AE234. Aircraft Propulsion Quiz 6 19D170030

Q1. $M = 1.5$ @ altitude $= 15$ km.

$T_{amb} = 205$ K $\quad P_{amb} = 11.6$ kPa, $C_p = 1$ kJ/kg-K.

$T_{o,amb} = 297.23 \quad P_{o,amb} = 42.58$ kPa.

$\eta_c = 0.9 \quad \eta_t = 0.9. \quad Q_R = 45$ MJ/kg.

$\pi_c = 12$ turobjet $\quad T_{max} = 1400$ K turbojet $\quad T_{max} = 2500$ K ramjet

→ ~~as ther~~ Turbojet calculations

→ As there are no losses in the diffuser we have,

$P_{t2} = P_{o,amb} = 42.58$ kPa. and $\pi_c = \dfrac{P_{t3,\,isen}}{P_{t2}}$

$\therefore P_{t3,is} = 511.013$ kPa.

also $T_{t3,is} = T_{t2}(\pi_c)^{\frac{\gamma-1}{\gamma}} = (T_{o,amb})(12)^{\frac{0.4}{1.4}} = 604.547$ K

But $T_{t3,non\text{-}isen} = T_{t2}\left[1 + \dfrac{1}{\eta_c}\left[\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right]\right] = \cancel{1299.062}$ 638.693

Now we know $T_{max} = 1400$ K $= T_{t4}$.

$\therefore f = \dfrac{C_p T_{t4}}{Q_R}\left(1 - \dfrac{T_{t3}}{T_{t4}}\right) = \dfrac{1000}{45\times10^6}(1400 - 638.693)$

$\boxed{f = 0.0169} = \dfrac{\dot{m}_f}{\dot{m}_a}$ $\quad$ But now specific thrust is,

and. $TSFC = \dfrac{f}{F/\dot{m}}$ $\quad$ before that, we have that. $\dfrac{\dot{w}}{\dot{m}_a C_p T_{t2}}$ which is.

$\therefore \dfrac{\dot{w}}{\dot{m}_a \cancel{C_p T_{t2}}} = \cancel{C_p} T_{t4}\,\eta_t\left(1 - \pi_t^{\frac{1-\gamma}{\gamma}}\right) - \dfrac{1}{\eta_c}\left[\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right]$

As we have perfect expansion case $\pi_c = \pi_t = 12$

$$\therefore \frac{\dot{w}}{\dot{m}_a C_p T_{t2}} = \frac{\cancel{1000} \times 1400 \times 0.9 \times \left(1 - (12)^{\frac{-0.4}{1.4}}\right)}{297.23} - \frac{1}{0.9}\left(12^{\frac{0.4}{1.4}} - 1\right)$$

$= \cancel{640511.7887} -$

$2.1549 - 1.1488 = 1.0060811$

$$\frac{\dot{w}}{\dot{m}_a} = 299037.4854 \ \frac{J}{kg} = 299.037 \ \frac{kJ}{kg}$$

But heat added $= \frac{q_b}{\dot{m}_a} = f Q_R = 45 \times 10^6 \times 0.0169$

$= 760.5 \ \frac{kJ}{kg}$

$\therefore$ heat lost as KE of jet $= 461.463 \ \frac{kJ}{kg} = \frac{q_{loss}}{\dot{m}_a} = \frac{V_{jet}^2}{2}$

$\therefore V_{jet} = \cancel{960.69}$. 960.69 m/s.

Now incoming ~~thrust~~ velocity $= M \cdot a = 1.5 \times \sqrt{1.4 \times 287.1 \times 205}$

$= 430.57$

$\therefore$ Specific thrust $= \frac{F}{\dot{m}_a} = V_{jet} - V_a = 530.115$

$\therefore \text{TSFC} = \frac{0.0169}{530.115} = 3.187 \times 10^{-5}$ for Turbojet

Ramjet Calculations :

Specific Thrust $= M_a a_a \left(\sqrt{\frac{T_e}{T_a}} - 1\right)$

Now here $T_e = T_{max}$ which is equal to 2500 K

specific thrust $= 1.5 \times \sqrt{1.4 \times 287.1 \times 205} \times \left(\sqrt{\frac{2500}{205}} - 1\right)$

$= 1073.058$

Now, TSFC for ramjet

$$= \frac{f \dot{m}_a}{\dot{m}_a (V_e - V_a)} = \frac{f}{V_a\left(\sqrt{\tau_b} - 1\right)}$$

where $\tau_b = \frac{T_{t4}}{T_{t3}} = \frac{T_e}{T_{amb}}$

and $f = \frac{C_p T_{t3}}{Q_R}(\tau_b - 1) = 0.0489$ >>>

$$TSFC = \frac{a_0}{\gamma - 1} \times \frac{1}{Q_R} \times \frac{1+\frac{\gamma-1}{2}M_0^2}{M_0} \times \left(\sqrt{\tau_b}+1\right)$$

$\tau_b = \frac{T_{t4}}{T_{t3}}$ now here T_{t3} is the temperature after the isentropic combustion which is equal to Toamb as we have perfectly isentropic compression. And T_{t4} is T_{max} which is given by the 2500K

$$\therefore \tau_b = \frac{2500}{297.23} = 8.41099. \quad \sqrt{\tau_b}+1 = 3.9$$

$$\therefore TSFC = \frac{\sqrt{1.4 \times 287.1 \times 205}}{1.4-1} \times \frac{10^{-6}}{45} \times \frac{1+0.2(1.5)^2}{1.5} \times 3.9$$

$$= \frac{287.0499}{0.4} \times \frac{10^{-6}}{45} \times \frac{1.45}{1.5} \times 3.9 = 60.121 \times 10^{-6}$$

$$= 6.01 \times 10^{-5}.$$

$\therefore \boxed{TSFC = 6.01 \times 10^{-5} \text{ for Ramjet.}}$

units for TSFC are in $\boxed{\frac{\text{seconds}}{\text{metre}}}$

(Dimensional analysis

$$TSFC = \frac{f}{\frac{F}{ma}} = \frac{\text{unitless}}{\frac{N}{kg/s}} = \frac{kg/s}{N} = \frac{kg/s}{kg\ m/s^2} = \frac{\cancel{kg}}{\cancel{s}} \times \frac{s^{\cancel{2}}}{\cancel{kg}\,m} = \frac{s}{m}$$

Final answers :

TSFC for turbojet $= 3.187 \times 10^{-5} \ \frac{s}{m}$

TSFC for Ramjet $= 6.01 \times 10^{-5} \ \frac{s}{m}$

Another approach is for ramjets.

$$TSFC = \frac{f}{(1+f)V_e - V_a}$$