AE 236: Compressible Fluid Mechanics

(Module IV: Variable Area Flow)

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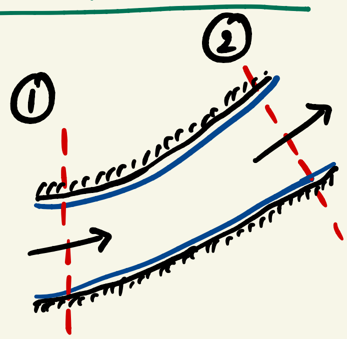
Quasi - 1D flow



$\frac{dA}{dx}$ is small

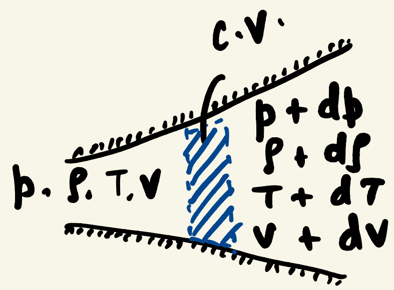
Assumptions

1. Flow is isentropic (unless otherwise mentioned)
2. Neglect effects of friction and heat transfer
3. Steady flow

Effects of dA on dV (or dM)

$\boxed{\text{Mass}}$ $\rho A V$ = mass flow rate = constant

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \quad — \quad (1)$$



$\boxed{\text{Energy}}$ $C_p T + \frac{V^2}{2} = \text{constant}$

$$C_p dT + V dV = 0 \quad — \quad (2)$$

$\boxed{\text{State}}$ $p = \rho R T$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T} \quad — \quad (3)$$

$\boxed{\text{Isentropic}}$ p/ρ^{γ} = constant

$$\frac{dp}{p} = \gamma \frac{d\rho}{\rho} \quad — \quad (4)$$

Eqn. 2 can be written as

$$\frac{dT}{T} + \frac{V^2}{C_p T}\frac{dV}{V} = 0 \quad — \text{⑤}$$

Substitute 4 and 5 in 3

$$(\gamma-1)\frac{d\rho}{\rho} + \frac{V^2}{C_p T}\frac{dV}{V} = 0$$

$$\frac{V^2}{C_p T} = (\gamma-1) M^2 \Rightarrow \frac{dT}{T} + (\gamma-1) M^2 = 0 \quad — \text{⑥}$$

$$\Rightarrow \cancel{(\gamma-1)}\frac{d\rho}{\rho} + \cancel{(\gamma-1)} M^2 \frac{dV}{V} = 0$$

$$\frac{d\rho}{\rho} = -M^2\frac{dV}{V} \quad — \text{⑦}$$

Substitute 7 in 1

$$-M^2\frac{dV}{V} + \frac{dA}{A} + \frac{dV}{V} = 0$$

$$\boxed{\frac{dA}{A} = (M^2-1)\frac{dV}{V}}$$

or

$$\boxed{\frac{dA}{dV} = (M^2-1)\frac{A}{V}}$$

Cases

1. $M<1$, dA and dV have opposite signs

$$dA > 0 \Leftrightarrow dV < 0$$
$$dA < 0 \Leftrightarrow dV > 0$$

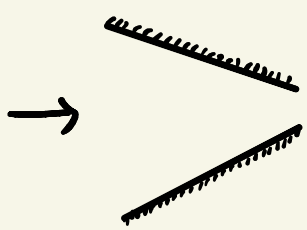
2. $M>1$, dA & dV have the same sign

$$dA < 0 \Leftrightarrow dV < 0$$
$$dA > 0 \Leftrightarrow dV > 0$$

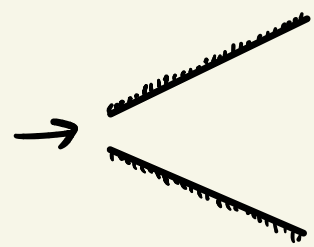
3. $M = 1$, sonic flow, $\frac{dA}{dV} = 0$

$\Rightarrow$ A reaches an extremum (minimum)

$M=1$ can only exist at A_{min}



subsonic nozzle / supersonic diffuser



subsonic diffuser / supersonic nozzle

$$V = Ma$$

$$\frac{dV}{V} = \frac{dM}{M} + \frac{da}{a} = \frac{dM}{M} + \frac{1}{2}\frac{dT}{T}$$

$$\frac{dT}{T} = -(\gamma-1)M^2\frac{dV}{V}$$

From eqn. 6

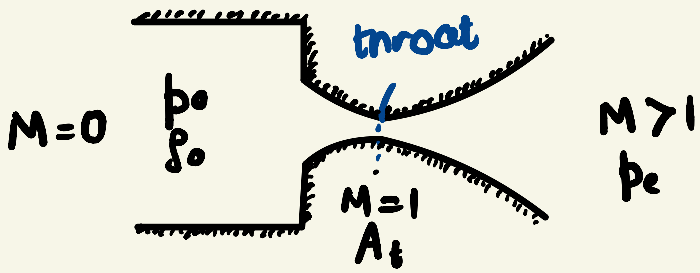
$$\frac{dM}{M} = \left[1+\frac{(\gamma-1)}{2}M^2\right]\frac{dV}{V}$$

$$\boxed{\frac{dA}{A} = \frac{(M^2-1)}{\left[1+\frac{\gamma-1}{2}M^2\right]}\frac{dM}{M}}$$

Cases

1. $M<1$, dA and dM have opposite signs
2. $M>1$, dA and dM have the same sign
3. $M=1$, $dA=0$, A is a minimum (A_t)

To accelerate a subsonic flow to a supersonic flow, we need a convergent-divergent nozzle (c-d nozzle) or a de Laval nozzle



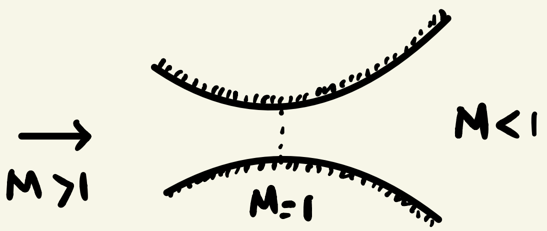
(a) Pressure decreases across the nozzle

(b) $M=1$ at the throat

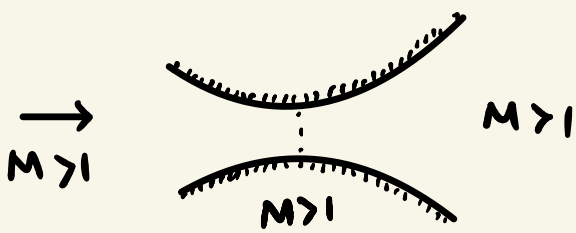
Supersonic flow is achieved only if p_0/p_e is 'large enough'. If p_0/p_e is not large, v increases till flow reaches throat ($M<1$) and then decreases like a venturi; ie, flow is subsonic throughout.

If initial flow is supersonic, two things happen

Case 1

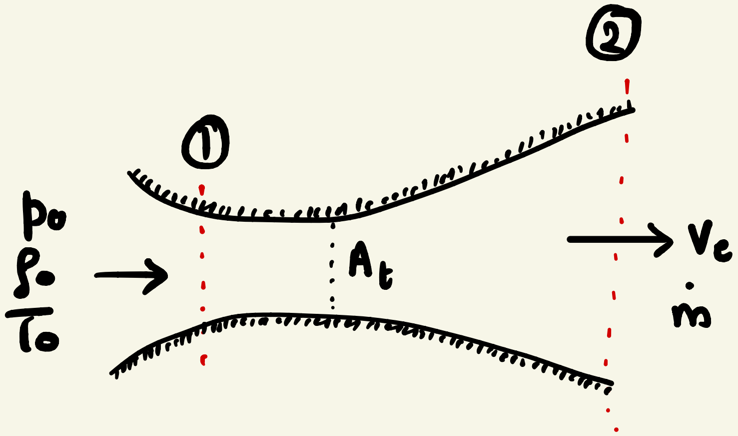


Case 2



Finding V_e, $\dot{m}$, A_t

$$\dot{m} = \rho_1 V_1 A_1 = \text{constant}$$



$$V_1^2 + \frac{2a_1^2}{(\gamma-1)} = \frac{2a_0^2}{\gamma-1} \quad — (1)$$

Flow isentropic

$$\frac{a_1}{a_0} = \left(\frac{T_1}{T_0}\right)^{1/2} = \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{2\gamma}} = \left(\frac{\rho_1}{\rho_0}\right)^{\frac{\gamma-1}{2}}$$

$$V_1 = \left\{\left(\frac{2}{\gamma-1}\right) a_0^2 \left[1 - \left(\frac{a_1}{a_0}\right)^2\right]\right\}^{1/2}$$

$$\boxed{V_1 = \left\{\left(\frac{2\gamma}{\gamma-1}\right) \frac{p_0}{\rho_0} \left[1 - \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2}} \quad — (*)$$

$$\dot{m} = \rho_0 V_1 A_1 \frac{\rho_1}{\rho_0}$$

$$\boxed{\dot{m} = \rho_0 A_1 \left(\frac{p_1}{p_0}\right)^{\frac{1}{\gamma}} \left\{\left(\frac{2\gamma}{\gamma-1}\right)\left(\frac{p_0}{\rho_0}\right)\left[1 - \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2}} \quad (**)$$

At station 2

$$\dot{m} = \rho_0 A_2 \left(\frac{p_2}{p_0}\right)^{\frac{1}{\gamma}} \left\{\left(\frac{2\gamma}{(\gamma-1)}\right)\left(\frac{p_0}{\rho_0}\right)\left[1 - \left(\frac{p_2}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2}$$

$$\boxed{\frac{A_2}{A_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} \left\{\frac{1 - \left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{p_2}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}\right\}^{1/2}} \quad — (***)$$

connects area and pressure ratio

Critical conditions (p^*, T^*, ρ^*)

A convenient reference point in presenting the flow equations in a variable area duct.

Point in the flow at which $M=1$ (may not actually exist)

Energy equation at critical condition gives ($V^* = a^*$)

$$\boxed{V^{*2} = \frac{2}{\gamma+1} a_0^2}$$

Also, $\left(\frac{a^*}{a_0}\right)^2 = \frac{T^*}{T_0} = \frac{2}{\gamma+1}$

$$\Rightarrow \frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}, \quad \frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$

For air ($\gamma = 1.4$) Remember these !!!

$$\frac{T^*}{T_0} = 0.833 \qquad \frac{p^*}{p_0} = 0.528 \qquad \frac{\rho^*}{\rho_0} = 0.634$$

Now (**) at critical condition gives

$$\dot{m} = \rho_0 A^* \left(\frac{p^*}{p_0}\right)^{\frac{1}{\gamma}} \left\{\left(\frac{2\gamma}{\gamma-1}\right)\left(\frac{p_0}{\rho_0}\right)\left[1 - \left(\frac{p^*}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2}$$

$$= \rho_0 A^* \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \left\{\left(\frac{2\gamma}{\cancel{\gamma-1}}\right)\left(\frac{p_0}{\rho_0}\right)\left[\frac{\cancel{\gamma-1}}{\gamma+1}\right]\right\}^{1/2}$$

$$\Rightarrow \boxed{A^* = \frac{\dot{m}}{\sqrt{\gamma p_0 \rho_0}} \left(\frac{2}{\gamma+1}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}} \quad — (\text{**}\text{**})$$

The area at any section of the duct, in terms of the critical area A^* is

$$\frac{A}{A^*} = \left(\frac{p_*}{p}\right)^{\frac{1}{\gamma}} \left\{ \frac{1 - \left(\frac{p_*}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}} \right\}^{1/2}$$

$$\Rightarrow \boxed{\frac{A}{A^*} = \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(\frac{\gamma-1}{2}\right)^{\frac{1}{2}}}{\left\{\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right\}^{\frac{1}{2}}}} \quad - \text{**}$$

Let us say, a nozzle is designed for a given $\dot{m}$, with an overall pressure ratio of p_e/p_0

Discharge velocity is obtained as

$$V_e = \left\{\left(\frac{2\gamma}{\gamma-1}\right)\left(\frac{p_0}{\rho_0}\right)\left[1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{\frac{1}{2}} \quad - (1)$$

Exit area A_e is found from

$$\dot{m} = \rho_0 A_e \left(\frac{p_e}{p_0}\right)^{\frac{1}{\gamma}} \left\{\left(\frac{2\gamma}{\gamma-1}\right)\left(\frac{p_0}{\rho_0}\right)\left[1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{\frac{1}{2}} \quad - (2)$$

Throat area A^* is found from

$$\frac{A_e}{A^*} = \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(\frac{\gamma-1}{2}\right)^{\frac{1}{2}}}{\left\{\left(\frac{p_e}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right\}^{\frac{1}{2}}} \quad - (3)$$

A/A^* in terms of M

Energy equation gives

$$v = M a_0 \left[1 + \frac{\gamma-1}{2} M^2\right]^{-\frac{1}{2}}$$

$$\frac{\rho}{\rho_0} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{-1}{\gamma-1}}$$

$$\frac{\dot{m}}{A} = \rho v = \rho_0 v \cdot \frac{\rho}{\rho_0}$$

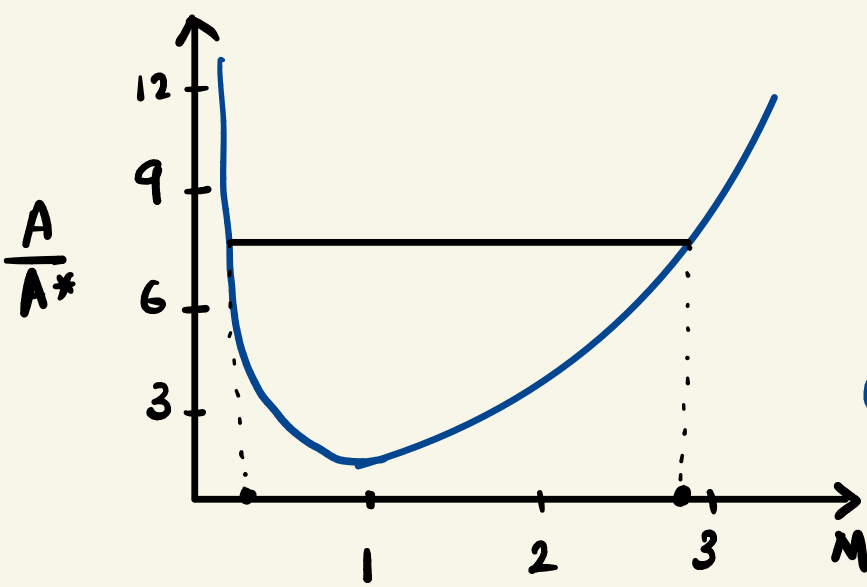
$$= \frac{\rho_0 a_0 M}{\left\{1 + \frac{\gamma-1}{2} M^2\right\}^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$\Rightarrow \boxed{\frac{A_2}{A_1} = \left(\frac{M_1}{M_2}\right)\left\{\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2}\right\}^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$\dot{m}$ constant

For $A_1 = A^*$, $A_2 = A$, $M_2 = M$, $M_1 = 1$, we have

$$\boxed{\frac{A}{A^*} = \frac{1}{M}\left\{\left(\frac{2}{\gamma+1}\right)\left[1 + \frac{\gamma-1}{2} M^2\right]\right\}^{\frac{\gamma+1}{2(\gamma-1)}}} \quad — \quad ***$$



(a) $\frac{A}{A^*}$ reaches a minimum at $M=1$

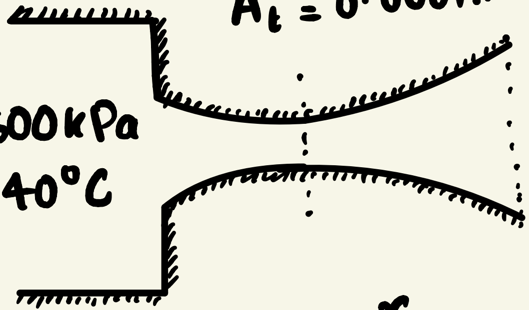
(b) For a given A/A^* there are two M

Problems

① H_2 $\qquad A_t = 0.0001\,m^2 = A^*$ $\qquad$ Find $\dot{m}$, A_e ?

$p_0 = 600\,kPa$

$T_0 = 40^\circ C$



$p_e = 130\,kPa$

$$\frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.528$$

$$\frac{p_e}{p_0} = \frac{130}{600} = 0.2167 < \frac{p^*}{p_0}$$

$\Rightarrow$ M = 1 at throat and flow is supersonic through the nozzle diverging section

$$\dot{m} = \rho_0 A^* \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \left\{ \left(\frac{2\gamma}{\cancel{\gamma-1}}\right) \frac{p_0}{\rho_0} \left(\frac{\cancel{\gamma-1}}{\gamma+1}\right) \right\}^{\frac{1}{2}}$$

$$\rho_0 = \frac{p_0}{RT} \qquad R = \frac{8314.3}{2.016} = 4124.2$$

$\Rightarrow$ $\rho_0 = 0.4648\,kg/m^3$

$\dot{m} = 0.03622\,kg/s$

$$\frac{A_e}{A^*} = \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(\frac{\gamma-1}{2}\right)^{\frac{1}{2}}}{\left\{ \left(\frac{p_e}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma+1}{\gamma}} \right\}^{\frac{1}{2}}}$$

$A_e = 0.0006416\,m^2$

Operating characteristics of nozzles

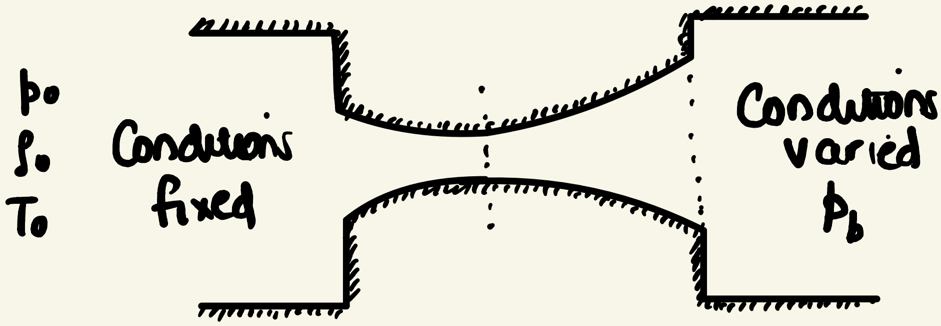
Nozzle – a shape that accelerates gas flow

Diffuser – a shape that decelerates gas flow

Assumptions

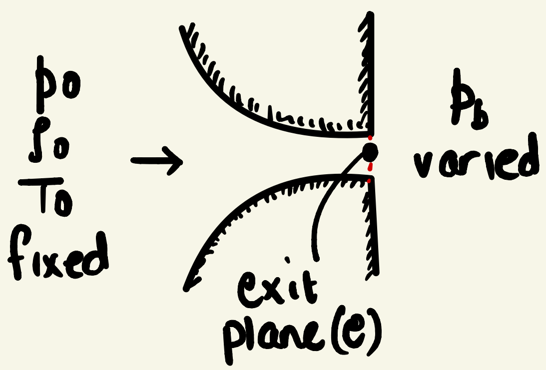
(1) Upstream conditions are stagnation conditions

(2) Only conditions downstream of nozzle varied



Downstream pressure is called **back pressure** (p_b)

Case 1: convergent nozzle



$p_b = p_0 \Rightarrow$ no flow ($M_e = 0$)

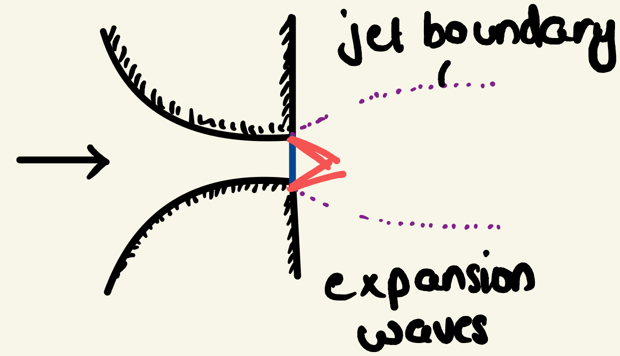
$p_b < p_0 \Rightarrow$ flow ($M_e \neq 0$)

When $p_b = p^* = p_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$. $M_e = 1$

For $p_b > p^*$, reduction in p_b increases the mass flow

For $p_b \leq p^*$, reduction in p_b have no effect on the flow in the nozzle, mass flow rate remains constant and $M_e = 1 \Rightarrow$ Nozzle is **choked**

When $p_b < p^*$, expansion from p_e to p_b takes place outside the nozzle through a series of expansion waves



$\boxed{p_b > p^*}$

$$p_e = p_b$$

discharge velocity

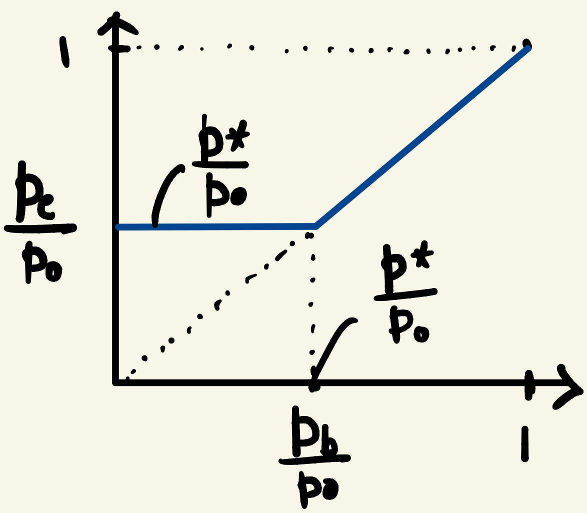
$$V_e = \left\{ \left(\frac{2\gamma}{\gamma-1}\right)\left(\frac{p_0}{\rho_0}\right)\left[1 - \left(\frac{p_b}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{\frac{1}{2}}$$

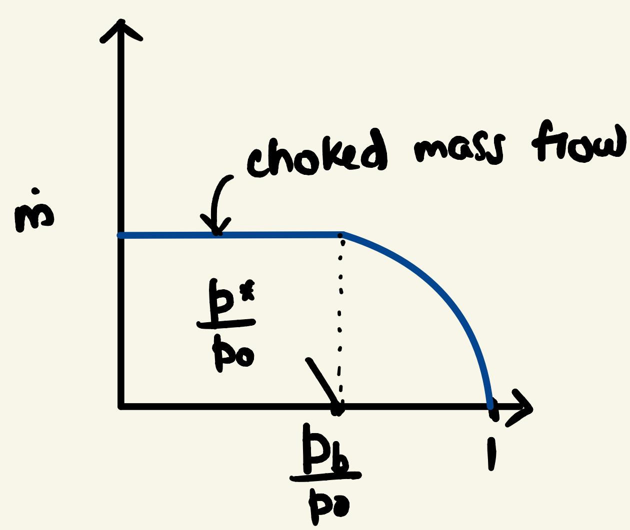
$$\dot{m} = \rho_0 A_e \left(\frac{p_b}{p_0}\right)\left\{ \left(\frac{2\gamma}{\gamma-1}\right)\left(\frac{p_0}{\rho_0}\right)\left[1 - \left(\frac{p_b}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{\frac{1}{2}}$$

$\boxed{p_b \leq p^*}$

$$\left.\begin{aligned} p_e &= p_0 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \\ V_e &= \sqrt{\left(\frac{2\gamma}{\gamma+1}\right)\frac{p_0}{\rho_0}} \\ \dot{m} &= \sqrt{\gamma p_0 \rho_0}\, A_e \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \end{aligned}\right\}$$

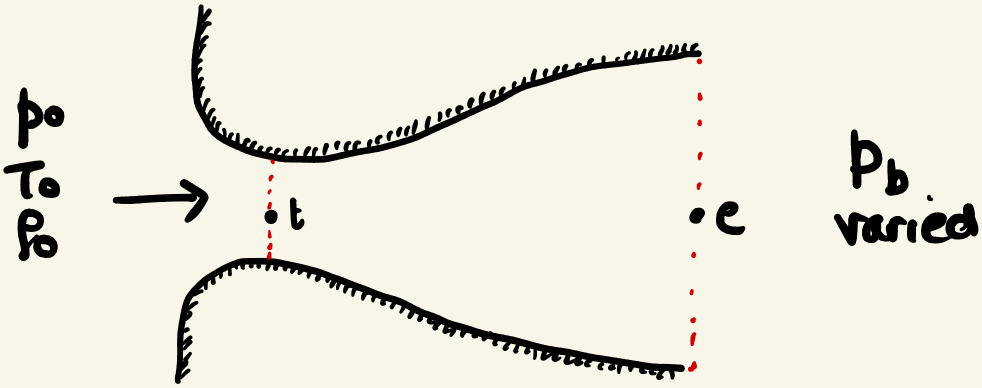
independent of p_b and invariant for $p_b \leq p^*$

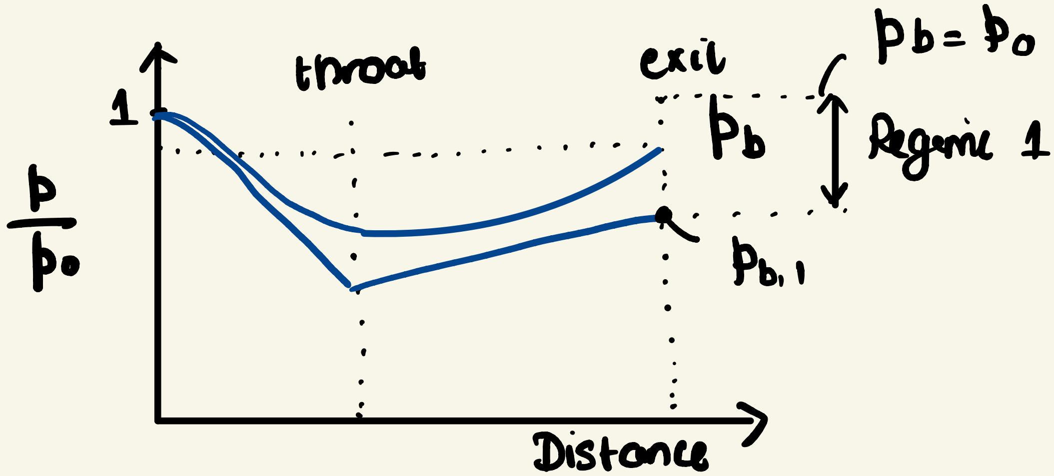




Case 2: Convergent-divergent nozzle

Regime 1 : $p_t > p^*$. Flow subsonic throughout





Nozzle operates like a venturi until p_b decreases enough to make $M_t = 1$

$$p_e = p_b$$

$$\frac{A_e}{A^*} = \left(\frac{p_t}{p_b}\right)^{\frac{1}{\gamma}} \left[\frac{1 - \left(\frac{p_t}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{p_b}{p_0}\right)^{\frac{\gamma-1}{\gamma}}} \right]^{\frac{1}{2}}$$

$$\left(\frac{p_t}{p_0}\right)^{\frac{1}{\gamma}} \left\{ 1 - \left(\frac{p_t}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \right\}^{\frac{1}{2}} = \left(\frac{A_e}{A^*}\right)\left(\frac{p_b}{p_0}\right)^{\frac{1}{\gamma}} \left\{ 1 - \left(\frac{p_b}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \right\}$$

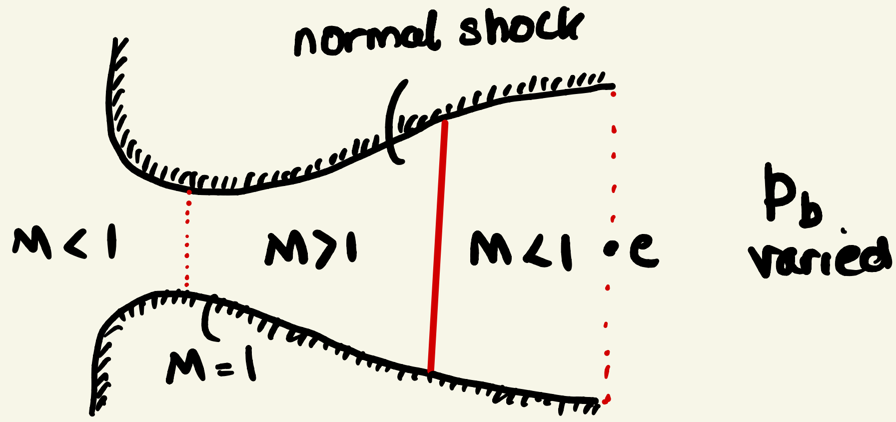
When $M_t = 1$

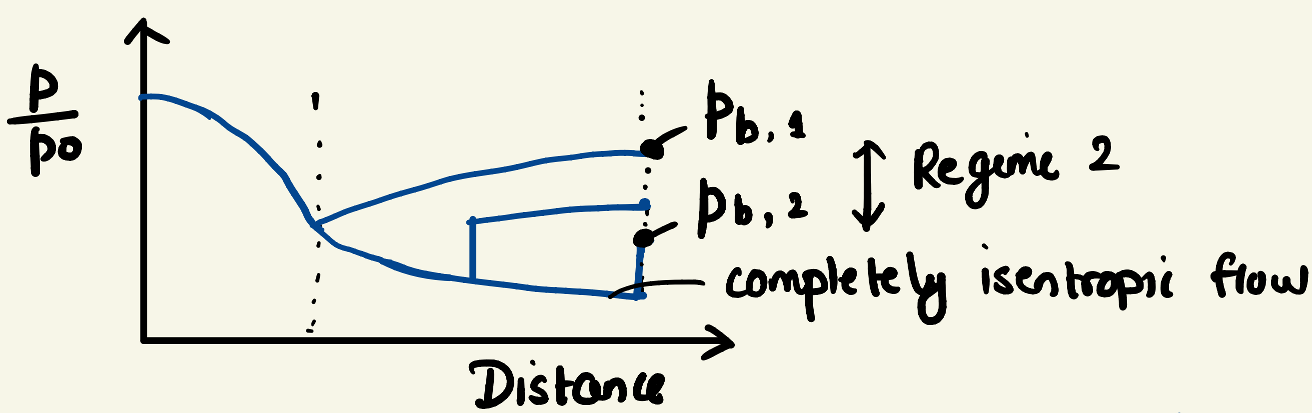
$$p_t = p^* = p_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\left(\frac{A_e}{A^*}\right)\left(\frac{p_{b,1}}{p_0}\right)^{\frac{1}{\gamma}}\left\{1-\left(\frac{p_{b,1}}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right\}^{\frac{1}{2}} = \left(\frac{\gamma-1}{\gamma+1}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$

$p_{b,1} = p_b$ at which $p_t = p^*$

Regime 2: From the end of regime 1 until there is a normal shock at the exit ($p_b = p_{b,2}$)





For $p_b < p_{b,1}$ there will be a normal shock in the divergent portion and this shock will move downstream as p_b is decreased until it reaches the exit at $p_b = p_{b,2}$

As p_b decreases in this regime, we have stronger normal shocks as M upstream is larger.

When the shock reaches the exit plane, flow in the nozzle is isentropic throughout and p_e/p_0 is the design pressure ratio and M_e is the design Mach number

In this limit, we have

$$\left(\frac{p_e}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma+1}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left(\frac{\gamma-1}{2}\right)\left(\frac{A^*}{A_e}\right)^2$$

$$\frac{1}{M_e}\left\{\left(\frac{2}{\gamma+1}\right)\left[1+\left(\frac{\gamma-1}{2}\right)M_e^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}\right\} = \frac{A_e}{A^*}$$

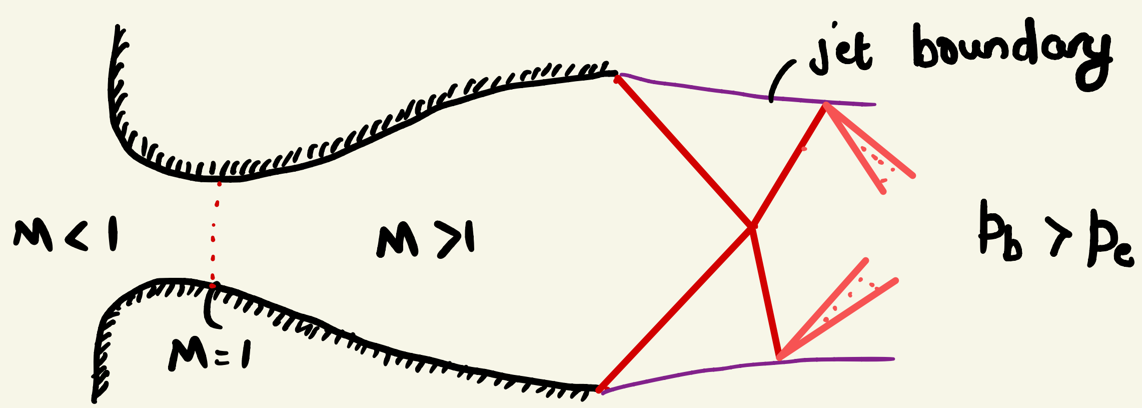
Also, from normal shock relationships

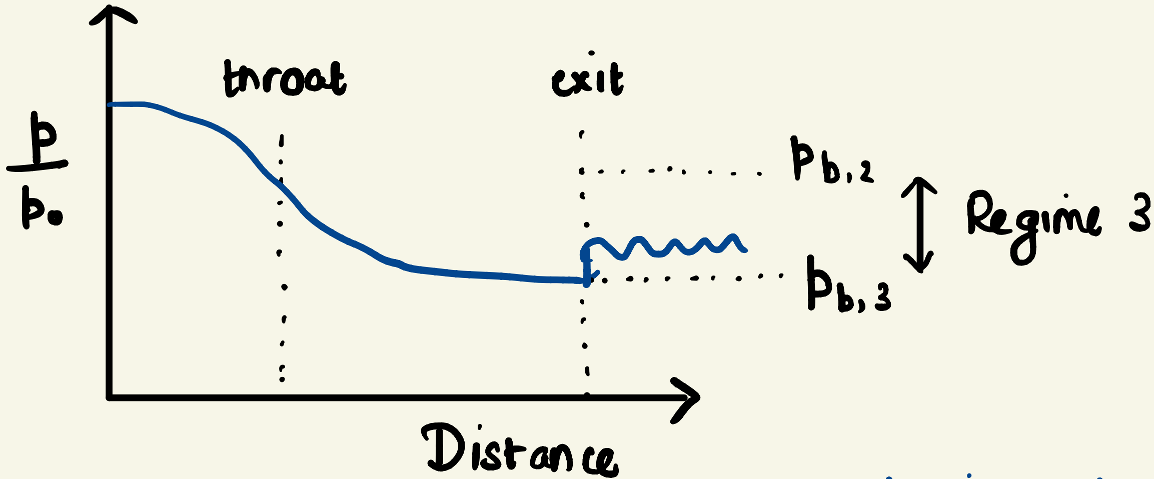
$$\frac{p_{b,2}}{p_e} = \frac{2\gamma M_e^2 - (\gamma-1)}{\gamma+1}$$

$$\frac{p_{b,2}}{p_0} = \frac{p_{b,2}}{p_e}\cdot\frac{p_e}{p_0}$$

Once shock reaches the exit, further reductions in p_b will not affect the nozzle

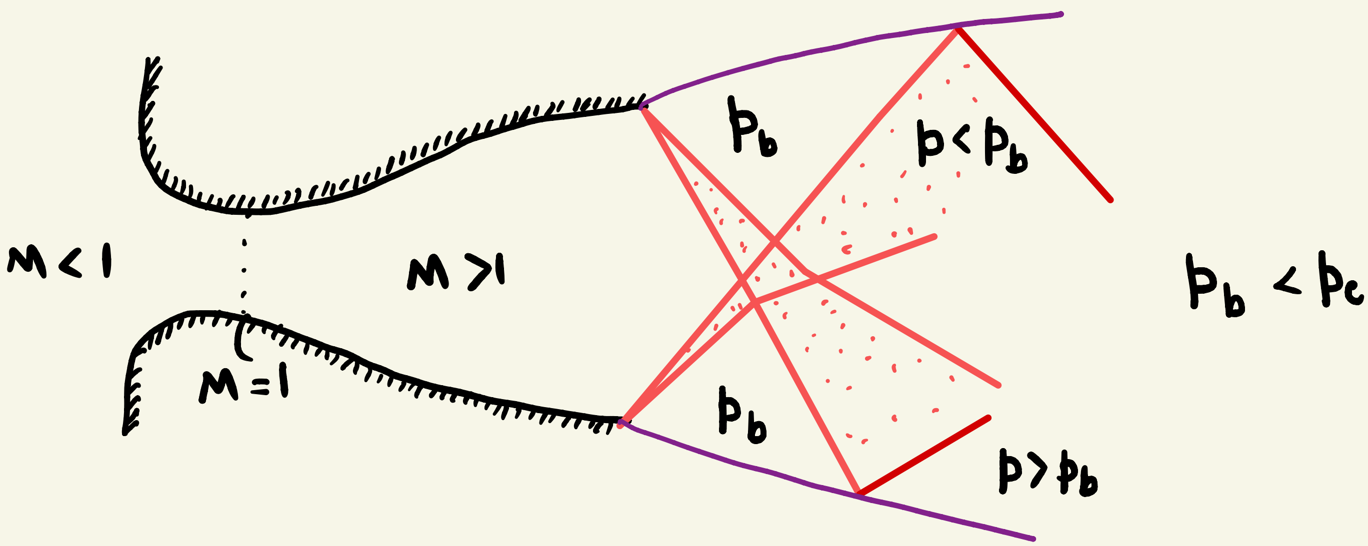
Regime 3 (over expanded regime): p_b at which oblique shocks are present at the nozzle exit

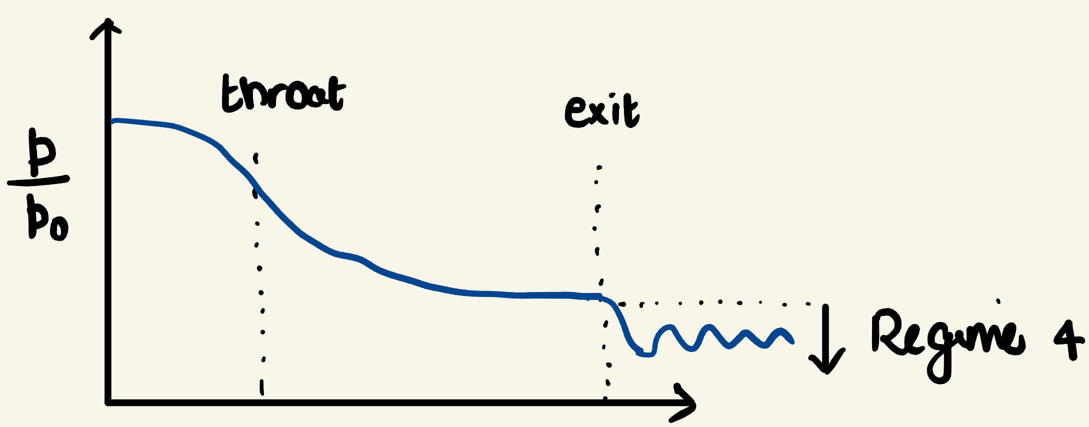




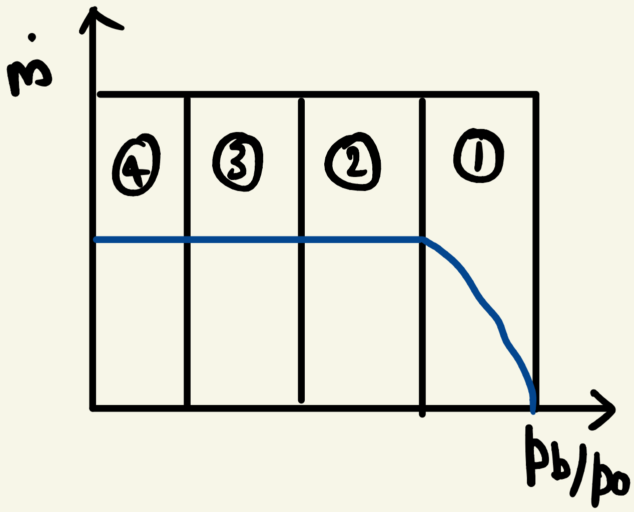
As p_b decreases and approaches p_e, oblique shock becomes weaker until it vanishes at $p_b = p_e$. The nozzle is now operating at design conditions and there are no waves inside or outside the nozzle ($p_b = p_{b,3}$)

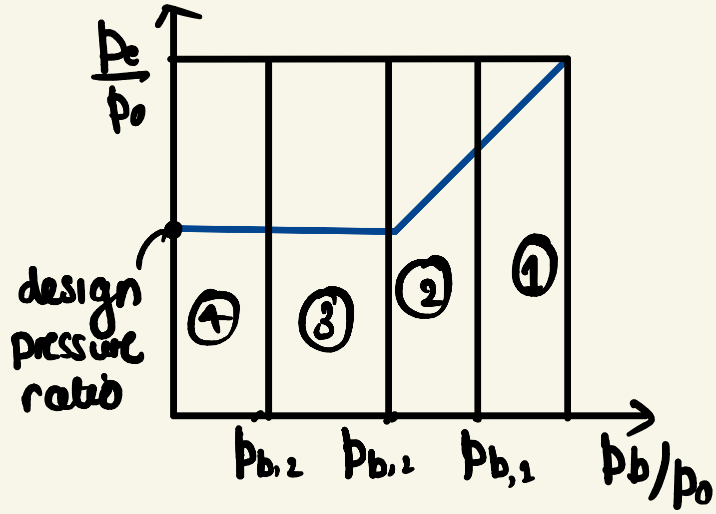
Regime 4 (underexpanded regime): p_b at which expansion waves are present at the nozzle exit





$$\left(\frac{p_b}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p_b}{p_0}\right)^{\frac{\gamma+1}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left(\frac{\gamma-1}{2}\right) \left(\frac{A^*}{A_e}\right)^2$$





Problems

(2) Air flow in a c-d nozzle. Reservoir : $p_0 = 600\,kPa$, $T = 40^\circ C$. $p_{b,design} = 100\,kPa$

Find (a) A_e/A_t (b) V_e (c) p_b at which normal shock is present in the exit plane

(a) $\dfrac{p_e}{p_0} = \dfrac{100}{600} = 0.1667$

From isentropic tables, $M_e = 1.83$

At $M = 1.83$, isentropic tables $\dfrac{A_e}{A^*} = 1.472$

(b) $\dfrac{a_e}{a_0} = 0.7739$ (from tables)

$a_0 = \sqrt{\gamma R T_0} = 354\,m/s$

$v_e = \dfrac{v_e}{a_e} \cdot \dfrac{a_e}{a_0} \cdot a_0 = 502.1\,m/s$

(c) From normal shock tables at $M_e = 1.83$

$$\frac{p_2}{p_1} = 3.74$$

$$\Rightarrow p_b = 3.74 \cdot 100 = 374 \text{ kPa}$$

(3) c-d nozzle : $p_0 = 800 \text{ kPa}$, $T_0 = 40^\circ C$, $M = 2.7$

$A^* = 0.08 m^2$

1. A_e ?

Isentropic tables at $M_e = 2.7$ $\quad \frac{A}{A^*} = 3.183$

$$A_e = 3.183 \times 0.08 = 0.255 m^2$$

2. $\dot{m}_{design}$?

$$\dot{m} = \rho^* V^* A^*$$

$$\rho_0 = \frac{p_0}{RT_0} = 8.91 \text{ kg/m}^3$$

$$\frac{\rho^*}{\rho_0} = 0.63394 \Rightarrow \rho^* = 5.65 \text{ kg/m}^3$$

$$V^* = a^* = \sqrt{\gamma R T^*}$$

$$\frac{T^*}{T_0} = 0.83055 \Rightarrow T^* = 260 K$$

$$V^* = 323.2 \text{ m/s}$$

$$\dot{m} = 5.65 \times 323.2 \times 0.08$$

$$= 146 \text{ kg/s}$$

3. Design back pressure

$$\left.\frac{p_o}{p_b}\right|_{M=2.7} = 23.283 \Rightarrow p_{b,design} = 34.36 \text{ kPa}$$

4. Lowest p_b for which only subsonic flow

$$\frac{A}{A^*} = 3.183 \Rightarrow \frac{p_o}{p_b} = 1.025 \text{ for the subsonic case}$$

$$\left. p_b \right|_{subsonic} = \frac{800}{1.025} = 780.5 \text{ kPa}$$

5. p_b at which normal shock in exit plane

From normal shock tables at $M = 2.7$

$$\frac{p_b}{p_{b,design}} = 8.33832$$

$$\Rightarrow p_b = 8.33832 \times 34.36 = 286.5 \text{ kPa}$$

6. p_b below which no shock waves exist inside the nozzle.

$$p_b < 286.5 \text{ kPa}$$

7. Range of p_b over which oblique shock waves exist

$$34.36 \text{ kPa} < p_b < 286.5 \text{ kPa}$$

8. Range of p_b over which expansion waves exit

$$p_b < 34.36 \text{ kPa}$$

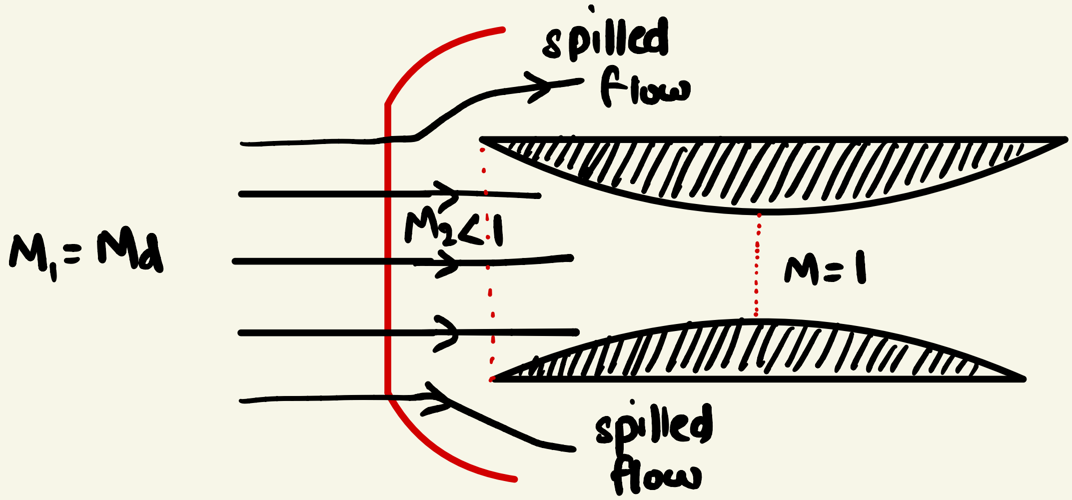
Convergent - Divergent Supersonic Diffusers

Ideally, we want shockless diffusion to $M<1$ before flow reaches engine

Q. What happens at off-design conditions?

Fixed diffusers : A_i, A_t are fixed $\Rightarrow \frac{A^*}{A}$ fixed.

A^*/A is larger at $M < M_d$ which for a fixed diffuser results in a larger mass flow rate trying to enter it. As a result, normal shock is present before the inlet which persists even at $M = M_d$ although the shock is now closer to the intake



Q. How to get rid of the shock in a fixed diffuser?

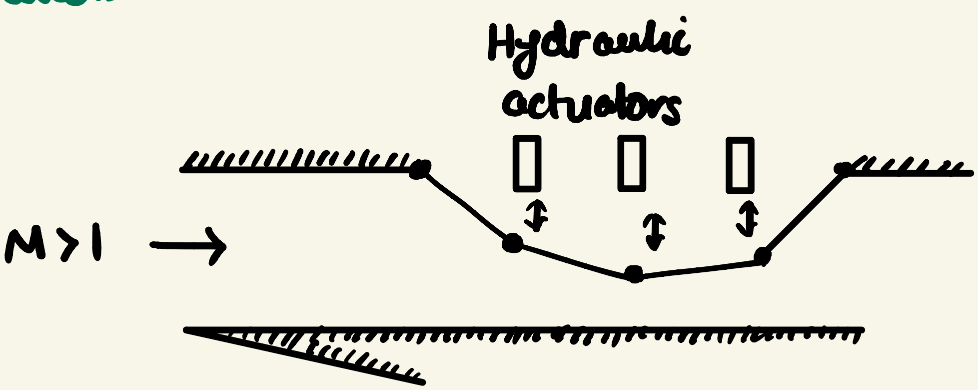
Overspeeding: Increase freestream M until shock reaches the inlet and is swallowed. Shock swallowing happens when M_2 has the design A/A^* value. ($M_1 = M_0$). All of the mass flow is ingested

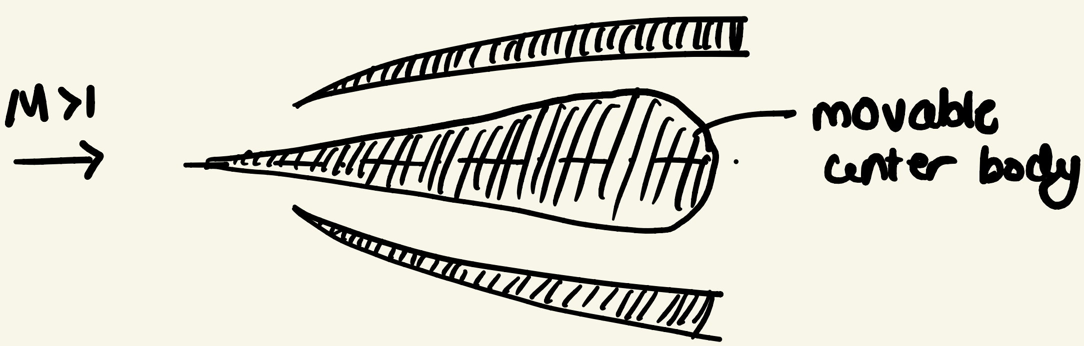
Once the shock is swallowed, the flight Mach number can be decreased which moves the shock upstream towards the throat until at $M = M_d$, the shock reaches throat and vanishes.

In reality, it would not be practical to operate under these design conditions because the slightest decrease in M would cause the shock to be disgorged, i.e., move out of the diffuser

Because of poor performance at off-design conditions and requirement of overspeeding, fixed c-d diffusers are seldom used.

Solution: Diffuser with a variable area throat

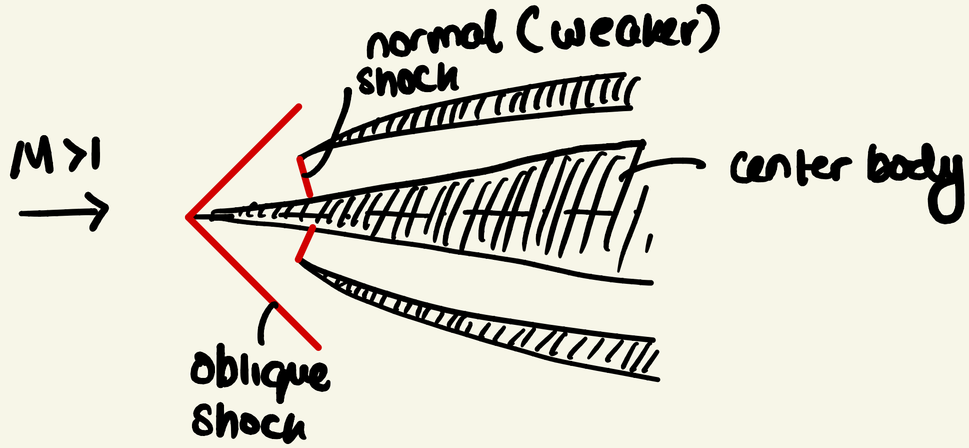




With a variable A/A^*, the throat can be opened to allow the shock to be swallowed and then the throat area can be decreased until the shock disappears at all Mach numbers ⇒ no overspeeding required.

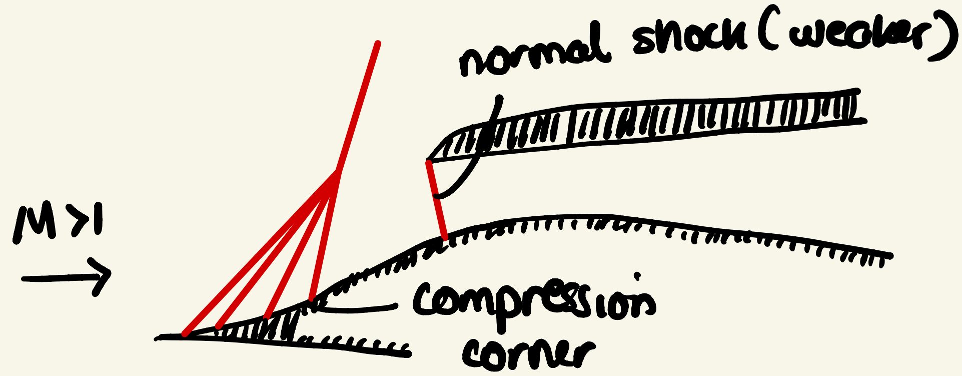
Other improvements:

1. Oblique shock diffusers



Much of the deceleration done through oblique shocks less stagnation pressure los

2. Curved compressive corner



Measure of diffuser performance

$$\text{Stagnation pressure recovery ratio} = \frac{p_0 \text{ in diffuser}}{p_0 \text{ in freestream}}$$

Problems

④ Jet aircraft : $M = 1.7$, fixed A/A^* c-d diffuser.
Find ideal A/A^* and M to which aircraft must be taken to swallow the shock.

At $M = 1.7 \Rightarrow \frac{A}{A^*} = 1.338$

Subsonic M which has $\frac{A}{A^*} = 1.338$ is $M_2 = 0.5$

$\Rightarrow$ Mach number before the shock, $M_1 =$ 2.64

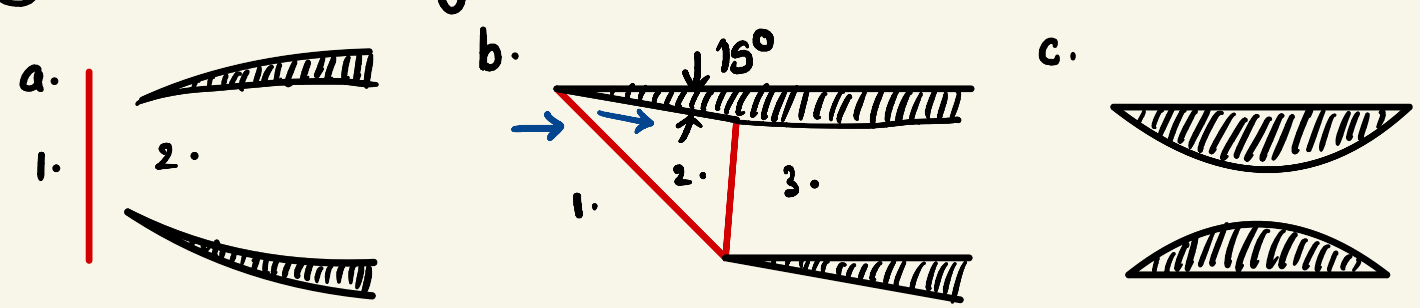
⑤ Jet aircraft : $M_{cruise} = 2.5$, variable area diffuser
what is $\frac{A^*_{cruise}}{A^*_{M=1.3}}$ assuming fixed A_i?

At $M = 2.5$ $\quad A/A^* = 2.6367$

$M = 1.3$ $\quad A/A^* = 1.0663$

$$\frac{A^*_{cruise}}{A^*_{M=1.3}} = \frac{1.0663}{2.6367} = 0.404$$

⑥ Aircraft cruising at $M = 3$. $p_0 = 400$ kPa



Find p_0 at exit of these diffusers.

a. At $M = 3$ $\quad \frac{p_{o2}}{p_{o1}} = 0.3283$

$$\Rightarrow p_{o2} = 0.3283 \times 400 = 131.3 \text{ kPa}$$

b. $\delta = 15^\circ$, $M = 3$, $M_{n_1} = 1.6$, $M_2 = 2.255$

$$\frac{p_{o2}}{p_{o1}} = 0.8952$$

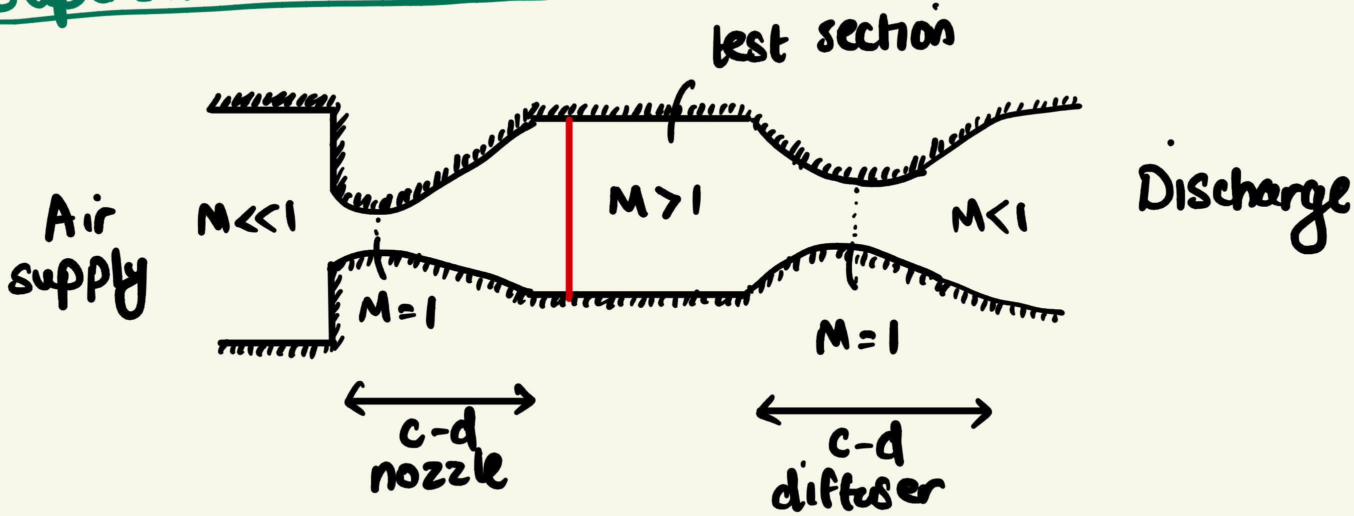
At $M_2 = 2.255$, normal shock tables give

$$\frac{p_{o3}}{p_{o2}} = 0.6168$$

$$\Rightarrow p_{o3} = 0.8952 \times 0.6168 \times 400 = 220.9 \text{ kPa}$$

c. p_o fixed $= 400$ kPa.

Supersonic wind tunnel



The tunnel is started by either increasing the pressure ahead of the nozzle or decreasing the pressure behind the diffuser.

A normal shock wave exists in the nozzle during startup and moves downstream to the test section. Because of p_o loss across the shock, A_t of diffuser $>$ A_t of nozzle to swallow the starting shock.

Problems

⑦ Wind tunnel : $M_d = 3$, variable area diffuser.

Find $\dfrac{A^*_{diffuser}\big|_{design}}{A^*_{diffuser}\big|_{shock}}$

At design condition, $\dfrac{A^*}{A} = 0.236$

Using normal shock tables at $M_d = 3$

$$M_2 = 0.475$$

Using isentropic tables at $M = 0.475$

$$\frac{A^*}{A} = 0.72$$

Required ratio $= \dfrac{0.236}{0.72} = 0.328$