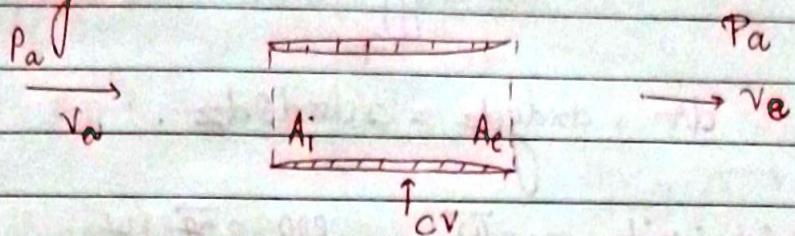


Thrust :-

Force on fluid \rightarrow Reaction \rightarrow Generate a force
 ↑
 Push the air back [Air pushes back.]

Simple engine -



$$\dot{m} = \rho_a A_i v_a$$

$$\text{Thrust} = \dot{m} (v_e - v_a) = T$$

- pressure at exit need not equal to P_a
- v_e & v_a are usually not uniform/ finite.

A diagram of a nozzle with varying cross-sections. The inlet velocity is v_a and the exit velocity is v_e . The jet radius is δ_j . The momentum flux is $P_m = \dot{m} \bar{v}_j = \rho \bar{v}_j^2 \delta_j$. The entrainment velocity is \bar{v}_j .

$$P_m = \rho v_e^2 \delta_0 = \rho \bar{v}_j^2 \delta_j$$

$$\dot{m} = \rho v_e \delta_0 = \rho \bar{v}_j \delta_j$$

$$P_m = \dot{m} \bar{v}_j = \rho \bar{v}_j^2 \delta_j \Rightarrow \bar{v}_j = \sqrt{\frac{P_m}{\rho \delta_j}}$$

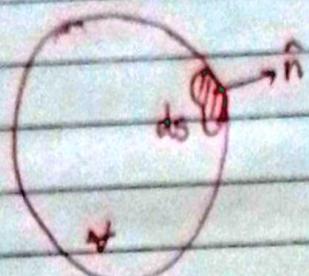
$\delta_j \uparrow \rightarrow$ jet widens $\Rightarrow \bar{v}_j \downarrow$

$$\dot{m} = \sqrt{P_m \rho \delta_j} \uparrow - (\text{entrainment})$$

- Due to wing effect, rel. & pressure different from v_a/P_a \rightarrow errors in estimation

- Energy is added through combustion \rightarrow fuel is consumed.
 $m_e = m_{in} + m_f$

Newton's laws for CV:-



Inside the vol., we've fluid, ρ, \vec{v}
 $m = \iiint_V \rho dV$

$$dV = dx dy dz = r dr d\theta dz$$

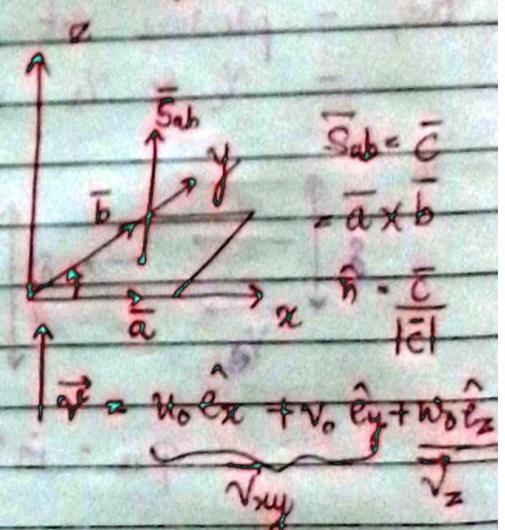
Momentum of fluid inside $\rightarrow \bar{P_m} = \iiint_V \rho \vec{v} dV$

the vol.

Avg. vel. of fluid in $\vec{v} \rightarrow \bar{v} = \frac{\bar{P_m}}{m} = \frac{\iiint_V \rho \vec{v} dV}{\iiint_V \rho dV}$

$$d\vec{s} = ds \hat{n}$$

How much flow goes through \bar{s}_{ab} ?
 $\bar{v}_z \parallel \bar{s}_{ab}, \bar{v}_{xy} \perp \bar{s}_{ab}$



$\bar{v} \cdot \bar{s}_{ab} \leftarrow$ volumetric flow through \bar{s}_{ab}

$\bar{v} \cdot \bar{s}_{ab} \leftarrow$ mass flow rate through \bar{s}_{ab}

flow rate \rightarrow well-defined surface area * normal velocity
mass flow rate $\rightarrow m = \iint \bar{v} \cdot d\vec{s}$ $\begin{cases} > 0 & \text{outflow} \\ < 0 & \text{inflow} \end{cases}$
across a surface

$$\dot{m}(t + \Delta t) - m(t) = \iint \bar{v} \cdot d\vec{s} \Delta t$$

$$\lim_{\Delta t \rightarrow 0} \frac{m(t + \Delta t) - m(t)}{\Delta t} = - \iint \bar{v} \cdot d\vec{s}$$

$$\frac{dm}{dt} = -\oint \rho \vec{v} \cdot d\vec{s}$$

$$\frac{d}{dt} (\iiint \rho dV) = -\oint \rho \vec{v} \cdot d\vec{s}$$

for a fixed C.V., (boundaries do not move)

$$\iiint \frac{\partial \rho}{\partial t} dV + \oint \rho \vec{v} \cdot d\vec{s} = 0 \quad \text{--- (1)}$$

going back to engine.

$\oint \rho \vec{v} \cdot d\vec{s} = -\rho V_a \sin \theta + \rho V_e \cos \theta$

$$\iiint \frac{\partial \rho}{\partial t} dV = 0 \quad \text{--- (steady state)}$$

∴ By cont. eqn (1), $\rho V_a \sin \theta = \rho V_e \cos \theta$

For fuel coming inside the engine,

$$\oint \rho \vec{v} \cdot d\vec{s} = -\rho V_a \sin \theta + \rho V_e \cos \theta - m_f \quad \text{if in steady state}$$

$$\therefore \rho V_e \cos \theta = \rho V_a \sin \theta + m_f$$

$$\dot{m} = \dot{m}_a + \dot{m}_f$$

$$f = \frac{\dot{m}_f}{\dot{m}_a} \ll 1$$

$$\dot{m} = \dot{m}_a(1+f)$$

Momentum conserv.

$$P_m = \iiint \rho \vec{v} dV$$

$$\dot{m} = \rho \vec{v} \cdot d\vec{s}$$

$$\dot{P}_m = \dot{m} \vec{V} = \rho \vec{v} \vec{V} \cdot d\vec{s}$$

Over At,

$$T_m = \oint_{\text{eff}} g \bar{v}^2 \cdot d\bar{s} \leftarrow \text{loss of momentum of fluid in CV due to advection.}$$

Momentum to be affected by \rightarrow pressure, shear stress, body forces, interaction with other fluids

Pressure force at ds part = $-pd\bar{s}$

Total pressure force = $-\oint pd\bar{s}$

$$\frac{\partial}{\partial t} \left[\oint g \bar{v} d\bar{A} \right] = - \oint g \bar{v} \bar{v} \cdot d\bar{s} - \oint p d\bar{s} + F_{\text{other}}$$

For steady state,

$$F_{\text{other}} = \oint g \bar{v} \bar{v} \cdot d\bar{s} + \oint p d\bar{s}$$

Momentum
cons.

$$\oint g \bar{v} \bar{v} \cdot d\bar{s} = -g_i V_a^2 \sin \theta + g_e V_e^2 \sin \theta$$

$$\oint p d\bar{s} = -P_a S + P_e S + P_a [S - S_e]$$

$$= [P_e - P_a] S_e$$

$$F = \oint g \bar{v} \bar{v} \cdot d\bar{s} + \oint p d\bar{s} = [S_e V_e^2 S_e - S_i V_a^2 S_i] + (P_e - P_a) S_e$$

$$F = S_e V_e^2 S_e - S_i V_a^2 S_i + (P_e - P_a) S_e$$

$$m_a = S_i V_a S_i \Rightarrow S_i V_a^2 S_i = m_a V_a$$

$$m_e = S_e V_e S_e \Rightarrow S_e V_e^2 S_e = m_e V_e$$

$$\begin{aligned} T &= F = m_e V_e - m_a V_a + (P_e - P_a) S_e \\ &= m_a \{ (1+f) V_e - V_a \} + (P_e - P_a) S_e \end{aligned}$$

Considering the CV only till A_e , i.e. no effect of the aircraft on the engine flows

$$\therefore \text{uninstalled thrust} \rightarrow T = mV_e - mV_a + (p_e - p_a)A_e$$

$$\text{Installed thrust} \rightarrow T_{\text{inst}} = T - T_{\text{losses}}$$

$$\text{Ram Drag} \rightarrow D_0 = mV_a \equiv \rho_a V_a^2 S \sin \approx \rho_a V_a^2 A \sin$$

$$\text{gross thrust} \rightarrow T_g = mV_e + (p_e - p_a)A_e$$

$$T = T_g - D_0$$

Fuels :- i) Piston engines \rightarrow Avgas \rightarrow aviation gasoline
 \rightarrow added (tetraethyl lead) $< 0.56 \text{ gm/l}$
 $\varrho_R = 44 \text{ MJ/kg} \quad | \quad \beta = 0.71 \text{ kg/l}$
 $= 31.6 \text{ MJ/l}$

ii) Jet engines \rightarrow kerosene \rightarrow Jet A, Jet A-1 (JP-1A)
 \rightarrow Jet B (cold climate), TS-1 (russian)

JP-4 - 50 gasoline + 50 kerosene

JP-8 - for military purpose

$$\varrho_R = 43 \text{ MJ/kg} \quad | \quad \beta = 0.745 - 0.84 \text{ kg/l}$$

$$\text{iii) Diesel} \rightarrow \varrho_R = 45.6 \text{ MJ/kg} = 38.6 \text{ MJ/l}$$

$$H_2 \rightarrow \varrho_R = 120 - 140 \text{ MJ/kg}$$

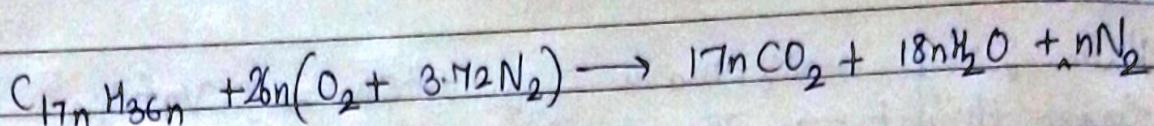
$$\begin{aligned} (\text{gas}) &\rightarrow \varrho_R = 0.01 \text{ MJ/l} \\ (\text{liq.}) &\rightarrow \varrho_R = 8.5 - 10 \text{ MJ/l} \quad \leftarrow \text{storage net. requirements} \end{aligned}$$

Fuel requirements :-

How much air do we need to burn 1kg of fuel (kerosene) ? $\left[\frac{m_a}{m_f} \right]$

$$C_{17n}C_{36n} \approx 85\% C, 15\% H$$

26 x 342



$$f_{st} = \frac{m_f}{m_a} = \frac{1}{14.75} = 0.068 \quad (\text{for stoichiometric comb'})$$

$f < f_{st}$ → fuel lean

used for Ramjets,

$f > f_{st}$ → fuel rich

afterburning Turbojets

$$f = 0.005 - 0.02 \quad \leftarrow \text{for most gas-Turbines}$$

$$f \ll 1$$

For subsonic engines, $p_e = p_a$

$$\therefore T = m_a [V_e - V_a + f V_e]$$

$$T \approx m_a [V_e - V_a], \quad f \ll V_e - V_a$$

$$\tau = \frac{V_a}{V_e}, \quad 1 - \tau \gg f$$

Near take-off τ is very small $\Rightarrow 1 - \tau \approx 1$

$$T = m_a V_e [(1 - \tau) + f]$$

$$T \approx m_a V_e (1 - \tau)$$

τ shd not be too large

$$\tau \ll 1 - f$$

$$\therefore \tau \ll 0.98$$

$$v_a \approx 250 \text{ m/s}$$

$$\gamma \approx 1/2$$

$$v_e \approx 500 \text{ m/s}$$



Efficiency :- $\frac{\text{inf} Q_R}{T} \rightarrow T \rightarrow$ pushing the vehicle at speed v_a

Power gained $\rightarrow T v_a$

work $= F \times \text{dist.}$, over $A\Delta t$, dist. $= v_a A\Delta t$, $F = T$,

$$W = F \text{ dist.} = T v_a A\Delta t$$

$$P = T v_a$$

Total power from fuel $\rightarrow \frac{\text{inf} Q_R}{T}$,

fuel also had KE $\rightarrow \frac{1}{2} \frac{\text{inf} v_a^2}{Q_R}$

$$\frac{\frac{1}{2} \text{inf} v_a^2}{\text{inf} Q_R} = \frac{v_a^2}{2 Q_R} \approx \frac{250^2 \times 250}{2 \times 40 \times 10^6} = 8 \times 10^{-5} \ll 1$$

overall efficiency :-

$$\text{efficiency} = \frac{\text{useful work}}{\text{Total energy input}}$$

$$\eta_T = \frac{T v_a}{\text{inf} Q_R} = \frac{m a [v_e - v_a] v_a}{\text{inf} Q_R}$$

$$= m a v_e^2 (1 - \frac{v_a}{v_e}) \frac{v_a}{v_e}$$

$$f \frac{\text{inf} Q_R}{m a}$$

$$\eta_T = \frac{(1-\gamma)\gamma}{f(Q_R/v_e^2)}$$

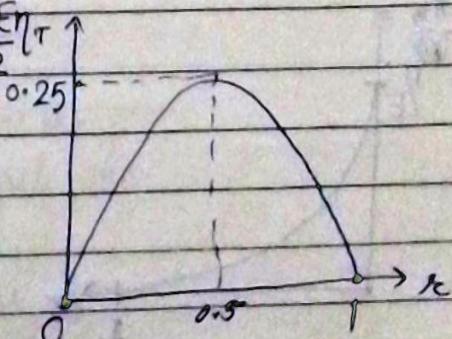
$$F = \frac{f Q_R}{\frac{1}{2} v_e^2} = \frac{2 f Q_R}{v_e^2} = \frac{2 \times 10^{-2} \times 40 \times 10^6}{500 \times 500}$$

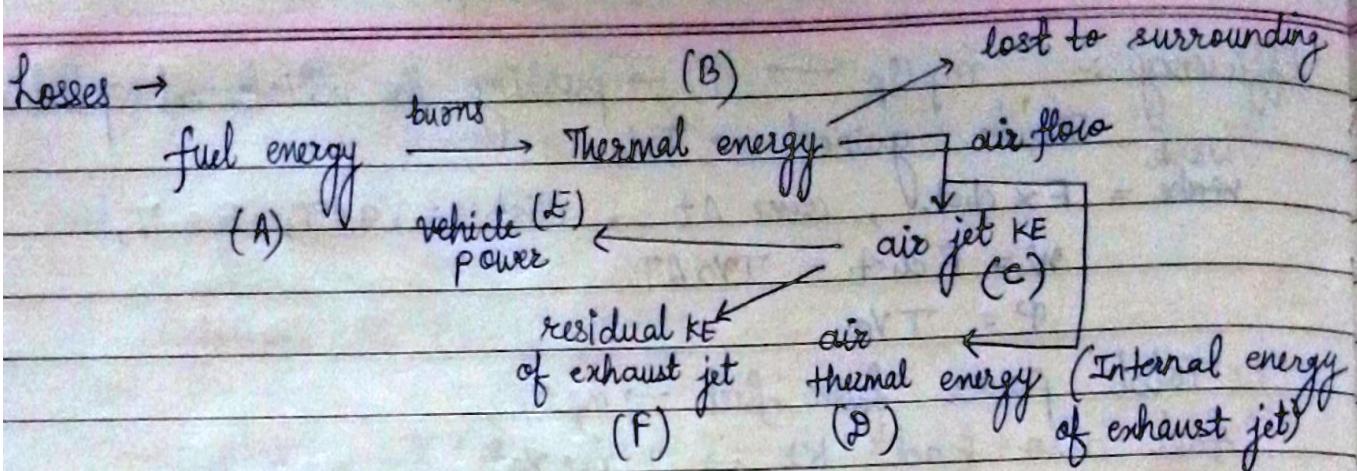
$$= 3.2$$

$$\therefore \eta_T = \frac{2\gamma(1-\gamma)}{E}$$

$$E \rightarrow 4-10$$

$$\eta_{T/\text{opt.}} = \frac{1}{2E} \rightarrow 0.05 - 0.125$$





$$U_e = V_e - V_a$$

Propulsive efficiency $\rightarrow \eta_p = \frac{\text{Power to vehicle}}{\text{Vehicle + Residual jet power}}$

$$\eta_p = \frac{TV_a}{TV_a + \frac{1}{2} \dot{m} U_e^2} = \frac{TV_a}{\frac{1}{2} \dot{m} (V_e^2 - V_a^2)}$$

$$= \frac{\dot{m} (V_e - V_a) V_a}{\frac{1}{2} \dot{m} (V_e^2 - V_a^2)} = \frac{2 V_a}{V_e + V_a} = \frac{2 \gamma}{1 + \gamma}$$

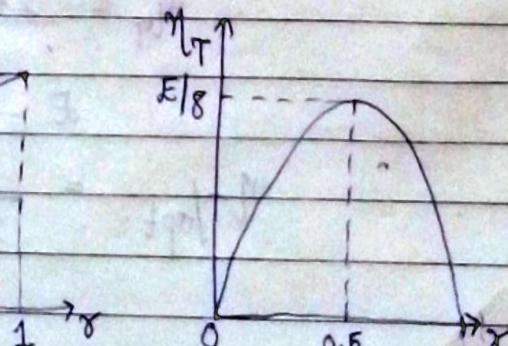
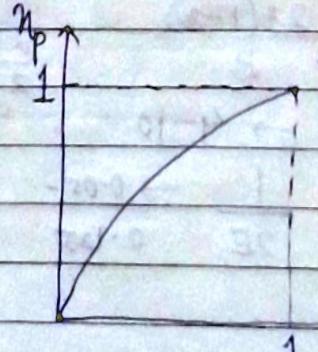
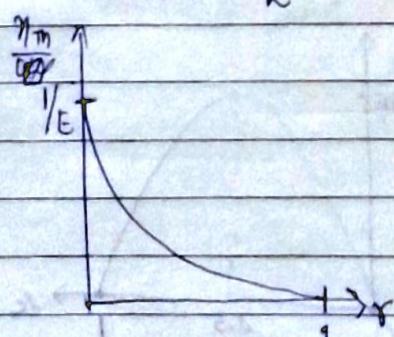
Overall efficiency $\Rightarrow (A) \rightarrow (E)$

Propulsive efficiency $\rightarrow (C) \rightarrow (E)$

jet KE $\Rightarrow (A) \rightarrow (C)$

$$\eta_m = \frac{\text{jet KE}}{\text{fuel energy}}, \quad \eta_m = \frac{\frac{1}{2} \dot{m} (V_e^2 - V_a^2)}{\dot{m} U_e^2} = \frac{\dot{m} V_e^2 (1 - \gamma^2)}{2 f \dot{m} U_e^2}$$

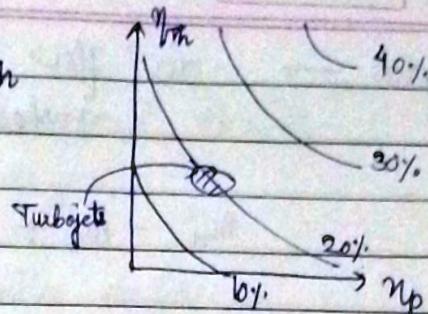
$$\eta_m = \frac{1 - \gamma^2}{E}$$



$$\therefore \eta_T = \eta_p \eta_{Th}$$

Limiting factor -

$$V_e = 2Va,$$



Most subsonic jets $\rightarrow M = 0.85$, $h \sim 35,000 \text{ ft}$, $T_a = 216 \text{ K}$
 $V_a \sim 250 \text{ m/s}$

$f \sim 0.005 - 0.02 \leftarrow$ cannot reduce f further while sustaining combustion.

$$\eta_T = \frac{V_e^2}{f}$$

$$\dot{m}_{fQR} = \frac{1}{2} [\dot{m} V_e^2] + \dot{m} c T_e - \left(\frac{1}{2} \dot{m} V_a^2 + \dot{m} c T_a \right)$$

$$= \frac{1}{2} \dot{m} [V_e^2 - V_a^2] + \dot{m} c (T_e - T_a)$$

$$1 = \underbrace{\frac{1}{2} \dot{m} (V_e^2 - V_a^2)}_{\dot{m}_{fQR}} + \underbrace{\dot{m} c (T_e - T_a)}_{\dot{m}_{fQR}}$$

$$\eta_{Th} = 1 - \underbrace{\left(\frac{c (T_e - T_a)}{fQR} \right)}_{\dot{m}_{fQR}}, \text{ the term reducing } \eta_{Th}$$

Can $(T_e - T_a)$ be reduced or energy $c(T_e - T_a)$ be utilised in thrust?

$$T = \underbrace{\dot{m} (V_e - V_a)}_{\text{Can we } \uparrow ?}, \text{ fixed (restricted by } \eta_p, \sigma)$$

Core engine \rightarrow air flow rate $\rightarrow \dot{m}_c$
 $f_c = \frac{\dot{m}_f}{\dot{m}_c}$, reacts with fuel \dot{m}_f
 $f = 0.005 - 0.02$
 gains energy \rightarrow runs the fan
 \rightarrow jet

Bypass \rightarrow air flow rate $\rightarrow \dot{m}_b = \alpha \dot{m}_c$
pushed by fan.

$$\dot{m}_a = \dot{m}_b + \dot{m}_c = \dot{m}_c (1 + \alpha)$$

$$\text{core thrust}, T_c = \dot{m}_c [(1+f_c) V_e - V_a]$$

$$\text{Bypass thrust}, T_b = \dot{m}_b [V_{e_2} - V_a]$$

$$\text{If } V_{e_2} = V_e \Rightarrow T_b = \alpha \dot{m}_c [V_e - V_a]$$

$$\tau = T_c + T_b = \dot{m}_c [(1+\alpha+f_c) V_e - (1+\alpha) V_a]$$

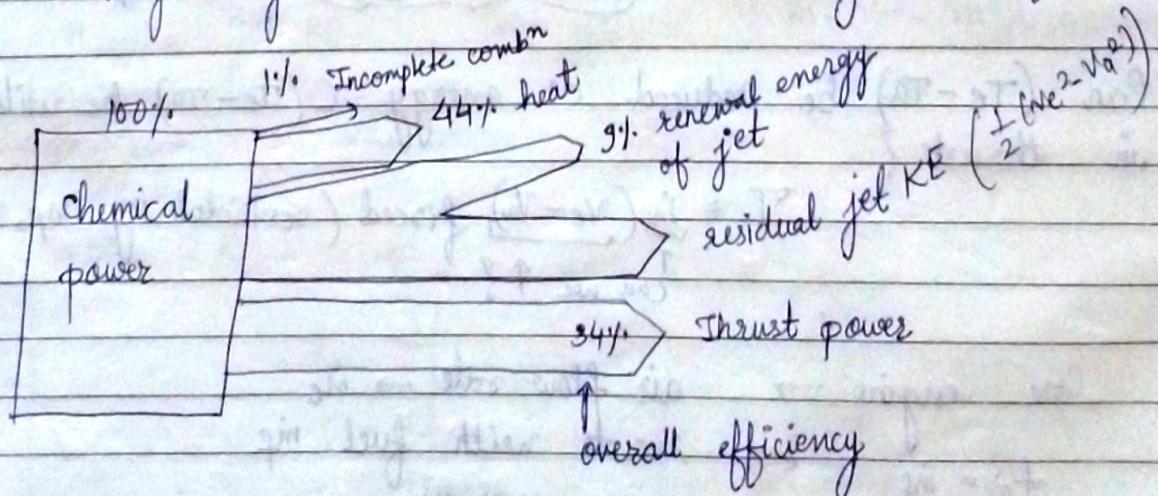
$$= \dot{m}_c (1+\alpha) \left\{ \left[1 + \frac{f_c}{1+\alpha} \right] V_e - V_a \right\}$$

$$= \dot{m}_a [(1+f) V_e - V_a] \quad \text{where } f = \frac{f_c}{1+\alpha}$$

$$\eta \propto \frac{V_a^2}{f} \propto \frac{V_a^2(1+\alpha)}{f_c}$$

$$\therefore \alpha \uparrow \Rightarrow \eta \uparrow$$

very large $\alpha \Rightarrow \dot{m}_b > \dot{m}_c \Rightarrow \text{drag} \uparrow$ (rel. of fan tip)



specific fuel consumption :-

$$\dot{T}_{SFC} = \frac{\dot{m}_f}{T} \xrightarrow{\text{thrust}} \text{fuel needed for unit thrust}$$

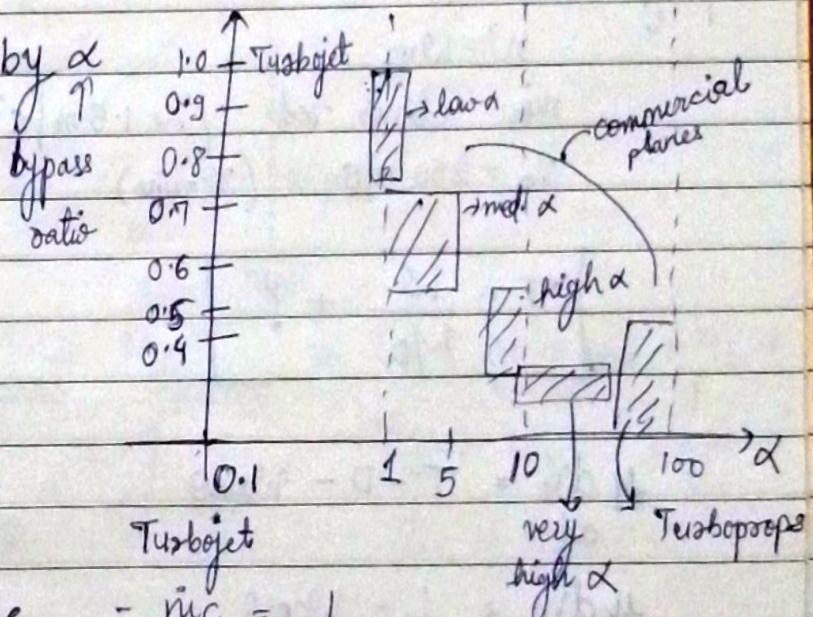
$$\dot{B}_{SFC} = \frac{\dot{m}_f}{P} \xleftarrow{\text{Brake}} \text{Propeller engines.}$$

$$\frac{\dot{m}_f}{T} = \frac{\dot{m}_f}{\dot{m}_a(V_e - V_a)} = \frac{f}{V_e - V_a} = \frac{10^{-2}}{200} \sim 5 \times 10^{-5} \text{ kg/N-s}$$

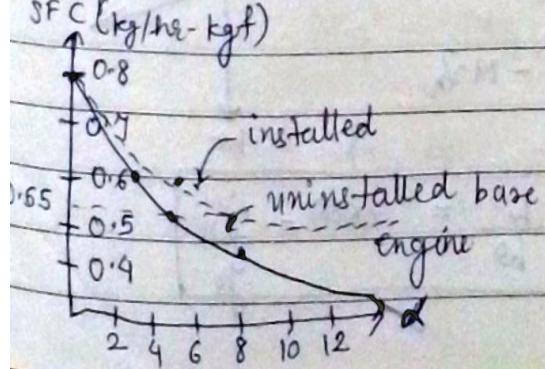
SFC shd be as small as possible.

$$\eta_T = \frac{T V_a}{\dot{m}_f g_R} = \frac{1}{SFC \ g_R} \alpha_{Va}, \quad SFC \propto \frac{1}{\eta_T} \propto \frac{1}{\alpha_{Va}}$$

Improve η_T by α

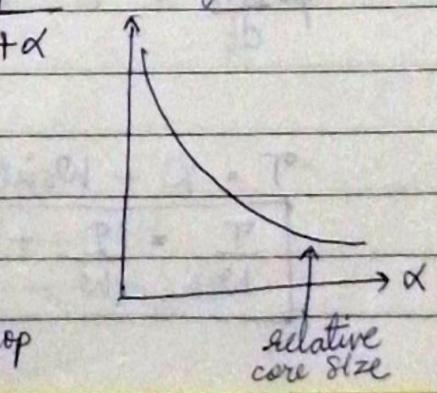


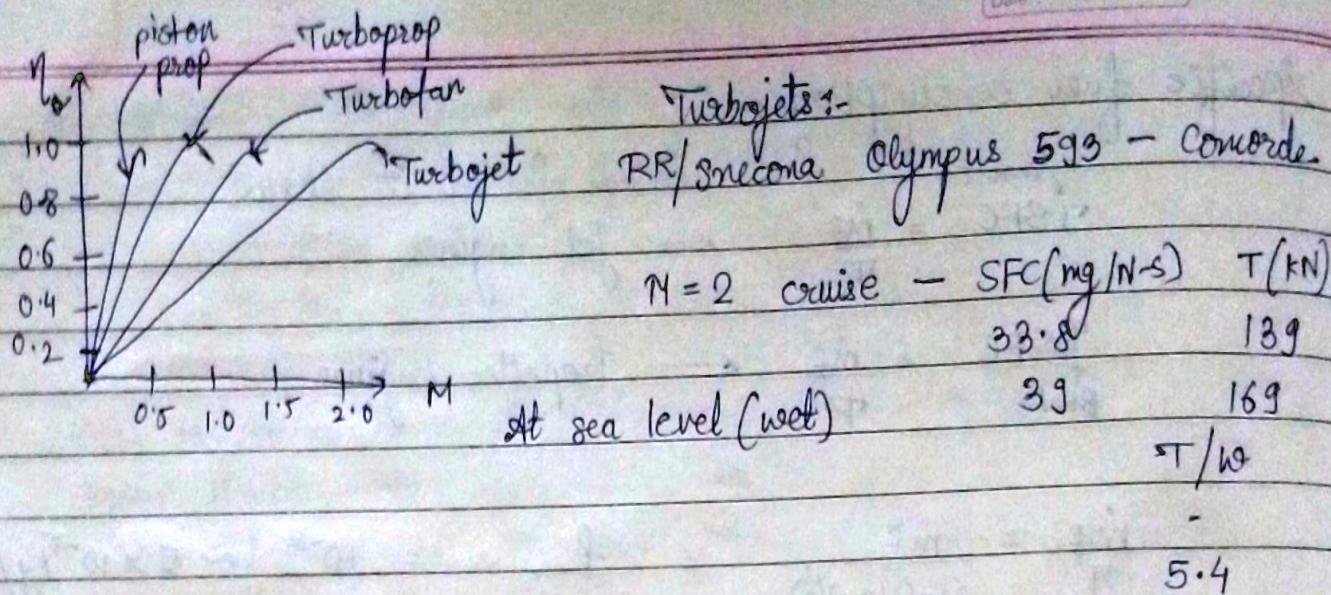
$$\text{Turbofans : Relative core size.} = \frac{\dot{m}_c}{\dot{m}_a} = \frac{1}{1+\alpha}$$



$$[1 \text{ kgf} = 9.8 \text{ N}]$$

$$\alpha \leftarrow \infty$$





PW J-58 (JP-7)	SR-71	SFC	T (kN)	T/W
M = 3.2 (cruise west condn)		54	151 (bare)	5.2

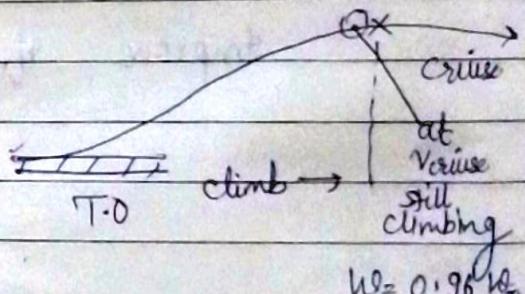
Top of climb :-

$$W = W_{T_0}$$

min. climb rate ($\sim 1.5 \text{ m/s}$)

$$V_a = 250 \text{ m/s} \quad (V_{\text{cruise}})$$

$$\boxed{T = \frac{W}{V_a} + ?}$$



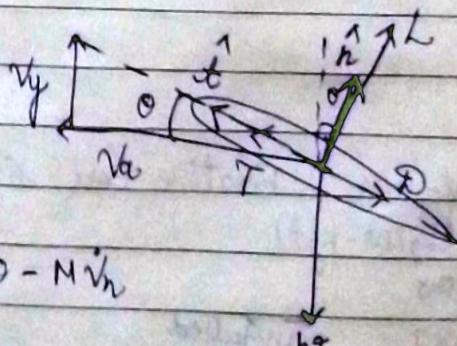
$$\mu \frac{dV_t}{dt} = T - D - W \sin \theta$$

$$\mu \frac{dV_n}{dt} = L - W \cos \theta$$

$$L = W \cos \theta - N \dot{V}_n$$

$$T = D + W \sin \theta + N \dot{V}_t$$

$$\boxed{\frac{T}{W} = \frac{D}{W} + \sin \theta + \frac{\dot{V}_t}{g_0}, \quad \frac{L}{W} = \cos \theta - \frac{\dot{V}_n}{g_0}}$$



$$\tan \theta = \frac{V_y}{\sqrt{a}} = \frac{1.5}{250}$$

$$\theta \approx \tan \theta = 0.006 \text{ radian} \approx 0.34^\circ$$

$$L \approx W, \quad \frac{T}{W} = \frac{D}{W} + \theta + \frac{V_t}{g}$$

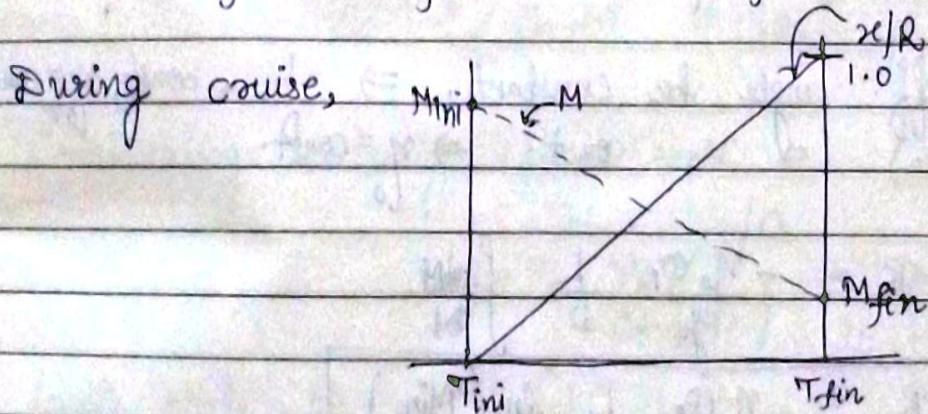
$$\frac{T}{W} = \frac{1}{L/D} + \theta$$

assume negligible

For specific 947, $T \Big|_{\substack{\text{Take off} \\ \text{Top of climb}}} = 1.11 T \Big|_{\text{cruise}}$

$W_{TO}, T \rightarrow \text{Range, endurance}$

$$R = \int dx \equiv \int v dt, \quad T = \int dt$$



$$M_{\text{Take-off}}(T_0) = M_{\text{cruise}} + M_{\text{f fuel consumed}} + M_{\text{aircraft structure}} + M_L + M_{\text{f climb}} + M_{\text{payload}}$$

\uparrow (say) \uparrow 10% (say) \uparrow (4% of T_0)

$$M_{\text{ini}} = M_{TO} - M_{\text{f climb}}$$

$$M_{\text{fin}} = M_{TO} - M_{\text{f climb}} - M_{\text{cruise}}$$

$$R = \int v dt$$

over dt

distance covered $\rightarrow v dt$
fuel consumed $\rightarrow -m_f dt$

$$dM = - \dot{m}_f dt$$

$$dx = v dt.$$

$$\therefore \frac{dN}{dx} = - \frac{\dot{m}_f}{v}$$

$$dx = - \sqrt{\frac{dM}{\dot{m}_f}}, \quad \dot{m}_f = \frac{V_a}{n_0 \rho_R} \cdot \frac{w}{w/D} = \frac{V_a \cdot M_0}{n_0 \rho_R L/B}$$

$$dx = \frac{V}{\dot{m}_f} = \frac{n_0 \rho_R}{M_0} \cdot \frac{L}{D}$$

$$\int_{x_{in}}^{x_{fin}} dx = \int_{t_{in}}^{t_{fin}} - \frac{n_0 \rho_R}{g_0} \frac{L}{D} \frac{dM}{M}$$

$$R = - \int_{t_{in}}^{t_{fin}} \frac{n_0 \rho_R}{g_0} \left(\frac{L}{D} \right) \frac{dM}{M}$$

Let 'x' - eff. angle be constant. $\Rightarrow \frac{L}{D} = \text{const}, n_0 = \text{const}$.
 $n_0 = n_0 (V_a) \Rightarrow V_a = \text{const.} \Rightarrow n_0 = \text{const.}$

$$\therefore R = - \frac{n_0 \rho_R}{g_0} \frac{L}{D} \int \frac{dM}{M}$$

$$R = \frac{n_0 \rho_R}{g_0} \frac{L}{D} \ln \left(\frac{M_{in}}{M_{fin}} \right)$$

$$T = \frac{n_0 \rho_R}{g_0} \frac{L}{D} \ln \left[\frac{M_{in}}{M_{fin}} \right]$$

$$TSFC = \frac{\dot{m}_f}{T} = \frac{V_a}{n_0 \rho_R}$$

$$\therefore R = \frac{1}{TSFC} \frac{V_a}{g_0} \frac{L}{D} \ln \left[\frac{M_{in}}{M_{fin}} \right]$$

$$R \propto n_0 \propto \frac{1}{TSFC}$$

fuel fraction $\rightarrow \frac{M_{\text{fuel}}}{M_{\text{TO}}} \approx \frac{M_{\text{ini}} - M_{\text{fin}}}{M_{\text{ini}}} = 1 - \frac{M_{\text{fin}}}{M_{\text{ini}}}$

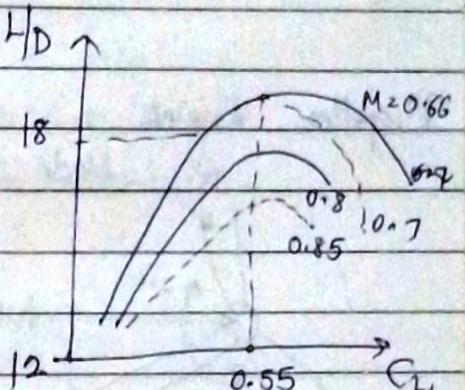
$$\frac{M_{\text{ini}}}{M_{\text{fin}}} = \exp \left[\frac{-TSFC}{L/D} - \frac{g_0 R}{V_a} \right] = 1 - \exp \left[-\frac{TSFC}{L/D} \frac{g_0 R}{V_a} \right]$$

Range - $dx = -V_a \frac{dN}{\eta_{\text{if}}} = -V_a \frac{dN}{T(\text{SFC})}$

$$T = D = \frac{W}{L/D} = \frac{M g_0}{L/D}$$

$$R = \int dx = \int_{t_{\text{in}}}^{\infty} -V_a \frac{L/D}{SFC} \frac{g_0}{M} dM$$

$$R = V_a \left(\frac{L}{D} \right) \frac{1}{g_0 SFC} \ln \left[\frac{M_{\text{ini}}}{M_{\text{fin}}} \right]$$



Considering velocity effect too,

$$M = V_a / c_s, \quad c_s = \sqrt{\gamma R T_a}$$

$$\frac{ML}{D} = \frac{V_a}{c_s} \left(\frac{L}{D} \right)$$

$$\text{Length scale, } R_o = V_a \left(\frac{L}{D} \right) \frac{1}{g_0 SFC}$$

$$R = R_o \ln \left[\frac{M_{\text{ini}}}{M_{\text{fin}}} \right]$$

$$\ln \left[\frac{M_{\text{ini}}}{M_{\text{fin}}} \right] = \frac{R}{R_o} \Rightarrow \frac{h_{\text{lin}}}{h_{\text{effin}}} = \exp \left[\frac{R}{R_o} \right]$$

$$\frac{h_{\text{lin}}}{h_{\text{effin}}} = 1 + \frac{h_{\text{lt}}}{h_{\text{effin}}}$$

$$h_{\text{engine(empty)}} + h_{\text{L2}}$$

$$\frac{h_f}{h_e + h_{\text{L2}}} = \exp \left[\frac{R}{R_o} \right] - 1$$

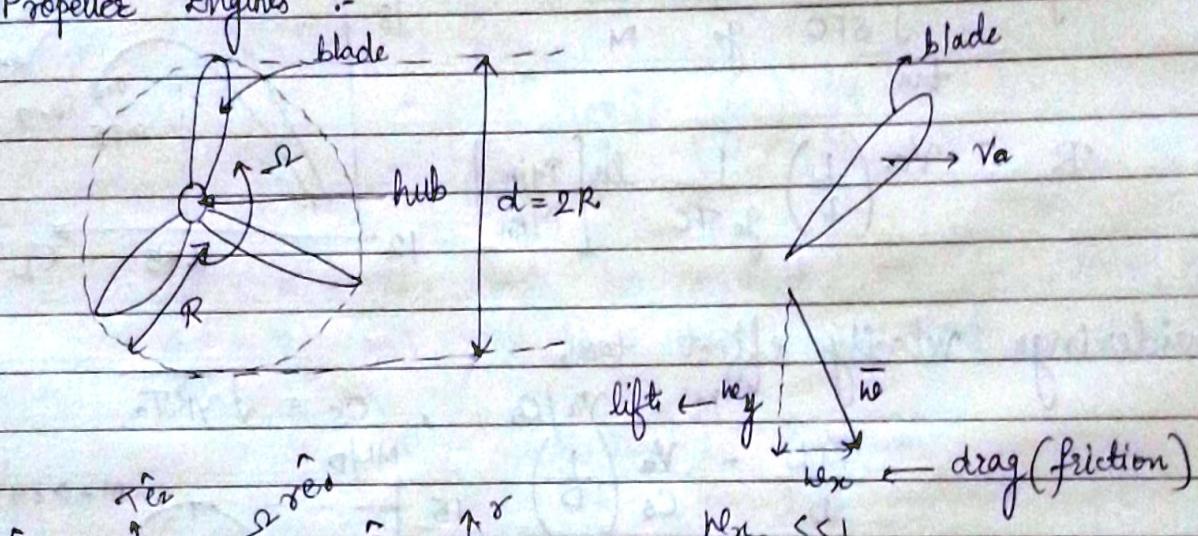
$$\frac{h_f}{w_e} = \left[1 + \frac{w_e}{w_f} \right] \left[\exp\left(\frac{R}{R_0}\right) - 1 \right]$$

$$\frac{g_f}{R w_e} = \left[1 + \frac{w_e}{w_f} \right] \frac{1}{R} \left[\exp\left(\frac{R}{R_0}\right) - 1 \right]$$

For the same engine,

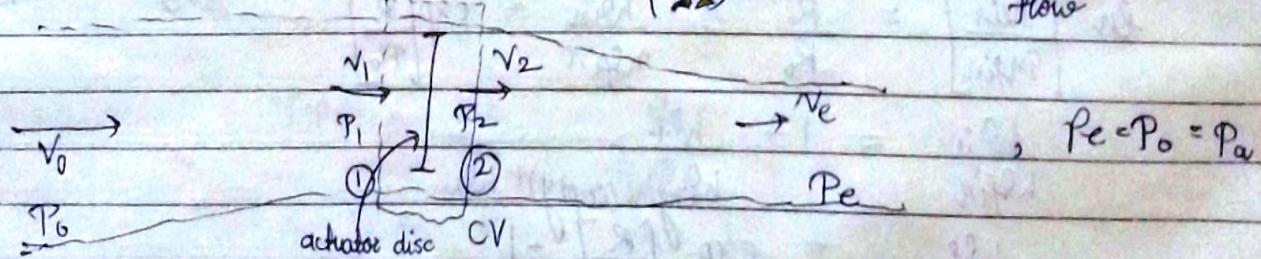
small fuel consumption \Rightarrow large R_0 & small $\ln \left[\frac{w_{in}}{w_{fin}} \right]$
 \Rightarrow small R_0 & large $\ln \left[\frac{w_{in}}{w_{fin}} \right]$

Propeller Engines :-



over one rotation, blades are smeared everywhere. imagine an actuator disk.

For time interval $<$ rotation time $(\frac{2\pi}{N_e})$ \rightarrow blades are like unsteady flow



Streamtube passing through the actuator disk.

$$P_{\text{useful}} = T V_0 \quad \& \quad P_{\text{induced}} = T w$$

$$T_{ss} = P_{\text{to}}^{2/3} (2gA)^{1/3}, \quad w_0 (V_0=0) = \sqrt{\frac{T}{2gA}}$$

$$\dot{m} = S_0 A_0 V_0 = S_1 A_1 V_1 = S_2 A_2 V_2 = \dot{m}_e A_e V_e$$

$$A_1 = A_2 = \frac{\pi d^2}{4} = A_d$$

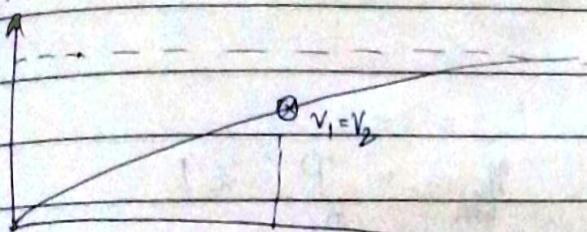
Assumpⁿ :- Incompressible flow

$$S_0 = S_1 = S_2 = S_e = S.$$

$$A_0 V_0 = A_1 V_1 = A_2 V_2 = A_e V_e$$

$$\tau = \dot{m} (V_e - V_0)$$

A_0



$$\tau = \dot{m} V_e - \dot{m} V_0$$

$$= S A_e V_e V_e - S A_0 V_0 V_0$$

$$= S (A_e V_e^2 - A_0 V_0^2)$$

Push force $\rightarrow P_2 > P_1$

Inviscid, irrotational, steady Bernoulli eqⁿ -

$$P_0 + \frac{1}{2} \rho V_0^2 = \phi_1 + \frac{1}{2} \rho V_1^2 \quad \leftarrow \text{upstream}$$

$$P_2 + \frac{1}{2} \rho V_2^2 = P_e + \frac{1}{2} \rho V_e^2 \quad \leftarrow \text{downstream}$$

$$\tau = \dot{m} V_2 - \dot{m} V_1 \neq (P_2 - P_1) A_d \quad \leftarrow \text{for CV.}$$

$$\dot{m} = \rho V_1 A_d = \rho V_2 A_d$$

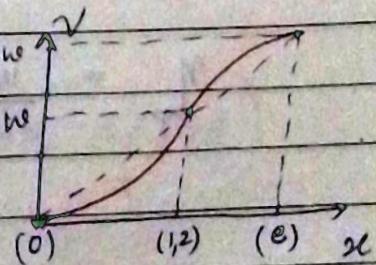
$$\dot{m}(V_2 - V_1) = 0 \Rightarrow \tau = (P_2 - P_1) A_d$$

$$\therefore P_2 - P_1 = \frac{1}{2} \rho (V_e^2 - V_0^2)$$

$$\therefore \tau = \frac{1}{2} \rho (V_e^2 - V_0^2) A_d = S A_d V_1 (V_e - V_0)$$

$$\therefore V_1 = \frac{V_e + V_0}{2} = V_2$$

$$\text{if } V_1 = V_0 + w \Rightarrow V_e = V_0 + 2w$$



$$w_e = \frac{1}{2} \left[-N_0 + \sqrt{N_0^2 + \left(\frac{\Omega T}{SA} \right)} \right]$$

$$\therefore T = 2gAd w_e (N_0 + w_e)$$

$$\begin{aligned} \text{Power} &= \frac{1}{2} m (v_e^2 - v_{0e}^2) = \frac{1}{2} m (v_e - v_{0e})(v_e + v_{0e}) \\ &= \frac{1}{2} m \times 2w_e \times 2(v_{0e} + w_e) = 2m w_e (v_{0e} + w_e) \\ &= 2gAd v_{1e} w_e (N_1) = 2gAd v_1^2 w_e = 2gAd (v_{0e} + w_e)^2 w_e \end{aligned}$$

$$P, \text{Power} = T(N_0 + w_e)$$

$$P_{\text{shaft}} > P$$

$$\eta_{\text{sh}} = \frac{P}{P_{\text{sh}}} < 1$$

$$\eta_{\text{power}} = \frac{T v_{0e}}{P_{\text{shaft}}} = \eta_{\text{sh}} \frac{T v_{0e}}{P} = \eta_{\text{sh}} \frac{T v_{0e}}{T v_1}$$

$$N_1 = \frac{v_{0e} + v_e}{2} \Rightarrow \eta_p = \eta_{\text{sh}} \left[\frac{2v_{0e}}{v_{0e} + v_e} \right]$$

$$\text{Ideal case} \Rightarrow \eta_{\text{sh}} = 1 \Rightarrow \eta_{\text{power}} = \frac{2v_{0e}}{v_{0e} + v_e}$$

$$T = \frac{P}{v_0 + w_e} = \frac{\eta_{\text{sh}} P_{\text{sh}}}{v_0 + w_e} \quad \text{engine output}$$

$$v_e \uparrow \Rightarrow T \downarrow$$

$$P = T v_0 + T w_e$$

↑ ↑
useful induced
power power

$$\eta_p = \frac{\text{useful power}}{\text{total power}}$$

$$P_1 = P_a + \frac{1}{2} \rho v_a^2 - \frac{1}{2} \rho v_1^2$$

$$= P_a + \frac{1}{2} \rho [v_a^2 - (v_a + w)^2] = P_a - \frac{1}{2} \rho w v_a \left[\frac{w+2}{v_a} \right]$$

$$= P_a - \rho w v_a \left[\frac{w+1}{2 v_a} \right]$$

$$w = \gamma v_a \quad \left| \begin{array}{l} \gamma = \frac{v_a}{v_e} = \frac{v_a}{v_a + 2w} = \frac{1}{1 + 2\epsilon} \end{array} \right.$$

$$\therefore P_1 = P_a - \rho v_a^2 \epsilon \left(1 + \frac{\epsilon}{2} \right)$$

$$\therefore \frac{P_1}{P_a} = 1 - \frac{\rho v_a^2 \epsilon}{P_a} \left(1 + \frac{\epsilon}{2} \right)$$

$$= 1 - \gamma M_a^2 \epsilon \left(1 + \frac{\epsilon}{2} \right)$$

$$P_2 = P_a + \frac{1}{2} \rho v_c^2 - \frac{1}{2} \rho v_1^2$$

$$= P_a + \rho w v_a \left[1 + \frac{3}{2} \epsilon \right]$$

$$\therefore \frac{P_2}{P_a} = 1 + \gamma M_a^2 \epsilon \left(1 + \frac{3}{2} \epsilon \right)$$

$$\therefore \frac{P_2}{P_1} = \frac{1 + \gamma M_a^2 \epsilon \left[1 + \frac{3}{2} \epsilon \right]}{1 + \gamma M_a^2 \epsilon \left[1 + \frac{\epsilon}{2} \right]}$$

$$\epsilon \ll 1$$

$$\therefore \frac{P_2}{P_1} \approx 1 + \gamma M_a^2 \epsilon \left[1 + \frac{3}{2} \epsilon \right] + \gamma M_a^2 \epsilon \left(1 + \frac{\epsilon}{2} \right)$$

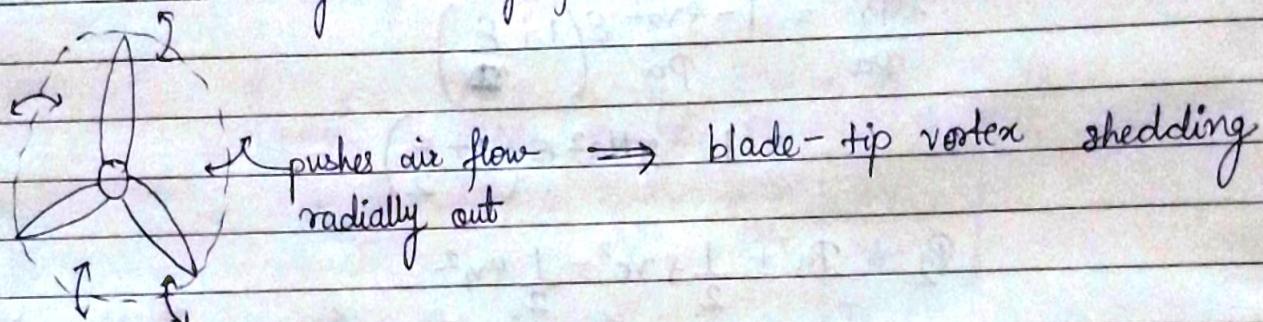
$$\therefore \frac{P_2}{P_1} \approx 1 + 2\gamma Ma^2 \epsilon (1+\epsilon) \leftarrow \text{pressure ratio}$$

The pressure \uparrow from $P_a \rightarrow P_1$, then \downarrow from $P_2 \rightarrow P_a$,

$$P_2 - P_1 = \frac{T}{Ad} \leftarrow \text{disk loading}$$

$$P_{0a} = P_{01}, \quad P_{0e} = P_{02}$$

- Assumpⁿ:
- net effect is pushing air back.
 - neglect drag friction losses. (or swirl)



- neglect blade-tip shedding vortices.

Efficiency:-

$$\frac{P_{sh}}{1} \rightarrow \text{propeller} \rightarrow \text{fluid jet} \rightarrow \frac{P_{jet}}{\frac{1}{2} \rho (V_e^2 - V_a^2)} \downarrow \eta_p$$

$$\eta_p = \frac{TV_a}{P_{jet}} = \frac{2Na}{V_a + V_e}$$

$$\eta_{prop} = \frac{P_j}{P_{sh}}$$

$$\eta_T = \eta_{prop} \eta_p = \frac{TV_a}{P_{sh}}$$

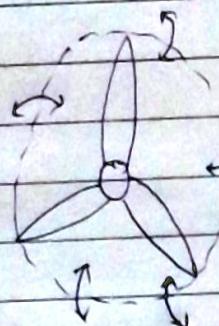
$$\therefore \frac{P_2}{P_1} \approx 1 + 2\gamma Ma^2 E(1+\epsilon) \leftarrow \text{pressure ratio}$$

The pressure \uparrow from $P_a \rightarrow P_1$, then \downarrow from $P_2 \rightarrow P_a$,

$$P_2 - P_1 = \frac{T}{Ad} \leftarrow \text{disk loading}$$

$$P_{0a} = P_{01}, P_{0e} = P_{02}$$

- Assumpn:-
- net effect is pushing air back.
 - neglect drag friction losses. (or swirl)



pushes air flow \Rightarrow blade-tip vortex shedding
radially out

- neglect blade-tip shedding vortices.

Efficiency:-

$$\frac{P_{sh}}{P_{sh}} \rightarrow \text{propeller} \rightarrow \text{fluid jet} \rightarrow \frac{P_{jet}}{2} \ln \left(\frac{V_e^2 - V_a^2}{V_a^2} \right)$$

η_{prop}

η_p

TVa

$$\eta_p = \frac{TVa}{P_{jet}} = \frac{2Va}{V_a + V_e}$$

$$\eta_{prop} = \frac{P_j}{P_{sh}}$$

$$\eta_T = \eta_{prop} \eta_p = \frac{TVa}{P_{sh}}$$

$$T = P_{sh} \left(Ad \frac{g}{g_{sh}} \right) = \frac{\eta_{propeller}}{V_a}$$



$$P_{jet} = T \left(\frac{V_e + V_a}{2} \right) = T (V_a + w) = T V_a + T w$$

$$\eta_p = \frac{T V_a}{P_{jet}} = \frac{2 V_a}{V_e + V_a} = \frac{1}{1 + w/V_a}$$

Vehicle power Thrust induced power

$$P_{jet} \approx T \propto w$$

$$\frac{T}{4gAd} = \frac{w}{V_a} \left(1 + \frac{w}{V_a} \right)$$

$$\therefore \left(\frac{w}{V_a} \right)^2 + \left(\frac{w}{V_a} \right) - \frac{T}{4gAd} = 0$$

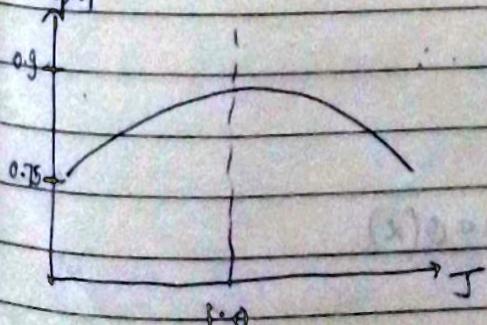
$$\therefore w = \frac{1}{2} \left[-1 \pm \sqrt{1 + \frac{T}{4gAd}} \right]$$

$$w = \frac{V_a}{2} \left[\sqrt{\frac{1+T}{4gAd}} - 1 \right]$$

T/gAd must be larger (at least not small due to larger wt & touching to ground)

Advance ratio $\rightarrow J = \frac{V_a}{n_d}$ ← vehicle speed $= \pi V_a$
 ← diameter w/r
 $\frac{1}{2} \pi n D$ $\frac{1}{2} \pi n D$ ← tip speed

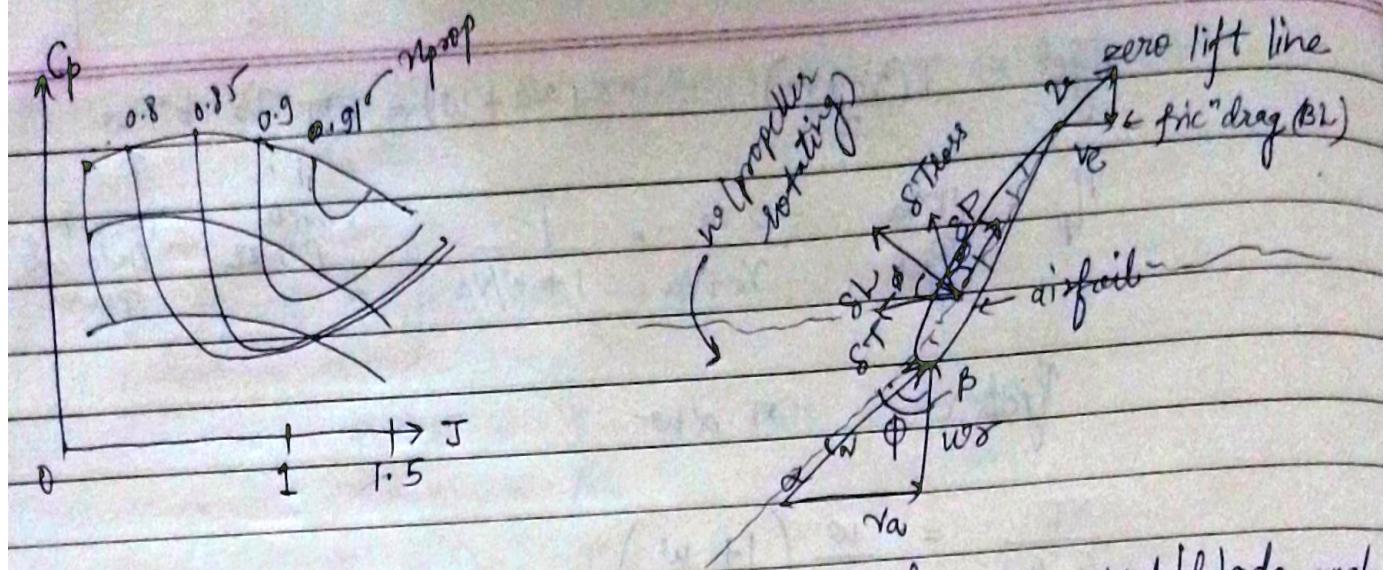
Power co-eff., $C_p = \frac{P_{sh}}{g n^3 d^5}$



$$\eta_{prop.} = \frac{\pi}{2} \frac{J^3}{C_p} \frac{\eta_p^3}{(1-\eta_p)}$$

McCroskey propellers, 2pm ≈ 2700 , $n = 45$

$$\tan \phi = \frac{V_a}{w\alpha}$$



$\tan \phi = \frac{V_a}{w\alpha}$, α - angle of attack, β - pitch/blade angle
 $\alpha = \beta - \phi$

$$S_h = \frac{1}{2} \rho V^2 S C_L = q C_L S S$$

$$S_D = q C_D S S$$

$$S_T = S_L \cos \phi - S_D \sin \phi$$

$$S_{T_{loss}} = S_L \sin \phi + S_D \cos \phi$$

$$\text{Torque to drive propeller} = \int S_{T_{loss}} w\alpha$$

$$= \int (C_L \sin \phi - C_D \cos \phi) q \cancel{\alpha} C_w \alpha d\sigma$$

$$\eta_{prop} = \frac{S_T V_a}{S_{T_{loss}} w\alpha} = \frac{q ds [C_L \cos \phi - C_D \sin \phi]}{q ds [C_L \sin \phi + C_D \cos \phi] w\alpha}$$

$$\eta_{prop} \approx \left[\frac{C_L - \tan \phi}{C_D} \right] \frac{V_a}{w\alpha}$$

$$\cdot \frac{C_L}{C_D} \alpha f(\alpha)$$

$$\text{Let } \frac{C_L}{C_D} = \tan \theta \Rightarrow \theta = \theta(\alpha)$$

Propeller Analysis - Blade Element Theory



$$\therefore \eta_{prop} = -\tan\phi (\tan(\alpha - \phi))$$

$$\eta_{prop, opt} \rightarrow \frac{d\eta_{prop}}{d\phi} = 0 \Rightarrow \phi_{opt} = \frac{\alpha}{2}$$

$$\left(\frac{V_a}{w_e} \right)_{opt.} = -\frac{C_D}{C_L} + \sqrt{\left(\frac{C_D}{C_L} \right)^2 + 1}$$

$$\eta_{prop, opt} = \tan^2\left(\frac{\alpha}{2}\right) = \left(\frac{V_a}{w_e} \right)_{opt.}^2$$

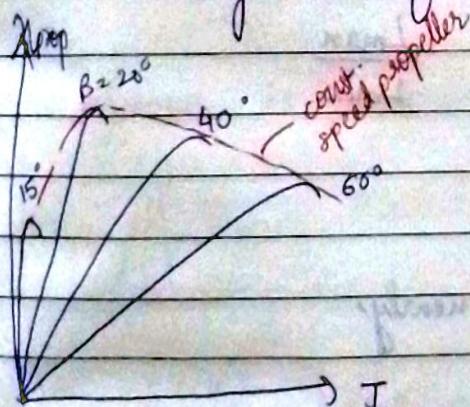
$$J = \frac{\pi^2}{\pi} \left(\frac{V_a}{w_e \max} \right)$$

$$\therefore J_{opt.} = \pi \left[1 - \frac{C_D}{C_L} + \frac{1}{2} \left(\frac{C_D}{C_L} \right)^2 \right]$$

for opt. performance of propeller, η_{prop} is high. can be achieved,

\Rightarrow change $\beta \rightarrow \beta_{hub} + \beta_{tip}$

change in airfoil shape \rightarrow hubs & tips have different airfoil shapes.



$$V_{tip} = \sqrt{V_a^2 + \left(\frac{wd}{2} \right)^2}$$

$$M_t = \sqrt{M_a^2 + M_w^2}$$

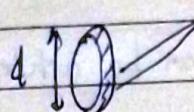
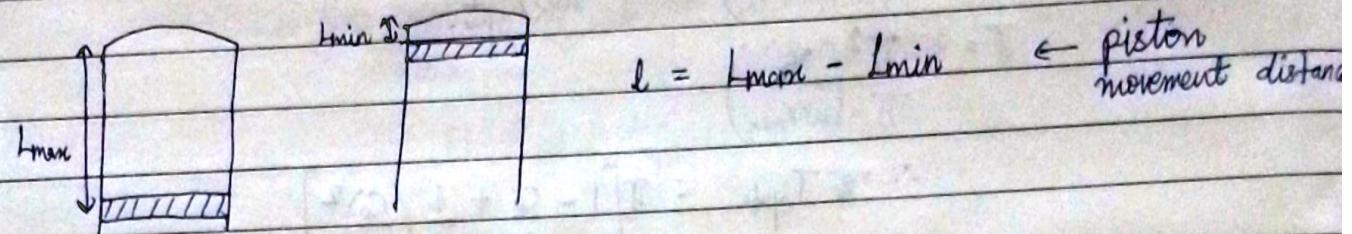
$$M_a = \frac{V_a}{a}, M_w = \frac{wd}{2a}$$

Piston Engines :-

Piston movement → rotates a shaft → rotates the propeller

IC/ Reciprocating engine:-

Combustion → Piston → shaft $\xrightarrow{\text{Nsh}}$ Propeller $\xrightarrow{\text{prop}}$ Thrust $\xrightarrow{\text{vehicle power}}$

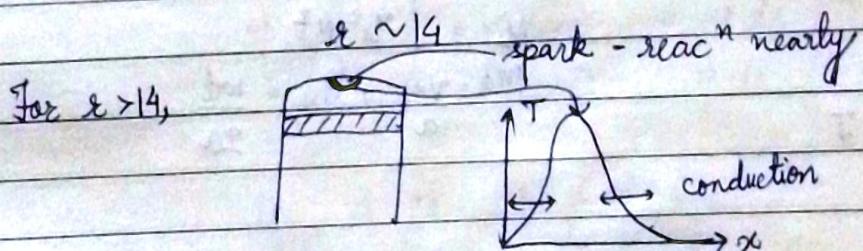


$$A_d = \frac{\pi d^2}{4}$$

$$\frac{V_0}{(P_0)} = \frac{\pi d^2}{4} l_{\max} \rightarrow \frac{V}{(P_c)} = \frac{\pi d^2}{4} l_{\min}$$

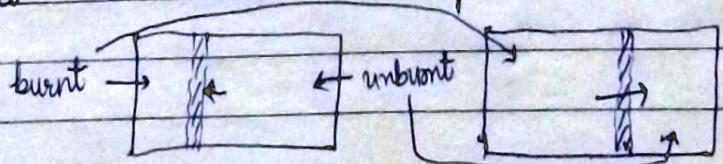
$$\text{Compression ratio} - \quad r = \frac{V_0}{V_c} \approx \frac{l_{\max}}{l_{\min}}$$

$$\frac{P_c}{P_0} = f(r)$$



If conduction dominates \Rightarrow surrounding gas heats up & starts reacting

fuel - air mixture \Rightarrow premixed comb.ⁿ



flame speed - s_f

combustion - fuel-air mixture should get heated up to overcome the activation energy.

Reacⁿ spreads everywhere quickly.

If T_f - Time for reacⁿ to spread all over the box

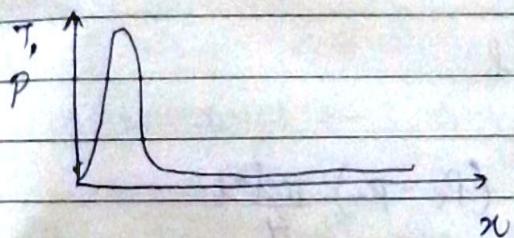
T_r - time for the reacⁿ to complete $\propto \frac{1}{w^r}$

$$w_r = f(P)$$

high P (moderately) $\leftarrow T_r$ is small, but large enough to spread flame everywhere

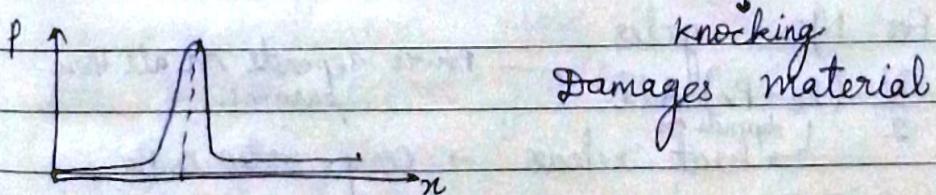
low P \leftarrow reacⁿ is too slow for proper combustion or to generate sufficient energy to heat up the remaining gas.

very high P - reacⁿ is too fast before surrounding gas heats upto react, this region has reacted.



very high P diff. \rightarrow shock wave
as the shock passes, gas heats up & reacts.

This wave is known as Detonation wave.



Measured in terms of octane no. Higher octane rating can be compressed to higher pressures.

Airplane Octane - AvGas > 100

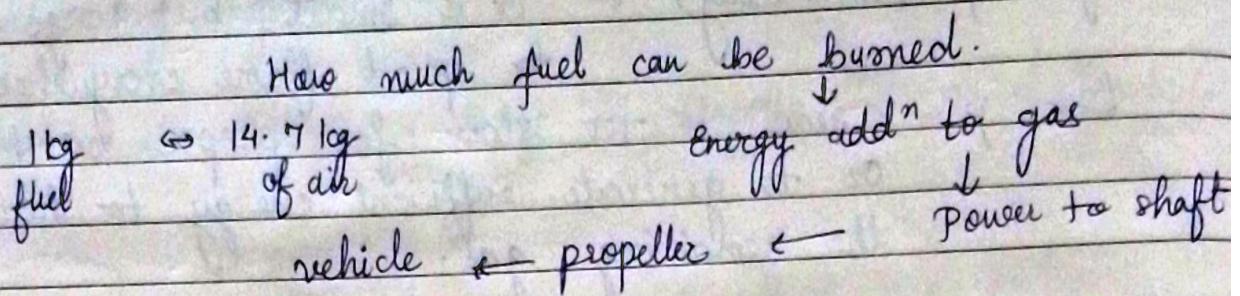
why compress?

We can Intake \Rightarrow burn \Rightarrow TT PT \Rightarrow drive shaft

But why don't we do this & compress?

Reason - $p \uparrow \rightarrow$ reacⁿ rates \uparrow
to push the piston $\rightarrow p_r > p_{amb}$.

$p_r \rightarrow S \Rightarrow$ How much air can be packed into cylinder
 \downarrow fixes



Let P_e - post combⁿ pressure, $A_d = \frac{\pi d^2}{4}$

$$F = (P_e - P_\infty) \frac{\pi d^2}{4}, \text{ disp.} \rightarrow l$$

$$\therefore \text{Work} = (P_e - P_\infty) \frac{\pi d^2}{4} l$$

$$1 \text{ cycle} - 2 \text{ revolutions of shaft} - (P_e - P_\infty) \frac{\pi d^2}{4} l$$

$$N (\text{rpm}) \approx 2500 - 6000$$

For $N/2$ cycles

$$\text{Power} = \frac{N}{2} (P_e - P_\infty) A_d l$$

power depends on all these params.
 \downarrow depends on heat release \rightarrow comp. ratio & P_∞

Mainly power \propto air intake (P_0, S_0, T_0)
rpm, cylinder size, 'l'

Gagg & Ferrar :-

$$P = P_{st} \overset{\text{power}}{\rightarrow} [1.1325 - 0.182]$$

$$\delta = S/S_{st}$$

Simpler model \rightarrow atmp. \rightarrow $P = P_{st} \overset{\text{power}}{\rightarrow} \delta$

Power → rpm — higher rpm → higher power
 → [S/P in the cylinder] [super charger / turbo
 → volume [$\frac{\pi d^2 l}{4}$] charger]

Quantitative analysis - Thermodynamics:-

System ← identifiable / well-defined collection of particles.
 ← does not change unless disturbed from outside
 (egbm)

what if the system is disturbed? ← stays the same (stable)
 ← changes (unstable egbm)

surroundings ← everything else (except the system)

Interactions betⁿ the system & surroundings happen through the boundary.

- boundary can move
- system cannot gain/lose mass.

State - internal configuration of a system that's at egbm.

Properties - measurable params.

- ↳ extensive - depends on size
- ↳ intensive - independent of size

Quasi-steady process - changes are gradual

- constraint changes a little, system changes are slower than → adjusts instantaneously.
 the relaxation processes

- gradual change in constraints, such that all the intermediate states are well-defined

Thermodynamics can only tell us about end states. Since most real processes are transients.

I Law - For any process (involving no net effects external to the system except disp. of a mass in a gravitational field betⁿ specified levels), the magnitude of the mass is fixed by the end states of the system & it is independent of the details of the process.

II law - Among all allowed states of a system with given values of energy (E), no. of particles (N) & constraints, one & only one is a stable eqbm state. Such a state can be reached from any of the said allowable states without any net effects on the envnt.

Postulate II - If a funcⁿ (called entropy S) of the extensive parameters of any composite system, defined for all eqbm states & having the following property: "values assumed by the extensive params in the absence of an internal constraint are those that maximize the entropy over the manifold of constrained eqbm states."

Postulate III - The entropy of a composite system is additive over the constitute subsystems. The entropy is continuous & differentiable & is a monotonically increasing funcⁿ of energy.

$$Tds = dv + pdv$$

$$Tds = dh - vdp$$



$$ds = \frac{1}{T} de + \frac{p}{T} d\nu - \sum_i \frac{u_i}{T} dN_i$$

Basic eqn:-

$$u_2 - u_1 = Cv(T_2 - T_1)$$

$$h_2 - h_1 = Cp(T_2 - T_1)$$

$$\delta_2 - \delta_1 = Cv \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

$$\delta_2 - \delta_1 = Cp \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$C_p - Cv = R$$

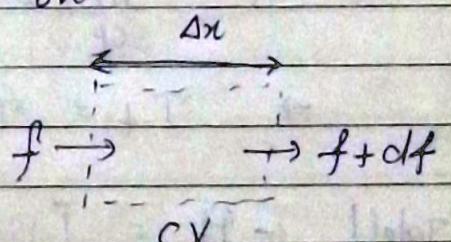
* Slow down gas flow

$$\frac{\partial}{\partial t} \left[g(e + \frac{1}{2} u^2) \right] + \frac{\partial}{\partial x} \left[gu \left(h + \frac{1}{2} u^2 \right) \right] = \frac{\partial q_{cond}}{\partial x} + \dot{q}''' - \dot{w}_{sh}'''$$

$$u = 0 \Rightarrow \frac{\partial}{\partial t} (ge) = \frac{\partial q_c}{\partial x} + \dot{q}''' - \dot{w}_{sh}'''$$

$$de = \dot{sq} - \dot{s}_{sh}''' = \dot{sq} - \dot{s}_{sh}$$

$u \neq 0$ - steady



$$d \left[gu \left(h + \frac{1}{2} u^2 \right) \right] = \dot{sq} - \dot{s}_{sh}$$

$h_t = h + \frac{1}{2} u^2 - [\text{enthalpy when flow stops} - \text{total / stagnation enthalpy}]$

$$d \left[gu h_t \right] = \dot{sq} - \dot{s}_{sh}$$

$$\frac{\partial s}{\partial t} + \frac{\partial}{\partial x} (gu) = 0 \rightarrow \text{steady} \Rightarrow d(gu) = 0$$

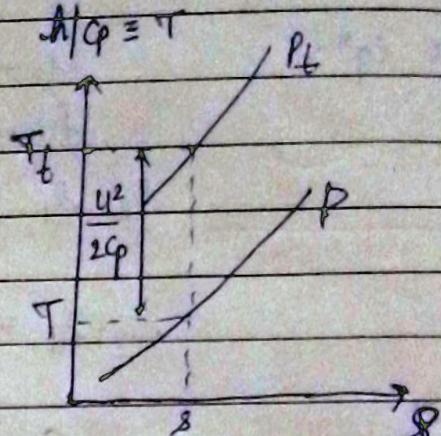
$\frac{dm}{A} = gu = \text{const.}$

$$C_p dT_t = dh_t = \frac{\delta q}{m/A} - \frac{\delta u_{\text{sh}}}{m/A} = T ds - s_{\text{sh}}$$

\uparrow \uparrow
 δq δu_{sh}

$$T_t - T = \frac{u^2}{C_p}$$

$$P_t = P + \frac{\rho u^2}{2} \leftarrow (\text{No})$$



$$dh = T ds + v dp$$

$$C_p dT = T ds + \frac{1}{P} dp \quad (ds = T ds - pdv)$$

$$C_p dT = T ds + RT \frac{dp}{P}$$

$$\frac{\gamma}{\gamma-1} \frac{dT}{T} = \frac{ds}{R} + \frac{dp}{P} \rightarrow \frac{P_f}{P_i} = \left(\frac{T_f}{T_i} \right)^{\frac{\gamma}{\gamma-1}} \exp \left(\frac{-\Delta s}{R} \right)$$

$$\text{Isentropic} \rightarrow \Delta s = 0 \Rightarrow \frac{P_t}{P} = \left(\frac{T_t}{T} \right)^{\frac{\gamma}{\gamma-1}}$$

$$T_t = \frac{h_t}{C_p} = \frac{h + \frac{1}{2} u^2}{C_p} = \frac{h}{C_p} + \frac{\gamma-1}{2\gamma R T} u^2$$

$$\Rightarrow T_t = T \left[1 + \frac{\gamma-1}{2} M^2 \right]$$

$$\text{Total pressure} \leftarrow \frac{P_t}{P} = \left[1 + \frac{\gamma-1}{2} M^2 \right]^{\frac{\gamma}{\gamma-1}}$$

$$dh_t = T ds - s_{\text{sh}} \leftarrow \text{flow eqn}$$

$$dh = T ds + v dp \leftarrow \text{not valid when there's flow}$$

$$dh_t = T_t ds + v_t dp_t$$

$$T ds - s_{\text{sh}} = T_t ds + \frac{1}{P_t} dp_t$$

W.D. by the flow
 $\Rightarrow \dot{m}_{\text{fresh}} > 0$
 $\Rightarrow \frac{dp_t}{dt} < 0$

$$= T_t ds + RT_t \frac{dp_t}{dt}$$

$$\frac{dp_t}{T_t} = - \left[1 - \frac{T_t}{T} \right] \frac{ds}{R} - \frac{\dot{m}_{\text{fresh}}}{RT_t}$$

W.D. on the flow
 $\Rightarrow \dot{m}_{\text{fresh}} < 0$
 $\Rightarrow \frac{dp_t}{dt} > 0$

\downarrow
heat add $\Rightarrow ds > 0 \Rightarrow \frac{dp_t}{dt} < 0 \Rightarrow u_e \downarrow$

$$\sigma T_t = T_t + \frac{u^2}{2c_p}, \quad 1 - \frac{T_t}{T} = \frac{u^2/2c_p}{T + u^2/2c_p} = \frac{u^2}{2c_p T_t} > 0$$

$$\left(1 - \frac{T_t}{T} \right) \propto \frac{u^2}{2c_p T_t} = \frac{(y-1) M^2 / 2}{1 + \frac{y-1}{2} M^2} \propto M^2$$

$$M \downarrow \Rightarrow \text{low } p_t \text{ loss} \quad (M_b \sim 0.3) \quad (M_a \sim 0.85)$$

Flow shd be slowed down before burner $\xrightarrow{\text{reason 1}}$ combustion
 $\xrightarrow{\text{reason 2}} p_t$

How to slow down the flow,

$$\dot{m} = \rho A u = \frac{P}{RT} A M \sqrt{YRT} = \frac{P}{\sqrt{T}} \sqrt{\frac{Y}{R}} M A$$

$$= \frac{P_0}{\sqrt{T_0}} \left[1 + \frac{y-1}{2} M^2 \right]^{\frac{y+1}{2(y-1)}} M \sqrt{\frac{Y}{R}} A$$

$$\dot{m} = A \frac{P_0}{\sqrt{T_0}} f(M, Y, R)$$

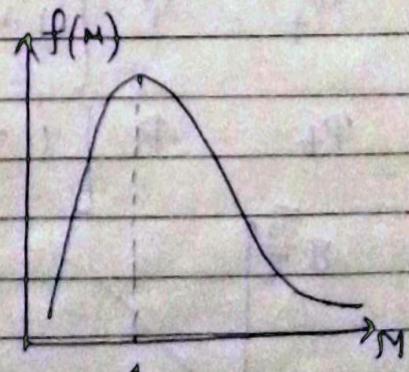
\downarrow energy $f(M, Y, R)$

$$\dot{m} = A \frac{P_0}{\sqrt{T_0}} f(M, Y, R)$$

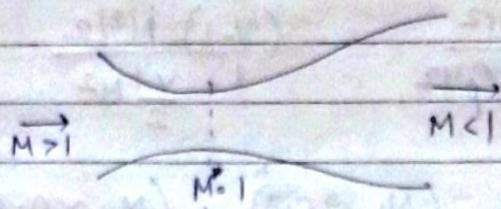
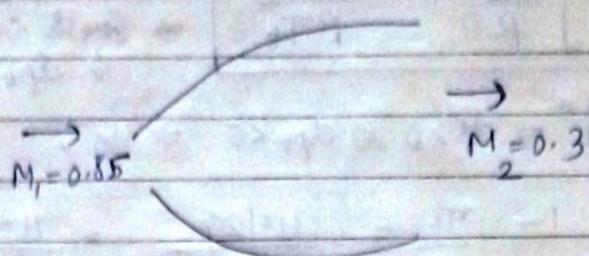
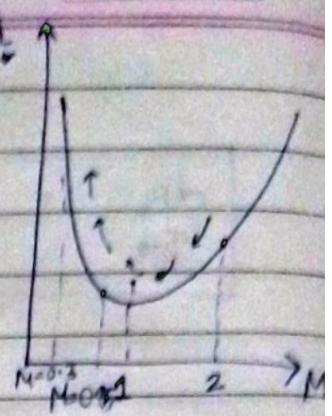
\uparrow nozzle diffuser prop. \downarrow gas prop.

$$f_* = f \left(1, Y, R \right) = \left(\frac{y+1}{2} \right)^{\frac{y+1}{2(y-1)}} \sqrt{\frac{Y}{R}}$$

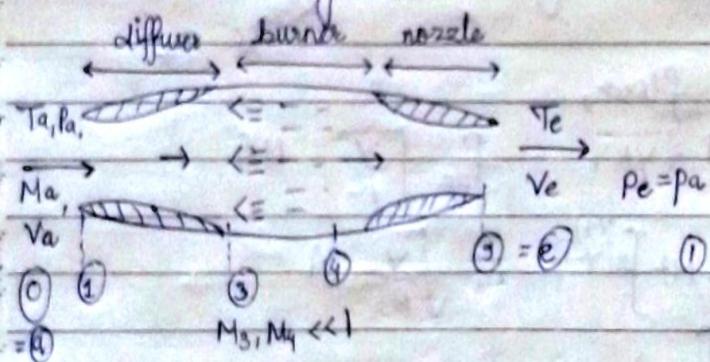
$$\rightarrow \dot{m} = A_* \frac{P_t}{\sqrt{T_t}} f_*$$



$$A = \frac{\dot{m}}{f(M)} \frac{\sqrt{T_t}}{P_t}, \quad \frac{A}{A^*} = \frac{f_a}{f(M)} \cdot \frac{A}{A^*}$$



RAMJET Engines :-



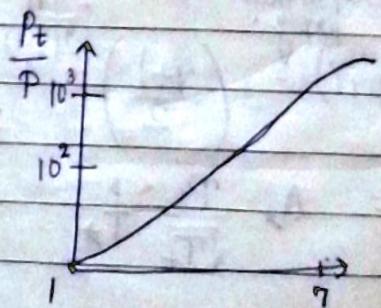
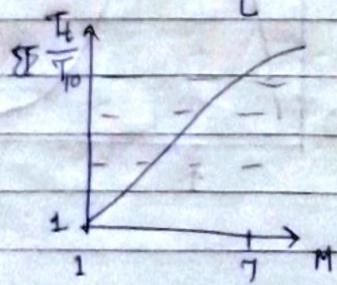
Stagnation states while analyzing the burner

(1) - (3) → static - stagnation

$$\frac{T_0, P_0}{M_0} \left(\frac{T_a, P_a}{M_a} \right) = \frac{T_{t_0}, P_{t_0}}{T_{t_3}, P_{t_3}}$$

$$T_t = T \left[1 + \frac{y-1}{2} M^2 \right] = T [1 + 0.2 M^2]$$

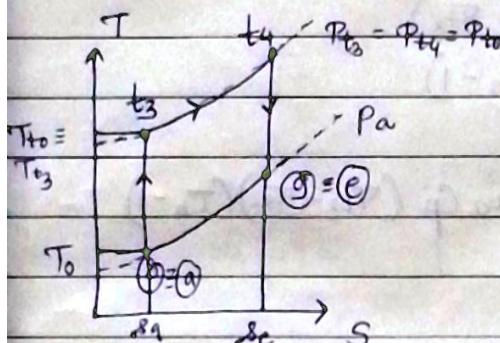
$$P_t = P \left[1 + \frac{y-1}{2} M^2 \right]^{\frac{y}{y-1}} = P [1 + 0.2 M^2]^{3.5}$$



$\textcircled{1} \rightarrow \textcircled{3}$ - Isentropic compression
 $t_0 \rightarrow t_3$ $(T_0, P_0) \rightarrow (T_{t_3}, P_{t_3})$

$\textcircled{3} - \textcircled{4}$ - Isobaric combustion
 $t_3 - t_4$ $(T_{t_3}, P_{t_3}) \rightarrow (T_{t_4}, P_{t_4})$

$\textcircled{4} - \textcircled{9}$ - Isentropic expansion
 $(T_{t_4}, P_{t_4}) \rightarrow (T_9, P_9) = (T_e, P_a)$



$$\frac{P_{t_3}}{P_a} = \left[\frac{1 + \frac{\gamma-1}{2} Ma^2}{2} \right]^{\frac{1}{\gamma-1}}$$

$$\frac{P_{t_4}}{P_e} = \left[\frac{1 + \frac{\gamma-1}{2} Ne^2}{2} \right]^{\frac{1}{\gamma-1}}$$

$$M_e = Ma$$

$$\frac{P_{t_3}}{P_0} = \frac{P_{t_4}}{P_e} \Rightarrow \frac{T_{t_3}}{T_0} = \frac{T_{t_4}}{T_e}$$

$T_0 \approx 220 \text{ K}$ | $T_{t_4} \leftarrow \text{max. / peak temperature}$
 (fixed by altitude) | limited by material safety
 (fixed by tech.)

$$T_e = \frac{T_{t_4} \cdot T_a}{T_{t_3}}$$

$$T_b = \frac{T_{t_4}}{T_{t_3}} \Rightarrow T_e = T_b T_a$$

Burner:- $\dot{m}_f Q_f = \dot{m} C_p (T_{t_4} - T_{t_3})$

- assume $f = \frac{\dot{m}_f}{\dot{m}_a} \ll 1$ & $C_p(\gamma, R)$ are not changing

due to combustion.

$\frac{f Q_f}{C_p T_{t_3}}$	$= T_b - 1$
-----------------------------	-------------

$$\bar{C}_b = 1 + \frac{f \rho R}{c_p T_3} \leftarrow \text{flow enthalpy}$$

Surplus

Thermal energy of jet -

$$q_{\text{thj}} = \dot{m}_a C_p (T_e - T_a)$$

$$q_{\text{loss}} = \dot{m}_a C_p T_a (\bar{C}_b - 1)$$

Combustion heat release

$$q_b = \dot{m}_a C_p (T_{t_4} - T_{t_3})$$

$$= \dot{m}_a C_p T_{t_3} (\bar{C}_b - 1)$$

WD \leftrightarrow Energy that was utilized.

$$w = q_b - q_{\text{loss}} = \dot{m}_a C_p (T_{t_3} - T_a) (\bar{C}_b - 1)$$

Efficiency \rightarrow

$$\eta = \frac{w}{q_b} = 1 - \frac{T_a}{T_{t_3}}$$

$$\eta = 1 - \frac{T_a}{T_{t_3}} = 1 - \left(\frac{P_a}{P_{t_3}} \right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{1}{1 + \frac{\gamma-1}{2} Ma^2}$$

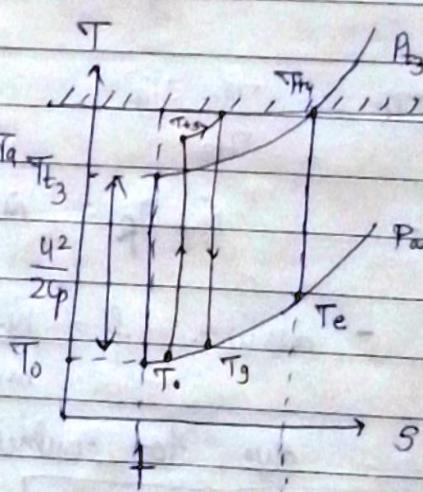
$$= \frac{(\gamma-1) Ma^2 / 2}{1 + \frac{\gamma-1}{2} Ma^2}$$

$\eta \uparrow \leftarrow Ma \uparrow$

$$Ma \uparrow \Rightarrow \frac{u_a^2}{2 C_p} = (\gamma-1) \frac{u_a^2}{2 R \gamma} = \frac{\gamma-1}{2} Ma^2 \frac{T_a}{T_{t_3}}$$

$$q_b = \dot{m}_a C_p T_{t_3} (\bar{C}_b - 1)$$

If Ma is too large, not much heat addⁿ is possible $\rightarrow \bar{C}_b$ is small.



η is high but not much work output.

How high can T_a be?

When is work highest?

$$w = q_b - q_{loss} = m a c_p T_{t_3} (T_b - 1) (T_{t_3} - T_a)$$

$$\frac{w}{m a c_p T_a} = \frac{T_{t_3}}{T_a} (T_b - 1) - (T_b - 1) \equiv \frac{T_{t_4}}{T_a} - \frac{T_{t_3}}{T_a} - \frac{T_e}{T_a} + 1$$

$$\frac{P_{t_4}}{P_e} = \frac{P_{t_3}}{P_a} \Rightarrow \frac{T_{t_4}}{T_e} = \frac{T_{t_3}}{T_a} \rightarrow$$

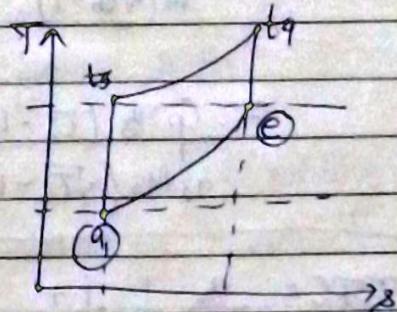
$$\frac{w}{m a c_p T_a} = \left[\frac{T_{t_4}}{T_a} + 1 \right] - \frac{T_{t_3}}{T_a} - \frac{T_{t_4}}{T_b} \equiv f(T_b)$$

$$\frac{d}{dT_b} \left[\frac{w}{m a c_p T_a} \right] = 0 \Rightarrow -\frac{1}{T_a} + \frac{T_{t_4}}{T_b^2} = 0$$

$$\Rightarrow T_{t_3} = \sqrt{T_a T_{t_4}} \rightarrow T_e = T_b$$

$$\eta = 1 - \frac{T_a}{T_{t_3}}$$

$$\eta_{opt} = 1 - \frac{T_a}{\sqrt{T_a T_{t_4}}} = 1 - \sqrt{\frac{T_a}{T_{t_4}}}$$



To improve efficiency \Rightarrow increase T_{t_4} .

Optimum T_a ?

$$\frac{T_{t_3}}{T_a} = 1 + \frac{\gamma - 1}{2} Ma^2 = \sqrt{\frac{T_{t_4}}{T_a}}$$

$$\Rightarrow Ma = \sqrt{\frac{2}{\gamma - 1} \left[\sqrt{\frac{T_{t_4}}{T_a}} - 1 \right]}$$

Thrust analysis -

$$T = \dot{m}_a (V_e - V_a) = \dot{m}_a V_a \left[\frac{V_e}{V_a} - 1 \right]$$

If $M_a = M_\infty$,

$$T = \dot{m}_a M_\infty a_\infty \left[\sqrt{\frac{T_e}{T_a}} - 1 \right]$$

normalised

thrust $\rightarrow \frac{T}{\dot{m}_a a_\infty} = M_\infty \left[\sqrt{T_b} - 1 \right]$

$\frac{T}{\dot{m}_a}$ \leftarrow Thrust per unit mass $\rightarrow m/s$

\uparrow Normalised with ambient speed of sound.

$$T_b = \frac{T_{tq}}{T_3} = \frac{T_{tq}}{T_0} \left[1 + \frac{\gamma - 1}{2} M_\infty^2 \right]^{-1}$$

$$\text{TSFC} = \frac{\dot{m}_a}{T} = \frac{f_{\text{final}}}{\dot{m}_a (V_e - V_a)} = \frac{f}{V_e - V_a}$$

$$= \frac{f}{V_a (\sqrt{T_b} - 1)}$$

$$f Q_R = C_p T_b (T_b - 1)$$

$$= C_p T_b (T_b - 1)$$

$$a_\infty M_\infty (\sqrt{T_b} - 1)$$

$$C_p T_3 = \frac{\gamma R T_0}{\gamma - 1} \left(1 + \frac{\gamma - 1}{2} M_\infty^2 \right)$$

$$= \frac{a_\infty^2}{\gamma - 1} \left[1 + \frac{\gamma - 1}{2} M_\infty^2 \right]$$

$$\boxed{\text{TSFC} = \frac{a_\infty}{\gamma - 1} \frac{1}{Q_R} \frac{1 + \frac{\gamma - 1}{2} M_\infty^2}{M_\infty} (\sqrt{T_b} + 1)}$$

TSFC increases with T_{tq} .

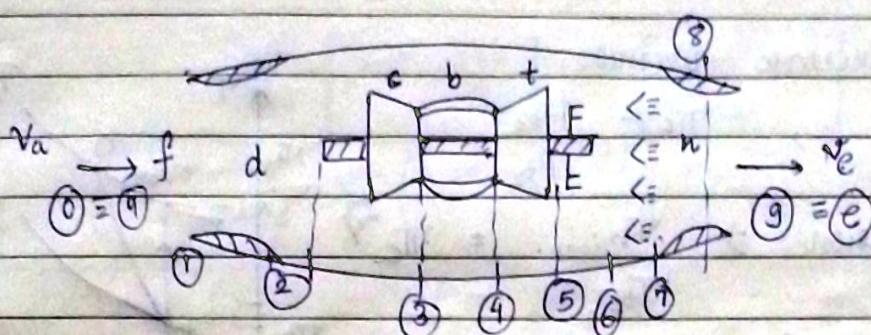
Super charger in piston engine is used to increase 'p'.

Thrust when $V=0 \Rightarrow P_{t_3} = P_{to} :-$

Use compressor to increase P_{t_3} ($P_{t_3} > P_{to}$).

Turbine extracts energy after the burner to drive the compressor

$$\frac{dp_t}{P_t} = - \left[1 - \frac{T}{T_t} \right] \frac{ds}{R} - \frac{s_{insh}}{RT_t}$$



Gasturbine \rightarrow Turbojet engine with afterburners

Gas Turbine Cycle:-

RAMJET engine $\rightarrow M \approx 2-4$

Compression was mostly from flow K.E.

In above diagram, $2-3 \rightarrow$ compressor - adding work
 $4-5 \rightarrow$ Turbine - extract work

Stagnation states $\rightarrow t_2 \rightarrow t_3$ in compressor
 $t_4 \rightarrow t_5$ in turbine

$$i_{ec} = m_a C_p (T_{t_3} - T_{t_2}) \underset{\text{const.}}{\equiv} m_a C_p T_{t_2} \left[\frac{T_{t_3}}{T_{t_2}} - 1 \right]$$

$$i_{et} = m_a C_p (T_{t_4} - T_{t_5}) \equiv m_a C_p T_{t_4} \left[1 - \frac{T_{t_5}}{T_{t_4}} \right]$$

$$h_{net} = h_{t_1} - h_{c_0} = \dot{m} a C_p T_{t_2} \left\{ \frac{T_{t_4}}{T_{t_2}} \left[1 - \frac{T_{t_5}}{T_{t_4}} \right] - \left[\frac{T_{t_3} - 1}{T_{t_2}} \right] \right\}$$

$$T_{t_2} = T_2 \left[1 + \frac{\gamma - 1}{2} M_2^2 \right] \approx T_{t_2} = T_0 \left[1 + \frac{\gamma - 1}{2} M_0^2 \right]$$

As altitude increases in the flight, T_{t_2} reduces, inspite of M_a increase.

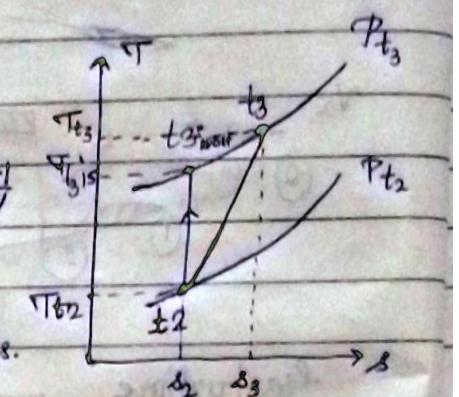
Compressor \rightarrow increase pressure

$$P_{t_2} \rightarrow P_{t_3}, \quad \pi_c = \frac{P_{t_3}}{P_{t_2}}$$

$$\text{Isentropic compression} \Rightarrow \frac{T_{t_3 \text{isen.}}}{T_{t_2}} = \frac{\pi_c}{\gamma}$$

$$h_{t_3} - h_{t_2} = (h_{t_3 \text{isen.}} - h_{t_2}) + \Delta h_{\text{diss.}}$$

$$h_{t_3} = h_{t_3 \text{isen.}} + \Delta h_{\text{diss.}}$$



Compressor Efficiency :- $\eta_c = \frac{\text{ideal work input}}{\text{actual work input}} = \frac{h_{t_3 \text{isen.}} - h_{t_2}}{h_{t_3} - h_{t_2}}$

$$\text{Expts.} \rightarrow \eta_c = \frac{T_{t_3 \text{isen.}} - T_{t_2}}{T_{t_3} - T_{t_2}} = \frac{T_{t_3 \text{isen.}} / \pi_c - 1}{T_{t_3} / \pi_c - 1} \quad \text{Isentropic relations}$$

$$\frac{T_{t_3 \text{isen.}}}{T_{t_2}} = \pi_c^{\gamma-1/\gamma} \Rightarrow \boxed{\frac{T_{t_3}}{T_{t_2}} = 1 + \frac{1}{\eta_c} \left[\pi_c^{\gamma-1/\gamma} - 1 \right]}$$

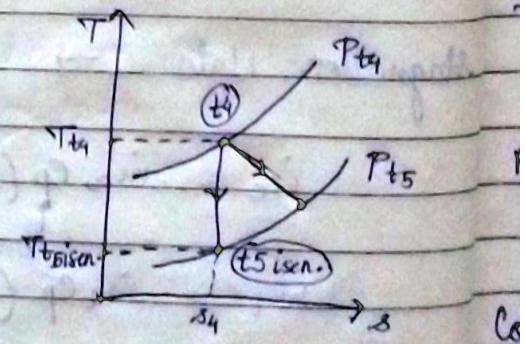
As π_c increases, η_c decreases.

Turbine -

$$\pi_t = \frac{P_{t_4}}{P_{t_5}}$$

$$T_{t_5 \text{isen.}} = \frac{1}{\pi_t^{\gamma-1/\gamma}}$$

$$h_{t_5} - h_{t_4} = (h_{t_5 \text{isen.}} - h_{t_4}) + \Delta h_{\text{diss.}}$$



$$h_{t5} = h_{t4} - (h_{t4} - h_{t5 \text{ isen}}) + \Delta h_{\text{diss}}$$

Efficiency - $\eta_t = \frac{\text{actual work extracted}}{\text{ideal work extracted}} = \frac{h_{t4} - h_{t5}}{h_{t4} - h_{t5 \text{ isen}}}$

$$\eta_t = \frac{T_{t4} - T_{t5}}{T_{t4} - T_{t5 \text{ isen}}} = \frac{1 - \pi_{t5}/\pi_{t4}}{1 - \pi_t^{(y-1)}}$$

$$\frac{T_{t5}}{T_{t4}} = 1 - \eta_t \left[1 - \pi_t^{-\frac{(y-1)}{y}} \right]$$

$$\frac{\dot{m}_c C_p}{T_{t2}} = \frac{T_{t4}}{T_{t2}} \left[1 - \frac{T_{t5}}{T_{t4}} \right] = \left[\frac{T_{t3}}{T_{t2}} - 1 \right]$$

$$\frac{\dot{m}_c}{\dot{m}_c C_p T_{t2}} = \frac{T_{t4}}{T_{t2}} \cdot \eta_t \left[1 - \pi_t^{-\frac{(y-1)}{y}} \right] - \frac{1}{\eta_c} \left[\pi_t^{\frac{y-1}{y}} - 1 \right]$$

What is the gasturbine usage?

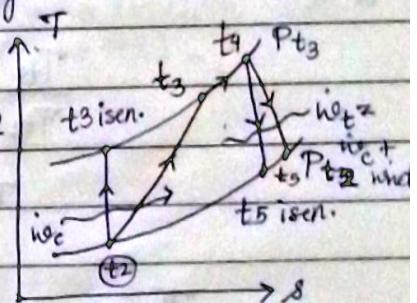
→ shaft work
→ jet

combination (fan + jet) $\rightarrow P_{t5} > P_{t2}$

shaft power only.

$$\dot{q}_b = \dot{m}_c C_p (T_{t4} - T_{t3}) \equiv m_f \dot{q}_R = \dot{m}_c \dot{q}_{rf}$$

$$f = \frac{C_p (T_{t4} - T_{t3})}{\dot{q}_R}$$

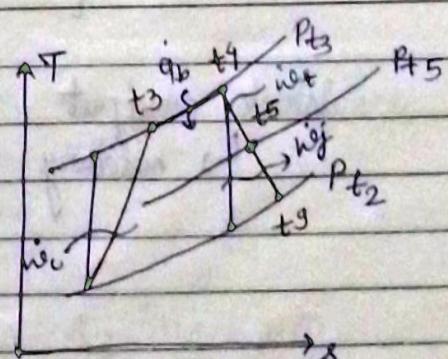


Including jet

$$\dot{w}_{ej} = \frac{v_{jet}^2}{2}$$

$$\text{Fan } \dot{w}_t = \dot{w}_c + \dot{w}_{ej}$$

compressor fan



Core engine \rightarrow compressor - burner - turbine

$$\Omega = \frac{\dot{W}_{net}}{m_a C_p T_{t_2}} = \frac{T_{t_4}}{T_{t_2}} \eta_t \left[1 - \pi_c^{-\frac{y-1}{y}} \right] - \frac{1}{\eta_c} \left[\pi_c^{\frac{y-1}{y}} - 1 \right]$$

$$\Omega = \Omega \left(\frac{T_{t_4}}{T_{t_2}}, \eta_c, \eta_t, \pi_c, \pi_t, \gamma \right)$$

$$\Omega = \Omega(\pi_c, \pi_t) \text{ if others are fixed}$$

Simple Gas-Turbine \rightarrow shaft power only

$$\Rightarrow T_t = \pi_c T_c$$

$$\Omega = \Omega(\pi_c) \quad \gamma = \frac{y-1}{y}$$

$$\Omega = \frac{T_{t_4}}{T_{t_2}} \eta_t \left(1 - \pi_c^{-\gamma} \right) - \frac{1}{\eta_c} \left(\pi_c^\gamma - 1 \right)$$

When is work output maximized?

$$\frac{d\Omega}{d\pi_c} = 0 \quad \frac{T_{t_4}}{T_{t_2}} \eta_t \gamma \pi_c^{-\gamma-1} - \frac{1}{\eta_c} \gamma \pi_c^{\gamma-1} = 0$$

$$\frac{T_{t_4}}{T_{t_2}} \eta_t \gamma \pi_c^{-\gamma-1} = \frac{1}{\eta_c} \gamma \pi_c^{\gamma-1}$$

$$\pi_c|_{opt.}^{2\gamma} = \frac{T_{t_4}}{T_{t_2}} \eta_t \eta_c \rightarrow \pi_c|_{opt.} = \left[\frac{T_{t_4}}{T_{t_2}} \eta_t \eta_c \right]^{\frac{1}{2\gamma}}$$

$$\pi_c|_{opt.} = \left[\frac{T_{t_4}}{T_{t_2}} \eta_t \eta_c \right]^{\frac{1}{2\gamma}} \frac{1}{2(y-1)}$$

Work output to be maximized $\rightarrow \pi_c \approx 13$.

\hookrightarrow military engines $\rightarrow \pi_c \approx 12-14$