TOF Problems For General Conics

Lambert's Example for Ellipse

A spacecraft is **observed** twice, 90^o **apart**, for which $\mathbf{h_1}$ is **2298 km**, & h_2 is **6476 km**. If 'a' is $\mathbf{12 \times 10^6}$ m, determine (1) Δt between two **observations** and (2) e.

$$r_1 = 6378 + 2298 = 8676 km, \quad r_2 = 6378 + 6476 = 12584 km$$

$$\theta = 90^o; \quad d = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos 90^o} = 15.5 \times 10^6 m,$$

$$a = 12 \times 10^6 m, \quad \cos\alpha = 1 - \frac{r_1 + r_2 + d}{2a} = -0.5317;$$

$$\alpha = 2.145 rad; \quad \cos\beta = 1 - \frac{r_1 + r_2 - d}{2a} = 0.75;$$

$$\beta = 0.724 rad; \quad \sqrt{\frac{a^3}{\mu}} = 2082 s, \quad TOF = 2580 s = 43 \min$$

Lambert's Example for Ellipse

(2) **Eccentricity** of the orbit:

$$\psi = \alpha - \beta = 1.421 rad, \quad \frac{\cos E_B}{\cos E_A} = \frac{a - r_2}{a - r_1} = \frac{-854000}{3324000}$$

$$\frac{\cos E_B}{\cos E_A} = -0.26; \quad \tan E_A = \frac{1}{\sin\psi}\left(\cos\psi - \frac{\cos E_B}{\cos E_A}\right)$$

$$\tan E_A = 0.4138, \quad E_A = 0.3923 rad; \quad e = \frac{a - r_1}{a \cos E_A}$$

$$e = 0.2997$$

Lambert's Example for Parabola

An object is **sighted** twice at an angular separation of 170.5^o at distances of **6378** km and **920,000** km respectively. If it is **known** that object is on a **parabolic** path, determine time **elapsed** between two **observations.**

$$\mu = 3.986 \times 10^{14} m^2 / s^2$$

$$r_1 = 6.378 \times 10^6 m; \quad r_2 = 9.2 \times 10^8 m; \quad d = 9.26291 \times 10^8 m$$

$$TOF = 665697 s = 11094.9 \min = 184.9 h (\text{Exact: } 182.2 h)$$

Lambert's Example for Hyperbola

An object is **sighted** twice at an angular **separation** of 135.5^o at distances of **6378** km and **920,000 km** respectively. If it is **known** that object is on a **hyperbolic** path, and has a = -16.89×10^6 m, determine Δt**.**

$$\boxed{\mu = 3.986\times10^{14}\, m^2/s^2}$$

$$r_1 = 6.378\times10^6\, m;\quad r_2 = 9.2\times10^8\, m;\quad d = 9.2456\times10^8\, m$$

$$\sinh\frac{\alpha}{2} = 5.234;\quad \sinh\frac{\beta}{2} = 0.164;\quad \alpha = 4.7146;\quad \beta = 0.3265$$

$$TOF = 177513s = 2958.6\min = 49.3h$$

Lambert's Example – 'a' Estimation

A spacecraft is **observed** twice**, 90°** & 42.9 min **apart**, for which $\mathbf{h_1}$ is **2298 km**, & $\mathbf{h_2}$ is **6476 km**. Determine 'a'.

$$r_1 = 6378 + 2298 = 8676 km, \quad r_2 = 6378 + 6476 = 12584 km$$

$$d = 15.5 \times 10^6 m, \quad r_1 + r_2 + d = 37.02 \times 10^6 m; \quad r_1 + r_2 - d = 6.03 \times 10^6 m$$

Lambert's Example – 'a' Estimation

Let us take a **guess** for 'a' midway between **'r_1' & 'r_2'** as 10.8×10^6 m. With this, we get $\alpha = 2.36$ & $\beta = 0.765$, $(1/n) = 1778$s and **$\Delta t = 46.9$ min.**

Next, we take **$a = 13\times10^6$ m**, for which we get **$\Delta t = 40.6$ min**. We can now set up an iteration **to refine it.**