

**MA 214: Introduction to Numerical Analysis**  
Indian Institute of Technology Bombay  
Quiz 1

Time: 1 hour

Instructors: S. Baskar and S. Sivaji Ganesh

Marks: 15

Date: 23-01-2018

**Instructions:**

- (1) Write your Name, Roll Number, and Tutorial Batch clearly on your answer book as well as every supplement you may use. A penalty of -1 mark will be awarded for failing to do so.
- (2) Number the pages of your answer book and make a question-page index on the front page.
- (3) The answer to each question should start on a new page. If the answer for a question is split into two parts and written in two different places, the first part alone will be corrected.
- (4) Only scientific calculators are allowed. Any kind of programming device is not allowed.
- (5) Formulas used need not be proved but needs to be stated clearly.
- (6) The question paper contains 4 questions. Answer all the questions.

- (1) Let  $f(x)$  be continuous on  $[a, b]$ , let  $x_1, \dots, x_n$  be points in  $[a, b]$ , and let  $g_1, \dots, g_n$  be negative real numbers. Show that

$$\sum_{i=1}^n f(x_i)g_i = f(\xi) \sum_{i=1}^n g_i, \text{ for some } \xi \in [a, b].$$

[4 Marks]

- (2) Let  $x_A = -3.14$  and  $y_A = 0.512$  be obtained from the numbers  $x_T$  and  $y_T$ , respectively, using 4-digit rounding. For any such values of  $x_T$  and  $y_T$ , find the smallest interval that contains  $x_T/y_T$ . [4 Marks]
- (3) Define the notion of number of significant digits. Obtain the number of significant digits of  $x_A = 0.0140191$  present in  $x = 0.0140185$ . [4 Marks]

- (4) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  be continuously differentiable functions such that
- there exists a constant  $M > 0$  such that  $|f'(x)| \geq M$  and  $|g'(x)| \leq M$  for all  $x \in \mathbb{R}$ ,
  - the condition number of  $f$  is less than 1, and
  - the condition number of  $g$  is greater than 1.

Show that  $|g(x)| < |f(x)|$  for all  $x \in \mathbb{R}$ .

[3 Marks]

— End of the Question Paper —