Constant Specific Thrust Problems

Problem No. 01

Consider a rocket with **following** parameters.

$\mathbf{m_0 = 74}$ **Tons,** $m_p = 54$ Tons, $I_{sp} = 240s$, $g_0 = 9.81 m/s^2$, $V_0 = 90$ m/s, $\boldsymbol{\theta_0 = 3^o}$ and $n_0 = 2.0$.

Determine the velocity at $\theta_b = 90^o$ & **time** taken for it, along with its **feasibility.**

Solution No. 01

The **solution** is as follows.

$$k' = \frac{90}{\left[\tan 1.5 + \tan^3 1.5\right]} = 3434.6 m/s$$

$$V_b = k' \times \left[\tan 45 + \tan^3 45\right] = 6869.2 m/s$$

$$\Delta t = \frac{3434.6}{9.81}\left[1 + \frac{1}{3} - \tan 1.5 - \frac{1}{3}\tan^3 1.5\right] = 457.6 s$$

$$\frac{m_0}{m_b} = e^{\left(\frac{n_0 \bar{g}}{g_0 I_{sp}}\right)\Delta t} = e^{\frac{2\times 457.6}{240}} = 45.318 \rightarrow m_p = 72.4\ \text{T (Not feasible)}$$

Problem No. 02

As the **configuration** in the previous example is **infeasible**, let us now arrive at a **value** of 'n_0' that will make the mission **feasible**.

In this regard, we **restate** the problem as follows; Determine, '$\mathbf{n_0}$', the velocity at $\theta_b = 90^o$ and total **time** taken, if 54T of **propellant** is to be consumed.

Solution No. 02

The **applicable** design procedure is as **follows**.

$$\frac{m_0}{m_b} = 3.7 = e^{\left(\frac{n_0}{240}\right)\times\Delta t} \rightarrow n_0 \times \Delta t = 314$$

$$\Delta t = \frac{k'}{9.81}\left[\frac{1-(0.026)^{n_0-1}}{n_0-1} + \frac{1-(0.026)^{n_0+1}}{n_0+1}\right]$$

$$k' = \frac{90}{\left[(0.026)^{n_0-1} + (0.026)^{n_0+1}\right]}; \quad V_b = 2k'$$

n_0 is **obtained** through iteration.

Assume a value of $n_0 < 2$ & evaluate Δt from 2nd.

Use this Δt to obtain new n_0 from 1st, which will give a new Δt and so on. (Converged $n_0 = 1.673$).