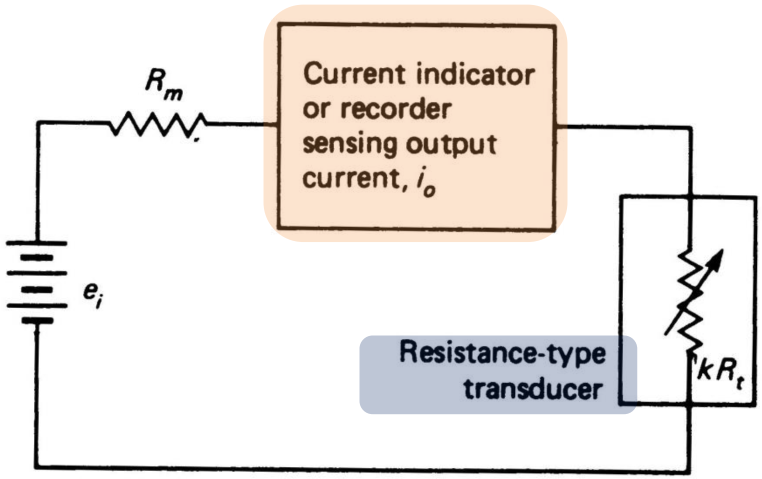
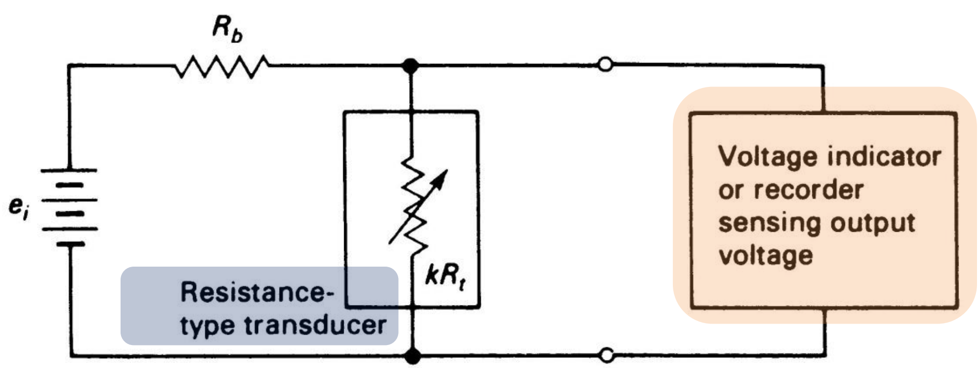
AE 242
Aerospace Measurements Laboratory

Sensors – electrical resistance change

In many transducers physical quantity, manifest as change in resistance. Change in resistance can be measured as change in current in the circuit or voltage across the transducer. Current variation can be indicated by current indicator and voltage variation can be indicated by voltage indicator.



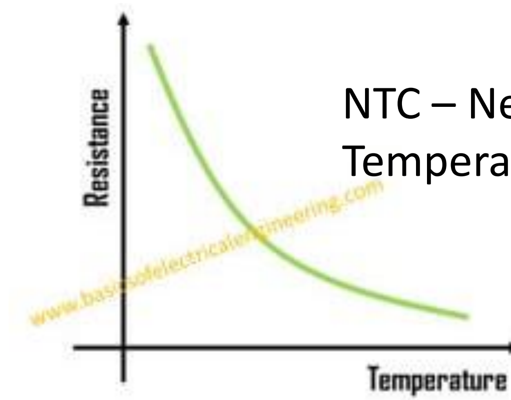
Current sensitive circuit



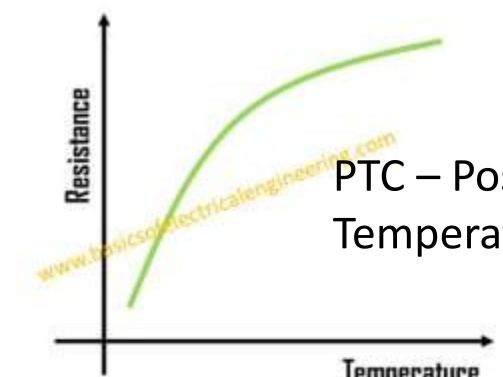
Voltage sensitive circuit

Sensors – Thermistor

It is a semiconductor material and its resistance changes as temperature changes. Sensitivity of these materials is very high. Resistance can increase (PTC) or decrease with increase (NTC) in temperature.



NTC – Negative Temperature Coefficient



PTC – Positive Temperature Coefficient

Current sensitive circuits

Let a transducer element whose variation in resistance can be expressed by a factor k. Factor k can vary from 0.0 – 1.0 (0 % to 100%)

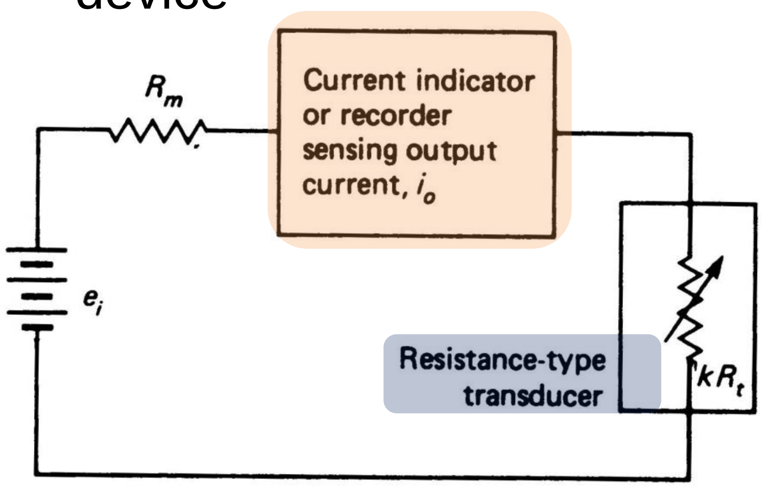
By Ohm's law,
Current in the circuit

$$i_0 = \frac{e_i}{kR_t + R_m}$$

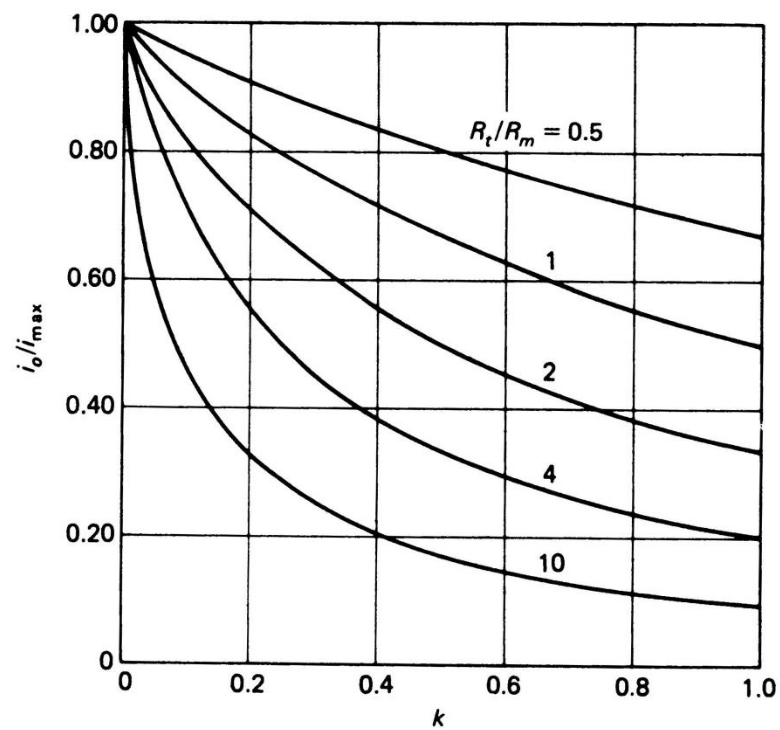
Current is maximum when k = 0

$$\frac{i_0}{i_{max}} = \frac{i_0 R_m}{e_i} = \frac{1}{1 + k(R_t / R_m)}$$

R_m is the resistance of the measuring device



Current sensitive circuit



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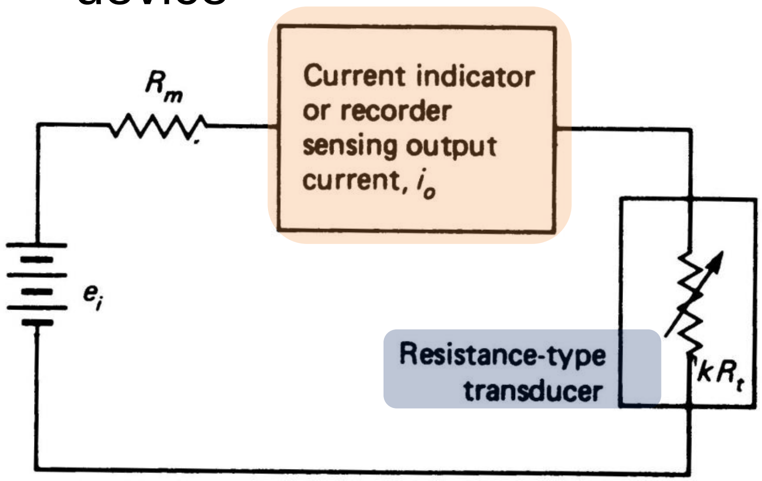

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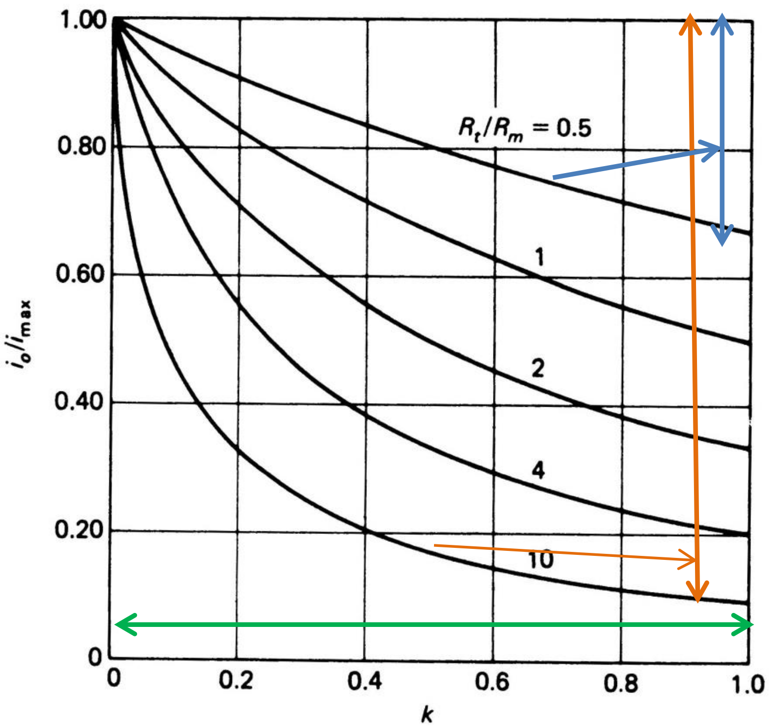


Voltage sensitive circuits

Let a transducer element whose variation in resistance can be expressed by a factor k. Factor k can vary from 0.0 – 1.0 (0 % to 100%), R_b is ballast resistance and R_t is transducer element

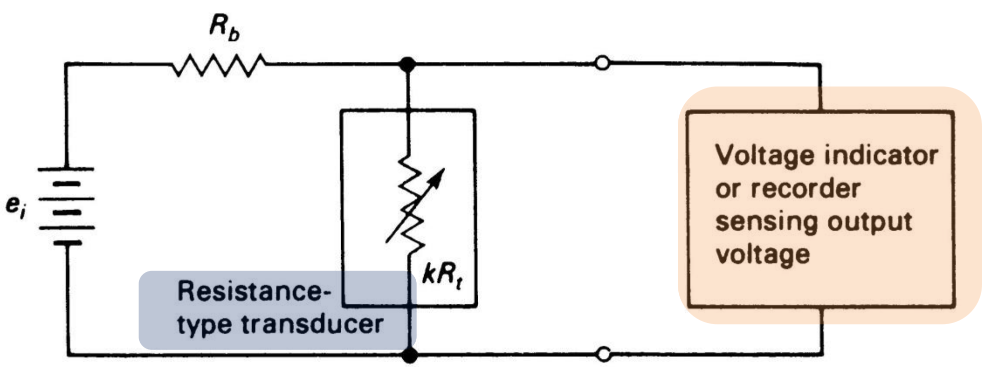
By Ohm's law,
Current in the circuit

$$i = \frac{e_i}{R_b + kR_t}$$

Voltage across transducer

$$e_0 = i(kR_t) = \frac{e_i kR_t}{R_b + kR_t} \quad \text{or} \quad \frac{e_0}{e_i} = \frac{kR_t / R_b}{1 + kR_t / R_b}$$

e_0/e_i is the measure of input and kR_t/R_b is the measure of output



Voltage sensitive circuit

Voltage sensitive circuits

η as sensitivity, ratio of change in output to change in input

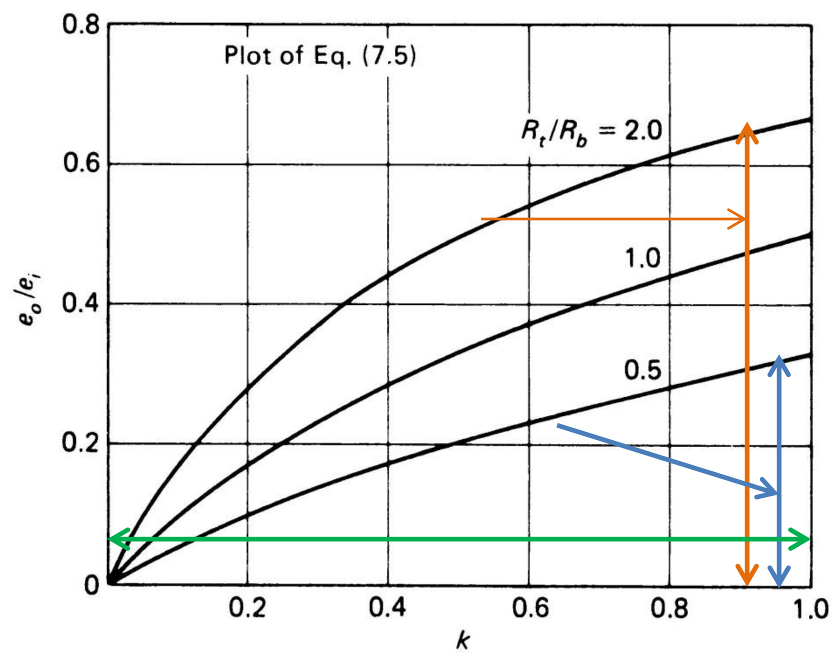
$$\eta = \frac{de_0}{dk} = \frac{e_i R_t R_b}{(R_b + kR_t)^2}$$

Sensitivity can be studied with respect to ballast resistance, a user variable

$$\frac{d\eta}{dR_b} = \frac{e_i R_t (kR_t - R_b)}{(R_b + kR_t)^3}$$

Derivative will be zero when 1) $R_b = \infty$ minimum sensitivity, and 2) $R_b = kR_t$ maximum sensitivity. R_b is a fixed quantity and sensitivity will maximum for a certain value of kR_t

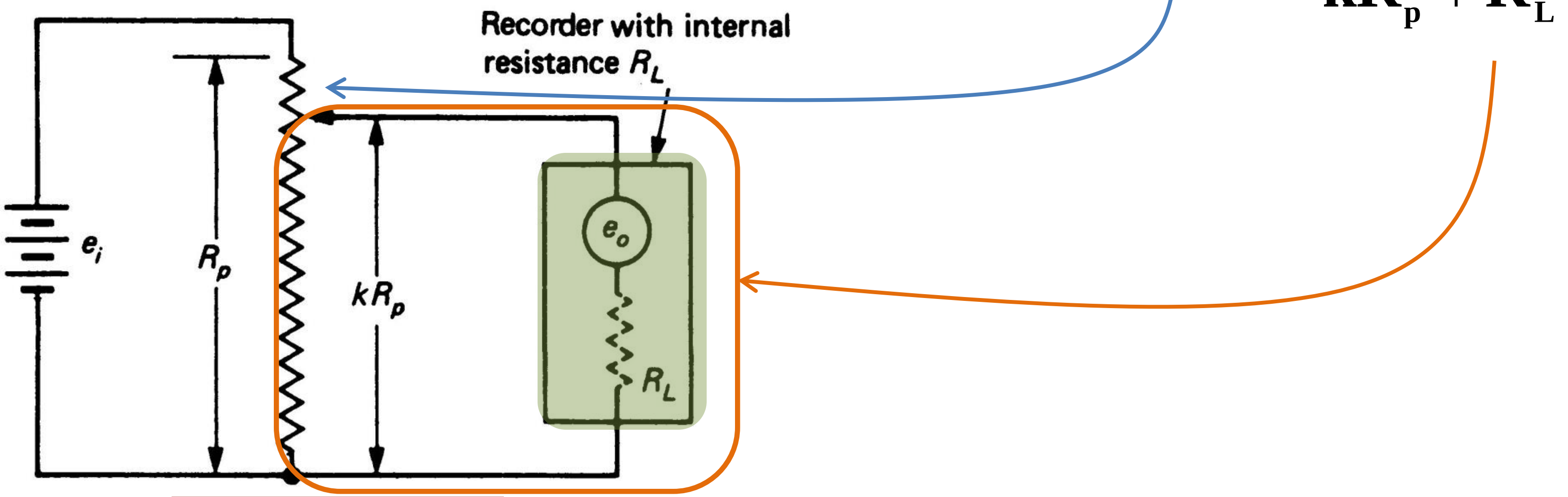
$$\frac{e_0}{e_i} = \frac{kR_t / R_b}{1 + kR_t / R_b}$$

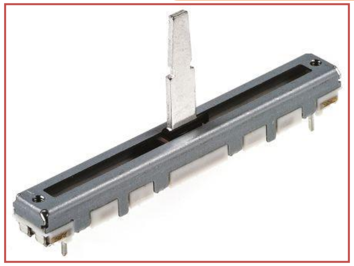


Voltage sensitive circuits

Potentiometer type in which total resistance remain same. Potential can be measured with a low impedance or high impedance indicator. When the indicator impedance is very high relative to R_p, negligible current will be drawn from the source and voltage output is proportional to the position of wiper (for a linear potentiometer). When R_L is comparable with R_P, the output is non-linear.

$$R = R_p(1-k) + \frac{kR_pR_L}{kR_p + R_L}$$





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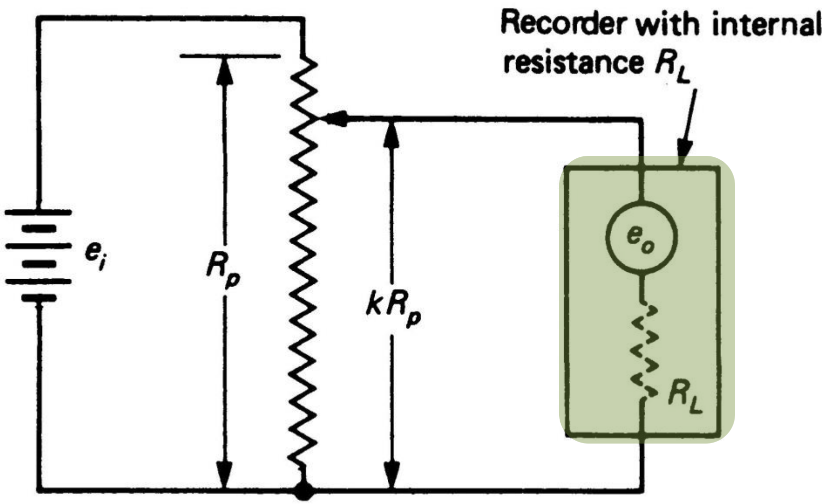
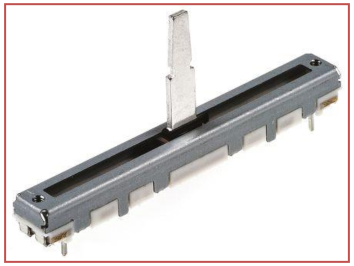
$$\mathbf{R} = \mathbf{R_p}(1-\mathbf{k}) + \frac{\mathbf{kR_pR_L}}{\mathbf{kR_p} + \mathbf{R_L}}$$

$$\mathbf{i} = \frac{\mathbf{e_i}}{\mathbf{R}} = \frac{\mathbf{e_i}(\mathbf{kR_p} + \mathbf{R_L})}{\mathbf{kR_p^2}(1-\mathbf{k}) + \mathbf{R_pR_L}}$$

$$\mathbf{e_0} = \mathbf{e_i} - \mathbf{iR_p}(1-\mathbf{k})$$

$$\frac{\mathbf{e_0}}{\mathbf{e_i}} = \frac{\mathbf{k}}{1 + (\mathbf{R_p}/\mathbf{R_L})\mathbf{k} - (\mathbf{R_p}/\mathbf{R_L})\mathbf{k}^2}$$



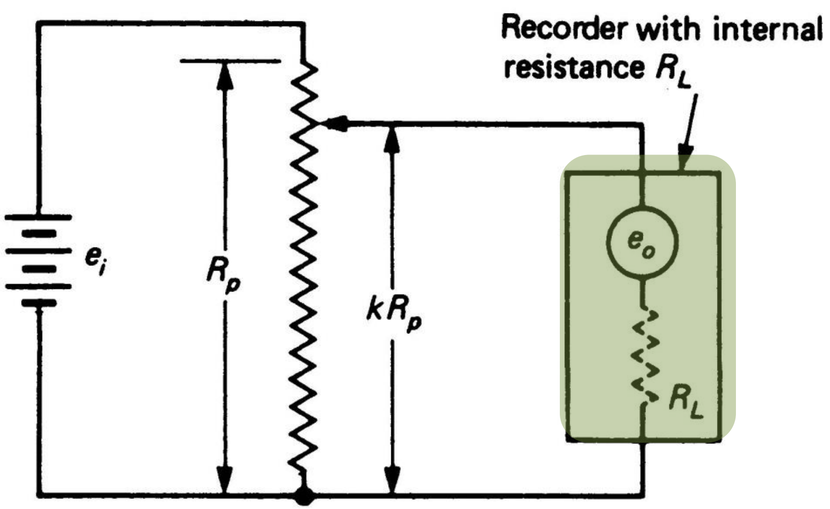


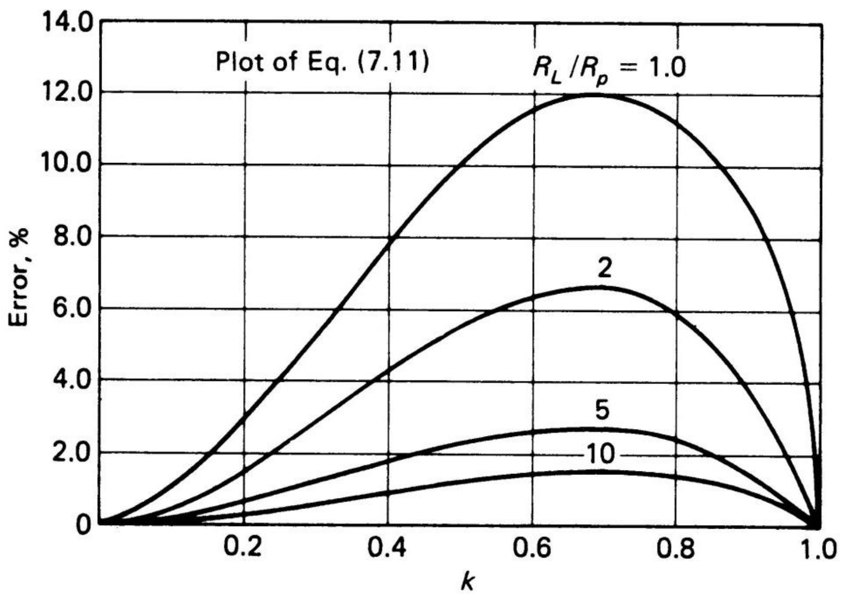
Voltage sensitive circuits

At end point i.e. k=0 and k=1, error is zero. At other values of k, the output will be always less compared to actual value.

$$\mathbf{Error} = \mathbf{e}_{\mathbf{o\ ideal}} - \mathbf{e}_{\mathbf{o}}$$

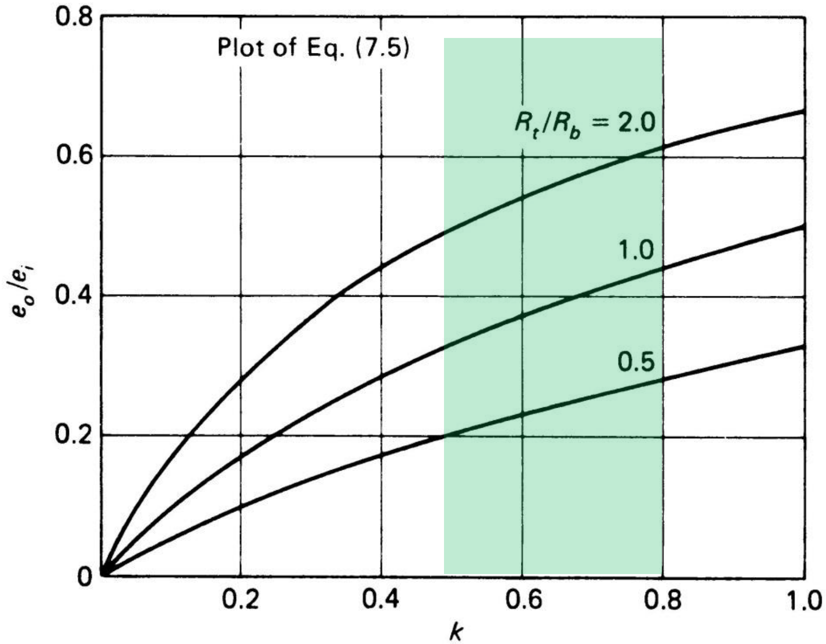
$$\mathbf{Error} = \mathbf{e}_{\mathrm{i}}\left[\mathbf{k} - \frac{\mathbf{k}}{\mathbf{k}(1-\mathbf{k})(\mathbf{R}_{\mathrm{p}}/\mathbf{R}_{\mathrm{L}})\mathbf{k}+1}\right] = \mathbf{e}_{\mathrm{i}}\left[\frac{\mathbf{k}^2(1-\mathbf{k})}{\mathbf{k}(1-\mathbf{k})+(\mathbf{R}_{\mathrm{p}}/\mathbf{R}_{\mathrm{L}})}\right]$$

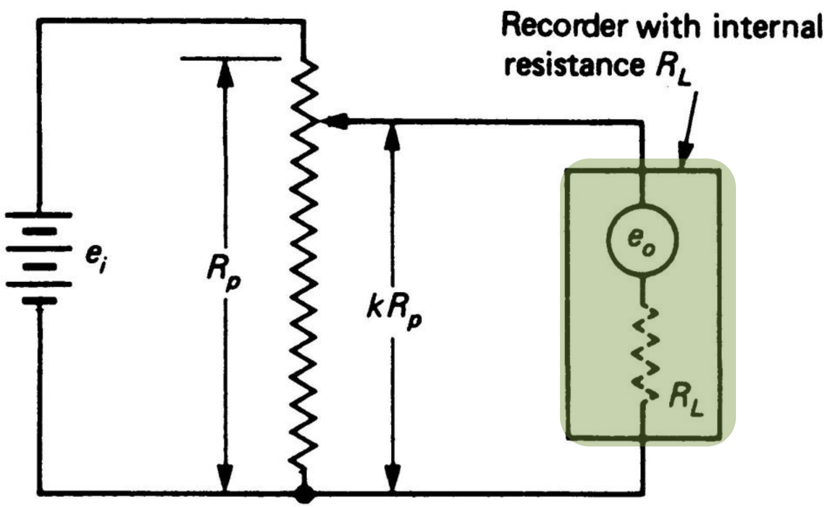


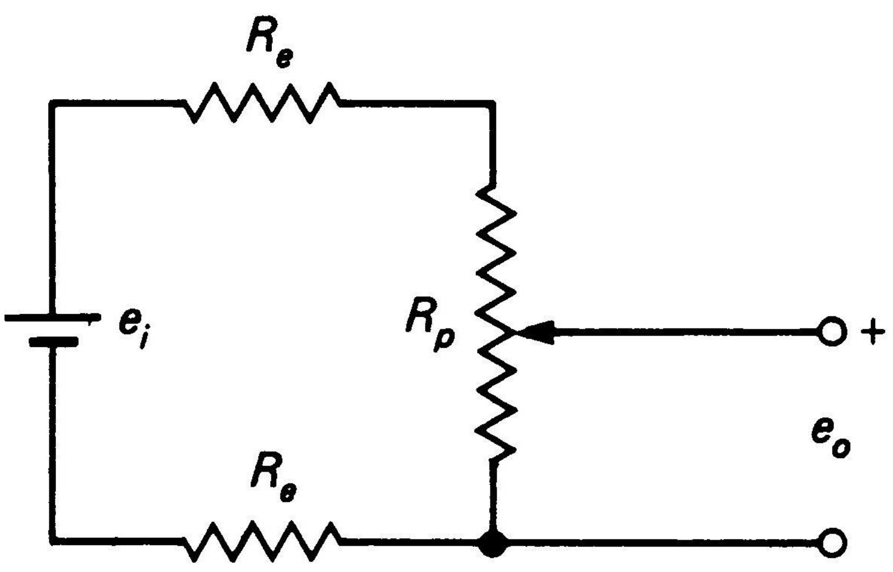


Voltage sensitive circuits

Non-linearity can be reduced by introducing end resistors. This shifts the operation range and in this range output is assumed linear. Introduction of these resistors reduces the range of output voltage, this can be improved by higher input voltage.

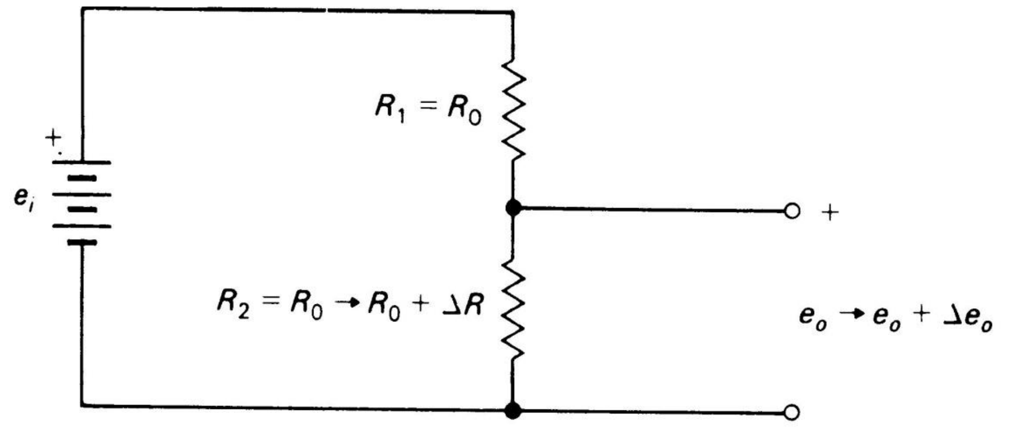






Small change in transducer resistance

Some resistance transducers show very small change in their resistance. Strain gage resistance vary about 0.0001%. For the circuit given, initial $R_1 = R_2 = R_0$



$$\mathbf{e}_0 = \frac{\mathbf{R}_2}{\mathbf{R}_1 + \mathbf{R}_2}\mathbf{e}_\mathbf{i} = \frac{\mathbf{R}_0}{\mathbf{R}_0 + \mathbf{R}_0}\mathbf{e}_\mathbf{i} = \frac{\mathbf{e}_\mathbf{i}}{2}$$

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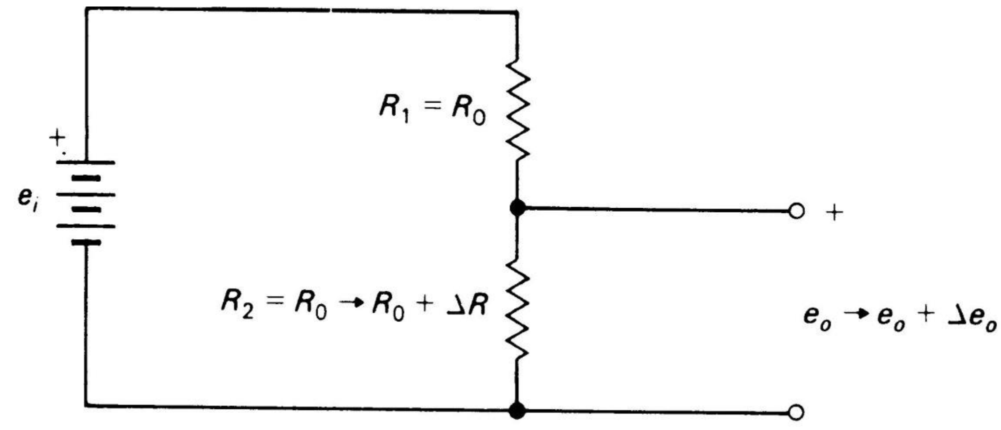

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$R_2 = R_0$ changes by a small amount ΔR, and Output is

$$\mathbf{e}_0 + \Delta\mathbf{e}_0 = \frac{\mathbf{R}_0 + \Delta\mathbf{R}}{\mathbf{R}_0 + (\mathbf{R}_0 + \Delta\mathbf{R})}\mathbf{e}_\mathbf{i}$$

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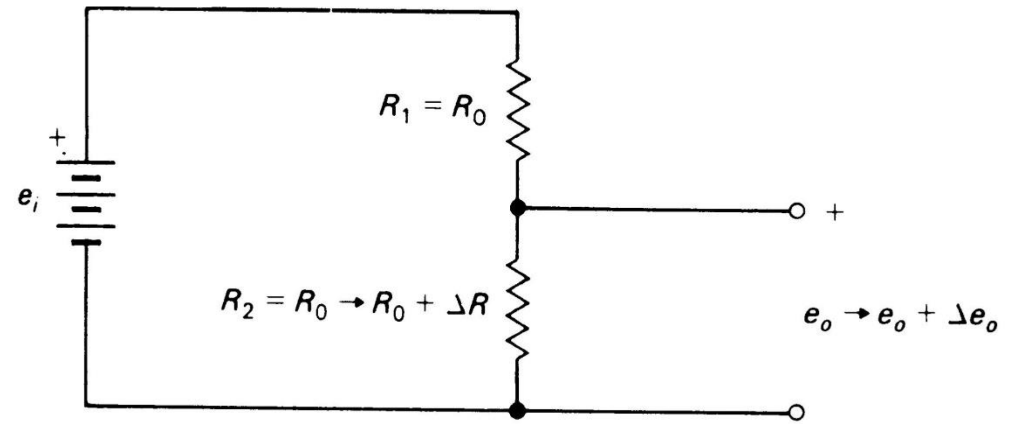

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$$\mathbf{e}_0 + \Delta\mathbf{e}_0 = \frac{1}{2}\left(\frac{1 + \Delta\mathbf{R}/\mathbf{R}_0}{1 + \Delta\mathbf{R}/2\mathbf{R}_0}\right)\mathbf{e}_\mathbf{i} = \frac{\mathbf{e}_\mathbf{i}}{2}\left(1 + \frac{\Delta\mathbf{R}/2\mathbf{R}_0}{1 + \Delta\mathbf{R}/2\mathbf{R}_0}\right)$$

$$= \frac{\mathbf{e}_\mathbf{i}}{2} + \frac{\mathbf{e}_\mathbf{i}}{2}\frac{\Delta\mathbf{R}}{2\mathbf{R}_0}\left(\frac{1}{1 + \Delta\mathbf{R}/2\mathbf{R}_0}\right)$$

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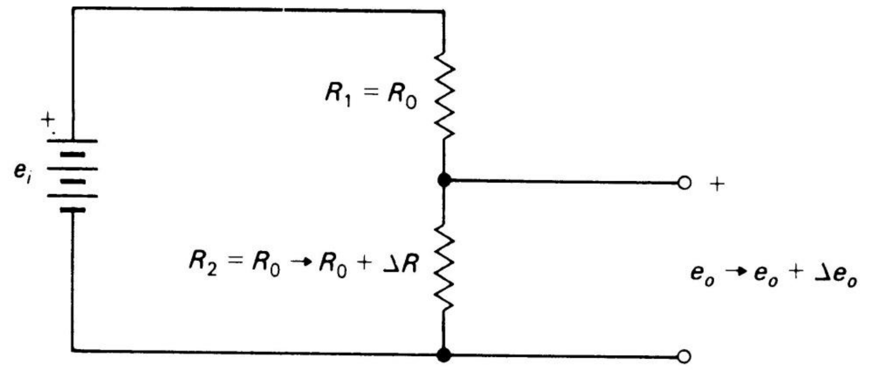

$\Delta R/2R_0 << 1$ the output can be approximated as

$$\mathbf{e}_0 + \Delta\mathbf{e}_0 \approx \mathbf{e}_0 + \frac{\Delta\mathbf{R}}{4\mathbf{R}_0}\mathbf{e_i}$$

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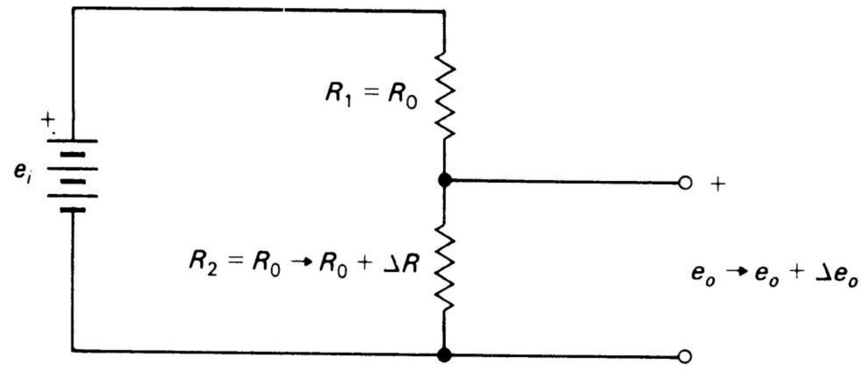

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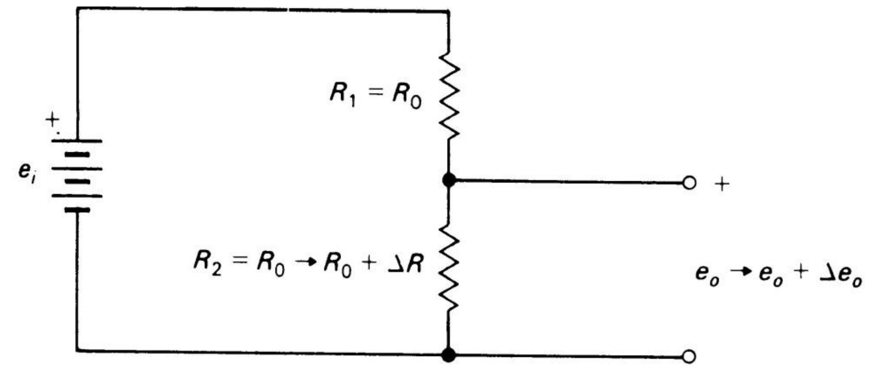
For small variation in resistance it will be linear and it is advantageous. Variation in small resistance and the output is at disadvantage. For a 120 Ω strain gage change in resistance is 240 μΩand it will change the output in micro volts

$$\frac{\Delta\mathbf{e}_0}{\mathbf{e}_0} = \frac{(\Delta\mathbf{R}/4\mathbf{R}_0)\mathbf{e_i}}{\mathbf{e_i}/2} = \frac{\Delta\mathbf{R}}{2\mathbf{R}_0} = 10^{-6}$$

Measurement is $e_0 + \Delta\, e_0$ and this will need a very precise instrument.

Small change in transducer resistance

Difficulty is to resolve voltage change which is a small fraction of output voltage.



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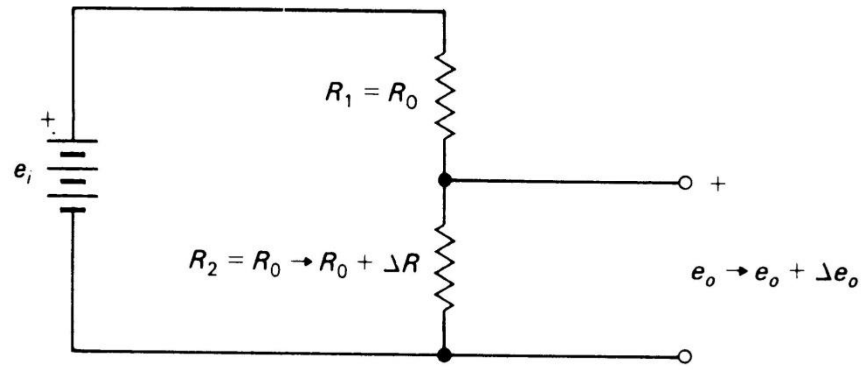
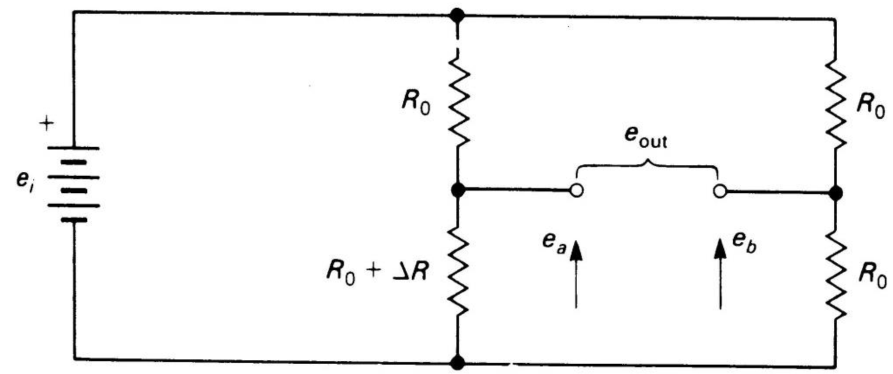


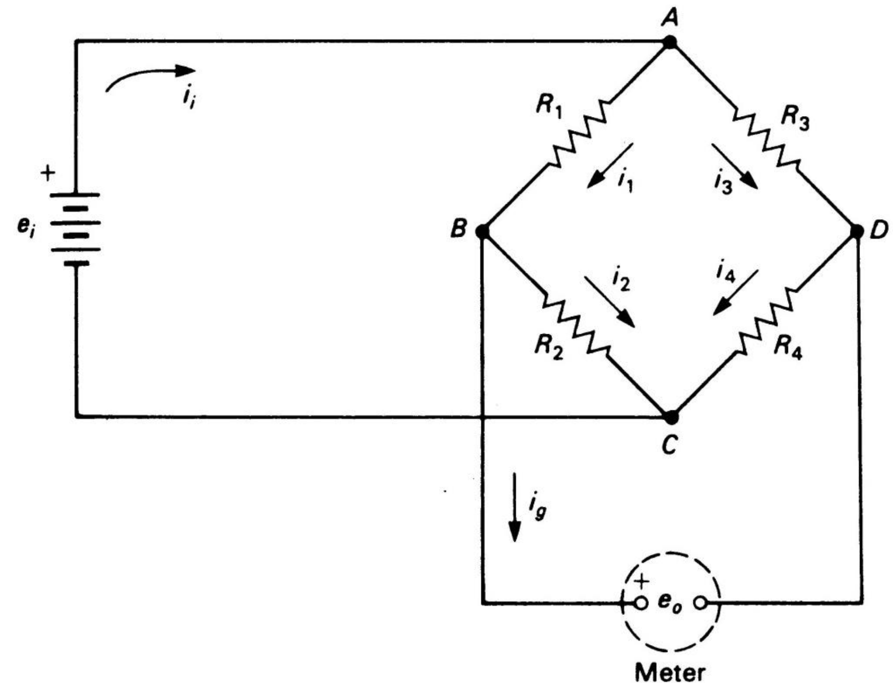
The difficulty can be removed by measuring only the difference and amplifying it.



$$\mathbf{e_{out}} = \mathbf{e_a} - \mathbf{e_b} = \Delta \mathbf{e}_0 = \frac{\Delta \mathbf{R}}{4\mathbf{R}_0}\mathbf{e_i}$$

Wheatstone bridge

Consist of four arms of resistors, a detector and power supply source. Two arms are voltage divider and the detector finds the potential difference. Bridge is balanced when potential difference is zero and no current flow through detector. When bridge is balanced:



$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \text{or} \quad \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

For the Wheatstone resistance bridge to be balance, the ratio of resistances of any two adjacent arms must equal the ratio of resistances of the remaining two arms, taken in the same sense.