MA 214: Introduction to numerical analysis (2021–2022)

Tutorial 10

(April 13, 2022)

(1) – (3) Obtain the LDL^t factorizations of the matrices:

$$A_1 = \begin{pmatrix} 4 & 1 & 1 & 1 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 2 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 4 & 1 & -1 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & -1 & 5 & 2 \\ 0 & 0 & 2 & 4 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 6 & 2 & 1 & -1 \\ 2 & 4 & 1 & 0 \\ 1 & 1 & 4 & -1 \\ -1 & 0 & -1 & 3 \end{pmatrix}.$$

(4) – (5) Obtain the LL^t factorizations of the matrices:

$$A_4 = \begin{pmatrix} 4 & 0 & 2 & 1 \\ 0 & 3 & -1 & 1 \\ 2 & -1 & 6 & 3 \\ 1 & 1 & 3 & 8 \end{pmatrix}, \qquad A_5 = \begin{pmatrix} 4 & 1 & 1 & 1 \\ 1 & 3 & 0 & -1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 1 & 4 \end{pmatrix}.$$

(6) Use the LDL^t factorization to solve the system:

$$\begin{aligned} 2x_1 - x_2 \phantom{{}- x_3} &= 3, \\ -x_1 + 2x_2 - x_3 &= -3, \\ - x_2 + 2x_3 &= 1. \end{aligned}$$

(7) Use the LL^t factorization to solve the above system.

(8) – (9) Find all values of $\alpha \in \mathbb{R}$ such that the following matrices are positive definite:

$$A_8 = \begin{pmatrix} 2 & \alpha & -1 \\ \alpha & 2 & 1 \\ -1 & 1 & 4 \end{pmatrix}, \qquad A_9 = \begin{pmatrix} \alpha & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 4 \end{pmatrix}.$$