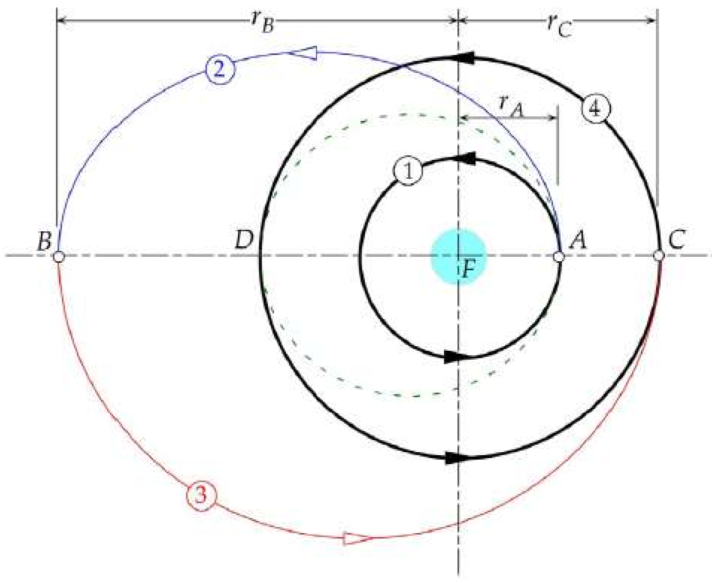
Bielliptic Hohmann Transfer



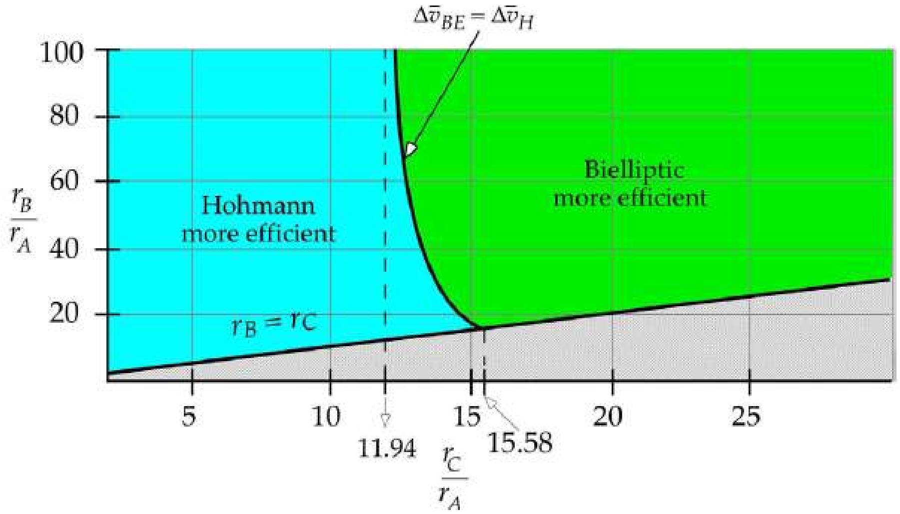
- $\Delta V_{total})_{bielliptical} < \Delta V_{total})_{Hohmann}$

- $$\Delta \bar{V}_H = \frac{1}{\sqrt{\alpha}} - \frac{\sqrt{2}(1-\alpha)}{\sqrt{\alpha(1+\alpha)}} - 1$$

$$\Delta \bar{V}_{BE} = \sqrt{\frac{2(\alpha+\beta)}{\alpha\beta}} - \frac{1+\sqrt{\alpha}}{\sqrt{\alpha}} - \sqrt{\frac{2}{\beta(1+\beta)}}\,(1-\beta)$$

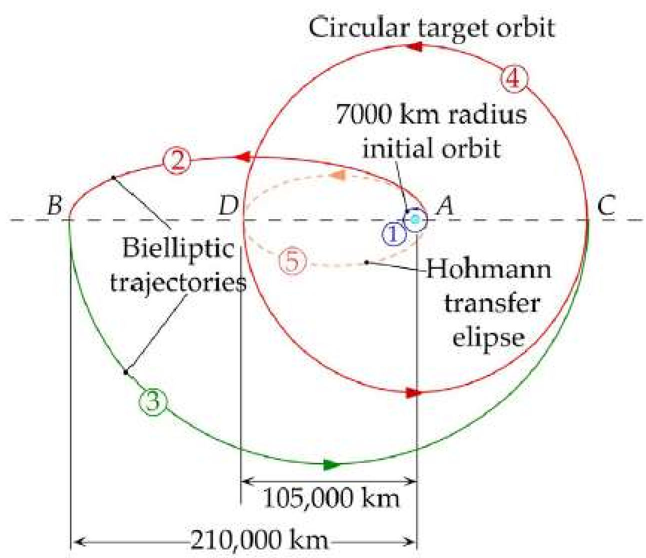
- $$\Delta \bar{V}_H = \left.\frac{\Delta V_{total}}{V_0}\right)_{Hohmann}, \quad \Delta \bar{V}_{BE} = \left.\frac{\Delta V_{total}}{V_0}\right)_{bielliptical}$$

$$V_0 = \sqrt{\frac{\mu}{r_A}}, \quad \alpha = \frac{r_C}{r_A}, \quad \beta = \frac{r_B}{r_A}$$



Example

Find the total delta-v requirement for bielliptic Hohmann transfer from a geocentric circular orbit of 7000 km radius to one of 105,000 km radius. Let the apogee of the first ellipse be 210,000 km. Compare the delta-v schedule and total flight time with that for an ordinary single Hohmann transfer ellipse (see Fig. 6.9).



Details

$r_A = 7000$ Km , $r_B = 210000$ Km , $r_C = r_D = 105000$ Km

Orbit 1: $v_A)_1 = \sqrt{\frac{\mu}{r_A}}$

Orbit 2: $V_A)_2 = \dfrac{\|h_2\|}{r_A}$, $V_B)_2 = \dfrac{\|h_2\|}{r_B}$

Orbit 3: $V_B)_3 = \dfrac{\|h_3\|}{r_B}$, $V_C)_3 = \dfrac{\|h_3\|}{r_C}$

Orbit 4: $V_C)_4 = V_D)_4 = \sqrt{\dfrac{\mu}{r_D}}$

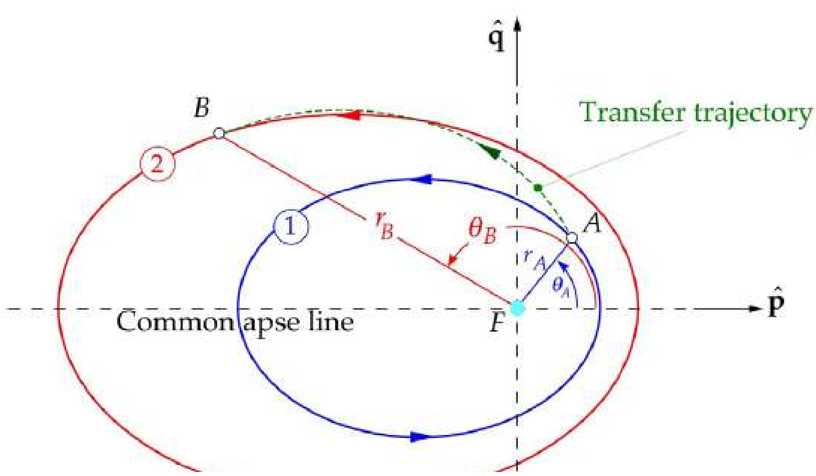
$$\Delta V_{total})_{bielliptical} = \Delta V_A + \Delta V_B + \Delta V_C$$

$$t_{bielliptical} = \frac{1}{2}\left(\frac{2\pi}{\sqrt{\mu}} a_2^{3/2} + \frac{2\pi}{\sqrt{\mu}} a_3^{3/2}\right)$$

$V_A)_5 = \dfrac{\|h_5\|}{r_A}$, $V_D)_5 = \dfrac{\|h_5\|}{r_D}$

$$\Delta V_{total})_{Hohmann} = \Delta V_A + \Delta V_D$$

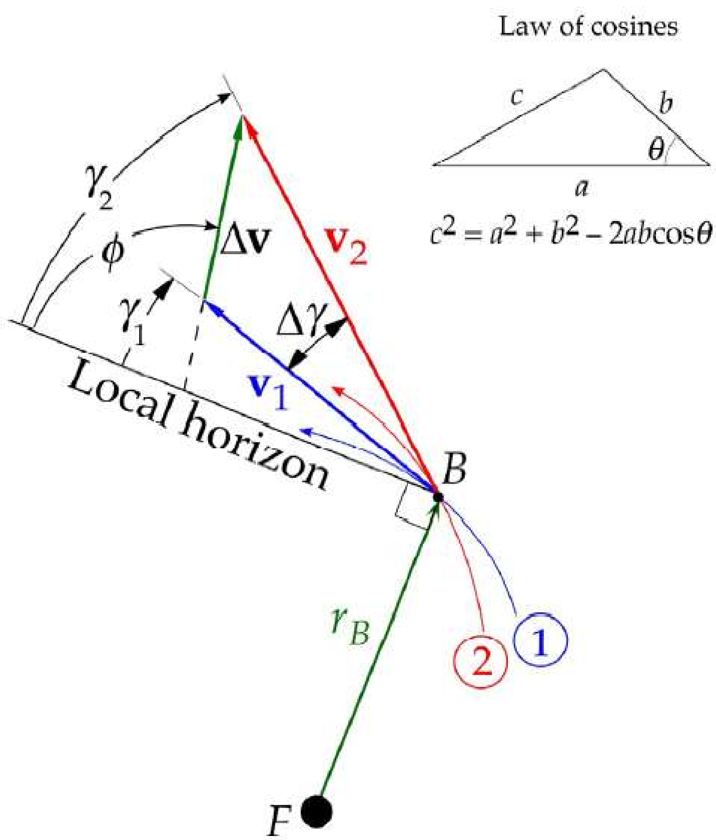
Non-Hohmann Transfers with a Common Apse Line



- $r_A = \frac{\|h_2\|^2}{\mu} \frac{1}{1+\|e\|\cos\theta_A}$, $r_B = \frac{\|h_2\|^2}{\mu} \frac{1}{1+\|e\|\cos\theta_B}$

- $\|e\| = \frac{r_A - r_B}{r_A\cos\theta_A - r_B\cos\theta_B}$

$$\|h\| = \sqrt{\mu r_A r_B}\sqrt{\frac{\cos\theta_A - \cos\theta_B}{r_A\cos\theta_A - r_B\cos\theta_B}}$$



- $\|\Delta V\| = \sqrt{(V_2 - V_1)\cdot(V_2 - V_1)} = \sqrt{\|V_1\|^2 + \|V_2\|^2 - 2\|V_1\|\|V_2\|\cos\underline{\Delta\gamma}}$

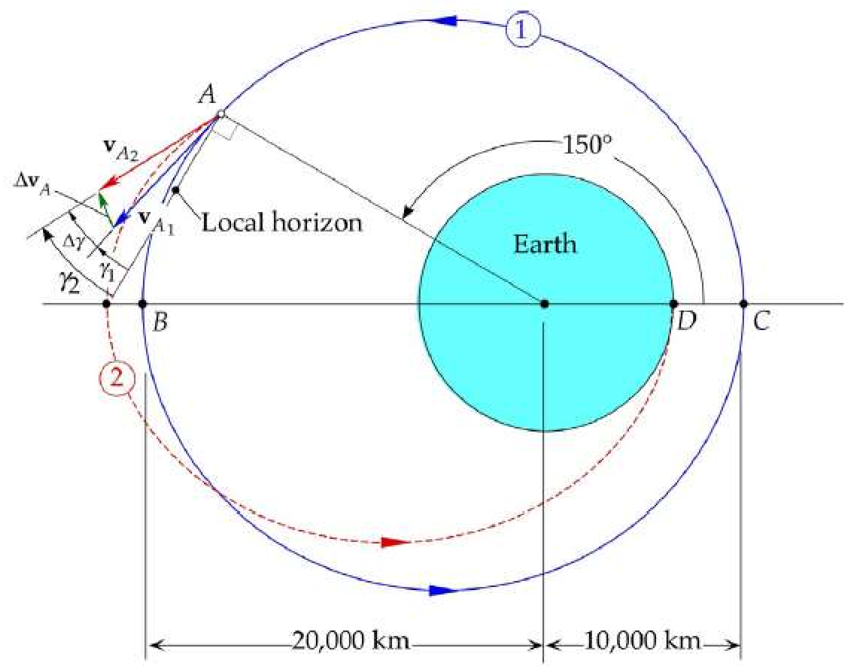
- $\tan\phi = \frac{\Delta V_r}{\Delta V_1}$

- $\varepsilon = \dfrac{\mathbf{v}\cdot\mathbf{v}}{2} - \dfrac{\mu}{r}$

- $\Delta\varepsilon = \mathbf{v}\cdot\Delta\mathbf{v} + \dfrac{1}{2}\Delta\mathbf{v}\cdot\Delta\mathbf{v} = \|\mathbf{v}\|\|\Delta\mathbf{v}\|\cos\Delta\gamma + \dfrac{1}{2}\|\Delta\mathbf{v}\|^2$

Example

A geocentric satellite in orbit 1 of Fig. 6.15 executes a delta-v maneuver at A, which places it on orbit 2, for reentry at D. Calculate Δv at A and its direction relative to the local horizon.



Details

$r_B = 20000\ \text{km}, \quad r_C = 10000\ \text{km}, \quad r_D = 6378\ \text{km}$

Orbit 1: $\|e_1\| = \dfrac{r_B - r_C}{r_B + r_C}, \quad r_A = \dfrac{\|h_1\|^2}{\mu}\dfrac{1}{1+\|e_1\|\cos\theta_A}$

$$V_{\perp A})_1 = \frac{\|h_1\|}{r_A}, \quad V_{r_A})_1 = \frac{\mu}{\|h_1\|}\|e_1\| \sin\theta_A$$

$$V_A)_1 = \sqrt{V_{\perp A})_1^2 + V_{r_A})_1^2}, \quad \gamma_1 = \tan^{-1}\left(\frac{V_{r_A})_1}{V_{\perp A})_1}\right)$$

Orbit 2: $V_{\perp A})_2 = \dfrac{\|h_2\|}{r_A}, \quad V_{r_A})_2 = \dfrac{\mu}{\|h_2\|}\|e_2\| \sin\theta_A$

$$V_A)_2 = \sqrt{V_{\perp A})_2^2 + V_{r_A})_2^2}, \quad \gamma_2 = \tan^{-1}\left(\frac{V_{r_A})_2}{V_{\perp A})_2}\right)$$