The accuracy in problem (1) - (4) is expected within 10^{-2}.

Problem 1: *Use the Newton-Raphson method with $p_0 = -1.5$ to solve*

$$cos(x + \sqrt{2}) + x(x/2 + \sqrt{2}) = 0$$

Solution: We have

$$f(x) = cos(x + \sqrt{2}) + x(x/2 + \sqrt{2})$$

On differentiating the above function, we get

$$f'(x) = -sin(x + \sqrt{2}) + x + \sqrt{2}$$

Based on the Newton-Raphson method, we can write

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

Calculations:

Based on the Newton-Raphson method, we can write

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

Calculations:

i	p_i	$f(p_i)$	$f'(p_i)$	$\lvert p_i - p_{i-1}\rvert$
0	-1.5	0.00000226	-0.00010518	-
1	-1.47855076	0.00000071	-0.00004438	0.02144924
2	-1.46246535	0.00000023	-0.00001872	0.01608541
3	-1.45040194	0.00000007	-0.00000790	0.01206342
4	-1.44135464	0.00000002	-0.00000333	0.00904729

It is clear that after the 4^{th} iteration $|p_n - p_{n-1}| < 10^{-2}$.
Hence, our solution to the above equation would be

$$p_4 = -1.44135464$$

We can also note that the exact solution of the above quation is $-\sqrt{2} = -1.41421356$.

Problem 2: *Use the Newton-Raphson method with* $p_0 = -0.5$ *to solve*

$$e^{6x} + 3(ln2)^2 e^{2x} - (ln8)e^{4x} - (ln2)^3 = 0$$

Solution: Note that we have $f(x) = e^{6x} + 3(\ln 2)^2 e^{2x} - (\ln 8)e^{4x} - (\ln 2)^3$

Problem 2: *Use the Newton-Raphson method with $p_0 = -0.5$ to solve*

$$e^{6x} + 3(ln2)^2 e^{2x} - (ln8)e^{4x} - (ln2)^3 = 0$$

Solution: Note that we have $f(x) = e^{6x} + 3(\ln 2)^2 e^{2x} - (\ln 8)e^{4x} - (\ln 2)^3$

$$\Rightarrow f'(x) = 6e^{6x} + 6(ln2)^2 e^{2x} - 4(ln8)e^{4x}$$

First note that

$$f(x) = (e^{2x} - ln2)^3$$

so that our only solution is

$$x = \frac{ln(ln2)}{2} \approx -0.1833$$

Now we start applying Newton-Raphson method with $p_0 = -0.5$ and $p = \frac{ln(ln2)}{2} \approx -0.1833$

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

2

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

i	p_i	$f(p_i)$	$f'(p_i)$	p_{i+1}	$e_i = p_{i+1} - p$
0	-0.5	-0.03441303	0.23352789	-0.35263844	0.31674354
1	-0.35263844	-0.00790145	0.11757765	-0.28543642	0.16938198
2	-0.28543642	-0.00210282	0.05564489	-0.24764649	0.10217996
3	-0.24764649	-0.00058753	0.02564886	-0.22473983	0.06439003
4	-0.22473983	-0.00016808	0.01165791	-0.21032222	0.04148337
5	-0.21032222	-0.00004872	0.00525552	-0.20105165	0.02706576
6	-0.20105165	-0.00001424	0.00235736	-0.1950131	0.01779519
7	-0.1950131	-0.00000418	0.00105402	-0.19104778	0.01175664
8	-0.19104778	-0.00000123	0.00047031	-0.18843034	0.00779132
9	-0.18843034	-0.00000036	0.00020957	-0.18669676	0.00517388
10	-0.18669676	-0.00000011	0.0000933	-0.18554604	0.0034403

We reach our desired accuracy at the iteration $i = 8$ and hence we can stop there. Thus $x \approx -0.19$ is the solution by Newton-Raphson method.

Problem 3: *Use the modified Newton-Raphson method in problem (1) above.*

Solution: We have

$$f(x) = cos(x + \sqrt{2}) + x(x/2 + \sqrt{2}) \tag{01}$$

And $$f(-\sqrt{2}) - cos(0) = 0$$

$$f(x) = cos(x + \sqrt{2}) + x(x/2 + \sqrt{2}) \tag{01}$$

And $$f(-\sqrt{2}) - cos(0) = 0$$

On differentiating the equation (01), we get

$$f'(x) = -sin(x + \sqrt{2}) + x + \sqrt{2} \tag{02}$$

And $$f'(-\sqrt{2}) = 0$$

On differentiating the equation (02), we get

$$f''(x) = -cos(x + \sqrt{2}) + 1 \tag{03}$$

And $$f''(-\sqrt{2}) = 0$$

On differentiating the equation (03), we get

$$f'''(x) = sin(x + \sqrt{2}) \tag{04}$$

On differentiating the equation (03), we get

$$f'''(x) = sin(x + \sqrt{2}) \tag{04}$$

And $$f'''(-\sqrt{2}) = 0$$

On differentiating the equation (04), we get

$$f''''(x) = cos(x + \sqrt{2}) \tag{05}$$

And $f''''(-\sqrt{2}) = 1 \neq 0$ Hence $-\sqrt{2}$ is a zero of f.

Using modified Newton-Raphson method, we get

$$g(x) = x - \frac{f(x)f'(x)}{f'(x)^2 - f(x)f''(x)}$$

$$p_1 = p_0 - \frac{f(p_0)f'(p_0)}{f'(p_0)^2 - f(p_0)f''(p_0)}$$

Hence we get the following

i	p_i	$\lvert p_i - p_{i-1}\rvert$
0	-1.5	-
1	-1.4142346	0.0857654
2	-1.4142416	0.0000070

After 2^{nd} iteration $|p_i - p_{i-1}| < 10^{-2}$ and $p_2 = -1.414216$.

Problem 4: *Use the modified Newton-Raphson method in problem (2) above.*

Solution: We have

$$f(x) = e^{6x} + 3(ln2)^2e^{2x} - 3(ln2)e^{4x} - (ln2)^3$$

Problem 4: *Use the modified Newton-Raphson method in problem (2) above.*

Solution: We have

$$f(x) = e^{6x} + 3(ln2)^2 e^{2x} - 3(ln2)e^{4x} - (ln2)^3$$

By observation, we get a root of $f(x)$ as

$$x = ln(ln2)^{1/2} = -0.18325646$$

Also $$f'(x) = 6(e^{6x} + (ln)^2 e^{2x} - 2(ln2)e^{4x})$$
and $$f''(x) = 12(3e^{6x} + (ln2)^2 e^{2x} - 4(ln2)e^{4x})$$
We have

$$p_0 = -0.5 \text{ and } p_{n+1} = p_n - \frac{f(p_n)f'(p_n)}{f'(P_n)^2 - f(p_n)f''(p_n)}$$

Hence we get the following

Also $f'(x) = 6(e^{6x} + (ln)^2 e^{2x} - 2(ln2)e^{4x})$

and $$f''(x) = 12(3e^{6x} + (ln2)^2e^{2x} - 4(ln2)e^{4x})$$

We have

$$p_0 = -0.5 \text{ and } p_{n+1} = p_n - \frac{f(p_n)f'(p_n)}{f'(P_n)^2 - f(p_n)f''(p_n)}$$

Hence we get the following

i	p_i	$\lvert p_i - p_{i-1} \rvert$
0	-0.5	-
1	-0.26536892	0.23463107
2	-0.18964449	0.07572442
3	-0.18329709	0.00634739

where p_3 would be the root.

Problem 5: *For* $p_0 = 0.5$ *and* $p_n = \dfrac{2 - e^{p_{n-1}} + p_{n-1}^2}{3}$, *generate first five terms of the sequence* $\{\hat{p}_n\}$ *using the Aitken's* Δ^2*-method.*

Solution: Given that $p_0 = 0.5$ and $p_n = \dfrac{2 - e^{p_{n-1}} + p_{n-1}^2}{3}$.

of the sequence $\{\hat{p}_n\}$ using the Aitken's Δ^2-method.

Solution: Given that $p_0 = 0.5$ and $p_n = \dfrac{2 - e^{p_{n-1}} + p_{n-1}^2}{3}$.
Using Aitken's method, we have

$$\hat{p}_n = p_n \frac{(p_{n+1} - p_n)^2}{p_{n+2} - 2p_{n+1} + p_n}$$

So we get the following

i	p_i	$\hat{p}_i$
0	0.5	0.25868
1	0.20043	0.25761
2	0.27275	0.25753
3	0.25361	0.25753
4	0.25855	0.25753
5	0.25727	0.25753
6	[illegible]	[illegible]

i	p_i	$\hat{p}_i$
0	0.5	0.25868
1	0.20043	0.25761
2	0.27275	0.25753
3	0.25361	0.25753
4	0.25855	0.25753
5	0.25727	0.25753
6	0.25760	0.25753

Clearly, the first five terms of the sequence $\{\hat{p}_n\}$ using Aitken's Δ^2-method are 0.25868, 0.25761, 0.25753, 0.25753, 0.25753. Here the sequence $\{\hat{p}_n\}$ using Aitken's method converged at $n = 2$ itself as compared to the given sequemce which converges at $n = 8$.

Problem 6: *Find appropriate polynomials of degree at most one and at most*

3	0.25361	0.25753
4	0.25855	0.25753
5	0.25727	0.25753
6	0.25760	0.25753

Clearly, the first five terms of the sequence $\{\hat{p}_n\}$ using Aitken's Δ^2-method are 0.25868, 0.25761, 0.25753, 0.25753, 0.25753. Here the sequence $\{\hat{p}_n\}$ using Aitken's method converged at $n = 2$ itself as compared to the given sequemce which converges at $n = 8$.

Problem 6: *Find appropriate polynomials of degree at most one and at most two interpolating $f(x) = cos\, x$ on $x_0 = 0, x_1 = 0.6, x_2 = 0.9$ to approximate $cos(0.45)$. Find the absolute errors.*

Solution: We have $cos(0.95) = 0.900$ and

$$y_0 = f(x_0) = cos(0) = 1$$
$$y_1 = f(x_1) = cos(0.6) = 0.825$$
$$y_2 = f(x_2) = cos(0.9) = 0.622$$

The Lagrange interpolating polynomials of degree at most one will be as follows

$$P_1 = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1$$

Putting known values we get

$$P_1(x) = 1 - 0.292x \text{ and } P_1(0.45) = 0.869$$

$$\therefore \text{ Absolute error} = |0.900 - 0.869| = 0.031$$

The Lagrange interpolating polynomials of degree at most two will be as follows

$$y_1 = f(x_1) = cos(0.6) = 0.825$$
$$y_2 = f(x_2) = cos(0.9) = 0.622$$

The Lagrange interpolating polynomials of degree at most one will be as follows

$$P_1 = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1$$

Putting known values we get

$$P_1(x) = 1 - 0.292x \text{ and } P_1(0.45) = 0.869$$

$$\therefore \text{ Absolute error} = |0.900 - 0.869| = 0.031$$

The Lagrange interpolating polynomials of degree at most two will be as follows

$$P_2(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2$$

Putting known values we get

$$P_1(x) = 1 - 0.292x \text{ and } P_1(0.45) = 0.869$$

$$\therefore \text{ Absolute error} = |0.900 - 0.869| = 0.031$$

The Lagrange interpolating polynomials of degree at most two will be as follows

$$P_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2$$

Putting known values we get

$$P_2(x) = -0428x^2 - 0.035x + 1 \text{ and } P_2(0.45) = 0.898$$

$$\therefore \text{ Absolute error} = |0.900 - 0.898| = 0.002$$

Problem 7: *Repeat the above problem for* $f(x) = \sqrt{1+x}$.

Solution: We have $f(x) = \sqrt{1+x}$, $x_0 = 0$, $x_1 = 0.6$, $x_2 = 0.9$. Hence we get

Problem 7: *Repeat the above problem for* $f(x) = \sqrt{1+x}$.

Solution: We have $f(x) = \sqrt{1+x}$, $x_0 = 0$, $x_1 = 0.6$, $x_2 = 0.9$. Hence we get

$$\begin{aligned} y_0 &= f(x_0) = 1 \\ y_1 &= f(x_1) = 1.264911 \\ y_2 &= f(x_2) = 1.378405 \end{aligned}$$

The Lagrange interpolating polynomial of degree at most one will be as follows

$$P_1(x) = \frac{(x - x_1)}{(x_0 - x_1)} y_0 + \frac{(x - x_0)}{(x + 1 - x_0) y_1}$$

For $x \in [0, 0.6]$ we get $P_1(x) = 0.441518x + 1$ and $P_1(0.45) = 1.196863$.

$$\therefore \text{ Absolute error} = |1.196863 - 1.204159| = 0.007296$$

The Lagrange interpolating polynomial of degree at most two will be as follows

$$P_2(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2$$

Putting values of x_0, x_1, x_2 we get the value of $P_2(x)$ at $x = 0.45$ as $P_2(0.45) = 1.203424$.

$$\therefore \text{ Absolute error} = |1.203424 - 1.204159| = 0.000735$$

Problem 8: *Use aprropriate Lagrange polynomials of degrees one, two and three to find $f(8.4)$ with the following data:*

Problem 8: *Use aprropriate Lagrange polynomials of degrees one, two and three to find $f(8.4)$ with the following data:*

$$f(8.1) = 16.94410$$
$$f(8.3) = 17.56492$$
$$f(8.6) = 18.50515$$
$$f(8.7) = 18.82091$$

Solution: Let

$$y_0 = f(x_0) = f(8.1) = 16.94410$$
$$y_1 = f(x_1) = f(8.3) = 17.56492$$
$$y_2 = f(x_2) = f(8.6) = 18.50515$$
$$y_3 = f(x_3) = f(8.7) = 18.82091$$

The Lagrange interpolating polynomial of degree one will be as follows

$$f(8.3) = 17.56492$$
$$f(8.6) = 18.50515$$
$$f(8.7) = 18.82091$$

Solution: Let

$$\begin{aligned}
y_0 &= f(x_0) = f(8.1) = 16.94410 \\
y_1 &= f(x_1) = f(8.3) = 17.56492 \\
y_2 &= f(x_2) = f(8.6) = 18.50515 \\
y_3 &= f(x_3) = f(8.7) = 18.82091
\end{aligned}$$

The Lagrange interpolating polynomial of degree one will be as follows

$$P_1(x) = \frac{(x - x_2)}{(x_1 - x_2)} y_1 + \frac{(x - x_1)}{(x_2 - x_1)} y_2$$

Putting values of x_1, x_2 we get the value of $P_1(x)$ at $x = 8.4$ as

$$f(x) = P_1(8.4) = 17.87833$$

The Lagrange interpolating polynomials of degree two will be as follows

$$P_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2$$

Putting values of x_0, x_1, x_2 we get the value of $P_2(x)$ at $x = 8.4$ as

$$f(x) = P_2(8.4) = 17.87713$$

And

$$Q_2(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}y_3$$

Putting values of x_1, x_2, x_3 we get the value of $Q_2(x)$ at $x = 8.4$ as

$$f(x) = Q_2(8.4) = 17.877155$$

where THERE'S SOMETHING MISSING BEFORE THIS, DIY

$$L_i(x) = \frac{(x - x_0) \cdots (x - x_n)}{(x_i - x_0) \cdots (x_i - x_n)}$$

Putting values of x_0, x_1, x_2, x_3 we get the value of $P_3(x)$ at $x = 8.4$ as

$$f(x) = P_3(8.4) = 17.8771425$$

Problem 9: *Use appropriate Lagrange polynomials of degree one, two and three to find $f(0.25)$ with the following data:*

$$\begin{aligned} f(0.1) &= 0.29004986 \\ f(0.2) &= -0.56079734 \\ f(0.3) &= -0.81401972 \\ f(0.4) &= -1.0526302 \end{aligned}$$

Solution: Note that $0.1 < 0.2 < 0.25 < 0.3 < 0.4$

For degree 1, we use $f(0.2) = -0.56079734$, $f(0.3) = -0.81401972$

$$L_0(x) = (x - 0.3)/(0.2 - 0.3)$$
$$L_1(x) = (x - 0.2)/(0.3 - 0.2)$$
$$P_1(x) = f(0.2)L_0(x) + f(0.3)L_1(x)$$

So $$P_1(0.25) = -0.68740853$$

For degree two, using $0.1, 0.2$ and 0.3

$$\begin{aligned}
L_0(0.25) &= \frac{(0.25-0.2)(0.25-0.3)}{(0.1-0.2)(0.1-0.3)} = -0.125 \\
L_1(0.25) &= \frac{(0.25-0.1)(0.25-0.3)}{(0.2-0.3)(0.2-0.3)} = 0.75 \\
L_2(0.25) &= \frac{(0.25-0.1)(0.25-0.2)}{} = -0.375
\end{aligned}$$

For degree 1, we use $f(0.2) = -0.56079734$, $f(0.3) = -0.81401972$

$$L_0(x) = (x - 0.3)/(0.2 - 0.3)$$
$$L_1(x) = (x - 0.2)/(0.3 - 0.2)$$
$$P_1(x) = f(0.2)L_0(x) + f(0.3)L_1(x)$$

So $$P_1(0.25) = -0.68740853$$

For degree two, using $0.1, 0.2$ and 0.3

$$L_0(0.25) = \frac{(0.25 - 0.2)(0.25 - 0.3)}{(0.1 - 0.2)(0.1 - 0.3)} = -0.125$$
$$L_1(0.25) = \frac{(0.25 - 0.1)(0.25 - 0.3)}{(0.2 - 0.3)(0.2 - 0.3)} = 0.75$$
$$L_2(0.25) = \frac{(0.25 - 0.1)(0.25 - 0.2)}{(0.3 - 0.1)(0.3 - 0.2)} = -0.375$$

So $$P_2(0.25) = -0.0790843775$$

For degree three, use $0.1, 0.2, 0.3$ and 0.4

$$(0.2-0.3)(0.2-0.3)$$

$$L_2(0.25) = \frac{(0.25-0.1)(0.25-0.2)}{(0.3-0.1)(0.3-0.2)} = -0.375$$

So
$$P_2(0.25) = -0.0790843775$$

For degree three, use $0.1, 0.2, 0.3$ and 0.4

$$
\begin{aligned}
L_0(0.25) &= \frac{(0.25-0.2)(0.25-0.3)(0.25-0.4)}{(0.1-0.2)(0.1-0.3)(0.1-0.4)} = -0.5625\\
L_1(0.25) &= \frac{(0.25-0.1)(0.25-0.3)(0.25-0.4)}{(0.2-0.3)(0.2-0.3)(0.2-0.4)} = 0.5625\\
L_2(0.25) &= \frac{(0.25-0.1)(0.25-0.2)(0.25-0.4)}{(0.3-0.1)(0.3-0.2)(0.3-0.4)} = 0.5625\\
L_3(0.25) &= \frac{(0.25-0.1)(0.25-0.2)(0.25-0.3)}{(0.4-0.1)(0.4-0.2)(0.4-0.3)} = -0.0625
\end{aligned}
$$

So
$$P_3(0.25) = -0.5443921625$$