MA 214: Introduction to numerical analysis (2021–2022)

Tutorial 7

(March 16, 2022)

(1) Use the composite trapezoidal rule with $n = 6$ to approximate the following integrals:

$$\int_0^2 \frac{2}{x^2+4}dx \quad \text{and} \quad \int_0^\pi x^2 \cos x dx.$$

(2) Use the composite Simpson's rule to approximate the above integrals.

(3) Suppose that $f(0) = 1$, $f(0.5) = 2.5$, $f(1) = 2$ and $f(0.25) = f(0.75) = \alpha$. Find α if the composite trapezoidal rule with $n = 4$ gives the value 1.75 for $\displaystyle\int_0^1 f(x)dx$.

(4) Use adaptive quadrature to compute the following integral with accuracy within 10^{-2}:

$$\int_1^3 e^{2x} \sin 3x dx.$$

(5) Use adaptive quadrature to compute the following integral with accuracy within 10^{-3}:

$$\int_0^5 \left(2x\cos(2x) - (x-2)^2\right) dx.$$

(6) Let $T(a,b)$, $T(a, \frac{a+b}{2}) + T(\frac{a+b}{2}, b)$ be the single and double applications of the trapezoidal ruleto $\int_a^b f(x)dx$. Derive the relationship between

$$\left| T(a,b) - T\left(a, \frac{a+b}{2}\right) - T\left(\frac{a+b}{2}, b\right) \right|$$

and

$$\left| \int_a^b f(x)dx - T\left(a, \frac{a+b}{2}\right) - T\left(\frac{a+b}{2}, b\right) \right|.$$

(7) Approximate the following integrals using Gaußian quadrature with $n = 2$ and compare your results to the exact values of the integrals:

$$\int_1^{1.5} x^2 \ln x dx \quad \text{and} \quad \int_0^1 x^2 e^{-x} dx.$$

(8) Repeat the above problem with $n = 4$.

(9) Determine constants a, b, c, d that produce a quadrature formula

$$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

that has degree of precision 3.