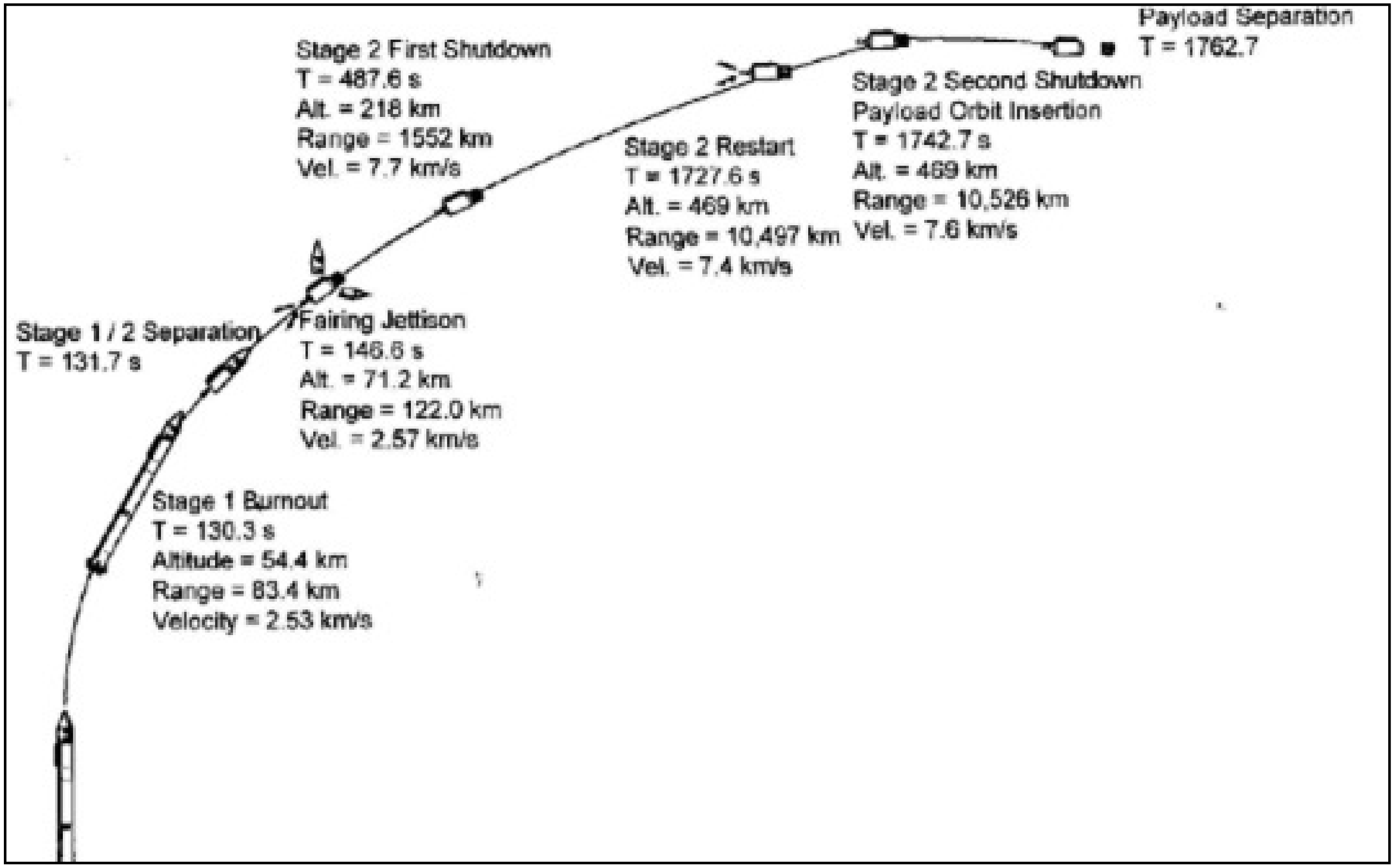
Inclined / Curvilinear Trajectories

Typical Ascent Trajectory



Inclined Motion Concept

In reality, **vertical** motion is used **only** for a very small **part** of the overall ascent **mission** and for the most part, **ascent** trajectory is inclined and **curvilinear** in nature.

This is mainly because one of the **terminal** constraint is that the **inclination** should be zero or **close** to it, with respect to the **local** horizon.

Inclined Motion Concept

Further, considering **Earth's** curvature, the rocket **needs** to undergo large **flight** path angle changes **(>90^0)**.

This requirement calls for a **different** methodology of **trajectory** design & solution**.**

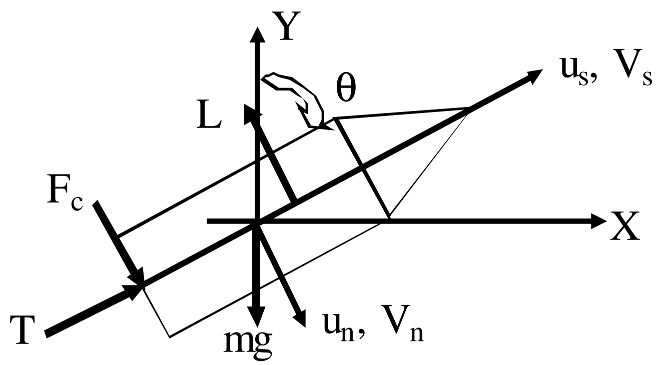
Effect of Inclination

A **curvilinear** flight path requires **motion** in a plane and therefore, needs a **2-D** model for the motion.

Also, as **thrust** is used mainly for **ΔV** and is always along the **flight** path, a normal **force** is needed to produce the **curvilinear** path.

Effect of Inclination

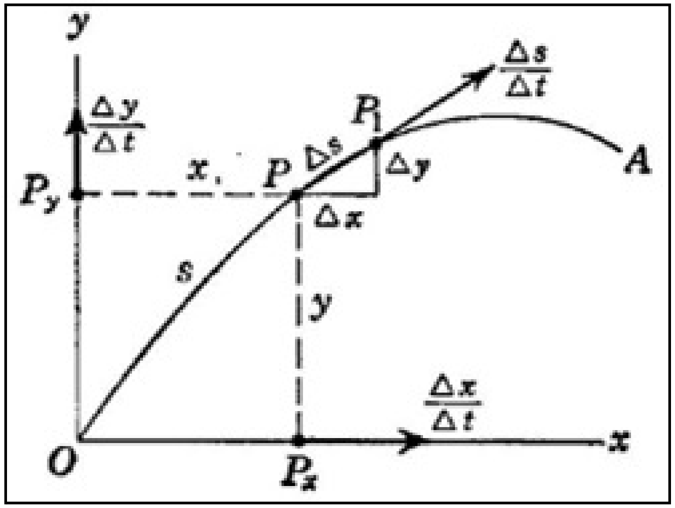
Consider a rocket having **inclination** with the vertical, as shown **below.**



$u_s, u_n \rightarrow$ Unit Vectors along s, n directions

Curvilinear Velocity Model

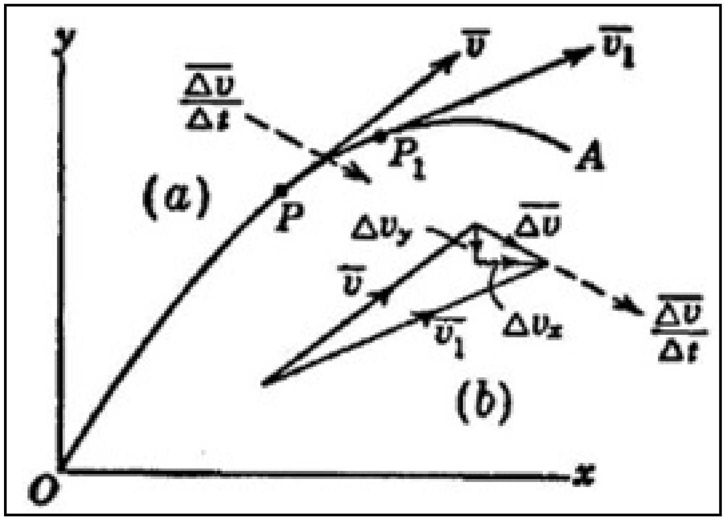
The **schematic** of a planar **motion**, along with the **expression** for velocity are as **follows**.



$$V = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \to \vec{V} = \dot{s} \cdot \hat{u}_s$$

Curvilinear Acceleration Model

The **corresponding** acceleration model and its **expression** are as given **below**.



$$\bar{a} = \frac{d}{dt}\left(\bar{V}\right) = \frac{d}{dt}\left(V\hat{u}_s\right)$$

$$\bar{a} = \dot{V}\bar{u}_s + V\frac{d\hat{u}_s}{dt} = \dot{V}\bar{u}_s + V\dot{\theta}\bar{u}_n$$

$$\bar{a} = \bar{a}_s + \bar{a}_n$$

Planar Motion Equations

The **scalar** equations of planar **motion**, in the absence of **'L'** and **'F_c',** are as follows.

$$a_z = \frac{dV}{dt} = -\frac{\dot{m}}{m} g_0 I_{sp} - \tilde{g}\cos\theta; \quad \tilde{g} \rightarrow \text{Average Constant Gravity}$$

$$a_n = V\frac{d\theta}{dt} = \tilde{g}\sin\theta; \quad V, \theta \rightarrow \text{Trajectory Parameters}$$

Trajectory Features

In this case, the resulting **trajectory** is called **'gravity turn'** trajectory, as **'g'** alone is **responsible** for turning of the velocity **vector**.

Conceptually, we can **solve** for any **2** of the **3** quantities (V, m (or dm/dt), θ), provided **third** one is given.

Gravity Turn Solution Aspects

Thus, **gravity** turn equations can be **solved** either as trajectory **design** or as a vehicle **design** problem**.**

However, the problem is **complex** because we only have initial & **terminal** conditions, which can be **met** from many different profile **geometries**.

Further, we **note** that as equations are **time** varying and nonlinear, **most** such solutions are **numerical** in nature, as analytical **solutions** are not feasible in **general** case.

Conceptual Design Solutions

However, in the **early** design stage, **analytical** solutions are found to be more **useful** for gross vehicle **sizing.**

It should be noted that such **closed** form solutions help **greatly** in quick **assessment** of performance of **many** concepts, before selecting **best** for detailed **analyses.**

However, **such** solutions are **possible** only under simplifying **assumptions** that limit their **applicability**.

Constrained Closed Form Scenarios

Among the **many** possibilities of generating approximate **closed** form solutions, there are **three** which are simple, **elegant** and also have practical **utility**.

In this regard, we note that as **most** launch vehicles use **some** kind of control during **ascent**, it is possible to **specify** a constant value for either **($d\theta/dt$),** (T/m) or V.

Constrained Solution Featues

It is to be **noted** that constant (**dθ/dt**) helps manage the mission **time** better.

Further, constant (**T/m**) helps to manage the **structural** weight better.

Lastly, constant **V** case is used to manage **propellant.**

Summary

To **summarize**, gravity based rotation of the **velocity** vector is a convenient **way** of achieving the **desired** trajectory profile, while **minimizing** the propellant.

Further, **simplified** scenarios help in **arriving** at closed form solutions that **aid** in quick performance **assessment**.