MA 214: Introduction to Numerical Analysis

Indian Institute of Technology Bombay

Quiz 2

Time: **1 hour**
Instructors: **S. Baskar** and **S. Sivaji Ganesh**

Marks: **15**
Date: **13-02-2019**

Instructions:

(1) Write your **Name, Roll Number, and Tutorial Batch** clearly on your answer book as well as every supplement you may use. A **penalty of -1 mark will be awarded** for failing to do so.

(2) Number the pages of your answer book and make a question-page index on the front page.

(3) The answer to each question should start on a new page. If the answer for a question is split into two parts and written in two different places, the first part alone will be corrected.

(4) Only scientific calculators are allowed. Any kind of programing device is not allowed.

(5) Formulas used need not be proved but **needs to be stated clearly**.

(6) The question paper contains 4 questions. **Answer all the questions.**

(1) Using modified Gaussian elimination method with partial pivoting, and 4-digit rounding arithmetic, solve the following system of linear equations:

$$\begin{aligned} 6x_1 + 2x_2 + 2x_3 &= -2 \\ 2x_1 + \frac{2}{3}x_2 + \frac{1}{3}x_3 &= 1 \\ x_1 + 2x_2 - x_3 &= 0. \end{aligned}$$

[4 Marks]

(2) Obtain a Crout factorization of the matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

State clearly the assumptions that you make on the matrix A (and on its entries $a_{ij} \in \mathbb{R}$, $i, j = 1, 2, 3$) at each step in the process of obtaining the Crout factorization. **[4 Marks]**

(3) Let $A(\alpha)$ be a matrix depending on a parameter $\alpha \in \mathbb{R}$ given by

$$A(\alpha) = \begin{pmatrix} 0.1\alpha & 0.1\alpha \\ 1.0 & 2.5 \end{pmatrix}.$$

For each $\alpha \in \mathbb{R}$, compute the condition number of $A(\alpha)$. Determine an α_0 such that the condition number of $A(\alpha_0)$ is the minimum of the set $\{\kappa(A(\alpha)) : \alpha \in \mathbb{R}\}$. In the computation of condition numbers, use the matrix norm that is subordinate to the maximum vector norm on $\mathbb{R}^2$. **[3 Marks]**

(4) Write the formula for the Jacobi iterative sequence of the system

$$\begin{aligned} x_1 - 2x_2 &= 0, \\ 2x_1 - x_2 &= 3. \end{aligned}$$

Find an initial guess $\boldsymbol{x}^{(0)}$ such that the corresponding Jacobi iterative sequence does not converge to the exact solution of the system and justify your answer. **[4 Marks]**

— End of the Question Paper —