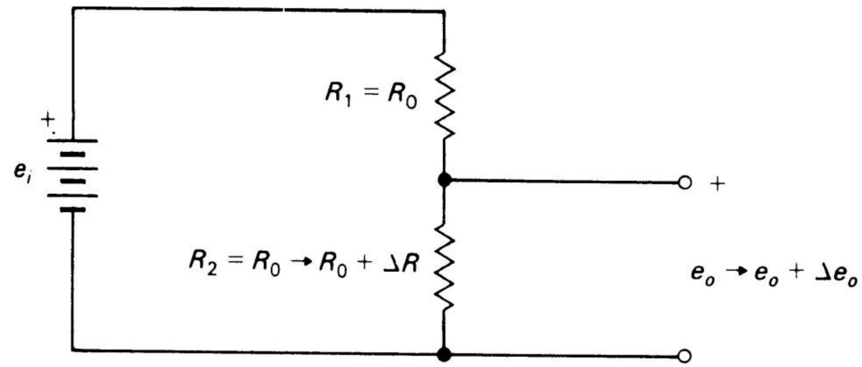
AE 242
Aerospace Measurements Laboratory

Small change in transducer resistance

$$\mathbf{e}_0 + \Delta\mathbf{e}_0 = \frac{\mathbf{e_i}}{2} + \frac{\mathbf{e_i}}{2}\frac{\Delta\mathbf{R}}{2\mathbf{R}_0}\left(\frac{1}{1+\Delta\mathbf{R}/2\mathbf{R}_0}\right)$$



$\Delta R/2R_0$ << 1 the output can be approximated as

$$\mathbf{e}_0 + \Delta\mathbf{e}_0 \approx \mathbf{e}_0 + \frac{\Delta\mathbf{R}}{4\mathbf{R}_0}\mathbf{e_i}$$

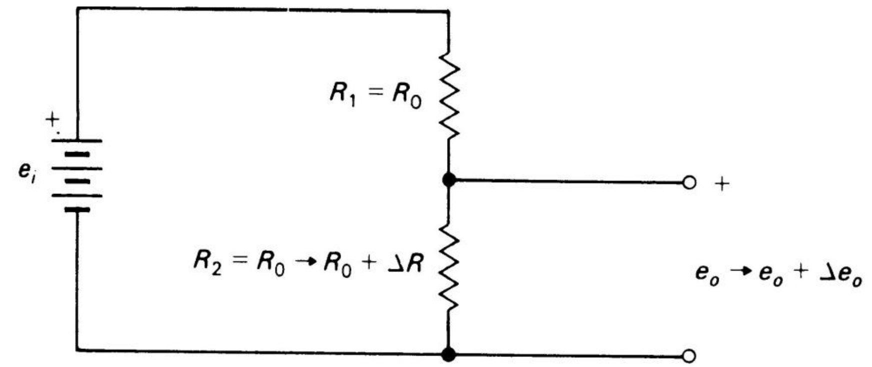
For small variation in resistance it will be linear and it is advantageous. Small variation in resistance results into small output, it is at disadvantage. For a 120 Ω strain gage change in resistance is 240 μΩand it will change the output in micro volts

$$\frac{\Delta\mathbf{e}_0}{\mathbf{e}_0} = \frac{(\Delta\mathbf{R}/4\mathbf{R}_0)\mathbf{e_i}}{\mathbf{e_i}/2} = \frac{\Delta\mathbf{R}}{2\mathbf{R}_0} = 10^{-6}$$

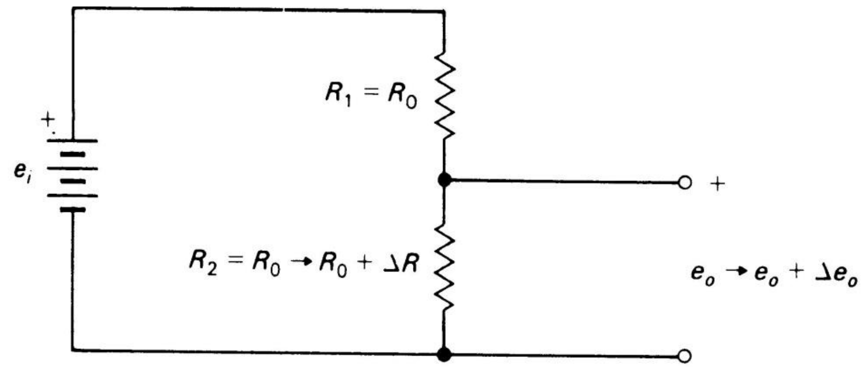
Measurement is $e_0 + \Delta e_0$ and this will need a very precise instrument.

Small change in transducer resistance

Difficulty is to resolve voltage change which is a small fraction of output voltage.



Small change in transducer resistance



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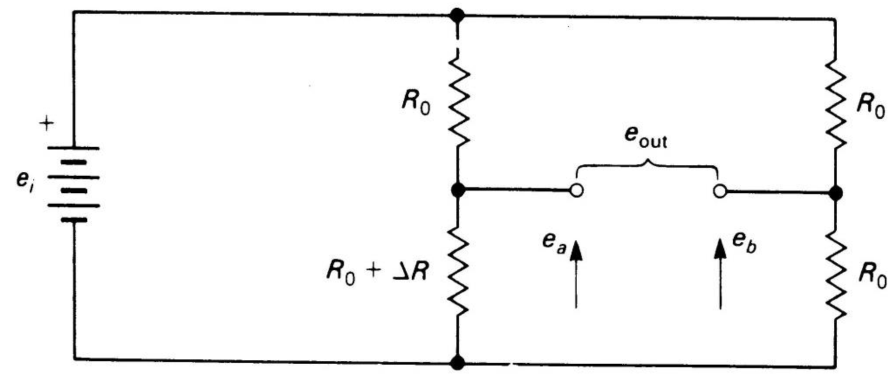
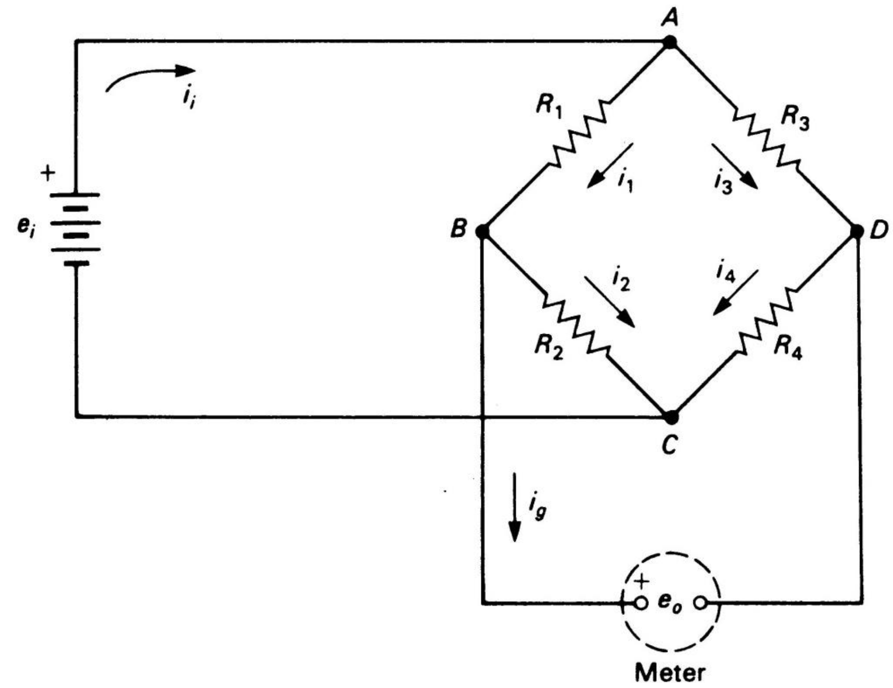


The difficulty can be removed by measuring only the difference and amplifying it.

$$\mathbf{e}_{\mathbf{out}} = \mathbf{e}_{\mathbf{a}} - \mathbf{e}_{\mathbf{b}} = \Delta\mathbf{e}_0 = \frac{\Delta\mathbf{R}}{4\mathbf{R}_0}\mathbf{e}_{\mathbf{i}}$$

Wheatstone bridge

Consist of four arms of resistors, a detector and power supply source. Two arms are voltage divider and the detector (meter) finds the potential difference. Bridge is balanced when potential difference is zero and no current flow through detector . When bridge is balanced:

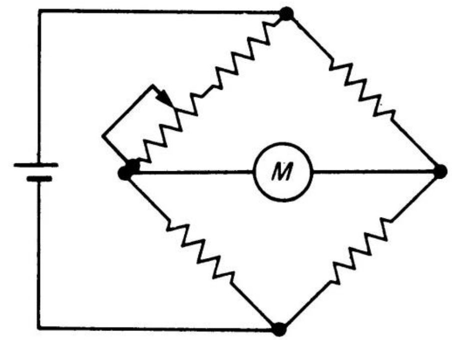


$$\frac{\mathbf{R}_1}{\mathbf{R}_2} = \frac{\mathbf{R}_3}{\mathbf{R}_4} \qquad \text{or} \qquad \frac{\mathbf{R}_1}{\mathbf{R}_3} = \frac{\mathbf{R}_2}{\mathbf{R}_4}$$

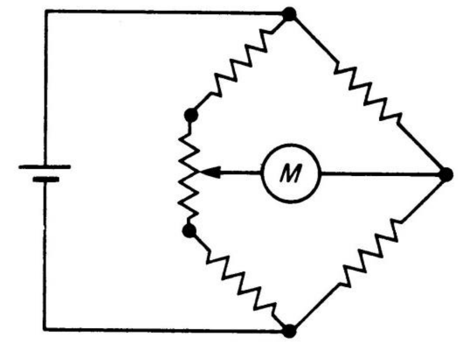
For the Wheatstone resistance bridge to be balance, the ratio of resistances of any two adjacent arms must equal the ratio of resistances of the remaining two arms, taken in the same sense.

Arrangements to balance bridge

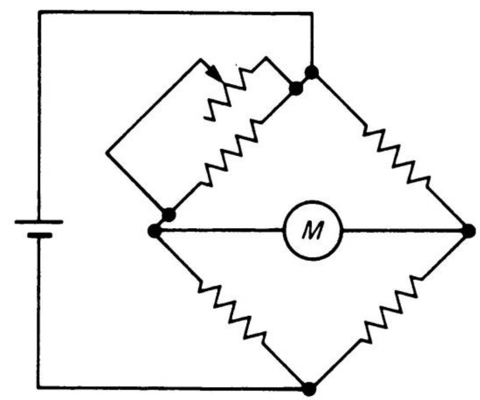
Bridge balancing is required prior to measurement. In case of null balance it is used for measurement. Series balance is used for large variation in resistance and shunt balance is used for small variation. Series or shunt balance depends on the bridge sensitivity. For deflection bridge initially it is balanced and the transducer output is measured by a meter.



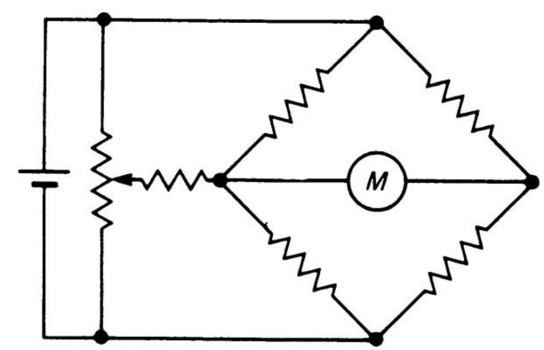
(a) Series balance



(b) Differential series balance



(c) Shunt balance

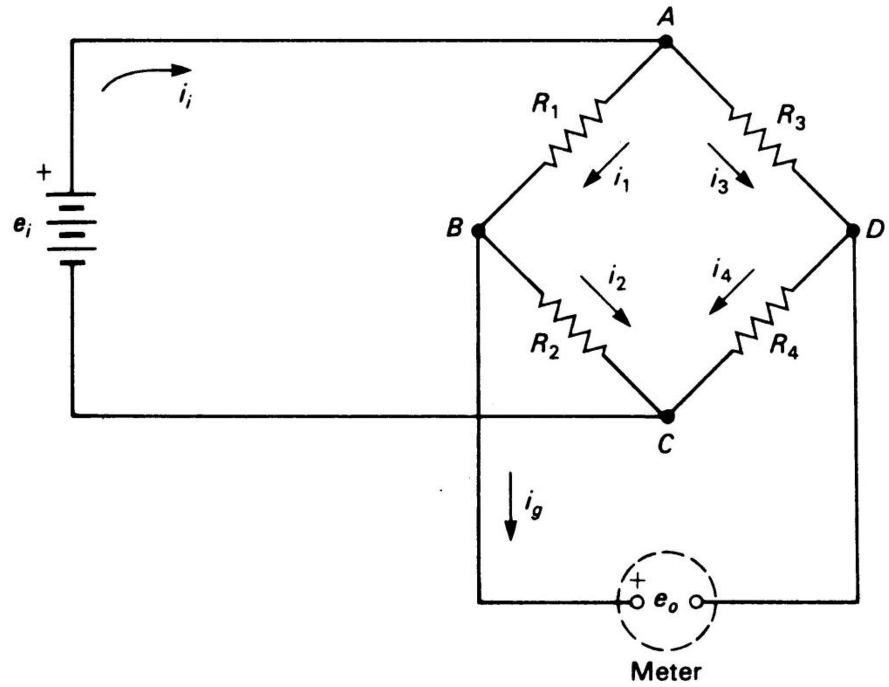


(d) Differential shunt balance

Voltage sensitive bridge

The output is connected to a high impedance measuring instrument. Output is potential difference between point B & D

$$\mathbf{e_o} = \mathbf{e_B} - \mathbf{e_D}$$



Using voltage divider relationship

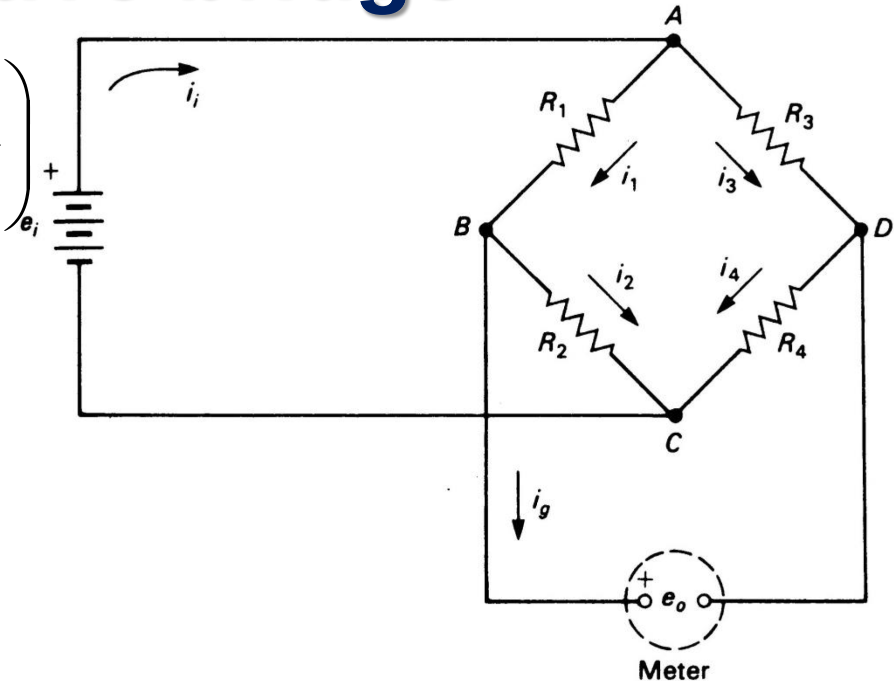
$$\mathbf{e}_0 = \mathbf{e_i}\left(\frac{\mathbf{R}_2}{\mathbf{R}_1 + \mathbf{R}_2} - \frac{\mathbf{R}_4}{\mathbf{R}_3 + \mathbf{R}_4}\right)$$

Resistance R_2 changes by a small amount ΔR, output changes by Δe_0

$$\mathbf{e}_0 + \Delta\mathbf{e}_0 = \mathbf{e_i}\left(\frac{(\mathbf{R}_2 + \Delta\mathbf{R}_2)\mathbf{R}_3 - \mathbf{R}_4\mathbf{R}_1}{(\mathbf{R}_1 + \mathbf{R}_2 + \Delta\mathbf{R}_2)(\mathbf{R}_3 + \mathbf{R}_4)}\right)$$

Voltage sensitive bridge

$$\mathbf{e}_0 + \Delta\mathbf{e}_0 = \mathbf{e}_i\left(\frac{(\mathbf{R}_2 + \Delta\mathbf{R}_2)\mathbf{R}_3 - \mathbf{R}_4\mathbf{R}_1}{(\mathbf{R}_1 + \mathbf{R}_2 + \Delta\mathbf{R}_2)(\mathbf{R}_3 + \mathbf{R}_4)}\right)$$

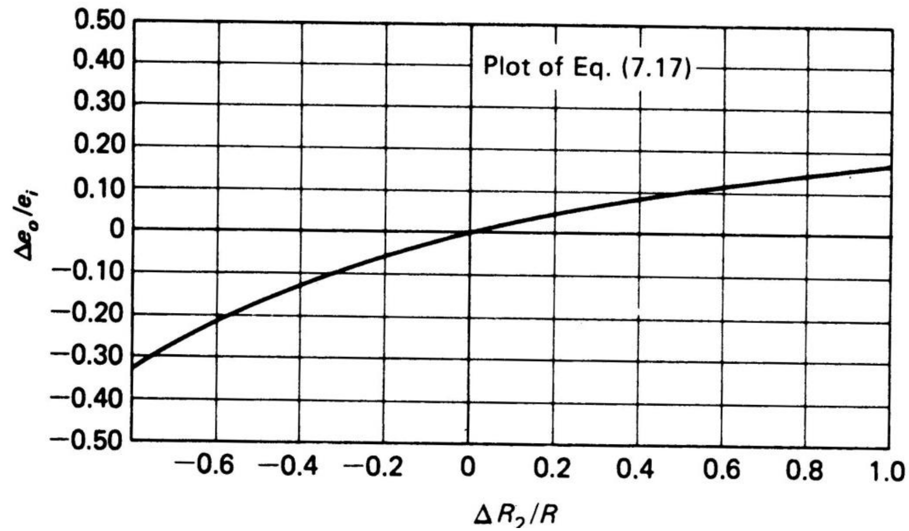


Assuming all resistances equal and this will also result in $e_0 = 0$, then

$$\frac{\Delta\mathbf{e}_0}{\mathbf{e}_i} = \frac{\Delta\mathbf{R}_2 / \mathbf{R}_2}{4 + 2(\Delta\mathbf{R}_2 / \mathbf{R}_2)}$$

The bridge is inherently non-linear and it can be assumed linear for small variation. In most strain gages $\Delta R_2/2R << 1$ and the linearised output is

$$\frac{\Delta\mathbf{e}_0}{\mathbf{e}_i} = \frac{\Delta\mathbf{R}_2}{4\mathbf{R}}$$



1% change in resistance will result into error of -0.5%

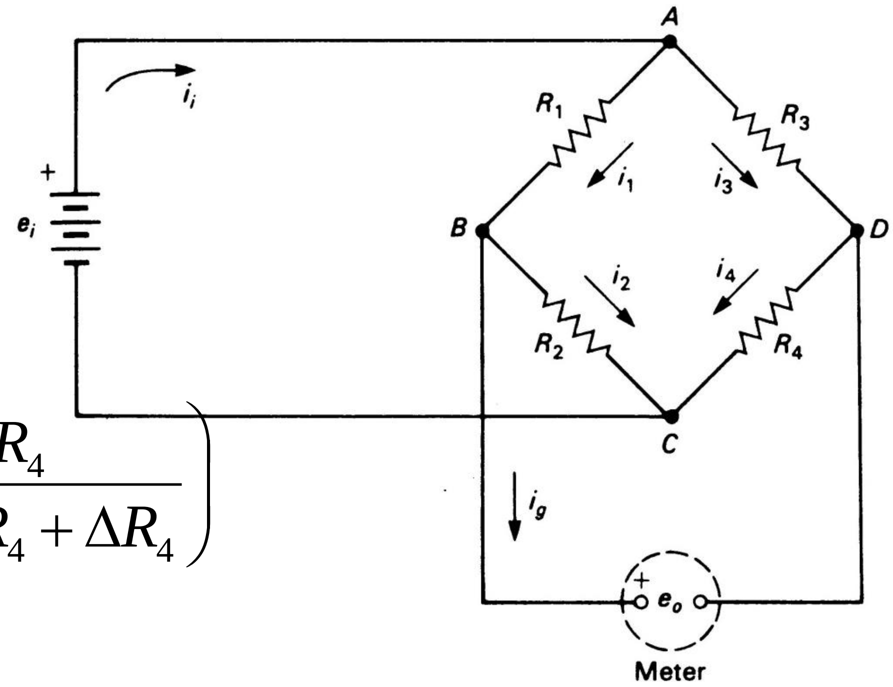
Voltage sensitive bridge

Special case when bridge is linear

Using voltage divider relationship

$$e_0 = e_i\left(\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right)$$

$$e_0 = e_i\left(\frac{R_2+\Delta R_2}{R_1+\Delta R_1+R_2+\Delta R_2} - \frac{R_4+\Delta R_4}{R_3+\Delta R_3+R_4+\Delta R_4}\right)$$



Voltage sensitive bridge

Special case when bridge is linear

Using voltage divider relationship

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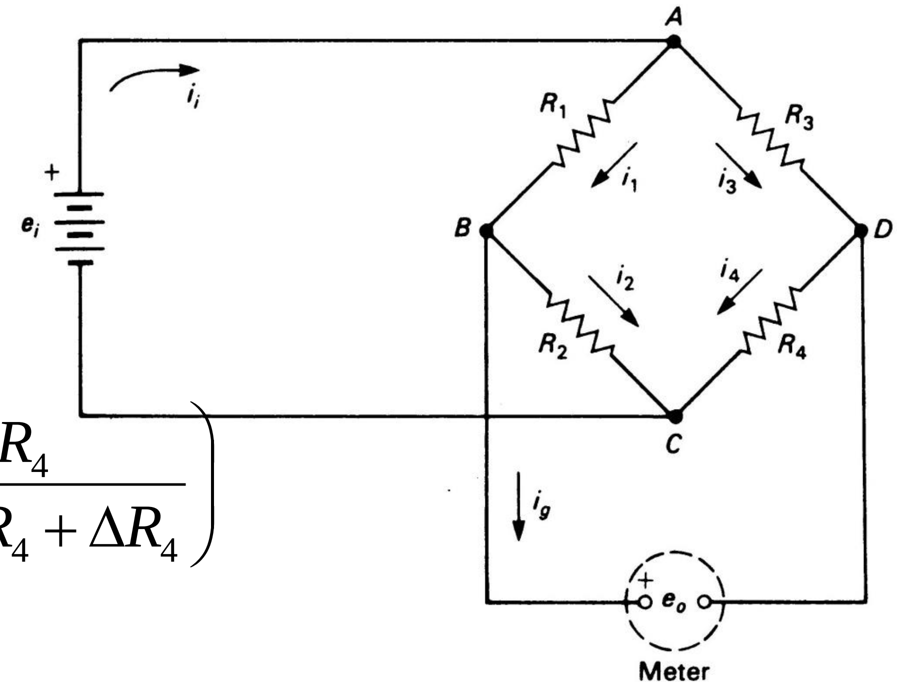


$$R_1 = R_2 = R_3 = R_4 = R$$

$$\Delta R_1 = \Delta R_4 = -\Delta R; \qquad \Delta R_2 = \Delta R_3 = \Delta R$$

$$e_0 = e_i\left(\frac{R_2+\Delta R}{R_1-\Delta R+R_2+\Delta R} - \frac{R_4-\Delta R}{R_3+\Delta R+R_4-\Delta R}\right)$$

$$e_0 = -e_i\left(\frac{\Delta R}{R}\right)$$

Strict linear relationship

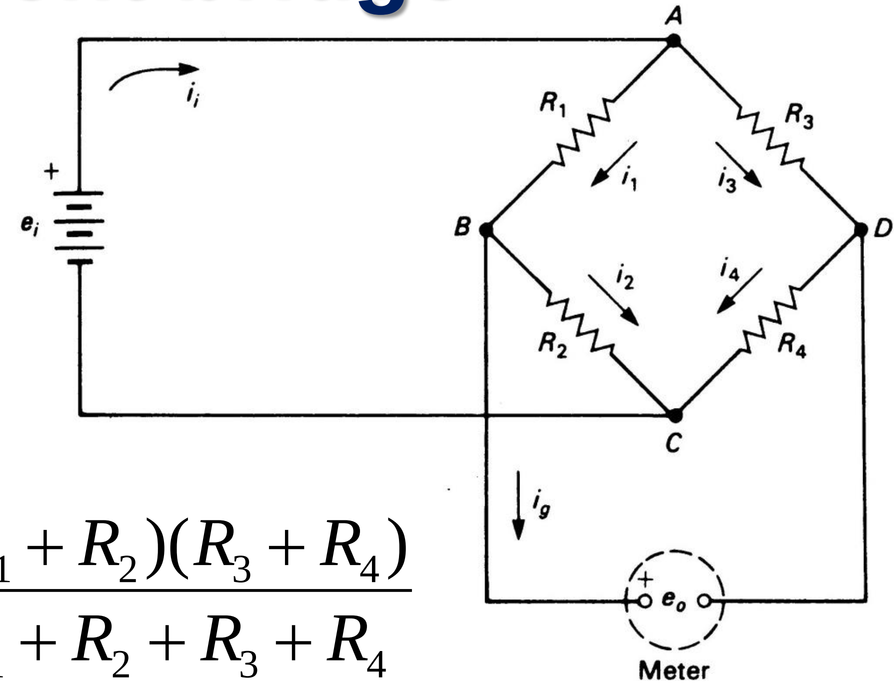
$$\Delta R_1 = \Delta R_4 = \Delta R; \qquad \Delta R_2 = \Delta R_3 = -\Delta R$$

$$e_0 = e_i\left(\frac{R_2-\Delta R}{R_1+\Delta R+R_2-\Delta R} - \frac{R_4+\Delta R}{R_3-\Delta R+R_4+\Delta R}\right)$$

$$e_0 = -e_i\left(\frac{\Delta R}{R}\right)$$

Constant current bridge

When resistance changes current i_i changes in case of constant voltage bridge (excitation voltage). In case of constant current bridge, current i_i is maintained by the power supply i.e. excitation voltage changes as the resistance changes.



$$i_i = \frac{e_i}{R_1 + R_2} + \frac{e_i}{R_3 + R_4}$$

or

$$e_i = i_i \frac{(R_1 + R_2)(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4}$$

Potential difference between point B and D

$$e_0 = e_i \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right)$$

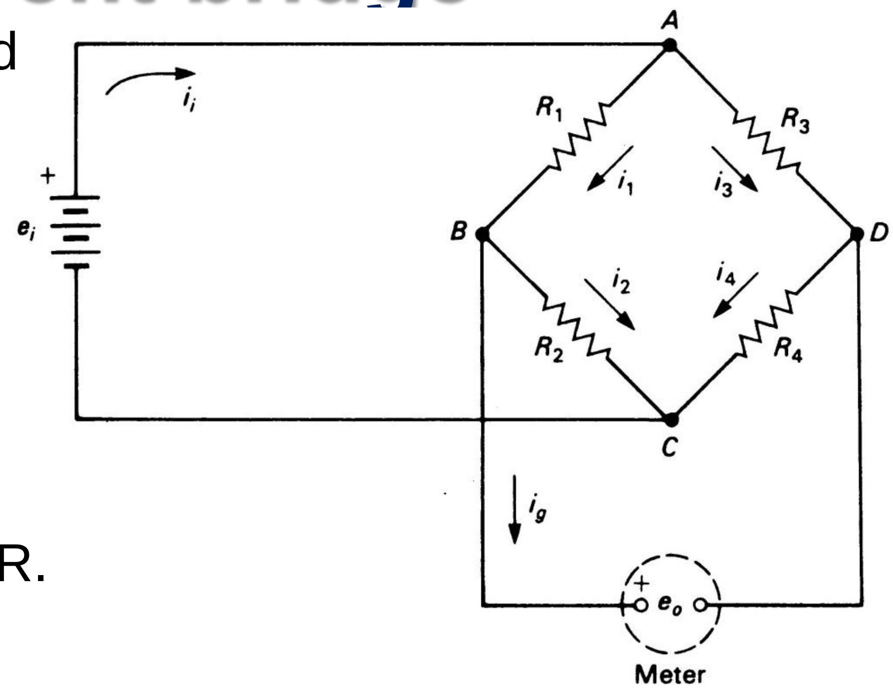
$$e_0 = i_i \frac{(R_1 + R_2)(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4} \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right)$$

For constant current, output is

$$e_0 = i_i \frac{R_2 R_3 - R_1 R_4}{R_1 + R_2 + R_3 + R_4}$$

Constant current bridge

Resistance R_2 is the measuring arm and the resistance changes by ΔR. Output can be written as



$$e_0 + \Delta e_0 = i_i \frac{(R_2 + \Delta R)R_3 - R_1 R_4}{R_1 + (R_2 + \Delta R) + R_3 + R_4}$$

For the case when all the resistance = R.

$$\Delta e_0 = i_i \left[\frac{\Delta R}{4 + \Delta R / R} \right]$$

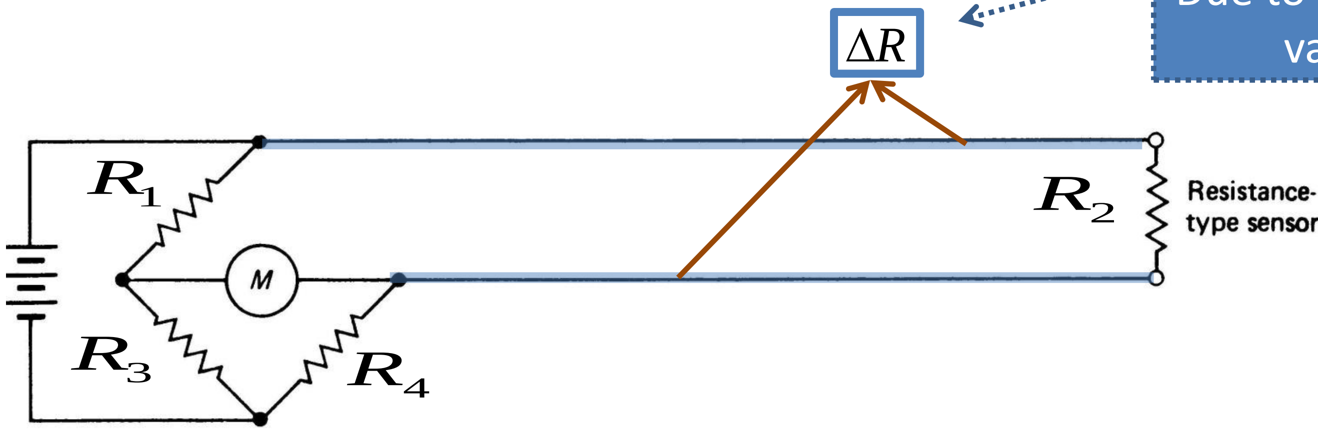
Constant current has better linearity compared to constant voltage

In case of constant voltage $$\frac{\Delta e_0}{e_i} = \frac{\Delta R_2 / R_2}{4 + 2(\Delta R_2 / R_2)}$$

Compensation for leads

Frequently sensor and bridge are separated by appreciable distance, connecting leads will be also of same length. These wires add resistance and temperature variation along these leads can add errors. This can be compensated by a compensating lead which is subjected to same temperature variations as the connecting wire.

Due to temperature variation



$$\frac{R_1}{R_3} = \frac{R_2 + 2\Delta R}{R_4}$$

Compensation for leads

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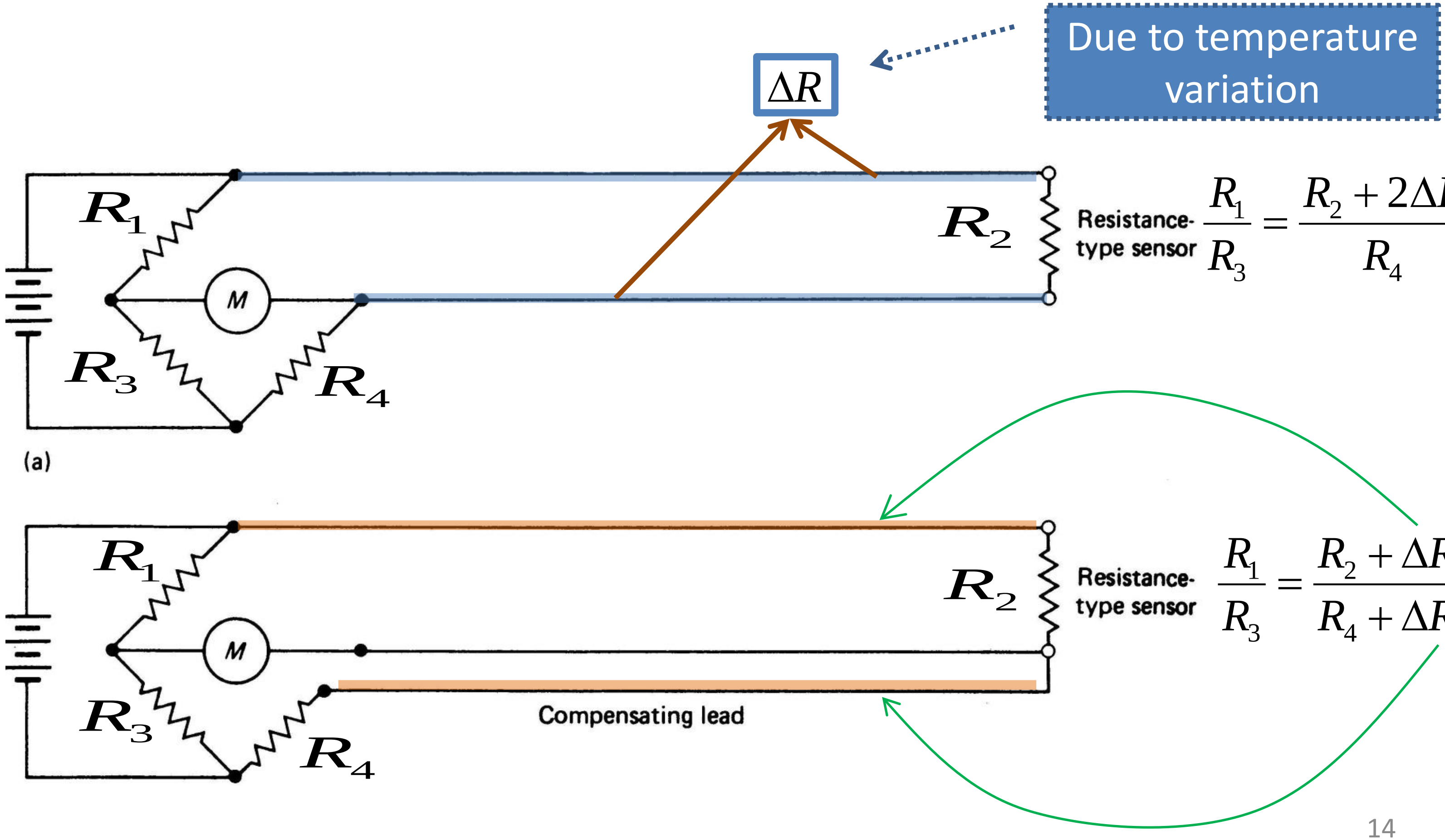
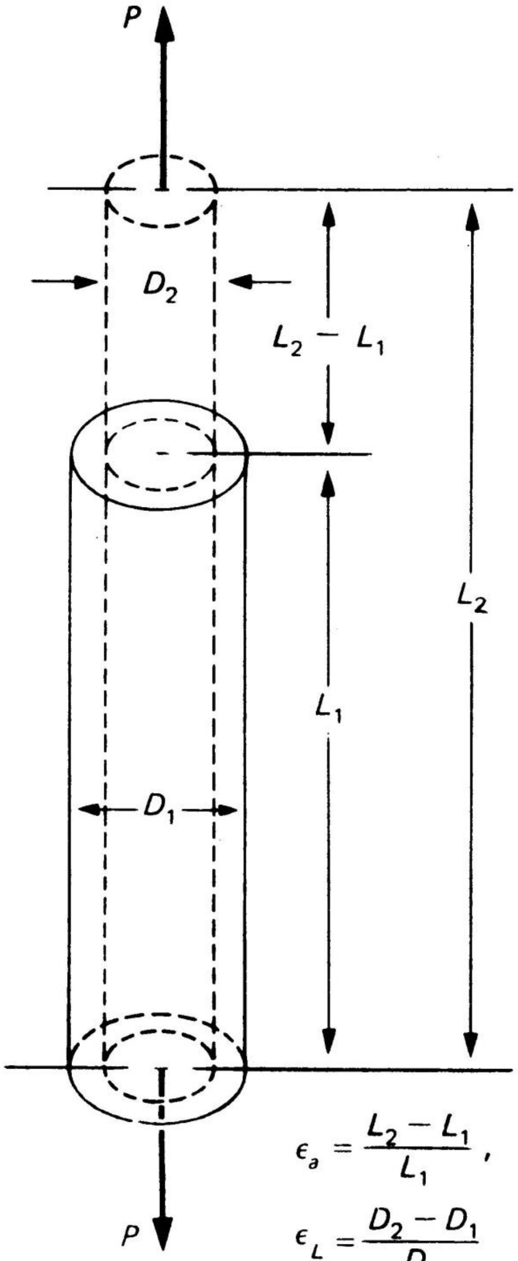


$$\frac{R_1}{R_3} = \frac{R_2 + 2\Delta R}{R_4}$$

$$\frac{R_1}{R_3} = \frac{R_2 + \Delta R}{R_4 + \Delta R}$$

Strain measurement

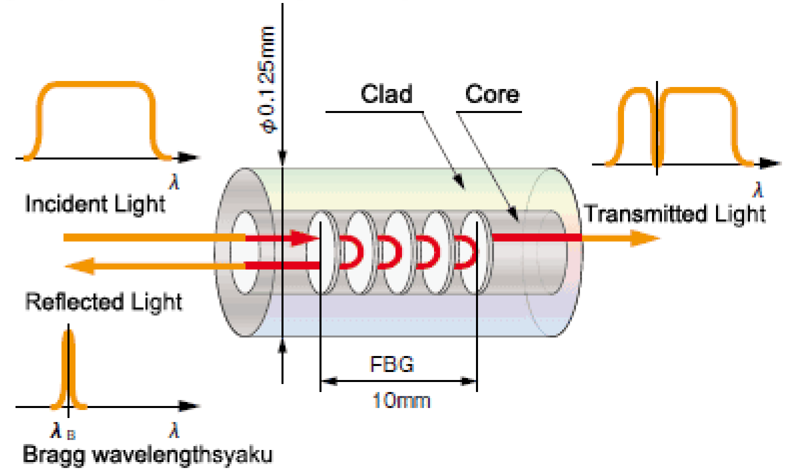
Strain is deformation per unit length. It is very small number and generally multiplied by 10^{-6} and called as micro strain or parts per million (ppm). Axial stress will cause axial strain and also lateral strain and generally it is related by Poisson's ratio. For tensile axial strain it will be compressive and vice versa. Strain can be measured directly or indirectly. One of the direct method is fixing strain gages over the location of interest.



$$\epsilon_a = \frac{L_2 - L_1}{L_1},$$

$$\epsilon_L = \frac{D_2 - D_1}{D_1}$$

Strain measurement



https://tml.jp/e/knowledge/special_ins/fiber_measurement.html

Extensometer

Distance between two points is measured by fixing a displacement sensor. Change in length is determined and strain is calculated.

Low cost, easy to use and useful when strain gage fixing is difficult.

Output can be mechanical or electronic.

FBG (Fiber Bragg Grating)
Fiber Bragg Gratings are a type of reflective structure constructed in a part of an optical fiber with a periodic variation in the refractive index of the fiber core along the longitudinal direction (approx. 10nm) of the fiber. When light passes through the FBG, only the light of a particular wavelength is reflected. Spectrum center wavelength of reflected light is called Bragg's wavelength.

FBG Sensing
The change in a mechanical strain or tensile force applied to the FBG optical fiber makes a change in the wavelength of the reflected light. The mechanical strain or tensile force is known by measuring the change in the wavelength.

Electrical resistance strain gage

When a length of wire is mechanically stretched, cross-section changes and the resistance of the wire changes. For a conductor of length L. cross-sectional area CD^2, D is characteristic dimension and C is constant depending on the cross-section. Resistance of the conductor for the given resistivity ρ

$$\boxed{R = \frac{\rho L}{A}} \qquad \boxed{A = CD^2} \qquad \boxed{R = \frac{\rho L}{A} = \frac{\rho L}{CD^2}}$$

When the conductor is strained each of the quantity can change

$$\boxed{dR = \frac{\rho dL}{A} + \frac{L d\rho}{A} - \frac{\rho L dA}{A^2}} \qquad \boxed{dR = \frac{(L d\rho + \rho dL)}{CD^2} - \frac{2C\rho L D dD}{(CD^2)^2}}$$

$$\boxed{\frac{dR}{R} = \frac{dL}{L} - 2\frac{dD}{D} + \frac{d\rho}{\rho}} \qquad \boxed{\frac{dR/R}{dL/L} = 1 - 2\frac{dD/D}{dL/L} + \frac{d\rho/\rho}{dL/L}}$$

Poisson' ratio $\boxed{\nu = -\frac{dD/D}{dL/L}}$

Piezoresistance $\boxed{\frac{d\rho/\rho}{dL/L}}$

Electrical resistance strain gage

Gage factor $$F = \frac{dR/R}{dL/L} = \frac{dR/R}{\varepsilon_a} = 1 + 2\nu + \frac{d\rho/\rho}{dL/L}$$

Gage factor is function of Poisson's ratio and for 0.3 it will be 1.6. Commonly available strain gages have gage factor of ~ 2. Gage factor relates the change in resistance with strain.

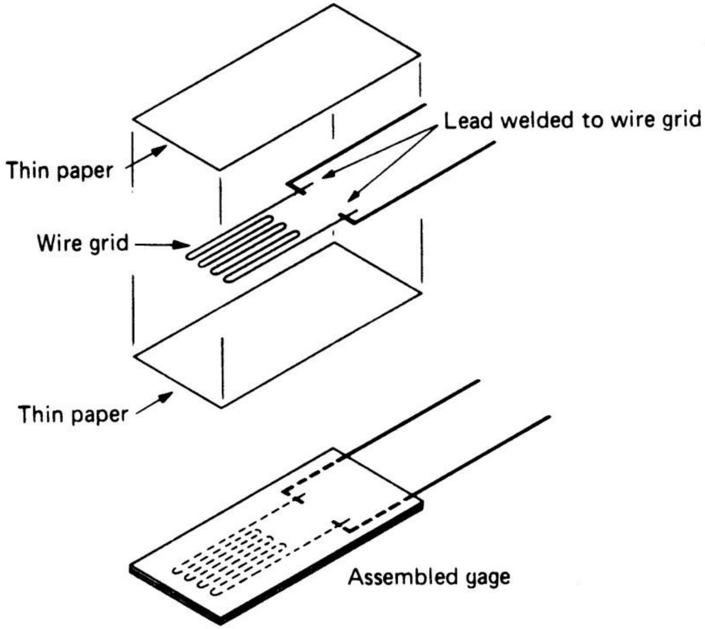
$$\varepsilon_a = \frac{1}{F}\frac{\Delta R}{R} \qquad \Delta R = F\varepsilon_a R$$

Gage factor and resistance value is generally supplied by the manufacturer. Gage factor determination is a destructive test. It is statistically determined and given to the user.

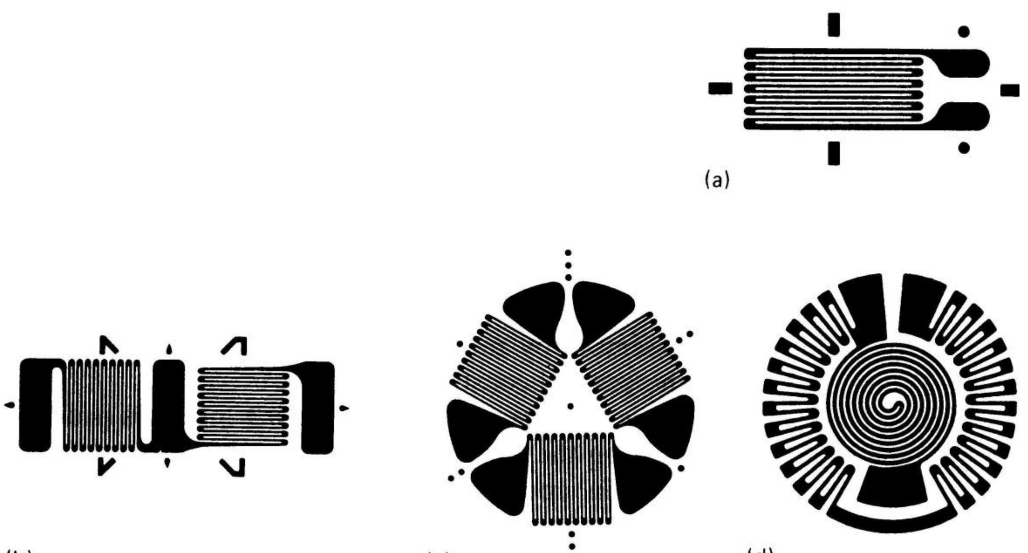
Semiconductor strain gages have higher gage factor, highly sensitive to temperature.

Type of strain gage

Wire type : Thin wire is wound in a grid form and it is glued between two thin paper sheets. Does not have good accuracy. It is obsolete.



Foil type : Thin metal is etched on a thin backing material. Available in various sizes, resistance, shapes and angular configuration.



(a) (b) (c) (d)

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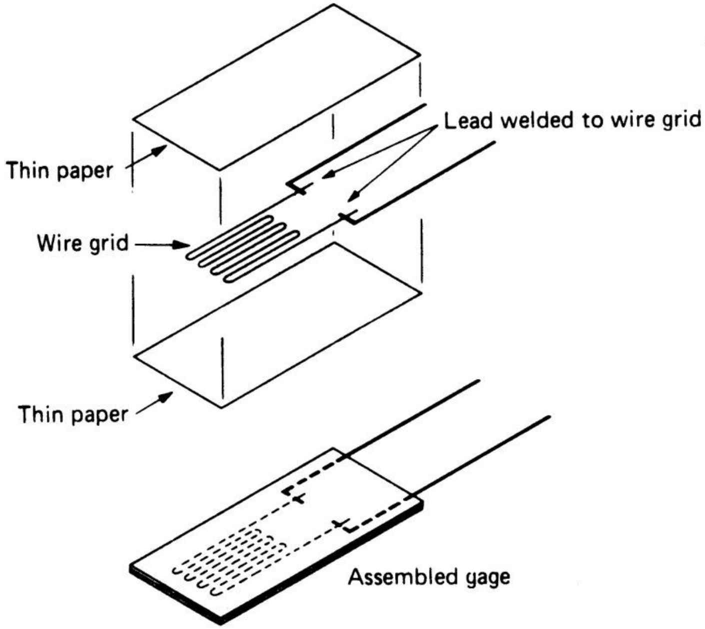

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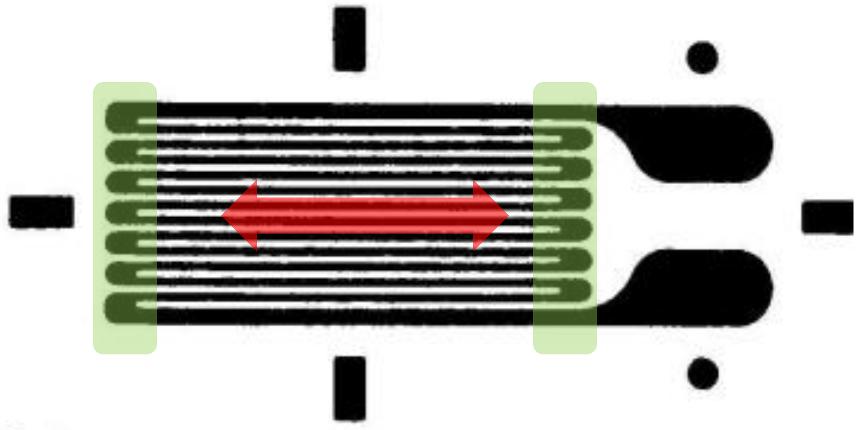
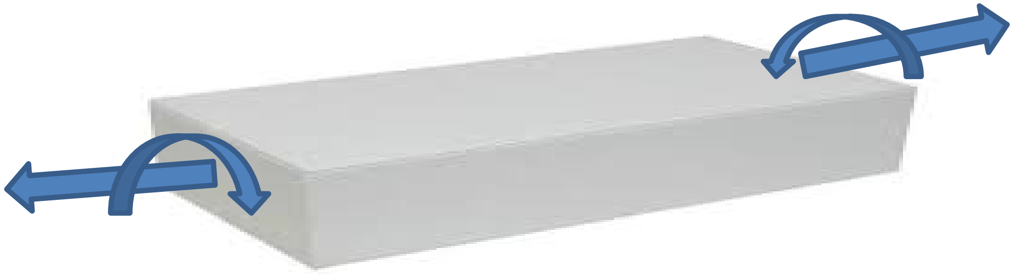


Rectangular block

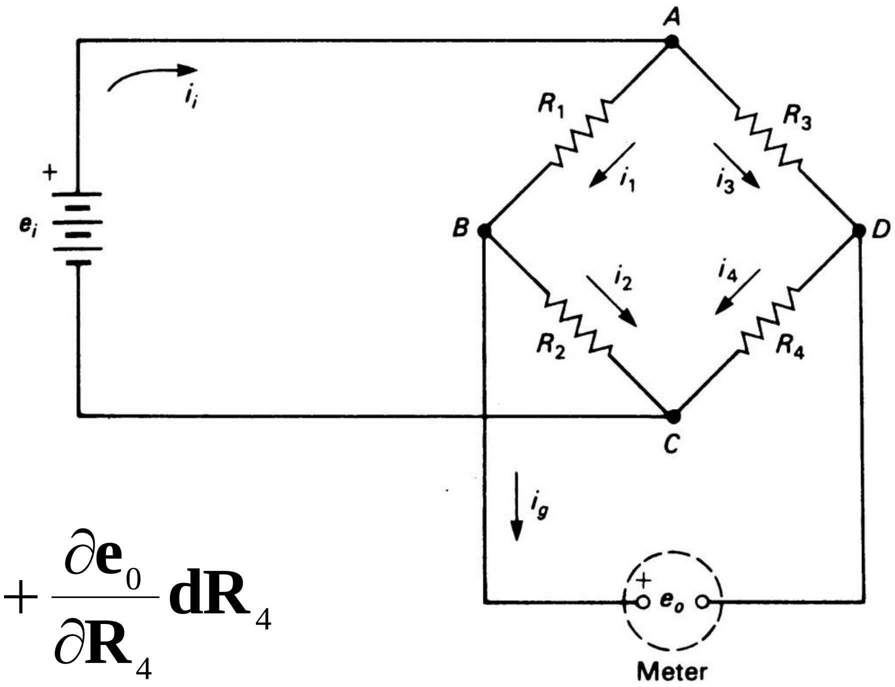


Total strain = axial strain + bending strain

Wheatstone bridge output

For a constant voltage, bridge output

$$\mathbf{e}_0 = \mathbf{e}_\mathbf{i}\left(\frac{\mathbf{R}_1\mathbf{R}_4 - \mathbf{R}_2\mathbf{R}_3}{(\mathbf{R}_1 + \mathbf{R}_2)(\mathbf{R}_3 + \mathbf{R}_4)}\right)$$



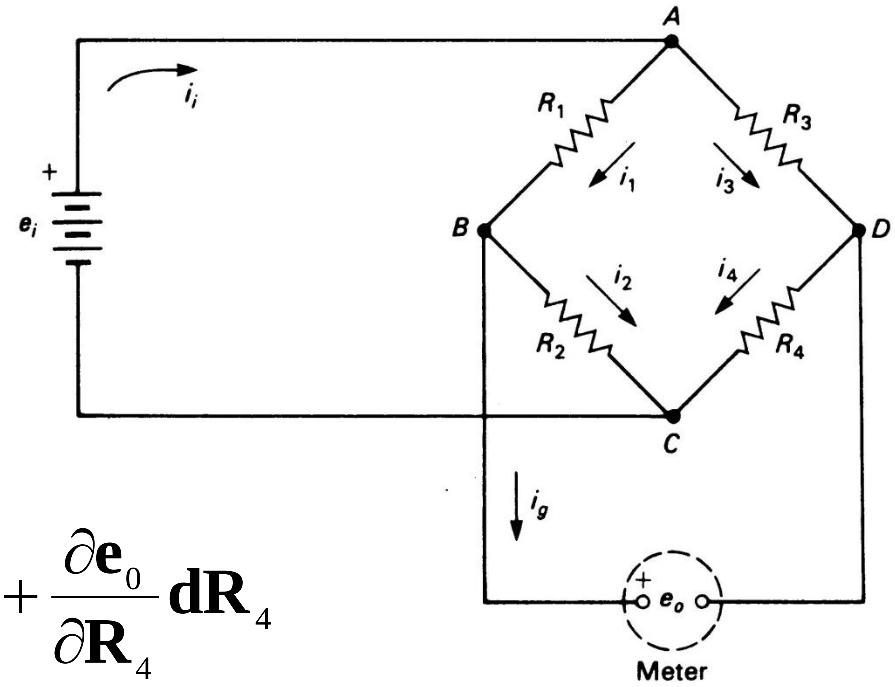
Assuming resistance of each arm is varying

$$\mathbf{de}_0 = \frac{\partial \mathbf{e}_0}{\partial \mathbf{R}_1}\mathbf{dR}_1 + \frac{\partial \mathbf{e}_0}{\partial \mathbf{R}_2}\mathbf{dR}_2 + \frac{\partial \mathbf{e}_0}{\partial \mathbf{R}_3}\mathbf{dR}_3 + \frac{\partial \mathbf{e}_0}{\partial \mathbf{R}_4}\mathbf{dR}_4$$

Wheatstone bridge output

For a constant voltage, bridge output

$$\mathbf{e}_0 = \mathbf{e_i}\left(\frac{\mathbf{R}_1\mathbf{R}_4 - \mathbf{R}_2\mathbf{R}_3}{(\mathbf{R}_1+\mathbf{R}_2)(\mathbf{R}_3+\mathbf{R}_4)}\right)$$



Assuming resistance of each arm is varying

$$\mathbf{de}_0 = \frac{\partial \mathbf{e}_0}{\partial \mathbf{R}_1}\mathbf{dR}_1 + \frac{\partial \mathbf{e}_0}{\partial \mathbf{R}_2}\mathbf{dR}_2 + \frac{\partial \mathbf{e}_0}{\partial \mathbf{R}_3}\mathbf{dR}_3 + \frac{\partial \mathbf{e}_0}{\partial \mathbf{R}_4}\mathbf{dR}_4$$

Evaluating partial derivatives and by substitution

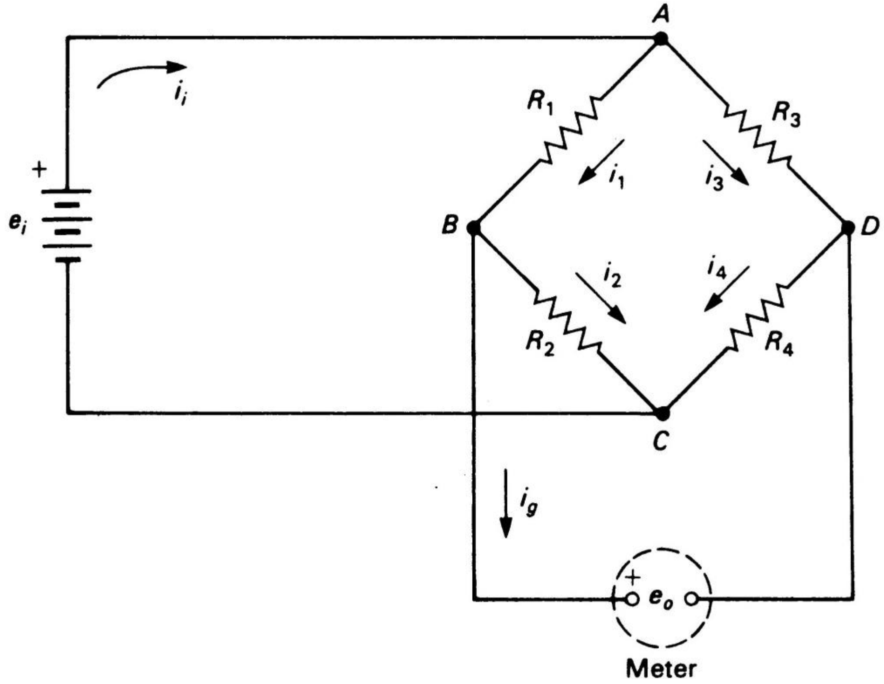
$$\frac{\mathbf{de}_0}{\mathbf{e_i}} = \frac{\mathbf{R}_2\mathbf{dR}_1}{(\mathbf{R}_1+\mathbf{R}_2)^2} - \frac{\mathbf{R}_1\mathbf{dR}_2}{(\mathbf{R}_1+\mathbf{R}_2)^2} - \frac{\mathbf{R}_4\mathbf{dR}_3}{(\mathbf{R}_3+\mathbf{R}_4)^2} + \frac{\mathbf{R}_3\mathbf{dR}_4}{(\mathbf{R}_3+\mathbf{R}_4)^2}$$

dR_1, dR_2, dR_3 and dR_4 are various resistance changes in each of bridge arm

Wheatstone bridge output

$$\frac{de_0}{e_i} = \frac{R_2 dR_1}{(R_1 + R_2)^2} - \frac{R_1 dR_2}{(R_1 + R_2)^2}$$

$$- \frac{R_4 dR_3}{(R_3 + R_4)^2} + \frac{R_3 dR_4}{(R_3 + R_4)^2}$$

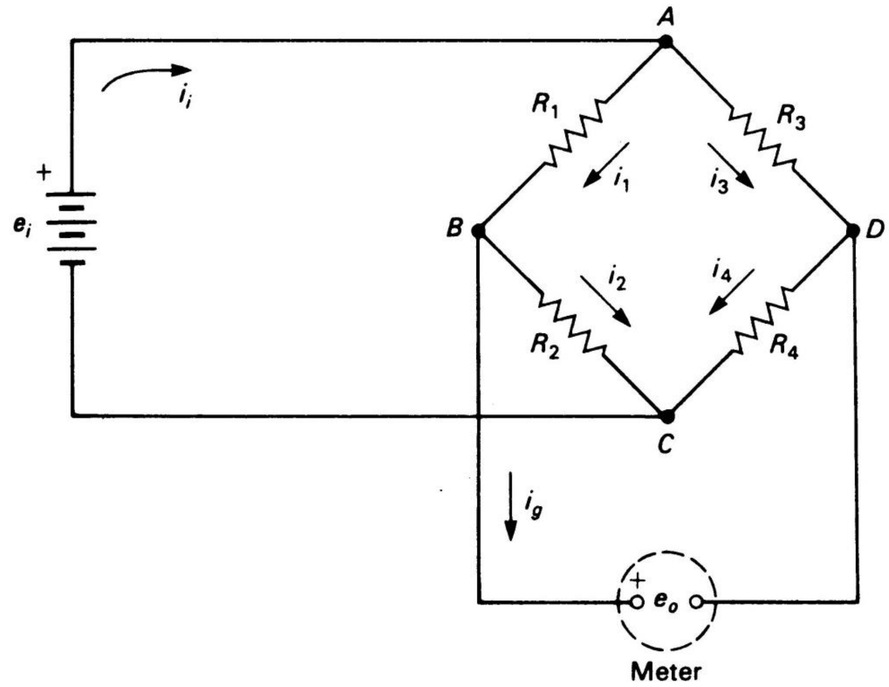


If R_1, R_2, R_3 and R_4 are all equal to R

$$\frac{de_0}{e_i} = \frac{dR_1 - dR_2 - dR_3 + dR_4}{4R}$$

Wheatstone bridge output

$$\frac{de_0}{e_i} = \frac{R_2 dR_1}{(R_1 + R_2)^2} - \frac{R_1 dR_2}{(R_1 + R_2)^2} - \frac{R_4 dR_3}{(R_3 + R_4)^2} + \frac{R_3 dR_4}{(R_3 + R_4)^2}$$



If R_1, R_2, R_3 and R_4 are all equal to R

$$\frac{de_0}{e_i} = \frac{dR_1 - dR_2 - dR_3 + dR_4}{4R}$$

Change in resistance is related to strain : $\frac{dR_n}{R_n} = F\varepsilon_n$

Output voltage in terms of strain : $\frac{de_0}{e_i} = \frac{F}{4}[\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4]$

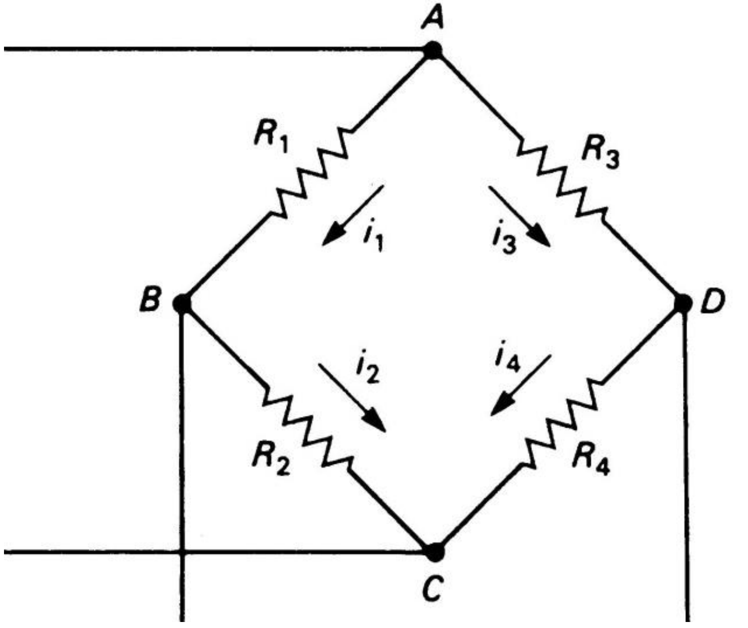
If one arm is active: $\frac{de_0}{e_i} = \frac{F}{4}\varepsilon$

Wheatstone bridge

Full bridge - When all the arms are active.

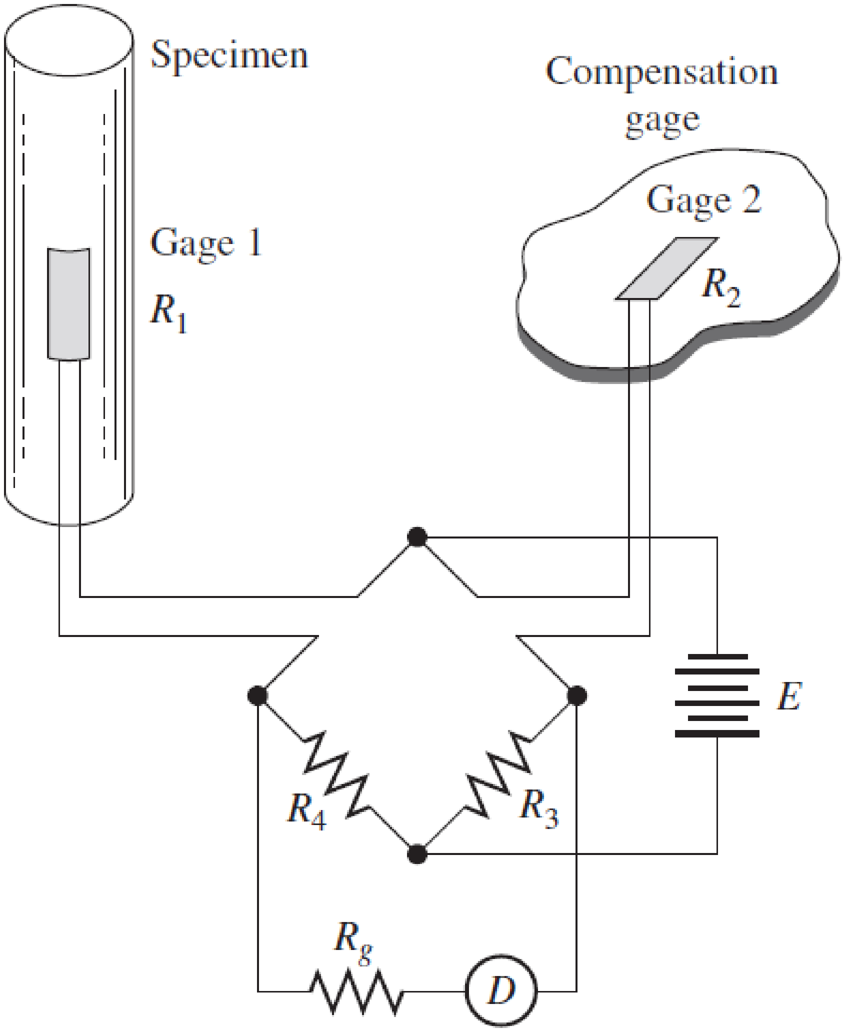
Half bridge - When two arms are active.

Quarter bridge - When only one arm is active. Dummy gage is used for temperature compensation.



Wheatstone bridge

Quarter bridge - When only one arm is active. Dummy gage is used for temperature compensation.



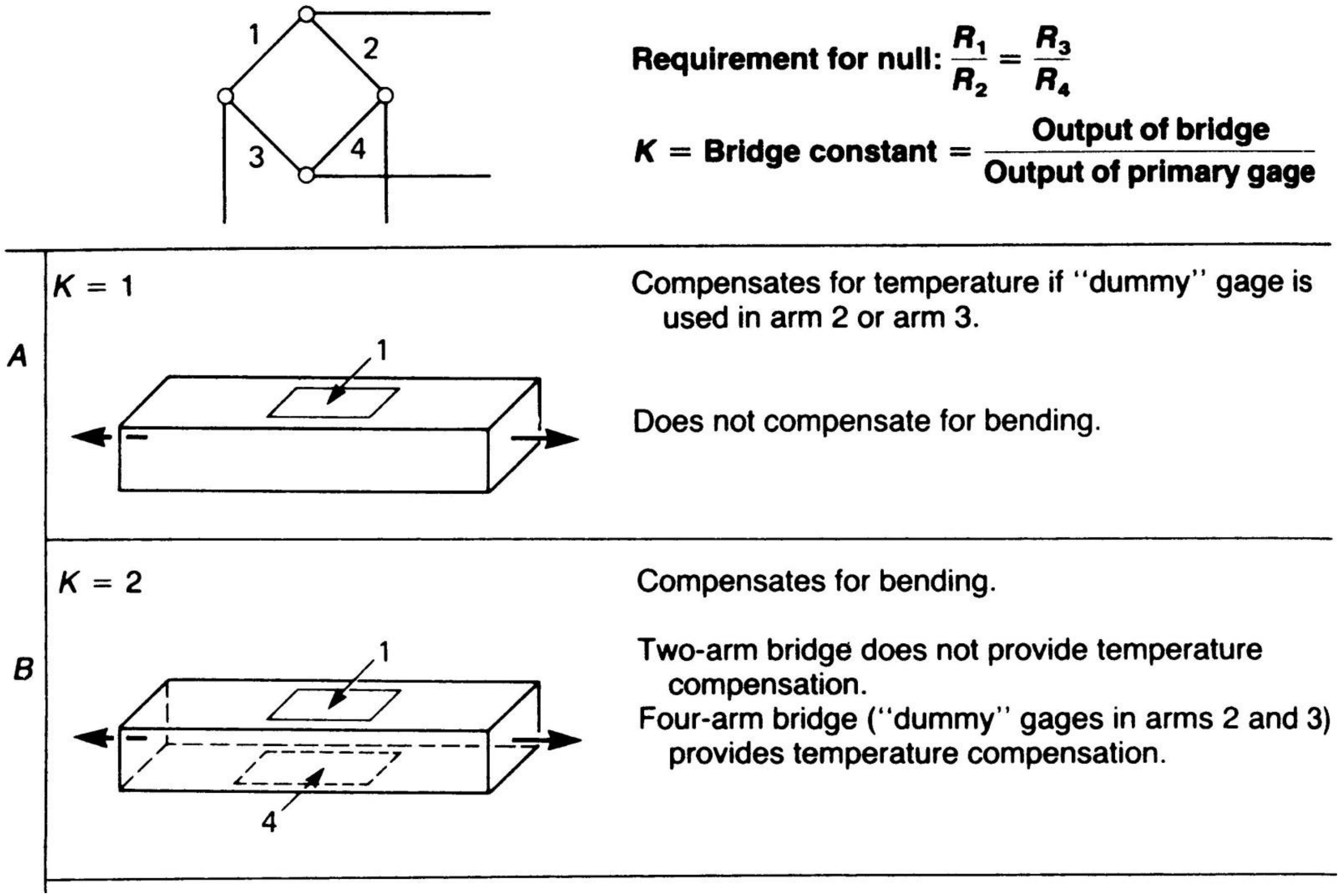
Gage 2 is fixed to same specimen material and also subjected to same temperature. Only gage 1 is strained when load is applied.

For isotropic material orientation of compensation strain gage is immaterial. For orthotropic or anisotropic material orientation of compensation gage should be same as specimen gage.

Strain gage orientation

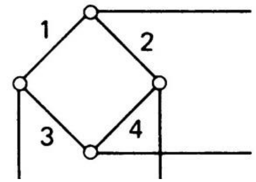
Using the equation shown, effect of strain gage orientation can be explained:

$$\frac{de_0}{e_i} = \frac{F}{4}[\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4]$$



Strain gage orientation

$$\frac{de_0}{e_i} = \frac{F}{4}[\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4]$$

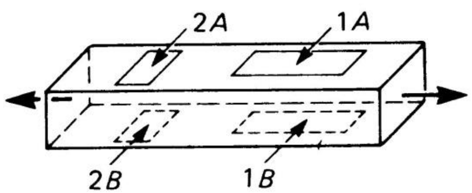


Requirement for null: $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$

K = Bridge constant = $\dfrac{\text{Output of bridge}}{\text{Output of primary gage}}$

C

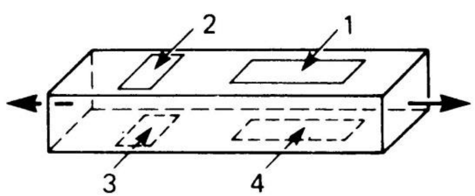
$K = 1 + \nu$



Two-arm bridge compensates for temperature and bending.

D

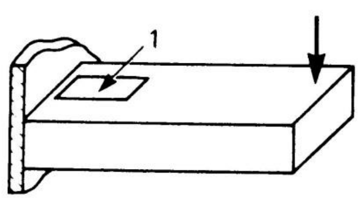
$K = 2(1 + \nu)$



Four-arm bridge compensates for temperature and bending.

E

$K = 1$

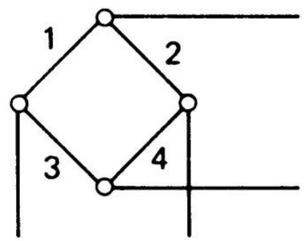


Temperature compensation accomplished when "dummy" gage is used in arm 2 or arm 3.

Bridge is also sensitive to axial and torsional components of loading.

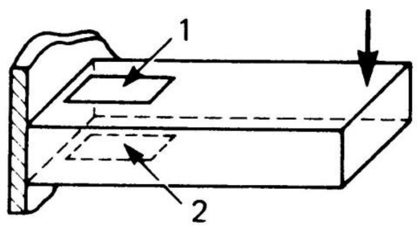
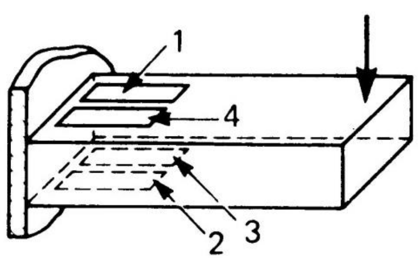
Strain gage orientation

$$\frac{de_0}{e_i} = \frac{F}{4}[\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4]$$



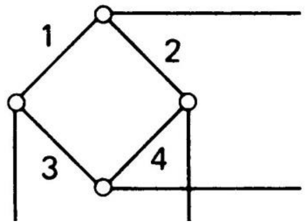
Requirement for null: $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

K = Bridge constant = $\frac{\text{Output of bridge}}{\text{Output of primary gage}}$

F	$K = 2$ 	Temperature effects and axial and torsional components are compensated.
G	$K = 4$ 	Four-arm bridge. Temperature effects and axial and torsional components are compensated.

Strain gage orientation

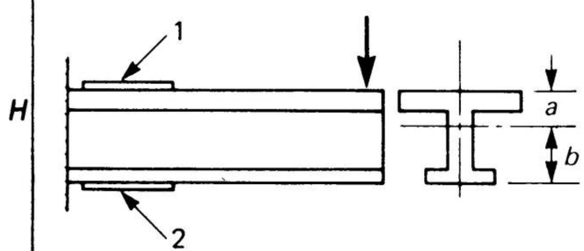
$$\frac{de_0}{e_i} = \frac{F}{4}[\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4]$$



Requirement for null: $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

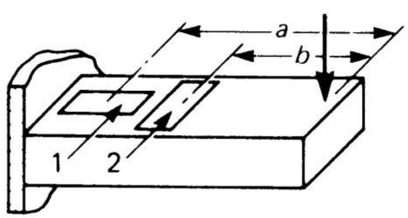
K = Bridge constant = $\frac{\text{Output of bridge}}{\text{Output of primary gage}}$

$K = \frac{a + b}{a}$



Temperature effects and axial and torsional components are compensated.

$K = 1 + \left(\frac{b}{a}\right)\nu$

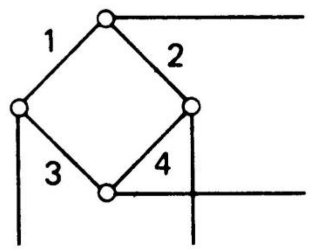


Temperature effects are compensated.

Axial and torsional load components are not compensated.

Strain gage orientation

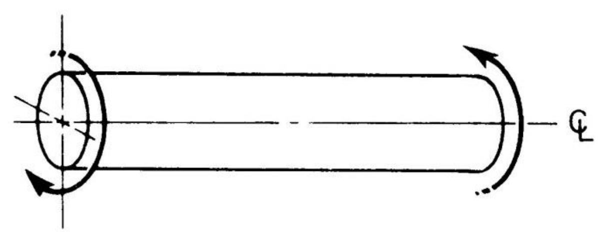
$$\frac{de_0}{e_i} = \frac{F}{4}[\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4]$$



Requirement for null: $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

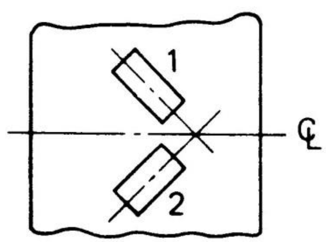
K = Bridge constant = $\frac{\text{Output of bridge}}{\text{Output of primary gage}}$

Torsion



J

$K = 2$



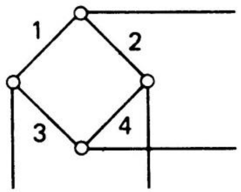
Two-arm bridge.

Temperature and axial load components are compensated.

Bending components are accentuated.

Strain gage orientation

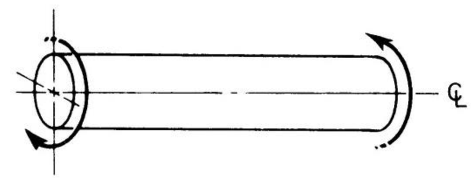
$$\frac{de_0}{e_i} = \frac{F}{4}[\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4]$$



Requirement for null: $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

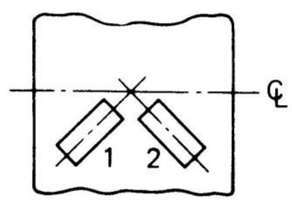
K = Bridge constant = $\frac{\text{Output of bridge}}{\text{Output of primary gage}}$

Torsion



K

$K = 2$



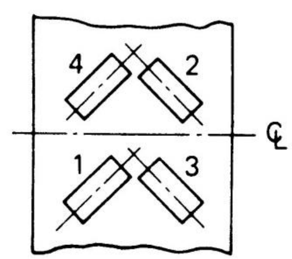
Two-arm bridge.

Temperature effects and axial load components ar compensated.

Relatively insensitive to bending.

L

$K = 4$



Four-arm bridge.

Sensitive to torsion only.

(Gages 1 and 3 are on opposite sides of the shaft from gages 2 and 4.)