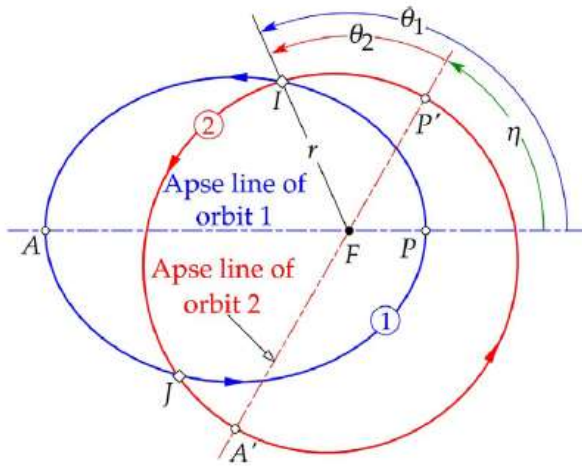


Apside Line Rotation

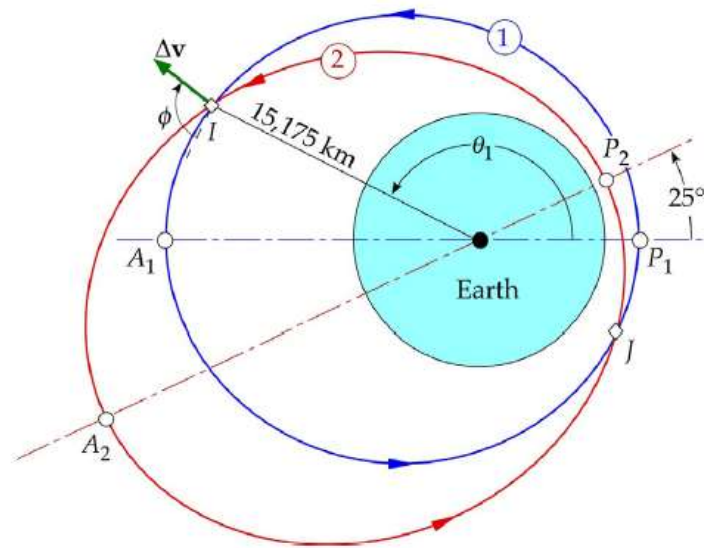


- $\eta = \theta_2 - \theta_1$
- Case 1: η , $\|h\|$ and $\|e\|$ are given for both the orbits.
- $r_{I1} = \frac{\|h_1\|^2}{\mu} \frac{1}{1 + \|e_1\| \cos \theta_1}$, $r_{I2} = \frac{\|h_2\|^2}{\mu} \frac{1}{1 + \|e_2\| \cos \theta_2}$
- $r_{I1} = r_{I2}$
- $\|e_1\| \|h_2\|^2 \cos \theta_1 - \|e_2\| \|h_1\|^2 \cos \theta_2 = \|h_1\|^2 - \|h_2\|^2$
- $\underbrace{(\|e_1\| \|h_2\|^2 - \|e_2\| \|h_1\|^2 \cos \eta)}_a \cos \theta_1 + \underbrace{(-\|e_2\| \|h_1\|^2 \sin \eta)}_b \sin \theta_1 = \underbrace{\|h_1\|^2 - \|h_2\|^2}_c$
- $A = d + \cos^{-1} \left(\frac{1}{1 + \|e\| \cos \theta} \right)$ $\theta = \tan^{-1} \left(\frac{1}{1 + \|e\| \cos \theta} \right)$

$$v_1 = \sqrt{\mu} \left(\frac{c}{a} \cos \psi \right), \quad \psi = \cos^{-1} \left(\frac{b}{a} \right)$$

Example

An earth satellite is in an 8000 km by 16,000 km radius orbit (orbit 1 of Fig. 6.18). Calculate the delta-v and the true anomaly θ_1 required to obtain a 7000 km by 21,000 km radius orbit (orbit 2) whose apse line is rotated 25° counterclockwise. Indicate the orientation ϕ of Δv to the local horizon.



Details

$$\|e_1\| = \frac{r_{A_1} - r_{P_1}}{r_{A_1} + r_{P_1}}, \quad \|e_2\| = \frac{r_{A_2} - r_{P_2}}{r_{A_2} + r_{P_2}}$$

$$r_{P_1} = \frac{\|h_1\|^2}{\mu} \frac{1}{1 + \|e_1\|}, \quad r_{P_2} = \frac{\|h_2\|^2}{\mu} \frac{1}{1 + \|e_2\|}$$

$$\phi = \tan^{-1} \left(\frac{b}{a} \right), \quad \theta_1 = \phi \pm \cos^{-1} \left(\frac{c}{a} \cos \phi \right)$$

$$\|r\| = \frac{\|h_1\|^2}{\mu} \frac{1}{1 + \|e_1\| \cos \theta_1}$$

$$V_{\perp I)_1} = \frac{\|h_1\|}{\|v\|}, \quad V_{r_I)_1} = \frac{\mu}{\|h_1\|} \|e_1\| \sin \theta_1$$

$$V_I)_1 = \sqrt{V_{\perp I)_1^2 + V_{r_I)_1^2}, \quad \gamma_1 = \tan^{-1} \left(\frac{V_{r_I)_1}{V_{\perp I)_1} \right)$$

$$V_{\perp I)_2} = \frac{\|h_2\|}{\|v\|}, \quad V_{r_I)_2} = \frac{\mu}{\|h_2\|} \|e_2\| \sin(\theta_1 - 25^\circ)$$

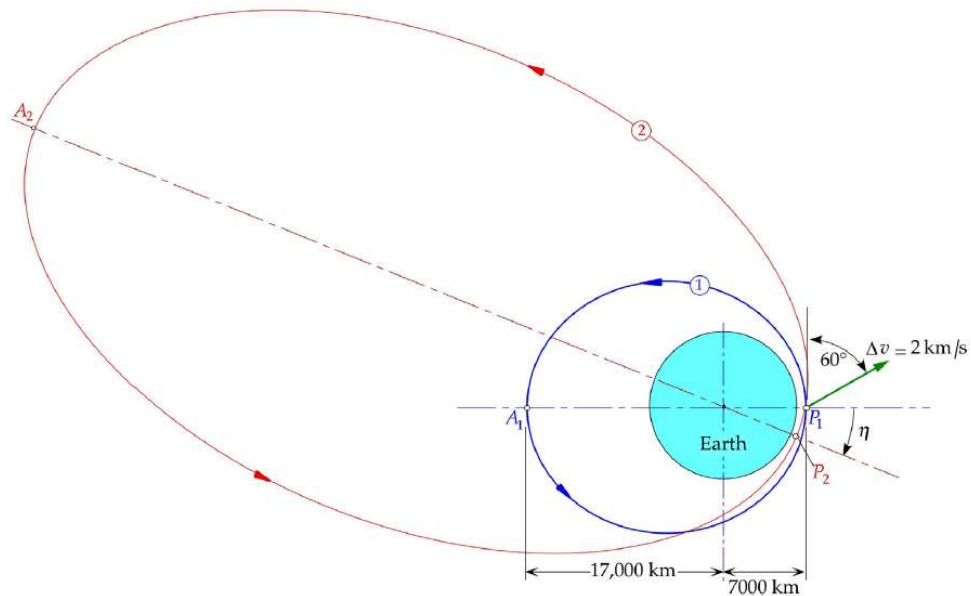
$$V_I)_2 = \sqrt{V_{\perp I)_2^2 + V_{r_I)_2^2}, \quad \gamma_2 = \tan^{-1} \left(\frac{V_{r_I)_2}{V_{\perp I)_2} \right)$$

- Case 2: Impulsive manoeuvre takes place at a given true anomaly θ_1 on orbit 1.
- $\|h_2\| = \|r\| [V_{\perp I)_1} + \Delta V_{\perp}] = \|h_1\| + \|r\| \Delta V_{\perp}$
- $V_{r_I)_2} = V_{r_I)_1} + \Delta V_r$
- $\frac{\mu}{\|h_2\|} \|e_2\| \sin \theta_2 = V_{r_I)_1} + \Delta V_r$
- $\sin \theta_2 = \frac{1}{\|e_2\|} \frac{(\|h_1\| + \|r\| \Delta V_{\perp})(\mu \|e_1\| \sin \theta_1 + \|h_1\| \Delta V_r)}{\mu \|h_1\|}$
- $\cos \theta_2 = \frac{1}{\|e_2\|} \frac{(\|h_1\| + \|r\| \Delta V_{\perp})^2 \|e_1\| \cos \theta_1 + (2\|h_1\| + \|r\| \Delta V_{\perp}) \|r\| \Delta V_{\perp}}{\|h_1\|^2}$

$$- \tan \theta_2 = \frac{[V_{\perp I})_1 + \Delta V_{\perp}][V_{r I})_1 + \Delta V_r]}{[V_{\perp I})_1 + \Delta V_{\perp}]^2 \|e_1\| \cos \theta_1 + \frac{\mu}{\|r\|} [2V_{\perp I})_1 + \Delta V_{\perp}] \Delta V_{\perp}}$$

Example

An earth satellite in orbit 1 of Fig. 6.19 undergoes the indicated delta-v maneuver at its perigee. Determine the rotation η of its apse line as well as the new perigee and apogee.



Details

$$\|e_1\| = \frac{r_{A_1} - r_{P_1}}{r_{A_1} + r_{P_1}}$$

$$r_{P_1} = \frac{\|h_1\|^2}{\mu} \frac{1}{1 + \|e_1\|}$$

$$V_{\perp P_1})_1 = \frac{\|h_1\|}{r_{P_1}}, \quad V_{r P_1})_1 = 0$$

$$\Delta V_{\perp} = \Delta V \cos 60^\circ, \quad \Delta V_r = \Delta V \sin 60^\circ$$

$$\|h_2\| = \|h_1\| + \|r\| \Delta V_{\perp}$$

$$\tan \theta_2 = \dots$$

$$\|e_2\| = \dots$$

$$r_P = \frac{\|h_2\|^2}{\mu} \frac{1}{1 + \|e_2\|}, \quad r_{A_2} = \frac{\|h_2\|^2}{\mu} \frac{1}{1 - \|e_2\|}$$