1. TRUE (0.5 marks)

The 3 conservation equations have 4 unknowns for a given gas (γ fixed) and given state (p_1, T_1, ρ_1 fixed) : ρ_2, p_2, h_2, V_2

If the gas is ideal and calorically perfect, we have a fourth equation $p = \rho R T$ and enthalpy is related to temperature through the constant c_p, $h = c_p T$ (0.5 marks)

2.

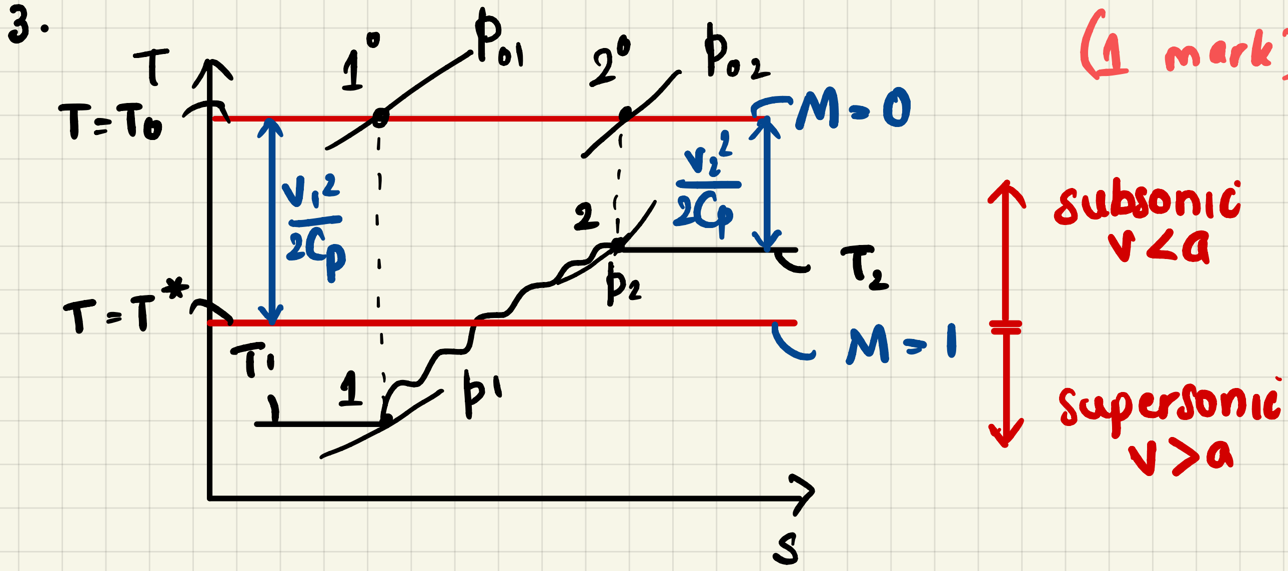
$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1}$$ shock strength

For air, $\gamma = 1.4 \Rightarrow \frac{p_2}{p_1} = 29$ (0.4 marks)

For helium, $\gamma = 1.67 \Rightarrow \frac{p_2}{p_1} = 31$ (0.4 marks)

Shock strength greater in helium (0.2 marks)

3.



(1 mark)

$T = \text{const} \Rightarrow a = \text{constant}$

Distance between two lines with $T = \text{constant}$ and $T_0 = \text{constant}$ gives $V^2/2C_p$

Fixed vertical distance $\Rightarrow V = \text{constant}$

$\Rightarrow$ lines of const. M are horizontal on T-S diagram

Line corresponding to $M=1$ should lie between states 1 (supersonic, $V>a$) and 2 (subsonic $V<a$)

(1 mark)

4.

$U_s = 550\,m/s$

$p_{01} = p_1 = 10^5\,Pa$

$T_{01} = T_1 = 289\,K$

$p_2 = ?$

$T_2 = ?$

$V_2 = ?$

$p_{02} = ?$

$T_{02} = ?$

(a) $a_1 = \sqrt{1.4 \times 287 \times 289} = 340.76\,m/s$

$\Rightarrow M_s = 1.61$

From shock tables,

$M_2' = 0.6655 \qquad \frac{p_2}{p_1} = 2.8575 \qquad \frac{T_2}{T_1} = 1.3949$

$$\Rightarrow \boxed{\begin{aligned} p_2 &= 2.86\ bar \\ T_2 &= 403.1\,K \end{aligned}}$$

(1 mark)

$a_2 = \sqrt{1.4 \cdot 287 \cdot 403.1} = 402.45\,m/s$

$V_2 = M_1 a_1 - M_2' a_2$

$\Rightarrow \boxed{V_2 = 280.8\,m/s}$

(1 mark)

(b) Actual Mach number downstream $M_2 = \frac{V_2}{a_2} = 0.698$

$p_{01} = p_1 = 1\ bar$

$p_{02} = \left(\frac{p_{02}}{p_2}\right)_{M_2} \cdot p_2 = \left(\frac{1}{0.7222}\right) \cdot 2.86$

$\Rightarrow \boxed{p_{02} = 3.96\ bar > p_{01}}$

(0.5 + 0.25 marks)

$T_{01} = T_1 = 289\,K$

$T_{02} = \left(\frac{T_{02}}{T_2}\right)_{M_2} \cdot T_2 = 1.0974 \cdot 403.1$

$\Rightarrow \boxed{T_{02} = 442.4\,K > T_{01}}$

(0.5 + 0.25 marks)

(c) From our understanding of normal shock waves, we expect $T_{02} = T_{01}$ and $p_{02} < p_{01}$. The inconsistency stems from the unsteadiness of the shock which changes the stagnation conditions

(0.5 marks)

To resolve the inconsistency and compute the correct entropy change and losses across the shock, we need to look at p_0 and T_0 for the stationary shock.

$p_1 = p_1' = 1$ bar $\quad$ $\vec{V_1'} = 550$ m/s $\quad$ | $\quad$ $\vec{V_2'} = U_s - V_2$ $\quad$ $p_2' = p_2$

$T_1 = T_1' = 289$ K $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $T_2' = T_2$

$$p_{01}' = \left(\frac{p_{01}'}{p_1'}\right)_{M_s} \cdot p_1' = \left(\frac{1}{0.2318}\right) \cdot 1 = 4.31 \text{ bar}$$

$$p_{02}' = \left(\frac{p_{02}'}{p_2'}\right)_{M_2'} \cdot p_2' = 1.3463 \cdot \quad = 3.85 \text{ bar}$$

we have $p_{02}' < p_{01}'$ (1 mark)

$$T_{01}' = \left(\frac{T_{01}'}{T_1'}\right)_{M_s} \cdot T_1' = 1.5185 \cdot 289 = 438.8 \text{ K}$$

$$T_{02}' = \left(\frac{T_{02}'}{T_2'}\right)_{M_2'} \cdot T_2' = 1.0884 \cdot 403.1 = 438.8 \text{ K}$$

we have $T_{02}' = T_{01}'$ (1 mark)