AE 234/711 Aircraft Propulsion
Tutorial 1

US Standard Atmosphere 1976 *(expressions from Gudmundsson)*

For Troposphere: $0 \leq h \leq 36,089\ ft \equiv 11\ km$,

Temperature ratio: $\theta = 1 - \dfrac{h}{145442}$

Pressure ratio: $\delta = \theta^{5.2561}$

For Lower Stratosphere: $11\ km \equiv 36,089\ ft \leq h \leq 65,617\ ft \equiv 20\ km$,

Temperature ratio: $\theta = 0.751865$

Pressure ratio: $\delta = 0.223361\ \exp\left[-\left(\dfrac{h - 36089}{20806}\right)\right]$

where, θ and δ are the temperature and pressure normalised by their sea level values. The altitude h is in feet. The sea-level conditions for the standard atmosphere are $T_{sl} = 288.15\ K$, $p_{sl} = 101325\ Pa$ and $\rho_{sl} = 1.225 kg/m^3$.

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1. An earlier version of Boeing 777 had a take-off mass of 250 tons, and reaches a speed of 250 kmph at take-off. If the wing planform area is $450m^2$. Assume that the drag polar is $C_D = 0.018 + 0.055C_L^2$.
   - Calculate the lift coefficient, lift and drag at take-off for standard sea-level conditions.
   - Bengaluru is at an altitude of approximately 3,020 feet. What would be the lift coefficient and drag at take-off in Bengaluru for the same take-off weight?
   - For what speed do we have the minimum thrust requirement for steady and level flight. Calculate that speed, mach number, and the minimum thrust value at the following altitudes: sea level, 5 km, 11 km.
   - Repeat the above for cruise flight at minimum power condition. How is this speed related to the above speed?
2. The city of Mumbai, while at the sea level, is on the coast. One should then consider the effect of humidity on air density. As per wikipedia, the mean relative humidity in Mumbai appears to vary from $60 - 90$% over one year. The mean temperature can be taken as 25^oC. Compare the take-off conditions (lift coefficient and drag) for Mumbai with the standard sea-level case from the earlier problem.

Following Gudmundsson, for a given relative humidity (RH in %age) value, one can obtain the actual air density as follows

$$\rho = \rho_{std}\left(\frac{1+x}{1+x\mathcal{R}_{H_2O}/\mathcal{R}}\right) \equiv \rho_{std}\left(\frac{1+x}{1+1.609x}\right)$$

where the $\mathcal{R}$ is the specific gas constant for air, $\mathcal{R}_{H_2O}$ is the specific gas constant for water vapour, and the x is the humidity ratio in kg water vapour per kg of air

$$x = \left(\frac{RH}{100}\right) 0.003878\ e^{0.0656\ T_c}$$

Here, T_c is the local temperature value in degrees celsius.

3. Derive expressions for the thrust and efficiencies of an air-breathing engine.

4. Consider an aircraft engine (a turbojet) in take-off condition with the following parameters: $\dot{m}_a = 100$ kg/s, $\dot{m}_f = 2$ kg/s, $\mathcal{Q}_R = 42$ MJ/kg.

   - Plot the propulsive, thermal and overall efficiencies for this engine for exhaust speed in the range 0 m/s to $1,000$ m/s. Consider the flight velocity to be $V_a = 200$ m/s. Show the optimum efficiency values.
   - Plot the propulsive, thermal and overall efficiencies for this engine in terms of the vehicle speed for an exhaust speed of $V_e = 500$ m/s. Show the optimum efficiency values.

5. Express the propulsive efficiency η_p in terms of the velocity ratio $s = V_e/V_a$. Plot η_p vs s.

6. Verify that the fuel-air ratio for the stoichiometric combustion of a hydrocarbon fuel of the approximate composition $C_{17n}H_{36n}$ is nearly 0.068.