AE 236 : Compressible Fluid Mechanics

(Module I : Isentropic Flow)

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What is a compressible flow ?

Density changes appreciably with pressure

$$\rho = f(p, T)$$

↓

Temperature changes appreciably

compressibility

$$\frac{1}{\rho}\frac{d\rho}{dp}$$

Primarily gases (gas dynamics)

⇒ We have to worry about thermodynamics in addition to the flow dynamics

Practical scenarios

Gas turbines — flow through blading and nozzles

Steam turbines

Reciprocating engines

Natural gas transmission lines

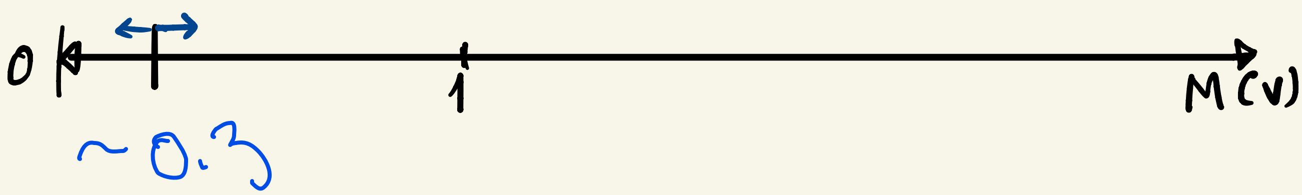
Combustion chambers

High vs. low speeds

Speed of sound is the reference chosen to compare velocities

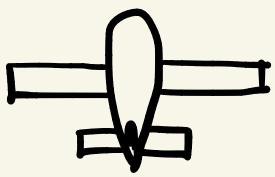
Propeller	Transport	Concorde	Reentry
Subsonic	Transonic	Supersonic	Hypersonic

incompressible | compressible

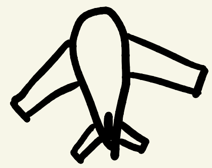


Aircrafts

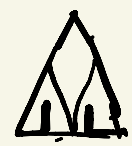
Propeller (low speed)
rounded nose
unswept wings

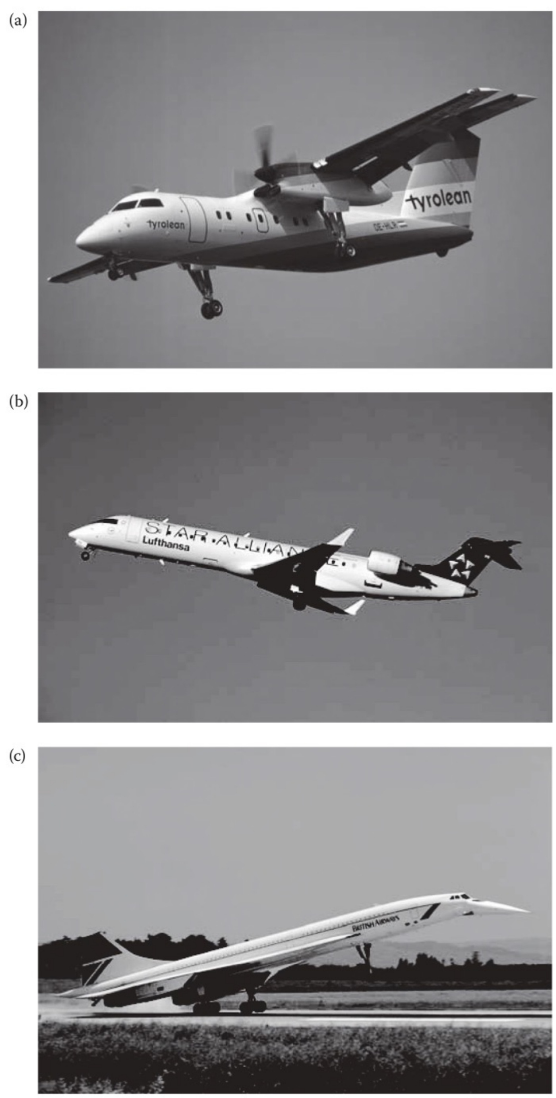


Transport (higher speeds)
rounded nose
swept wings



Concorde (even higher speeds)
sharp nose
more sweep





low subsonic

transonic

M

supersonic

Fundamental assumptions

1. Gas is continuous   mean free path λ < dimensions of flow

2. No chemical changes   no reactions, ionization, dissociation

3. Gas is perfect

$p = \rho R T$ perfect gas equation

$$R = \frac{R_u}{m} = \frac{8314.3 \text{ J/kg}}{28.966}$$

$$= 287.04 \text{ J/kgK} \quad \text{(for air)}$$

$$\boxed{\begin{array}{l}\text{calorically perfect} \\ C_p, C_v - \text{constants} \\ \gamma = C_p/C_v \\ C_p - C_v = R\end{array}}$$

Thermally perfect $\Rightarrow$ C_p, C_v are $f(T)$

4. Gravitational effects are negligible
   gravitational potential omitted, no buoyancy

5. Magnetic & electric effects not considered

6. Effect of viscosity negligible $F = \mu A \frac{dV}{dy}$
   except close to bodies and 'shocks'. μ: small $\frac{dV}{dy}$: also small

Under these assumptions, the flow is completely described in terms of the following variables

(a) V (b) p (c) ρ (d) T.

(I) State equation
(II) Conservation of mass (continuity)
(III) " " momentum (Newton's law)
(IV) " " energy

} 4 eqns. connecting the 4 variables

Conservation laws

$\boxed{\text{Mass}}$ ~~Rate of increase of mass~~ in c.v. = Rate of mass entry − Rate of mass exit

$$\rho v A = \text{constant}$$

$\boxed{\text{Momentum}}$ Net force on gas in the c.v. = ~~Rate of increase of momentum in the c.v~~ + momentum rate exit − momentum rate entry.

$$\frac{V^2}{2} + \int \frac{dp}{\rho} = \text{constant}$$

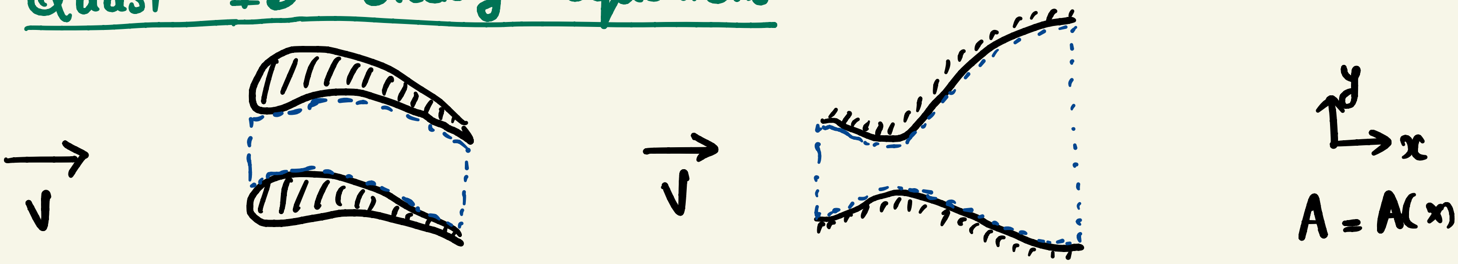
$\boxed{\text{Energy}}$ ~~Rate of increase of internal energy~~ + Rate of exit of enthalpy & k.e. − Rate of entry of enthalpy & k.e.

= Rate heat is transferred to c.v − ~~Rate work done by the gas in c.v.~~ 0

$$\underbrace{h_1 + \frac{V_1^2}{2}}_{h_{t1}} + q = \underbrace{h_2 + \frac{V_2^2}{2}}_{h_{t2}}$$

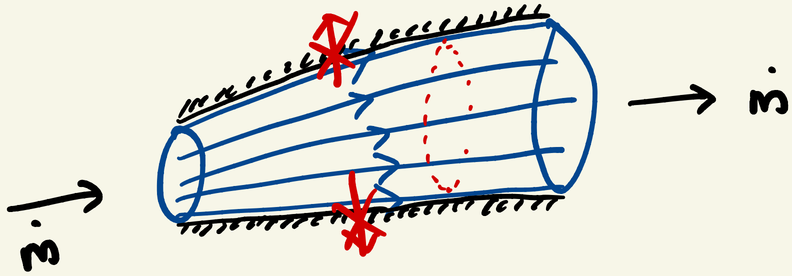
Additional assumptions]
1. flow is steady
2. not considering cases where gas does work

Quasi 1D steady equations



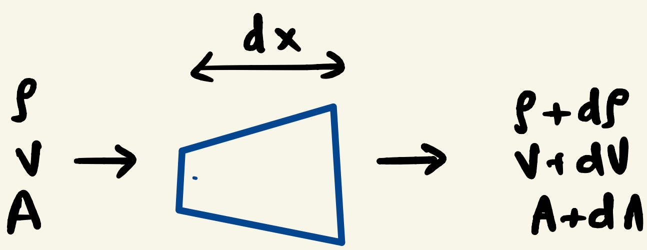
Area and curvature vary slowly with x

Stream tube:



Any duct can be treated as a stream tube

Differential control volume:



We neglect $dV \cdot d\rho$ and other second order terms

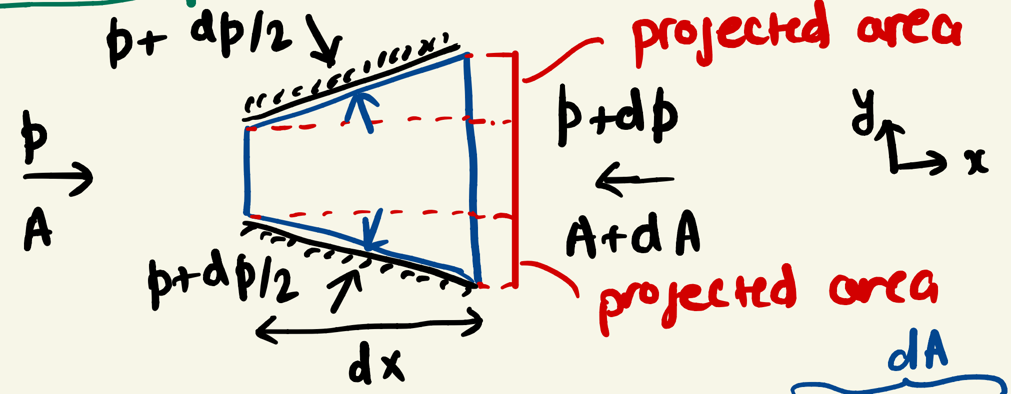
Continuity equation

$$\rho V A = (\rho + d\rho)(V + dV)(A + dA)$$

$$0 = \rho V dA + \rho A dV + A V d\rho$$

$$\boxed{\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0}$$

Momentum equation (Euler's equation)



Net forces

$$pA - (p+dp)(A+dA) + \underbrace{\frac{1}{2}[p + (p+dp)][\overbrace{(A+dA) - A}^{dA}]}_{} - dF_\mu$$

$$= -\cancel{p\,dA} - A\,dp + \cancel{p\,dA} - dF_\mu$$

$$= -A\,dp - dF_\mu$$

Momentum flux

$$\rho V A[(V+dV) - V] = \rho V A\,dV$$

We have

not considered

$$-A\,dp - \cancel{dF_\mu}^{\,0} = \rho V A\,dV$$

Euler's equation

$$\Rightarrow \boxed{-\frac{dp}{\rho} = V\,dV}$$

If velocity increases pressure decreases and vice versa

$$\frac{V^2}{2} + \int \frac{dp}{\rho} = \text{constant}$$

Need to know $\rho = \rho(p)$ to integrate

"Barotropic flow"

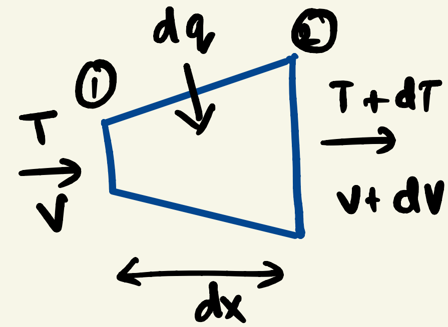
eg: For incompressible flow: $\rho = \text{constant}$

$$\frac{V^2}{2} + \frac{p}{\rho} = \text{constant}$$

Bernoulli's equation

Steady flow energy equation

$$h_2 + \frac{V_2^2}{2} = h_1 + \frac{V_1^2}{2} + q - \cancelto{0}{w}$$



$$h = C_p T$$

$$C_p T_2 + \frac{V_2^2}{2} = C_p T_1 + \frac{V_1^2}{2} + q$$

$$C_p T + \frac{V^2}{2} + dq = C_p(T + dT) + \frac{(V + dV)^2}{2}$$

$$\boxed{C_p\, dT + V\, dV = dq}$$

Adiabatic flow: $dq = 0$

$$C_p\, dT + V\, dV = 0$$

$$C_p T + \frac{V^2}{2} = \text{const}$$

if velocity increases temperature decreases and vice versa

State equation

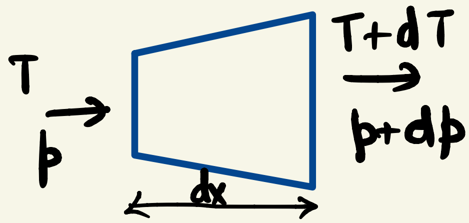
$$\frac{p}{\rho T} = R = \text{constant}$$

$$\Rightarrow \frac{p}{\rho T} = \frac{p + dp}{(\rho + d\rho)(T + dT)}$$

$$= \frac{p}{\rho T}\left[\left(1 + \frac{dp}{p}\right)\left(1 - \frac{d\rho}{\rho}\right)\left(1 - \frac{dT}{T}\right)\right]$$

$$\boxed{\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0}$$

Entropy relations



$$C_p - C_v = R$$
$$C_p / C_v = \gamma$$
$$\Downarrow$$
$$C_p = \frac{R\gamma}{\gamma - 1}$$
$$C_v = \frac{R}{\gamma - 1}$$

$$T\,ds = dh - dp/\rho$$
$$= C_p\,dT - dp/\rho$$
$$ds = C_p \frac{dT}{T} - R\frac{dp}{p}$$
$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$
$$\frac{s_2 - s_1}{C_p} = \ln\frac{T_2}{T_1} - \frac{\gamma-1}{\gamma}\ln\frac{p_2}{p_1}$$
$$= \ln\left[\left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Isentropic flow : $ds = s_2 - s_1 = 0$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}}$$

Relation b/n p & T

From state relation, we have

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\cdot\frac{\rho_1}{\rho_2} \Rightarrow \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{p_2}{p_1}\cdot\frac{\rho_1}{\rho_2}$$

$$\Rightarrow \boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}}$$

Relation b/n p & ρ

Equivalence of momentum and energy equations:

$$ds = c_p \ln\frac{(T+dT)}{T} - R\ln\left(\frac{p+dp}{p}\right)$$

$$= c_p \frac{dT}{T} - R\frac{dp}{p}$$

$$\frac{ds}{c_p} = \frac{dT}{T} - \frac{\gamma-1}{\gamma}\frac{dp}{p}$$

Isentropic flow: $ds = 0$

$$c_p\, dT = \frac{RT}{p} dp = \frac{dp}{\rho}$$

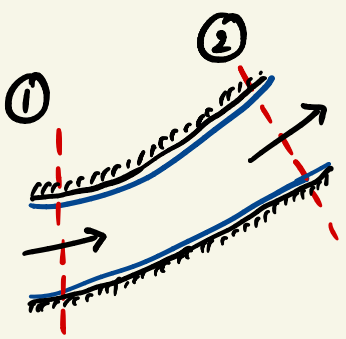
Also, $c_p\, dT + V dV = 0$

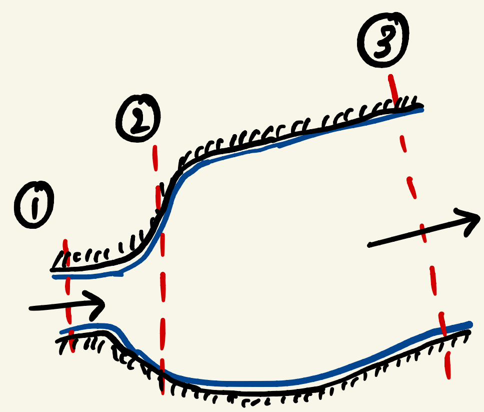
Substituting, we get

$\frac{dp}{\rho} = -V dV$, the Euler's equation

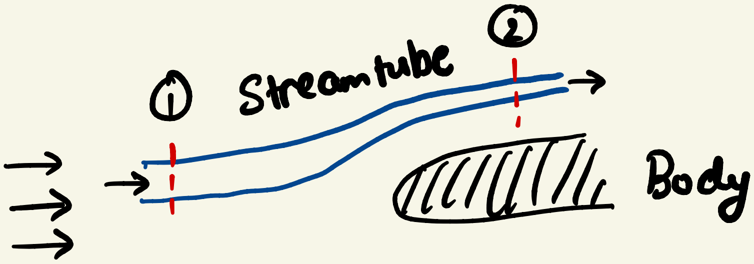
Momentum and energy equations give the same information for an isentropic flow

Examples of quasi 1D flow situations





Flow not 1D at station 2



Mach number

$$M = \frac{\text{gas velocity}}{\text{speed of sound}} = \frac{V}{a}$$

$M < 1$ subsonic

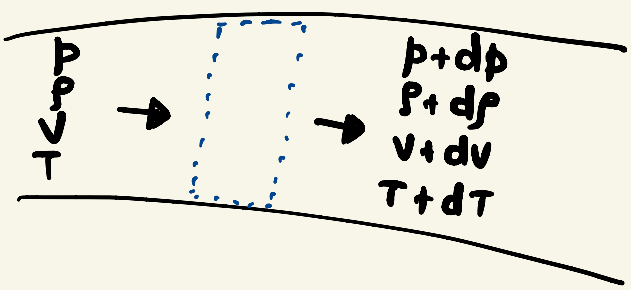
$M \sim 1$ transonic

$M > 1$ supersonic

$M > 5$ hypersonic

$$a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma R T}.$$

Isentropic flow in a Streamtube



Pressure

$$\frac{dp}{\rho} = -V\,dV \Rightarrow \frac{dp}{p} = -\frac{\gamma \rho V^2}{\gamma p}\frac{dV}{V}$$

$$\frac{dp}{p} = -\gamma \frac{V^2}{a^2}\frac{dV}{V} \quad \text{or}$$

$$\boxed{\frac{dp}{p} = -\gamma M^2 \frac{dV}{V}}$$

Fractional change in pressure depends on M^2

Temperature

$$C_p\, dT = -V\, dV$$

$$\frac{dT}{T} = -\frac{\gamma-1}{R\gamma}\frac{V^2}{T}\frac{dV}{V} = -(\gamma-1)M^2\frac{dV}{V}$$

$$\boxed{\frac{dT}{T} = -(\gamma-1)M^2\frac{dV}{V}}$$

Density

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T} \Rightarrow \frac{d\rho}{\rho} = \frac{dp}{p} - \frac{dT}{T}$$

$$\frac{d\rho}{\rho} = -\gamma M^2\frac{dV}{V} + (\gamma-1)M^2\frac{dV}{V}$$

$$\boxed{\frac{d\rho}{\rho} = -M^2\frac{dV}{V}}$$

M is the parameter that determines the importance of compressibility

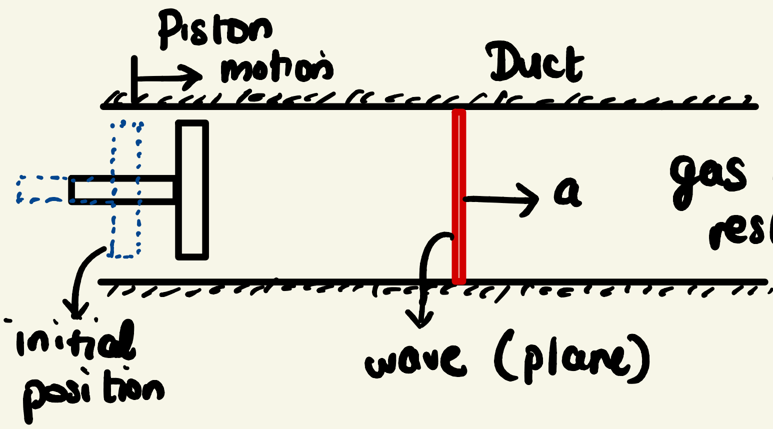
$$\frac{d\rho/\rho}{dV/V} = -M^2$$

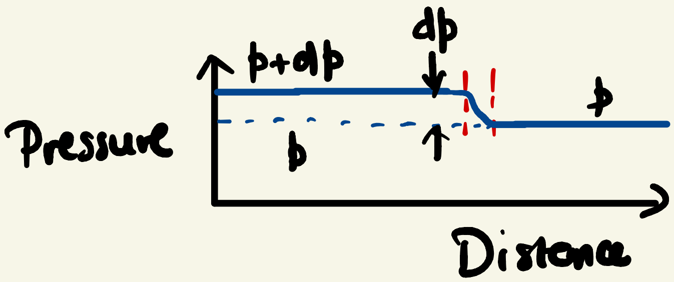
$M = 0.1$ $d\rho = 1\%\, dV$
$M = 0.33$ $d\rho = 10\%\, dV$
$M = 0.4$ $d\rho = 16\%\, dV$

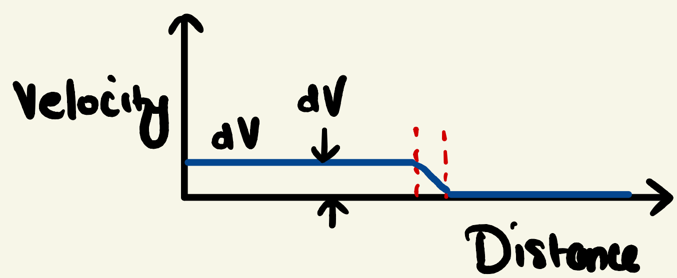
The Mach number at which compressibility becomes important depends on the flow situation and required accuracy

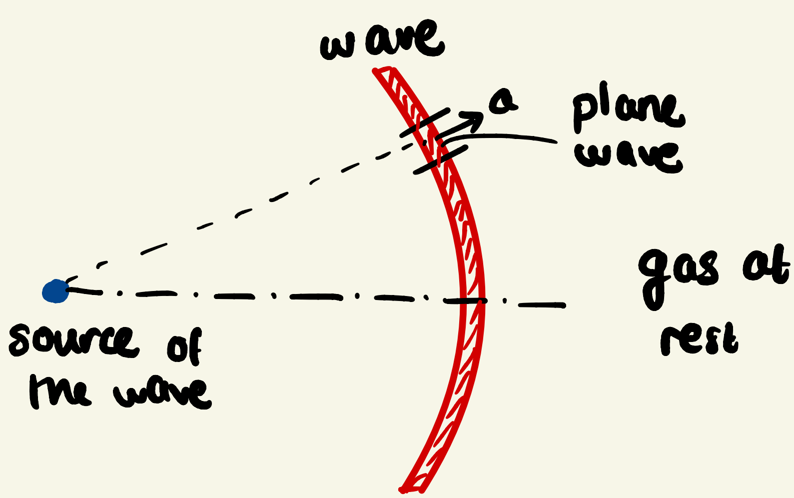
Speed of sound

Speed at which very weak pressure waves are transmitted through the gas

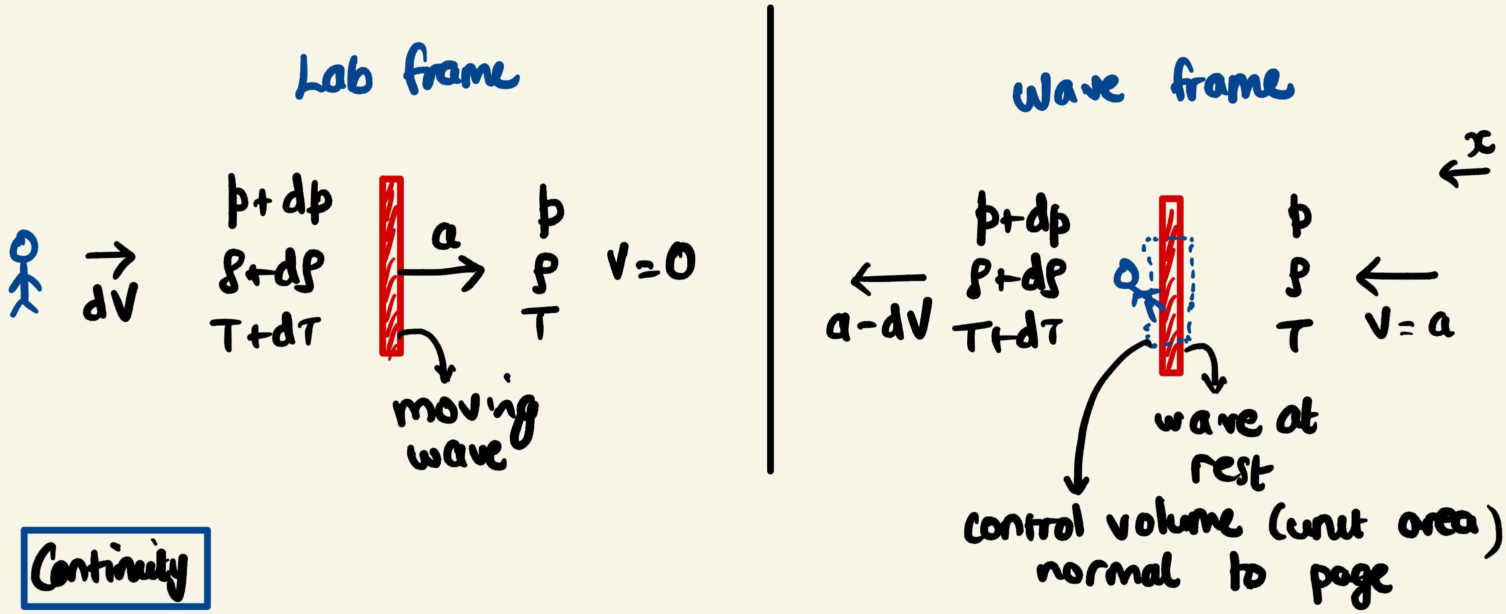








Plane wave is a small, effectively plane portion of a spherical wave moving outward through the gas from a point of disturbance



Continuity

$$\frac{\dot{m}}{A} = \rho a = (\rho+d\rho)(a-dV)$$

$$\Rightarrow \quad 0 = d\rho\, a - \rho\, dV$$

$$d\rho = \frac{\rho}{a} dV \qquad \text{— } (1)$$

Momentum

$$pA - (p+dp)A = \dot{m}\left[(a-dV) - a\right]$$

$$-dp = \frac{\dot{m}}{A}(-dV)$$

$$dp = \rho a\, dV \qquad \text{— } (2)$$

$$(2)/(1) \quad \Rightarrow \quad \frac{dp}{d\rho} = \frac{\rho a}{\rho / a} = a^2$$

$$\boxed{a = \sqrt{\frac{dp}{d\rho}}}$$

To evaluate, we need to know the process gas undergoes in passing through the wave.

Wave is weak $\Rightarrow$ dV, dT are small

i.e. gradients of velocity and temperature negligible

$\Rightarrow$ heat transfer and viscous effects on the flow through the wave are negligible.

$\Rightarrow$ Gas undergoes an isentropic process

$$\frac{p}{\rho^\gamma} \;\; \frac{p}{\rho^\gamma} = \text{constant} = C$$

$$\Rightarrow \left.\frac{dp}{d\rho}\right)_s = \gamma C \rho^{\gamma-1} = \frac{\gamma p}{\rho} = \gamma R T$$

$$\boxed{a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma R T}}$$

Speed of sound depends only on the absolute temperature for a given gas

Also,

$$\boxed{a = \sqrt{\gamma \frac{R_u}{m} T.}}$$

m = molar mass of the gas

γ does not vary greatly between gases

Speed of sound at a given T is approximately inversely proportional to $\sqrt{m}$

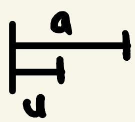
Mach waves

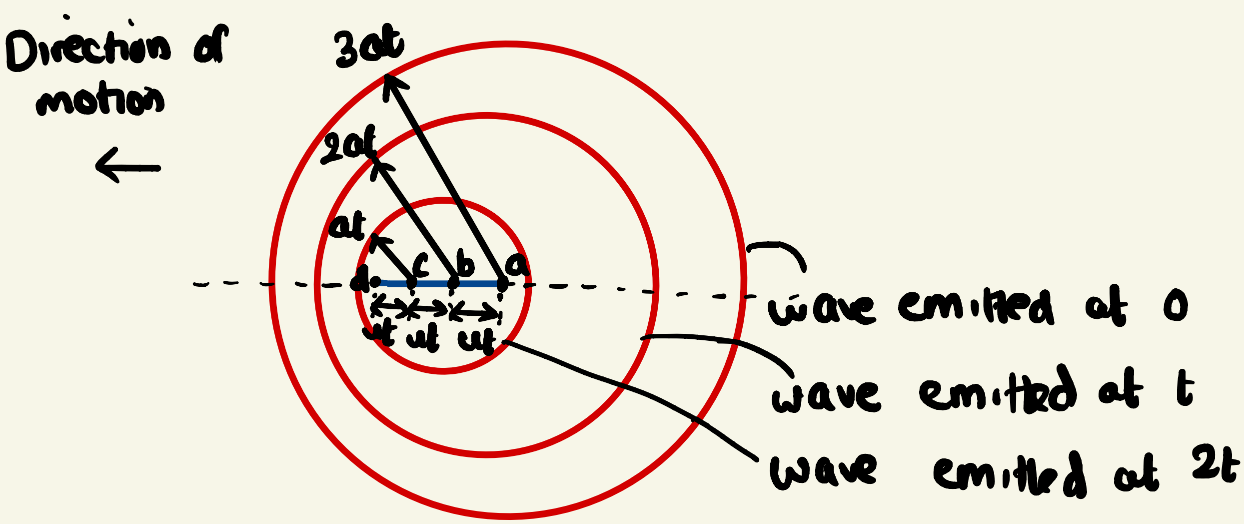
For the gas to move smoothly over a body, disturbances propagate ahead of the body to 'warn' the gas of the approach of the body.

Pressure of the body surface > Pressure of gas

weak pressure waves spread out from the body at the speed of sound

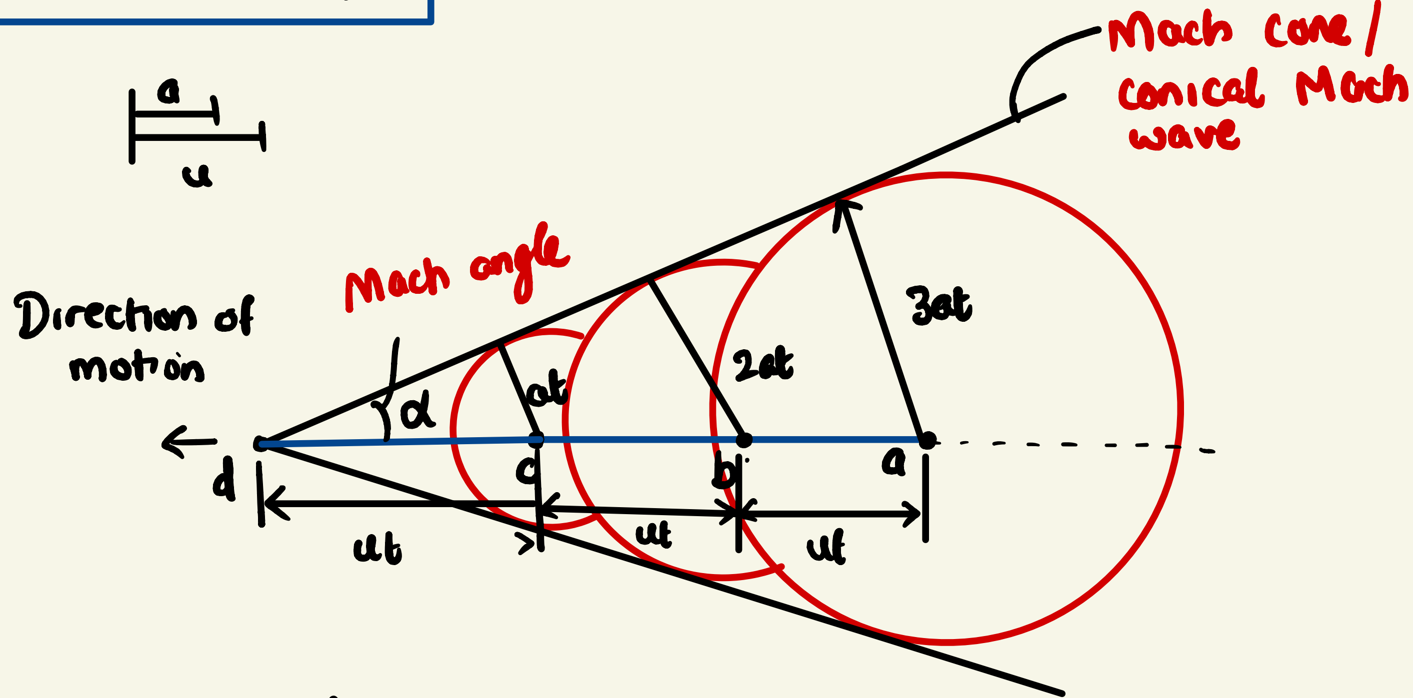
$\boxed{\text{Case 1: } M < 1}$





waves emitted at times 0, t, 2t are shown at 3t

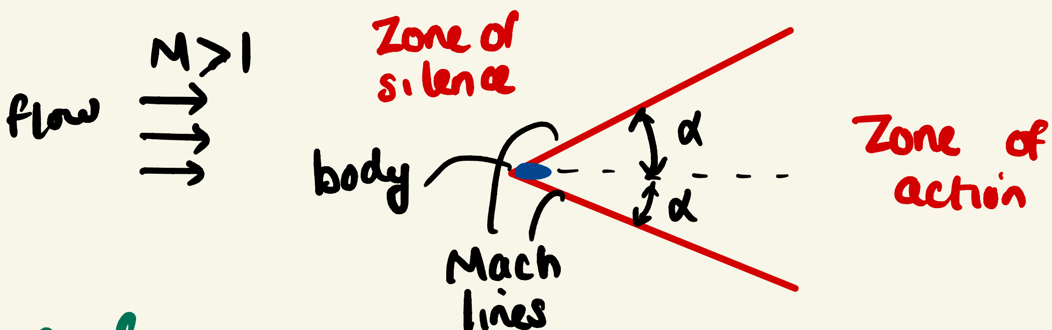
Case 2 : $M > 1$



Waves emitted at times 0, t, 2t are shown at 3t

All waves lie within the cone. Only the gas that lies in this cone is 'aware' of the presence of the body

Jumps in flow variables when flow reaches the cone



Mach angle.

$$\sin\alpha = \frac{at}{vt} = \frac{1}{M}$$

$$\boxed{\alpha = \sin^{-1}\left(\frac{1}{M}\right)}$$

Used in the measurement of Mach number of a gas flow

Problems

① $M_{max} = 0.91$ @ sea level. Find V_{max} at

(a) $T = 5^\circ C$ (b) $T = 45^\circ C$

$$V_{max} = M_{max} \cdot a$$

$$a = \sqrt{\gamma R T} \qquad R = 287.04$$

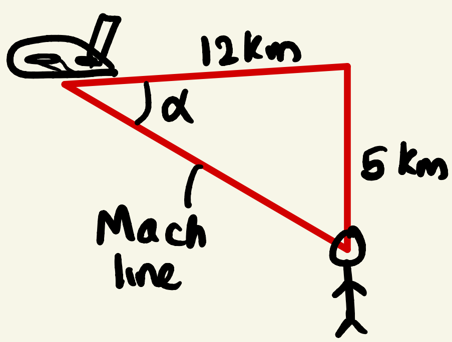
(a) $$V_{max}\big|_{5^\circ C} = 0.91 \cdot \sqrt{1.4 \times 287 \times 278}$$

$$= 304 \text{ m/s}$$

(b) $$V_{max}\big|_{45^\circ C} = 325 \text{ m/s}$$

Better to set land speed records on a hot summer day

② Find V if T @ 5km $= 271.9$ K



$$a = \sqrt{1.4 \cdot 287.04 \cdot 271.9}$$

$$= 330.6 \text{ m/s}$$

$$\tan\alpha = \frac{5}{12} = 0.417 = \frac{1}{\sqrt{M^2-1}}$$

$$\Rightarrow M = \sqrt{1 + (1/0.417)^2} = 2.6$$

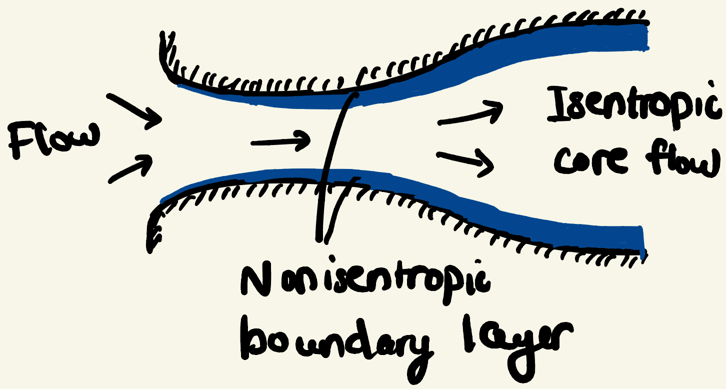
$$V = 2.6 \times 330.6 = 859.6 \text{ m/s}$$

1D isentropic flow

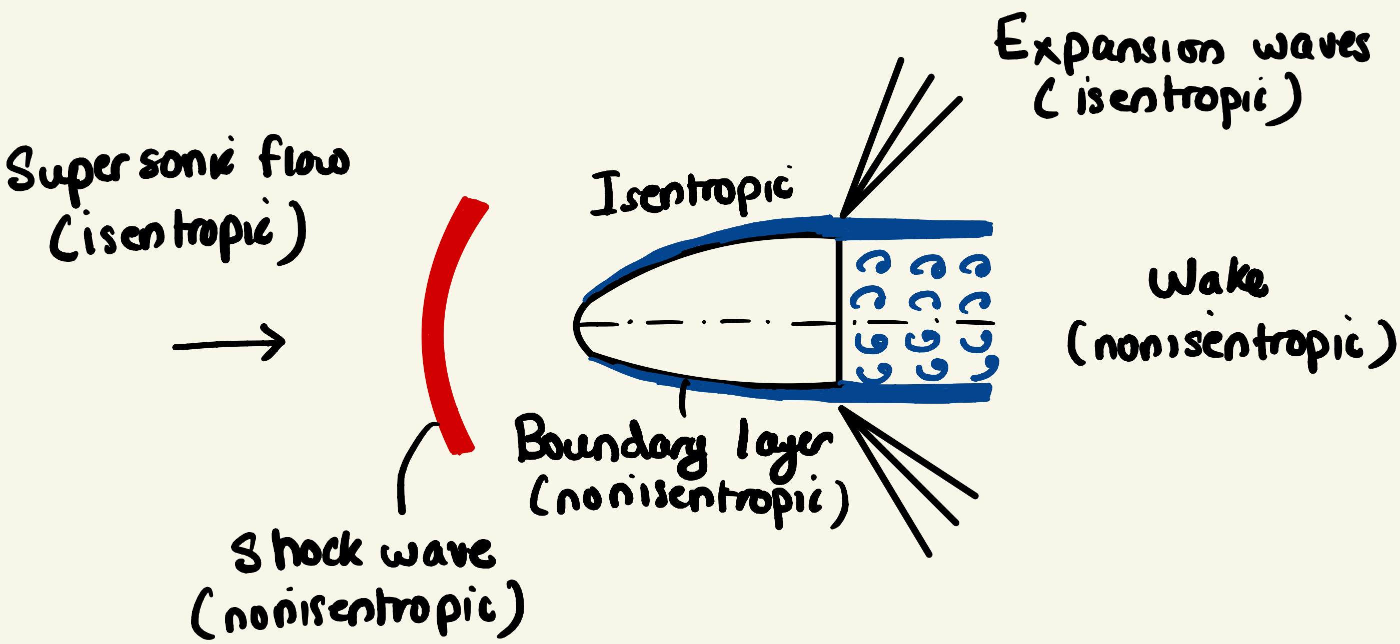
Effects of viscosity and heat transfer are restricted to the boundary layers, wakes and shock waves

$\Rightarrow$ rest of the flow isentropic

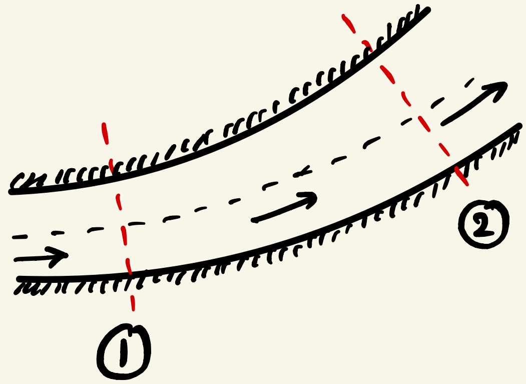
Internal flow



External flow



Governing equations



$$\frac{p}{\rho^{\gamma}} = \text{constant}$$

$$\Rightarrow \frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

Also $\dfrac{p_1}{\rho_1 T_1} = \dfrac{p_2}{\rho_2 T_2} \Rightarrow \dfrac{T_2}{T_1} = \dfrac{p_2}{p_1} \cdot \dfrac{\rho_1}{\rho_2}$

$$\frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma-1}$$

Now $a = \sqrt{\gamma R T}$

$$\Rightarrow \boxed{\frac{a_2}{a_1} = \left(\frac{T_2}{T_1}\right)^{1/2} = \left(\frac{\rho_2}{\rho_1}\right)^{\frac{\gamma-1}{2}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{2\gamma}}}$$

Energy equation

$$C_p T_1 + \frac{V_1^2}{2} = C_p T_2 + \frac{V_2^2}{2}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{1 + V_1^2/2C_pT_1}{1 + V_2^2/2C_pT_2}$$

$$\frac{V^2}{2C_pT} = \frac{\gamma-1}{2} M^2$$

$$\Rightarrow \boxed{\frac{T_2}{T_1} = \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}}$$

This equation applies in adiabatic flow

L 1

$$\boxed{\frac{p_2}{p_1} = \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{\gamma}{\gamma-1}}} \quad (2)$$

$$\boxed{\frac{\rho_2}{\rho_1} = \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{1}{\gamma-1}}} \quad (3)$$

These two equations apply in isentropic flows only

Continuity equation

$$\rho_1 V_1 A = \rho_2 V_2 A_2$$

$$\Rightarrow \boxed{\left(\frac{\rho_2}{\rho_1}\right)\cdot\left(\frac{V_2}{V_1}\right) = \frac{A_1}{A_2}} \quad (4)$$

Eqns. 1 – 4 sufficent to determine all features of 1D isentropic flow (memorize these!)

Why did we not use momentum equation?

$$\frac{V_2^2}{2} - \frac{V_1^2}{2} + \int_1^2 \frac{dp}{\rho} = 0$$

$$\boxed{\begin{aligned} \frac{p}{\rho^\gamma} &= \frac{p_1}{\rho_1^\gamma} \\ \frac{1}{\rho} &= \left(\frac{p_1}{p}\right)^{\frac{1}{\gamma}} \cdot \frac{1}{\rho_1} \end{aligned}}$$

$$\int_1^2 \frac{dp}{\rho} = \int_1^2 \frac{dp}{(p/p_1)^{\frac{1}{\gamma}} \rho_1}$$

$$= \frac{\gamma}{\gamma-1} \left(\frac{p_1}{\rho_1}\right) \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\Rightarrow V_2^2 - V_1^2 + \frac{2}{\gamma-1} a_1^2 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = 0$$

$$\Rightarrow \frac{V_2^2}{a_2^2} \cdot \frac{a_2^2}{a_1^2} - M_1^2 + \frac{2}{\gamma-1} \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = 0$$

$$\Rightarrow \left[1 + \frac{\gamma-1}{2} M_2^2\right] \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 1 + \frac{\gamma-1}{2} M_1^2$$

or

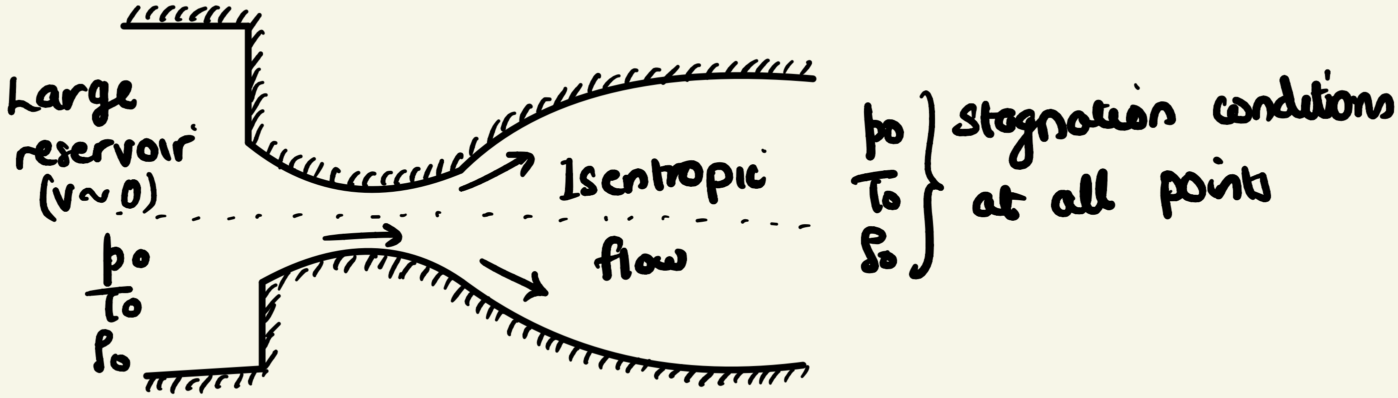
$$\boxed{\frac{p_2}{p_1} = \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}\right]^{\frac{\gamma}{\gamma-1}}}$$

which is an expression we derived without using momentum equation

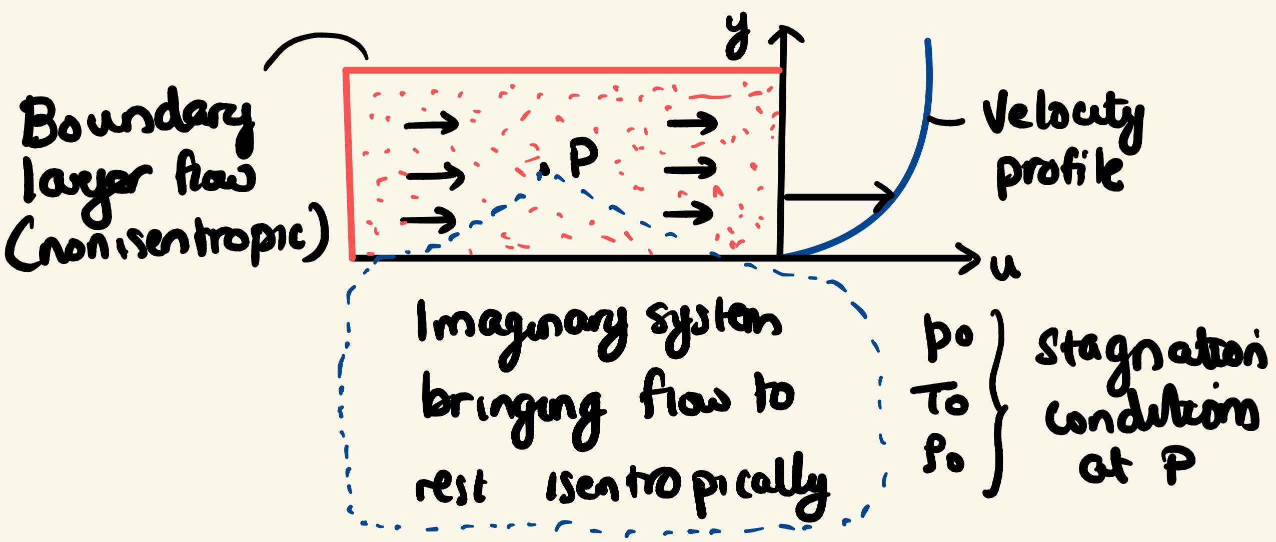
Energy equation is simpler to apply than the momentum equation.

Stagnation conditions

Stagnation conditions are those that would exist if the flow at any point in a fluid stream was isentropically brought to rest



For this isentropic flow, the stagnation conditions at all points in the flow will be those existing at the zero velocity point



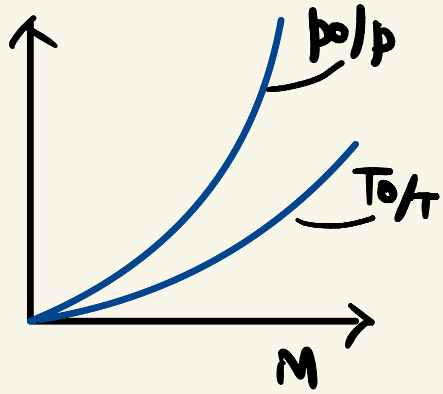
Even for nonisentropic flows, we can imagine stagnation conditions at any local point that would exist if the local flow were brought to rest isentropically

$$\boxed{\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2}$$

$$\boxed{\frac{p_0}{p} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}}$$

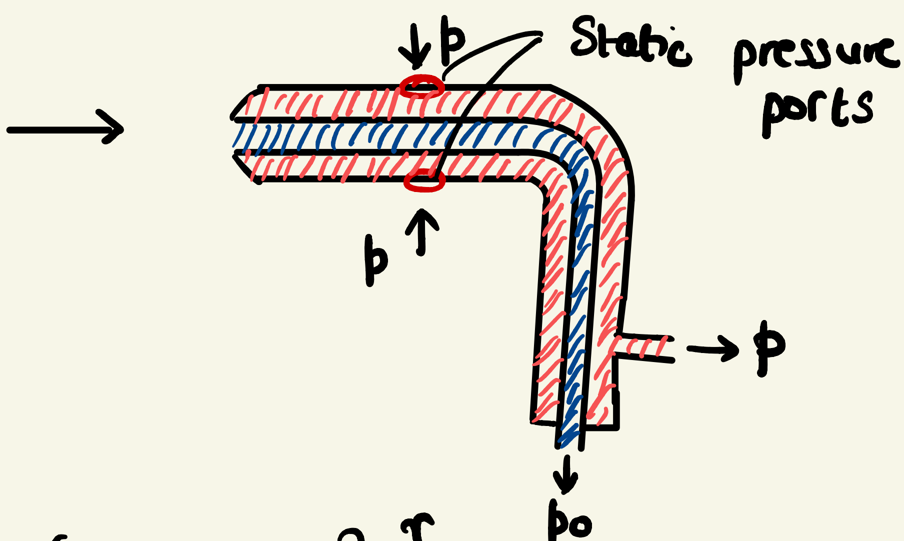
$$\boxed{\frac{\rho_0}{\rho} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{1}{\gamma-1}}}$$

For air, $\gamma = 1.4$



Pitot static tube

$\boxed{M < 1}$ Pitot-static probe can be used to measure Mach number in subsonic flow.



$$\frac{p_0}{p} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \quad \boxed{M = \sqrt{\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}}$$

For incompressible flows,

$$V_{inc} = \sqrt{\frac{2(p_0 - p)}{\rho}}$$

What is the error in using incompressible formula?

For compressible flows,

$$p_0 - p = p\left[\frac{p_0}{p} - 1\right]$$

$$= \left(\frac{1}{2}\rho V^2\right)\left(\frac{2p}{\rho V^2}\right)\left\{\left[1 + \left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}} - 1\right\}$$

$$= \frac{1}{2}\rho V^2\left\{\left(\frac{2}{\gamma M^2}\right)\left[\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]\right\}$$

$$V_{com} = \sqrt{\frac{2(p_0 - p)}{\rho}}\left\{\left(\frac{2}{\gamma M^2}\right)\left[\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]\right\}^{\frac{-1}{2}}$$

Error using incompressible formula

$$\epsilon = \left|\frac{(V_{com} - V_{inc})}{V_{com}}\right| = \left|1 - \frac{V_{inc}}{V_{com}}\right|$$

$$\Rightarrow \boxed{\epsilon = \left|1 - \left\{\left(\frac{2}{\gamma M^2}\right)\left[\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]\right\}^{\frac{1}{2}}\right|}$$

For $M \sim 0.3$ $\epsilon < 1\%$

For $M \sim 0.6$ $\epsilon \sim 5\%$

Problems

③ Subsonic: $p = 96\,kPa$, $T = 300\,K$, $p_0 - p = 32\,kPa$

Find V for (a) incompressible (b) compressible flow

$$\rho = p/RT = 1.115\ kg/m^3$$

(a) $$V_{inc} = \sqrt{\frac{2(32 \cdot 10^3)}{1.115}} = 239.6\ m/s$$

(b) $$\frac{p_0 - p}{p} = \frac{p_0}{p} - 1 = \frac{32}{96}$$

$$\Rightarrow \frac{p_0}{p} = 1.33 = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow M = 0.654.$$

$$V_{com} = Ma = M\sqrt{\gamma RT}$$

$$= 0.654\sqrt{1.4 \cdot 287 \cdot 300}$$

$$= 225.7\ m/s$$

To determine velocity in incompressible flow, only $\Delta p\ (p_0 - p)$ has to be measured

For compressible flows, to find the Mach number p_0 & p have to be separately measured, and for velocity, the temperature also has to be measured to find speed of sound.

Critical conditions

Conditions that would exist if M is isentropically changed to 1. (V^*, p^*, T^*, ρ^*)

$$\boxed{\frac{T^*}{T} = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]}$$

$$\boxed{\frac{a^*}{a} = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{1}{2}}}$$

$$\boxed{\frac{p^*}{p} = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\boxed{\frac{\rho^*}{\rho} = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{1}{\gamma-1}}}$$

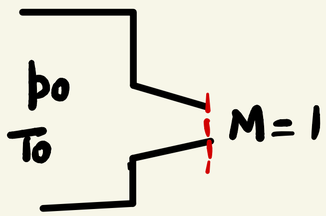
Relations b/n stagnation and critical points

$$\left.\begin{aligned}
\frac{T^*}{T_0} &= \frac{2}{\gamma+1} = \left(\frac{a^*}{a_0}\right)^2 = 0.833 \\
\frac{p^*}{p_0} &= \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.528 \\
\frac{\rho^*}{\rho_0} &= \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} = 0.634
\end{aligned}\right\}$$

Remember these

Problems

(4)



$p_o = 300\,kPa$, $T_o = 50^\circ C$

Find p, T, V @ exit plane for air $(\gamma = 1.4)$

$$p^* = p_o \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 158.5\,kPa.$$

$$T^* = T_o \left(\frac{2}{\gamma+1}\right)$$

$$= 269.2\,K. \quad = -3.8^\circ C$$

$$V = Ma = a$$

$$= \sqrt{1.4 \times 287.04 \times 269.2}$$

$$= 328.9\ m/s$$

↳ For a monoatomic gas (He) , $\gamma = 1.67$

$$p^* = 146.1\,kPa$$

$$T^* = 242.2\,K = -30.8^\circ C$$

$$V_{He} = \sqrt{1.67 \cdot \left(\frac{8314.3}{4}\right) \cdot 242.2}$$

$$= 916.1\ m/s$$

Isentropic tables

M	$\frac{p_0}{p}$	$\frac{T_0}{T}$	$\frac{\rho_0}{\rho}$	$\frac{a_0}{a}$	$\frac{A}{A^*}$
0.1	.	.	.	.	.
.					
1.8	✓	✓			

Problems

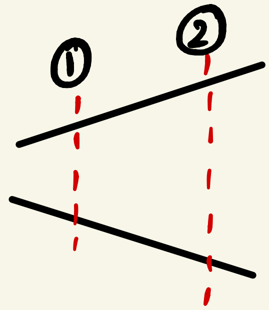
(5) At some point, $V_1 = 600\,m/s$, $p_1 = 70\,kPa$, $T_1 = 5°C$ (steady flow of air)
At some other point, $p_2 = 30\,kPa$
Find: M_2, T_2, V_2 assuming 1D isentropic flow

$$M_1 = 600 / \sqrt{1.4 \times 287.04 \times 278}$$
$$= 1.80$$

From gas tables @ $M_1 = 1.8$
$p_0/p_1 = 5.75$
$T_0/T_1 = 1.65$

known $\frac{p_2}{p_1} = \frac{p_0/p_1}{p_0/p_2}$ known

$$\Rightarrow \frac{p_0}{p_2} = 13.42$$



Inspecting gas tables for this p_0/p, we get

$M_2 = 2.345$ and $T_0/T_2 = 2.10$

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_0/T_1}{T_0/T_2} = \frac{1.65}{2.1} \Rightarrow T_2 = 218.4\,K$$

known (beside T_0/T_1) known (beside T_1)

$$V_2 = 2.345 \times \sqrt{1.4 \times 287.04 \times 218.4} = 694.7\,m/s$$