AE 234 Aircraft Propulsion
Quiz 2

- This exam is for 90 minutes, and counts for 20 points.
- You should scan the answer sheet and upload it on moodle within the given time.
- This is an open book exam.

We consider the GE-90 turbofan engine that powers the Boeing 777 airplanes. At cruise conditions (Mach 0.85 at an altitude of 35,000 ft), the bypass ratio is 8.1 and turbine entry temperature is 1380 K. The fan pressure ratio is 1.65. The core flow goes through a booster after the fan before entering the high pressure compressor. The booster and high pressure compressor have pressure ratios of 1.14 and 21.5, respectively.

Let the fuel energy content be $\mathcal{Q}_R = 42MJ/kg$. The combustion product gas that enters the turbines has the same molecular weight as freestream air, but has a different ratio of specific heats: $\gamma = 1.33$.Let the fan and the low pressure compressor have an efficiency of 0.90. The high pressure compressor efficiency is 0.88. The low and high pressure turbines efficiencies is 0.90.

Consider the core and bypass exhausts to be perfectly expanded.

Question 1 . . . . *4 points*

Draw the schematic of a turbofan engine similar to the description of the GE 90 engine. Clearly label the components and state the processes happening in each of the component using a sentence (or two) each.

Question 2 . . . . *8 points*

Clearly label the station numbers as per the standard numbering scheme. Obtain the flow properties (p, v, T, s) at each of these stations.

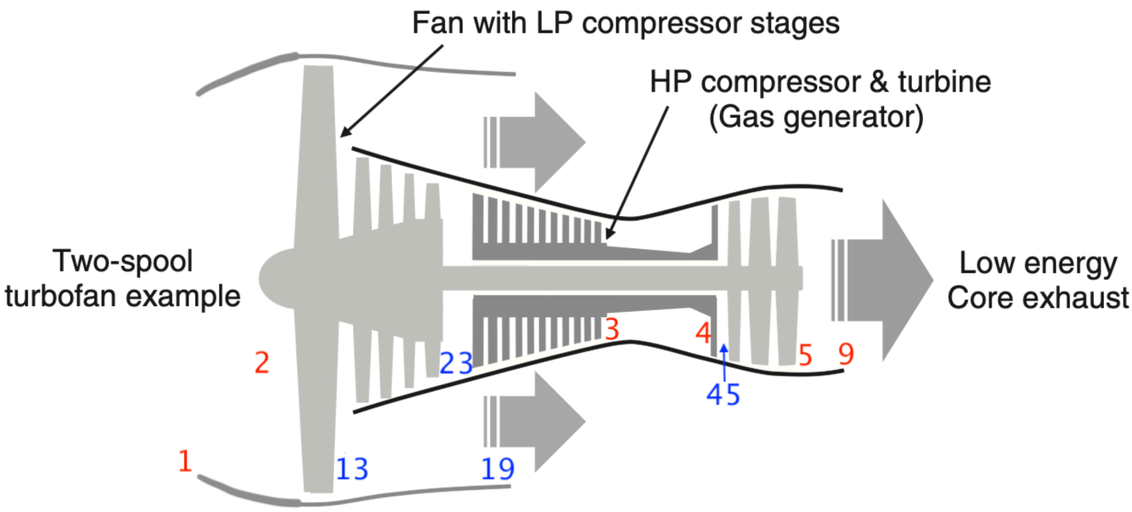
Question 3 . . . . *6 points*

Give the expressions that enable the plotting of various processes in this cycle on $p-v$ and $T-s$ axes. Based on the calculated properties, draw the thermodynamic cycles for the core and bypass streams on $p-v$ and $T-s$ axes. Try to make the plots quantitively accurate.

Question 4 . . . . *2 points*

Calculate the engine specific fuel consumption.

Question 1



Question 2

Bypass

Work done by fan on the bypass flow is: $\dot{W}_f = \alpha \dot{m}_c \left(h_{t_{13}} - h_{t_2}\right)$.
For the given pressure ratio ($\pi_f = 1.65$), we have

$$\eta_f = \frac{h_{t_{13_{is}}} - h_{t_2}}{h_{t_{13}} - h_{t_2}} = \frac{\pi_f^{\kappa} - 1}{\tau_f - 1}, \text{ where } \kappa = \frac{\gamma - 1}{\gamma} \text{ and } \tau_f = \frac{T_{t_{13}}}{T_{t_2}}$$

Giving us

$$T_{t_{13}} = T_{t_2}\left(1 + \frac{1}{\eta_f}\left\{\pi_f^{\kappa} - 1\right\}\right) \tag{1}$$

Core

For the core, the flow passes through fan and booster before reaching the high pressure compressor. This fan+booster is considered low pressure compressor (LPC). Work done by LPC on the core flow is: $\dot{W}_{lpc} = \dot{m}_c \left(h_{t_{23}} - h_{t_2}\right)$.
For the given pressure ratio ($\pi_{lpc} = \pi_f \pi_{booster} = 1.65 \times 1.14 = 1.881$), we have

$$\eta_{lpc} = \frac{h_{t_{23_{is}}} - h_{t_2}}{h_{t_{23}} - h_{t_2}} = \frac{\pi_{lpc}^{\kappa} - 1}{\tau_{lpc} - 1}, \text{ where } \tau_{lpc} = \frac{T_{t_{23}}}{T_{t_2}}$$

Giving us

$$T_{t_{23}} = T_{t_2}\left(1 + \frac{1}{\eta_{lpc}}\left\{\pi_{lpc}^{\kappa} - 1\right\}\right) \tag{2}$$

Both these are driven by the low pressure turbine (LPT), with the work balance

$$
\begin{aligned}
\dot{W}_{lpt} &= \dot{W}_f + \dot{W}_{lpc} \\
\dot{m}_c c_{p4} \left(T_{t_4} - T_{t_{45}}\right) &= \alpha \dot{m}_c c_p \left(T_{t_{13}} - T_{t_2}\right) + \dot{m}_c c_p \left(T_{t_{23}} - T_{t_2}\right) \\
T_{t_{45}} &= T_{t_4} - \frac{c_p}{c_{p4}} \left\{ \alpha \left(T_{t_{13}} - T_{t_2}\right) + \left(T_{t_{23}} - T_{t_2}\right) \right\}
\end{aligned}
$$

Going by the turbine efficiency expression

$$
\eta_{lpt} = \frac{h_{t_4} - h_{t_{45}}}{h_{t_4} - h_{t_{45_{is}}}} = \frac{1 - 1/\tau_{lpt}}{1 - \pi_{lpt}^{-\kappa_4}}, \text{ where } \kappa_4 = \frac{\gamma_4 - 1}{\gamma_4} \text{ and } \tau_{lpt} = \frac{T_{t_4}}{T_{t_{45}}}
$$

Giving us

$$
\pi_{lpt} = \left(1 - \frac{1}{\eta_{lpt}} \left(1 - \frac{1}{\tau_{lpt}}\right)\right)^{-1/\kappa_4} \tag{3}
$$

Before going to the high pressure components, we look at the burner/combustor:

$$
\begin{aligned}
&\dot{m}_f Q_R = \dot{m}_c \left\{ (1 + f)\, c_{p4} T_{t_4} - c_p T_{t_3} \right\} \\
&\implies f = \frac{c_{p_4} T_{t_4} - c_p T_{t_3}}{Q_R - c_{p_4} T_{t_4}}
\end{aligned}
$$

Here, $c_{p_4} T_{t_4} \approx 0.04 Q_R$, which can be neglected.

We now see the high pressure components

$$
\dot{m}_c (1 + f)\, c_{p4} \left(T_{t_{45}} - T_{t_5}\right) = \dot{W}_{hpt} = \dot{W}_{hpc} = \dot{m}_c c_p \left(T_{t_3} - T_{t_{23}}\right)
$$

Remember that

$$
\begin{aligned}
\eta_{hpc} &= \frac{h_{t_{3_{is}}} - h_{t_{23}}}{h_{t_3} - h_{t_{23}}} = \frac{\pi_{hpc}^{\kappa} - 1}{\tau_{hpc} - 1}, \text{ where } \tau_{hpc} = \frac{T_{t_3}}{T_{t_{23}}} \\
\implies h_{t_3} - h_{t_{23}} &= \frac{1}{\eta_{hpc}} h_{t_{23}} \left(\pi_{hpc}^{\kappa} - 1\right)
\end{aligned}
$$

So, we can write

$$
T_{t_5} = T_{t_{45}} - \frac{c_p T_{t_{23}}}{(1 + f)\, c_{p4}} \left(\pi_{hpc}^{\kappa} - 1\right)
$$

Going by the turbine efficiency expression

$$
\eta_{hpt} = \frac{h_{t_{45}} - h_{t_5}}{h_{t_{45}} - h_{t_{5_{is}}}} = \frac{1 - 1/\tau_{hpt}}{1 - \pi_{hpt}^{-\kappa_4}}, \text{ where } \tau_{hpt} = \frac{T_{t_{45}}}{T_{t_5}}
$$

Giving us

$$
\pi_{hpt} = \left(1 - \frac{1}{\eta_{hpt}} \left(1 - \frac{1}{\tau_{hpt}}\right)\right)^{-1/\kappa_4} \tag{4}
$$

From these, we can get most of the properties at all the stations. We now need to find entropy and density. Density can be obtained from $p_t v_t = RT_t$.

The entropy of the static and stagnation states are the same (since the stagnation state is related to the static state through an isentropic process). So, for a process going from a state i (initial) to f (final), we have

$$c_p \mathrm{d}T_t \equiv \mathrm{d}h_t = \mathrm{d}e_t + \mathrm{d}\left(p_t v_t\right) = T_t \mathrm{d}s - p_t \mathrm{d}v_t + \mathrm{d}\left(p_t v_t\right)$$
$$c_p \mathrm{d}T_t = T_t \mathrm{d}s + v_t \mathrm{d}p_t = T_t \mathrm{d}s + RT_t \frac{\mathrm{d}p_t}{p_t}$$
$$\implies \mathrm{d}s = c_p \frac{\mathrm{d}T_t}{T_t} - R\frac{\mathrm{d}p_t}{p_t}$$
$$s_f - s_i = c_p \ln \frac{T_{t_f}}{T_{t_i}} - R \ln \frac{p_{t_f}}{p_{t_i}}$$

So, the properties after each stage are:

	T (K)	p (kPa)	v (m^3/kg)	s (J/K)
a	218.81	23.84	2.6	0
t2	250.4	38.2	1.9	0
t13	290.4	61.18	1.36	13.9
t23	302.6	69.75	1.25	17.5
t3	784.8	1499.57	0.15	94.4
t4	1380	1499.57	0.26	661.5
t45	969.4	297.47	0.94	717.2
t5	649.2	47.11	3.96	782.4

The pressure drop is 5.04 across the HPT and 6.31 across the LPT. The fuel-air ratio is 0.02.

Question 3

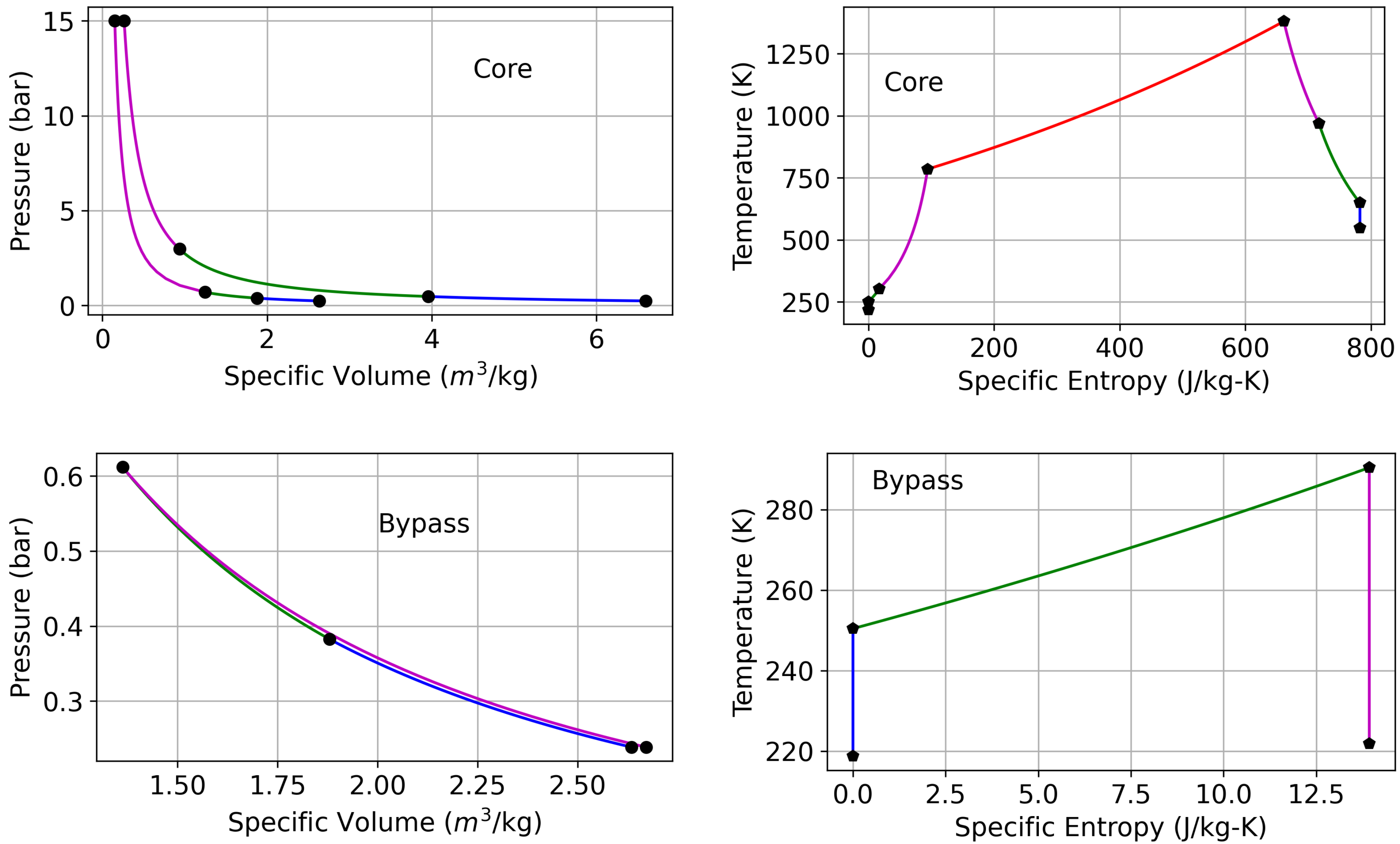
The figures are on the next page.

Question 4

Assuming an expansion to the ambient pressure (p_a),

$$V_b = \sqrt{2\left(h_{t_{13}} - h_{19}\right)} = \sqrt{2c_p T_{t_{13}} \left\{1 - \left(\frac{p_a}{p_{t_{13}}}\right)^{\kappa}\right\}} = 371.1\ m/s \qquad (M_b = 1.24)$$
$$V_c = \sqrt{2\left(h_{t_5} - h_9\right)} = \sqrt{2c_{p_4} T_{t_5} \left\{1 - \left(\frac{p_a}{p_{t_5}}\right)^{\kappa_4}\right\}} = 483.3\ m/s \qquad (M_c = 1.06)$$

The mach numbers are high because we have neglected the diffuser and nozzle losses. The ratio of speeds ($\beta = V_b/V_c$) comes to 0.77. The resulting specific thrust is $131.4\ N - s/kg$, with a TSFC of $16.72\ mg/N - s$.


Core
Pressure (bar)
Specific Volume (m^3/kg)
0
5
10
15
0
2
4
6
Core
Temperature (K)
Specific Entropy (J/kg-K)
250
500
750
1000
1250
0
200
400
600
800
Bypass
Pressure (bar)
Specific Volume (m^3/kg)
0.3
0.4
0.5
0.6
1.50
1.75
2.00
2.25
2.50
Bypass
Temperature (K)
Specific Entropy (J/kg-K)
220
240
260
280
0.0
2.5
5.0
7.5
10.0
12.5