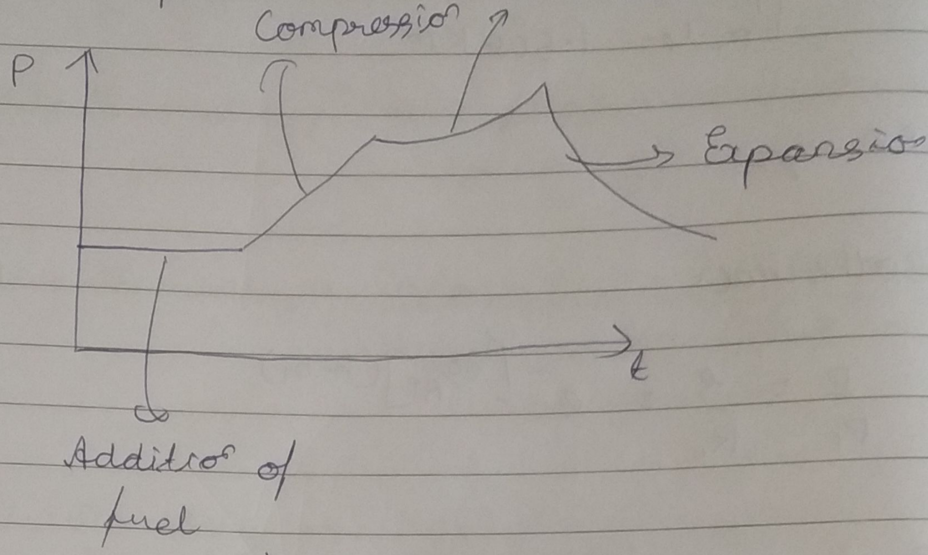
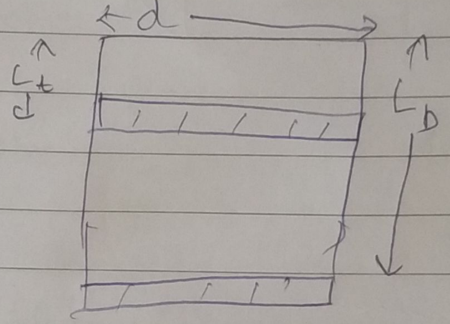
* Piston Prop





$M = (M_f + M_a) = \rho V$

$V = A \cdot L_b = \frac{\pi d^2}{4} L_b$

Compression ratio $\rightarrow r = \frac{A L_b}{A L_t} = \frac{L_b}{L_t}$

$L_b - L_t$ = Stroke Length

For isentropic compression,

$$P_b V_b^{\gamma} = P_t V_t^{\gamma}$$

$$P_t = P_b r^{\gamma}$$

Work done by piston = $(p - p_\infty) A h$

$h \rightarrow$ Stroke length

$$\text{Power} = (\bar{p} - p_\infty)\, A \cdot L_b \left[1 - \frac{1}{r}\right] \frac{N_{rpm}}{120} \cdot K$$

$K \rightarrow$ No. of cylinders
$\bar{p} \rightarrow$ Mean effective pressure

$$\text{Heat release} = M_f Q_R = \frac{f M}{(1+f)} Q_R$$

$$= \frac{f Q_R}{(1+f)} \frac{P_\infty}{RT} L_b A$$

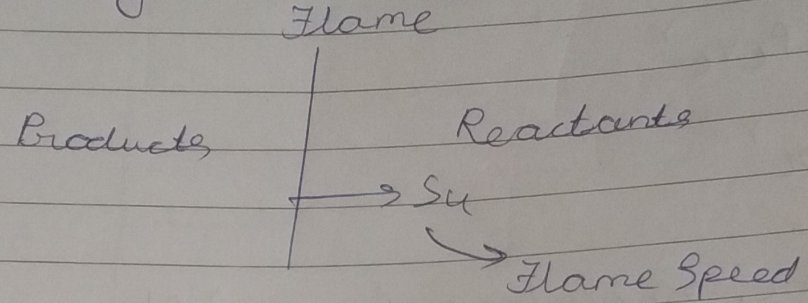
$$\text{Total heat} = M \cdot \frac{N_{rpm}}{120} \cdot K$$

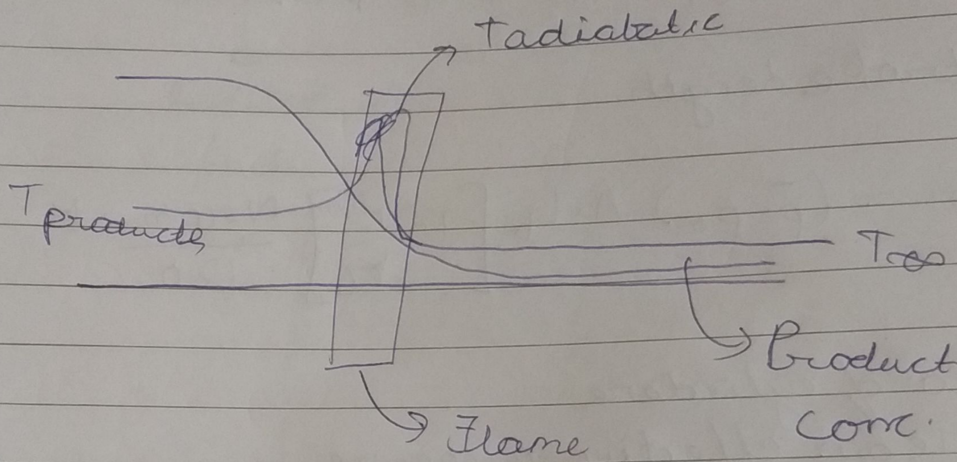
$$\frac{P}{H_{tot}} = \frac{\Delta \bar{P}}{P_\infty} \frac{RT}{f Q_R} \left[1 - \frac{1}{r}\right] (1+f) \qquad (\eta)$$

$$\Delta \bar{P} = \bar{P} - P_\infty$$

$\eta \uparrow$ with $r \uparrow$, $f \downarrow$

During combustion,





If we consider $\theta = \frac{T - T_\infty}{T_p - T_\infty}$

$\theta > 1 \rightarrow$ Flame region

Let flame thickness be δ_f and area be A.

$\dot{M} = V \omega_r$ → Reaction rate

$\dot{M} = A \delta_f \omega_r$

$\dot{M} = \rho S_u A$

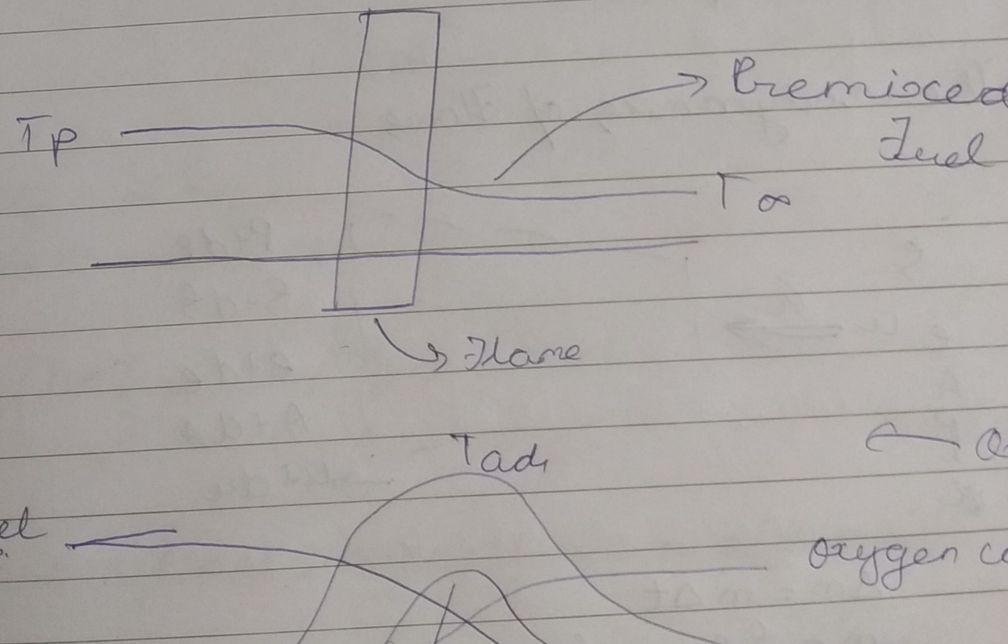
$$S_u = \frac{1}{\rho} \delta_f \omega_r$$

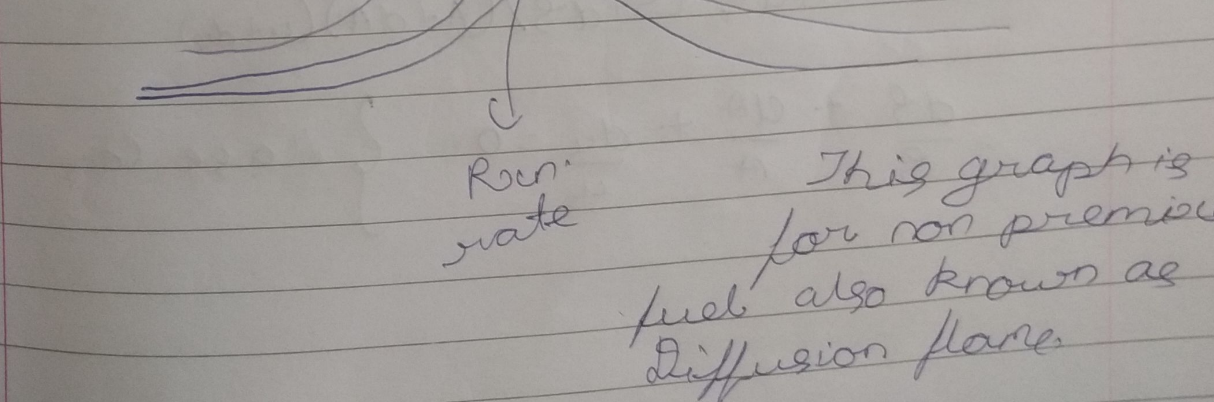
Let $\omega_r \propto p^n$

$S_u \propto p^{n-1}$ as $\rho \propto p$

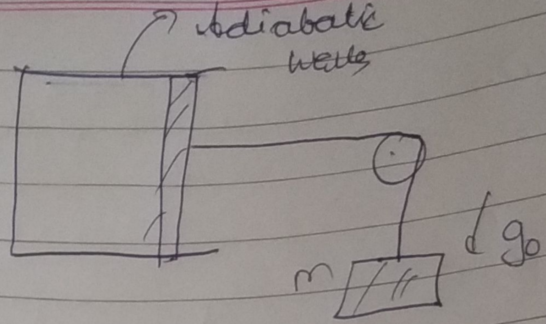
At low pressure, we get a steady combustion but at high pressure, we get detonation also known as knocking.

* Flame Temp. Correction





This graph is for non premixed fuel also known as Diffusion flame.



$\Delta Q = 0$

$\Delta E = -\Delta W$

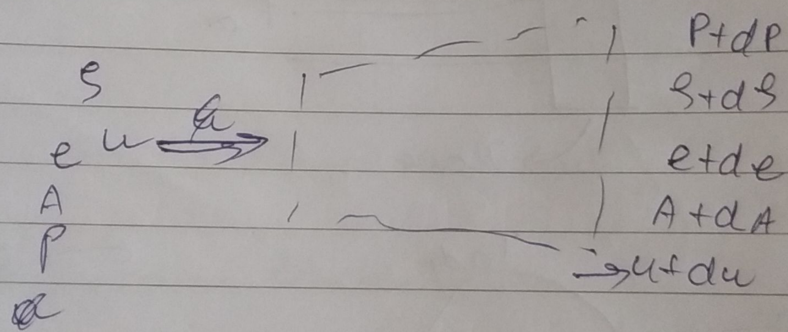
$\Delta W = m g_0 \Delta h$

$\boxed{\Delta E = \Delta Q - \Delta W} \rightarrow$ First Law

ΔW from POV of system $= p\Delta V$

$dh = d(e + pv)$

* Thermodynamics of Flows



$\Delta m = \dot{m}\,\Delta t$

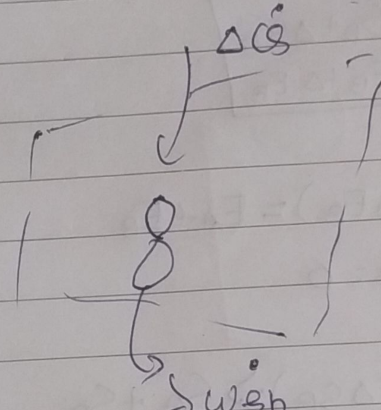
$\dot{m} = \rho A u = (\rho + d\rho)(A + dA)(u + du)$

$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{du}{u} = 0$ } Mass Cons.

$$\dot m (u+du) = \dot m u + pA - (p+dp)(A+dA) + \left(p+\frac{dP}{2}\right) dA$$

$$\boxed{\rho u\,du = -dP}$$

$$\dot m\left[\frac{1}{2}(u+du)^2 + (e+de)\right] = \dot m\,[u+e] + \Delta\dot Q - \delta\dot w_{sh} + puA - (p+dP)(u+du)(A+dA) + \left(p+\frac{dP}{2}\right)\left(u+\frac{du}{2}\right) dA$$



$$\Delta\dot Q = \dot m\,\delta q$$
$$\Delta\dot w_{sh} = \dot m\,\delta w_{sh}$$

$$d\left(h + \frac{1}{2}u^2\right) = \delta q - \delta w_{sh}$$

If we define stagnation enthalpy

$h_t = h + \frac{1}{2}u^2$ then

$dh_t = \delta q - \delta w_{sh}$

* Stability of a System

→ measured by Entropy

T_A

S_A	S_B
E_A	E_B

T_B

$(T_B > T_A)$

$\downarrow$

$S_A + \Delta S_A$	$S_B + \Delta S_B$
$E_A + \Delta E_A$	$E_B + \Delta E_B$

$$(E_A + \Delta E_A) + (E_B + \Delta E_B) = E_A + E_B$$

$$\Delta E_A + \Delta E_B = 0$$

$$(S_A + \Delta S_A) + (S_B + \Delta S_B) \geq S_A + S_B$$

$$\Delta S_A + \Delta S_B > 0$$

$$\frac{\partial S}{\partial E} > 0$$

If we take $S = \alpha(E)E$

$$\Delta S_A + \Delta S_B = \alpha_A \Delta E_A + \alpha_B \Delta E_B$$

If $\Delta E_A > 0$

$\alpha_A \geq \alpha_B \quad \Longleftrightarrow \quad T_B > T_A$

$\cancel{A} \cdot \alpha \propto \frac{1}{T}$

$$\Delta S = \frac{\Delta E}{T}$$

$\delta q = T\,dS \rightarrow$ For a rev. process

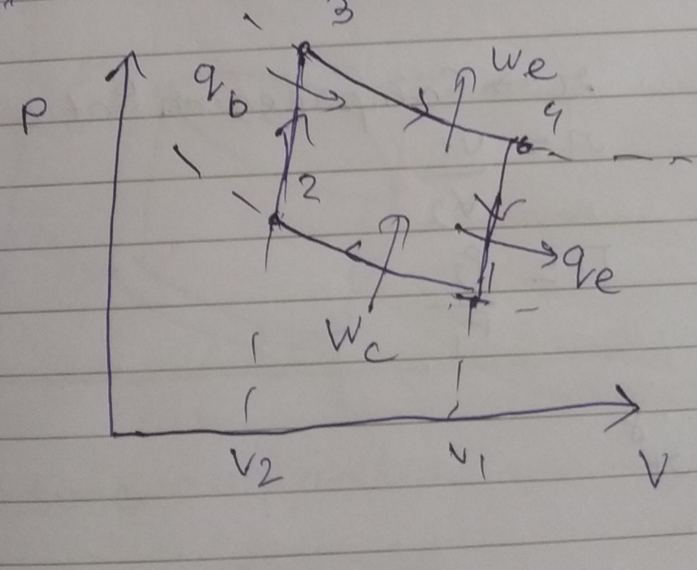
For irrev., $dS > \frac{1}{T}\delta q$

Free Energy $\rightarrow e - TS$ Helmholtz

Free Energy $\rightarrow h - TS$ Gibbs

* Otto Cycle

1. Intake
2. Compression
3. Combustion
4. Expansion
5. Exhaust



Heat added $\rightarrow q_b$

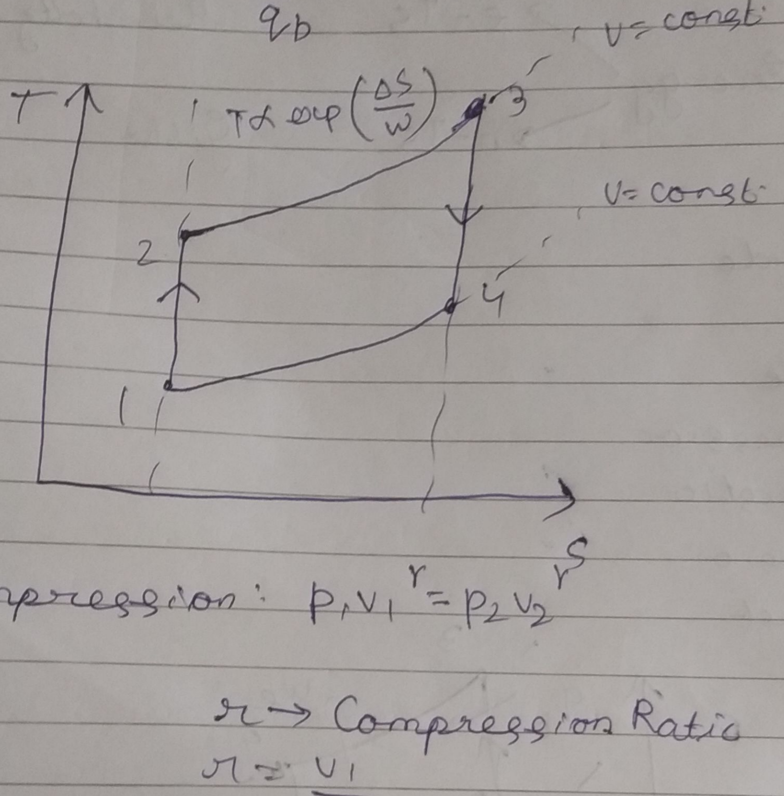
Work $= w_e - w_c$

$$\eta = \frac{w_e - w_c}{q_b}$$

$$q_b = q_e + (w_e - w_c)$$

$$w_e - w_c = q_b - q_e$$

$$\eta_{th} = 1 - \frac{q_e}{q_b}$$



Compression: $p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$

$r \rightarrow$ Compression Ratio

$$r = \frac{v_1}{v_2}$$

$$\Pi = \frac{P_2}{P_1}$$

$$\Pi = r^{\gamma}$$

$$q_b = e_3 - e_2$$
$$= C_v (T_3 - T_2)$$

$q_l = C_v (T_4 - T_1)$

$\eta_{th} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$

$\frac{T_2}{T_1} = \frac{T_3}{T_4}$

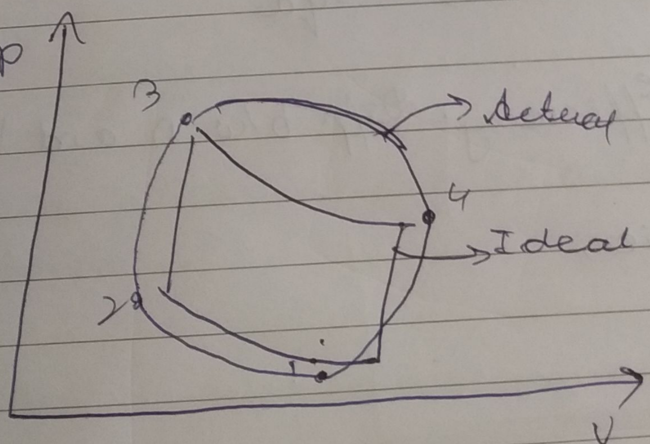
$\downarrow$

$\frac{T_4}{T_1} = \frac{T_3}{T_2}$

$\eta_{th} = 1 - \frac{T_1}{T_2}$

$= 1 - \frac{1}{r^{\gamma - 1}}$

* Actual Cycle



Due to some dissipation, we will get

$T \, ds_d$

$de = T \, ds_d - P dV$

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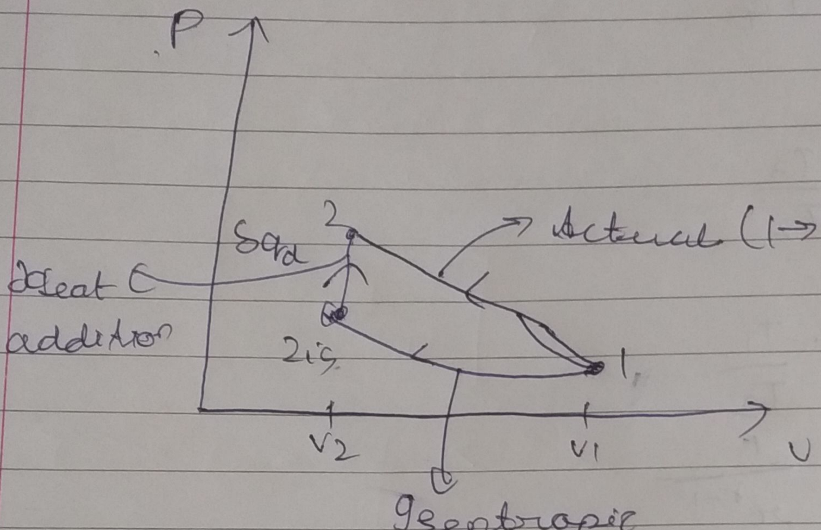
So we can model the actual process $PV^n = \text{const.}$
(Polytropic)

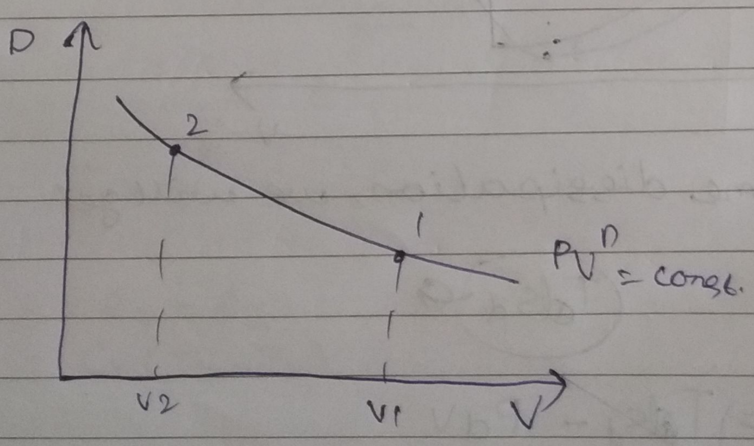
$\frac{\text{Isentropic work}}{\text{Actual work}}$ = Isentropic / Adiabatic Efficiency



Isentropic work = δW_{is}
Actual Work = $\delta W_{is} + \delta q_d$

- Polytropic Efficiency: Diff. b/w n and γ.

For compression,



$$P_2 V_2^n = P_1 V_1^n$$

$$P_2 = P_1 r^n$$

$$W = \int_1^2 P\,dV = e_2 - e_1$$

$$W = C_v [T_2 - T_1] = C_v T_1 [r^{n-1} - 1]$$

Isentropic $\Rightarrow n = \gamma$

$$W_{is} = C_v T_1 [r^{\gamma-1} - 1]$$

$$\eta = \frac{W_{is}}{W} = \frac{r^{\gamma-1} - 1}{r^{n-1} - 1}$$

Polytropic Efficiency $= \frac{\gamma - 1}{n - 1}$

* Piston Engines

Multiple cylinders can be used with the drawbacks of:

→ Heating
→ Noise
→ Space
→ Weight

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* Jet Propulsion

→ Fuel droplet vaporization

$$\tau_v \left[h (T_f - T_d) \pi d^2 \right] \sim L_v \frac{4}{3} \pi r^3 \rho_d$$

(Spherical droplet of radius r, dia d)
L_v = Vaporisation Const.
$h(T_f - T_d)$ = Heat Transfer
τ_v = Time for vaporisation

$$\boxed{\tau_v \sim \frac{L_v d \rho_d}{6 h (T_f - T_d)}}$$

$$\tau_v + \tau_m + \tau_r < \tau_{res}$$

τ_v → Vaporisation Time
τ_m → Mixing Time
τ_r → Rxn. Time
τ_{res} → Residence Time

$$\tau_{res} = \frac{L_c}{V_c}$$

So aircraft engines slow down air to increase τ_{res}.

- Isochoric combustion produces a detonation wave

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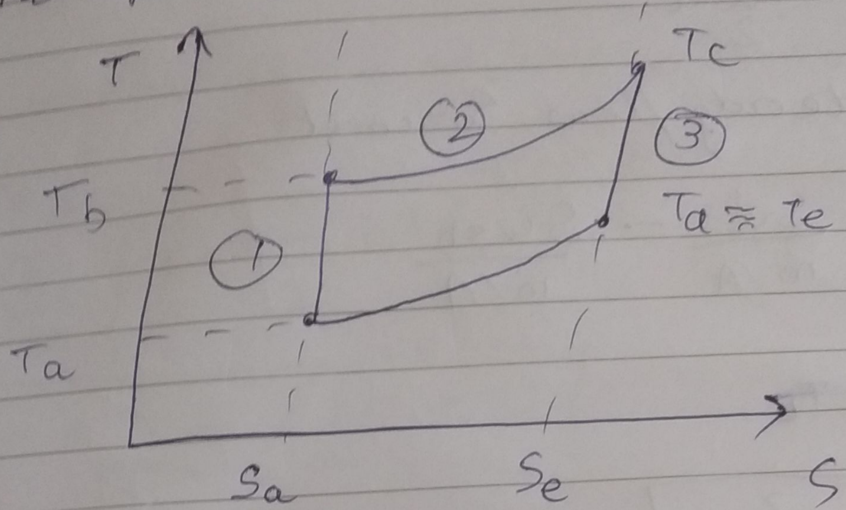
→ Thermodynamic Cycle

1. Isentropic Compression
2. Isobaric Combustion
3. Isentropic Expansion



Brayton Cycle

$\cancel{\frac{\partial}{\partial t}}$

→ Slowing down Flow

$$\frac{\partial}{\partial t}\left[\rho\left(e+\frac{1}{2}u^2\right)\right]+\frac{\partial}{\partial x}\left(\rho u\left(h+\frac{1}{2}u^2\right)\right)=\frac{\partial q_{cond}}{\partial x}+\dot{q}'''-\dot{w}_{sh}'''$$

$u=0$

$$\frac{\partial}{\partial t}(\rho e)=\frac{\partial q_c}{\partial x}+\dot{q}'''-\dot{w}_{sh}'''$$

$$d\dot{e}=\delta\dot{q}-\delta\dot{w}_{sh}\equiv\delta\dot{q}-\delta\dot{w}$$

$u \neq 0$

$$d\left[\rho u\left(h+\frac{1}{2}u^2\right)\right] = \delta\dot{q} - \delta\dot{w}_{sh}$$

$$h_t = h + \frac{1}{2}u^2$$

For steady flow, $\rho u = $ const.

$$dh_t = \frac{\delta\dot{q}}{\dot{m}/A} - \frac{\delta\dot{w}_{sh}}{\dot{m}/A}$$

$$h_t = C_p T_t$$

$$\boxed{T_t - T = \frac{u^2}{2C_p}}$$

$$\frac{P_t}{P} = \left(\frac{T_t}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_t = T\left[1 + \frac{\gamma-1}{2}M^2\right]$$

$$\boxed{\frac{P_f}{P_i} = \left(\frac{T_f}{T_i}\right)^{\frac{\gamma}{\gamma-1}} e^{-\Delta S/R}}$$

$$\frac{dP_t}{P_t} = -\left[1-\frac{T}{T_t}\right]\frac{ds}{R} - \frac{\delta w_{sh}}{RT_t}$$

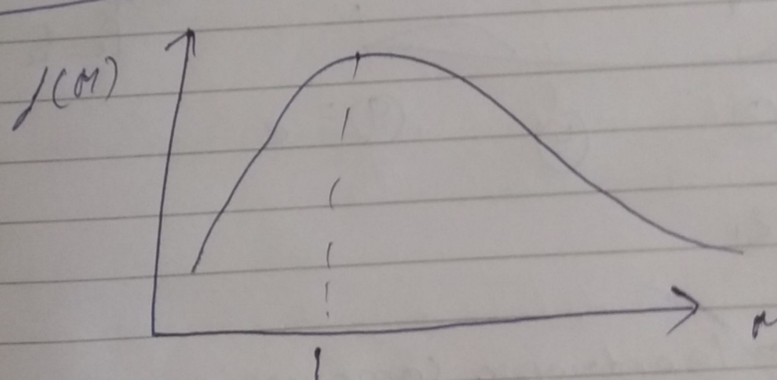
$$1-\frac{T}{T_t} = \frac{u^2}{2c_pT_t} > 0$$

If work is done on flow, $\delta w_{sh} < 0$ $dP_t > 0$

$$\dot{m} = \rho A u$$

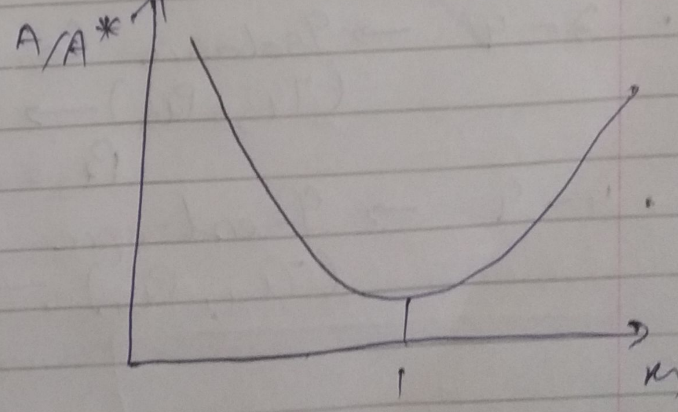
$$= \frac{P_{\cancel{t}}}{\sqrt{T_{\cancel{t}}}}\left[1+\frac{\gamma-1}{2}M^2\right]^{-\frac{(\gamma+1)}{2(\gamma-1)}} M\sqrt{\frac{\gamma}{R}}\,A$$

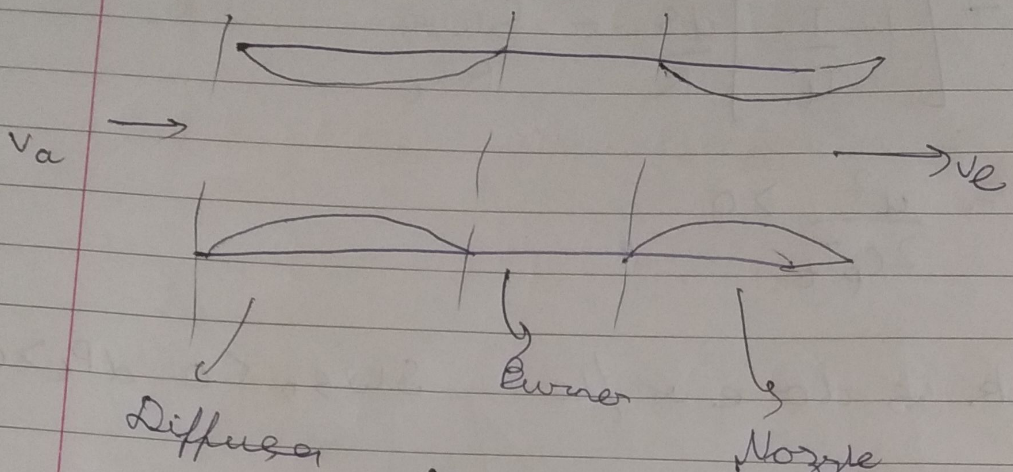
$$\boxed{\dot{m} = \frac{A P_t}{\sqrt{T_t}}\, f(M,\gamma,R)}$$



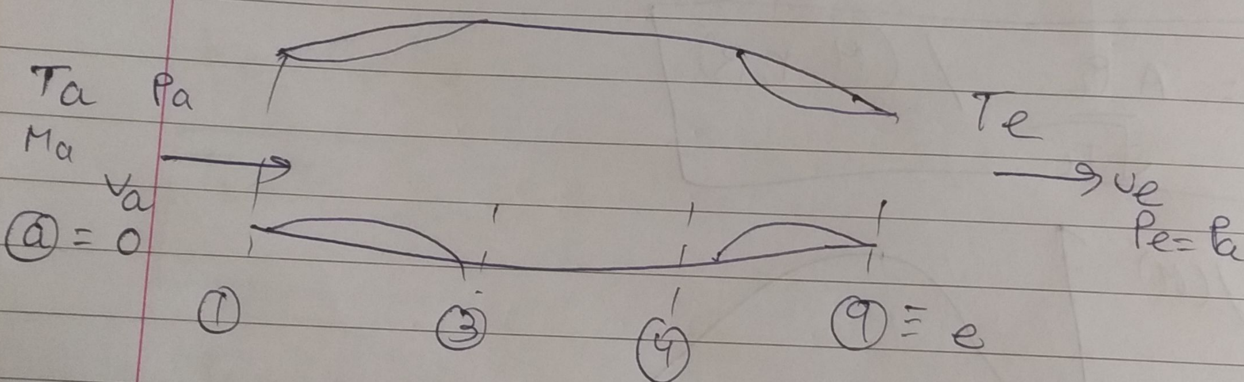
when $M=1$,
$A = A^*$
$f = f^*$

$$\frac{A}{A^*} = \frac{f^*}{f(M)}$$





(Ramjet)



$M_3, M_4 << 1$

- $0 - 3 \rightarrow$ Isentropic compression
  $(T_0, P_0) \rightarrow (T_{t_3}, P_{t_3})$

- $3 - 4 \rightarrow$ Isobaric Combustion
  $(T_{t_3}, P_{t_3}) \rightarrow (T_{t_4}, P_{t_4})$
  $P_{t_3} = P_{t_4}$

- $4 - 9 \rightarrow$ Isentropic Expansion
  $(T_{t_4}, P_{t_4}) \rightarrow (T_9, P_9) \equiv (T_e, P_a)$

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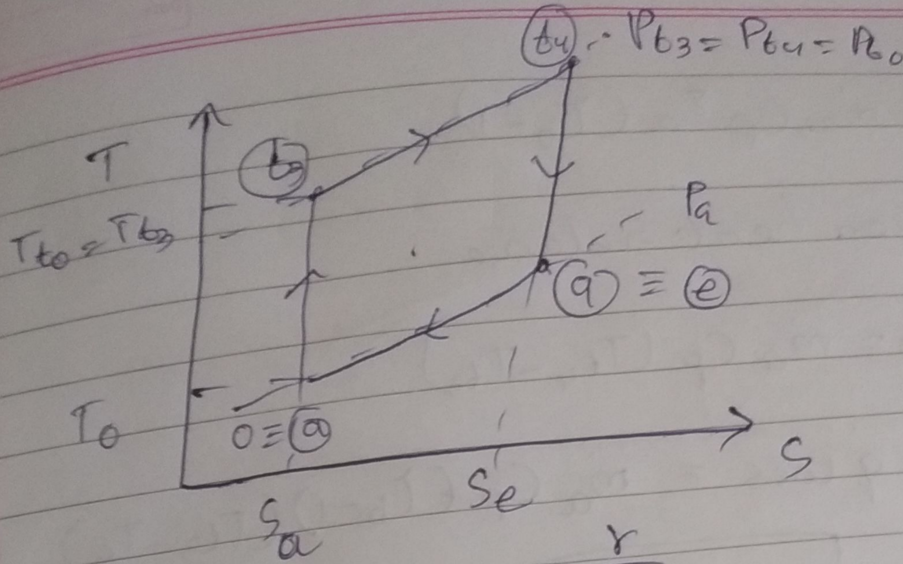

$$\frac{P_{t3}}{P_a} = \left[1 + \frac{\gamma-1}{2} M_a^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{t4}}{P_e} = \left[1 + \frac{\gamma-1}{2} M_e^2\right]^{\frac{\gamma}{\gamma-1}}$$

$M_e = M_a$

$T_{t4} \rightarrow$ Max. Temp.

$$T_e = \frac{T_{t4}}{T_{t3}} T_a \qquad \tau_b = \frac{T_{t4}}{T_{t3}}$$

At burner,

$\dot{m}_f Q_R = \dot{m}_a C_p (T_{t4} - T_{t3})$

$$\frac{f Q_R}{C_p T_{t3}} = \tau_b - 1$$

Thermal energy of jet (Surplus)

$q = \dot{m}_a C_p (T_e - T_a)$

$$q_{loss} = \dot{m}_a C_p T_a (\tau_b - 1)$$

Combustion heat release

$$q_b = \dot{m}_a C_p (T_{t4} - T_{t3})$$

$$W = q_b - q_{loss} = \dot{m}_a C_p (\tau_b - 1)(T_{t3} - T_a)$$

Efficiency $\rightarrow$ $\boxed{\eta = \frac{W}{q_b} = 1 - \frac{T_a}{T_{t3}}}$

Using M we see that η increases with M_a

$$T_{t3} - T_0 = \frac{u_a^2}{2C_p} = \frac{(\gamma-1)}{2} M_a T_a$$

$$\frac{W}{\dot{m}_a C_p T_a} = \frac{T_{t4}}{T_a} - \frac{T_{t3}}{T_a} - \frac{T_e}{T_a} + 1$$

Here T_{t4}, T_a are fixed

$$T_e = \frac{T_{t4}}{T_3} T_a$$

$$\frac{W}{\dot{m}_a C_p T_a} = f(T_{t3})$$

We can differentiate to get max work

$\boxed{T_{t3} = \sqrt{T_a T_{t4}}}$ $\boxed{T_e = T_{t3}}$

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$$\eta_{max} = 1 - \sqrt{\frac{T_a}{T_{t4}}}$$

To improve efficiency → Increase T_{t4}

$$M_a\ (\text{optimum}) = \sqrt{\frac{2}{\gamma-1}\left[\sqrt{\frac{T_{t4}}{T_a}} - 1\right]}$$

$$T = \dot{m}_a (v_e - v_a)$$

$$\boxed{\frac{T}{\dot{m}_a a_0} = M_0 \left[\sqrt{\tau_b} - 1\right]}$$

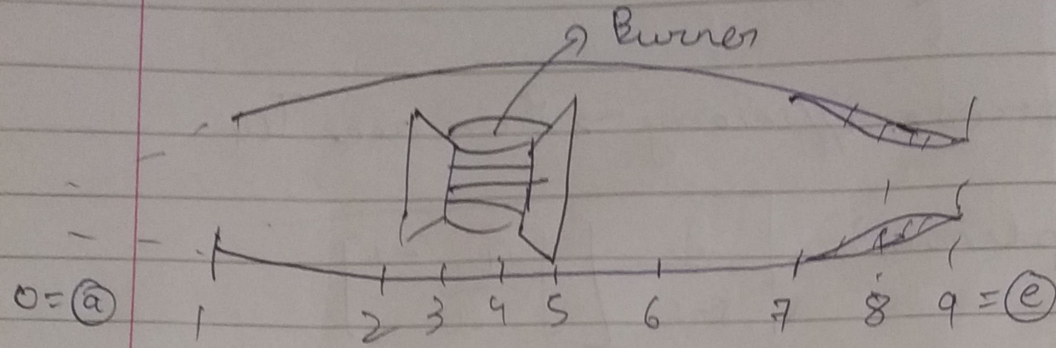
$$TSFC = \frac{1}{v_a(\sqrt{\tau_b} - 1)}$$

$$= \frac{a_0}{\gamma-1} \frac{1 + \frac{\gamma-1}{2} M_0^2}{M_0} (\sqrt{\tau_b} + 1) \frac{1}{Q_R}$$

Since $\eta = 1 - \frac{T_a}{T_{t3}} = 1 - \left(\frac{P_a}{P_{t3}}\right)^{\frac{\gamma-1}{\gamma}}$

We can use compressor to increase P_{t3}.
we also use a turbine and the combustion is used after turbine (also known as afterburner)

* Gas Turbine Cycle



2-3 → Compressor (Adding work)
4-5 → Turbine (extracting work)

$$\dot{W}_c = \dot{m}_a C_p (T_{t_3} - T_{t_2}) = \dot{m}_a C_p T_{t_2} \left[\frac{T_{t_3}}{T_{t_2}} - 1\right]$$
↳ Compressor

$$\dot{W}_t = \dot{m}_a C_p (T_{t_4} - T_{t_5}) = \dot{m}_a C_p T_{t_4} \left[1 - \frac{T_{t_5}}{T_{t_4}}\right]$$

$$\dot{W}_{net} = \dot{W}_t - \dot{W}_c$$

In compressor

$$T_{t_2} = T_2 \left[1 + \frac{\gamma-1}{2} M_2^2\right] \simeq T_{t_0} = T_0 \left[1 + \frac{\gamma-1}{2} M_0^2\right]$$

Compressor increases pressure from P_{t_2} to P_{t_3}

$$\pi_c = \frac{P_{t_3}}{P_{t_2}}$$

Isentropic Compression ⇒ $\dfrac{T_{t_3 is}}{T_{t_2}} = \pi_c^{\frac{\gamma-1}{\gamma}}$

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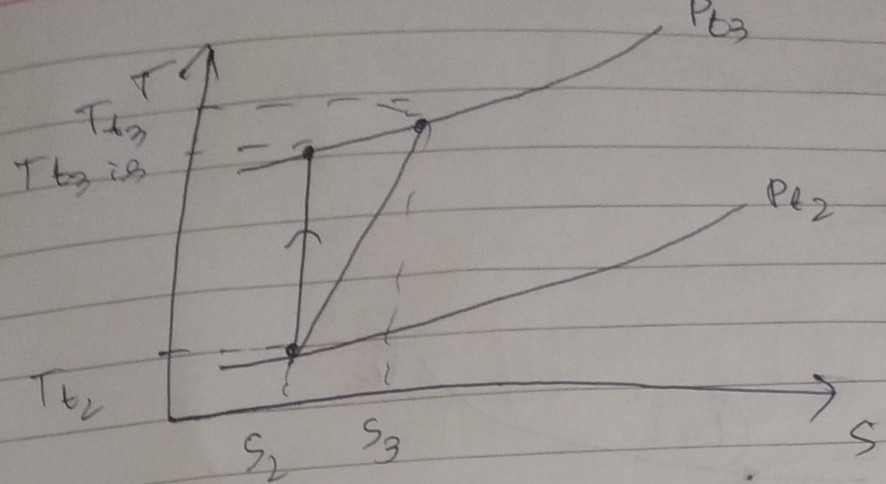


$$h_{t3} - h_{t2} = (h_{t3\,is} - h_{t2}) + \Delta h_{dissip}$$

$$h_{t3} = h_{t3\,is} + \Delta h_{diss}$$

Compressor Efficiency

$$\eta_c = \frac{\text{Ideal Work input}}{\text{Actual \quad ''}}$$

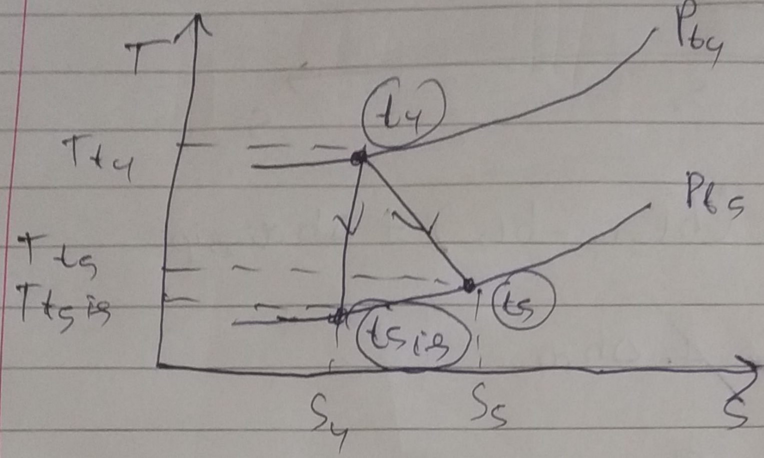
$$= \frac{h_{t3\,is} - h_{t2}}{h_{t3} - h_{t2}}$$

$$= \frac{\frac{T_{t3\,is}}{T_{t2}} - 1}{\frac{T_{t3}}{T_{t2}} - 1}$$

$$\boxed{\frac{T_{t3}}{T_{t2}} = 1 + \frac{1}{\eta_c}\left[\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

* In turbine,

$$\Pi_t = \frac{P_{t4}}{P_{t5}}$$



$$h_{t5} - h_{t4} = (h_{t5 is} - h_{t4}) + \Delta h_{diss}$$

$$\frac{T_{t5 is}}{T_{t4}} = \frac{1}{\Pi_t^{\gamma-1/\gamma}}$$

$$\eta_t = \frac{\text{Actual work extracted}}{\text{Ideal work extracted}} = \frac{h_{t4} - h_{t5}}{h_{t4} - h_{t5 is}}$$

$$\eta_t = \frac{1 - \frac{T_{t5}}{T_{t4}}}{1 - \Pi_t^{-\frac{\gamma-1}{\gamma}}}$$

$$\frac{T_{t5}}{T_{t4}} = 1 - \eta_t \left[1 - \Pi_t^{-\frac{\gamma-1}{\gamma}} \right]$$

$$\frac{\dot{W}_{net}}{\dot{m}_a C_p T_{t_2}} = \frac{T_{t_4}}{T_{t_2}}\left[1 - \frac{T_{t_5}}{T_{t_4}}\right] - \left[\frac{T_{t_3}}{T_{t_2}} - 1\right]$$

$$= \frac{T_{t_4}}{T_{t_2}} \eta_t \left[1 - \pi_t^{-\frac{\gamma-1}{\gamma}}\right] - \frac{1}{\eta_c}\left[\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\dot{q}_b = \dot{m}_a C_p (T_{t_4} - T_{t_3}) = \dot{m}_f Q_R = \dot{m}_a f Q_R$$

$$\Omega = \frac{\dot{W}_{net}}{\dot{m}_a C_p T_{t_2}}$$

$\frac{d\Omega}{d\pi_c} = 0$ (For work output max) (For $\pi_t = \pi_c$

$\cancel{\pi_c^{2\gamma} = T_{t_4}}$

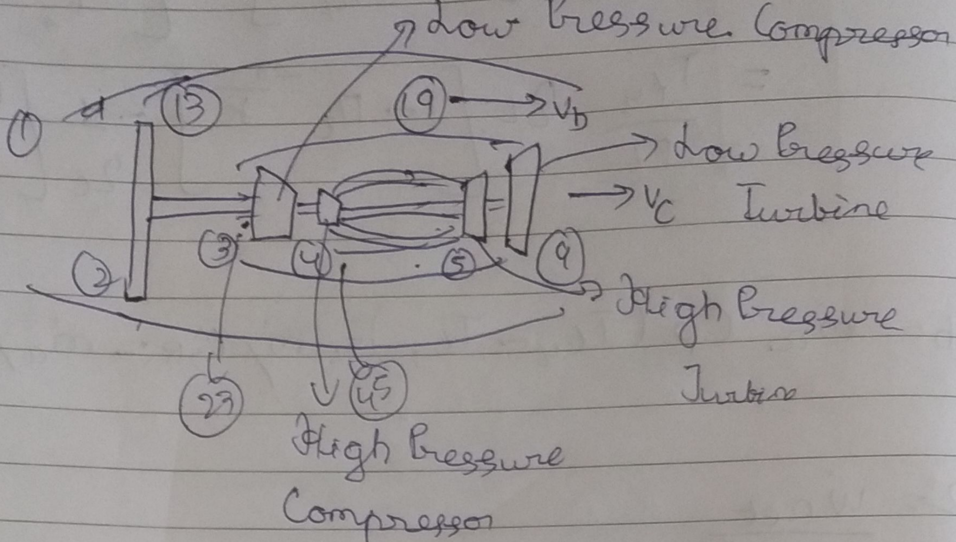
$$\pi_c = \left[\frac{T_{t_4}}{T_{t_2}} \eta_c \eta_t\right]^{\frac{\gamma-1}{2\gamma}} \quad \text{(Optimum)}$$

* Efficiency

$$\eta = \frac{\text{Net Work}}{\text{Heat}} = \frac{\dot{W}_{net}}{\dot{q}_b}$$

$$\boxed{\eta_{cycle} = 1 - \frac{T_{t_5} - T_{t_2}}{T_{t_4} - T_{t_3}}}$$

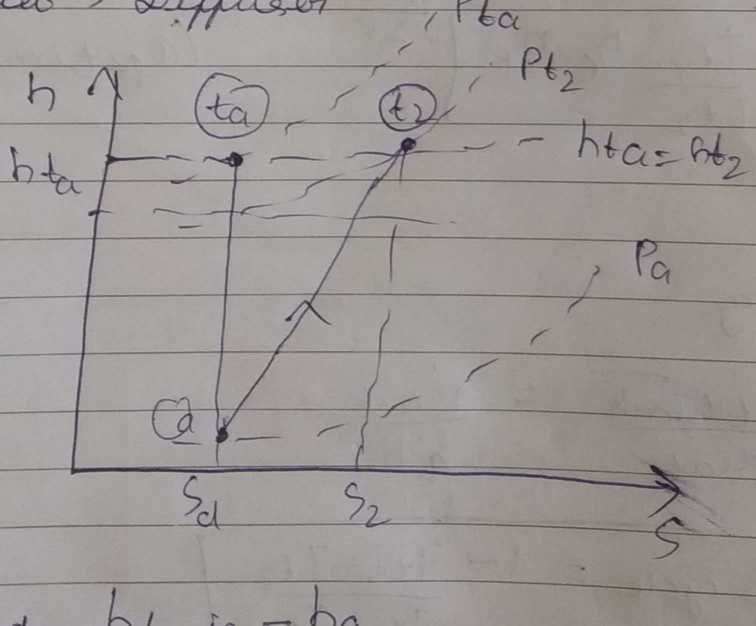
* 2-Spool Engine



$$\beta = \frac{v_b}{v_c}$$

$$\pi_f = \frac{P_{t_{13}}}{P_{t_2}}$$ (Pressure ratio + fan)

* Inlet → Diffuser



$$\eta_d = \frac{h_{t_2 is} - h_a}{h_{t_2} - h_a}$$

$h_{t2} = h_a + \frac{1}{2} a^2$

$= h_a \left[1 + \frac{\gamma - 1}{2} M_a^2 \right]$

$\frac{h_{t2}}{h_a} - 1 = \frac{\gamma - 1}{2} M_a^2$

$\frac{h_{t2\,is}}{h_a} = \left(\frac{T_{t2}}{T_a}\right)_{is} = \left(\frac{P_{t2}}{P_a}\right)^{\frac{\gamma-1}{\gamma}}$

$\eta_d = \frac{\left(\frac{P_{t2}}{P_a}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\gamma-1}{2} M_a^2}$

$\frac{P_{t2}}{P_a} = \left[1 + \eta_d \frac{\gamma-1}{2} M_a^2 \right]^{\frac{\gamma-1}{\gamma}}$

For $\eta_d = 1$ we get the isentropic relatio

$\pi_d = \frac{P_{t2}}{P_{ta}} = \left[\frac{1 + \eta_d \frac{\gamma-1}{2} M_a^2}{1 + \frac{\gamma-1}{2} M_a^2} \right]^{\frac{\gamma}{\gamma-1}}$

$T = \dot{m}_c [v_c - v_a] + \underbrace{\alpha \dot{m}_c}_{\dot{m}_b} [\underbrace{\beta v_c}_{v_b} - v_a]$

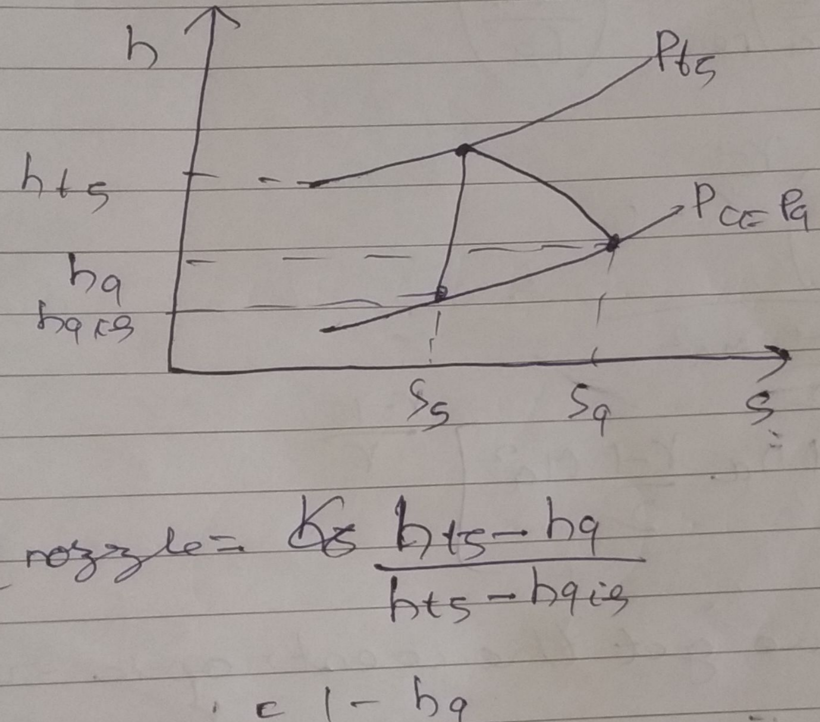
$h_{ta} = h_a + \frac{1}{2} v_a^2 = h_a + \frac{1}{2} v_c^2$

$$V_c = \left[2h_{t9}\left(1-\left(\frac{P_9}{P_{t9}}\right)^{\frac{\gamma-1}{\gamma}}\right)\right]^{1/2}$$

$P_9 = P_a$

$h_{t9} = h_{t5}$

From 5-9, we have a nozzle



$$\eta_{nozzle} = \frac{h_{t5} - h_9}{h_{t5} - h_{9is}}$$

$$= \frac{1 - \frac{h_9}{h_{t5}}}{1 - \frac{h_{9is}}{h_{t5}}}$$

$$\frac{h_{9is}}{h_{t5}} = \frac{h_{9is}}{h_{t9}} = \left(\frac{P_9}{P_{t5}}\right)^{\frac{\gamma-1}{\gamma}}$$

NPR (Nozzle Pressure Ratio) $= \frac{P_{t5}}{P_9}$

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$$\frac{h_9}{h_{t9}} = \frac{\left(P_{t5}/P_{t9}\right)^{\frac{\gamma-1}{\gamma}}}{(NPR)^{\frac{\gamma-1}{\gamma}}} = \frac{h_9}{h_{t5}} = \left(\frac{\Pi_9}{NPR}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\eta_n = \frac{1 - \frac{h_9}{h_{t5}}}{1 - \frac{h_{9\,is}}{h_{t5}}} = \frac{1 - \left(\frac{\Pi_n}{NPR}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{1}{NPR}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$\boxed{\eta_n = \frac{(NPR)^{\frac{\gamma-1}{\gamma}} - \Pi_n^{\frac{\gamma-1}{\gamma}}}{(NPR)^{\frac{\gamma-1}{\gamma}} - 1}}$$

For HP compressor and HP Turbine

$$\dot m_c (h_{t3} - h_{t23}) = \dot m_c (h_{t4} - h_{45})$$

$$\boxed{h_{t3} - h_{t23} = h_{t4} - h_{t45}}$$

For bypass,

$$\dot m_b (h_{t13} - h_{t2}) = \alpha \dot m_c (h_{t13} - h_{t2})$$

~~For ...~~

For LPC

$$\dot m_c \{(h_{t23} - h_{t2}) + \alpha (h_{t13} - h_{t2})\} = \dot m_c \underbrace{(1+f)}_{\approx 1} \{h_{t45} - h_{t5}\}$$

$$\boxed{(h_{t23} - h_{t2}) + \alpha (h_{t13} - h_{t2}) = (h_{t45} - h_{t5})}$$

For burner,

$$\dot{m}_f Q_R = \dot{m}_c (1+f) h_{t4} - \dot{m}_c h_{t3}$$

$$\boxed{f Q_R = (1+f) h_{t4} - h_{t3} \approx h_{t4} - h_{t3}}$$

$$h_{t5} = h_9 + \frac{1}{2} v_c^2$$

$$h_{t13} = h_{19} + \frac{1}{2} v_b^2$$

$$h_{t3} - h_{t2} + \alpha \left[h_{19} + \frac{1}{2} v_b^2 - h_{t2} \right] = h_{t3} + f Q_R - h_9 - \frac{1}{2} v_c^2$$

$$f Q_R = \left(\frac{1}{2} v_c^2 + \frac{1}{2} \alpha v_b^2 \right) + (\alpha h_{19} + h_9) - (1+\alpha) h_{t2}$$

$$h_{t2} = h_a + \frac{1}{2} v_a^2$$

$$\boxed{f Q_R = \underbrace{\frac{1}{2}(v_c^2 - v_a^2) + \frac{1}{2} \alpha (v_b^2 - v_a^2)}_{\text{Jet KE wrt freestream}} + \underbrace{(\alpha h_{19} + h_9)}_{\text{Jet thermal energy}}}$$

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$$\eta_{th} = 1 - \frac{(\alpha h_{1a} + h_a)}{f Q_R}$$

$$\frac{\gamma}{\gamma-1} \frac{dT_t}{T_t} = \frac{ds}{R} + \frac{dP_t}{P_t} = \frac{1}{e_c} \frac{dP_t}{P_t} \geq \frac{dP_t}{P_t}$$

$e_c \rightarrow$ Polytropic Efficiency

$$\frac{\gamma}{\gamma-1} e_c = \frac{n}{n-1}$$

$$P_t \propto T_t^{\frac{\gamma e_c}{\gamma-1}}$$

$$\eta_c = \frac{\pi_c^{K_{is}} - 1}{\pi_c^{K_c} - 1}$$

$$K_{is} = \frac{\gamma-1}{\gamma} \qquad K_c = \frac{\gamma-1}{\gamma e_c}$$

$e_c \rightarrow$ Polytropic efficiency
$\eta_c \rightarrow$ Isentropic efficiency

For Turbine,

$$\frac{\gamma}{\gamma-1} \frac{dT_t}{T_t} = \frac{ds}{R} + \frac{dP_t}{P_t} = e_t \frac{dP_t}{dt}$$

$$K_t = \frac{(\gamma-1)e_t}{\gamma}$$

$$\eta_t = \frac{\pi_t^{k_t} - 1}{\pi_t^{k_r} - 1}$$