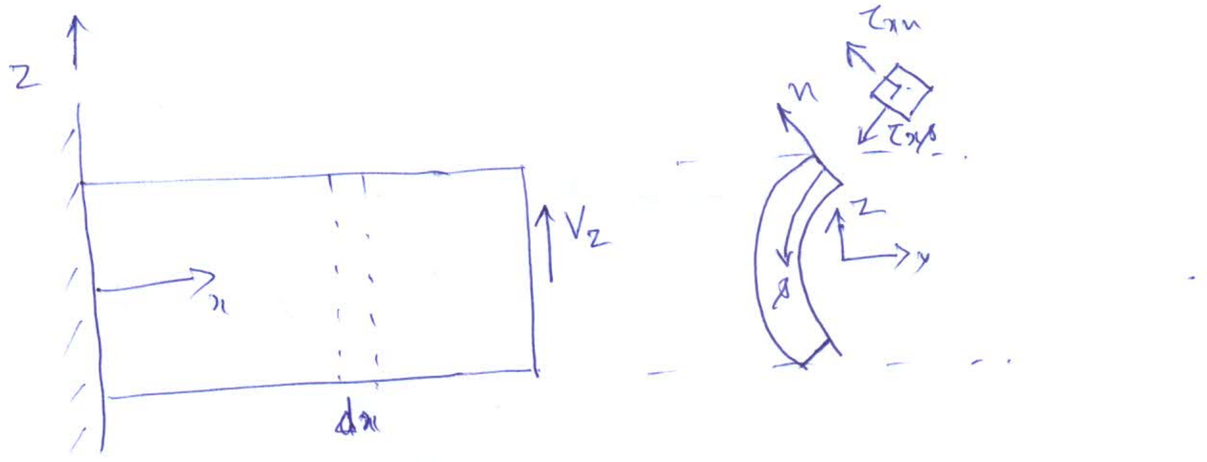
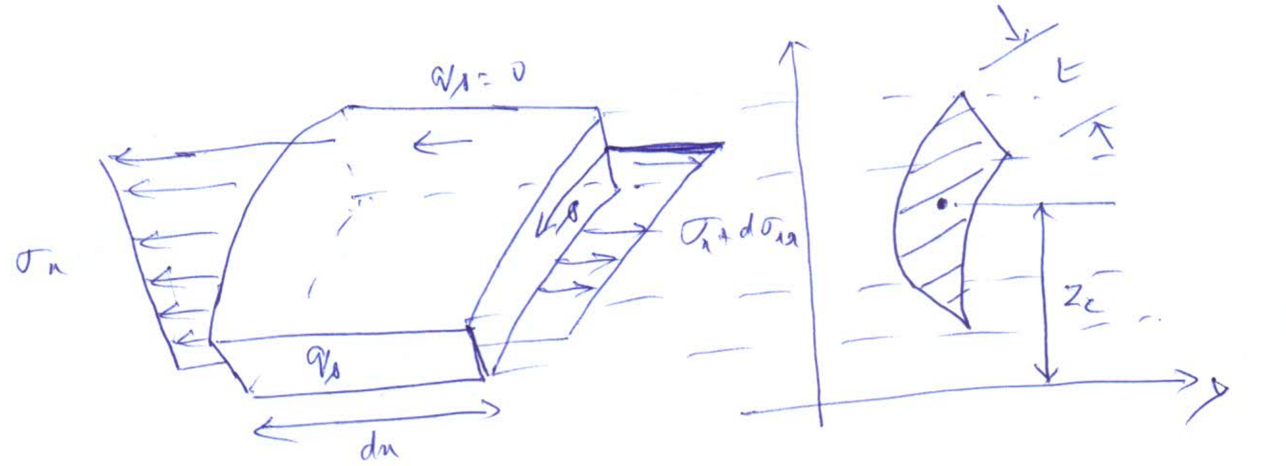
Symmetric thin walled sections





$$d\sigma_{xx} + q\,dx = 0$$

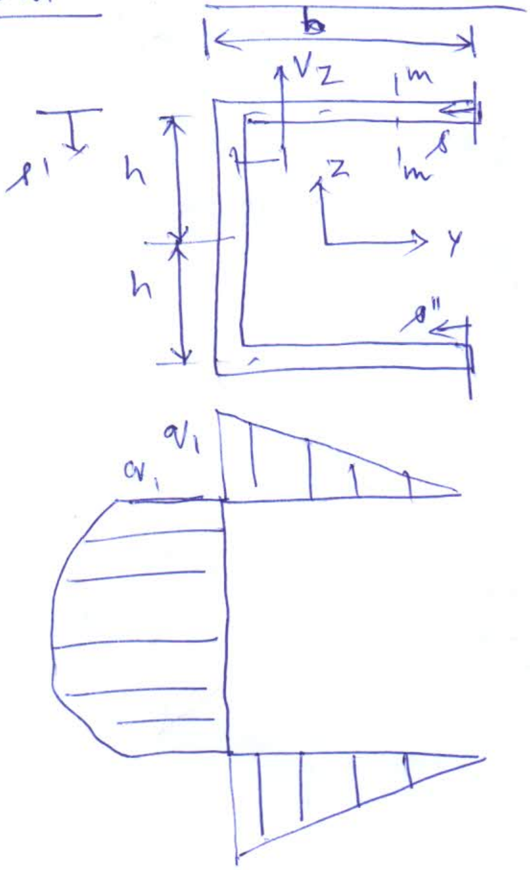
$$\iint \frac{d\sigma_{xx}}{dx}\,dA = -q_s \qquad \sigma_{xx} = \frac{M_y z}{I_y} \qquad \frac{dM_y}{dx} = V_z$$

$$q_s = \frac{-V_z}{I_y}\iint_{As} z\,dA$$

$$q_s = \frac{-V_z}{I_y} Q \qquad Q = \iint_{As} z\,dA = A_s z_c$$

Unsymmetric thin-walled sections

Beam with channel section



$$q = \frac{-V_z Q}{I_y} \qquad I_y = \ldots$$

$$A_s = t \cdot s \qquad z_c = h$$

$$q_s = \frac{-V_z t h s}{I_y} \qquad 0 \le s \le b.$$

linear distribution & max @ $s = b$

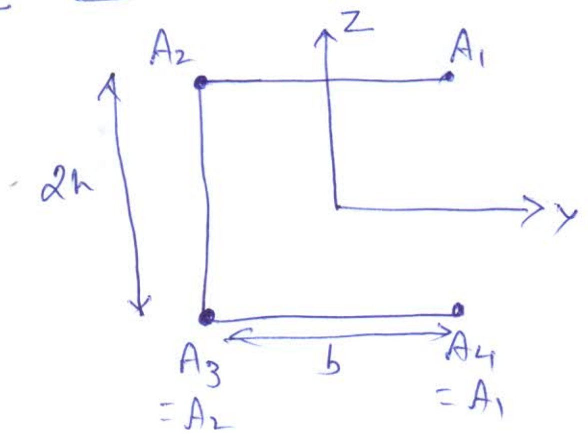
shear flow on vertical web

$$q_s = \frac{-V_z \left[tbh + ts'(h - s'/2) \right]}{I_y}$$

$$@\ s' = 0 \quad q_s = -\frac{V_z t b h}{I_y}$$

$$q_s = -\frac{V_z t s(-h)}{I_y} = \frac{V_z t s'' h}{I_y}$$

stringer web-sections



In general,

$$q_i = \frac{-V_z Q_i}{I_y}$$

$$I_y = 2h^2 (A_1 + A_2)$$

$$Q_i = \sum_{k=1}^{i} z_k A_k$$

$$q_1 = \frac{-V_z A_1 h}{2h^2 (A_1 + A_2)}$$

$$q_2 = \frac{-V_z (A_1 + A_2) h}{2h^2 (A_1 + A_2)} = -\frac{V_z}{2h} \qquad q_3 = \frac{-V_z (A_1 h + A_2 h - A_2 h)}{2h^2 (A_1 + A_2)}$$

~~$q_4 =$~~

Unsymmetric thin-walled sections

$$\sigma_{xx} = (k_y M_z - k_{yz} M_y) y + (k_z M_y - k_{yz} M_z) z$$

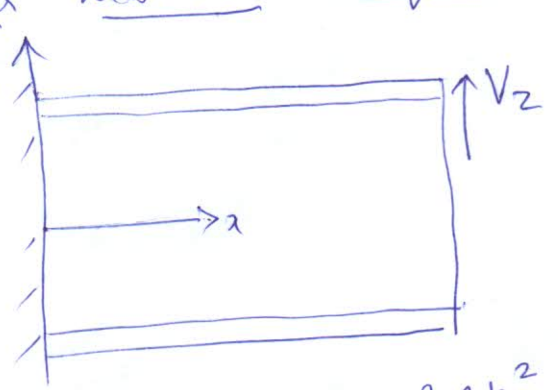
$$k_y = \frac{I_y}{I_y I_z - I_{yz}^2} \qquad k_z = \frac{I_z}{I_y I_y - I_{yz}^2} \qquad k_{yz} = \frac{I_{yz}}{I_y I_z - I_{yz}^2}$$

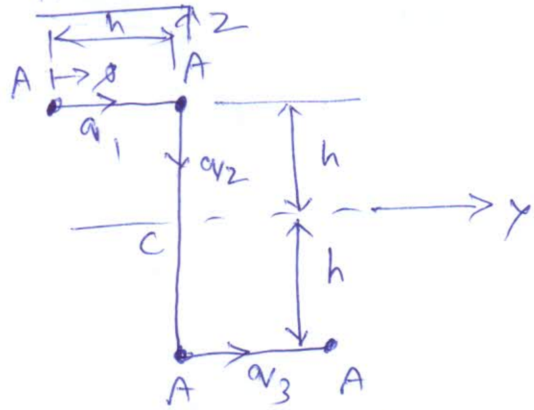
$$q_s = -\iint \frac{d\sigma_{xx}}{dx} dA$$

$$q_1 = -(k_y V_y - k_{yz} V_z) Q_z - (k_z V_z - k_{yz} V_y) Q_y$$

$$Q_z = \iint_{A_1} y \, dA \qquad Q_y = \iint_{A_1} z \, dA$$

Stringer web-beam unsymmetrical section





$I_y = 4Ah^2 \qquad I_z = 2Ah^2$

$$I_{yz} = \sum_i A_i y_i z_i = A_1(-h)(h) + 0 + 0 + A_1(y)(-h)$$

$$= -2Ah^2$$

$$\therefore k_y = \frac{1}{Ah^2} \qquad k_z = \frac{1}{2Ah^2} \qquad k_{yz} = \frac{-1}{2Ah^2}$$

$$q_1 = + k_{yz} V_z Q_z - k_z V_z Q_y$$

$$= k_{yz} V_z (h) - k_z V_z (-h) = 0$$

$Q_y = 2Ah$

$Q_z = -Ah$

$$q_2 = k_{yz} V_z Q_z - k_z V_z Q_y$$

$$= \frac{-1}{2Ah^2} V_z (-Ah) - \frac{1}{2Ah^2} (2Ah) V_z = -\frac{1}{2h} V_z$$

$$q_3 = q_1 = 0.$$

Shear center in open sections also called center of twist

apply torque only θ no bending; apply shear force only bending no $\underline{\theta}$.

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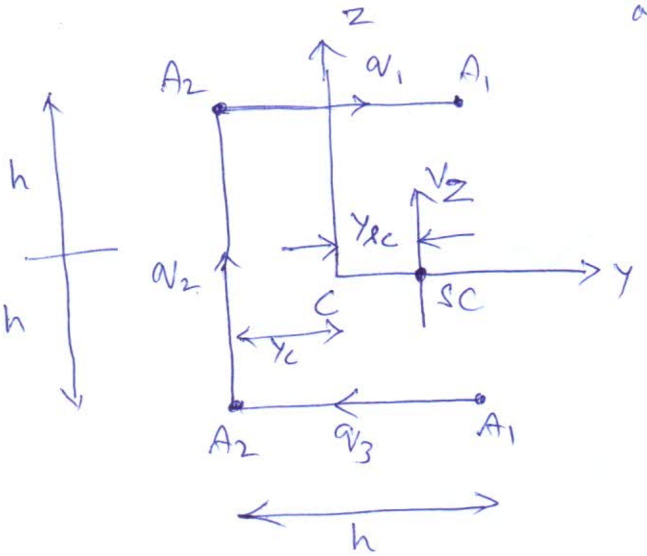


$$q_1 = V_z A_1 / 2h(A_1+A_2)$$

$$q_2 = \frac{V_z}{2h}$$

$$q_3 = \frac{V_z A_1}{2h(A_1+A_2)}$$

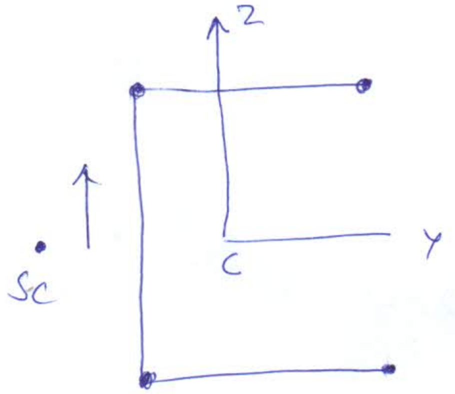
$$(y_{sc})V_z = -q_1(h)(h) - q_2(2h)(y_c) - q_3(h)(h)$$

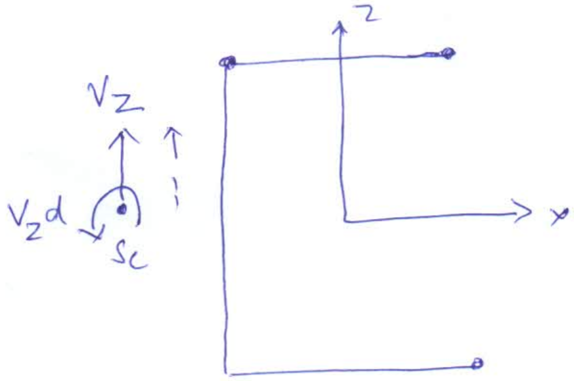
$$y_c = \frac{\sum A_i y_i}{\sum A_i}$$

$$y_c = \frac{A_1 h}{A_1+A_2}$$

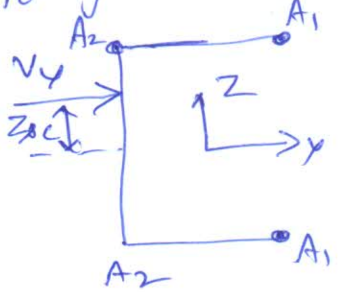
$$y_c = \frac{2A_1 h}{2(A_1+A_2)} = \frac{A_1 h}{A_1+A_2}$$

$$\Rightarrow y_{sc} = \frac{-2h A_1}{A_1+A_2}$$





To find vertical position z_{sc}



$$I_{yz} = 0.$$

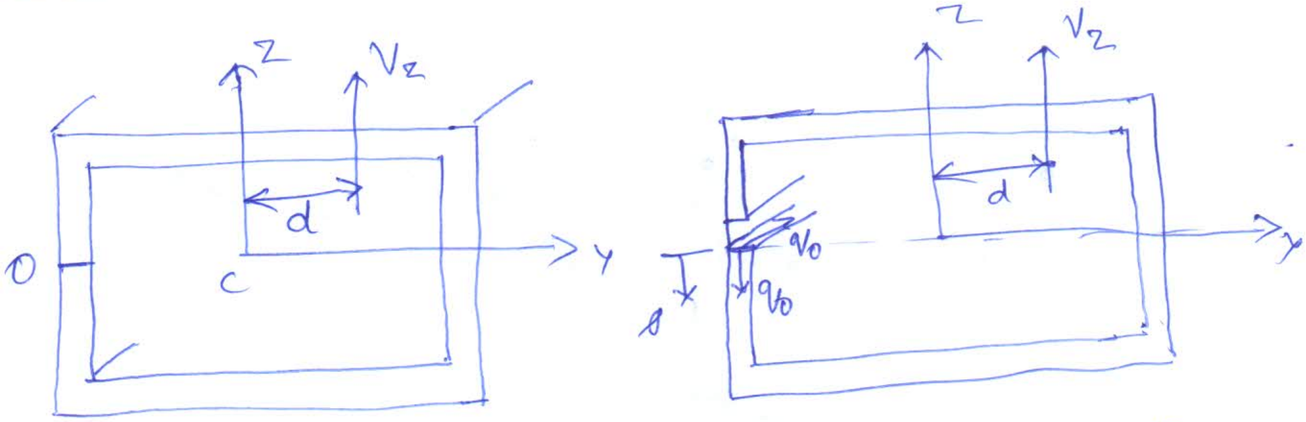
$$q_1 = \frac{-V_y Q_z}{I_y}$$

$$q_1 = \frac{-V_y A_1(h-y_c)}{I_y}$$

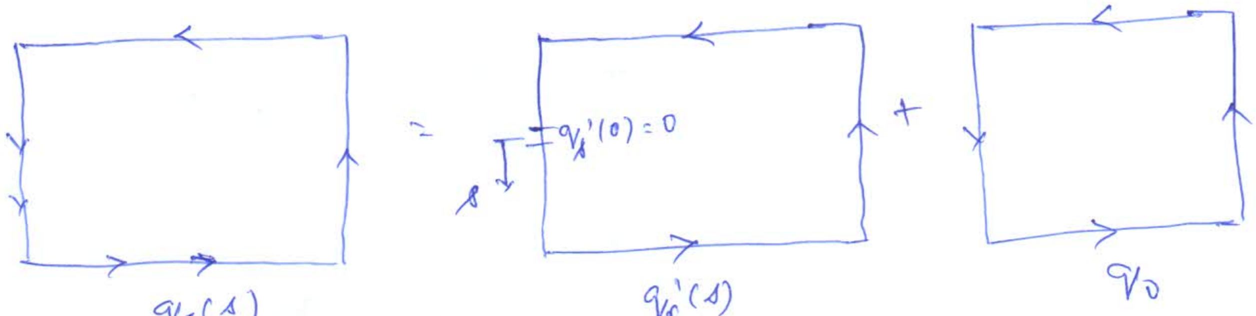
$$q_2 = q_1 - \frac{V_y A_2(-y_c)}{I_y} = 0.$$

Write moment equation like above and solve for z_{sc}.

Closed thin-walled sections and combined flexural & torsional shear flow

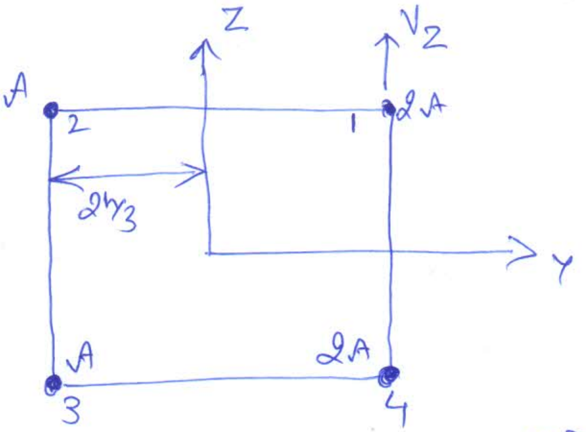


Closed-section considered as an open section but with a non-zero shear flow at point O.



$$q_s = q_s' + q_0$$

Now, $V_z \cdot d = 2\bar{A} q_0$ + moment due to q_s' about the x-axis



$I_{yz} = 0$ symmetry about y-axis.

$I_y = \frac{3}{2} A h^2 \quad I_z = \frac{4}{3} A h^2$

Assume cut between 1-2. $\Rightarrow q'_{12} = 0$

$$q'_{23} = -\frac{V_z \, A \cdot h/2}{3/2 \, A h^2} = -\frac{V_z}{3h}$$

$$q'_{34} = 0$$

$$q'_{41} = \frac{-V_z \cdot 2A \cdot (-h/2)}{3/2 \, A h^2} = \frac{2V_z}{3h}$$

Resulting moment of the total shear flow $q = q' + q_0$ must be equal to

$\bar{A} = h \times h$ area enclosed by the centerline

$$V_z \cdot 0 = q'_{23} \, h \cdot h + 2\bar{A} q_0$$

$$0 = q'_{23} h^2 + 2h^2 q_0$$

$$\Rightarrow q_0 = -\frac{1}{2} q'_{23} = \frac{V_z}{6h}$$

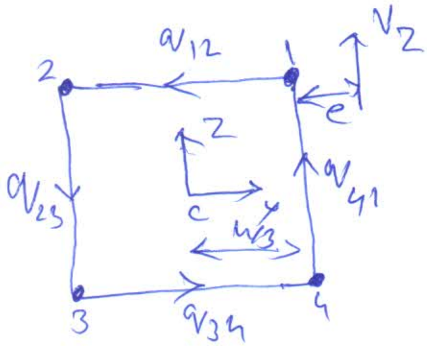
∴ Total shear flow

$$q_{12} = q'_{12} + q_0 = \frac{V_z}{6h}$$

$$q_{23} = q'_{23} + q_0 = -\frac{V_z}{6h}$$

$$q_{34} = q'_{34} + q_0 = \frac{V_z}{6h}$$

$$q_{41} = q'_{41} + q_0 = \frac{5V_z}{6h}$$



$$V_z \cdot e = q'_{23} h^2 + 2h^2 q_0$$

$$\therefore q_0 = \frac{V_z e}{2h^2} - \frac{q'_{23}}{2} = \frac{V_z}{6h^2}(h + 3e)$$

$\Rightarrow q_{12} = q_0 = \frac{V_z}{6h^2}(h+3e) \quad q_{41} = \frac{V_z}{6h^2}(5h+3e)$

$q_{23} = \frac{V_z}{6h^2}(-h+3e)$

$q_{34} = \frac{V_z}{6h^2}(h+3e)$

if V_z passes through shear centre $\theta = 0$.

$$\Rightarrow \theta = \frac{1}{2G\bar{A}}\left[q_{12}\frac{h}{t} + q_{23}\frac{h}{t} + q_{34}\frac{h}{t} + q_{41}\frac{h}{t}\right] = 0$$

$\Rightarrow e = -\frac{1}{2}h$ or $y_c = \frac{h}{3} + e = -h/6$