Department of Aerospace Engineering

ENDTERM: AE 238-AEROSPACE STRUCTURAL MECHANICS

Date: 26/04/2021

Time: 2 hours

Max Marks: 35

Instructions:

- **All questions are compulsory**.
- You are allowed to look at your notes, class slides and books. However, if you are found to have copied the solution from someone else, then both students will receive FR grade in the course.
- Take pictures of your answer-sheets and upload them on Moodle by **11:15 am.**
- If you face any problems with uploading, email your answer-sheets to susmitanaskar@iitb.ac.in. Answer-sheets received after 11:15 am. will not be evaluated.

Q.1 Consider a bar of length L = 1.5 m with elliptical cross section (in the x-y plane) with a = 10 mm, and $a = 3b$, where a is length of semi-major axis and b is the length of semi-minor axis. It is subjected to a torque T_0 at its ends.

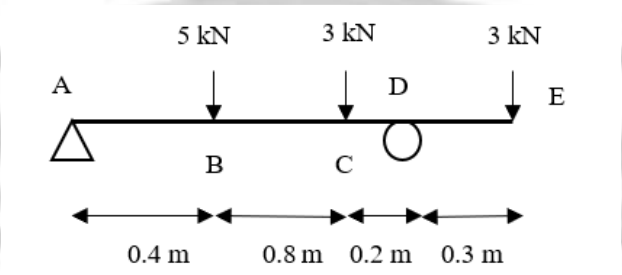
(a) Use the governing equation $\nabla^2\phi + 2G\alpha = 0$ to determine ϕ. Deduce an expression for torsional rigidity. State your assumptions clearly.

(b) If T_0 = 10 N-m, determine the magnitude of angle of twist.

(c) If the yield stress is $\sigma_y = 110$ MPa, calculate the maximum angle of twist at which material starts yielding. Use Tresca yield criterion.

5 marks

Q.2 A beam is loaded as shown in **Fig. 1.** The yield strength of the beam material is 320 MPa. Two different cross-sections are used. Each of them has an area of 393.75π mm^2. The first one is a hollow circular section with the inner diameter to outer diameter ratio equal to 0.75 and the second one is a square section. Use Rankine's theory to decide which of the two designs is safer. Find the factor of safety in both the cases.



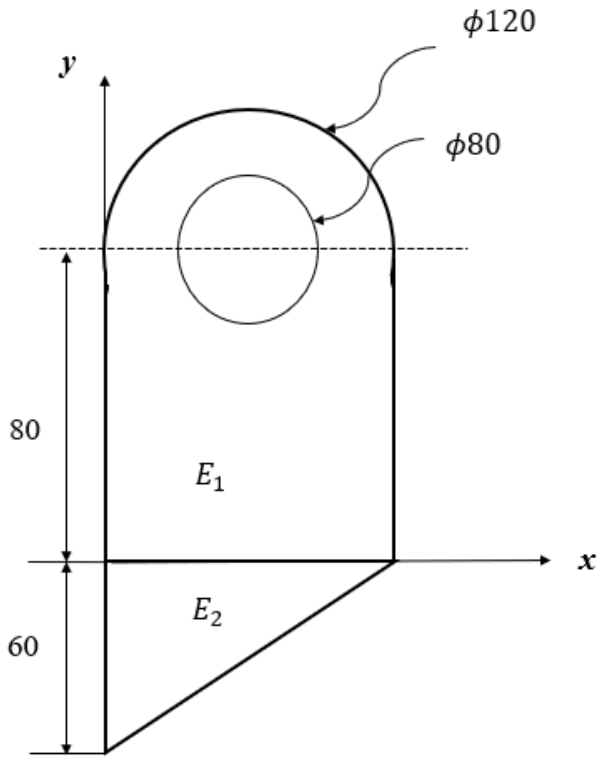
5 marks

Fig. 1

Q.3 A plane section made of two materials of moduli E_1 and E_2 respectively is shown in **Fig. 2.** A hole of diameter 80 mm has been cut from material 1. Given that E_1 = 180 GPa and E_2 = 77 GPa.

(a) Calculate the modulus weighted centroid location (with respect to E_1 as well as E_2).

(b) When $E_1 = E_2$, that is the elastic moduli is same everywhere, compute I^*_{xx}, I^*_{xy}, and I^*_{yy}.



All dimensions are in mm

Fig. 2

5 marks

Q. 4 The displacements in an elastic material are given by

$$u = -\frac{M(1-\nu^2)}{EI}xy,\ v = \frac{M(1+\nu)\nu}{2EI}y^2 + \frac{M(1-\nu^2)}{2EI}\left(x^2 - \frac{L^2}{4}\right),\ w = 0$$

where M, E, I and L are constant parameters.

(a) Determine the corresponding stress and strain fields.

(b) Show that this problem represents pure bending of a rectangular beam in the *x*-*y* plane.

5 marks

Q. 5 Consider the Z section shown in **Fig. 3**. Find out the bending stress at point A if the section is subjected to a bending moment *M* as shown in the figure (the units of *M*, *E* and the geometric dimensions are consistent).

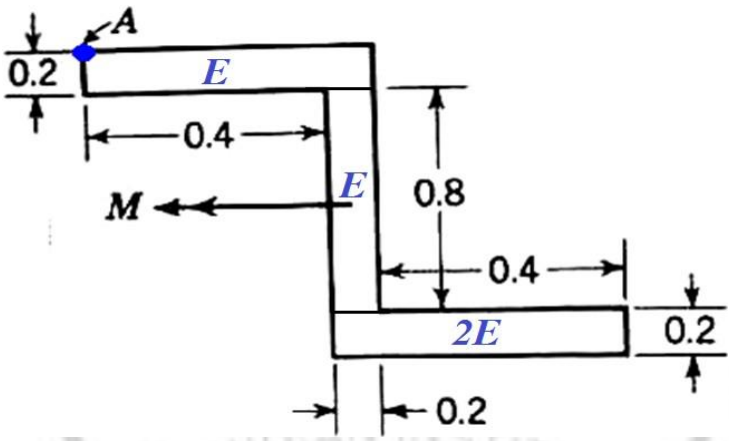


Fig. 3

7 marks

Q. 6 Consider the elliptical beam section shown in **Fig. 4**. It is subjected to a torque M_t. For torsional analysis of the section, the Prandtl stress function is given by

$$\psi = -\frac{M_t}{\pi ab}\left(\frac{y^2}{a^2}+\frac{z^2}{b^2}-1\right)$$

Use warping function formulation to find out the torsion constant (J), twist per unit length and the expressions for describing the stress field in the section.

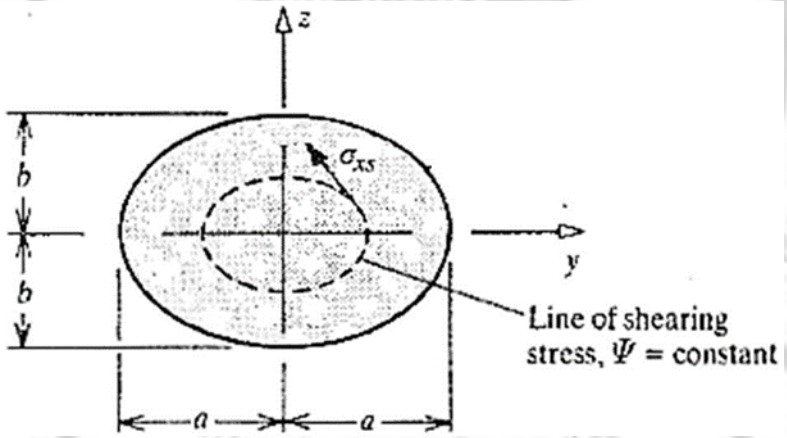


Fig. 4

8 marks