*AE 240/338 Quiz No. 2 Tuesday, March 29, 2016 Weightage: 10% Time: 19.00 – 2025 Hrs.*
*Venue: LC 101 Examination is closed notes. Only calculator is allowed.*

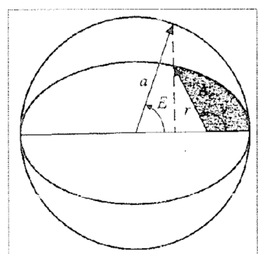
Q.1 (a) Prove that for an elliptic orbit, semi-major axis 'a' and eccentricity 'e', in terms of energy 'ε' and angular momentum 'h', are as follows. (Hint: Use geometric relations of an ellipse, as given below.) (2)

$$a = -\frac{\mu}{2\varepsilon}; \quad e = \sqrt{1 + \frac{2\varepsilon h^2}{\mu^2}};$$

$$r = \frac{p}{1 + e\cos\theta}; \quad p = h^2/\mu = a\left(1 - e^2\right), \quad \varepsilon = \frac{v^2}{2} - \frac{\mu}{r}; \quad r_p = a(1-e), \quad r_a = a(1+e); \quad h = r_p v_p$$

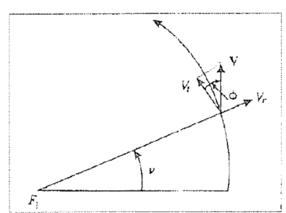
(b) Next, using the above relations, determine whether or not an orbit is formed for a satellite injected at an altitude of 200 km above Earth's surface, with a velocity of 7.5 km/s parallel to the local horizon. ($R_E$ = 6,378 km, $\mu_E = 3.986 \times 10^{14}\ m^3.s^{-2}$). (1)

Q.2 (a) Derive the expression for time of flight between any two points on an elliptic trajectory in terms of true anomaly 'θ (= ν)', using auxiliary circle approach and concepts of mean / eccentric anomaly. (Hint: Use concept of mean velocity & following figure. You can express 'θ' in terms of 'E' separately). (2)



(b) Next, using the above relation, find time to travel from $\theta_A = 200^o$ to $\theta_B = 260^o$ on a geocentric ellipse which has semi-major axis of 15,000 km and eccentricity of 0.63. (2)

Q.3 (a) Derive basic relations between elevation angle 'ϕ' and true anomaly 'θ' for a space object moving on a conic section trajectory, as shown below. (Hint: use conic section relations as given in Q.1, ν=θ). (1)



(b) Next, an incoming object is sighted at an altitude of 37,040 km above earth, with a velocity of 8000 m/s and -65° elevation angle. What kind of trajectory is described by the object? (1)

Q.4 Derive the expression for 'ΔV', to be given at perigee of a circular orbit, in order to raise the apogee from radius '$a_0$' to radius '$a_1$', as per the figure given below. (Hint: You may use relations given in Q.1). (1)