AE 711 Aircraft Propulsion
Tutorial 1

1. The International Standard Atmosphere describes the mean atmospheric. conditions at the latitude 45^oN (midway between the equator and the north pole). However, the Indian subcontinent is is between 8^oN and 36^oN. The International Tropical Reference Atmosphere (ITRA) [1] to describe the atmosphere in the tropics (30^oS - 30^oN). The below tables[2] summarise the two models.

    - Remember that the lift, drag and the mass flowrate (into the engine) are proportional to the density of ambient air. Estimate the error involved in estimating the properties in Indian atmosphere using ISA. At what altitude is this error maximum?
    - When we use tables, we obtain values at locations not mentioned in the table by linear interpolation. Select a few altitudes that are not mentioned in the table (5 km, 10 km, 15 km, etc.) and examine the error due to linear interpolation.

Parameters at base altitudes

ISA Model

b	$\mathcal{H}$ (km$'$)	z (km)	T (K)	L_b (K/km$'$)	p_b (Pa)
0	0	0.00	288.15	-6.5	101325.00
1	11	11.02	216.65	0.0	22632.06
2	20	20.06	216.65	$+1.0$	5474.89
3	32	32.16	228.65	$+2.8$	868.02
4	47	47.35	270.65	0.0	110.91
5	51	51.41	270.65	-2.8	66.94
6	71	71.80	214.65	-2.0	3.96
7	80	81.02	196.65		0.89

ITRA-86 Model

b	$\mathcal{H}$ (km$'$)	z (km)	T (K)	L_b (K/km$'$)	p_b (Pa)
0	0	0.00	300.15	-6.0	101000.00
1	6	6.01	264.15	-6.5	48861.38
2	16	16.04	199.15	$+2.3$	11102.42
3	46	46.34	268.15	0.0	134.87
4	51	51.41	268.15	-3.0	71.41
5	74	74.87	199.15	-0.6	2.43
6	80	81.02	195.55	0.0	0.86

Parameters at Sealevel

Quantity	Symbol	Unit	ISA Value	ITRA Value
Geopotential altitude	$\mathcal{H}_o$	km$'$	0	0
Geometric altitude	z_o	km	0	0
Kinetic temperature	T_o	K	288.5	300.15
Pressure	p_o	Pa	101325	101000
Density	ρ_o	kg/m^3	1.225	1.172
Acceleration due to gravity	g_o	m/s^2	9.80665	9.78852
Mean molecular mass	$\mathcal{M}_o$	kg/(kmol)	28.9644	28.9644

2. Given below are the expressions[3] for the US Standard Atmosphere (USSA), which is very close to the International Standard Atmosphere. The sealevel conditions are the same for ISA and US Standard Atmosphere.

$$\frac{T}{T_{sl}} = \begin{cases} 1 - \frac{h}{145442} & \text{if } 0 \le h \le 36,089\text{ft} \\ 0.751865 & \text{if } 36,089\text{ft} \le h \le 65,617\text{ft} \end{cases}$$

$$\frac{p}{p_{sl}} = \begin{cases} \left(\frac{T}{T_{sl}}\right)^{5.2561} & \text{if } 0 \le h \le 36,089\text{ft} \\ 0.223361 \exp\left[-\frac{h-36089}{20806}\right] & \text{if } 36,089\text{ft} \le h \le 65,617\text{ft} \end{cases}$$

[1] Ananthasayanam & Narasimha, Adv. Space Res. Vol 7, No. 10, pp (10)117-(10)131, 1987
[2] Oliviera, Proceedings of COBEM, 18th Int. Conf. of Mechanical Engg., Nov. 6-11, 2005, Ouro Preto, Brazil
[3] Gudmundsson, General Aviation Aircraft Design, 2014

These formulae are an alternative to tables, which give the properties at a few selected altitudes of importance. Both of these are given in the next page.

- What is the error in density if we use ISA when flying over USA. Using the tabulated values of the ISA Model, quantify the difference in terms of % error.
- When we use tables, we obtain values at locations not mentioned in the table by linear interpolation. Select a few altitudes that are not mentioned in the table (32,000 ft, 34,000 ft etc.) and examine the error due to linear interpolation.
- Using the table provided above, derive similar expressions for temperature and pressure for ITRA-86 upto 20 km.

3. An earlier version of Boeing 777 had a take-off mass of 250 tons, and reaches a speed of 250 kmph at take-off. If the wing planform area is $450m^2$. Assume that the drag polar is $C_D = 0.018 + 0.055C_L^2$.

   - Calculate the lift coefficient, lift and drag at take-off for standard sea-level conditions.
   - Bengaluru is at an altitude of approximately 3,020 feet. What would be the lift coefficient and drag at take-off in Bengaluru for the same take-off weight?
   - For what speed do we have the minimum thrust requirement for steady and level flight. Calculate that speed, mach number, and the minimum thrust value at the following altitudes: sea level, 5 km, 11 km.
   - Repeat the above for cruise flight at minimum power condition. How is this speed related to the above speed?

   Lift and Drag: Lift $=$ Weight $= 2452.5$ kN at all altitudes for this problem.
   Take-off

   - Standard Sea-level: $C_L = 1.85,\ C_D = 0.205,\ L/D = 9.0,\ D = 273$ kN
   - Bengaluru: $C_L = 2.02,\ C_D = 0.242,\ L/D = 8.3,\ D = 294$ kN

   Steady, level flight at Minimum Drag conditions:
   $C_L = \sqrt{C_{D_0}/k} = 0.57,\ C_D = 2C_{D_0} = 0.036,\ L/D = 15.9,\ D = 154$ kN

   - Standard Sea-level: $V = 124.7$ m/s $= 449$ km/h, $M = 0.37$
   - 5 km: $V = 160.9$ m/s $= 579.2$ km/h, $M = 0.50$
   - 11 km: $V = 228.8$ m/s $= 823.7$ km/h, $M = 0.78$

   Steady, level flight at Minimum Power conditions:
   $V_{min_P} = V_{min_D}/\sqrt[4]{3},\ C_L = \sqrt{3C_{D_0}/k}$
   $C_L = 0.99,\ C_D = 4C_{D_0} = 0.072,\ L/D = 13.8,\ D = 178$ kN

   - Standard Sea-level: $V = 94.8$ m/s $= 341$ km/h, $M = 0.28$
   - 5 km: $V = 122.2$ m/s $= 440.1$ km/h, $M = 0.38$
   - 11 km: $V = 273.9$ m/s $= 625.9$ km/h, $M = 0.59$

4. The city of Mumbai, while at the sea level, is on the coast. One should then consider the effect of humidity on air density. As per wikipedia, the mean relative humidity in Mumbai appears to vary from $60 - 90\%$ over one year. The mean temperature can be taken as 25^oC. Compare the take-off conditions (lift coefficient and drag) for Mumbai with the standard sea-level case from

the earlier problem.
For a given relative humidity (RH in %age) value, one can obtain the actual air density as[4]

$$\rho = \rho_{std}\left(\frac{1+x}{1+x\mathcal{R}_{H_2O}/\mathcal{R}}\right) \equiv \rho_{std}\left(\frac{1+x}{1+1.609x}\right)$$

where the $\mathcal{R}$ is the specific gas constant for air, $\mathcal{R}_{H_2O}$ is the specific gas constant for water vapour, and the x is the humidity ratio in kg water vapour per kg of air

$$x = \left(\frac{RH}{100}\right) 0.003878\ e^{0.0656\ T_c}$$

Here, T_c is the local temperature value in degrees celsius. Mean temperature of 25^oC

- Relative Humidity of 60%: $\rho = 1.216\ kg/m^3$, $C_L = 1.858$, $C_D = 0.208$, $L/D = 8.9$, $D = 274.4$ kN
- Relative Humidity of 90%: $\rho = 1.212\ kg/m^3$, $C_L = 1.865$, $C_D = 0.209$, $L/D = 8.9$, $D = 275.2$ kN

[4]Gudmundsson, General Aviation Aircraft Design, 2014