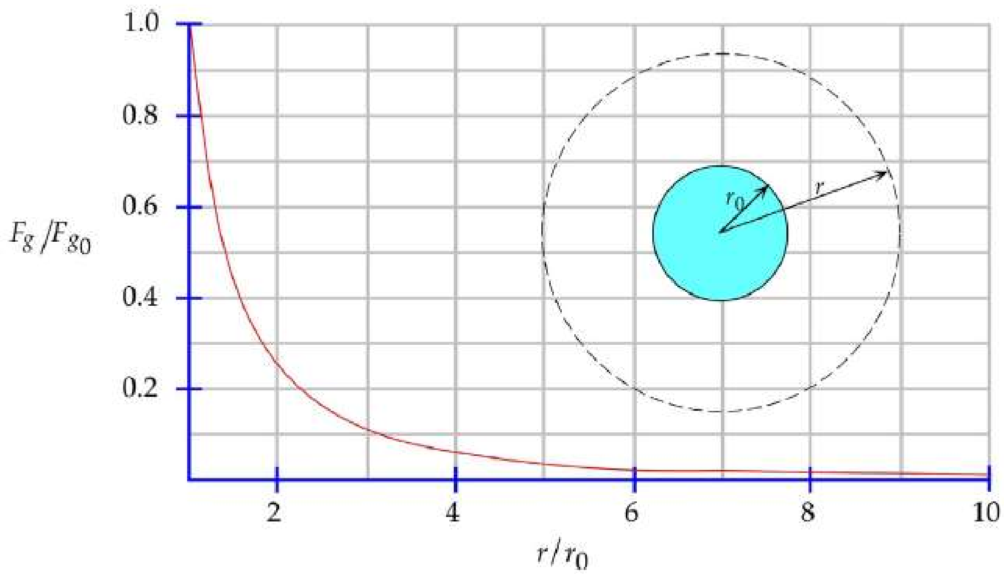
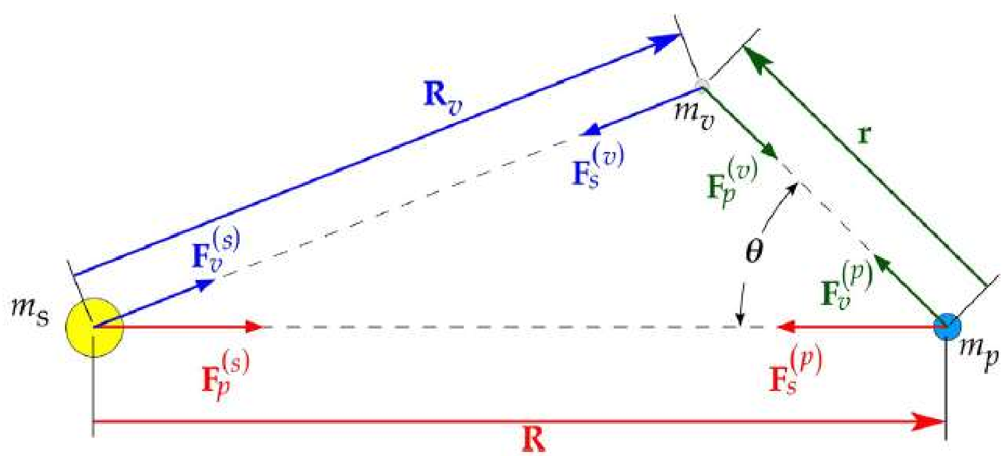
Sphere of Influence





- $F_p^{(v)} = \dfrac{-Gm_v m_p}{\|r\|^3}\, r$

$$F_s^{(v)} = \frac{-Gm_v m_s}{\|R_v\|^3}\, R_v$$

$$F_s^{(p)} = -\frac{G m_p m_s}{\|R\|^3} R$$

- $R_v = R + r$

- $$\|R_v\| = \left(\|R\|^2 + \|r\|^2 - 2\|R\|\|r\| \cos\theta\right)^{1/2}$$
$$= \|R\| \left[1 - 2\left(\|r\|/\|R\|\right)\cos\theta + \left(\|r\|/\|R\|\right)^2\right]^{1/2}$$

- $\|R_v\| = \|R\|$

- $m_v \ddot{R}_v = F_s^{(v)} + F_p^{(v)}$

- $$\ddot{R}_v = \underbrace{-\frac{G m_s}{\|R_v\|^3} R_v}_{A_s} \underbrace{- \frac{G m_p}{\|r\|^3} r}_{P_p}$$

- $$\frac{\|P_p\|}{\|A_s\|} = \frac{m_p}{m_s} \left(\frac{\|R\|}{\|r\|}\right)^2$$

- $m_p \ddot{R} = F_v^{(p)} + F_s^{(p)}$ $\left(F_v^{(p)} = -F_p^{(v)}\right)$

- $$\ddot{R} = \frac{G m_v}{\|r\|^3} r - \frac{G m_s}{\|R\|^3} R$$

- $$\ddot{R}_v - \ddot{R} = -\frac{G m_p}{\|r\|^3} r \left(1 + \frac{m_v}{m_p}\right) - \frac{G m_s}{\|R_v\|^3} \left[R_v - \left(\frac{\|R_v\|}{\|R\|}\right)^3 R\right]$$

r

$$r + \left[1 - \left(\frac{\|R_v\|}{\|R\|} \right)^3 \right] R$$

- $\ddot{r} = a_p + p_s$, $a_p = -\frac{G m_p}{\|r\|^3} r$, $p_s = -\frac{G m_s}{\|R\|^3} r$

- $\frac{\|p_s\|}{\|a_p\|} = \frac{m_s}{m_p} \left(\frac{\|r\|}{\|R\|} \right)^3$

- $\frac{\|p_s\|}{\|a_p\|} < \frac{\|P_p\|}{\|A_s\|}$

- $\frac{\|r\|}{\|R\|} < \left(\frac{m_p}{m_s} \right)^{2/5}$

- $\frac{r_{SOI}}{\|R\|} = \left(\frac{m_p}{m_s} \right)^{2/5}$

Example

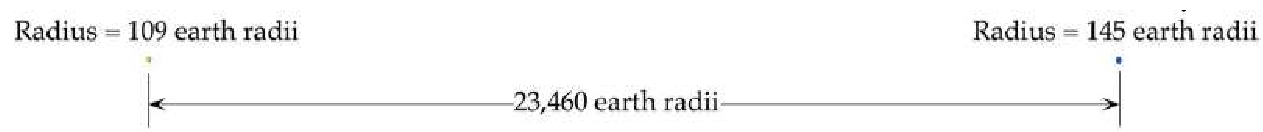
Calculate the radius of the earth's sphere of influence.

Details

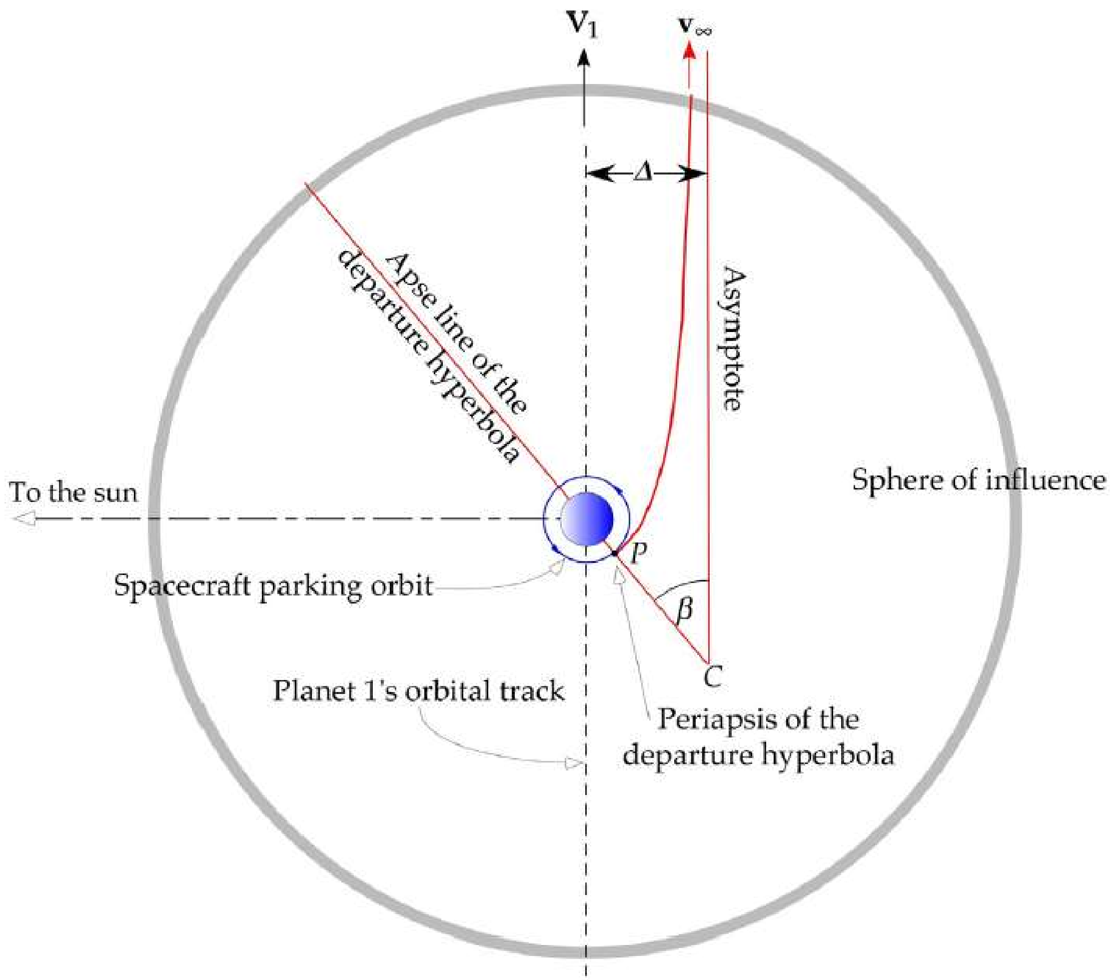
$$r_{SOI} = R_{Earth} \left(\frac{m_{Earth}}{m_{Sun}} \right)^{2/5}$$

Sun

Earth's SOI



Planetary Departure



- $V_\infty = \sqrt{\frac{\mu_{sun}}{R_1}}\left(\sqrt{\frac{2R_2}{R_1+R_2}} - 1\right)$

- $r_p = \frac{\|h\|^2}{\mu_1}\frac{1}{1+\|e\|}$

- $\|h\| = \frac{\mu_1}{V_\infty} \sqrt{\|e\|^2 - 1}$

- $\|e\| = 1 + \frac{r_p V_\infty^2}{\mu_1}$

- $\|h\| = r_p \sqrt{V_\infty^2 + \frac{2\mu_1}{r_p}}$

- $V_p = \frac{\|h\|}{r_p} = \sqrt{V_\infty^2 + \frac{2\mu_1}{r_p}}$

- $V_c = \sqrt{\frac{\mu_1}{r_p}}$

- $\Delta V = V_p - V_c = V_c \left(\sqrt{2 + \left(\frac{V_\infty}{V_c}\right)^2} - 1 \right)$

- $\beta = \cos^{-1}\left(\frac{1}{\|e\|}\right) = \cos^{-1}\left(\frac{1}{1 + \frac{r_p V_\infty^2}{\mu_1}}\right)$