AE 234/711 Aircraft Propulsion
Tutorial 2 - Efficiency, Multiple Exhausts, SFC, Range

1. In the last tutorial we calculated properties of air at different altitudes using approximate formulae. These formulae are an alternative to tables, which give the properties at a few selected altitudes of importance. Both of these are given in the next page.
   - Do the values in the table agree with the formulae? Quantify the difference between the two in terms of % *error*.
   - When we use tables, we obtain values at locations not mentioned in the table by linear interpolation. Select a few altitudes that are not mentioned in the table (32,000 ft, 34,000 ft etc.) and examine the error due to linear interpolation.
2. Consider the Rolls-Royce Trent 1000 or a GE GEnx engine that power Boeing 787. The vehicle flies at $M = 0.86$ at an altitude of $35,000$ ft with a jet velocity ≈ 350 m/s.
   - Calculate the propulsive efficiency.
   - What would the jet velocity have to be to increase the propulsive efficiency to 90%?
   - By what ratio would the mass flow through the engine need to increase to hold net thrust constant?
   - If the mass flow per unit area through the engine were held constant, estimate the increase in engine diameter needed for this improvement in η_p.
3. A turbojet-powered aircraft cruises at $V_a = 300\ m/s$ while the engine produces an exhaust speed of $600\ m/s$. The air mass flow rate is $100\ kg/s$ and the fuel mass flow rate is $2.5\ kg/s$. The fuel heating value is $\mathcal{Q}_R = 42\ MJ/kg$. Assuming that the nozzle is perfectly expanded, calculate
   - ram drag, gross thrust and uninstalled thrust in kN
   - TSFC ($mg/s - kN$), propulsive, thermal and overall efficiencies
   - aircraft range R for $L/D = 10$ and the $M_{init}/M_{final} = 1.25$
4. Thrust at take-off for a civil aircraft is required to be 0.3 times the maximum take-off weight. Consider the following three conditions of an aircraft similar to Boeing 747-400: take-off, top-of-climb and cruise. The maximum take-off weight for 747-400 is 3870 kN. Remember that $C_L \approx 1.6$ at take-off and $C_L \approx 0.5$ at cruise. Let the aircraft cruise at a Mach number of 0.85 and have a lift-to-drag ratio of 18. Assume the rate-of-climb to be 500 ft/min when the aircraft is nearing its cruise altitude at $35,000$ ft at a flight Mach number of 0.78. Assume the L/D at the top-of-climb to be the same as the cruise value. Compare the wing-loading and thrust of the aircraft at the three flight conditions.
5. Derive the expression for the range of an aircraft if its weight decreases from $\mathcal{W}_{ini}$ to $\mathcal{W}_{fin}$ during cruise. Consider the following flight scenarios:
   - Constant speed, attitude
   - Constant attitude, altitude
   - Constant altitude, speed

6. The unit of lbm/lbf − hr is common for TSFC. Show that 1lbm/lbf − hr $= 28.3$ gm/kN − s
7. We wish to investigate the range of a slender supersonic aircraft where its lift-to-drag ratio as a function of flight mach number is described by

$$\frac{\mathcal{L}}{\mathcal{D}} \approx 3\frac{M_a + 3}{M_a}$$

Varying the TSFC between $1.0 - 2.0$ lbm/hr−lbf, plot the variation of range, R, for the fligh mach number ranging between 2.0 and 4.0. You can assume that speed of sound is $333m/s$, and the aircraft initial to final weight ratio is $\mathcal{M}_0/\mathcal{M}_f = 2$.

US Standard Atmosphere 1976 *(expressions from Gudmundsson)*

For Troposphere: $0 \leq h \leq 36,089\ ft \equiv 11\ km$,

Temperature ratio: $\theta = 1 - \dfrac{h}{145442}$

Pressure ratio: $\delta = \theta^{5.2561}$

For Lower Stratosphere: $11\ km \equiv 36,089\ ft \leq h \leq 65,617\ ft \equiv 20\ km$,

Temperature ratio: $\theta = 0.751865$

Pressure ratio: $\delta = 0.223361\ \exp\left[-\left(\dfrac{h - 36089}{20806}\right)\right]$

where, θ and δ are the temperature and pressure normalised by their sea level values. The altitude h is in feet. The sea-level conditions for the standard atmosphere are $T_{sl} = 288.15\ K$, $p_{sl} = 101325\ Pa$ and $\rho_{sl} = 1.225kg/m^3$.

Altitude feet	km	Temperature K	Pressure 10^5 Pa	Density kg/m^3
0	0	288.15	1.013	1.225
31000	9.45	226.73	0.287	0.442
33000	10.05	222.82	0.260	0.336
35000	10.67	218.80	0.238	0.380
37000	11.28	216.65	0.214	0.344
39000	11.88	216.65	0.197	0.316
41000	12.50	216.65	0.179	0.287
51000	15.54	216.65	0.110	0.179

*Also known as the ICAO Standard Atmosphere.