Jacobi Constant

$$\left.\begin{aligned}
&\dot{x}\left(\ddot{x}-2\Omega\dot{y}-\Omega^2 x\right)=\left(-\frac{\mu_1}{\|r_1\|^3}(x+\pi_2 r_{12})-\frac{\mu_2}{\|r_2\|^3}(x-\pi_1 r_{12})\right)\dot{x}\\
&\dot{y}\left(\ddot{y}+2\Omega\dot{x}-\Omega^2 y\right)=\left(-\frac{\mu_1}{\|r_1\|^3}y-\frac{\mu_2}{\|r_2\|^3}y\right)\dot{y}\\
&\dot{z}\left(\ddot{z}=-\frac{\mu_1}{\|r_1\|^3}z-\frac{\mu_2}{\|r_2\|^3}z\right)\dot{z}
\end{aligned}\right\}$$

$\mu = G(m_1 + m$

$$\pi_1 = \frac{m_1}{m_1+m_2}, \quad \pi_2 = \frac{m_2}{m_1+m_2} \qquad \mu_1 = G m_1, \quad \mu_2 = G m_2$$

$$\ddot{x}\dot{x}+\ddot{y}\dot{y}+\ddot{z}\dot{z}-\Omega^2(x\dot{x}+y\dot{y}) = -\left(\frac{\mu_1}{\|r_1\|^3}+\frac{\mu_2}{\|r_2\|^3}\right)(\dot{x}x+\dot{y}y+\dot{z}z$$

$$\underbrace{\ddot{x}\dot{x}+\ddot{y}\dot{y}+\ddot{z}\dot{z}}\;-\Omega^2\underbrace{(x\dot{x}+y\dot{y})}_{\checkmark} = \underbrace{-\frac{\mu_1}{\|r_1\|^3}(\dot{x}x+\dot{y}y+}$$

$$\underbrace{-\frac{\mu_2}{\|r_2\|^3}(\dot{x}x+\dot{y}}$$

$$\ddot{x}\dot{x}+\ddot{y}\dot{y}+\ddot{z}\dot{z} = \frac{1}{2}\frac{d}{dt}\underbrace{\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)} = \frac{1}{2}\|$$

$$\dot{x}x + \dot{y}y = \frac{1}{2}\frac{d}{dt}(x^2+y^2)$$ ✓

$$\|r_1\|^2 = (x+\pi_2 r_{12})^2 + y^2 + z^2$$

$$2\|r_1\|\dot{\overline{\|r_1\|}} = 2(x+\pi_2 r_{12})\dot{x} + 2y\dot{y} + 2z\dot{z}$$

or, $$\dot{\overline{\|r_1\|}} = \underbrace{\frac{1}{\|r_1\|}\left(\pi_2 r_{12}\dot{x} + x\dot{x} + y\dot{y} + z\dot{z}\right)}$$

It follows, $$\frac{d}{dt}\frac{1}{\|r_1\|} = -\frac{1}{\|r_1\|^2}\dot{\overline{\|r_1\|}} = -$$

Similarly, $$\frac{d}{dt}\frac{1}{\|r_2\|} = -\frac{1}{\|r_2\|^2}\dot{\overline{\|r_2\|}} =$$

$$\frac{1}{2}\frac{d}{dt}\|v\|^2 - \frac{1}{2}\Omega^2\frac{d}{dt}\left(x^2+y^2\right) =$$

$$\frac{d}{dt}\left[\frac{1}{2}\|v\|^2 - \frac{1}{2}\Omega^2\left(x^2+y^2\right)\right.$$

$$\Rightarrow \underbrace{\frac{1}{2}\|v\|^2}_{\checkmark} - \underbrace{\frac{1}{2}\Omega^2\left(x^2+y^2\right)}_{\checkmark}$$

- Total energy of the sec

relative to the rotating f

$$\underbrace{\|v\|^2}_{\geq 0} = \Omega^2 \underbrace{(x^2+y^2)}_{} + \frac{2\mu}{\|r}$$

$$\|r\|^2 = x^2+y^2$$

$$\|r_1\| = \sqrt{(x+\pi_2 r_{12})^2 + y^2}$$

$$\|r_2\| = \sqrt{(x-\pi_1 r_{12})^2 + y^2}$$

Since $\|v\|^2$ cannot be neg

Since, $\|v\|$ [illegible]

that

$$\underbrace{\Omega^2(x^2+y^2)} + \underbrace{\frac{2\mu_1}{\|\vec{r}_1\|}} +$$

$$\|v\|^2 = 0$$

$$\Omega^2(\underbrace{x^2+y^2}) + \frac{2\mu_1}{\underbrace{\|\vec{r}_1\|}} + 2$$

Zero velocity curves corresp

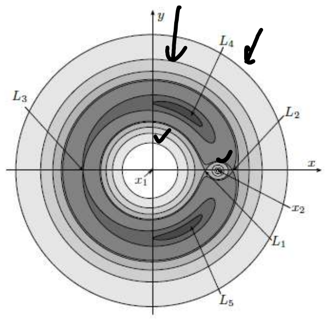
of the Jacobi constant.

Earth - Moon System

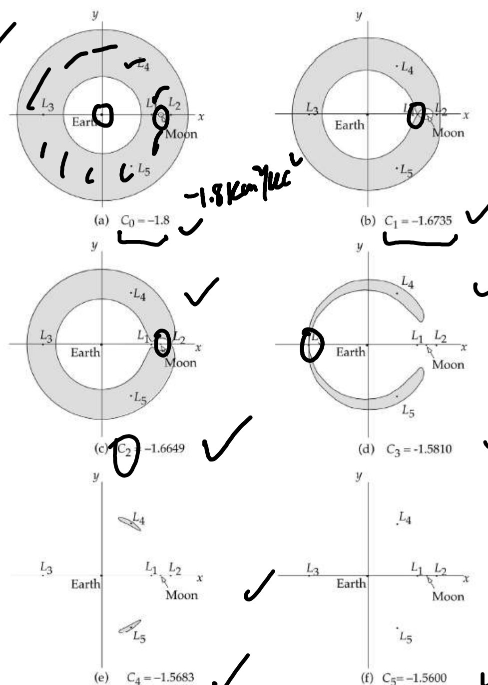
$$\Omega = \sqrt{\frac{G(m_1+m_2)}{r_{12}^3}}$$

$$\mu_1 = Gm_1 = 3$$

$$\mu_2 = Gm_2 = 1$$



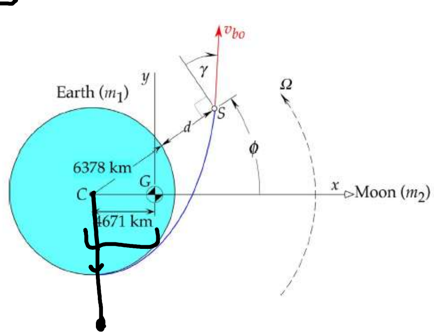
L_1, L_2, L_3



-1.8 km/sec

Example

The earth-orbiting spacecraft in Fig. 2.38 has a relative burnout velocity v_{bo} at an altitude of $d = 200$ km on a radial for which $\phi = -90°$. Find the value of v_{bo} for each of the six scenarios depicted in Fig. 2.37.



Details

$$\pi_1 = \frac{m_1}{m_1 + m_2} = 0.9878$$

$$\pi_2 = 1 - \pi_1 = 0.01215$$

$$x_1 = -\pi_1 r_{12} = -4670.6 \text{ km}$$

$\phi = -90^\circ$, $x = -4670.6$ km, $y = -6578$ km

$C = -1.8$ km^2/sec^2, $V_{bo} = 10.84518$ km/sec

$C = -1.6735$ km^2/sec^2, $V_{bo} = 10.85683$ km/sec

⋮

$C = -1.56$ km^2/sec^2, $V_{bo} = 10.86728$ km/sec

$$V_{esc} = \sqrt{\frac{2\mu}{r}} = 11.01 \text{ km/sec}$$