1. $T_1 = 253\,K$, $p_1 = 0.86 \times 10^5\ N/m^2$, $A_1 = 0.4\,m^2$

$V_1 = 120\ m/s$, $p_2 = 0.8 \times 10^5\ N/m^2$ $A_2 = 0.6\,m^2$

$V_2 = 330\ m/s$

$$\rho_1 = \frac{p_1}{RT_1} = 1.1744\ kg/m^3$$

$$\dot{m} = \rho_1 A_1 V_1 = 58.85\ kg/s$$

$$T = p_2 A_2 - p_1 A_1 - \dot{m}(V_2 - V_1)$$

$$= 25.54\,kN$$

2. $p_1 = 80\,kPa$, $T_1 = 300\,K$, $M_1 = 1.5$, $p_2 = 20\,kPa$

From isentropic tables at M_1

$$\frac{p_1}{p_{01}} = 0.2724 , \quad \frac{T_1}{T_{01}} = 0.6897$$

$$\frac{p_2}{p_{02}} = \frac{p_2}{p_1} \cdot \frac{p_1}{p_{01}} = 0.109$$

$$\Rightarrow M_2 = 2.1$$

$$\left.\frac{T_2}{T_{02}}\right)_{M_2} = 0.5314 \Rightarrow T_2 = \frac{T_2}{T_{02}} \cdot \frac{T_{02}}{T_1} \cdot T_1$$

$$= 231.15\,K$$

From Prandtl-Meyer function

$$\nu_1 = 11.905^\circ$$

$$\nu_2 = 29.097^\circ$$

$$\Rightarrow \text{Deflection } \delta = \nu_2 - \nu_1 = 17.192^\circ$$

3. $\frac{A_e}{A_t} = 5$, $T_0 = 500K$, $p_0 = 800kPa$

(i) For no flow, $P_b = p_0 = 800\,kPa$

when flow chokes at the throat

$$\frac{A}{A^*} = 5 \Rightarrow M = 0.115$$

$$\frac{p}{p_0} = 0.9907 \text{ or}$$

$$p_b = 0.9907 \times 800 = 792.56\,kPa$$

$$792.56\,kPa \leq p_b < 800\,kPa$$

(ii) For $\frac{A}{A^*} = 5$ and supersonic flow at exit, $M_1 = 3.18$

and $\frac{p}{p_0} = 2.195 \times 10^{-2}$

$\Rightarrow$ pressure before shock $= 17.56\,kPa$

From shock tables

$$M_2 = 0.4651 \text{ and } p_2/p_1 = 11.62$$

$$\Rightarrow p_b = 17.56 \times 11.62 = 204\,kPa$$

$$204\,kPa \leq p_b < 792.56\,kPa$$

(iii) $17.56\,kPa < p_b < 204\,kPa$

(iv) $p_b < 17.56\,kPa$

4. $d_1 = 0.4m$, $d_2 = 0.8m$, $p_1 = 200 \times 10^3 N/m^2$
$T_1 = 310K$, $V_1 = 265 m/s$

$$\dot{m} = \frac{p_1}{RT_1} \cdot \frac{\pi}{4} d_1^2 \cdot V_1$$

$$= 75.86 \ kg/s$$

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}} = 0.75$$

$$\frac{A_2}{A_1} = \frac{A_2/A^*}{A_1/A^*} \Rightarrow \frac{A_2}{A^*} = 4.248$$

$$\Rightarrow M_2 = 0.138$$

$$p_2 = \frac{p_2/p_0}{p_1/p_0} \cdot p_1 = 286.75 \ kPa$$

$$T_2 = \frac{T_2/T_0}{T_1/T_0} \cdot T_1 = 343.49 K$$

$$V_2 = M_2 \ a_2 = 51.25 \ m/s$$

5. $D_t = 0.0075 m$, $D_e = 0.015 m$, $p_0 = 7.5$ bar, $T_0 = 300K$
$f = 0.005$

$$\frac{A_e}{A^*} = \frac{D_e^2}{D_t^2} = 4$$

$\Rightarrow M_e = 2.94$ is the M at inlet to Fanno flow

$$\left.\frac{4fl^*}{D}\right|_M = 0.513$$

$$\Rightarrow L_{max} = l^* = 0.38475\,m$$

For Fanno flow, T_0 is a constant

$$\Rightarrow T_{02} = T_0^* = 300\,K$$

$$\left(\frac{p_0}{p_0^*}\right)_{2.91} = 3.9993 \Rightarrow p_{02} = p_0^* = 1.875\text{ bar}$$

$$\frac{p_2}{p_{02}} = \left(\frac{p}{p_0}\right)_1 = 0.5283 \Rightarrow p_2 = 0.9907\text{ bar}$$

$$\frac{T_2}{T_{02}} = \left(\frac{T}{T_0}\right)_1 = 0.8333 \Rightarrow T_2 = 250\,K$$

6. $V_s = 500\,m/s$, $p_a = 0.1 \times 10^6\,N/m^2$, $T_a = 290K$

$$M_1 = \frac{V_s}{\sqrt{\gamma R T_a}} = 1.465$$

$$\Rightarrow M_2 = 0.715,\ p_2/p_1 = 2.335,\ T_2/T_1 = 1.297$$

$$p_2 = 0.2335\,MPa$$

$$T_2 = 376.13\,K$$

$$V_2 = M_2\sqrt{\gamma R T_2} = 277.95\,m/s$$

$$V_b = V_1 - V_2 = 222.05\,m/s$$

$$M_b = \frac{V_b}{\sqrt{\gamma R T_2}} = 0.571;\ \left(\frac{T_0}{T}\right)_{M_b} = 0.939$$

$$\Rightarrow T_{0b} = 400.55\,K.$$

7. $T_1 = 300$ K, $p_1 = 55$ kPa, $V_1 = 60$ m/s, $\dot{m}_a/\dot{m}_f = 29$

$CV = 42 \times 10^6$ J/kg, $\gamma = 1.4$, $R = 287$ J/kgK

(i) $M_1 = \dfrac{V_1}{\sqrt{\gamma R T_1}} = 0.178$

$\left.\dfrac{T_1}{T_{01}}\right|_{M_1} = 0.9941 \Rightarrow T_{01} = 301.78$ K

1 kg fuel + 29 kg air releases 42×10^6 J

$\Rightarrow q = \dfrac{42}{30} \times 10^6 = 1.4$ MJ/kg

$= c_p (T_{02} - T_{01})$

$\Rightarrow T_{02} = 1694.8$ K

$\dfrac{T_{02}}{T_{01}} = \dfrac{T_{02}/T_0^*}{T_{01}/T_0^*} \Rightarrow \dfrac{T_{02}}{T_0^*} = 0.7289$

$\Rightarrow M_2 = 0.525$

(ii) $p_2 = \dfrac{p_2/p^*}{p_1/p^*} \cdot p_1 = 41.505$ kPa

$T_2 = \dfrac{T_2/T^*}{T_1/T^*} \cdot T_1 = 1574.75$ K

$V_2 = M_2 \sqrt{\gamma R T_2} = 795.45$ m/s

(iii) $T_{0\,max} = T_0^*$

$\left.\dfrac{T_0}{T_0^*}\right)_{M_1} = 0.1298 \Rightarrow T_{0\,max} = 2394.3$ K