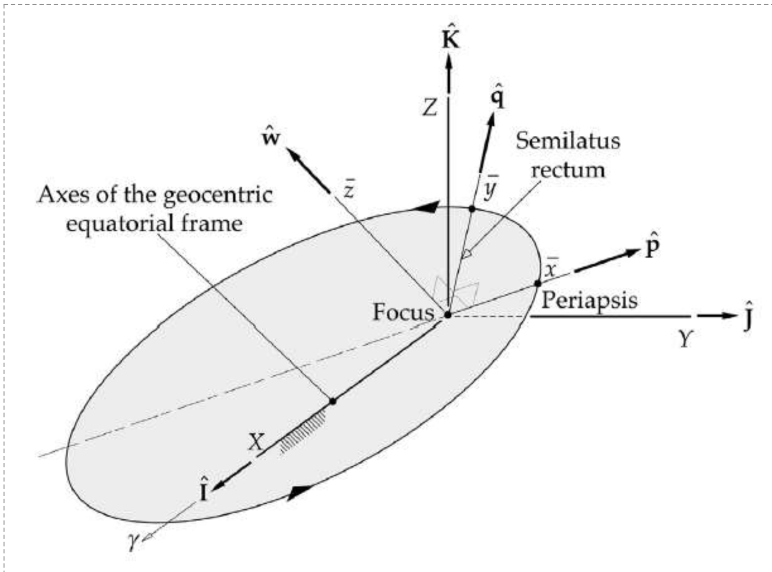


- $$r = \bar{x}\,\hat{p} + \bar{y}\,\hat{q} = \frac{\|h\|^2}{\mu}\frac{1}{1+\|e\|\cos\theta}(\cos\theta\,\hat{p} + \sin\theta\,\hat{q})$$

$$v = \dot{\bar{x}}\,\hat{p} + \dot{\bar{y}}\,\hat{q} = \frac{\mu}{\|h\|}\left[-\sin\theta\,\hat{p} + (\|e\| + \cos\theta)\,\hat{q}\right]$$

- $$r = \frac{\|h\|^2}{\mu}\frac{1}{1+\|e\|\cos\theta}\begin{bmatrix}\cos\theta\\ \sin\theta\\ 0\end{bmatrix}$$

$$v = \frac{\mu}{\|h\|}\begin{bmatrix}-\sin\theta\\ \|e\|+\cos\theta\\ 0\end{bmatrix}$$

- Direction cosine matrix of the transformation from XYZ to $\bar{x}\bar{y}\bar{z}$:

$$[Q]_{X\bar{x}} = R_3(\omega)R_1(i)R_3(\Omega)$$

$$[\mathbf{Q}]_{X\bar{x}} = \begin{bmatrix} -\sin\Omega\cos i\sin\omega + \cos\Omega\cos\omega & \cos\Omega\cos i\sin\omega + \sin\Omega\cos\omega & \sin i\sin\omega \\ -\sin\Omega\cos i\cos\omega - \cos\Omega\sin\omega & \cos\Omega\cos i\cos\omega - \sin\Omega\sin\omega & \sin i\cos\omega \\ \sin\Omega\sin i & -\cos\Omega\sin i & \cos i \end{bmatrix}$$

$$[\mathbf{Q}]_{\bar{x}X} = \begin{bmatrix} -\sin\Omega\cos i\sin\omega + \cos\Omega\cos\omega & -\sin\Omega\cos i\cos\omega - \cos\Omega\sin\omega & \sin\Omega\sin i \\ \cos\Omega\cos i\sin\omega + \sin\Omega co\omega & \cos\cos i\cos\omega - \sin\Omega\sin\omega & -\cos\Omega\sin i \\ \sin i\sin\omega & \sin i\cos\omega & \cos i \end{bmatrix}$$

- $r = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \quad V = \begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix}$

- $r = \begin{bmatrix} \bar{x} \\ \bar{y} \\ 0 \end{bmatrix} = [Q]_{X\bar{x}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

$V = \begin{bmatrix} \dot{\bar{x}} \\ \dot{\bar{y}} \\ 0 \end{bmatrix} = [Q]_{X\bar{x}} \begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix}$

Given the orbital elements h, e, i, Ω, ω, and θ, compute the state vectors $\mathbf{r}$ and $\mathbf{v}$ in the geocentric equatorial frame of reference. A MATLAB implementation of this procedure is listed in Appendix D.22. This algorithm can be applied to orbits around other planets or the sun.

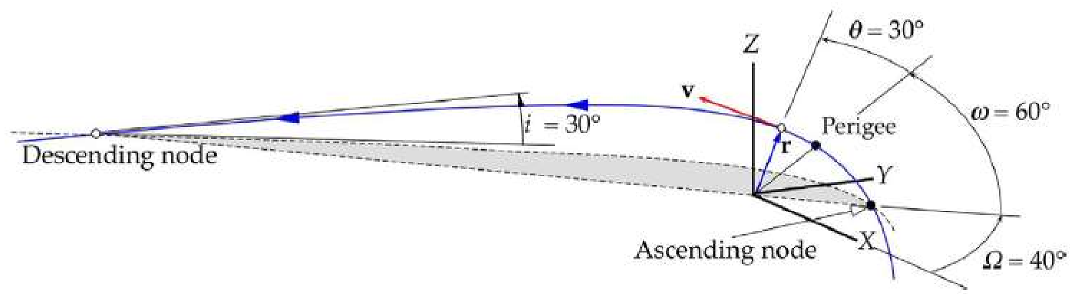
1. Calculate position vector $\{\mathbf{r}\}_{\bar{x}}$ in perifocal coordinates using Eq. (4.45).
2. Calculate velocity vector $\{\mathbf{v}\}_{\bar{x}}$ in perifocal coordinates using Eq. (4.46).
3. Calculate the matrix $[\mathbf{Q}]_{\bar{x}X}$ of the transformation from perifocal to geocentric equatorial coordinates using Eq. (4.49).
4. Transform $\{\mathbf{r}\}_{\bar{x}}$ and $\{\mathbf{v}\}_{\bar{x}}$ into the geocentric frame by means of Eq. (4.51).

Example

For a given earth orbit, the elements are $h = 80,000\,\text{km}^2/\text{s}$, $e = 1.4$, $i = 30°$, $\Omega = 40°$, $\omega = 60°$, and $\theta = 30°$. Using Algorithm 4.5, find the state vectors $\mathbf{r}$ and $\mathbf{v}$ in the geocentric equatorial frame.

Data:

Details



Effect of the Earth's Oblateness

- $\text{Oblateness} = \dfrac{\text{Equitorial radius} - \text{Polar radius}}{\text{Equitorial radius}}$

Table 4.3 Oblateness and second zonal harmonics

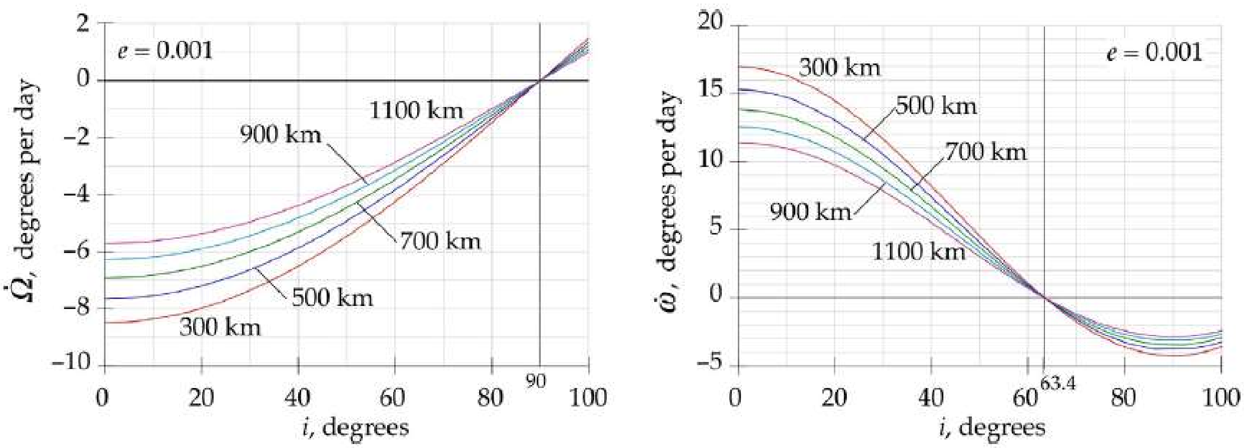
Planet	Oblateness	J_2
Mercury	0.000	$60(10^{-6})$
Venus	0.000	$4.458(10^{-6})$
Earth	0.003353	$1.08263(10^{-3})$
Mars	0.00648	$1.96045(10^{-3})$
Jupiter	0.06487	$14.736(10^{-3})$
Saturn	0.09796	$16.298(10^{-3})$
Uranus	0.02293	$3.34343(10^{-3})$
Neptune	0.01708	$3.411(10^{-3})$
(Moon)	0.0012	$202.7(10^{-6})$

- $\dot{\Omega} = -\left[\dfrac{3}{2}\dfrac{\sqrt{\mu}\, J_2 R^2}{(1-\|e\|^2)^2 a^{7/2}}\right]\cos i$

- $\dot{\omega} = -\left[\dfrac{3}{2}\dfrac{\sqrt{\mu}\, J_2 R^2}{(1-\|e\|^2)^2 a^{7/2}}\right]\left(\dfrac{5}{2}\sin^2 i - 2\right)$

$$\left[\quad (1-e^2)^2 a^2 \right] \left(\qquad \right)$$

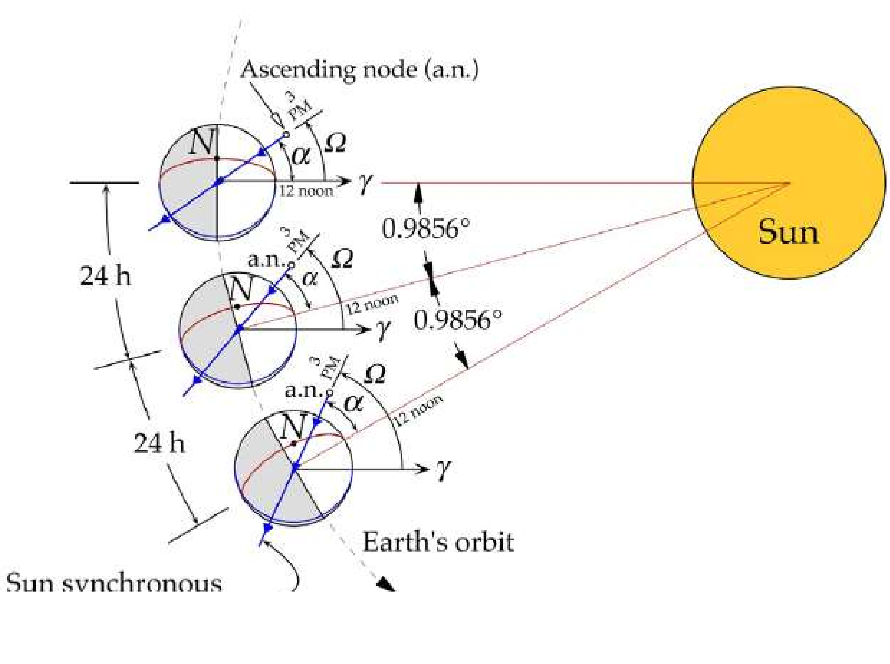
- $$\dot{\omega} = \dot{\Omega} \frac{\left(\frac{5}{2}\sin^2 i - 2\right)}{\cos i}$$



Example

A spacecraft is in a 280 km by 400 km orbit with an inclination of 51.43°. Find the rates of node regression and perigee advance.

Details

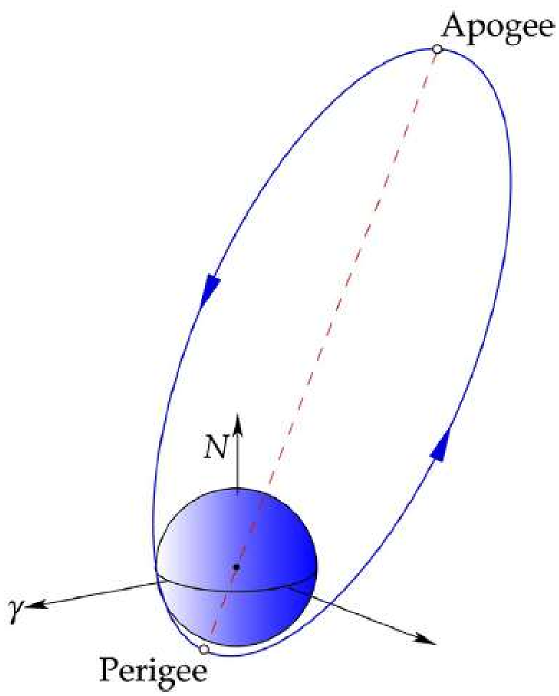


orbit

Example

A satellite is to be launched into a sun-synchronous circular orbit with a period of 100 min. Determine the required altitude and inclination of its orbit.

Details



Example

Determine the perigee and apogee for an earth satellite whose orbit satisfies all the following conditions: it is sun synchronous, its argument of perigee is constant, and its period is 3 h.

Details