

AE 236 : Compressible Fluid Mechanics

(Module II : Normal shock waves)

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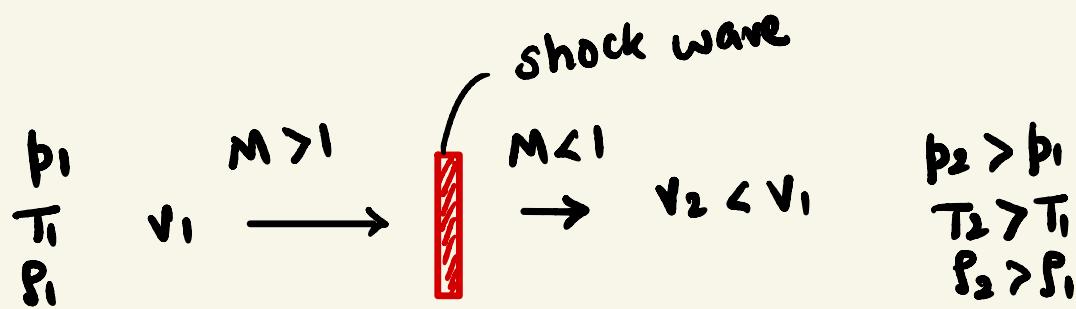
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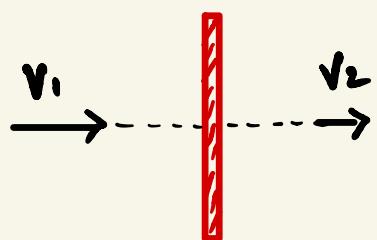


## 'Normal' shock waves

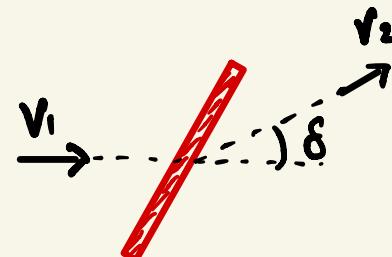
Extremely thin region in which the transition from the initial supersonic velocity, relatively low pressure state to the state that involves a relatively low velocity and high pressure is termed a **shock wave**



Thickness  $\sim$  few mean free paths



Normal shock wave

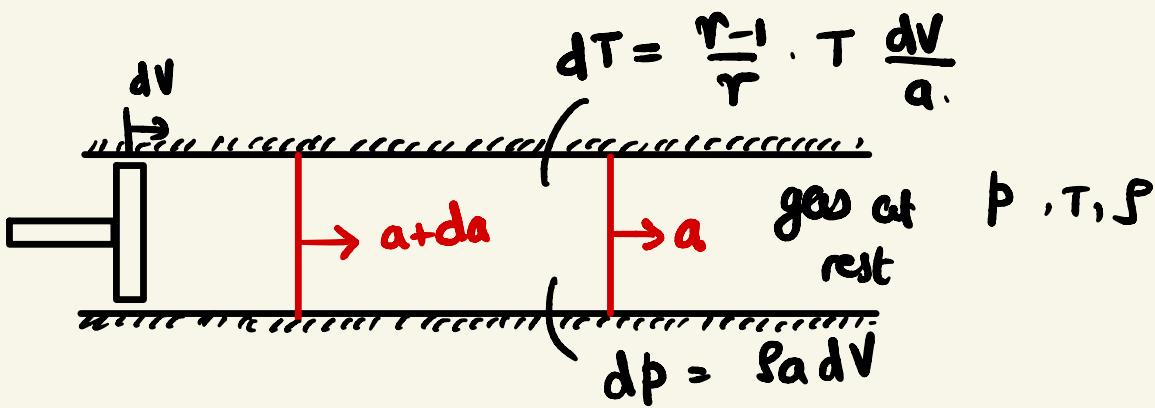


Oblique shock wave

A straight shock wave that is at right angles to the upstream flow is termed a **normal shock wave**

A straight shock wave that is at an angle to the upstream flow is termed an **oblique shock wave**

## How are shocks formed?



$$\cancel{C_p T} + \frac{a^2}{2} = C_p(T+dT) + \left(\frac{a-dV}{2}\right)^2$$

$$\Rightarrow \boxed{dT = \frac{r-1}{r} T \cdot \frac{dV}{a}}$$

Relative velocity of first wave =  $a$

Relative velocity of second wave =  $a+da+dV > a$

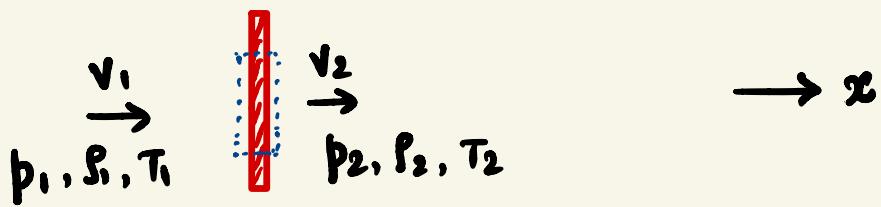
Waves following the first one are able to catch up to it, merge and form a single, stronger wave

$dp + dp + \dots + dp \rightarrow$  Large pressure change

$\Rightarrow$  Large temperature change (large gradients)

$\Rightarrow$  shock is not isentropic

# Stationary Normal Shock Wave Relations



## Continuity

$$\dot{m} = \rho_1 V_1 A = \rho_2 V_2 A$$

$$\frac{\dot{m}}{A} = \rho_1 V_1 = \rho_2 V_2$$

## Momentum

$$p_1 A - p_2 A = \dot{m} (V_2 - V_1)$$

$$p_1 - p_2 = \left( \frac{\dot{m}}{A} \right) (V_2 - V_1)$$

$$= \rho_1 V_1 (V_2 - V_1) = \rho_2 V_2 (V_2 - V_1)$$

$$V_1 V_2 - V_1^2 = \frac{p_1 - p_2}{\rho_1} \quad - (a)$$

$$V_2^2 - V_1 V_2 = \frac{p_1 - p_2}{\rho_2} \quad - (b)$$

(a) + (b) gives

$$V_2^2 - V_1^2 = (p_1 - p_2) \left[ \frac{1}{\rho_1} + \frac{1}{\rho_2} \right] - ①$$

## Energy

$$\frac{V_1^2}{2} + C_p T_1 = \frac{V_2^2}{2} + C_p T_2 = C_p T_0 = \text{const.}$$

where  $C_p = \frac{R\gamma}{\gamma-1}$  and  $C_p T = \frac{\gamma R T}{\gamma-1} = \frac{\gamma}{\gamma-1} \frac{P}{\rho}$ .

$$\Rightarrow V_1^2 + \frac{2\gamma}{\gamma-1} \frac{P_1}{\rho_1} = V_2^2 + \frac{2\gamma}{\gamma-1} \frac{P_2}{\rho_2}$$

$$\text{or } V_2^2 - V_1^2 = \frac{2\gamma}{\gamma-1} \left[ \frac{P_1}{\rho_1} - \frac{P_2}{\rho_2} \right] - \textcircled{2}$$

Using  $\textcircled{1}$  &  $\textcircled{2}$ , we get

$$\frac{2\gamma}{\gamma-1} \left[ \frac{P_2}{\rho_2} - \frac{P_1}{\rho_1} \right] = (P_2 - P_1) \left[ \frac{1}{\rho_1} + \frac{1}{\rho_2} \right] - \textcircled{3}$$

Let us define  $P_2/\rho_1$  as the strength of the shock

$\textcircled{3}$ .  $\rho_2/\rho_1$  gives

$$\frac{2\gamma}{\gamma-1} \left[ \frac{P_2}{\rho_1} - \frac{P_2}{\rho_1} \right] = \frac{\rho_1}{\rho_2} \left( \frac{P_2}{\rho_1} - 1 \right) \left[ \frac{P_2}{\rho_1} + 1 \right] \cdot \frac{P_2}{\rho_1}$$

$$\Rightarrow \frac{2\gamma}{\gamma-1} \left[ \frac{P_2}{\rho_1} - \frac{P_2}{\rho_1} \right] = \left( \frac{P_2}{\rho_1} - 1 \right) \left( \frac{P_2}{\rho_1} + 1 \right)$$

$$\Rightarrow \frac{P_2}{\rho_1} \left[ \frac{P_2}{\rho_1} - 1 + \frac{2\gamma}{\gamma-1} \right] = 1 - \frac{P_2}{\rho_1} + \frac{2\gamma}{\gamma-1} \frac{P_2}{\rho_1}$$

$$\Rightarrow \frac{s_2}{s_1} = \frac{\left(\frac{r+1}{r-1}\right) \frac{p_2}{p_1} + 1}{\left(\frac{r+1}{r-1}\right) + \frac{p_2}{p_1}} - \star$$

$$\frac{v_1}{v_2} = \frac{s_2}{s_1} \quad (\text{continuity})$$

$$\Rightarrow \frac{v_2}{v_1} = \frac{\frac{r+1}{r-1} + \frac{p_2}{p_1}}{\left(\frac{r+1}{r-1}\right) \frac{p_2}{p_1} + 1} - \star\star$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \cdot \frac{s_1}{s_2} \quad (\text{state equation})$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{\frac{r+1}{r-1} + \frac{p_2}{p_1}}{\frac{r+1}{r-1} + \frac{p_1}{p_2}} - \star\star\star$$

These relations are called Rankine - Hugoniot  
normal shock wave relations

From equation ①

$$V_2^2 - V_1^2 = (P_1 - P_2) \left[ \frac{1}{P_1} + \frac{1}{P_2} \right],$$

we see that

$$V_1^2 \left[ \left( \frac{V_2}{V_1} \right)^2 - 1 \right] = \frac{P_1}{P_2} \underbrace{\left( 1 - \frac{P_2}{P_1} \right)}_{f^n(P_2/P_1)} \underbrace{\left( 1 + \frac{P_1}{P_2} \right)}_{f^n(P_1/P_2)}$$

⇒ For a particular value of  $P_2/P_1$ , there is an associated particular value of

$$\frac{V_1^2}{P_1/P_2} = \frac{V_1^2}{\alpha^2/r} = r M_1^2$$

A particular shock strength is associated with a fixed  $M_1$ .

Entropy changes across a shock

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$= (R + C_V) \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

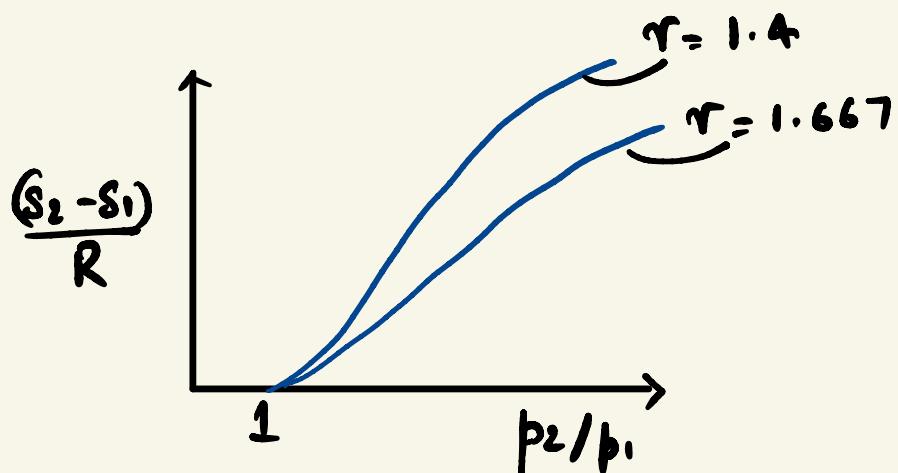
$$\frac{S_2 - S_1}{R} = \left[ 1 + \frac{1}{r-1} \right] \ln \left( \frac{T_2}{T_1} \right) - \ln \left( \frac{P_2}{P_1} \right)$$

$$= \left[ 1 + \frac{1}{r-1} \right] \ln \left[ \frac{P_2}{P_1} \frac{S_1}{S_2} \right] - \ln \left( \frac{P_2}{P_1} \right)$$

$$\frac{s_2 - s_1}{R} = \ln \left[ \left( \frac{p_2}{p_1} \right)^{\frac{1}{r-1}} \cdot \left( \frac{p_2}{p_1} \right)^{\frac{-r}{r-1}} \right]$$

$$\frac{s_2 - s_1}{R} = \ln \left\{ \left( \frac{p_2}{p_1} \right)^{\frac{1}{r-1}} \cdot \left[ \frac{(r+1) \frac{p_2}{p_1} + (r-1)}{(r+1) + (r-1) \frac{p_2}{p_1}} \right]^{\frac{-r}{r-1}} \right\}$$

From 2nd law of thermodynamics,  $\Delta S \geq 0$



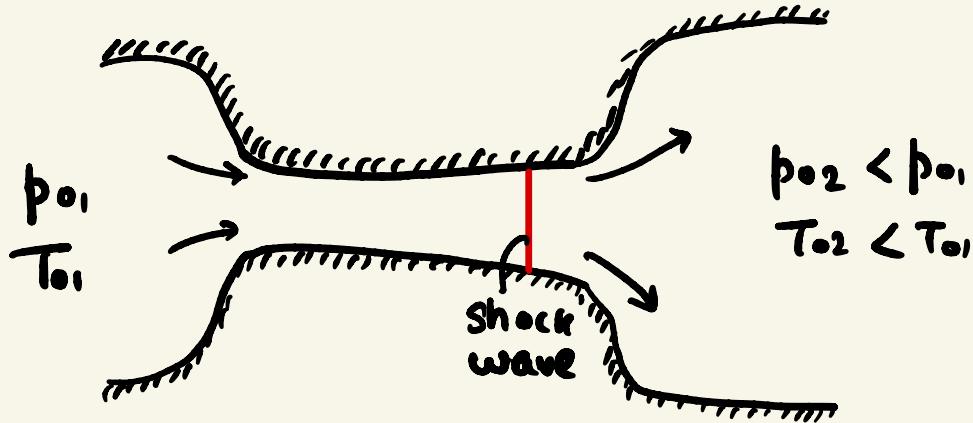
From the figure,  $\Delta S \geq 0$  would mean

$$\frac{p_2}{p_1} \geq 1$$

Shock waves are always compressive and there are no expansion shocks

$p_2 = p_1$  corresponds to the Mach wave

## Losses and stagnation pressure



$$S_{02} - S_{01} = C_p \ln \frac{T_{02}}{T_{01}} - R \ln \frac{p_{02}}{p_{01}}$$

Shock is adiabatic, i.e.,  $T_{02} = T_{01}$

$$\Rightarrow S_{02} - S_{01} = -R \ln \frac{p_{02}}{p_{01}}$$

Now  $S_{02} = S_2$  and  $S_{01} = S_1$  since flow is isentropic upstream and downstream

$$\Rightarrow \frac{S_2 - S_1}{R} = -\ln \frac{p_{02}}{p_{01}}$$

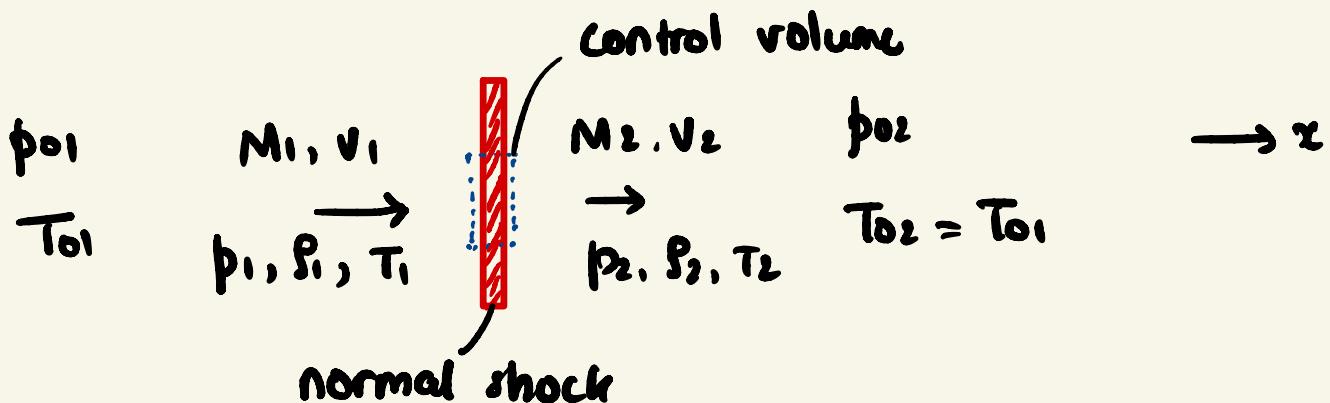
$$\text{or } \frac{p_{02}}{p_{01}} = e^{-\Delta S/R}$$

Since  $\Delta S \geq 0$ , we have

$$p_{02} \leq p_{01}$$

There is always a loss of stagnation pressure across a shock wave (represents losses in the flow)

## Normal shock relation in terms of $M_1$



### Continuity

$$\rho_1 v_1 = \rho_2 v_2$$

Divide by  $a_1$

$$\rho_1 M_1 = \rho_2 \frac{v_2}{a_2} \frac{a_2}{a_1}$$

$$\frac{\rho_2}{\rho_1} = \frac{a_1}{a_2} \cdot \frac{M_1}{M_2} \quad - \textcircled{1}$$

### Momentum

$$\rho_1 - \rho_2 = \rho_2 v_2^2 - \rho_1 v_1^2$$

$$P = \frac{a^2 \rho}{\gamma}$$

$$\text{Now } P = a^2 \rho / \gamma$$

$$\Rightarrow \frac{a_1^2 \rho_1}{\gamma} - \frac{a_2^2 \rho_2}{\gamma} = \rho_2 v_2^2 - \rho_1 v_1^2$$

Simplifying

$$\frac{\rho_2}{\rho_1} = \left( \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \right) \left( \frac{a_1}{a_2} \right)^2 \quad - \textcircled{2}$$

# Energy

$$v_1^2 + \frac{2}{r-1} a_1^2 = v_2^2 + \frac{2}{r-1} a_2^2$$

$$\Rightarrow \frac{r-1}{2} M_1^2 + 1 = \left(\frac{v_2}{a_1}\right)^2 \cdot \left(\frac{a_2}{a_1}\right)^2 \cdot \frac{r-1}{2} + \left(\frac{a_2}{a_1}\right)^2$$

Simplifying,

$$\left(\frac{a_2}{a_1}\right)^2 = \frac{2 + (r-1)M_1^2}{2 + (r-1)M_2^2} - \textcircled{3}$$

Eliminate  $P_2/P_1$  using  $\textcircled{1}$  &  $\textcircled{2}$

$$\frac{a_1}{a_2} \cdot \frac{M_1}{M_2} = \left( \frac{1 + rM_1^2}{1 + rM_2^2} \right) \cdot \left( \frac{a_1}{a_2} \right)^2$$

$$\frac{a_2}{a_1} = \left( \frac{1 + rM_1^2}{1 + rM_2^2} \right) \frac{M_2}{M_1} - \textcircled{4}$$

Eliminate  $a_2/a_1$  using  $\textcircled{3}$  &  $\textcircled{4}$

$$\frac{2 + (r-1)M_1^2}{2 + (r-1)M_2^2} = \left( \frac{1 + rM_1^2}{1 + rM_2^2} \right)^2 \left( \frac{M_2}{M_1} \right)^2$$

or

$$(r-1)(M_2^4 - M_1^4) - 2rM_2^2M_1^2(M_2^2 - M_1^2) + 2(M_2^4 - M_1^4) = 0$$

for nontrivial solution  $M_2 \neq M_1$ , we get

$$M_2^2 = \frac{(r-1)M_1^2 + 2}{2rM_1^2 - (r-1)} - \textcircled{5}$$

Substituting ③ in ③

$$\left(\frac{a_2}{a_1}\right)^2 = \frac{T_2}{T_1} = \left\{ \frac{[2rM_1^2 - (r-1)][2 + (r-1)M_1^2]}{(r+1)^2 M_1^2} \right\}$$

$$\begin{aligned} \frac{p_2}{p_1} &= \frac{p_1}{p_1} \cdot \frac{T_2}{T_1} = \frac{p_1}{p_1} \cdot \left(\frac{a_2}{a_1}\right)^2 \\ &= \frac{p_1}{p_1} \cdot \frac{1 + rM_1^2}{1 + rM_2^2} \cdot \frac{p_1}{p_2} = \frac{1 + rM_1^2}{1 + rM_2^2} \end{aligned}$$

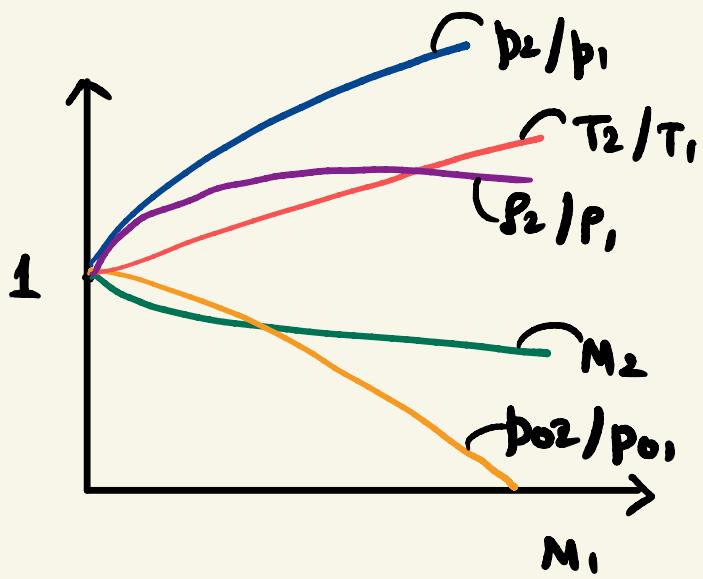
$$\frac{p_2}{p_1} = \frac{2rM_1^2 - (r-1)}{r+1}$$

$$\frac{p_2}{p_1} = \frac{p_2}{p_1} \cdot \frac{T_1}{T_2}$$

$$\frac{p_2}{p_1} = \frac{(r+1)M_1^2}{2 + (r-1)M_1^2}$$

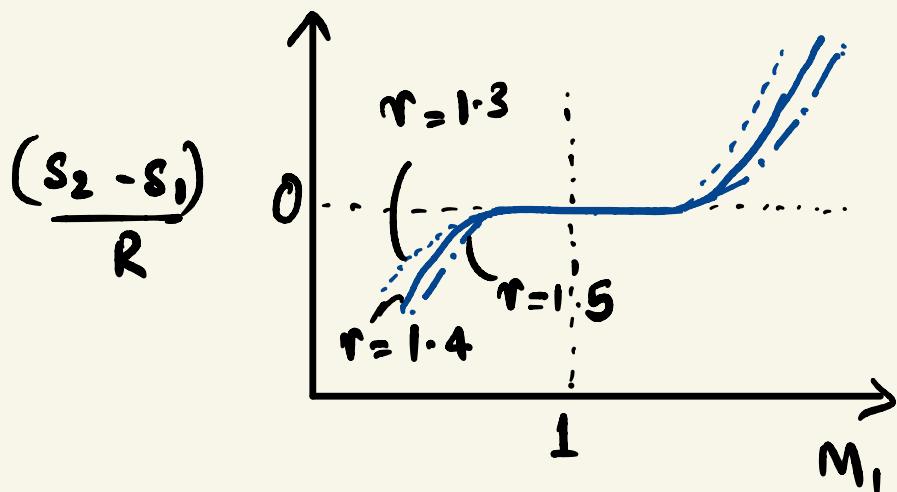
$$\frac{p_{02}}{p_{01}} = \frac{p_{02}/p_2}{p_{01}/p_1} \cdot \frac{p_2}{p_1}$$

$$\frac{p_{02}}{p_{01}} = \left\{ \frac{(r+1)}{2} \frac{M_1^2}{(1 + \frac{r-1}{2} M_1^2)} \right\}^{\frac{r}{r-1}} \left\{ \left( \frac{2r}{r+1} \right) M_1^2 - \left( \frac{r-1}{r+1} \right) \right\}^{\frac{-1}{r-1}}$$



$p_2/p_1$  and  $M_2$  become invariant;  $p_2/p_1$  and  $T_2/T_1$  tend to  $\infty$ ;  $p_{02}/p_{01}$  decreases as  $M_1 \rightarrow \infty$

$$\frac{s_2 - s_1}{R} = \ln \left\{ \left[ \frac{2r}{r+1} (M_1^2 - 1) + 1 \right]^{\frac{1}{r-1}} \left[ \frac{(r+1) M_1^2}{2 + (r-1) M_1^2} \right]^{\frac{-r}{r-1}} \right\}$$



From 2<sup>nd</sup> Law of Thermodynamics  $\Delta S \geq 0$

$$M_1 \geq 1$$

Normal shocks can only occur in a Supersonic flow

Also, from equation ⑥ &  $M_1 \geq 1$

$$M_2 \leq 1$$

Flow downstream of a normal shock is always subsonic

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1}$$

$$\frac{P_2}{P_1} = \frac{(\gamma + 1) M_1^2}{2 + (\gamma - 1) M_1^2}$$

$$\frac{T_2}{T_1} = \left\{ \frac{[2\gamma M_1^2 - (\gamma - 1)][2 + (\gamma - 1) M_1^2]}{(\gamma + 1)^2 M_1^2} \right\} - \star$$

$$M_2^2 = \frac{(\gamma - 1) M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)}$$

### limiting cases

①  $M_1 \rightarrow \infty$  (very strong normal shock)

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2}{\gamma + 1} ;$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma - 1) M_1^2}{(\gamma + 1)^2} ;$$

$$\frac{P_2}{P_1} = \frac{\gamma + 1}{\gamma - 1}$$

$$M_2^2 = \frac{\gamma - 1}{2\gamma}$$

strong shock  
solution

②  $M_1 \rightarrow 1$  (extremely weak shock)

Continuity is still valid and flow is isentropic

$$\frac{P_2}{P_1} \cdot \frac{a_2}{a_1} = \frac{M_1}{M_2}$$

where,

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{r-1}} \quad \text{and} \quad \frac{a_2}{a_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{2}}$$

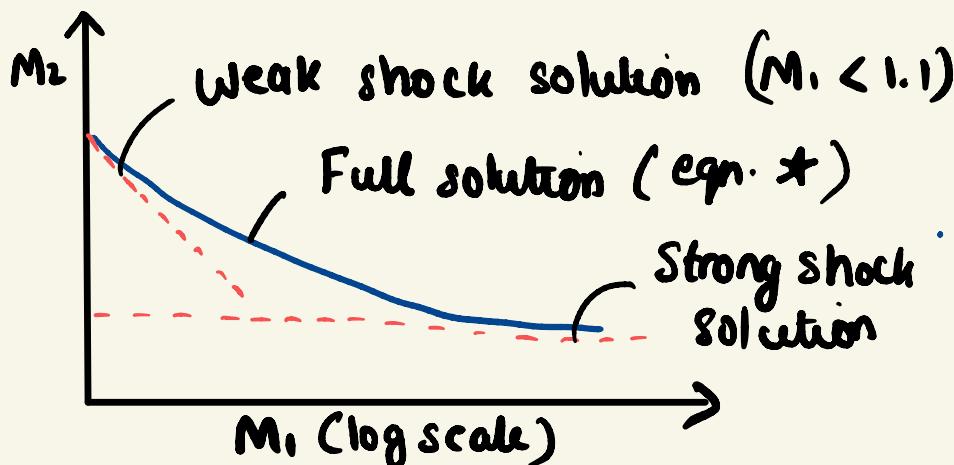
$$\Rightarrow \frac{M_2}{M_1} = \left(\frac{T_1}{T_2}\right)^{\frac{r+1}{2(r-1)}}$$

For this flow (isentropic)

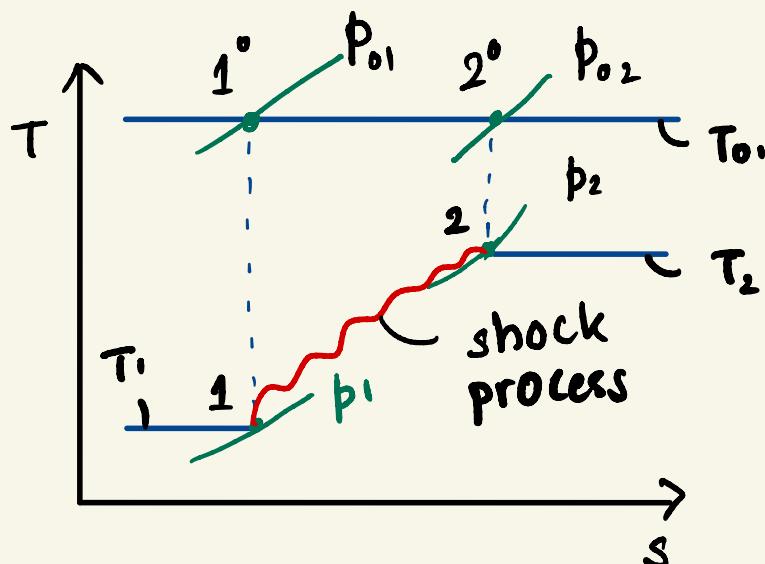
$$\frac{T_1}{T_2} = \frac{1 + \frac{(r-1)}{2} M_2^2}{1 + \frac{(r-1)}{2} M_1^2}$$

$$\Rightarrow \frac{M_2}{M_1} = \left\{ \frac{\left(1 + \frac{r-1}{2} M_2^2\right)}{\left(1 + \frac{r-1}{2} M_1^2\right)} \right\}^{\frac{r+1}{2(r-1)}}$$

weak  
shock  
solution



## T-s diagram of a normal shock



Since the shock process does not consist of equilibrium states, we denote it using squiggly lines

### Problems

① Aircraft :  $M_1 = 1.2$  flying over a building with glass window  $= 1 \times 2 \text{ m}^2$ . Max. force on window ?

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1}$$

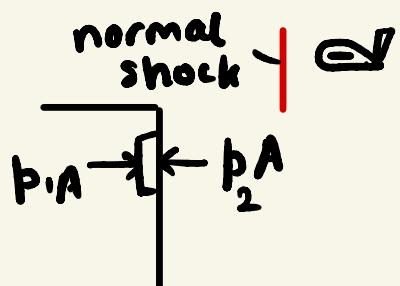
$$p_1 = 101.3 \text{ kPa}, \gamma = 1.4$$

$$\Rightarrow \frac{p_2}{p_1} = 1.513.$$

$$F_{\max} = (p_2 - p_1) A$$

$$= (1.513 \times 101.3 - 101.3) 2$$

$F_{\max} = 103.9 \text{ kN}$

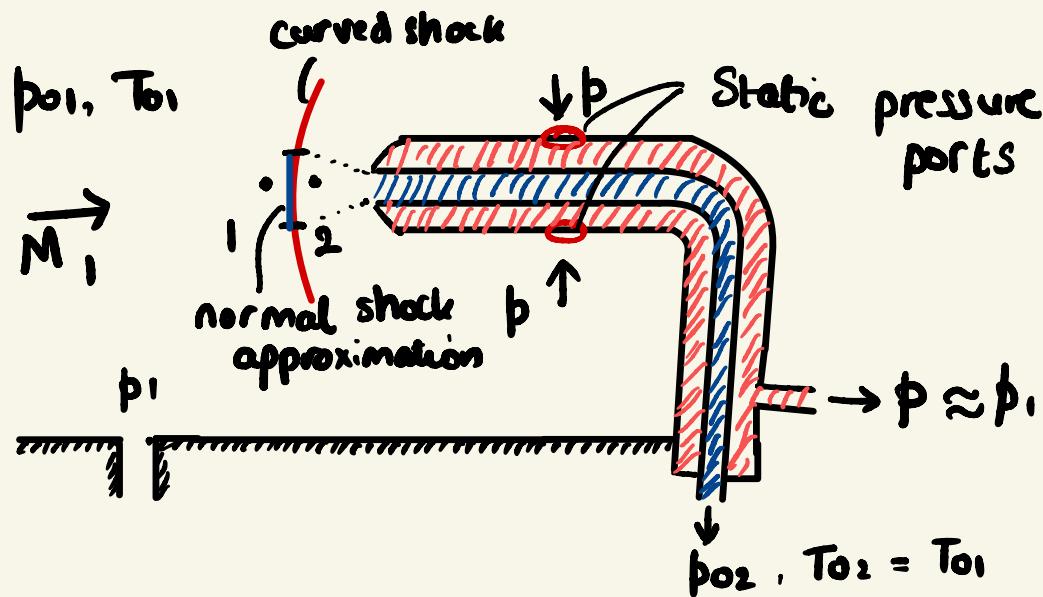


This force can potentially break the window as  $F_{\max} >$  weight of 100 people weighing 100 kg.

## Normal shock tables

$M_1$	$M_2$	$\frac{p_2}{p_1}$	$\frac{T_2}{T_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{a_2}{a_1}$	$\frac{p_{02}}{p_{01}}$	$\frac{\rho_{02}}{\rho_1}$

## Pitot probes in supersonic flow



Subsonic pitot tube equation gives  $M_2$   
We need  $M_1$

$$\frac{p_{02}}{p_1} = \underbrace{\frac{p_{02}}{p_2} \cdot \frac{p_2}{p_1}}_{f^n \text{ of } M_1}$$

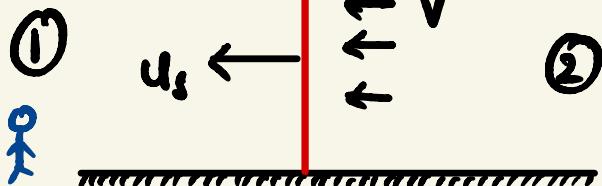
measured

Using isentropic and normal shock relations

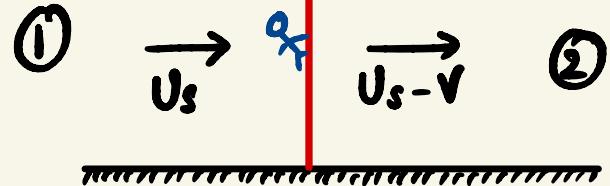
$$\Rightarrow \frac{p_{02}}{p_1} = \left[ \frac{\left( \frac{(r+1)}{2} M_1^2 \right)^{\frac{r}{r-1}}}{\left( \frac{2f M_1^2}{r+1} \right) - \left( \frac{r-1}{r+1} \right)} \right]^{\frac{1}{r-1}}$$

Rayleigh supersonic pitot tube equation

## Moving normal shock waves



Moving shock problem



Equivalent stationary shock problem

We have  $V_1 = U_s$  and  $V_2 = U_s - V$

$$M_1 = \frac{U_s}{a_1} = M_s$$

$$M_2 = \frac{U_s - V}{a_2} \Rightarrow V = M_1 a_1 - M_2 a_2$$

$$= \frac{U_s}{a_1} \cdot \frac{a_1}{a_2} - \frac{V}{a_2}, \text{ let } M_2' = \frac{V}{a_2}$$

$$M_2 = M_s \cdot \frac{a_1}{a_2} - M_2'$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_s^2 - (\gamma - 1)}{\gamma + 1}$$

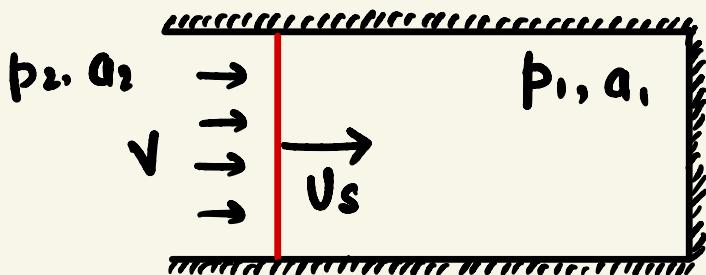
$$\frac{P_2}{P_1} = \frac{(\gamma + 1) M_s^2}{2 + (\gamma - 1) M_s^2}$$

$$\frac{T_2}{T_1} = \left( \frac{a_2}{a_1} \right)^2 = \frac{[2 + (\gamma - 1) M_s^2] [2\gamma M_s^2 - (\gamma - 1)]}{(\gamma \pi)^2 M_s^2}$$

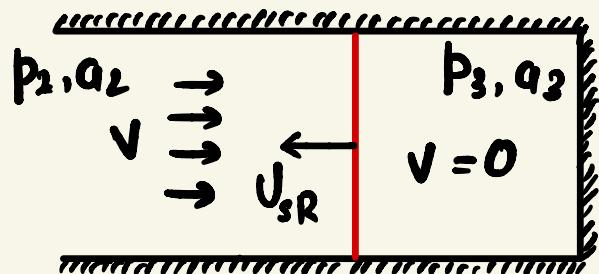
$$M_2' = \frac{2(M_s^2 - 1)}{[2\gamma M_s^2 - (\gamma - 1)]^{1/2} [2 + (\gamma - 1) M_s^2]^{1/2}}$$

## Reflections of moving shocks

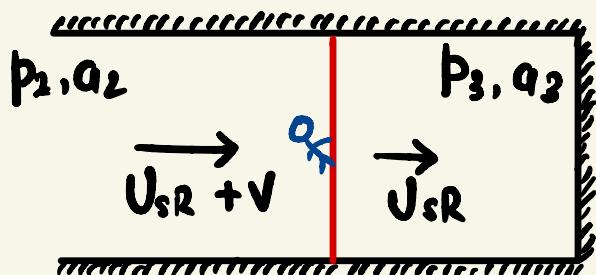
At a closed end, the velocity of gas MUST be 0.  
 ⇒ a normal shock is reflected off the closed end having a strength just sufficient to reduce the velocity to 0.



Moving shock



Reflected shock



Equivalent stationary shock problem

$$M_{R1} = \frac{U_{SR} + V}{a_2} = M_{SR} + M_2'$$

$$M_{R2} = \frac{U_{SR}}{a_3} = \frac{U_{SR}}{a_L} \cdot \frac{a_L}{a_3}$$

$$M_{R2} = M_{SR} \frac{a_2}{a_3}$$

known

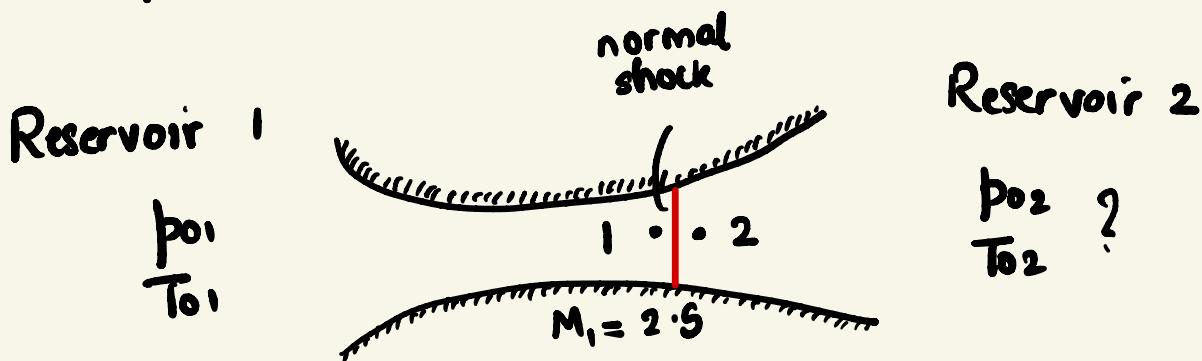
$$M_{R1} = M_{R2} \frac{a_3}{a_2} + M_2'$$

$\underbrace{\quad}_{f^n \text{ of } M_{R1}}$

solved iteratively  
in practice

## Problems

②  $p_{01} = 500 \text{ kPa}$ ,  $T_{01} = 35^\circ\text{C}$



From isentropic tables @  $M_1 = 2.5$

$$\frac{p_{01}}{p_1} = 17.085 \quad \frac{T_{01}}{T_1} = 2.25$$

$$\Rightarrow p_1 = 29.3 \text{ kPa} \quad T_1 = 136.9 \text{ K}$$

From shock tables @  $M_1 = 2.5$

$$M_2 = 0.513 \quad \frac{p_2}{p_1} = 7.215 \quad \frac{T_2}{T_1} = 2.1375$$

$$\Rightarrow p_2 = 208.8 \text{ kPa} \quad T_2 = 292.6 \text{ K}$$

From isentropic tables @  $M_2 = 0.513$

$$\frac{p_{02}}{p_2} = 1.194 \quad \frac{T_{02}}{T_2} = 1.052$$

$$\Rightarrow p_{02} = 249.4 \text{ kPa}$$

$$T_{02} = 308 \text{ K } (35^\circ\text{C})$$

③ Pitot probe:  $p_1 = 45 \text{ kPa}$ ,  $T_1 = -20^\circ\text{C}$ ,  $\Delta p = 350 \text{ kPa}$   
 $\hookrightarrow p_{02} - p_1$

Find  $M_1$  &  $V_1$ ?

$$\frac{p_{02}}{p_1} = \frac{(p_{02} - p_1) + p_1}{p_1} = 8.778$$

From the shock tables,

$$M_1 = 2.536$$

$$V_1 = 2.536 \cdot \sqrt{1.4 \cdot 287 \cdot 253}$$

$$V_1 = 809 \text{ m/s}$$

④ Shock  $p_2/p_1 = 1.15$  moving into still air  
 $(p_1 = 50 \text{ kPa}, T_1 = 30^\circ\text{C})$  (a) Find  $T$  &  $V$  of air  
behind the shock? (b) what if air not still  
but moving towards the shock at 100 m/s?

(a)



From shock tables.

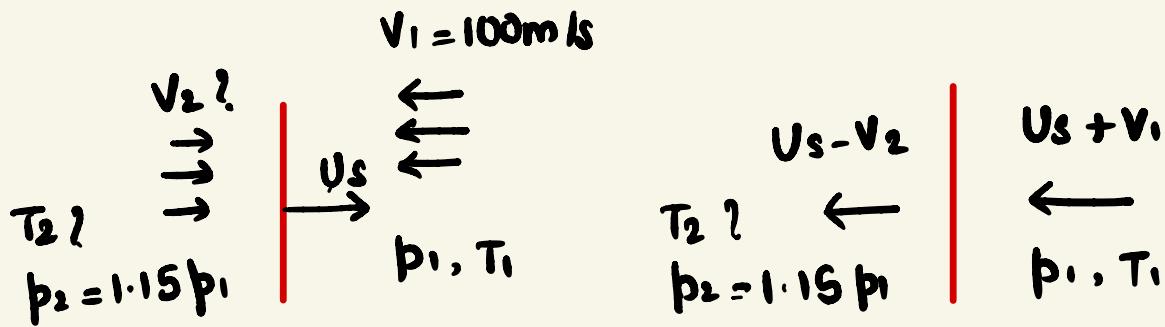
$$M_1 = 1.062 \quad M_2 = 0.943 \quad T_2/T_1 = 1.041$$

$$\Rightarrow T_2 = 315.4 \text{ K}$$

$$V_2 = M_1 a_1 - M_2 a_2$$

$$\Rightarrow V_2 = 35.1 \text{ m/s}$$

(b)



$$M_1 = \frac{U_s + V_1}{a_1}$$

$$M_2 = \frac{U_s - V_2}{a_2}$$

$$V_2 = U_s - M_2 a_2$$

$$= (M_1 a_1 - V_1) - M_2 a_2$$

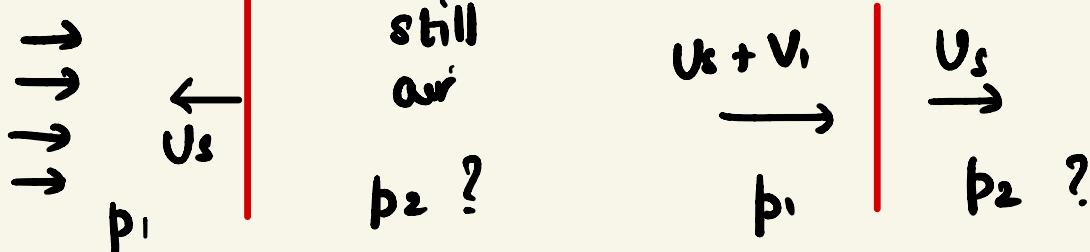
$$= -64.9 \text{ m/s}$$

$$T_2 = 315.4 \text{ K.}$$

Smart way is to think of the new problem from the point of view of the upstream flow

- ⑤ Air flowing out of a duct at 250 m/s,  $T = 0^\circ\text{C}$ ,  $p = 70 \text{ kPa}$ . Valve suddenly closed. Find pressure acting on valve immediately after closing it.

$$V_1 = 250 \text{ m/s}$$



$$M_1 = \frac{U_s + V_1}{a_1}$$

$$M_2 = \frac{U_s}{a_2} = \frac{U_s}{a_1} \cdot \frac{a_1}{a_2}$$

$$= \frac{U_s}{a_1} + \frac{V_1}{a_1} = M_2 \frac{a_2}{a_1} + \frac{V_1}{a_1}$$

$$a_1 = \sqrt{1.4 \cdot 287 \cdot 273} = 331.2 \text{ m/s}$$

$$\Rightarrow M_1 = M_2 \frac{a_2}{a_1} + 0.7548 \quad \text{--- (1)}$$

Solving by iteration

$M_1$	$M_2$	$a_2/a_1$	RHS of (1)
1.0	1.0	1.0	1.755
1.4	0.740	1.120	1.584
1.5	0.701	1.149	1.56
1.6	0.668	1.178	1.54.

Correct value of  $M_1 = 1.55$

$$p_2/p_1 = 2.636$$

$$\Rightarrow p_2 = 184.5 \text{ Pa}$$