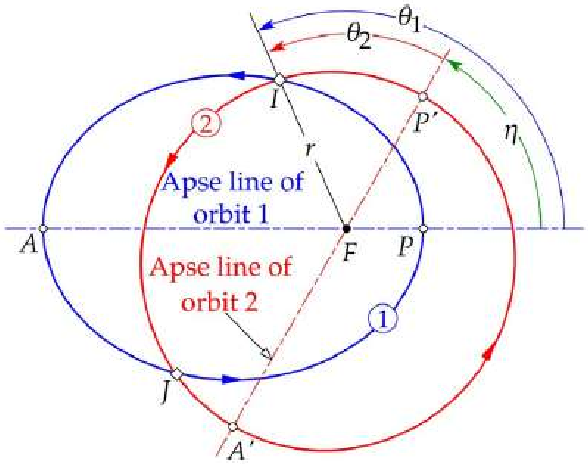
Apse Line Rotation



- $\eta = \theta_2 - \theta_1$

- Case 1: η, $\|h\|$ and $\|e\|$ are given for both the orbits.

- $r_I)_1 = \frac{\|h_1\|^2}{\mu} \frac{1}{1+\|e_1\|\cos\theta_1}$, $r_I)_2 = \frac{\|h_2\|^2}{\mu} \frac{1}{1+\|e_2\|\cos\theta_2}$

- $r_I)_1 = r_I)_2$

- $\|e_1\|\|h_2\|^2\cos\theta_1 - \|e_2\|\|h_1\|^2\cos\theta_2 = \|h_1\|^2 - \|h_2\|^2$

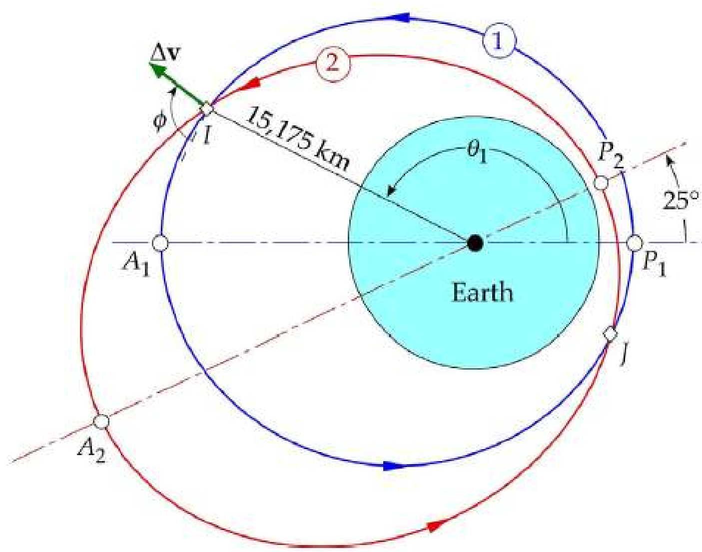
- $\underbrace{\left(\|e_1\|\|h_2\|^2 - \|e_2\|\|h_1\|^2\cos\eta\right)}_{a}\cos\theta_1 + \underbrace{\left(-\|e_2\|\|h_1\|^2\sin\eta\right)}_{b}\sin\theta_1 = \underbrace{\|h_1\|^2 - \|h_2\|^2}_{c}$

- $\theta = \phi + \cos^{-1}\left(c \cos\phi\right)$, $\phi = \tan^{-1}\left(b\right)$

$$\theta_1 = \phi \pm \cos^{-1}\left(\frac{c}{a}\cos\phi\right), \quad \phi = \tan^{-1}\left(\frac{b}{a}\right)$$

Example

An earth satellite is in an 8000 km by 16,000 km radius orbit (orbit 1 of Fig. 6.18). Calculate the delta-v and the true anomaly θ_1 required to obtain a 7000 km by 21,000 km radius orbit (orbit 2) whose apse line is rotated 25° counterclockwise. Indicate the orientation ϕ of Δv to the local horizon.



Details

$$\|e_1\| = \frac{r_{A_1} - r_{P_1}}{r_{A_1} + r_{P_1}}, \quad \|e_2\| = \frac{r_{A_2} - r_{P_2}}{r_{A_2} + r_{P_2}}$$

$$r_{P_1} = \frac{\|h_1\|^2}{\mu}\frac{1}{1+\|e_1\|}, \quad r_{P_2} = \frac{\|h_2\|^2}{\mu}\frac{1}{1+\|e_2\|}$$

$$\phi = \tan^{-1}\left(\frac{b}{a}\right), \quad \theta_1 = \phi \pm \cos^{-1}\left(\frac{c}{a}\cos\phi\right)$$

$$\|r\| = \frac{\|h_1\|^2}{\mu}\frac{1}{1+\|e_1\|\cos\theta_1}$$

$$V_{\perp_I})_1 = \frac{\|h_1\|}{\|r\|}, \quad V_{r_I})_1 = \frac{\mu}{\|h_1\|}\|e_1\|\sin\theta_1$$

$$V_I)_1 = \sqrt{V_{\perp_I})_1^2 + V_{r_I})_1^2}, \quad \gamma_1 = \tan^{-1}\left(\frac{V_{r_I})_1}{V_{\perp_I})_1}\right)$$

$$V_I)_2 = \frac{\|h_2\|}{\|r\|}, \quad V_{r_I})_2 = \frac{\mu}{\|h_2\|}\|e_2\|\sin(\theta_1 - 25^\circ)$$

$$V_I)_2 = \sqrt{V_{\perp_I})_2^2 + V_{r_I})_2^2}, \quad \gamma_2 = \tan^{-1}\left(\frac{V_{r_I})_2}{V_{\perp_I})_2}\right)$$

- Case 2: Impulsive manoeuvre takes place at a given true anomaly θ_1 on orbit 1.

- $\|h_2\| = \|r\|\left[V_{\perp_I})_1 + \Delta V_\perp\right] = \|h_1\| + \|r\|\Delta V_\perp$

- $V_{r_I})_2 = V_{r_I})_1 + \Delta V_r$

- $\frac{\mu}{\|h_2\|}\|e_2\|\sin\theta_2 = V_{r_I})_1 + \Delta V_r$

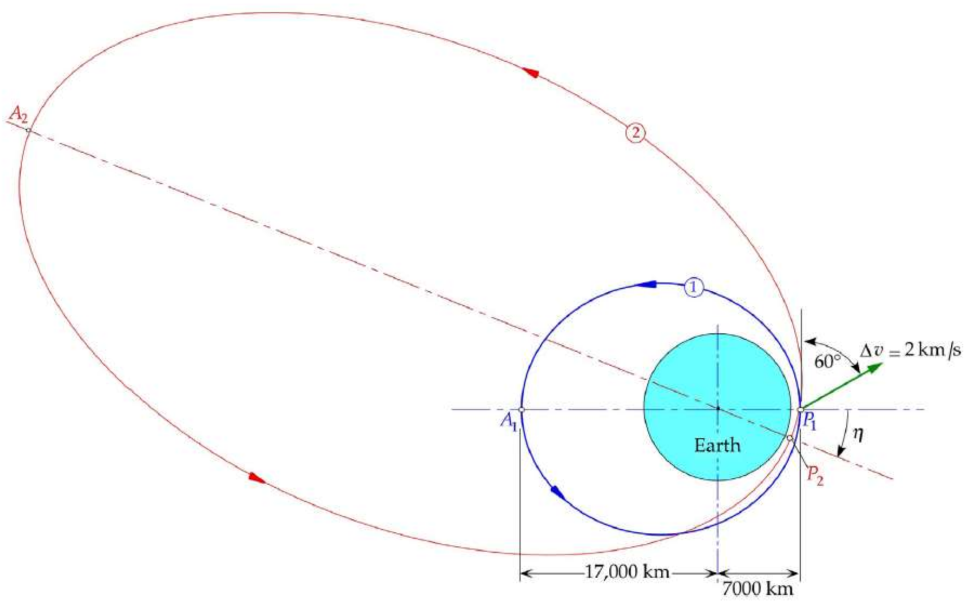
- $\sin\theta_2 = \frac{1}{\|e_2\|}\frac{(\|h_1\| + \|r\|\Delta V_\perp)(\mu\|e_1\|\sin\theta_1 + \|h_1\|\Delta V_r)}{\mu\|h_1\|}$

- $\cos\theta_2 = \frac{1}{\|e_2\|}\frac{(\|h_1\| + \|r\|\Delta V_\perp)^2\|e_1\|\cos\theta_1 + (2\|h_1\| + \|r\|\Delta V_\perp)\|r\|\Delta V_\perp}{\|h_1\|^2}$

$$-\tan\theta_2 = \frac{[V_{\perp_I})_1 + \Delta V_\perp][V_{r_I})_1 + \Delta V_r]}{[V_{\perp_I})_1 + \Delta V_\perp]^2 \|e_1\| \cos\theta_1 + \frac{\mu}{\|r\|}[2V_{\perp_I})_1 + \Delta V_\perp]\Delta V_\perp}$$

Example

An earth satellite in orbit 1 of Fig. 6.19 undergoes the indicated delta-v maneuver at its perigee. Determine the rotation η of its apse line as well as the new perigee and apogee.



Details

$$\|e_1\| = \frac{r_{A_1} - r_{P_1}}{r_{A_1} + r_{P_1}}$$

$$r_{P_1} = \frac{\|h_1\|^2}{\mu} \frac{1}{1 + \|e_1\|}$$

$$V_{\perp_{P_1}})_1 = \frac{\|h_1\|}{r_{P_1}}, \quad V_{r_{P_1}})_1 = 0$$

$$\Delta V_\perp = \Delta V \cos 60^\circ, \quad \Delta V_r = \Delta V \sin 60^\circ$$

$$\|h_2\| = \|h_1\| + \|r\| \Delta V_\perp$$

$$\tan\theta_2 = \ldots$$

$$\|e_2\| = \ldots$$

$$r_{P_2} = \frac{\|h_2\|^2}{\mu} \frac{1}{1+\|e_2\|} , \quad r_{A_2} = \frac{\|h_2\|^2}{\mu} \frac{1}{1-\|e_2\|}$$