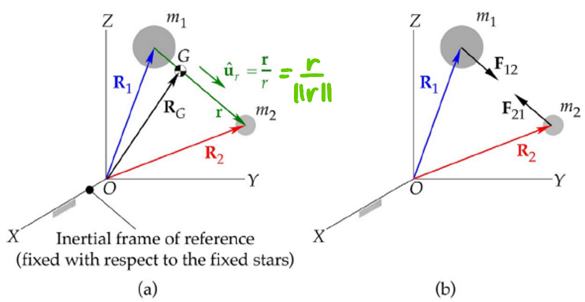
Lecture 5



- $R_1 = X_1\hat{I} + Y_1\hat{J} + Z_1\hat{K}$

  $R_2 = X_2\hat{I} + Y_2\hat{J} + Z_2\hat{K}$

- $R_G = \dfrac{m_1 R_1 + m_2 R_2}{m_1 + m_2}$ (position vector of the centre of mass / barycenter)

- $V_G = \dfrac{m_1 \dot{R}_1 + m_2 \dot{R}_2}{m_1 + m_2}$

- $a_G = \dfrac{m_1 \ddot{R}_1 + m_2 \ddot{R}_2}{m_1 + m_2}$

- Let $r = R_2 - R_1$

  $= (X_2 - X_1)\hat{I} + (Y_2 - Y_1)\hat{J} + (Z_2 - Z_1)\hat{K}$

- $F_{12} = \dfrac{G m_1 m_2}{\|r\|^2}\hat{u}_r$ $\left(\|r\| = \sqrt{(X_2-X_1)^2 + (Y_2-Y_1)^2 + (Z_2-Z_1)^2}\right)$

- $F_{21} = -\dfrac{G m_1 m_2}{\|r\|^2}\hat{u}_r$ (Newton's third law)

- $F_{21} = -\frac{G m_1 m_2}{\|r\|^2} \hat{u}_r$ (Newtons thira law)

- $F_{12} = m_1 \ddot{R}_1 = \frac{G m_1 m_2}{\|r\|^2} \hat{u}_r$ (Newton's second law)

- $F_{21} = m_2 \ddot{R}_2 = -\frac{G m_1 m_2}{\|r\|^2} \hat{u}_r$

- $a_G = 0 \Rightarrow R_G)_t = R_G)_0 + V_G t$

- G may serve as the origin of an inertial frame!

- $\ddot{R}_1 = \frac{G m_2}{\|r\|^3} r$

- $\ddot{R}_2 = -\frac{G m_1}{\|r\|^3} r$

$$R_1 = X_1 \hat{I} + Y_1 \hat{J} + Z_1 \hat{K}$$
$$R_2 = X_2 \hat{I} + Y_2 \hat{J} + Z_2 \hat{K}$$
$$r = (X_2 - X_1)\hat{I} + (Y_2 - Y_1)\hat{J} + (Z_2 - Z_1)\hat{K}$$
$$\ddot{X}_1 \hat{I} + \ddot{Y}_1 \hat{J} + \ddot{Z}_1 \hat{K} = \frac{G m_2}{\|r\|^3} \left[(X_2 - X_1)\hat{I} + (Y_2 - Y_1)\hat{J} + (Z_2 - Z_1)\hat{K} \right]$$
$$\ddot{X}_2 \hat{I} + \ddot{Y}_2 \hat{J} + \ddot{Z}_2 \hat{K} = -\frac{G m_1}{\|r\|^3} \left[(X_2 - X_1)\hat{I} + (Y_2 - Y_1)\hat{J} + (Z_2 - Z_1)\hat{K} \right]$$

$$\ddot{X}_2 \hat{I} + \ddot{Y}_2 \hat{J} + \ddot{Z}_2 \hat{K} = \frac{-Gm_1}{\|r\|^3}\left[(X_2 - X_1)\hat{I} + \ldots + (Z_2 - Z_1)\hat{K}\right]$$

$$\ddot{X}_1 = \frac{Gm_2}{\|r\|^3}(X_2 - X_1)$$

Similarly, one can find $\ddot{Y}_2, \ddot{Z}_2, \ddot{X}_1, \ddot{Y}_1, \ddot{Z}_1$.

- A conservative force, like gravity, can be obtained from its potential energy function:

$$F = -\nabla V$$

$$= -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(-\frac{Gm_1 m_2}{\|r\|}\right)$$

- $$V = -\frac{G m_1 m_2}{\sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}}$$

- $$F_{12} = -\left(\frac{\partial V}{\partial X_2}\hat{i} + \frac{\partial V}{\partial Y_2}\hat{j} + \frac{\partial V}{\partial Z_2}\hat{k}\right)$$

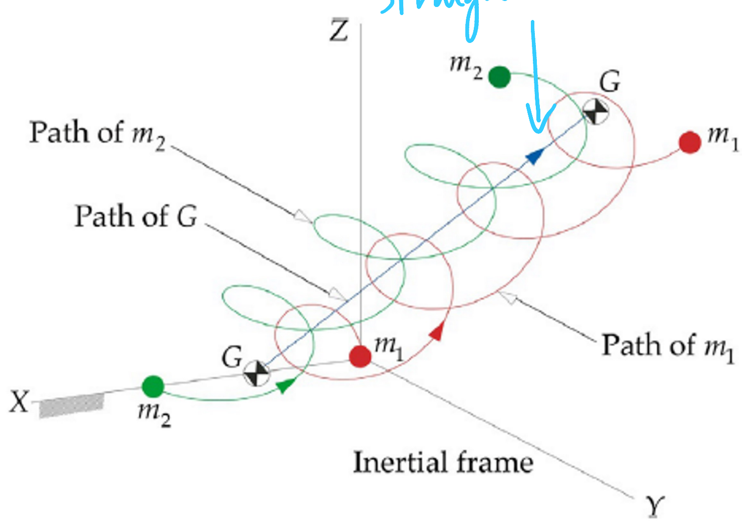
- $$F_{21} = -\left(\frac{\partial V}{\partial X_1}\hat{i} + \frac{\partial V}{\partial Y_1}\hat{j} + \frac{\partial V}{\partial Z_1}\hat{k}\right)$$

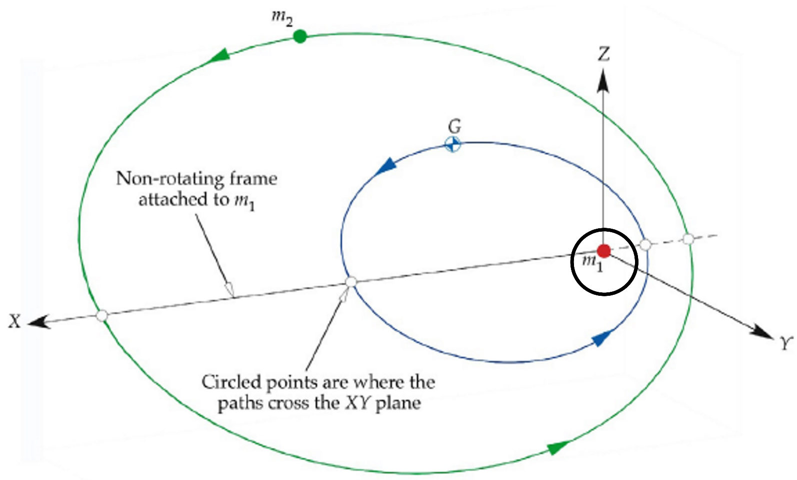
$$V = -\frac{G m_1 m_2}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

$$\frac{\partial V}{\partial x_2} = -G m_1 m_2 \cdot \frac{-1}{\cancel{2}} \cdot \frac{1}{\left[(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2\right]^3} \cdot \cancel{2}(x_2-x_1)$$

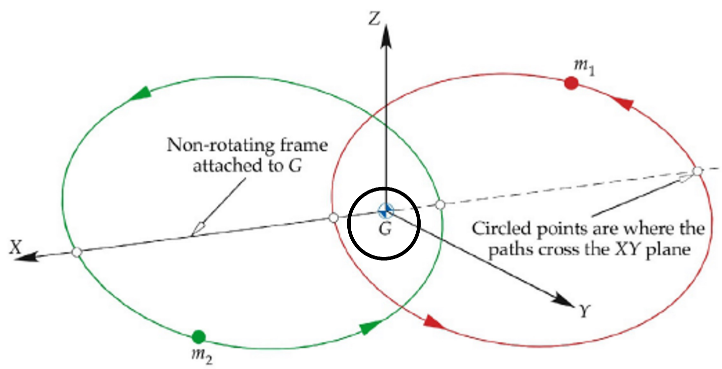
$$= \frac{G m_1 m_2}{\|r\|^3}(x_2-x_1)$$

straight line



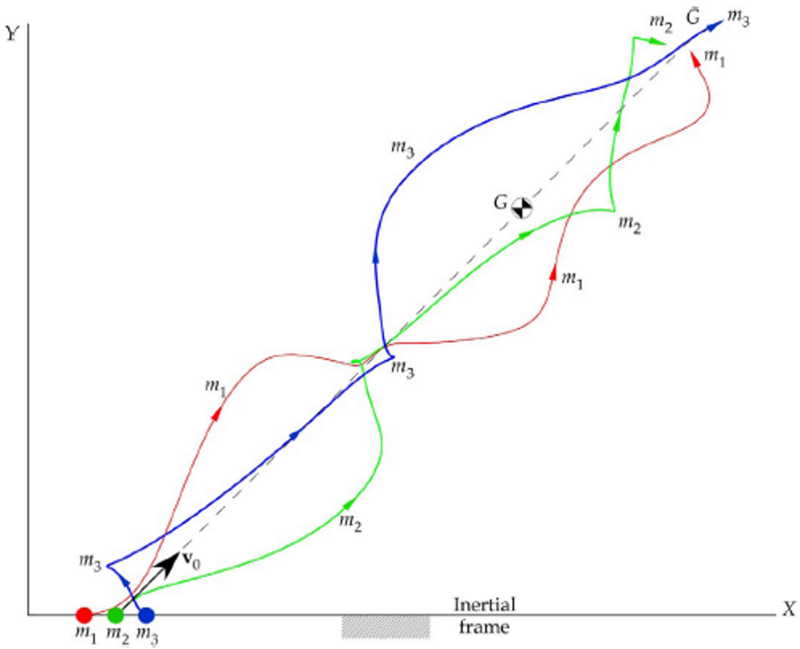


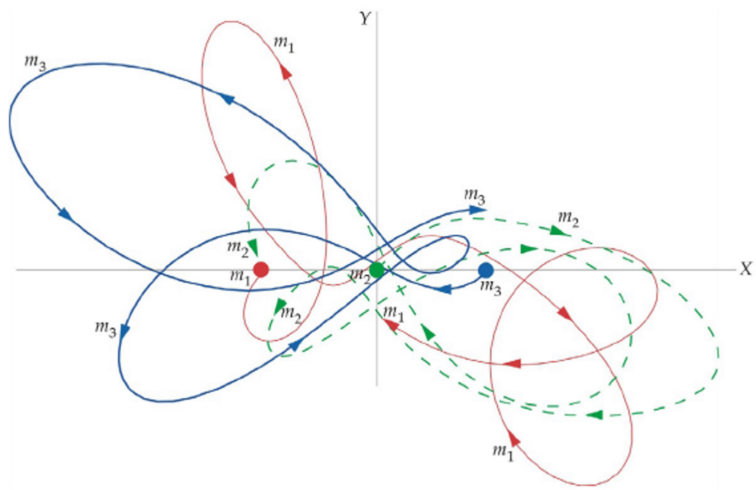
(a)



(b)

- The n-body problem has no closed-form solution, which is complex and chaotic in nature.





$$- \quad \ddot{r} = \ddot{R}_2 - \ddot{R}_1$$

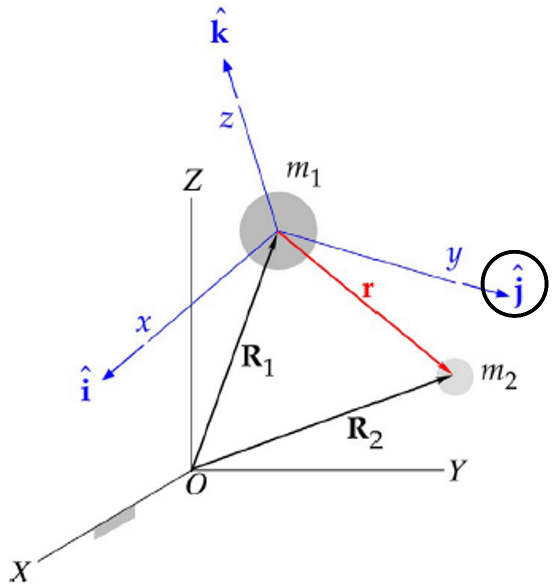
$$= -\frac{Gm_1}{\|r\|^2}\hat{u}_r - \frac{Gm_2}{\|r\|^2}\hat{u}_r$$

$$= -\frac{G(m_1+m_2)}{\|r\|^2}\hat{u}_r$$

$$= -\frac{\mu}{\|r\|^2} r$$

gravitational parameter (km/sec²)

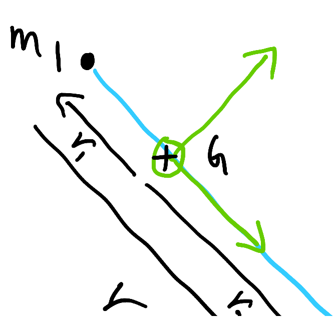
$$= -\frac{\mu}{\|r\|^3} r$$ paramet

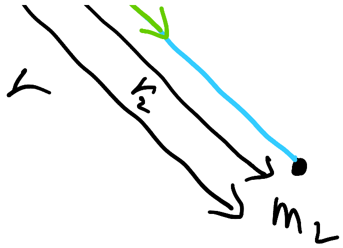


$$\ddot{r} = -\frac{\mu}{\|r\|^3} r$$

$$(\ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}) = -\frac{\mu}{\|r\|^3}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$\ddot{x} = -\frac{\mu}{\|r\|^3} x, \quad \ddot{y} = -\frac{\mu}{\|r\|^3} y, \quad \ddot{z} = -\frac{\mu}{\|r\|^3} z$$





- $m_2\ddot{r}_2 = -\dfrac{G m_1 m_2}{\|r\|^2}\,\hat{u}_r \,, \quad \hat{u}_r = \dfrac{r_2}{\|r_2\|}$

- $r = r_2 - r_1$

- $m_1 r_1 + m_2 r_2 = 0$

  $r_1 = -\dfrac{m_2}{m_1} r_2$

- $r = \left(\dfrac{m_1+m_2}{m_1}\right) r_2$

- $\ddot{r}_2 = \dfrac{-\overbrace{\mu'}^{\left(\frac{m_1}{m_1+m_2}\right)^3 \mu}}{\|r_2\|^3}\, r_2$

  $\ddot{} \quad \overbrace{\mu''}^{\left(\frac{m_2}{m_1+m_2}\right)^3 \mu} \quad r$

$$- \quad \ddot{r}_1 = -\frac{\mu''}{\|r_1\|^3} r_1 \longrightarrow \left(\overline{m_1 + m_2} \right)$$