Rocket Thrust Model

Propulsion Models

Thrust of a rocket **engine** is based on Newton's **3rd** law.

It also is **derivable** from conservation of **momentum**, as per the following **interpretation**.

Typically, a rocket engine **burns** the propellant and resulting **hot** gases are ejected in the **opposite** direction at a very **high** speed**.**

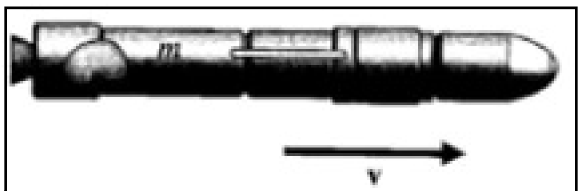
Propulsion Models

The **momentum,** so generated, creates equal & **opposite** momentum on the **rocket**, and appears as a **force**.

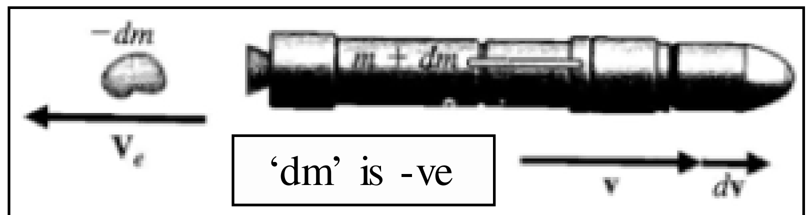
While **accurate** thrust models **require** detailed thermodynamic **laws/** internal flow models, basic **thrust model** is derived from **rocket** engine firing on test **bed**.

Reaction Jet Representation

Following **schematic** shows test bed scenario for a **rocket** burning propellant and **ejecting** hot gases.



Snapshot at time 't'



Snapshot at time 't+dt'

Thus, rocket '**loses**' a mass '**dm**' with the relative velocity '-V_e' and **gains** a net forward velocity '**dV**'.

Figures Courtesy Book 'Spaceflight Dynamics', by Wiesel

Thrust Expression

Following is the corresponding momentum **conservation** equation.

$$(m-dm)\left(\vec{V}+d\vec{V}\right)+dm\left(\vec{V}-\vec{V}_e\right)=m\vec{V}$$

$$\cancel{m\vec{V}}+md\vec{V}-\cancel{dm\vec{V}}-\cancel{dmd\vec{V}}+\cancel{dm\vec{V}}-dm\vec{V}_e=\cancel{m\vec{V}}$$

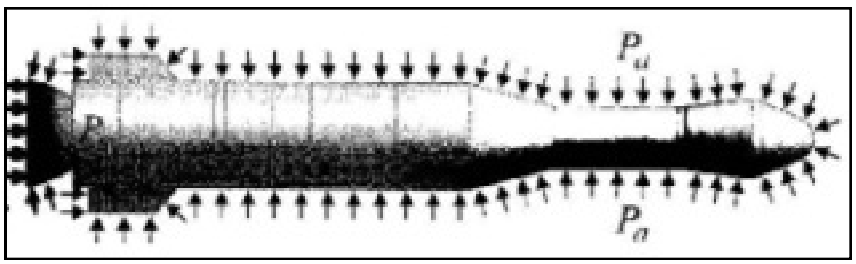
$$md\vec{V}-dm\vec{V}_e=0\rightarrow-\vec{T}dt-dm\vec{V}_e=0\rightarrow\vec{T}=-\dot{m}\vec{V}_e$$

On test **bed**, as both V, dV are **zero**, a net loss of **momentum** 'dmV_e' appears as incremental **impulse** 'Tdt', where 'T' is **thrust** in the direction opposite to $\mathbf{V_e}$.

Pressure Thrust Concept

The **thrust** component so obtained is **purely** due to exhaust **velocity** and, hence, is called ‘velocity ‘ **thrust**.

In addition, **there** is generally a difference in the **static** pressure (P_a) and pressure at nozzle **exit** (P_e), resulting in a **force**, as shown in the **schematic** below.



Figures Courtesy Book ‘Spaceflight Dynamics’, by Wiesel

Pressure Thrust Concept

This **force** is also in the axial **direction** and is called 'pressure' **thrust**, as it is due to pressure **difference**.

Therefore, the **resultant** total thrust (takes as **+ve** along the direction of V) expression for a **rocket** is as follows.

$$\boxed{\vec{T} = -\dot{m}\vec{V}_e + A_e\left(P_e - P_a\right)}$$

Here, A_e is the **cross-sectional** area of the **nozzle**.

Total Thrust Features

It is to be noted that the **1st** term is positive as (**dm/dt**) is taken as a **negative** quantity.

However, **2nd** term can be both **positive** or negative, depending on the **sign** of ‘($P_e - P_a$)’.

In **this** regard, we note that for **same**, P_e, we get slightly **better** performance from the **rocket** motor in **vacuum** (i.e. $P_a = 0$), than at **sea-level**.

Specific Impulse Concept

Conventionally, **thrust** is treated as a **combined** effect of propellant & **nozzle**, and is converted to a mechanical '**figure of merit**' called specific impulse or $\boldsymbol{I_{sp}}$.

Given below is the definition of **specific** impulse.

$$I_{sp}\ (\text{seconds}) = \frac{F_t}{|\dot{m}|\,g_0} = \frac{v_e}{g_0} - \frac{\Delta p A_e}{|\dot{m}|\,g_0}; \quad g_0 = 9.80665\ \text{m/s}^2$$

Typical Specific Impulse Values

$\mathbf{I_{sp}}$ is normally attributed to a **rocket** motor and depends on the **calorific** value of the propellant **used**.

Typical values for different **propellants** are as follows.

Solid: APCP (AP + HTPB/PBAN + Al) – 170 to 220

Liquid: (UDMH, Kerosene) + LO_2/N_2O_4 – 200 to 350

Cryogenic: (LH_2 + LO_2), LNG ~ 450

Nuclear: 300 – 500

Summary

Therefore, to **summarize**, thrust model, employed in **ascent** mission, is simplified **version** of the more accurate, but more **complex** internal flow based **models**.

However, it is **found** that we can still get a **fairly** good estimate of the **engine** performance using the **simplified** thrust and specific impulse **relations**.