot{m}_a C_p(T_{t3} - T_{t2})$~~

$\Rightarrow$ ~~$T_{t5} = 1010.14\,K$~~

This was correct!

~~$\eta_t = \frac{1 - T_{t5}/T_{t4}}{1 - \pi_t^{-\frac{\gamma-1}{\gamma}}} \Rightarrow 0.9 = \frac{1 - \frac{1010.14}{1400}}{1 - \pi_t^{-\frac{0.4}{1.4}}} \Rightarrow \pi_t = 3.654$~~

+1

$\therefore P_{t5} = 171.96\,kPa$

As ~~nozzle~~ is perfectly

$\pi_t = 21.89 = \frac{P_{t4}}{P_{t5}} \Rightarrow P_{t5} = 28.71\,kPa$; $P_e = 17.9\,kPa = P_5$

$\frac{T_{t5}}{T_{t4}} = 1 - \eta_t\left(1 - \pi_t^{-\frac{\gamma-1}{\gamma}}\right)$

$T_{t5} = \left(1 - 0.9\left(1 - 21.89^{\frac{-0.4}{1.4}}\right)\right)1400$

-1

-2

$T_{t5} = 661.73\,K$ ~~$= T_5(1 + 0.2(0.85)^2)$~~

~~$\therefore T_5 = 878.18\,K = T_e$~~ $\frac{1}{2}(v_e^2 - v_a^2) = c_p(T_{t4} - T_{t5} - T_{t3} + T_{t2})$

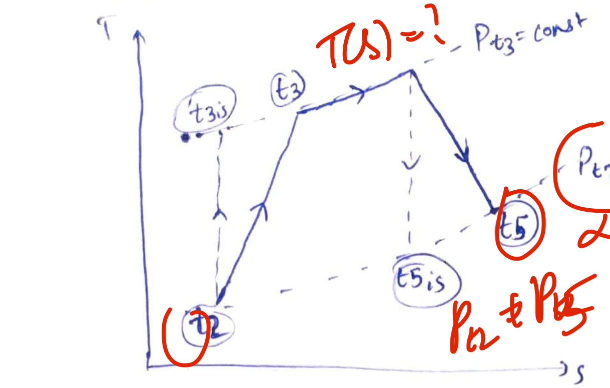
~~$\therefore v_e = 0.85\sqrt{1.4\times287\times578.18}$~~ $\therefore v_e =$ ~~$874\,m/s$~~ $873.61\,m/s$

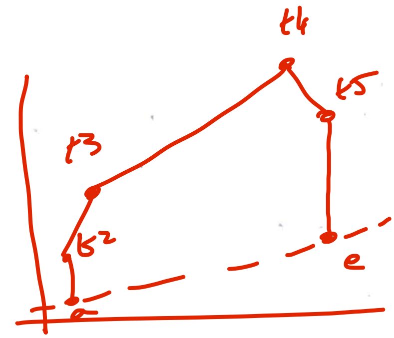
~~$v_e = 409.69\,m/s$~~

$\frac{P_{t5}}{P_e} = \frac{P_{t5}}{P_a} \rightarrow M_e \rightarrow T_e$

$\rightarrow V_e.$

Question 2





2 → 3 Non-isentropic compression

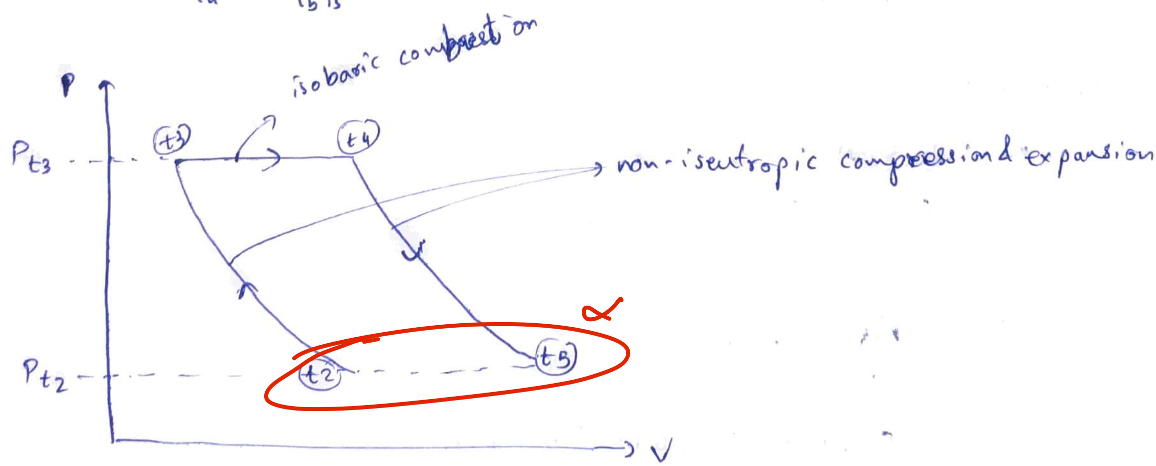
$$\eta_c = \frac{T_{t3is} - T_{t2}}{T_{t3} - T_{t2}} \qquad (T \text{ is in K})$$

3 → 4 Isobaric combustion.

$$\dot{m}_f Q_R = \dot{m}_a C_p (T_{t4} - T_{t3})$$ (Q_R is in J/kg, T is in K, $\dot{m}_f, \dot{m}_a$ are in kg/s, C_p is in J/kgK)

4 → 5 Non-isentropic expansion

$$\eta_T = \frac{T_{t4} - T_{t5}}{T_{t4} - T_{t5is}} \qquad (T \text{ is in K})$$



2

Question 3

$\dot{m}_f Q_R = \dot{m}_a C_p (T_{t4} - T_{t3})$

$\therefore f = \dfrac{1.005 (1400 - \cancel{1010.14} \; 637.82)}{45 \times 10^6}$

$= 0.017.$

$\therefore TSFC = \dfrac{f}{v_e - v_a}$

$= \dfrac{0.017}{873.61 \; \cancel{406.69} - 250.79}$

$= \cancel{1.0772} \; 0.27295 \times 10^{-4} \; (m/s)^{-1}$ or $\cancel{107.72} \; 27.3 \; gm/kN-s$

$r = \dfrac{v_a}{v_e} = \dfrac{\cancel{0.812}}{0.287}$; $E = \dfrac{2 f Q_R}{v_e^2} = \cancel{9.116}, 2.0047.$

$\therefore \eta_p = \dfrac{2r}{1+r} = 0.\cancel{4459} \; 4459 \equiv \cancel{75.93} \; 44.6 \; \%$

$\eta_{Th} = \dfrac{1 - r^2}{E} = \cancel{0.0686} \; 0.4577 \equiv \cancel{6.86} \; 45.77 \; \%.$

$\eta_{ov} = \eta_p \times \eta_{Th} = \cancel{0.0521} \; 0.2041 \equiv \cancel{5.21} \; 20.41 \; \%$

3

Question 4

$\dot{m}_a = \rho_a v_a A_a$

$= 0.287 \times 250.79 \times \pi \times 0.6^2$

$= 81.404 \; kg/s.$

$\therefore T = \dfrac{\dot{m}_f}{TSFC} = \dfrac{\dot{m}_a \times f}{TSFC} = \dot{m}_a (v_e - v_a)$

$= 81.404 (873.61 - 250.79)$

$= 50700 \; N \equiv 50.7 \; kN/engine.$

Question 5

Range for const. speed & attitude is given by:

$$R = \frac{v_a}{g_0 \, TSFC} \times \frac{L}{D} \times \log \frac{W_{ini}}{W_{fin}}$$

$W_{ini} = 170{,}600$ kg ; $\frac{L}{D} = 20$

$W_{fin} = 94850$ kg

$$\therefore \text{Range} = \frac{250.79 \times 10^{-3}}{9.81 \times 0.273 \times 10^{-4}} \times 20 \times \log \frac{170600}{94850}$$

1

Range = 10994.24 kms

Question 6

$$T_{t5} = 1400\left(1 - 0.9\left(1 - 21.89^{\frac{-0.33}{1.33}}\right)\right)$$

$T_{t5} = 814.09$ K

α

$$v_e = \sqrt{v_a^2 + 2C_p(T_{t4} - T_{t5} - T_{t3} + T_{t2})}$$

$$= \sqrt{250.79^2 + 2 \times 1005\,(1400 - 814.09 - 637.82 + 247.96)}$$

$v_e = 675.99$ m/s

$$T = \dot{m}_a(v_e - v_a)$$

$$= 81.404\,(675.99 - 250.79)$$

$$= 34612.98 \text{ N} = 34.61 \text{ kN/engine}$$

$$TSFC = \frac{f}{v_e - v_a}$$

$$f = \frac{C_{p4}T_{t4} - C_{p3}T_{t3}}{Q_R} \rightarrow f \text{ change}$$

2

$$= 3.9981 \times 10^{-5} (m/s)^{-1} = 39.98 \text{ gm/kN-s}$$

$$\text{Range} = \frac{250.79 \times 10^{-3}}{9.81 \times 3.9981 \times 10^{-5}} \times 20 \times \log \frac{170600}{94850}$$

Range = 7507.13 kms

Question 7

Increase $\pi_c|_{overall}$ to 45.

$\therefore \frac{P_{t3}}{P_2} = 45 \Rightarrow P_{t3} = 805.5\,kPa$; $P_{t2} = 28.71\,kPa$

$\therefore \pi_c = 28.06 = \pi_t$ α

$$T_{t3} = 247.96\left(1 + \frac{1}{0.9}\left(28.06^{\frac{0.4}{1.4}} - 1\right)\right) = 686.75\,K$$

~~P_{t4}~~ $P_{t4} = P_{t3} = 805.5\,kPa$. $T_{t4} = 1400\,K$;

α ~~$P_{t5} = 28.71\,kPa = P_{t2}$~~ α

$$T_{t5} = \left\{1 - 0.9\left(1 - 28.06^{-\frac{0.33}{1.33}}\right)\right\}1400 = 690.9\,K$$

$$v_e = \sqrt{250.79^2 + 2\times1005(1400 - 690.9 - 686.75 + 247.96)}$$

$$\simeq 778.6\,m/s$$

$$\frac{T}{\dot m_a} = v_e - v_a = 527.81\,N/kg/engine$$

(2.5) $\text{TSFC} = \frac{f}{v_e - v_a} = \frac{1005(1400 - 686.75)}{45\times10^6 \times 527.81}$ α $f = \frac{c_{p_4}T_{t4} - c_{p_3}T_{t3}}{Q_R}$

$$= 30.1799\times10^{-6}\,(m/s)^{-1} \simeq 30.18\,gm/kN\text{-}s$$