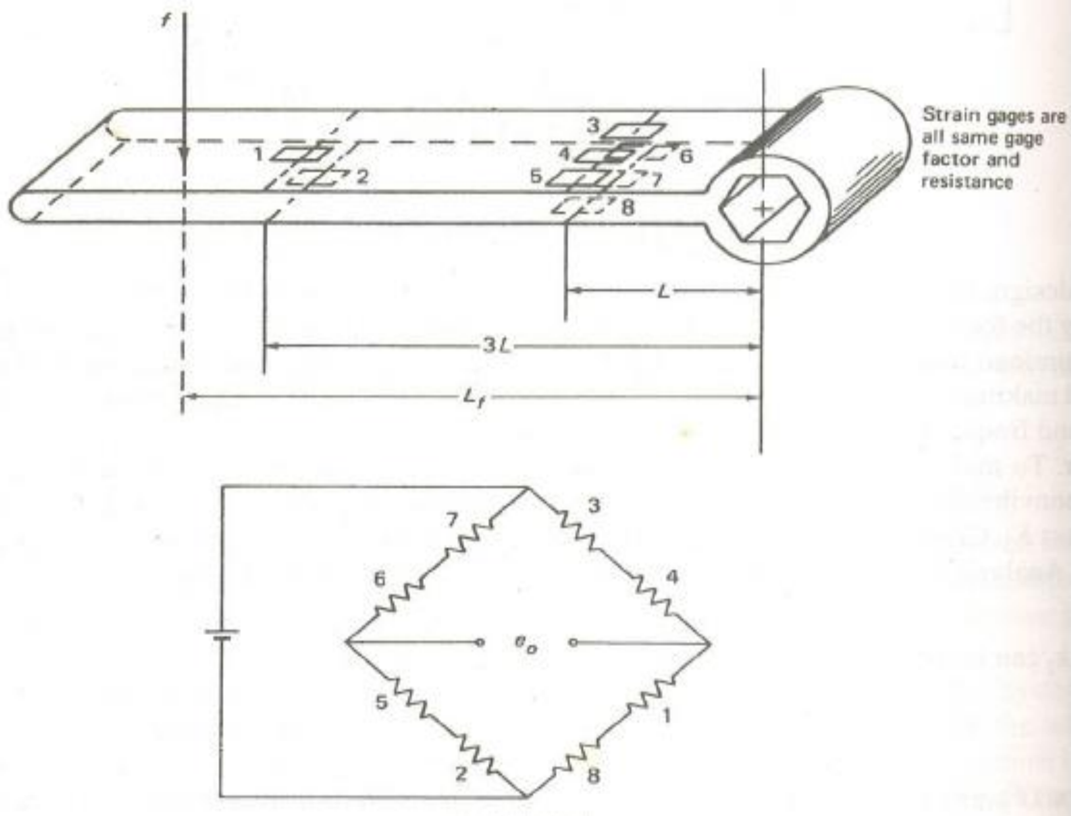
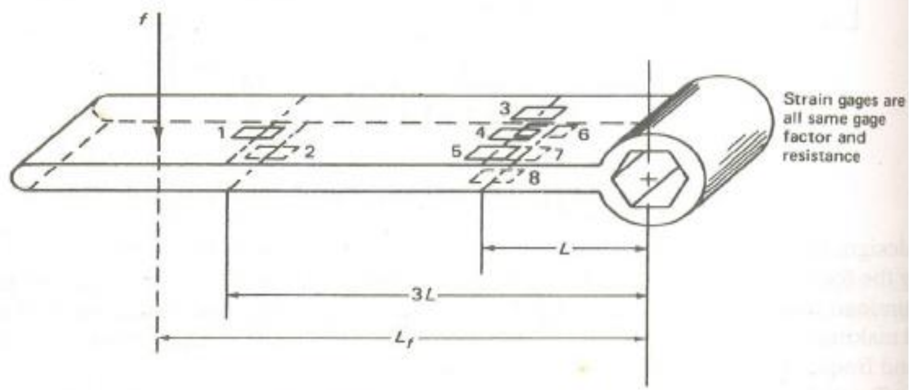
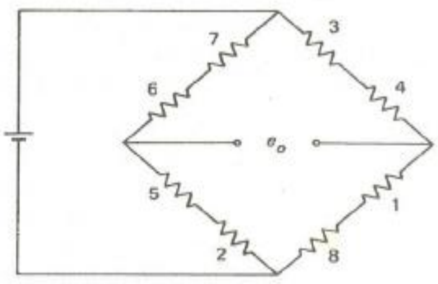
Problem (Time to solve - 10 min)

The torque wrench in the figure given below is claimed to produce an output voltage e_o proportional to the torque applied by force f to the nut, irrespective of the point of force application L_f, as long as $L_f > 3L$. Investigate the validity of this claim. Assume all the gauges have same gauge factor and same resistance(under no strain).







Arm A = 6,7
Arm B = 5,2
Arm C = 3,4
Arm D = 1,8

$$e_0 = \frac{e_i GF}{4}\left(\varepsilon_A - \varepsilon_B - \varepsilon_C + \varepsilon_D\right)$$

$$\varepsilon_3 = \varepsilon_4 = \varepsilon_5 = fK(L_f - L)$$

$$\varepsilon_6 = \varepsilon_7 = \varepsilon_8 = -fK(L_f - L)$$

$$\varepsilon_1 = fK(L_f - 3L) \qquad \varepsilon_2 = -fK(L_f - 3L)$$

$$\varepsilon_A = \varepsilon_6 + \varepsilon_7 = -2Kf(L_f - L)$$

$$\varepsilon_B = \varepsilon_5 + \varepsilon_2 = 2KfL$$

$$\varepsilon_C = \varepsilon_3 + \varepsilon_4 = 2Kf(L_f - L) \qquad \varepsilon_D = \varepsilon_1 + \varepsilon_8 = -2KfL$$

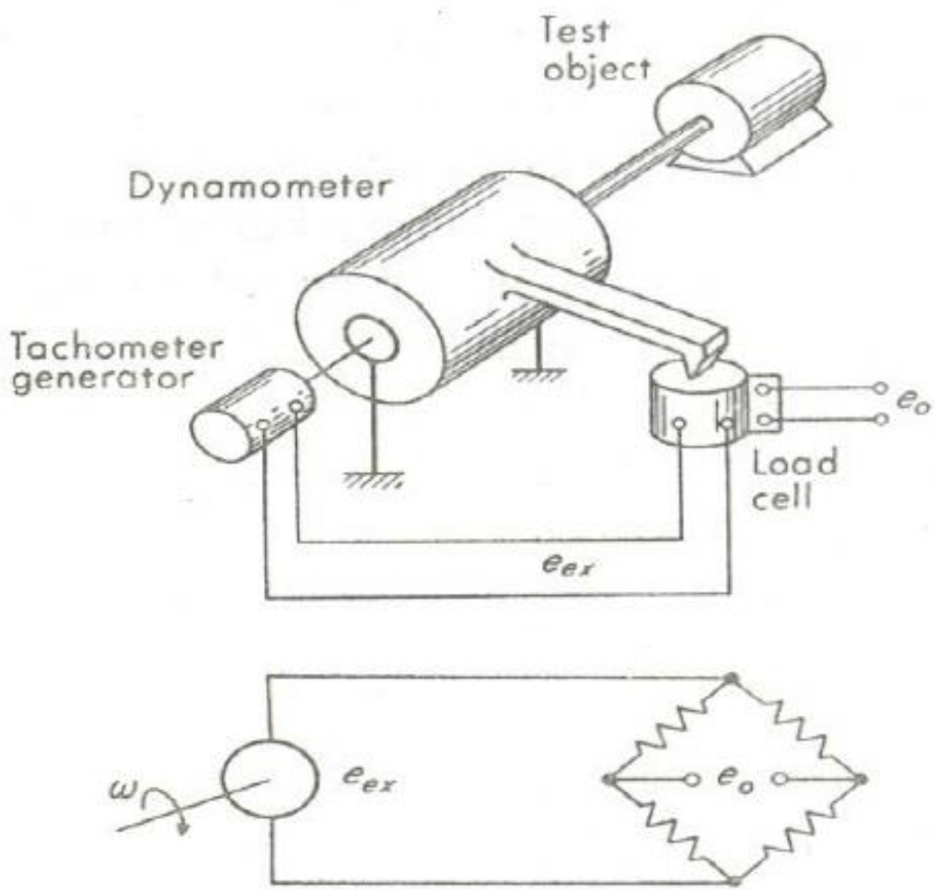
Substituting various strain values

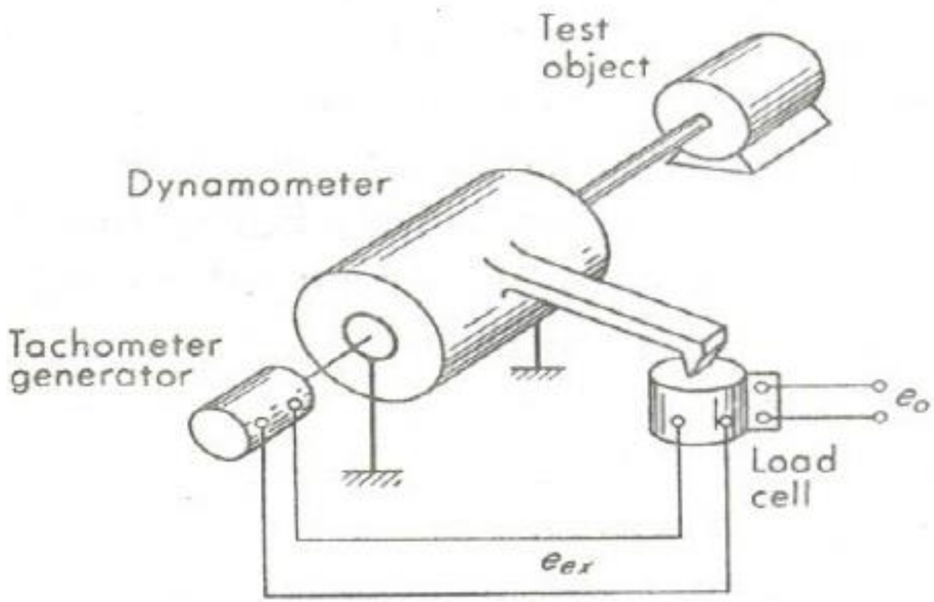
$$e_0 = \frac{e_i GF}{4} fK\left(\varepsilon_A - \varepsilon_B - \varepsilon_C + \varepsilon_D\right)$$

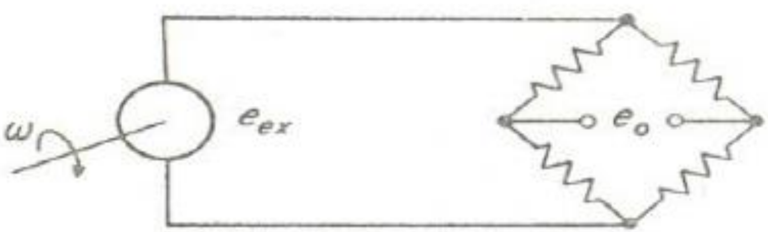
$$e_0 = \frac{e_i GF}{4} fK\left(-2(L_f - L) - 2L - 2(L_f - L) - 2L\right) \qquad e_0 = -\frac{e_i GF}{4} 4KfL_f$$

Problem (Time to solve - 10 min)

Suppose the tachometer generator in the system of the following figure puts out 6V/1000rev/min and the load cell produces 0.011mV/N.V. What will be the power calibration factor for e_o in horsepower per milli volt if the arm length is 300mm?







Techo generator Constant

$$K_T = \frac{6}{1000} V / RPM$$

$$K_T = \frac{6}{1000}\frac{60}{2\pi} V / rad / s = 0.057$$

$$e_{ex} = K_T \omega \qquad \omega = rad / s$$

Load cell constant K_L= 0.011 mV/V N

$$e_o = K_L F e_{ex}$$

Force applied on load cell

$$F = \frac{e_0}{e_{ex} K_L}$$

$$Power = torque \ \text{x} \ rotaional \ speed$$

$$Power = Fl\omega \qquad l \ is \ arm \ length = 0.3m$$

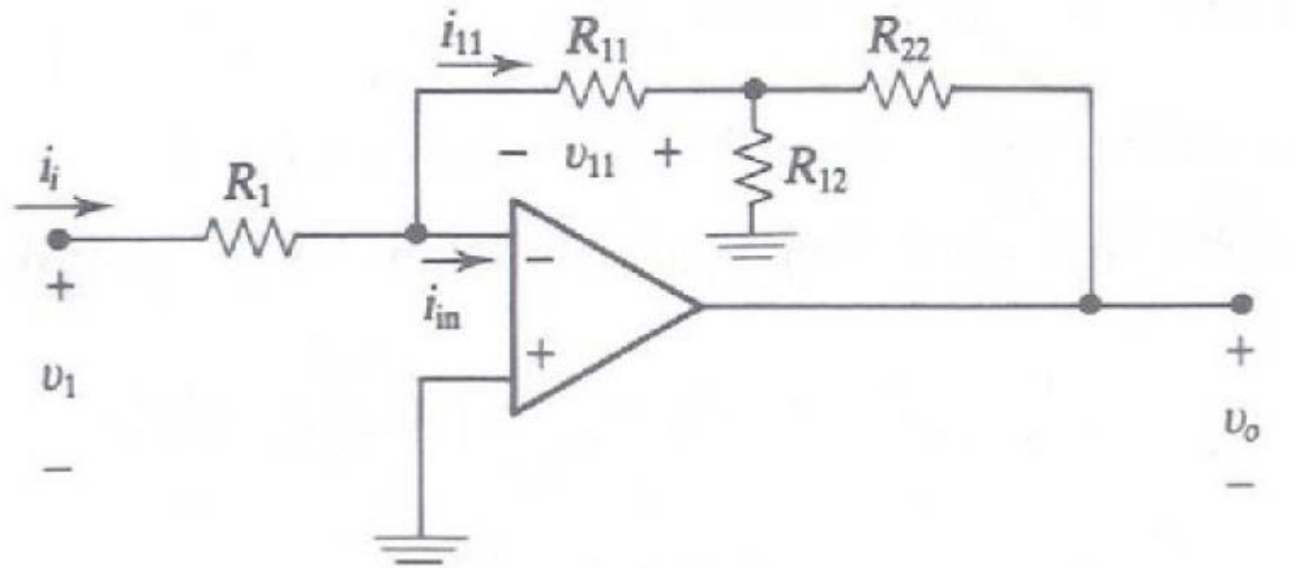
$$Power = \frac{e_0}{e_{ex} K_L} l \frac{e_{ex}}{K_T} = \frac{e_0}{K_T K_L} l = 478.46 e_0 \ \text{W} = \frac{478.46}{745.7} e_0 \ \text{HP}$$

$$Power = 0.641 e_0 \ \text{HP}$$

$$\text{Power constant} = 0.641$$

Problem (Time to solve - 10 min)

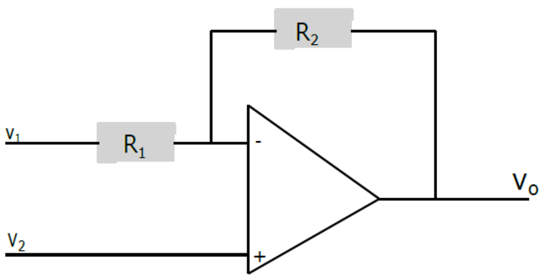
Find the gain of the circuits. Assume ideal op-amp.



In above circuit R_{11} and R_{12} are parallel due to virtual ground at non-inverting terminal.

$$v_0 = -\frac{\frac{1}{R_{11}} + \frac{1}{R_{12}} + R_{22}}{R_1} v_1$$

Q1) Design an OpAmp circuit. Input voltage range 0.3V-1.0V output voltage 0.5 V to 4.5 V.



$$v_0 = -\frac{R_2}{R_1}v_1 + \left(1+\frac{R_2}{R_1}\right)v_2$$

$$v_0 = v_2 + (v_2 - v_1)R \quad \text{eqn}\,1$$

$$\frac{R_2}{R_1} = R$$

Case 1		
v_1	v_2	v_o
0.3	?	0.5
1.0	?	4.5

From equation 1, we need to find the value of R and unknown voltage (refer two cases given as table). We can formulate problem as two equations and two unknowns.

$$0.5 = v_2 + (v_2 - 0.3)R$$
$$4.5 = v_2 + (v_2 - 1.0)R$$

$$R = -5.72$$
$$v_2 = 0.256V$$

Case 1 is not feasible. Resistance ratio is negative

Case 2		
v_1	v_2	v_o
?	0.3	0.5
?	1.0	4.5

$$0.5 = 0.3 + (0.3 - v_1)R$$
$$4.5 = 1.0 + (1.0 - v_1)R$$

$$R = 4.71$$
$$v_1 = 0.257V$$

Case 2 is feasible. Resistance ratio is positive

An object with a volume of $160cc$ is weighed on an equal arm balance. The standard mass required for balance is $0.5\,kg$ and has a volume of $50cc$. What is the value of correction necessary for air buoyancy?

$$error = volume\ difference\ X\ density$$

$$error = (V_{obj} - V_{std})\rho_{air}$$

$$\rho_{air} = 1.225X10^{-9}\,kg/mm^3$$

$$error = (160-90)X1.225X10^{-9}$$

$$error = 857.5X10^{-9}\,kg$$

Problem (Time to solve - 10 min)

A mercury thermometer has a capillary tube of 0.25 mm diameter. If the bulb is made of a zero expansion material, what volume must it have if a sensitivity of 4 mm/°C is desired? Assume operation near 20 °C.

For mercury, Linear expansion coeff. $= 60 \times 1\bar{0}^{6} m/m -^{\circ} C$.

$$Volume\ of\ a\ cuboid\ \ V = xyz$$

$$V + \Delta V = (x+\Delta x)(y+\Delta y)(z+\Delta z)$$

$$V + \Delta V = xyz + yz\Delta x + xz\Delta y + xy\Delta z$$

$$+ z\Delta x\Delta y + y\Delta x\Delta z + x\Delta y\Delta z + \Delta x\Delta y\Delta z$$

Ignoring higher order terms

$$\Delta V = yz\Delta x + xz\Delta y + xy\Delta z$$

$$\frac{\Delta V}{V} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$\Delta V_{\exp} = 3\Delta l_{\exp}$$

Volume of expanded fluid = capillary tube area x length of liquid in capillary tube

$$\Delta V = 3V\Delta l_{\exp} T$$

$$\Delta V = Change\ in\ volume\ per\ unit\ temperature$$

$$V = Volume\ of\ liquid$$

$$\Delta l_{\exp} = Coefficent\ of\ linear\ \exp ansion$$

$$T = Temperature$$

$$\Delta V_{cap} = A_{cap}\delta = \frac{\pi}{4} d_c^2 \delta$$

$$\Delta V_{cap} = Volume\ of\ liquid\ in\ capillary$$

$$A_{cap} = Capillary\ tube\ area$$

$$\delta = Lenght\ of\ liquid\ in\ capillary$$

$$d_c = Capillary\ tube\ diameter$$

Problem (Time to solve - 10 min)

A mercury thermometer has a capillary tube of 0.25 mm diameter. If the bulb is made of a zero expansion material, what volume must it have if a sensitivity of 4 mm/^{o}C is desired? Assume operation near 20 ^{o}C.

For mercury, Linear expansion coeff. $= 60 \times 1\bar{0}^{6} m/m -^{o} C$.

$$\Delta V = \Delta V_{cap}$$

$$\Delta V = 3V\Delta l_{\exp}T$$

$$\Delta V = 3V\Delta l_{\exp}T = \frac{\pi}{4} d_c^2 \delta \qquad \text{eqn 2}$$

$$\Delta l_{\exp} = Linear\ exapansion\ coeff = 60X10^{-6} m/m-^{o}C$$

$$T = Temperature$$

$$d_c = Capillary\ tube\ diameter = 0.25mm$$

$$\delta = Length\ of\ liquid\ in\ capillary = 4mm/^{o}C$$

Substituting above values in eqn 2

$$V = 1090.83mm^3$$

Problem (Time to solve - 10 min)

A balloon carrying a first order thermometer, with a 15s time constant, rises through the atmosphere at 6m/s. Assume temperature varies with altitude at 0.15 $^oC/30$m. The balloon radios temperature and altitude readings back to ground. At 3000m the balloon says the temperature is 0^{o}C. What is the true altitude at 0^{o}C occurs?

Ramp Input

$$q_0 = K\dot{q}_{is}(\tau e^{-t/\tau} + t - \tau)$$

During steady state $q_0 = K\dot{q}_{is}(t - \tau)$

$$Rate\ of\ \ change\ \ of\ \ temperature\ \dot{q}_{is} = lapse\ rate\ x\ speed$$

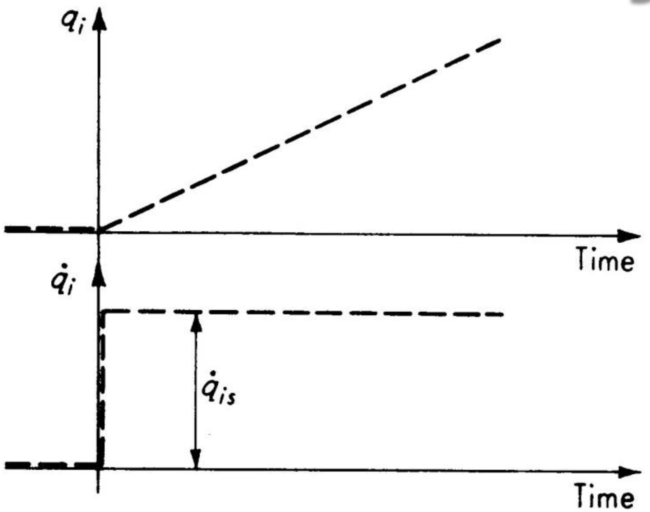
$$\dot{q}_{is} = \frac{0.15}{30}\, x\, 6 = 0.03^o C/\sec$$

At any instant there is dealy of τ sec (15 sec)

In τ sec balloon would have travelled = 15 x 6 = 90

0^oC would have occured at = 3000 - 90 = 2110 m

First-order system - Ramp response



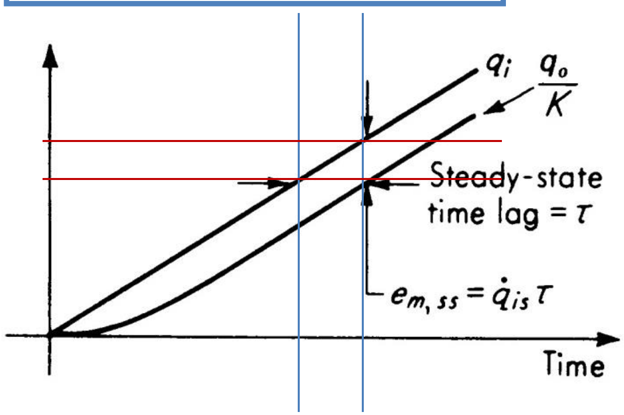
$$\mathbf{q}_i = \begin{cases} \mathbf{q}_0 = 0 & \mathbf{t} \leq 0 \\ \dot{\mathbf{q}}_{is}\mathbf{t} & \mathbf{t} \geq 0 \end{cases}$$

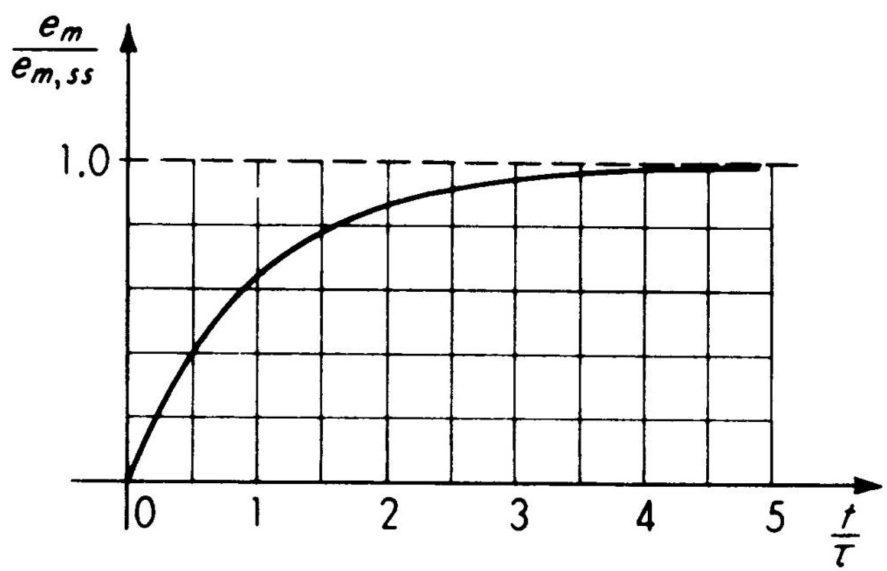
$$\mathbf{e}_m = \underbrace{-\dot{\mathbf{q}}_{is}\boldsymbol{\tau}\mathbf{e}^{-\mathbf{t}/\tau}}_{\text{Transient error } e_{m,t}} + \underbrace{\dot{\mathbf{q}}_{is}\boldsymbol{\tau}}_{\text{Steady state error } e_{m.ss}}$$

$$\mathbf{q}_0 = \mathbf{K}\dot{\mathbf{q}}_{is}(\boldsymbol{\tau}\mathbf{e}^{-\mathbf{t}/\tau} + \mathbf{t} - \boldsymbol{\tau})$$

First term gradually disappears – transient error. Second term persist for ever – steady state error.

$$\boxed{\frac{q_0}{K} = \dot{q}_{is}(\tau e^{-t/\tau} + t - \tau)}$$





Problem (Time to solve - 10 min)

A measuring instrument with a time constant of 0.4s and a static sensitivity of 0.01mV/^{o}C is used to measure the temperature of a medium, which changes from 15 to 80^{o}C. Taking the output as zero at 15^{o}C, find the time taken for the output voltage to reach 70% of the steady state value, if the temperature change occurs suddenly.

Ramp Input

$$\frac{q_0}{K} = q_i + (q_{ss} - q_i)(1 - e^{-t/\tau})$$

$$q_i = \text{input at } t = 0$$

$$q_{ss} = \text{Step input}$$

$$\text{Output} = 0 \ \text{ at } 15\ ^{\text{o}}C$$

Obtain time at which output will 0.7 of steady state output

$$0.7(q_{ss} - q_i) = (q_{ss} - q_i)(1 - e^{-t/\tau})$$

$$t = 0.48\,\text{sec}$$