

Q1. $U = (x^3 - 3y + z)i + (-zx + y^2)j + (2x^2 - 1.5z + yx)k$

for strain components we know, if $U = (u, v, w)$

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = 3x^2 \Big|_{-1, 1, -1} = \boxed{-3}$$

$$\epsilon_{yy} = \frac{\partial v}{\partial y} = 2y \Big|_{-1, 1, -1} = \boxed{2}$$

$$\epsilon_{zz} = \frac{\partial w}{\partial z} = x^2 - 1.5 \Big|_{-1, 1, -1} = \boxed{-0.5}$$

$$\epsilon_{xy} = \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = \frac{1}{2} (-3 + (-z)) \Big|_{-1, 1, -1} = \boxed{-2}$$

$$\epsilon_{yz} = \frac{1}{2} \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) = \boxed{0}$$

$$\epsilon_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 1 + 4xz^2 + y \Big|_{-1, 1, -1} = 1 + 4(-1)(-1)^3 + 1 = \boxed{6}$$

Q3. we have $\sigma_{xx} = 2x^3y^2$ $\sigma_{yy} = xy^4$

$$\tau_{xy} = -2x^2y^3.$$

We know that equilibrium conditions are (in 2D)

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + F_1 = 0; \quad \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + F_2 = 0$$

As we don't have any body forces, we get

$$\textcircled{1} \quad \frac{\partial}{\partial x}(2x^3y^2) + \frac{\partial}{\partial y}(-2x^2y^3) = 6x^2y^2 + (-6x^2y^2) = 0$$

$$\textcircled{2} \quad \frac{\partial}{\partial x}(-2x^2y^3) + \frac{\partial}{\partial y}(xy^4) = -4xy^3 + (4xy^3) = 0$$

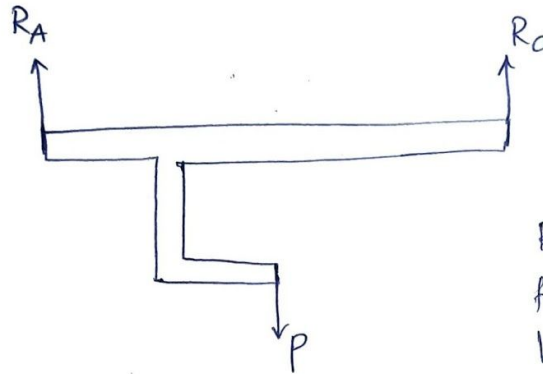
We can see that the stresses satisfy the equilibrium conditions.

Q4.

Sign convention

+ ↺ (- ↻)

Moments.



By normal force balance we have.

$$R_A + R_C = P$$

Now for rotational stability of the beam, taking moment about point A we get

$$(R_A \times 0) + (P \times \frac{L}{2}) + (R_C \times L) = 0 \Rightarrow \frac{PL}{2} = -R_C L$$

$$\Rightarrow \boxed{R_C = -\frac{P}{2}}$$

Similarly taking moments about C,

$$(R_A \times L) + (R_C \times 0) + (P \times \frac{L}{2}) = 0 \Rightarrow R_A = \frac{P}{2}$$

Hence we have both. $\boxed{R_A = R_C = \frac{P}{2}}$