

## MA 214: Introduction to numerical analysis (2021–2022)

### Tutorial 5

(February 16, 2022)

- (1) Let  $L_0, L_1, L_2$  and  $L_3$  be the Lagrange polynomials for distinct nodes  $x_0, x_1, x_2$  and  $x_3$ . Find all  $j \geq 0$  such that

$$x_0^j L_0(x) + x_1^j L_1(x) + x_2^j L_2(x) + x_3^j L_3(x) = x^j.$$

- (2) Let  $x_0, \dots, x_k$  be distinct nodes and define  $g(x) := f[x_0, x_1, \dots, x_k, x]$ . Prove that  $g[y_0, \dots, y_n] = f[x_0, \dots, x_k, y_0, \dots, y_n]$ .

- (3) If  $f(x) = g(x)h(x)$  then find a formula for the divided differences for  $f$  in terms of those of  $g$  and  $h$ .

- (4) Construct a Hermite polynomial  $H_3(x)$  for the following data for  $(x, f(x), f'(x))$ :  
(8.3, 17.56492, 3.116256), (8.6, 18.50515, 3.151762).

If the function here is  $f(x) = x \ln(x)$  then compute  $f(8.4)$  and the errors.

- (5) Construct a Hermite polynomial  $H_3(x)$  for the following data for  $(x, f(x), f'(x))$ :  
(0.8, 0.22363362, 2.1691753), (1, 0.65809197, 2.0466965).

If the function here is  $f(x) = \sin(e^x - 2)$  then compute  $f(0.9)$  and the errors.

- (6) Use 5-digit rounding arithmetic and compute the table for the values of  $\sin(x)$  and its derivative,  $\cos(x)$ , at 0.30, 0.32 and 0.35. Obtain the corresponding Hermite polynomial  $H$  and compute  $H(0.34)$ . Compare the actual error and the one predicted by the error formula.

- (7) Compute the natural cubic spline for the following data:

$x$	$-0.5$	$-0.25$	$0$
$f(x)$	$-0.0247500$	$0.3349375$	$1.1010000$

- (8) Compute the natural cubic spline for the following data:

$x$	$0.1$	$0.2$	$0.3$	$0.4$
$f(x)$	$-0.62049958$	$-0.28398668$	$0.00660095$	$0.24842440$

- (9) Compute the clamped cubic spline for the data in the above problem and  $f'(0.1) = 3.58502082$  and  $f'(0.4) = 2.16529366$ .