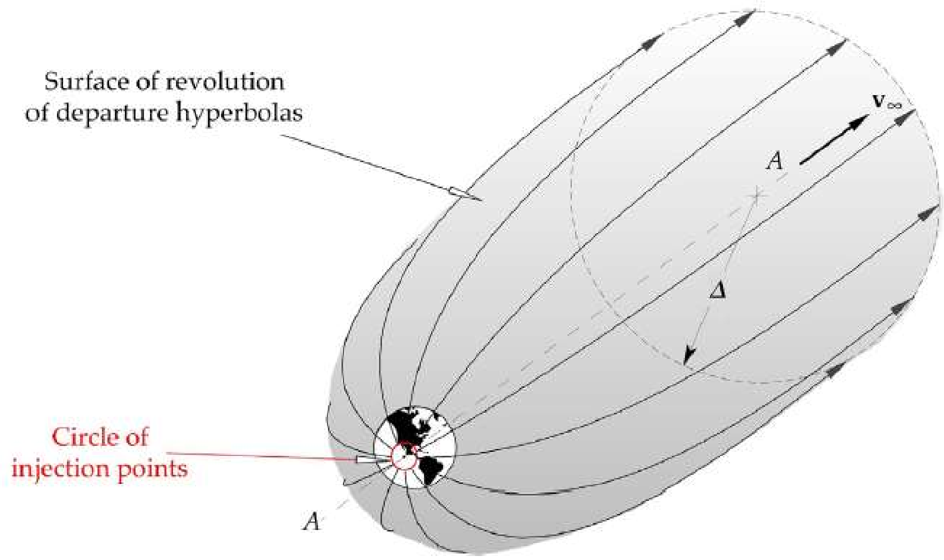
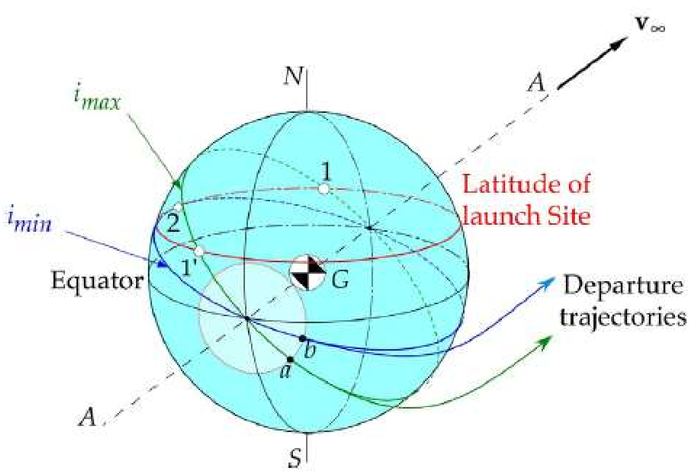
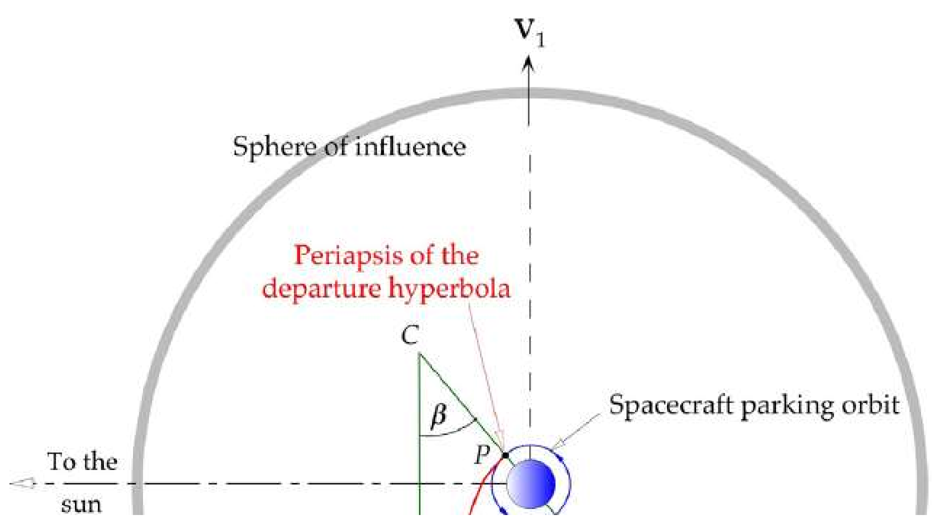
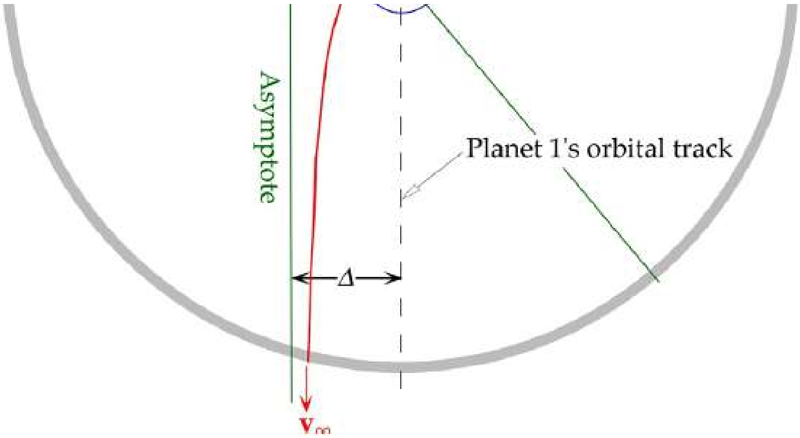

Surface of revolution
of departure hyperbolas
$\mathbf{v}_\infty$
A
Δ
Circle of
injection points
A


$\mathbf{v}_\infty$
N
A
i_{max}
1
Latitude of
launch Site
i_{min}
2
1'
Equator
G
Departure
trajectories
b
a
A
S


$\mathbf{V}_1$
Sphere of influence
Periapsis of the
departure hyperbola
C
β
Spacecraft parking orbit
P
To the
sun



Example

A spacecraft is launched on a mission to Mars starting from a 300-km circular parking orbit. Calculate (a) the delta-v required, (b) the location of perigee of the departure hyperbola, and (c) the amount of propellant required as a percentage of the spacecraft mass before the delta-v burn, assuming a specific impulse of 300 s.

Details

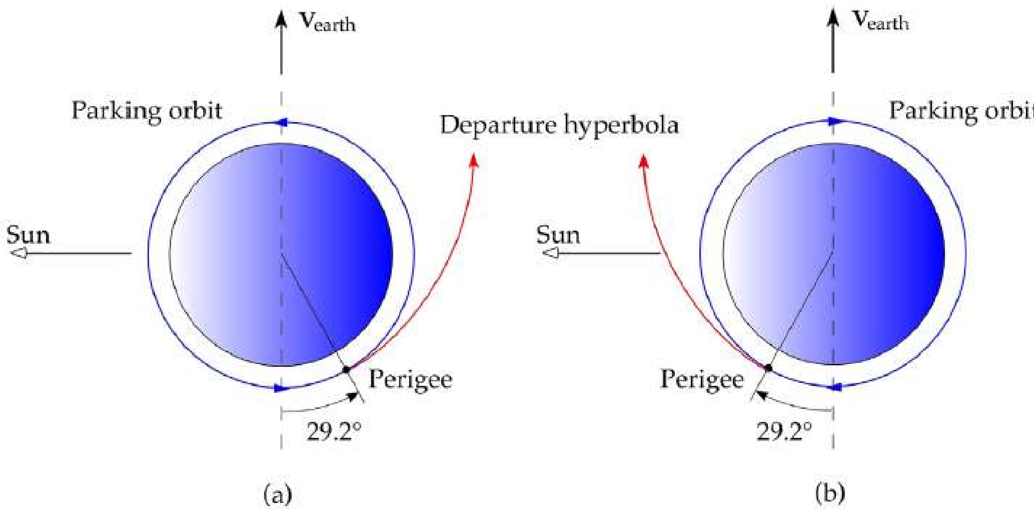
(a) $$V_\infty = \sqrt{\frac{\mu_{Sun}}{R_{Earth}}}\left(\sqrt{\frac{2R_{Mars}}{R_{Earth}+R_{Mars}}} - 1\right)$$

$$V_c = \sqrt{\frac{\mu_{Earth}}{r_{Earth}+300}}$$

$$\Delta V = V_c\left(\sqrt{2+\left(\frac{V_\infty}{V_c}\right)^2} - 1\right)$$

(b) $$\beta = \cos^{-1}\left(\frac{1}{1+\frac{r_p V_\infty^2}{\mu_{Earth}}}\right)$$

(c) $$\frac{\Delta m}{m} = 1 - e^{\frac{-\Delta v}{I_{sp} g_0}}$$



Sensitivity Analysis

- $$R_2 = \frac{\|h\|^2}{\mu_{sun}} \frac{1}{1-\|e\|}$$

- $$\|h\| = R_1 V_D^{(v)}, \quad \|e\| = \frac{R_2 - R_1}{R_2 + R_1}$$

- $$R_2 = \frac{R_1^2 (V_D^{(v)})^2}{2\mu_{sun} - R_1 (V_D^{(v)})^2}$$

- $$\delta R_2 = \frac{dR_2}{dV_D^{(v)}} \delta V_D^{(v)}$$
$$= \frac{4 R_1^2 \mu_{sun}}{[2\mu_{sun} - R_1 (V_D^{(v)})^2]^2} V_D^{(v)} \delta V_D^{(v)}$$

- $$\frac{\delta R_2}{R_2} = \frac{2}{1 - \frac{R_1 (V_D^{(v)})^2}{2\mu_{sun}}} \frac{\delta V_D^{(v)}}{V_D^{(v)}}$$

- $V_D^{(v)} = V_1 + V_\infty$

$$= V_1 + \sqrt{V_p^2 - \frac{2\mu_1}{r_p}}$$

- $\delta V_D^{(v)} = \dfrac{\partial V_D^{(v)}}{\partial r_p}\delta r_p + \dfrac{\partial V_D^{(v)}}{\partial V_p}\delta V_p$

$$= \frac{\mu_1}{V_\infty r_p^2}\delta r_p + \frac{V_p}{V_\infty}\delta V_p$$

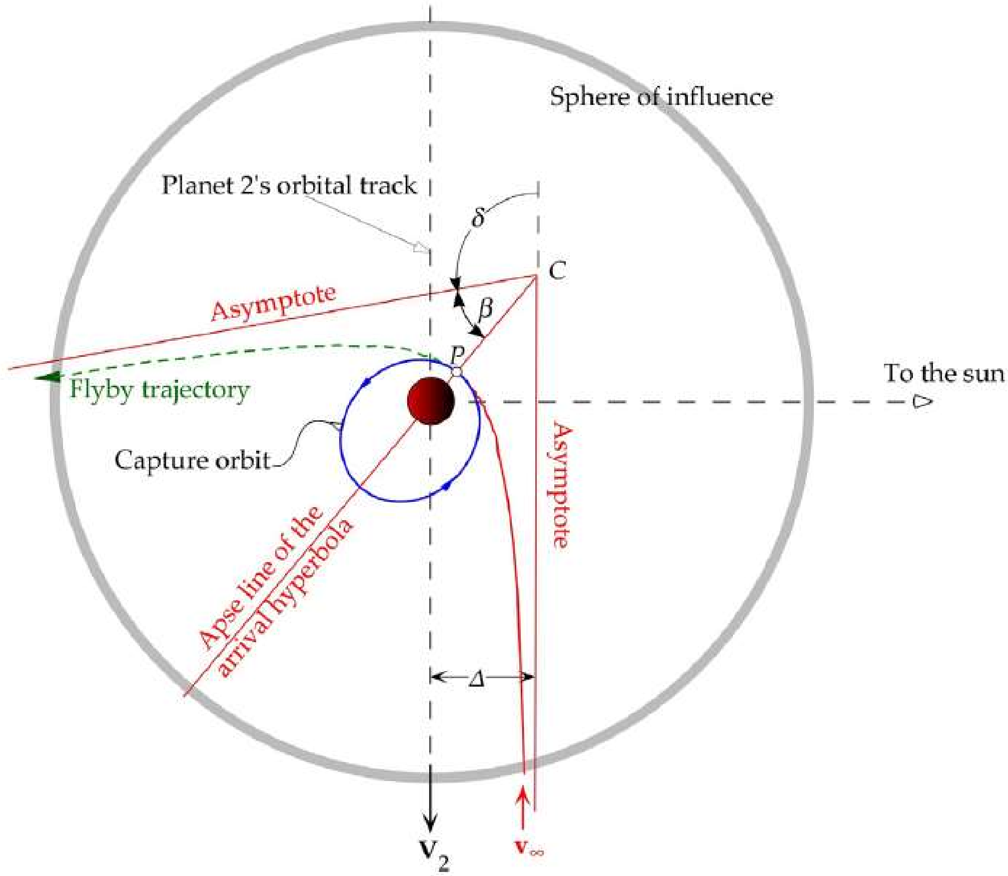
- $\dfrac{\delta V_D^{(v)}}{V_D^{(v)}} = \dfrac{\mu_1}{V_D^{(v)} V_\infty r_p}\dfrac{\delta r_p}{r_p} + \dfrac{V_\infty + \dfrac{2\mu_1}{r_p V_\infty}}{V_D^{(v)}}\dfrac{\delta V_p}{V_p}$

- $\dfrac{\delta R_2}{R_2} = \dfrac{2}{1 - \dfrac{R_1 (V_D^{(v)})^2}{2\mu_{sun}}}\left(\dfrac{\mu_1}{V_D^{(v)} V_\infty r_p}\dfrac{\delta r_p}{r_p} + \dfrac{V_\infty + \dfrac{2\mu_1}{r_p V_\infty}}{V_D^{(v)}}\dfrac{\delta V_p}{V_p}\right)$

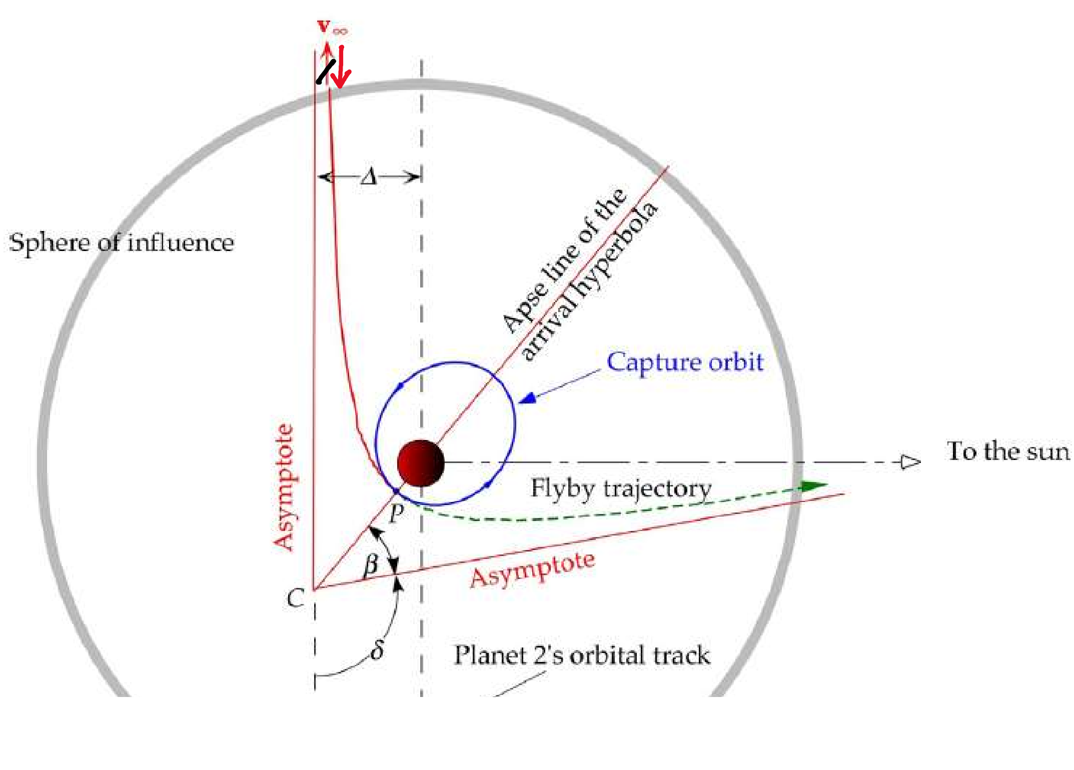
$$\mu_{sun} = 1.327(10^{11})\ \text{km}^3/\text{s}^2$$
$$\mu_1 = \mu_{earth} = 398{,}600\ \text{km}^3/\text{s}^2$$
$$R_1 = 149.6(10^6)\ \text{km}$$
$$R_2 = 227.9(10^6)\ \text{km}$$
$$r_p = 6678\ \text{km}$$

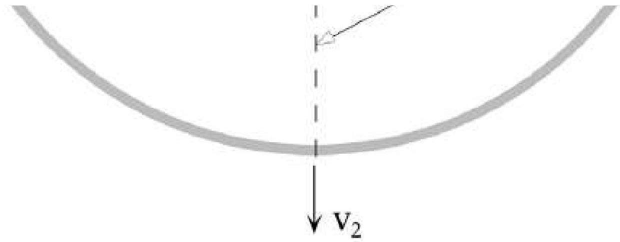
- $\dfrac{\delta R_2}{R_2} = 3.127\dfrac{\delta r_p}{r_p} + 6.708\dfrac{\delta V_p}{V_p}$

Planetary Rendezvous



$$-V_\infty = V_2 - V_A^{(v)}$$



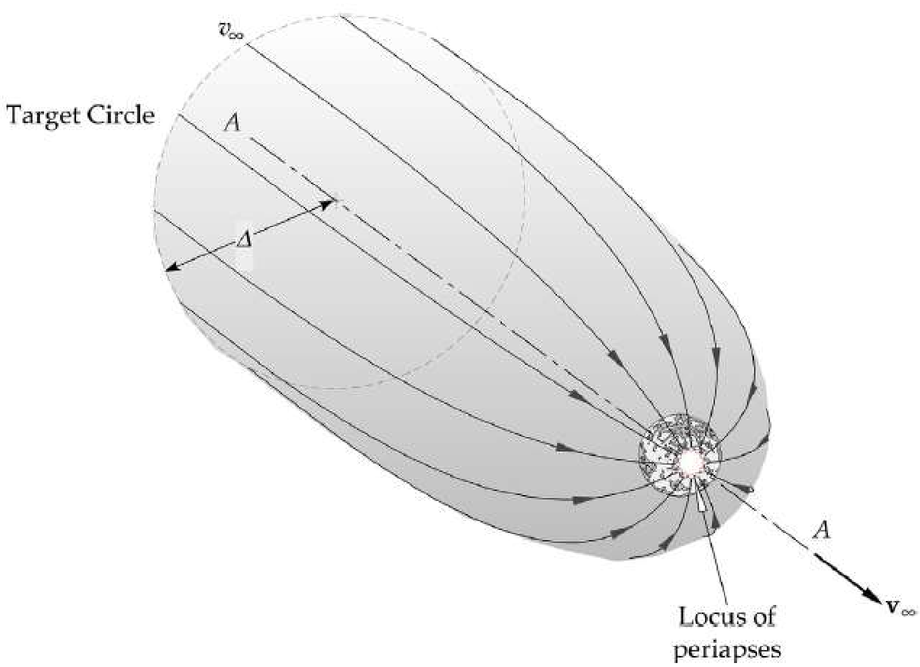


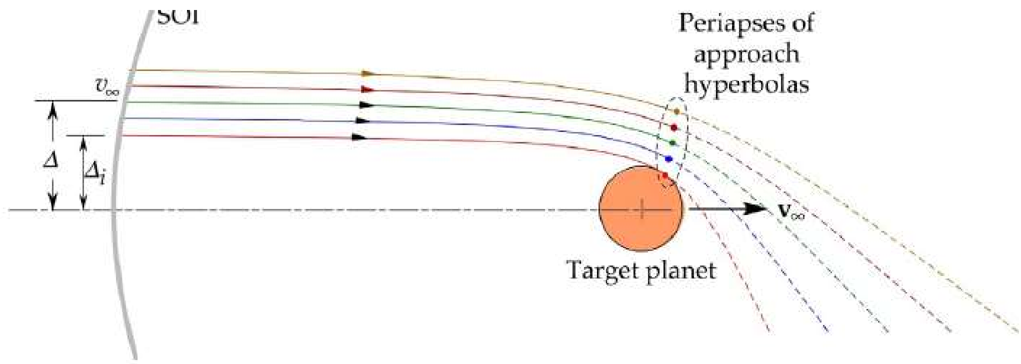
- $V_\infty = V_A^{(v)} - V_2$

- $\delta = 2\sin^{-1}\left(\dfrac{1}{1+\dfrac{r_p V_\infty^2}{\mu_2}}\right)$

- $\Delta = \dfrac{\|h\|^2}{\mu_2}\dfrac{1}{\sqrt{\|e\|^2-1}}$

$$= r_p\sqrt{1+\frac{2\mu_2}{r_p v_\infty^2}}$$





- $V_p)_{Hyperbola} = \sqrt{V_\infty^2 + \dfrac{2\mu_2}{r_p}}$

- $V_p)_{capture} = \sqrt{\dfrac{\mu_2(1+\|e\|)}{r_p}}$

- $\Delta V = V_p)_{Hyperbola} - V_p)_{capture}$

- $\dfrac{\Delta V}{V_\infty} = \sqrt{1+\dfrac{2}{\xi}} - \sqrt{\dfrac{1+\|e\|}{\xi}}$, $\xi = \dfrac{r_p V_\infty^2}{\mu_2}$

- $\dfrac{d}{d\xi}\dfrac{\Delta V}{V_\infty} = 0 \Rightarrow \xi = 2\,\dfrac{1-\|e\|}{1+\|e\|}$

- $\dfrac{d^2}{d\xi^2}\dfrac{\Delta V}{V_\infty} = \dfrac{\sqrt{2}}{64}\,\dfrac{(1+\|e\|)^3}{(1-\|e\|)^{3/2}}$

- $r_p = \dfrac{2\mu_2}{V_\infty^2}\,\dfrac{1-\|e\|}{1+\|e\|}$

- $\dfrac{r_p}{r_a} = \dfrac{1-\|e\|}{1+\|e\|}$

- $r_a = 2\mu_2$

$$\overline{\quad V_\infty^2 \quad}$$

$$- \Delta V = V_\infty \sqrt{\frac{1-\|e\|}{2}}$$

$$- \Delta = \sqrt{\frac{2}{1-\|e\|}}\, r_p$$

Example

After a Hohmann transfer from earth to Mars, calculate

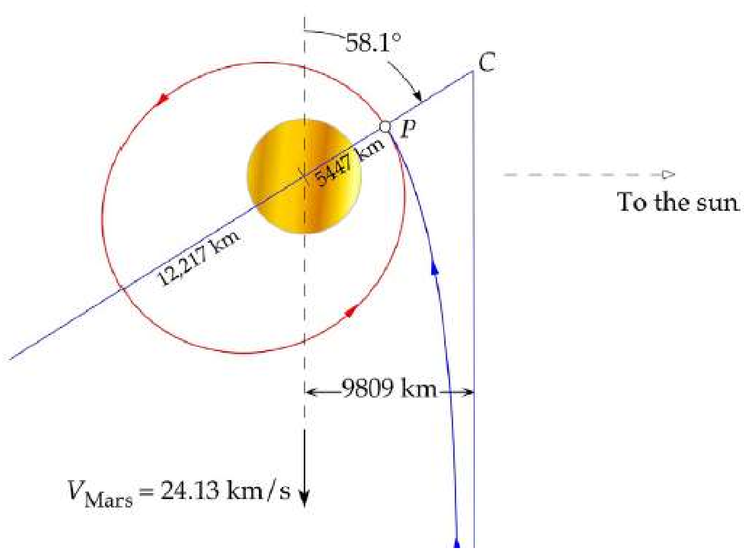
(a) the minimum delta-v required to place a spacecraft in orbit with a period of 7 h

(b) the periapsis radius

(c) the aiming radius

(d) the angle between periapsis and Mars' velocity vector.

Details



$$v_\infty = 2.648\ \text{km/s}$$