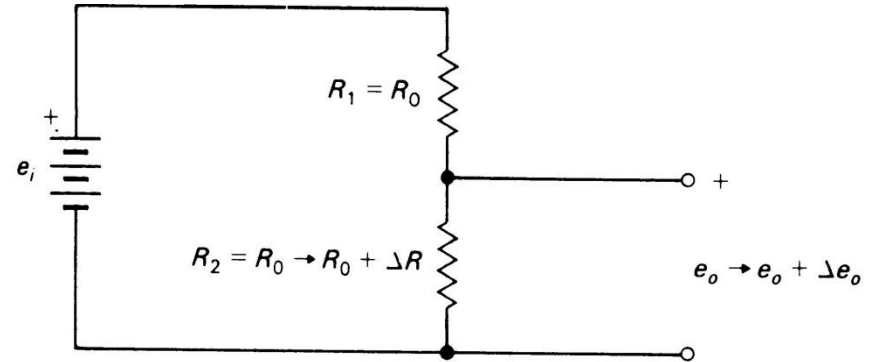


AE 242
Aerospace Measurements
Laboratory

Small change in transducer resistance

$$e_0 + \Delta e_0 = \frac{e_i}{2} + \frac{e_i}{2} \frac{\Delta R}{2R_0} \left(\frac{1}{1 + \Delta R / 2R_0} \right)$$



$\Delta R / 2R_0 \ll 1$ the output can be approximated as

$$e_0 + \Delta e_0 \approx e_0 + \frac{\Delta R}{4R_0} e_i$$

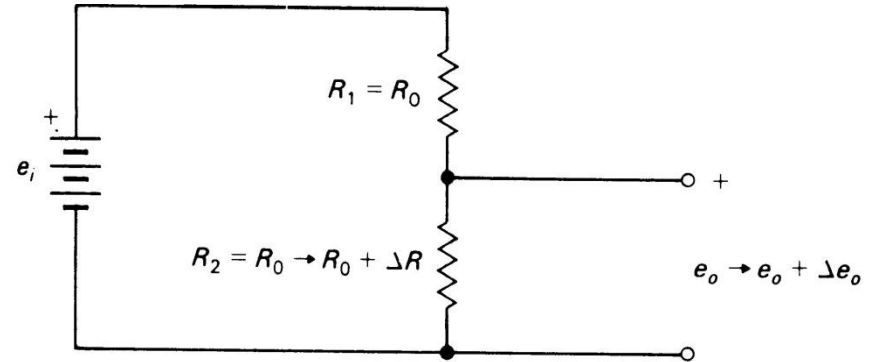
For small variation in resistance it will be linear and it is advantageous. Small variation in resistance results into small output, it is at disadvantage. For a 120Ω strain gage change in resistance is $240 \mu\Omega$ and it will change the output in micro volts

$$\frac{\Delta e_0}{e_0} = \frac{(\Delta R / 4R_0) e_i}{e_i / 2} = \frac{\Delta R}{2R_0} = 10^{-6}$$

Measurement is $e_0 + \Delta e_0$ and this will need a very precise instrument.

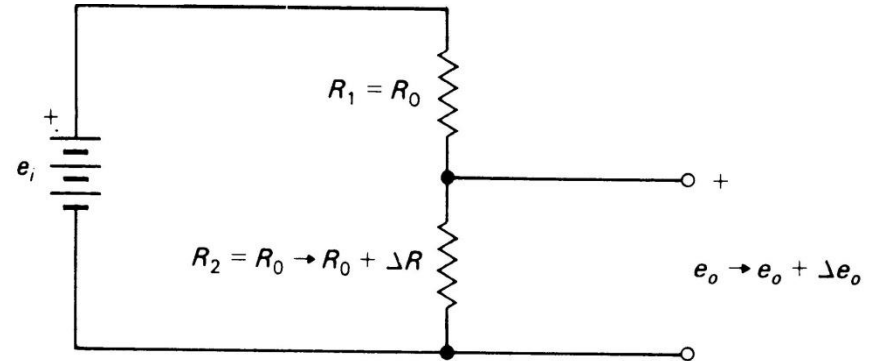
Small change in transducer resistance

Difficulty is to resolve voltage change which is a small fraction of output voltage.

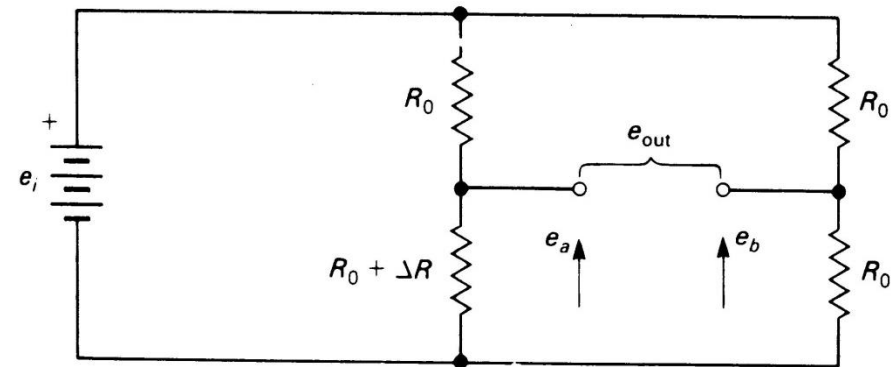


Small change in transducer resistance

Difficulty is to resolve voltage change which is a small fraction of output voltage.



The difficulty can be removed by measuring only the difference and amplifying it.

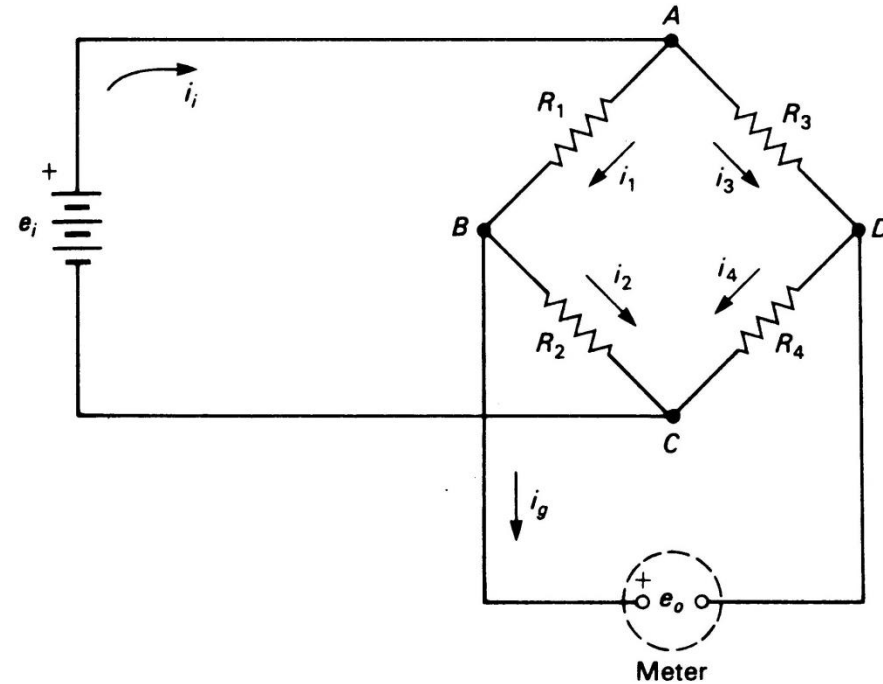


$$e_{out} = e_a - e_b = \Delta e_o = \frac{\Delta R}{4R_0} e_i$$

Wheatstone bridge

Consist of four arms of resistors, a detector and power supply source. Two arms are voltage divider and the detector (meter) finds the potential difference. Bridge is balanced when potential difference is zero and no current flow through detector. When bridge is balanced:

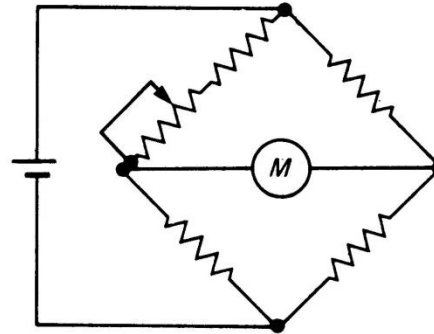
$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \text{or} \quad \frac{R_1}{R_3} = \frac{R_2}{R_4}$$



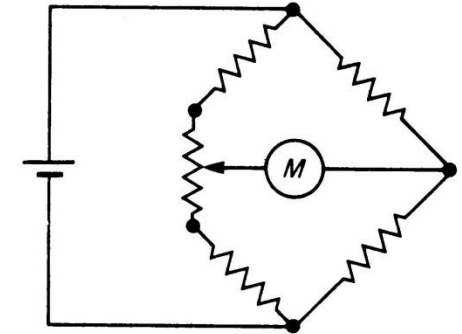
For the Wheatstone resistance bridge to be balance, the ratio of resistances of any two adjacent arms must equal the ratio of resistances of the remaining two arms, taken in the same sense.

Arrangements to balance bridge

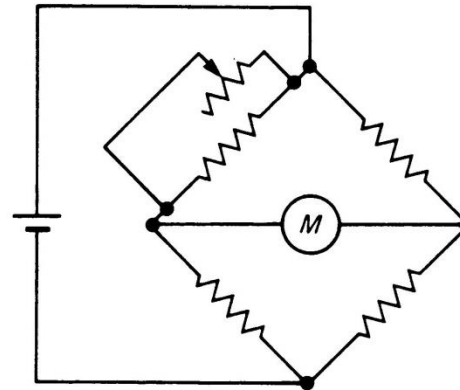
Bridge balancing is required prior to measurement. In case of null balance it is used for measurement. Series balance is used for large variation in resistance and shunt balance is used for small variation. Series or shunt balance depends on the bridge sensitivity. For deflection bridge initially it is balanced and the transducer output resulting to voltage is measured by a meter.



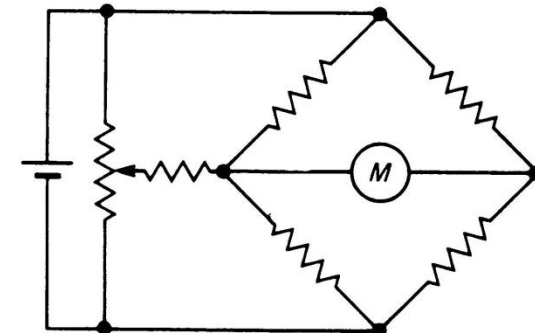
(a) Series balance



(b) Differential series balance



(c) Shunt balance



(d) Differential shunt balance

Voltage sensitive bridge

The output is connected to a high impedance measuring instrument.
Output is potential difference between point B & D

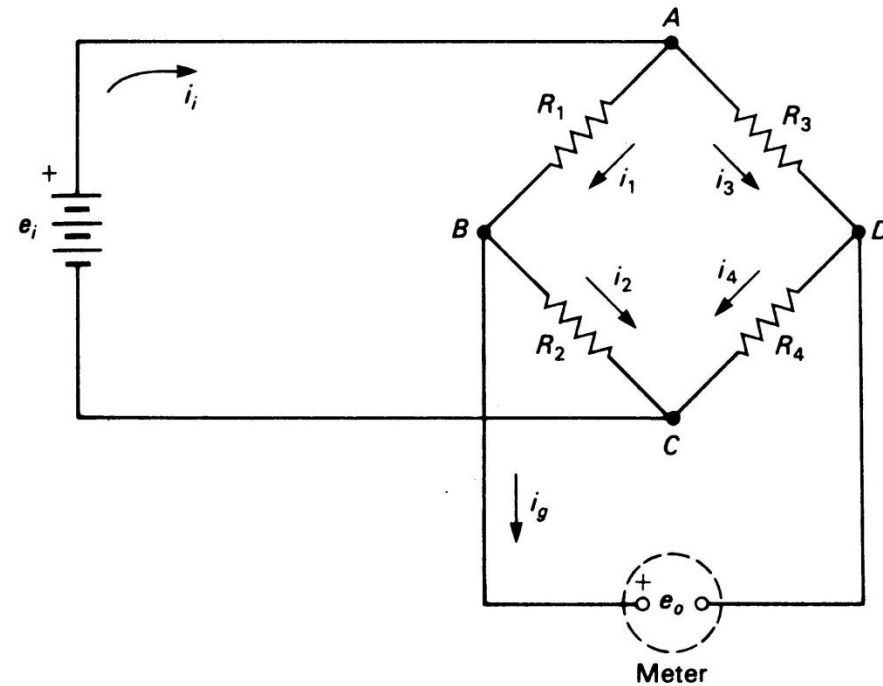
$$e_o = e_B - e_D$$

Using voltage divider relationship

$$e_o = e_i \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right)$$

Resistance R_2 changes by a small amount ΔR , output changes by Δe_o

$$e_o + \Delta e_o = e_i \left(\frac{(R_2 + \Delta R_2)R_3 - R_4R_1}{(R_1 + R_2 + \Delta R_2)(R_3 + R_4)} \right)$$



Voltage sensitive bridge

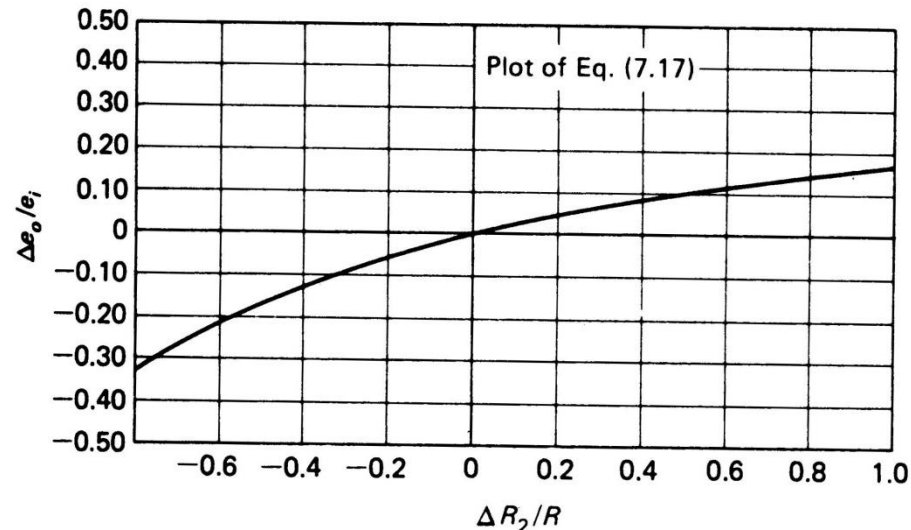
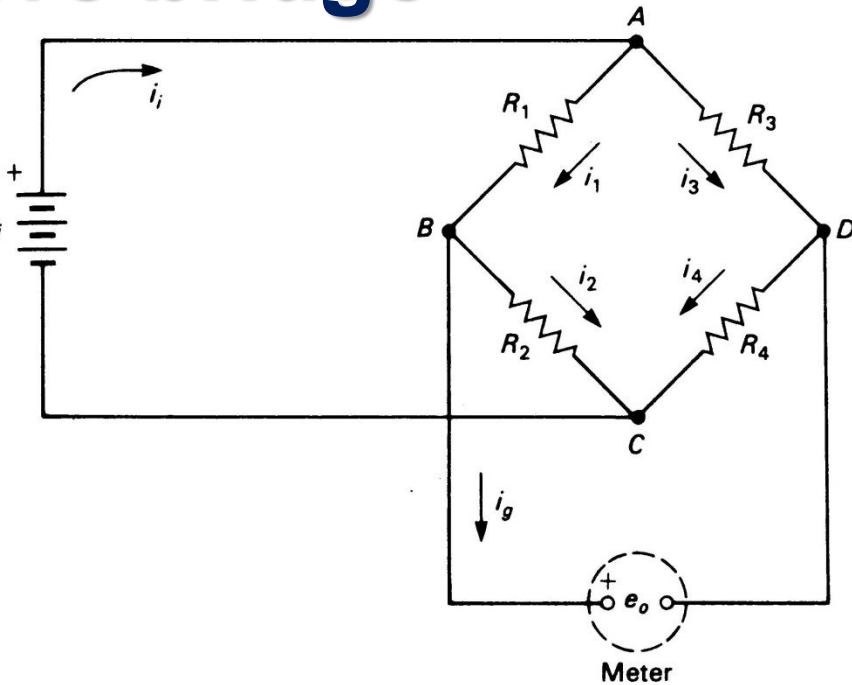
$$e_0 + \Delta e_0 = e_i \left(\frac{(\mathbf{R}_2 + \Delta \mathbf{R}_2) \mathbf{R}_3 - \mathbf{R}_4 \mathbf{R}_1}{(\mathbf{R}_1 + \mathbf{R}_2 + \Delta \mathbf{R}_2)(\mathbf{R}_3 + \mathbf{R}_4)} \right) + e_i$$

Assuming all resistances equal and this will also result in $e_0 = 0$, then

$$\frac{\Delta e_0}{e_i} = \frac{\Delta \mathbf{R}_2 / \mathbf{R}_2}{4 + 2(\Delta \mathbf{R}_2 / \mathbf{R}_2)}$$

The bridge is inherently non-linear and it can be assumed linear for small variation. In most strain gages $\Delta R_2 / 2R \ll 1$ and the linearised output is

$$\frac{\Delta e_0}{e_i} = \frac{\Delta \mathbf{R}_2}{4\mathbf{R}}$$



1% change in resistance will result into error of -0.5%

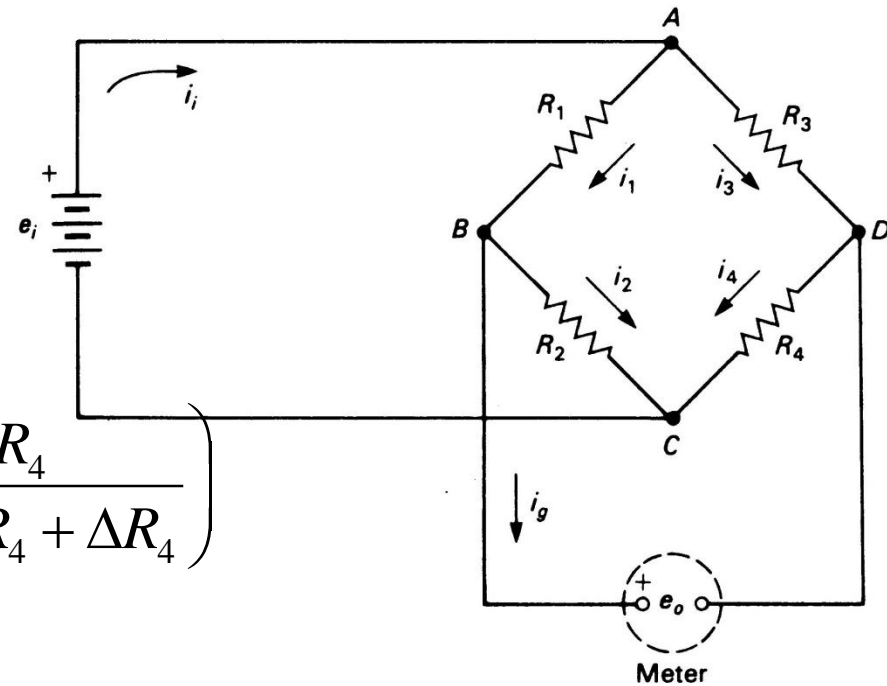
Voltage sensitive bridge

Special case when bridge is linear

Using voltage divider relationship

$$e_0 = e_i \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right)$$

$$e_0 = e_i \left(\frac{R_2 + \Delta R_2}{R_1 + \Delta R_1 + R_2 + \Delta R_2} - \frac{R_4 + \Delta R_4}{R_3 + \Delta R_3 + R_4 + \Delta R_4} \right)$$



Voltage sensitive bridge

Special case when bridge is linear

Using voltage divider relationship

$$e_0 = e_i \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right)$$

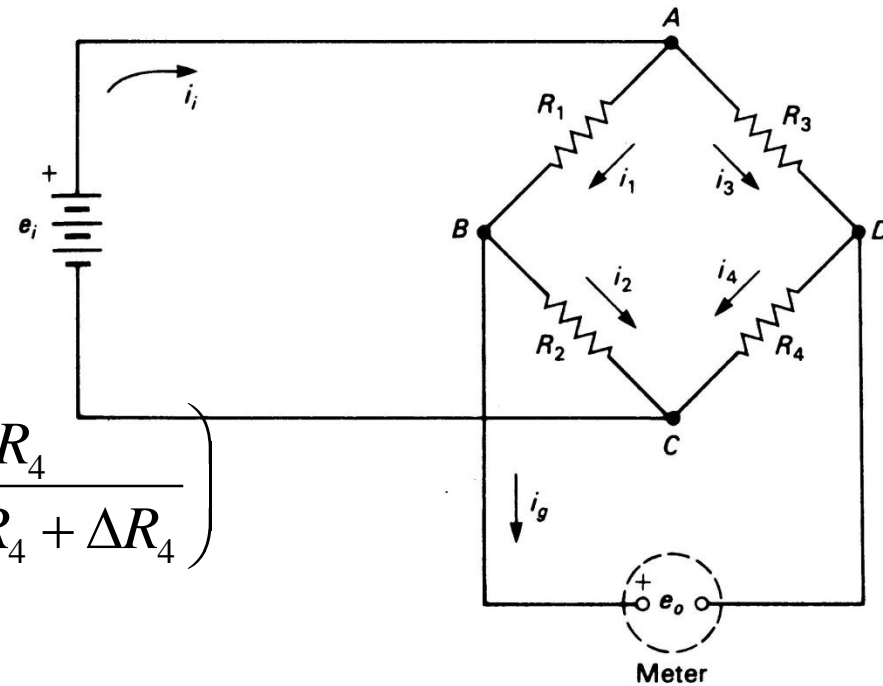
$$e_0 = e_i \left(\frac{R_2 + \Delta R_2}{R_1 + \Delta R_1 + R_2 + \Delta R_2} - \frac{R_4 + \Delta R_4}{R_3 + \Delta R_3 + R_4 + \Delta R_4} \right)$$

$$R_1 = R_2 = R_3 = R_4 = R$$

$$\Delta R_1 = -\Delta R_2 = \Delta R_3 = -\Delta R_4 = \Delta R$$

$$e_0 = e_i \left(\frac{R_2 + \Delta R}{R_1 + \Delta R + R_2 - \Delta R} - \frac{R_4 - \Delta R}{R_3 + \Delta R + R_4 - \Delta R} \right)$$

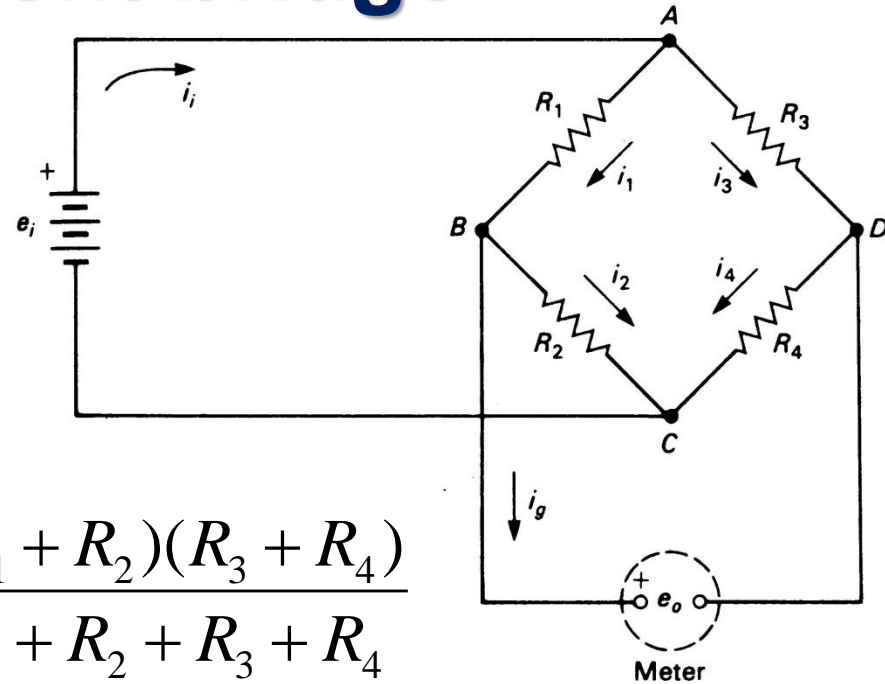
$$e_0 = e_i \left(\frac{\Delta R}{R} \right) \quad \text{Strict linear relationship}$$



Constant current bridge

When resistance changes current i_i changes in case of constant voltage bridge (excitation voltage). In case of constant current bridge, current i_i is maintained by the power supply i.e. excitation voltage changes as the resistance changes.

$$i_i = \frac{e_i}{R_1 + R_2} + \frac{e_i}{R_3 + R_4} \quad \text{or} \quad e_i = i_i \frac{(R_1 + R_2)(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4}$$



Potential difference between point B and D

$$e_0 = e_i \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right)$$

$$e_0 = i_i \frac{(R_1 + R_2)(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4} \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right)$$

For constant current, output is

$$e_0 = i_i \frac{R_2 R_3 - R_1 R_4}{R_1 + R_2 + R_3 + R_4}$$

Constant current bridge

Resistance R_2 is the measuring arm and the resistance changes by ΔR . Output can be written as

$$e_0 + \Delta e_0 = i_i \frac{(R_2 + \Delta R)R_3 - R_1 R_4}{R_1 + (R_2 + \Delta R) + R_3 + R_4}$$

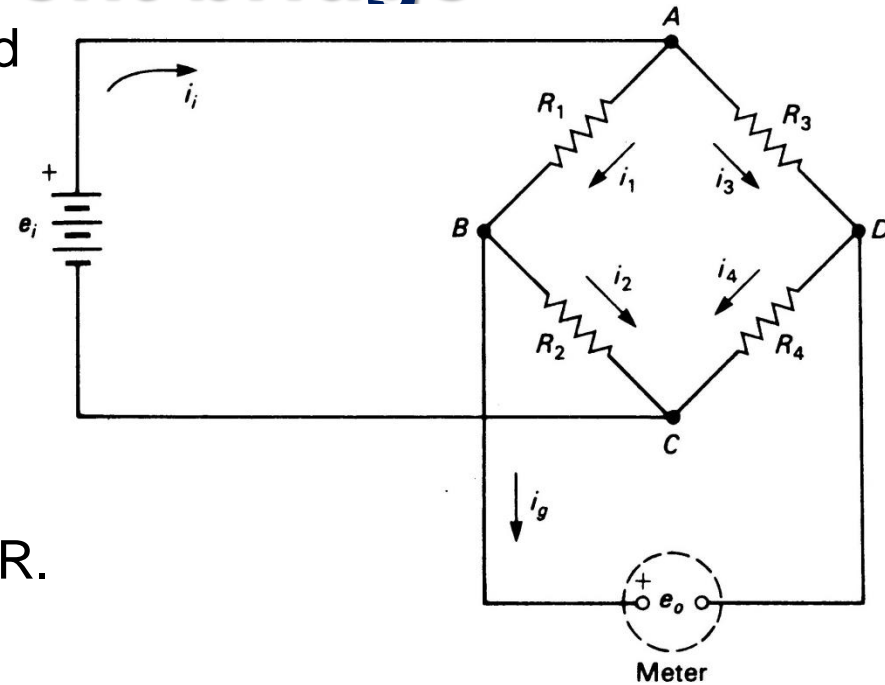
For the case when all the resistance = R .

$$\Delta e_0 = i_i \left[\frac{\Delta R}{4 + \Delta R / R} \right]$$

Constant current has better linearity compared to constant voltage

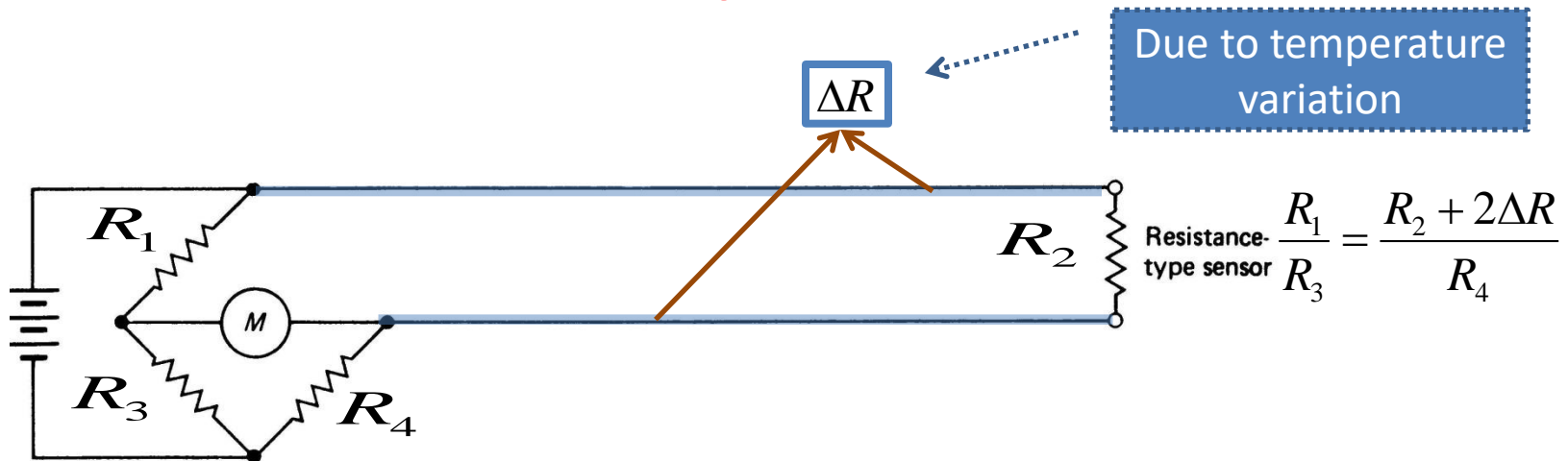
In case of constant voltage

$$\frac{\Delta e_0}{e_i} = \frac{\Delta R_2 / R_2}{4 + 2(\Delta R_2 / R_2)}$$



Compensation for leads

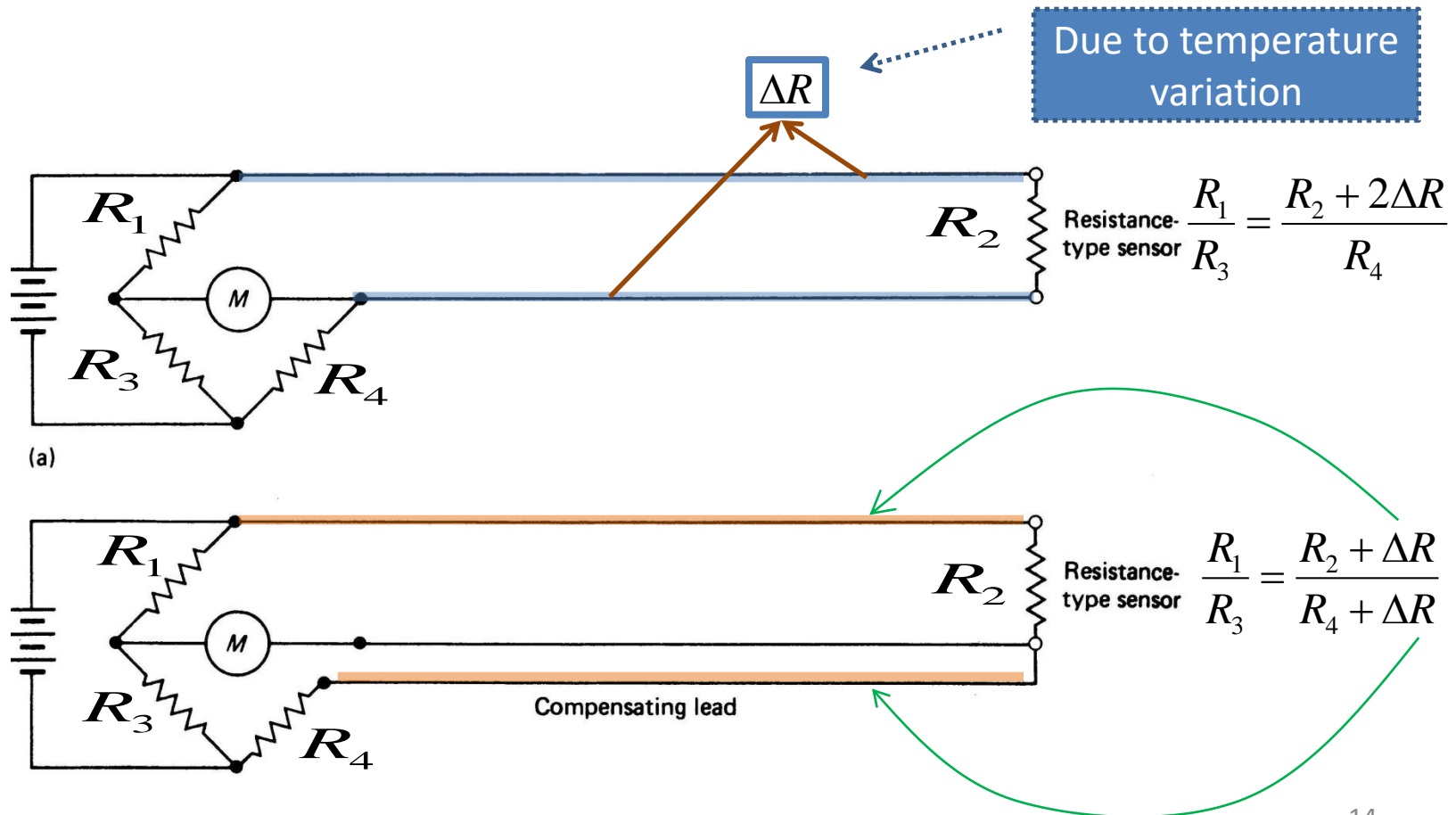
Frequently sensor and bridge are separated by appreciable distance, connecting leads will be also of same length. These wires add resistance and temperature variation along these leads can add to errors. This can be compensated by a compensating lead which is subjected to same temperature variations as the connecting wire.



Even when strain in arm 2 is zero. Bridge will be unbalanced due to ΔR resistance i.e. resistance change due to change in temperature.

Compensation for leads

Error due to change in temperature of connecting cable leading to change in resistance can be compensated by distributing ΔR in the two adjacent arms. This can be achieved by using three lead strain gage. All the three leads should have same length and electrical characteristics. Bridge will be always balanced when strain in arm 2 is zero.

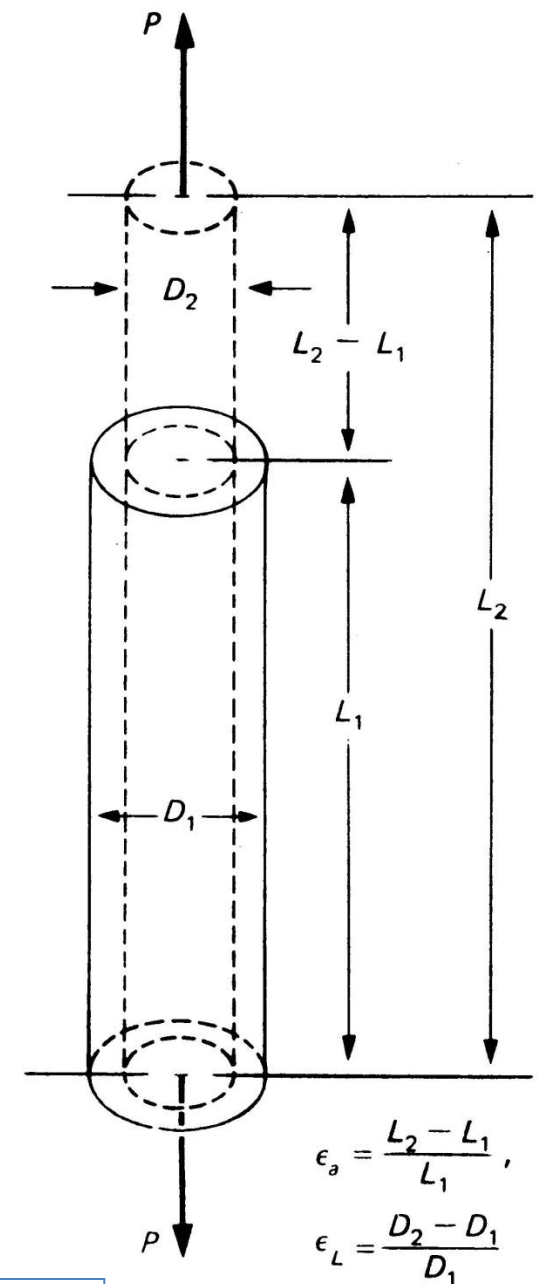


Strain measurement

Strain is deformation per unit length. It is very small number and generally multiplied by 10^{-6} and called as micro strain or parts per million (ppm). Axial stress will cause axial strain and also lateral strain and generally it is related by Poisson's ratio. For tensile axial strain it will be compressive and vice versa. Strain can be measured directly or indirectly. One of the direct method is fixing strain gages over the location of interest.



1 μ strain =
1mm elongation over 10^6 mm =
1mm elongation over 1000 m =
1mm elongation over 1 km



$$\epsilon_a = \frac{L_2 - L_1}{L_1},$$

$$\epsilon_L = \frac{D_2 - D_1}{D_1}$$

$$\epsilon_L = -\nu \epsilon_a$$

Electrical resistance strain gage

When a length of wire is mechanically stretched, cross-section changes and the resistance of the wire changes. For a conductor of length L , cross-sectional area CD^2 , D is characteristic dimension and C is constant depending on the cross-section. Resistance of the conductor for the given resistivity ρ

$$R = \frac{\rho L}{A}$$

$$A = CD^2$$

$$R = \frac{\rho L}{A} = \frac{\rho L}{CD^2}$$

When the conductor is strained each of the quantity can change

$$dR = \frac{\rho dL}{A} + \frac{L d\rho}{A} - \frac{\rho L dA}{A^2}$$

$$dR = \frac{(L d\rho + \rho dL)}{CD^2} - \frac{2C\rho L D dD}{(CD^2)^2}$$

$$\frac{dR}{R} = \frac{dL}{L} - 2 \frac{dD}{D} + \frac{d\rho}{\rho}$$

$$\frac{dR/R}{dL/L} = 1 - 2 \frac{dD/D}{dL/L} + \frac{d\rho/\rho}{dL/L}$$

Poisson' ratio $\nu = -\frac{dD/D}{dL/L}$

Piezoresistance $\frac{d\rho/\rho}{dL/L}$

Electrical resistance strain gage

Gage factor
$$F = \frac{dR/R}{dL/L} = \frac{dR/R}{\varepsilon_a} = 1 + 2\nu + \frac{d\rho/\rho}{dL/L}$$

Gage factor is function of Poisson's ratio and for 0.3 it will be 1.6.

Commonly available strain gages have gage factor of ~ 2 . Gage factor relates the change in resistance with strain.

$$\varepsilon_a = \frac{1}{F} \frac{\Delta R}{R}$$

$$\Delta R = F \varepsilon_a R$$

Gage factor and resistance value is generally supplied by the manufacturer. Gage factor determination is a destructive test. It is statistically determined and given to the user.

Table 7.3: Typical gauge materials and gauge factors

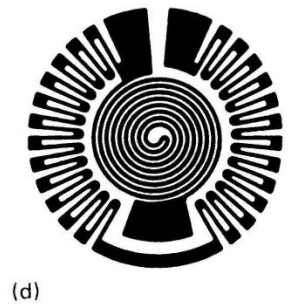
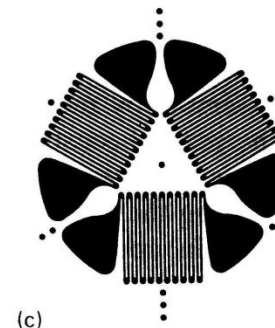
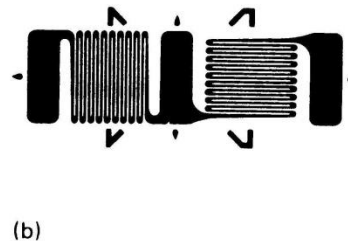
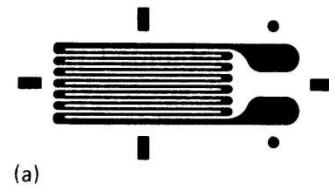
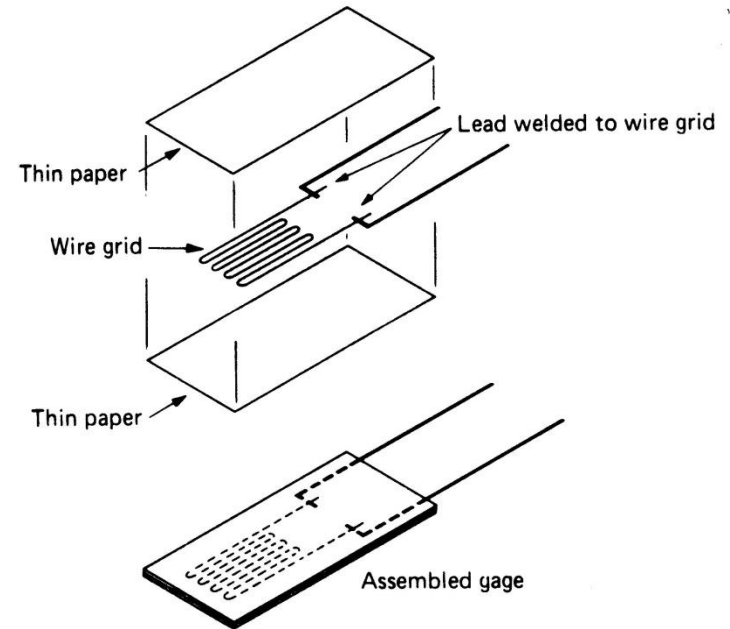
Material	Sensitivity (G)
Platinum (Pt 100%)	6.1
Platinum-Iridium (Pt 95%, Ir 5%)	5.1
Platinum-Tungsten (Pt 92%, W 8%)	4
Isoelastic (Fe 55.5%, Ni 36% Cr 8%, Mn 0.5%) [#]	3.6
Constantan / Advance / Copel (Ni 45%, Cu 55%) [#]	2.1
Nichrome V (Ni 80%, Cr 20%) [#]	2.1
Karma (Ni 74%, Cr 20%, Al 3%, Fe 3%) [#]	2
Armour D (Fe 70%, Cr 20%, Al 10%) [#]	2
Monel (Ni 67%, Cu 33%) [#]	1.9
Manganin (Cu 84%, Mn 12%, Ni%) [#]	0.47
Nickel (Ni 100%)	-12.1

[#] Isoelastic, Constantan, Advance, Copel, Nichrome V, Karma, Armour D, Monel, and Manganin are all trade names

Type of strain gage

Wire type : Thin wire is wound in a grid form and it is glued between two thin paper sheets. Does not have good accuracy. It is obsolete.

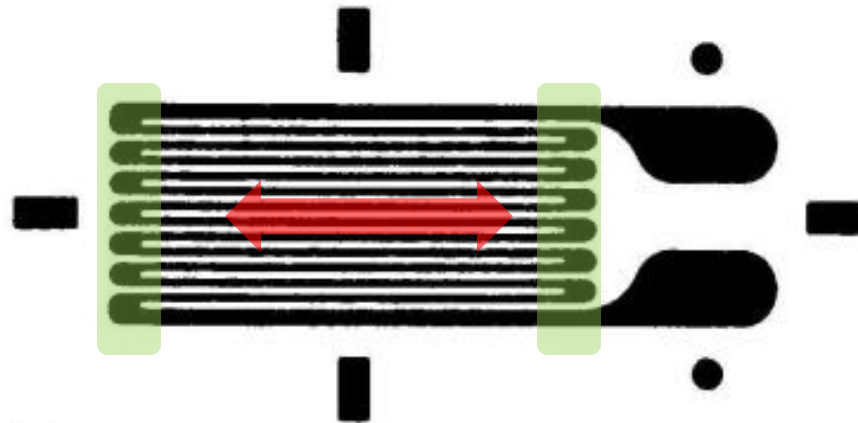
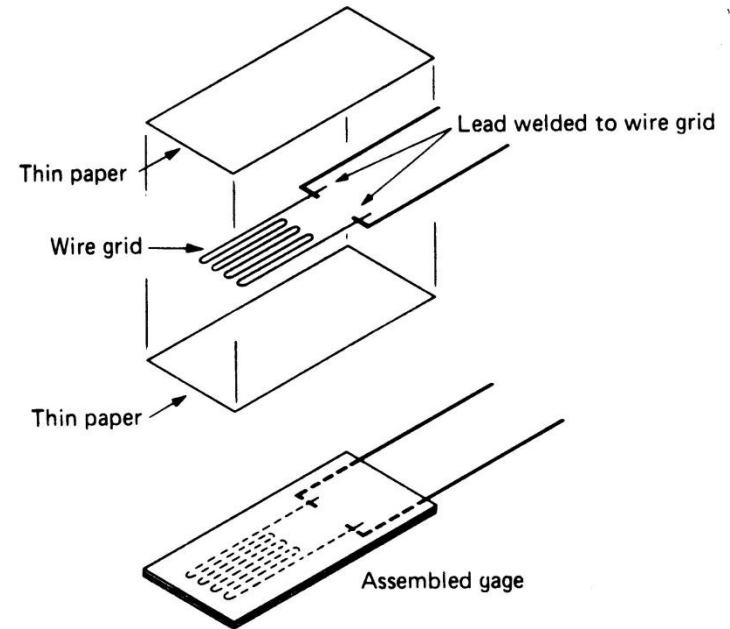
Foil type : Thin metal is etched on a thin backing material. Available in various sizes, resistance, shapes and angular configuration.



Type of strain gage

Wire type : Thin wire is wound in a grid form and it is glued between two thin paper sheets. Does not have good accuracy. It is obsolete.

Foil type : Thin metal is etched on a thin backing material. Available in various sizes, resistance, shapes and angular configuration.



Rectangular block



Total strain = axial strain + bending strain + thermal strain

Axial strain will be equal on top and bottom surface

Bending strain will be opposite on top and bottom surface

Thermal strain will be equal on top and bottom surface

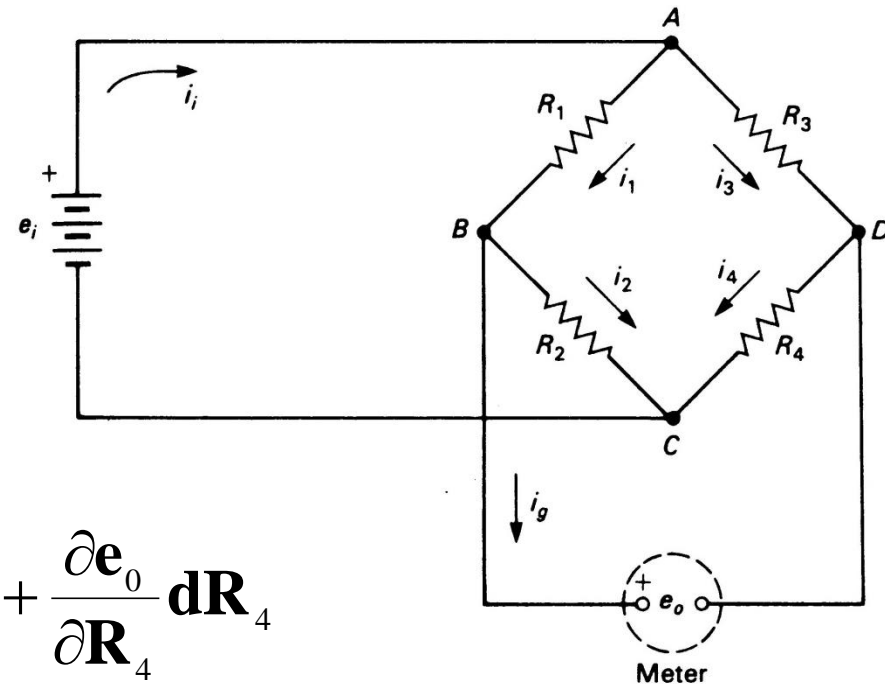
Wheatstone bridge output

For a constant voltage, bridge output

$$e_0 = e_i \left(\frac{R_1 R_4 - R_2 R_3}{(R_1 + R_2)(R_3 + R_4)} \right)$$

Assuming resistance of each arm is varying

$$de_0 = \frac{\partial e_0}{\partial R_1} dR_1 + \frac{\partial e_0}{\partial R_2} dR_2 + \frac{\partial e_0}{\partial R_3} dR_3 + \frac{\partial e_0}{\partial R_4} dR_4$$



Wheatstone bridge output

For a constant voltage, bridge output

$$e_0 = e_i \left(\frac{R_1 R_4 - R_2 R_3}{(R_1 + R_2)(R_3 + R_4)} \right)$$

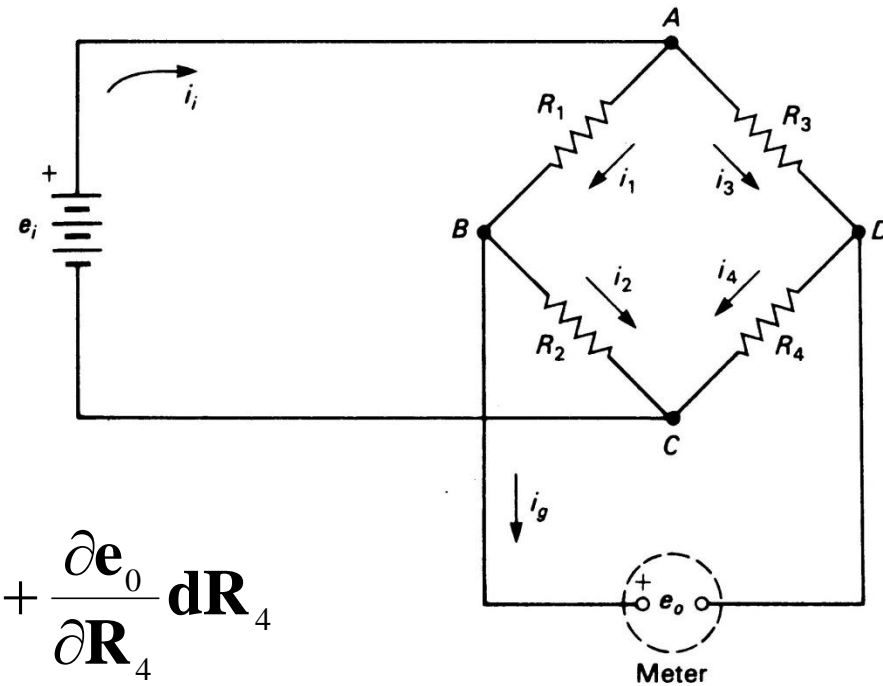
Assuming resistance of each arm is varying

$$de_0 = \frac{\partial e_0}{\partial R_1} dR_1 + \frac{\partial e_0}{\partial R_2} dR_2 + \frac{\partial e_0}{\partial R_3} dR_3 + \frac{\partial e_0}{\partial R_4} dR_4$$

Evaluating partial derivatives and by substitution

$$\frac{de_0}{e_i} = \frac{R_2 dR_1}{(R_1 + R_2)^2} - \frac{R_1 dR_2}{(R_1 + R_2)^2} - \frac{R_4 dR_3}{(R_3 + R_4)^2} + \frac{R_3 dR_4}{(R_3 + R_4)^2}$$

dR_1 , dR_2 , dR_3 and dR_4 are resistance changes in each of bridge arm

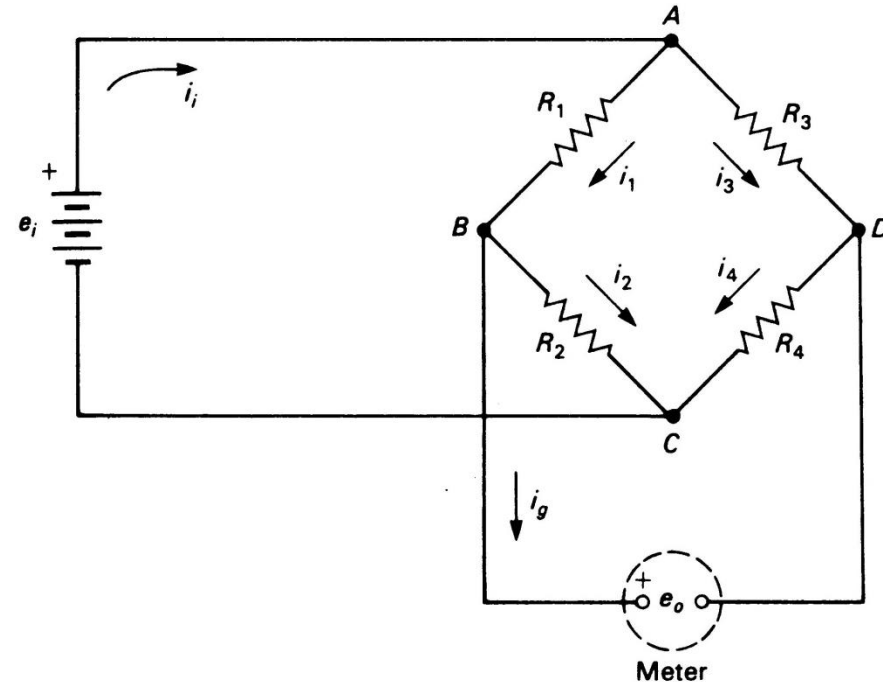


Wheatstone bridge output

$$\frac{de_0}{e_i} = \frac{R_2 dR_1}{(R_1 + R_2)^2} - \frac{R_1 dR_2}{(R_1 + R_2)^2} - \frac{R_4 dR_3}{(R_3 + R_4)^2} + \frac{R_3 dR_4}{(R_3 + R_4)^2}$$

If R_1 , R_2 , R_3 and R_4 are all equal to R

$$\frac{de_0}{e_i} = \frac{dR_1 - dR_2 - dR_3 + dR_4}{4R}$$



Wheatstone bridge output

$$\frac{de_0}{e_i} = \frac{R_2 dR_1}{(R_1 + R_2)^2} - \frac{R_1 dR_2}{(R_1 + R_2)^2} - \frac{R_4 dR_3}{(R_3 + R_4)^2} + \frac{R_3 dR_4}{(R_3 + R_4)^2}$$

If R_1 , R_2 , R_3 and R_4 are all equal to R

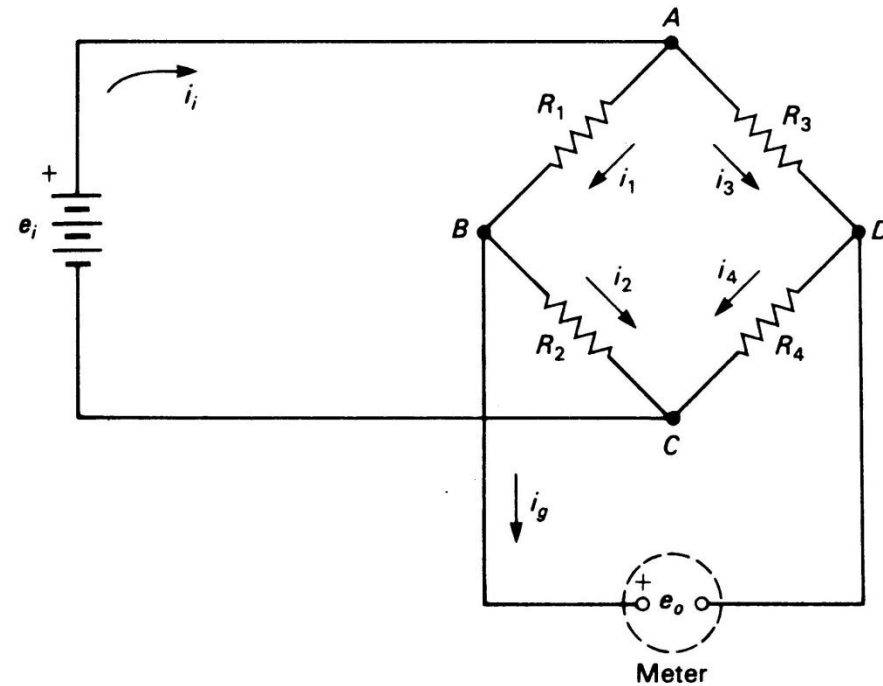
$$\frac{de_0}{e_i} = \frac{dR_1 - dR_2 - dR_3 + dR_4}{4R}$$

Change in resistance is related to strain : $\frac{dR_n}{R_n} = F \epsilon_n$

Output voltage in terms of strain : $\frac{de_0}{e_i} = \frac{F}{4} [\epsilon_1 - \epsilon_2 - \epsilon_3 + \epsilon_4]$

If one arm is active: $\frac{de_0}{e_i} = \frac{F}{4} \epsilon$

Quarter bridge



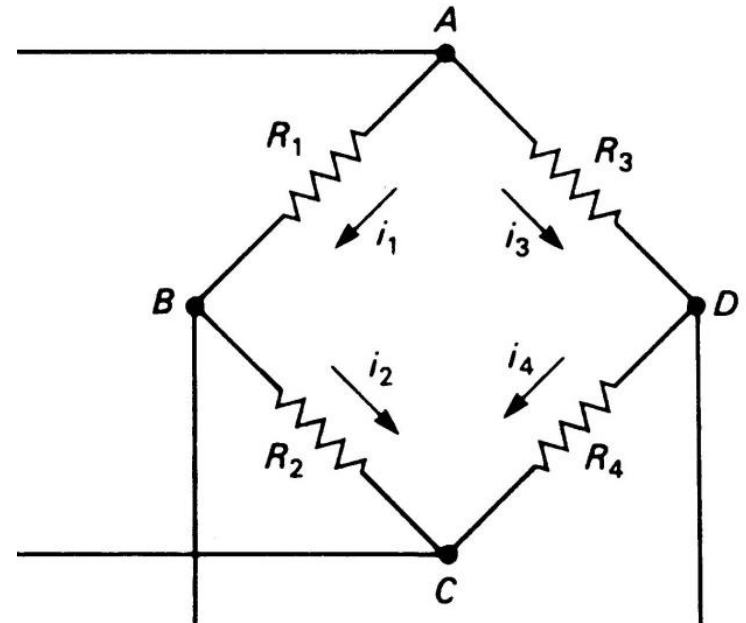
Wheatstone bridge

Full bridge - When all the arms are active. i.e. R_1 , R_2 , R_3 and R_4

Half bridge - When two arms are active.

Quarter bridge - When only one arm is active.

Active means that particular arm is acting as sensor

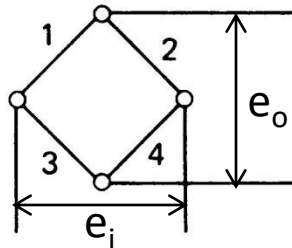


Dummy Gage: One of the resistance of wheatstone bridge is replaced with actual stain gage fixed to test material. It is not subjected to strain, other environmental conditions are same i.e. temperature.

Strain gage orientation

Using the equation shown, effect of strain gage orientation can be explained:

$$\frac{d\epsilon_0}{\epsilon_i} = \frac{F}{4} [\epsilon_1 - \epsilon_2 - \epsilon_3 + \epsilon_4]$$



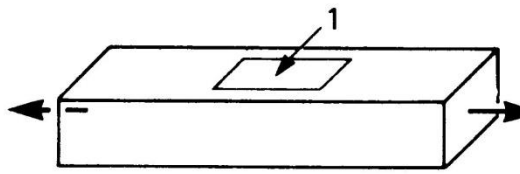
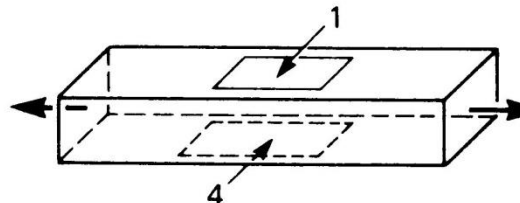
Requirement for null: $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

$K = \text{Bridge constant} = \frac{\text{Output of bridge}}{\text{Output of primary gage}}$

$\epsilon_a = \text{Axial strain}$

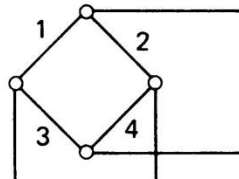
$\epsilon_b = \text{Bending strain}$

$\epsilon_T = \text{Thermal strain}$

A	<p>$K = 1$</p> 	<p>Compensates for temperature if "dummy" gage is used in arm 2 or arm 3.</p> <p>Does not compensate for bending.</p> <p>$\epsilon_1 = \epsilon_a$</p>	<p>$\epsilon_1 = \epsilon_a + \epsilon_T$</p> <p>$\epsilon_2 \text{ OR } \epsilon_3 = \epsilon_T$</p>
B	<p>$K = 2$</p> 	<p>Compensates for bending.</p> <p>Two-arm bridge does not provide temperature compensation.</p> <p>Four-arm bridge ("dummy" gages in arms 2 and 3) provides temperature compensation.</p> <p>$\epsilon_1 = \epsilon_a + \epsilon_b + \epsilon_T$</p> <p>$\epsilon_4 = \epsilon_a - \epsilon_b + \epsilon_T$</p>	<p>$\epsilon_1 = \epsilon_a + \epsilon_b + \epsilon_T$</p> <p>$\epsilon_2 = \epsilon_T$</p> <p>$\epsilon_3 = \epsilon_T$</p> <p>$\epsilon_4 = \epsilon_a - \epsilon_b + \epsilon_T$</p>

Strain gage orientation

$$\frac{de_0}{e_i} = \frac{F}{4} [\epsilon_1 - \epsilon_2 - \epsilon_3 + \epsilon_4]$$



Requirement for null: $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

$K = \text{Bridge constant} = \frac{\text{Output of bridge}}{\text{Output of primary gage}}$

C	$K = 1 + \nu$ 	<p>Two-arm bridge compensates for temperature</p> <div style="background-color: #f4a460; padding: 10px;"> $\epsilon_1 = \epsilon_a + \epsilon_T$ $\epsilon_2 = -\nu \epsilon_a + \epsilon_T$ </div>
D	$K = 2(1 + \nu)$ 	<p>Four-arm bridge compensates for temperature and bending.</p>
E	$K = 1$ 	<p>Temperature compensation accomplished when "dummy" gage is used in arm 2 or arm 3.</p> <p>Bridge is also sensitive to axial and torsional components of loading.</p>

$$\epsilon_1 = \epsilon_a + \epsilon_b + \epsilon_T$$

$$\epsilon_2 = -\nu \epsilon_a + \epsilon_T$$

$$\epsilon_3 = -\nu \epsilon_a + \epsilon_T$$

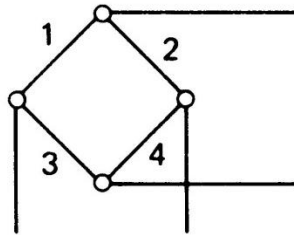
$$\epsilon_4 = \epsilon_a - \epsilon_b + \epsilon_T$$

$$\epsilon_1 = \epsilon_a + \epsilon_b + \epsilon_T$$

$$\epsilon_2 = \epsilon_T$$

Strain gage orientation

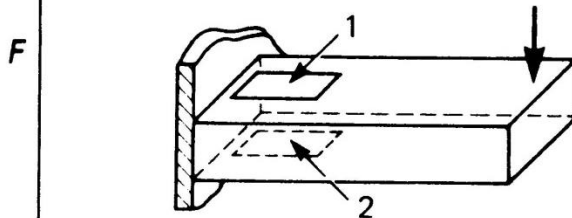
$$\frac{d\mathbf{e}_0}{\mathbf{e}_i} = \frac{\mathbf{F}}{4} [\boldsymbol{\varepsilon}_1 - \boldsymbol{\varepsilon}_2 - \boldsymbol{\varepsilon}_3 + \boldsymbol{\varepsilon}_4]$$



Requirement for null: $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

$K = \text{Bridge constant} = \frac{\text{Output of bridge}}{\text{Output of primary gage}}$

$K = 2$

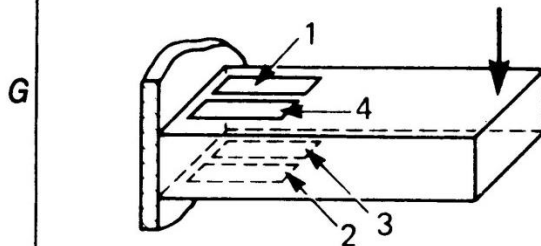


Temperature effects and axial and torsional components are compensated.

$$\boldsymbol{\varepsilon}_1 = \boldsymbol{\varepsilon}_a + \boldsymbol{\varepsilon}_b + \boldsymbol{\varepsilon}_T$$

$$\boldsymbol{\varepsilon}_2 = \boldsymbol{\varepsilon}_a - \boldsymbol{\varepsilon}_b + \boldsymbol{\varepsilon}_T$$

$K = 4$



Four-arm bridge.

Temperature effects and axial and torsional components are compensated.

$$\boldsymbol{\varepsilon}_1 = \boldsymbol{\varepsilon}_a + \boldsymbol{\varepsilon}_b + \boldsymbol{\varepsilon}_T$$

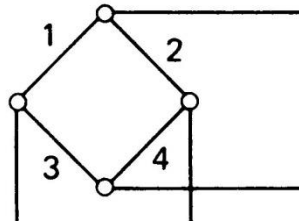
$$\boldsymbol{\varepsilon}_2 = \boldsymbol{\varepsilon}_a - \boldsymbol{\varepsilon}_b + \boldsymbol{\varepsilon}_T$$

$$\boldsymbol{\varepsilon}_3 = \boldsymbol{\varepsilon}_a - \boldsymbol{\varepsilon}_b + \boldsymbol{\varepsilon}_T$$

$$\boldsymbol{\varepsilon}_4 = \boldsymbol{\varepsilon}_a + \boldsymbol{\varepsilon}_b + \boldsymbol{\varepsilon}_T$$

Strain gage orientation

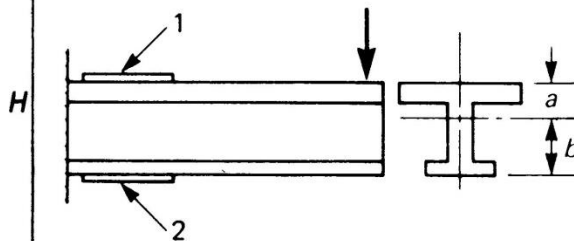
$$\frac{d\epsilon_0}{\epsilon_i} = \frac{F}{4} [\epsilon_1 - \epsilon_2 - \epsilon_3 + \epsilon_4]$$



Requirement for null: $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

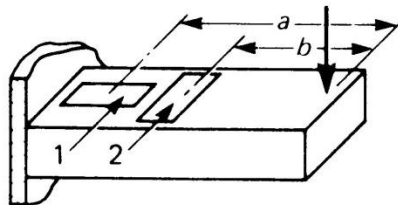
$K = \text{Bridge constant} = \frac{\text{Output of bridge}}{\text{Output of primary gage}}$

$$K = \frac{a+b}{a}$$



Temperature effects and axial and torsional components are compensated.

$$K = 1 + \left(\frac{b}{a}\right)\nu$$

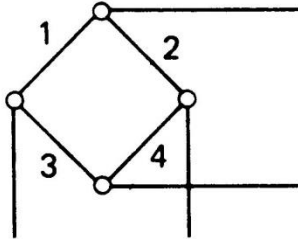


Temperature effects are compensated.

Axial and torsional load components are not compensated.

Strain gage orientation

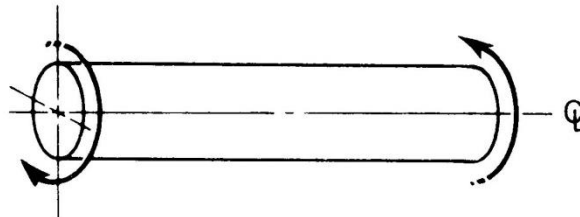
$$\frac{d\epsilon_0}{\epsilon_i} = \frac{F}{4} [\epsilon_1 - \epsilon_2 - \epsilon_3 + \epsilon_4]$$

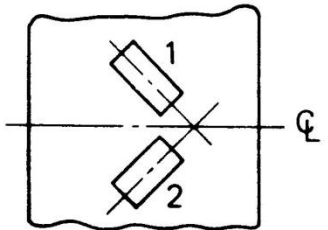


Requirement for null: $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

$K = \text{Bridge constant} = \frac{\text{Output of bridge}}{\text{Output of primary gage}}$

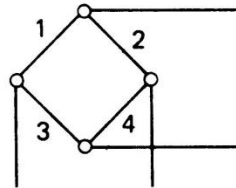
Torsion



J	<div style="display: flex; align-items: center;"> <div style="margin-right: 20px;">$K = 2$</div> <div style="text-align: center;">  </div> </div>	<p>Two-arm bridge.</p> <p>Temperature and axial load components are compensated.</p> <p>Bending components are accentuated.</p>
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Strain gage orientation

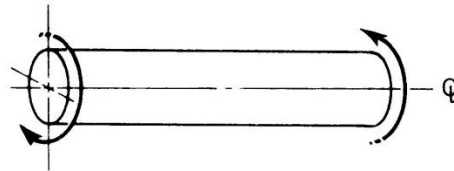
$$\frac{d\epsilon_0}{\epsilon_i} = \frac{F}{4} [\epsilon_1 - \epsilon_2 - \epsilon_3 + \epsilon_4]$$

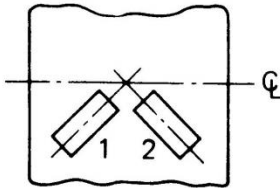
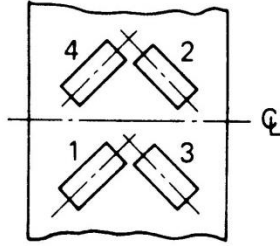


Requirement for null: $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

$K = \text{Bridge constant} = \frac{\text{Output of bridge}}{\text{Output of primary gage}}$

Torsion



K	<p>$K = 2$</p> 	<p>Two-arm bridge.</p> <p>Temperature effects and axial load components are compensated.</p> <p>Relatively insensitive to bending.</p>
L	<p>$K = 4$</p> 	<p>Four-arm bridge.</p> <p>Sensitive to torsion only.</p> <p>(Gages 1 and 3 are on opposite sides of the shaft from gages 2 and 4.)</p>