MA 214: Introduction to numerical analysis (2021–2022)

Tutorial 6

(March 09, 2022)

(1) Use the forward-difference and backward-difference formulae to determine each missing entry in the following table:

x	0.5	0.6	0.7
$f(x)$	0.4794255386	0.5646424734	0.6442176872
$f'(x)$	?	?	?

(2) Use the forward-difference and backward-difference formulae to determine each missing entry in the following table:

x	0.0	0.2	0.4
$f(x)$	0.0	0.7414027582	1.3718246976
$f'(x)$	?	?	?

(3) The data in the above problems were taken from the following functions. Compute the actual errors and find error bounds using the error formulae:

$$(1)\ \ f(x) = \sin x, \qquad (2)\ \ f(x) = e^x - 2x^2 + 3x - 1.$$

(4) If $f : [a, b] \to \mathbb{R}$ is continuously differentiable and $c_i \geqslant 0$, $\theta_i \in (a, b)$ for $i = 0, \dots, n$, then prove that there is a $\theta \in (a, b)$ such that

$$\sum_i c_i f'(\theta_i) = \left(\sum_i c_i \right) f'(\theta).$$

(5) Assume that for any sufficiently continuously differentiable function f, we have

$$f''(t) \approx Af(t+h) + Bf(t) + Cf(t-h)$$

where A, B, C are constants, depending on h, to be determined. Replace $f(t \pm h)$ by the Taylor expansions. Ignoring the terms involving h^3 or higher powers of h, solve for A, B, C. Write the approximate formula for $f''(t)$ obtained thus.

(6) Derive Simpson's $\dfrac{1}{3}$-rd rule with error term by using

$$\int_{x_0}^{x_2} f(x)dx = a_0 f(x_0) + a_1 f(x_1) + a_2 f(x_2) + k f^{(4)}(\xi).$$

Find a_0, a_1, and a_2 from the fact that the rule is exact for $f(x) = x^n$ when $n = 1, 2$, and 3. Then find k by applying the integration formula with $f(x) = x^4$.

(7) Approximate the following using the trapezoidal and Simpson's $\frac{1}{3}$-rd rule: Compute the actual error and compare it with the error given by the error formulae.
$$\int_{0.5}^{1} x^4 dx.$$

(8) Approximate the following using the trapezoidal and Simpson's $\frac{1}{3}$-rd rule: Compute the actual error and compare it with the error given by the error formulae.
$$\int_{0}^{0.5} \frac{2}{x-4} dx.$$

(9) The Trapezoidal rule applied to $\displaystyle\int_0^2 f(x)dx$ gives the value 4, and Simpson's $\frac{1}{3}$-rd rule gives the value 2. What is $f(1)$?
The Trapezoidal rule applied to $\displaystyle\int_0^2 f(x)dx$ gives the value 5, and the Midpoint rule gives the value 4. What value does Simpson's $\frac{1}{3}$-rd rule give?