Lecture 23

- $r_p = \frac{2\mu_2}{V_\infty^2} \frac{1-\|e\|}{1+\|e\|}$

- $\frac{r_p}{r_a} = \frac{1-\|e\|}{1+\|e\|}$

- $r_a = \frac{2\mu_2}{V_\infty^2}$

- $\Delta V = V_\infty \sqrt{\frac{1-\|e\|}{2}}$

- $\Delta = \sqrt{\frac{2}{1-\|e\|}}\, r_p$

Example

After a Hohmann transfer from earth to Mars, calculate
(a) the minimum delta-v required to place a spacecraft in orbit with a period of 7 h
(b) the periapsis radius
(c) the aiming radius
(d) the angle between periapsis and Mars' velocity vector.

Details

(a) $V_\infty = \Delta V_A = \sqrt{\frac{\mu_{sun}}{R_{Earth}}}\left(1 - \sqrt{\frac{2R_{Earth}}{R_{Earth}+R_{Mars}}}\right)$

$$a = \left(\frac{T\sqrt{\mu_{Mars}}}{2\pi}\right)^{2/3}$$

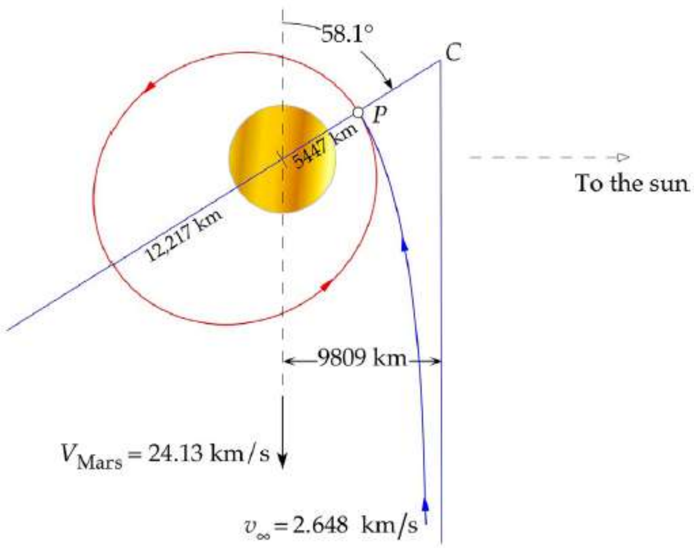
$$a = \frac{r_p}{1-\|e\|} = \frac{2\mu_{Mars}}{V_\infty^2}\frac{1}{1+\|e\|}$$

$\Delta V = \ldots$

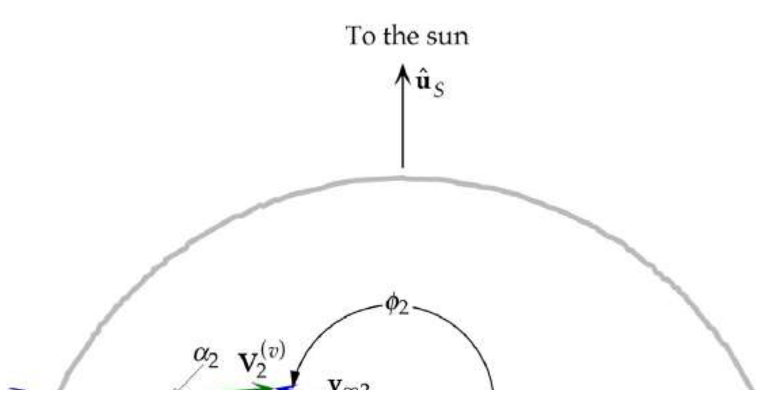
(b) $r_p = \ldots$

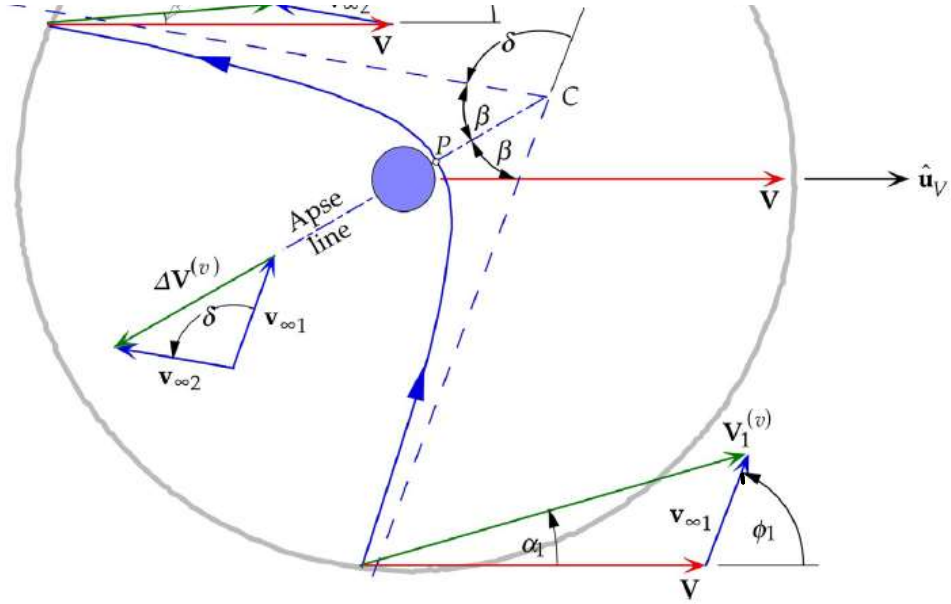
(c) $\Delta = \ldots$

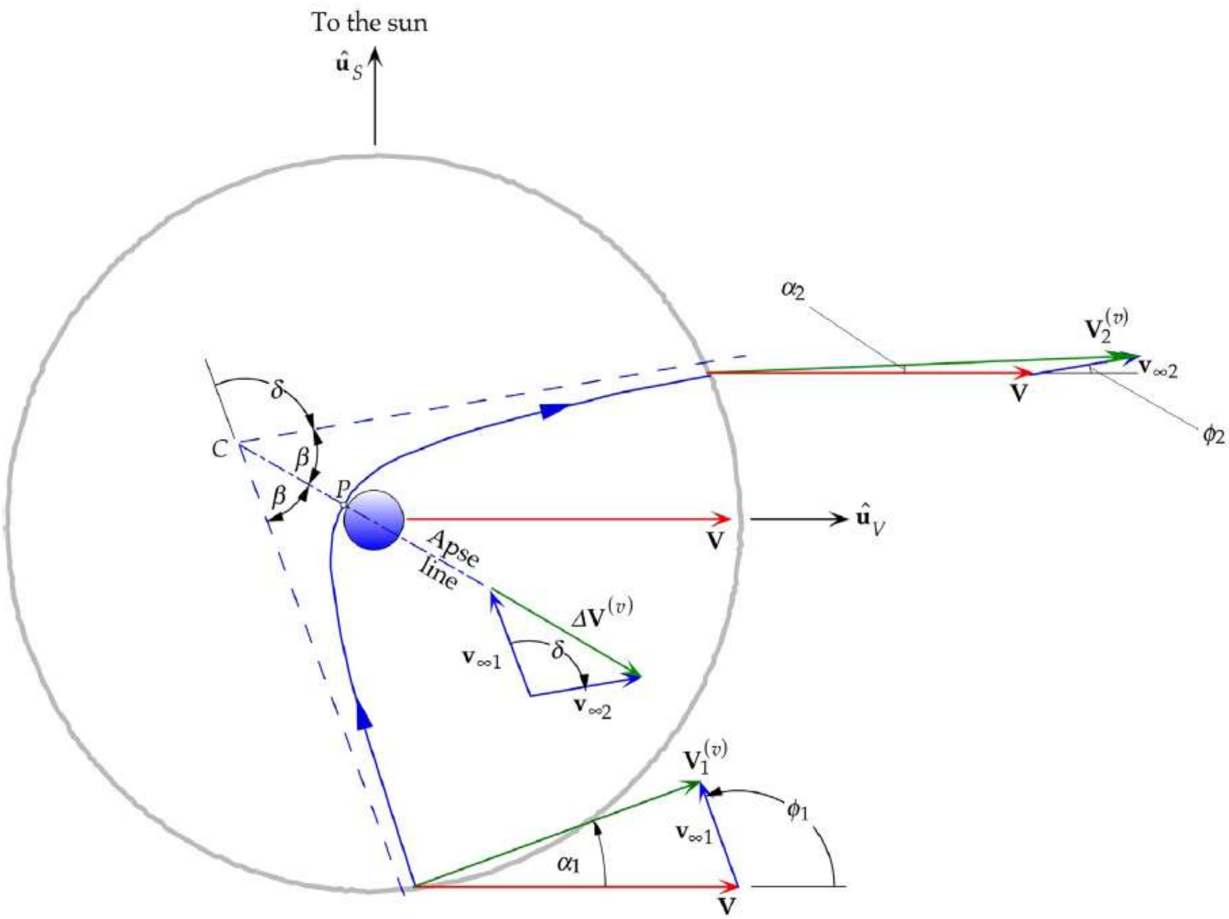
(d) $\beta = \ldots$



Planetary Flyby







- $V_1^{(v)} = V + V_{\infty_1}$

- $V_2^{(v)} = V + V_{\infty_2}$

- $\Delta V^{(v)} = V_2^{(v)} - V_1^{(v)}$

$$= V_{\infty_2} - V_{\infty_1}$$
$$= \Delta V_\infty$$

$$- V_1^{(v)} = \underbrace{V_1^{(v)})_V}_{\|V_1^{(v)}\|\cos\alpha_1} \hat{u}_V + \underbrace{V_1^{(v)})_S}_{\|V_1^{(v)}\|\sin\alpha_1} \hat{u}_S$$

$$- V_1^{(v)})_V = V_{\perp_1}^{(v)}$$
$$= \frac{\mu_{sun}}{\|h_1\|}(1+\|e_1\|\cos\theta_1)$$

$$- V_1^{(v)})_S = -V_{r_1}^{(v)}$$
$$= -\frac{\mu_{sun}}{\|h_1\|}\|e_1\|\sin\theta_1$$

$$- V = V\hat{u}_V \ , \ V = \sqrt{\frac{\mu_{sun}}{R}}$$

$$- V_{\infty_1} = V_{\infty_1})_V \hat{u}_V + V_{\infty_1})_S \hat{u}_S$$

$$- V_{\infty_1})_V = \|V_1^{(v)}\|\cos\alpha - V$$

$$- V_{\infty_1})_S = \|V_1^{(v)}\|\sin\alpha$$

$$- \|V_\infty\| = \sqrt{V_{\infty_1}\cdot V_{\infty_1}} = \sqrt{\|V_1^{(v)}\|^2 + V^2 - 2\|V_1^{(v)}\| V\cos\alpha_1}$$

$$- \phi_1 = \tan^{-1}\frac{V_{\infty_1})_S}{V_{\infty_1})_V}$$

$$= \tan^{-1}\left(\frac{\|V_1^{(v)}\|\sin\alpha_1}{\|V_1^{(v)}\|\cos\alpha_1 - V}\right)$$

$$- \phi_2 = \phi_1 + \delta$$

$$- \; V_{\infty_2} = \|V_\infty\| \cos\phi_2 \, \hat{u}_V + \|V_\infty\| \sin\phi_2 \, \hat{u}_S$$

$$- \; V_2^{(v)} = V + V_{\infty_2}$$
$$= V_2^{(v)})_V \hat{u}_V + V_2^{(v)})_S \hat{u}_S$$

$$- \; V_2^{(v)})_V = V + \|V_\infty\| \cos\phi_2$$

$$- \; V_2^{(v)})_S = \|V_\infty\| \sin\phi_2$$

$$- \; V_{\perp_2}^{(v)} = V_2^{(v)})_V$$

$$- \; V_{r_2}^{(v)} = -V_2^{(v)})_S$$

$$- \; \|h_2\| = R \underbrace{V_{\perp_2}^{(v)}}$$

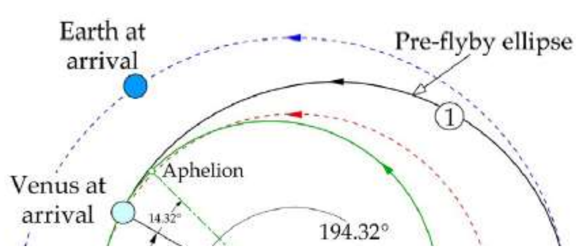
$$- \; R = \frac{\|h_2\|^2}{\mu_{sun}} \frac{1}{1 + \|e_2\| \cos\theta_2}$$

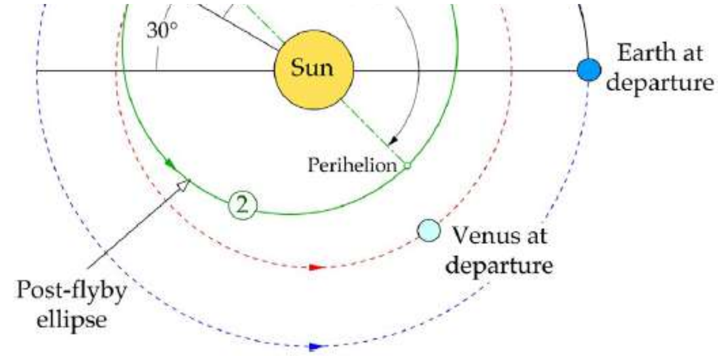
$$- \; V_{r_2}^{(v)} = \frac{\mu_{sun}}{\|h_2\|} \|e_2\| \sin\theta_2$$

Example

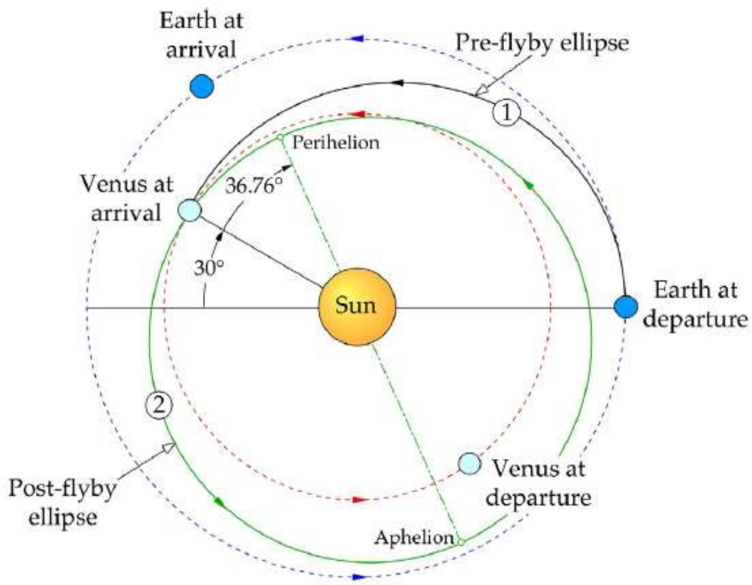
A spacecraft departs earth with a velocity perpendicular to the sun line on a flyby mission to Venus. Encounter occurs at a true anomaly in the approach trajectory of $-30°$. Periapsis altitude is to be 300 km.
(a) For an approach from the dark side of the planet, show that the postflyby orbit is as illustrated in Fig. 8.20.
(b) For an approach from the sunlit side of the planet, show that the postflyby orbit is as illustrated in Fig. 8.21.

(a)



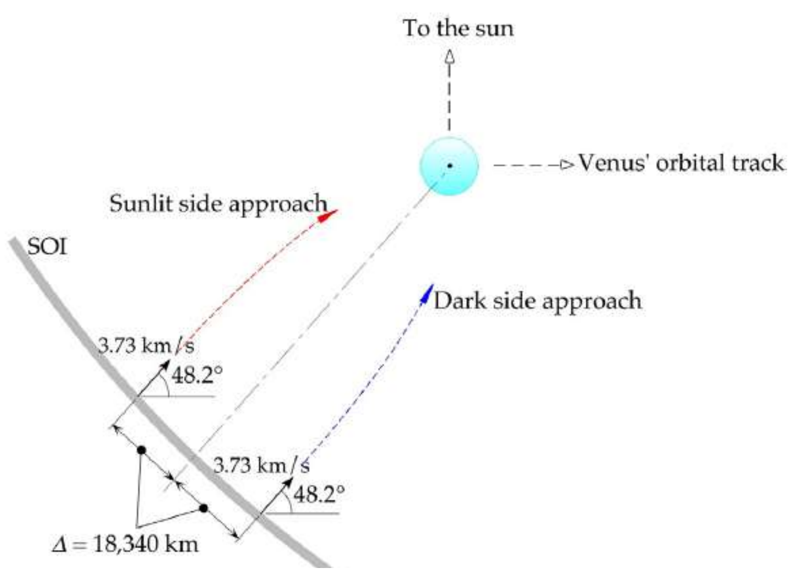


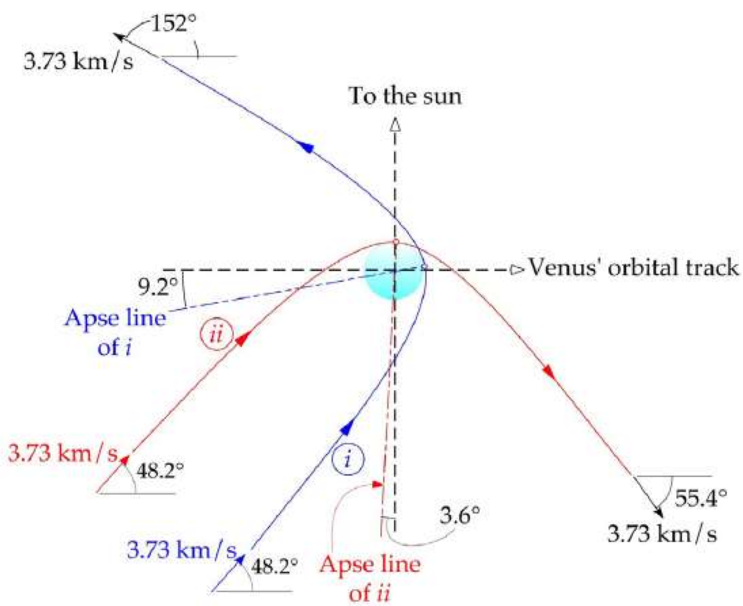
(b)

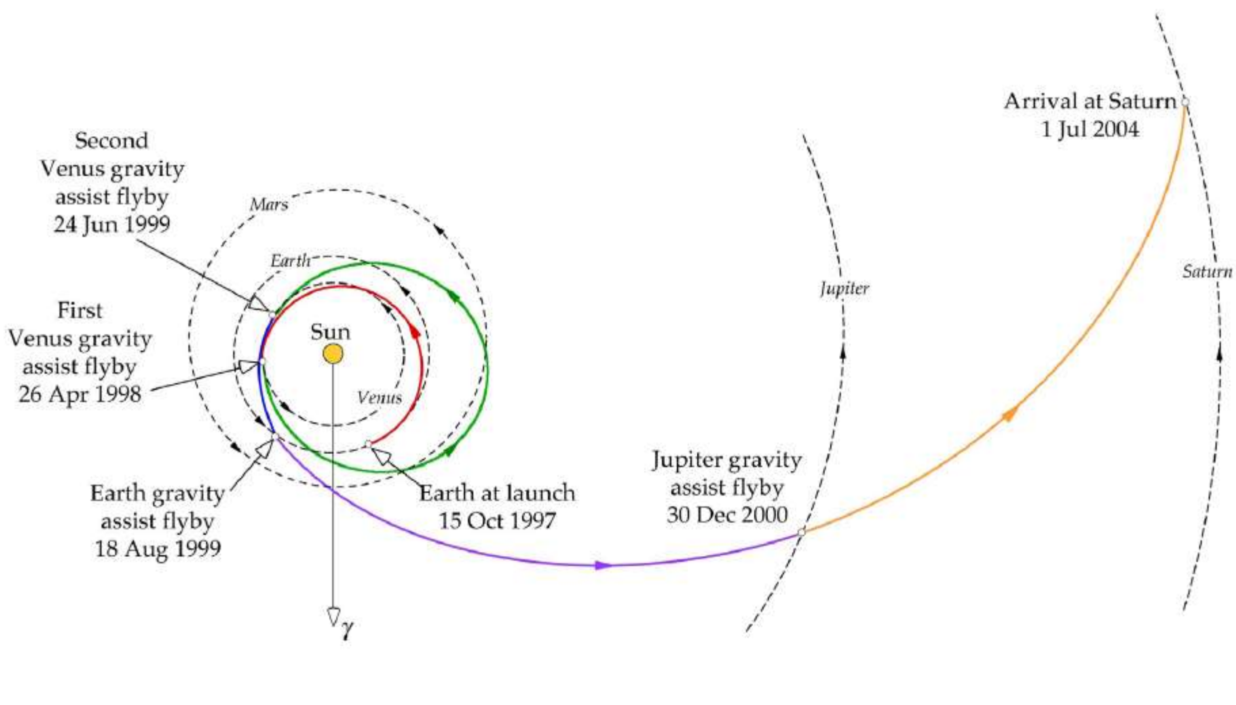


Details

This will be covered in next week's tutorial.




152°
3.73 km/s
To the sun
Venus' orbital track
9.2°
Apse line
of i
ii
3.73 km/s
48.2°
i
3.73 km/s
48.2°
Apse line
of ii
3.6°
55.4°
3.73 km/s


Second
Venus gravity
assist flyby
24 Jun 1999
Mars
Earth
First
Venus gravity
assist flyby
26 Apr 1998
Sun
Venus
Earth gravity
assist flyby
18 Aug 1999
Earth at launch
15 Oct 1997
γ
Jupiter
Jupiter gravity
assist flyby
30 Dec 2000
Arrival at Saturn
1 Jul 2004
Saturn