MA 214: Introduction to numerical analysis (2021–2022)

Tutorial 8

(March 23, 2022)

(1) Use Taylor polynomial P_4 and composite Simpson's rule with $n = 6$ to approximate the improper integral $\int_0^1 \frac{e^{2x}}{\sqrt[5]{x^2}} dx$.

(2) Use Taylor polynomial P_4 and composite Simpson's rule with $n = 6$ to approximate the improper integral $\int_0^1 \frac{\cos 2x}{x^{1/3}} dx$.

(3) Approximate the value of the improper integral $\int_1^\infty x^{-3/2} \sin \frac{1}{x} dx$.

(4) Use Euler's method with $h = 0.25$ to approximate the solution for the initial-value problem: $y' = 1 + (t - y)^2$, $2 \leqslant t \leqslant 3$ and $y(2) = 1$. Compare the results with $y(t) = t + \frac{1}{1-t}$.

(5) Use Euler's method with $h = 0.25$ to approximate the solution for the initial-value problem: $y' = \cos 2t + \sin 2t$, $0 \leqslant t \leqslant 1$ and $y(0) = 1$. Compare the results with $y(t) = \frac{1}{2} \sin 2t - \frac{1}{2} \cos 2t + \frac{3}{2}$.

(6) Consider the initial-value problem: $y' = -10y$, $0 \leqslant t \leqslant 2$, $y(0) = 1$ with solution $y(t) = e^{-10t}$. What happens when Euler's method is applied to this problem with $h = 0.1$? Does this behavior violate the error bound?

(7) Use Taylor's methods of order 2 and 4 with $h = 0.25$ to approximate the solution for the initial-value problem: $y' = 1 + \frac{y}{t}$, $1 \leqslant t \leqslant 2$, $y(1) = 2$.

(8) Use Taylor's methods of order 2 and 4 with $h = 0.25$ to approximate the solution for the initial-value problem: $y' = \cos 2t + \sin 3t$, $0 \leqslant t \leqslant 1$, $y(0) = 1$.

(9) Use Taylor's methods of order 2 and 4 with $h = 0.25$ to approximate the solution for the initial-value problem: $y' = te^{3t} - 2y$, $0 \leqslant t \leqslant 1$, $y(0) = 0$.