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Fundamentals of Engineering Thermodynamics

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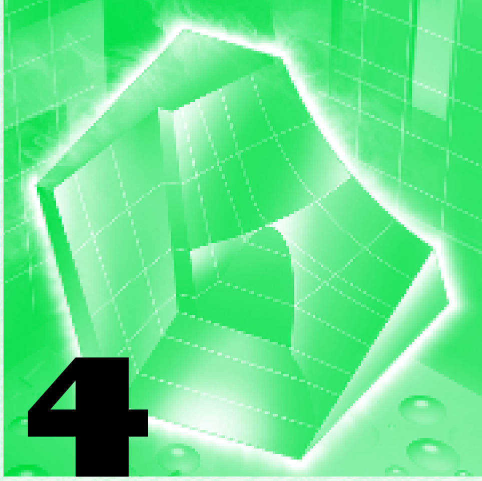
CHAPTER 4



Control Volume Analysis Using Energy

ENGINEERING CONTEXT The **objective** of this chapter is to develop and illustrate the use of the control volume forms of the conservation of mass and conservation of energy principles. Mass and energy balances for control volumes are introduced in Secs. 4.1 and 4.2, respectively. These balances are applied in Sec. 4.3 to control volumes at steady state and in Sec. 4.4 for transient applications.

◀ **chapter objective**

Although devices such as turbines, pumps, and compressors through which mass flows can be analyzed in principle by studying a particular quantity of matter (a closed system) as it passes through the device, it is normally preferable to think of a region of space through which mass flows (a control volume). As in the case of a closed system, energy transfer across the boundary of a control volume can occur by means of work and heat. In addition, another type of energy transfer must be accounted for—the energy accompanying mass as it enters or exits.

4.1 Conservation of Mass for a Control Volume

In this section an expression of the conservation of mass principle for control volumes is developed and illustrated. As a part of the presentation, the one-dimensional flow model is introduced.

▶ 4.1.1 Developing the Mass Rate Balance

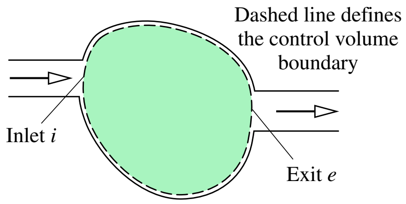
The mass rate balance for control volumes is introduced by reference to Fig. 4.1, which shows a control volume with mass flowing in at i and flowing out at e, respectively. When applied to such a control volume, the ***conservation of mass*** principle states

conservation of mass

$$\begin{bmatrix}\text{time } \textit{rate of change} \text{ of} \\ \text{mass contained within} \\ \text{the control volume } \textit{at time t}\end{bmatrix} = \begin{bmatrix}\text{time } \textit{rate} \text{ of flow} \\ \text{of mass } \textit{in} \text{ across} \\ \text{inlet } i \textit{ at time t}\end{bmatrix} - \begin{bmatrix}\text{time } \textit{rate} \text{ of flow} \\ \text{of mass } \textit{out} \text{ across} \\ \text{exit } e \textit{ at time t}\end{bmatrix}$$

Denoting the mass contained within the control volume at time t by $m_{cv}(t)$, this statement of the conservation of mass principle can be expressed in symbols as

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e \tag{4.1}$$



◀ **Figure 4.1** One-inlet, one-exit control volume.

mass flow rates

where dm_{cv}/dt is the time rate of change of mass within the control volume, and $\dot{m}_i$ and $\dot{m}_e$ are the instantaneous ***mass flow rates*** at the inlet and exit, respectively. As for the symbols $\dot{W}$ and $\dot{Q}$, the dots in the quantities $\dot{m}_i$ and $\dot{m}_e$ denote time rates of transfer. In SI, all terms in Eq. 4.1 are expressed in kg/s. For a discussion of the development of Eq. 4.1, see box.

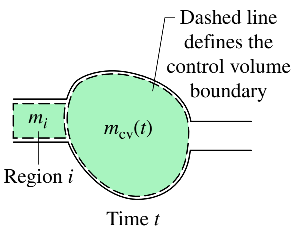
In general, there may be several locations on the boundary through which mass enters or exits. This can be accounted for by summing, as follows

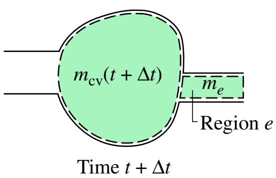
mass rate balance

$$\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e \tag{4.2}$$

Equation 4.2 is the ***mass rate balance*** for control volumes with several inlets and exits. It is a form of the conservation of mass principle commonly employed in engineering. Other forms of the mass rate balance are considered in discussions to follow.

DEVELOPING THE CONTROL VOLUME MASS BALANCE





For each of the extensive properties mass, energy, and entropy (Chap. 6), the control volume form of the property balance can be obtained by transforming the corresponding closed system form. Let us consider this for mass, recalling that the mass of a closed system is constant.

The figures in the margin show a system consisting of a fixed quantity of matter m that occupies different regions at time t and a later time $t + \Delta t$. The mass under consideration is shown in color on the figures. At time t, the mass is the sum $m = m_{cv}(t) + m_i$, where $m_{cv}(t)$ is the mass contained within the control volume, and m_i is the mass within the small region labeled i adjacent to the control volume. Let us study the fixed quantity of matter m as time elapses.

In a time interval Δt all the mass in region i crosses the control volume boundary, while some of the mass, call it m_e, initially contained within the control volume exits to fill the region labeled e adjacent to the control volume. Although the mass in regions i and e as well as in the control volume differ from time t to $t + \Delta t$, the *total* amount of mass is constant. Accordingly

$$m_{cv}(t) + m_i = m_{cv}(t + \Delta t) + m_e \tag{a}$$

or on rearrangement

$$m_{cv}(t + \Delta t) - m_{cv}(t) = m_i - m_e \tag{b}$$

Equation (b) is an *accounting* balance for mass. It states that the change in mass of the control volume during time interval Δt equals the amount of mass that enters less the amount of mass that exits.

Equation (b) can be expressed on a time rate basis. First, divide by Δt to obtain

$$\frac{m_{\text{cv}}(t + \Delta t) - m_{\text{cv}}(t)}{\Delta t} = \frac{m_i}{\Delta t} - \frac{m_e}{\Delta t} \tag{c}$$

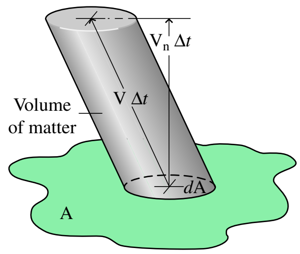
Then, in the limit as Δt goes to zero, Eq. (c) becomes Eq. 4.1, the instantaneous *control volume rate equation* for mass

$$\frac{dm_{\text{cv}}}{dt} = \dot{m}_i - \dot{m}_e \tag{4.1}$$

where dm_{cv}/dt denotes the time rate of change of mass within the control volume, and $\dot{m}_i$ and $\dot{m}_e$ are the inlet and exit mass flow rates, respectively, all at time t.

EVALUATING THE MASS FLOW RATE

An expression for the mass flow rate $\dot{m}$ of the matter entering or exiting a control volume can be obtained in terms of local properties by considering a small quantity of matter flowing with velocity V across an incremental area dA in a time interval Δt, as shown in Fig. 4.2. Since the portion of the control volume boundary through which mass flows is not necessarily at rest, the velocity shown in the figure is understood to be the velocity *relative* to the area dA. The velocity can be resolved into components normal and tangent to the plane containing dA. In the following development V_n denotes the component of the relative velocity normal to dA in the direction of flow.



▲ **Figure 4.2** Illustration used to develop an expression for mass flow rate in terms of local fluid properties.

The *volume* of the matter crossing dA during the time interval Δt shown in Fig. 4.2 is an oblique cylinder with a volume equal to the product of the area of its base dA and its altitude $\text{V}_\text{n}\,\Delta t$. Multiplying by the density ρ gives the amount of mass that crosses dA in time Δt

$$\begin{bmatrix}\text{amount of mass} \\ \text{crossing } d\text{A during} \\ \text{the time interval } \Delta t\end{bmatrix} = \rho(\text{V}_\text{n}\,\Delta t)\,d\text{A}$$

Dividing both sides of this equation by Δt and taking the limit as Δt goes to zero, the instantaneous mass flow rate across incremental area dA is

$$\begin{bmatrix}\text{instantaneous rate} \\ \text{of mass flow} \\ \text{across } d\text{A}\end{bmatrix} = \rho \text{V}_\text{n}\,d\text{A}$$

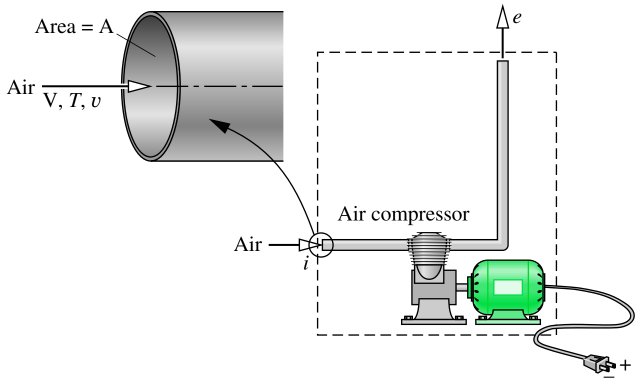
When this is integrated over the area A through which mass passes, an expression for the mass flow rate is obtained

$$\dot{m} = \int_\text{A} \rho \text{V}_\text{n}\,d\text{A} \tag{4.3}$$

Equation 4.3 can be applied at the inlets and exits to account for the rates of mass flow into and out of the control volume.

▶ 4.1.2 Forms of the Mass Rate Balance

The mass rate balance, Eq. 4.2, is a form that is important for control volume analysis. In many cases, however, it is convenient to apply the mass balance in forms suited to particular objectives. Some alternative forms are considered in this section.



◀ **Figure 4.3** Figure illustrating the one-dimensional flow model.

ONE-DIMENSIONAL FLOW FORM

one-dimensional flow

When a flowing stream of matter entering or exiting a control volume adheres to the following idealizations, the flow is said to be ***one-dimensional:***

- The flow is normal to the boundary at locations where mass enters or exits the control volume.
- *All* intensive properties, including velocity and density, are *uniform with position* (bulk average values) over each inlet or exit area through which matter flows.

METHODOLOGY UPDATE

In subsequent control volume analyses, we routinely assume that the idealizations of one-dimensional flow are appropriate. Accordingly the assumption of one-dimensional flow is not listed explicitly in solved examples.

▶ ***for example...*** Figure 4.3 illustrates the meaning of one-dimensional flow. The area through which mass flows is denoted by A. The symbol V denotes a single value that represents the velocity of the flowing air. Similarly T and v are single values that represent the temperature and specific volume, respectively, of the flowing air. ◀

When the flow is one-dimensional, Eq. 4.3 for the mass flow rate becomes

$$\dot{m} = \rho \mathrm{AV} \qquad \text{(one-dimensional flow)} \tag{4.4a}$$

or in terms of specific volume

$$\dot{m} = \frac{\mathrm{AV}}{v} \qquad \text{(one-dimensional flow)} \tag{4.4b}$$

volumetric flow rate

When area is in m^2, velocity is in m/s, and specific volume is in m^3/kg, the mass flow rate found from Eq. 4.4b is in kg/s, as can be verified. The product AV in Eqs. 4.4 is the ***volumetric flow rate.*** The volumetric flow rate is expressed in units of m^3/s.

Substituting Eq. 4.4b into Eq. 4.2 results in an expression for the conservation of mass principle for control volumes limited to the case of one-dimensional flow at the inlet and exits

$$\frac{dm_{\mathrm{cv}}}{dt} = \sum_i \frac{\mathrm{A}_i\mathrm{V}_i}{v_i} - \sum_e \frac{\mathrm{A}_e\mathrm{V}_e}{v_e} \qquad \text{(one-dimensional flow)} \tag{4.5}$$

Note that Eq. 4.5 involves summations over the inlets and exits of the control volume. Each individual term in either of these sums applies to a particular inlet or exit. The area, velocity, and specific volume appearing in a term refer only to the corresponding inlet or exit.

STEADY-STATE FORM

Many engineering systems can be idealized as being at ***steady state,*** meaning that *all* properties are unchanging in time. For a control volume at steady state, the identity of the matter within the control volume changes continuously, but the total amount present at any instant remains constant, so $dm_{cv}/dt = 0$ and Eq. 4.2 reduces to

steady state

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e \tag{4.6}$$

That is, the total incoming and outgoing rates of mass flow are equal.

Equality of total incoming and outgoing rates of mass flow does not necessarily mean that a control volume is at steady state. Although the total amount of mass within the control volume at any instant would be constant, other properties such as temperature and pressure might be varying with time. When a control volume is at steady state, *every* property is independent of time. Note that the steady-state assumption and the one-dimensional flow assumption are independent idealizations. One does not imply the other.

INTEGRAL FORM

We consider next the mass rate balance expressed in terms of local properties. The total mass contained within the control volume at an instant t can be related to the local density as follows

$$m_{cv}(t) = \int_V \rho \, dV \tag{4.7}$$

where the integration is over the volume at time t.

With Eqs. 4.3 and 4.7, the mass rate balance Eq. 4.2 can be written as

$$\frac{d}{dt}\int_V \rho \, dV = \sum_i \left(\int_A \rho \mathrm{V_n} \, dA\right)_i - \sum_e \left(\int_A \rho \mathrm{V_n} \, dA\right)_e \tag{4.8}$$

where the area integrals are over the areas through which mass enters and exits the control volume, respectively. The product $\rho \mathrm{V_n}$ appearing in this equation, known as the ***mass flux,*** gives the time rate of mass flow per unit of area. To evaluate the terms of the right side of Eq. 4.8 requires information about the variation of the mass flux over the flow areas. The form of the conservation of mass principle given by Eq. 4.8 is usually considered in detail in fluid mechanics.

mass flux

EXAMPLES

The following example illustrates an application of the rate form of the mass balance to a control volume *at steady state*. The control volume has two inlets and one exit.

EXAMPLE 4.1 Feedwater Heater at Steady State

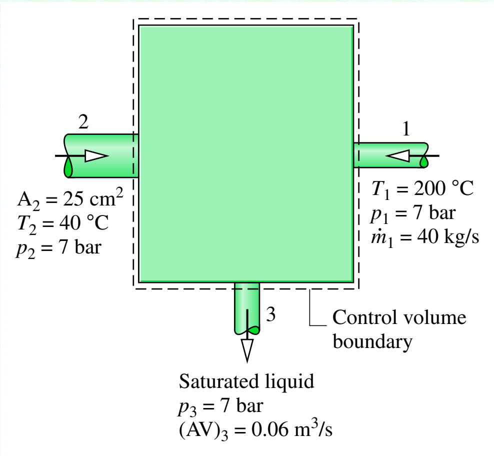
A feedwater heater operating at steady state has two inlets and one exit. At inlet 1, water vapor enters at $p_1 = 7$ bar, $T_1 = 200°C$ with a mass flow rate of 40 kg/s. At inlet 2, liquid water at $p_2 = 7$ bar, $T_2 = 40°C$ enters through an area $A_2 = 25\ \text{cm}^2$. Saturated liquid at 7 bar exits at 3 with a volumetric flow rate of 0.06 m^3/s. Determine the mass flow rates at inlet 2 and at the exit, in kg/s, and the velocity at inlet 2, in m/s.

SOLUTION

Known: A stream of water vapor mixes with a liquid water stream to produce a saturated liquid stream at the exit. The states at the inlets and exit are specified. Mass flow rate and volumetric flow rate data are given at one inlet and at the exit, respectively.

Find: Determine the mass flow rates at inlet 2 and at the exit, and the velocity V_2.

Schematic and Given Data:



◀ **Figure E4.1**

Assumption: The control volume shown on the accompanying figure is at steady state.

Analysis: The principal relations to be employed are the mass rate balance (Eq. 4.2) and the expression $\dot{m} = AV/v$ (Eq. 4.4b). At steady state the mass rate balance becomes

1 $$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 + \dot{m}_2 - \dot{m}_3$$

Solving for $\dot{m}_2$

$$\dot{m}_2 = \dot{m}_3 - \dot{m}_1$$

The mass flow rate $\dot{m}_1$ is given. The mass flow rate at the exit can be evaluated from the given volumetric flow rate

$$\dot{m}_3 = \frac{(AV)_3}{v_3}$$

where v_3 is the specific volume at the exit. In writing this expression, one-dimensional flow is assumed. From Table A-3, $v_3 = 1.108 \times 10^{-3}\ m^3/kg$. Hence

$$\dot{m}_3 = \frac{0.06\ m^3/s}{(1.108 \times 10^{-3}\ m^3/kg)} = 54.15\ kg/s$$

The mass flow rate at inlet 2 is then

$$\dot{m}_2 = \dot{m}_3 - \dot{m}_1 = 54.15 - 40 = 14.15\ kg/s$$

For one-dimensional flow at 2, $\dot{m}_2 = A_2V_2/v_2$, so

$$V_2 = \dot{m}_2 v_2 / A_2$$

State 2 is a compressed liquid. The specific volume at this state can be approximated by $v_2 \approx v_f(T_2)$ (Eq. 3.11). From Table A-2 at 40°C, $v_2 = 1.0078 \times 10^{-3}\ m^3/kg$. So

$$V_2 = \frac{(14.15\ kg/s)(1.0078 \times 10^{-3}\ m^3/kg)}{25\ cm^2}\left|\frac{10^4\ cm^2}{1\ m^2}\right| = 5.7\ m/s$$

1 At steady state the mass flow rate at the exit equals the sum of the mass flow rates at the inlets. It is left as an exercise to show that the volumetric flow rate at the exit does not equal the sum of the volumetric flow rates at the inlets.

Example 4.2 illustrates an unsteady, or *transient,* application of the mass rate balance. In this case, a barrel is filled with water.

EXAMPLE 4.2 Filling a Barrel with Water

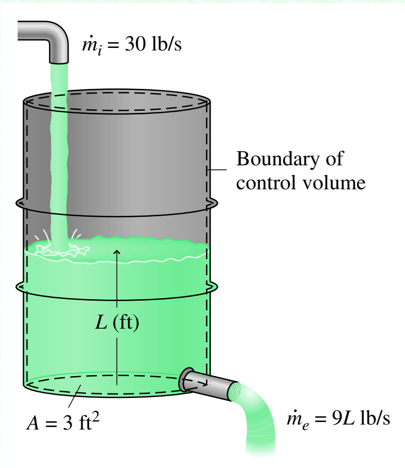
Water flows into the top of an open barrel at a constant mass flow rate of 7 kg/s. Water exits through a pipe near the base with a mass flow rate proportional to the height of liquid inside: $\dot{m}_e = 1.4\,L$, where L is the instantaneous liquid height, in m. The area of the base is 0.2 m^2, and the density of water is 1000 kg/m^3. If the barrel is initially empty, plot the variation of liquid height with time and comment on the result.

SOLUTION

Known: Water enters and exits an initially empty barrel. The mass flow rate at the inlet is constant. At the exit, the mass flow rate is proportional to the height of the liquid in the barrel.

Find: Plot the variation of liquid height with time and comment.

Schematic and Given Data:



Assumptions:

1. The control volume is defined by the dashed line on the accompanying diagram.
2. The water density is constant.

◀ **Figure E4.2*a***

Analysis: For the one-inlet, one-exit control volume, Eq. 4.2 reduces to

$$\frac{dm_{\text{cv}}}{dt} = \dot{m}_i - \dot{m}_e$$

The mass of water contained within the barrel at time t is given by

$$m_{\text{cv}}(t) = \rho A L(t)$$

where ρ is density, A is the area of the base, and $L(t)$ is the instantaneous liquid height. Substituting this into the mass rate balance together with the given mass flow rates

$$\frac{d(\rho A L)}{dt} = 7 - 1.4L$$

Since density and area are constant, this equation can be written as

$$\frac{dL}{dt} + \left(\frac{1.4}{\rho A}\right)L = \frac{7}{\rho A}$$

which is a first-order, ordinary differential equation with constant coefficients. The solution is

1 $$L = 5 + C\exp\left(-\frac{-1.4t}{\rho A}\right)$$

where C is a constant of integration. The solution can be verified by substitution into the differential equation.

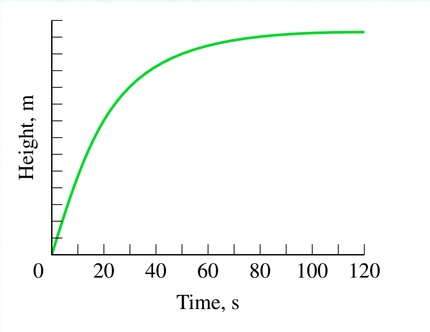
To evaluate C, use the initial condition: at $t = 0$, $L = 0$. Thus, $C = -5.0$, and the solution can be written as

$$L = 5.0[1 - \exp(-1.4t/\rho A)]$$

Substituting $\rho = 1000\ \mathrm{kg/m^2}$ and $A = 0.2\ \mathrm{m^2}$ results in

$$L = 5[1 - \exp(-0.007t)]$$

This relation can be plotted by hand or using appropriate software. The result is



◀ **Figure E4.2*b***

From the graph, we see that initially the liquid height increases rapidly and then levels out. After about 100 s, the height stays nearly constant with time. At this point, the rate of water flow into the barrel nearly equals the rate of flow out of the barrel. L → 5.

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1 Alternatively, this differential equation can be solved using *Interactive Thermodynamics: IT*. The differential equation can be expressed as

```
der(L,t) + (1.4 * L)/(rho * A) = 7/(rho * A)
rho = 1000 // kg/m^3
A = 0.2 // m^2
```

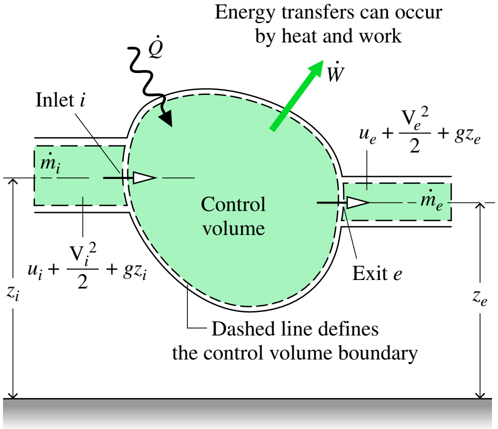
where der(L,t) is dL/dt, rho is density ρ, and A is area. Using the **Explore** button, set the initial condition at $L = 0$, and sweep t from 0 to 200 in steps of 0.5. Then, the plot can be constructed using the **Graph** button.

4.2 Conservation of Energy for a Control Volume

In this section, the rate form of the energy balance for control volumes is obtained. The energy rate balance plays an important role in subsequent sections of this book.

▶ 4.2.1 Developing the Energy Rate Balance for a Control Volume

We begin by noting that the control volume form of the energy rate balance can be derived by an approach closely paralleling that considered in the box of Sec. 4.1, where the control volume mass rate balance is obtained by transforming the closed system form. The present



◀ **Figure 4.4** Figure used to develop Eq. 4.9.

development proceeds less formally by arguing that, like mass, energy is an extensive property, so it too can be transferred into or out of a control volume as a result of mass crossing the boundary. Since this is the principal difference between the closed system and control volume forms, the control volume energy rate balance can be obtained by modifying the closed system energy rate balance to account for these energy transfers.

Accordingly, the *conservation of energy* principle applied to a control volume states:

$$\left[\begin{array}{c}\text{time } \textit{rate of change} \\ \text{of the energy} \\ \text{contained within} \\ \text{the control volume } \textit{at} \\ \textit{time t}\end{array}\right] = \left[\begin{array}{c}\text{net } \textit{rate} \text{ at which} \\ \text{energy is being} \\ \text{transferred in} \\ \text{by heat transfer} \\ \textit{at time t}\end{array}\right] - \left[\begin{array}{c}\text{net } \textit{rate} \text{ at which} \\ \text{energy is being} \\ \text{transferred out} \\ \text{by work } \textit{at} \\ \textit{time t}\end{array}\right] + \left[\begin{array}{c}\textit{net} \text{ rate of energy} \\ \text{transfer } \textit{into} \text{ the} \\ \text{control volume} \\ \text{accompanying} \\ \text{mass flow}\end{array}\right]$$

For the one-inlet one-exit control volume with one-dimensional flow shown in Fig. 4.4 the energy rate balance is

$$\frac{dE_{\text{cv}}}{dt} = \dot{Q} - \dot{W} + \underline{\dot{m}_i\left(u_i + \frac{V_i^2}{2} + gz_i\right)} - \underline{\dot{m}_e\left(u_e + \frac{V_e^2}{2} + gz_e\right)} \tag{4.9}$$

where E_{cv} denotes the energy of the control volume at time t. The terms $\dot{Q}$ and $\dot{W}$ account, respectively, for the net rate of energy transfer by heat and work across the boundary of the control volume at t. The underlined terms account for the rates of transfer of internal, kinetic, and potential energy of the entering and exiting streams. If there is no mass flow in or out, the respective mass flow rates vanish and the underlined terms of Eq. 4.9 drop out. The equation then reduces to the rate form of the energy balance for closed systems: Eq. 2.37.

EVALUATING WORK FOR A CONTROL VOLUME

Next, we will place Eq. 4.9 in an alternative form that is more convenient for subsequent applications. This will be accomplished primarily by recasting the work term $\dot{W}$, which represents the net rate of energy transfer by work across *all* portions of the boundary of the control volume.

Because work is always done on or by a control volume where matter flows across the boundary, it is convenient to separate the work term $\dot{W}$ into *two contributions:* One contribution is the work associated with the fluid pressure as mass is introduced at inlets and removed at exits. The other contribution, denoted by $\dot{W}_{cv}$, includes *all other* work effects, such as those associated with rotating shafts, displacement of the boundary, and electrical effects.

Consider the work at an exit e associated with the pressure of the flowing matter. Recall from Eq. 2.13 that the rate of energy transfer by work can be expressed as the product of a force and the velocity at the point of application of the force. Accordingly, the *rate* at which work is done at the exit by the normal force (normal to the exit area in the direction of flow) due to pressure is the product of the normal force, $p_e A_e$, and the fluid velocity, V_e. That is

$$\begin{bmatrix} \text{time rate of energy transfer} \\ \text{by work } \textit{from} \text{ the control} \\ \text{volume at exit } e \end{bmatrix} = (p_e A_e) V_e \tag{4.10}$$

where p_e is the pressure, A_e is the area, and V_e is the velocity at exit e, respectively. A similar expression can be written for the rate of energy transfer by work into the control volume at inlet i.

With these considerations, the work term $\dot{W}$ of the energy rate equation, Eq. 4.9, can be written as

$$\dot{W} = \dot{W}_{cv} + (p_e A_e) V_e - (p_i A_i) V_i \tag{4.11}$$

where, in accordance with the sign convention for work, the term at the inlet has a negative sign because energy is transferred into the control volume there. A positive sign precedes the work term at the exit because energy is transferred out of the control volume there. With $AV = \dot{m}v$ from Eq. 4.4b, the above expression for work can be written as

$$\dot{W} = \dot{W}_{cv} + \dot{m}_e(p_e v_e) - \dot{m}_i(p_i v_i) \tag{4.12}$$

where $\dot{m}_i$ and $\dot{m}_e$ are the mass flow rates and v_i and v_e are the specific volumes evaluated at the inlet and exit, respectively. In Eq. 4.12, the terms $\dot{m}_i(p_i v_i)$ and $\dot{m}_e(p_e v_e)$ account for the work associated with the pressure at the inlet and exit, respectively. They are commonly referred to as ***flow work.*** The term $\dot{W}_{cv}$ accounts for *all other* energy transfers by work across the boundary of the control volume.

flow work

▶ 4.2.2 Forms of the Control Volume Energy Rate Balance

Substituting Eq. 4.12 in Eq. 4.9 and collecting all terms referring to the inlet and the exit into separate expressions, the following form of the control volume energy rate balance results

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i\left(u_i + p_i v_i + \frac{V_i^2}{2} + gz_i\right) - \dot{m}_e\left(u_e + p_e v_e + \frac{V_e^2}{2} + gz_e\right) \tag{4.13}$$

The subscript "cv" has been added to $\dot{Q}$ to emphasize that this is the heat transfer rate over the boundary (control surface) of the *control volume*.

The last two terms of Eq. 4.13 can be rewritten using the specific enthalpy h introduced in Sec. 3.3.2. With $h = u + pv$, the energy rate balance becomes

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right) \tag{4.14}$$

The appearance of the sum $u + pv$ in the control volume energy equation is the principal reason for introducing enthalpy previously. It is brought in solely as a *convenience:* The algebraic form of the energy rate balance is simplified by the use of enthalpy and, as we have seen, enthalpy is normally tabulated along with other properties.

In practice there may be several locations on the boundary through which mass enters or exits. This can be accounted for by introducing summations as in the mass balance. Accordingly, the ***energy rate balance*** is

$$\frac{dE_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \sum_i \dot{m}_i \left(h_i + \frac{\mathrm{V}_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{\mathrm{V}_e^2}{2} + gz_e \right) \tag{4.15}$$

energy rate balance

Equation 4.15 is an *accounting* balance for the energy of the control volume. It states that the rate of energy increase or decrease within the control volume equals the difference between the rates of energy transfer in and out across the boundary. The mechanisms of energy transfer are heat and work, as for closed systems, and the energy that accompanies the mass entering and exiting.

METHODOLOGY UPDATE

Equation 4.15 is the most general form of the conservation of energy principle for control volumes used in this book. It serves as the starting point for applying the conservation of energy principle to control volumes in problem solving.

OTHER FORMS

As for the case of the mass rate balance, the energy rate balance can be expressed in terms of local properties to obtain forms that are more generally applicable. Thus, the term $E_{\text{cv}}(t)$, representing the total energy associated with the control volume at time t, can be written as a volume integral

$$E_{\text{cv}}(t) = \int_V \rho e \, dV = \int_V \rho \left(u + \frac{\mathrm{V}^2}{2} + gz \right) dV \tag{4.16}$$

Similarly, the terms accounting for the energy transfers accompanying mass flow and flow work at inlets and exits can be expressed as shown in the following form of the energy rate balance

$$\begin{aligned} \frac{d}{dt} \int_V \rho e \, dV = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} &+ \sum_i \left[\int_{\mathrm{A}} \left(h + \frac{\mathrm{V}^2}{2} + gz \right) \rho \mathrm{V}_{\mathrm{n}} \, d\mathrm{A} \right]_i \\ &- \sum_e \left[\int_{\mathrm{A}} \left(h + \frac{\mathrm{V}^2}{2} + gz \right) \rho \mathrm{V}_{\mathrm{n}} \, d\mathrm{A} \right]_e \end{aligned} \tag{4.17}$$

Additional forms of the energy rate balance can be obtained by expressing the heat transfer rate $\dot{Q}_{\text{cv}}$ as the integral of the *heat flux* over the boundary of the control volume, and the work $\dot{W}_{\text{cv}}$ in terms of normal and shear stresses at the moving portions of the boundary.

In principle, the change in the energy of a control volume over a time period can be obtained by integration of the energy rate balance with respect to time. Such integrations require information about the time dependences of the work and heat transfer rates, the various mass flow rates, and the states at which mass enters and leaves the control volume. Examples of this type of analysis are presented in Sec. 4.4. In Sec. 4.3 to follow, we consider forms that the mass and energy rate balances take for control volumes at steady state, for these are frequently used in practice.

4.3 Analyzing Control Volumes at Steady State

In this section steady-state forms of the mass and energy rate balances are developed and applied to a variety of cases of engineering interest. The steady-state forms obtained do not apply to the transient startup or shutdown periods of operation of such devices, but only to periods of steady operation. This situation is commonly encountered in engineering.

4.3.1 Steady-State Forms of the Mass and Energy Rate Balances

For a control volume at steady state, the conditions of the mass within the control volume and at the boundary do not vary with time. The mass flow rates and the rates of energy transfer by heat and work are also constant with time. There can be no accumulation of mass within the control volume, so $dm_{cv}/dt = 0$ and the mass rate balance, Eq. 4.2, takes the form

$$\underset{\text{(mass rate in)}}{\sum_i \dot{m}_i} = \underset{\text{(mass rate out)}}{\sum_e \dot{m}_e} \tag{4.18}$$

Furthermore, at steady state $dE_{cv}/dt = 0$, so Eq. 4.15 can be written as

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right) \tag{4.19a}$$

Alternatively

$$\underbrace{\dot{Q}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right)}_{\text{(energy rate in)}} = \underbrace{\dot{W}_{cv} + \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)}_{\text{(energy rate out)}} \tag{4.19b}$$

Equation 4.18 asserts that at steady state the total rate at which mass enters the control volume equals the total rate at which mass exits. Similarly, Eqs. 4.19 assert that the total rate at which energy is transferred into the control volume equals the total rate at which energy is transferred out.

Many important applications involve one-inlet, one-exit control volumes at steady state. It is instructive to apply the mass and energy rate balances to this special case. The mass rate balance reduces simply to $\dot{m}_1 = \dot{m}_2$. That is, the mass flow rate must be the same at the exit, 2, as it is at the inlet, 1. The common mass flow rate is designated simply by $\dot{m}$. Next, applying the energy rate balance and factoring the mass flow rate gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right] \tag{4.20a}$$

Or, dividing by the mass flow rate

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \tag{4.20b}$$

The enthalpy, kinetic energy, and potential energy terms all appear in Eqs. 4.20 as *differences* between their values at the inlet and exit. This illustrates that the datums used to assign values to specific enthalpy, velocity, and elevation cancel, provided the same ones are used at the inlet and exit. In Eq. 4.20b, the ratios $\dot{Q}_{cv}/\dot{m}$ and $\dot{W}_{cv}/\dot{m}$ are rates of energy transfer *per unit mass flowing through the control volume*.

The foregoing steady-state forms of the energy rate balance relate only energy transfer quantities evaluated at the *boundary* of the control volume. No details concerning properties *within* the control volume are required by, or can be determined with, these equations. When applying the energy rate balance in any of its forms, it is necessary to use the same units for all terms in the equation. For instance, *every* term in Eq. 4.20b must have a unit such as kJ/kg or Btu/lb. Appropriate unit conversions are emphasized in examples to follow.

▶ 4.3.2 Modeling Control Volumes at Steady State

In this section, we provide the basis for subsequent applications by considering the modeling of control volumes *at steady state*. In particular, several examples are given in Sec. 4.3.3 showing the use of the principles of conservation of mass and energy, together with relationships among properties for the analysis of control volumes at steady state. The examples are drawn from applications of general interest to engineers and are chosen to illustrate points common to all such analyses. Before studying them, it is recommended that you review the methodology for problem solving outlined in Sec. 1.7.3. As problems become more complicated, the use of a systematic problem-solving approach becomes increasingly important.

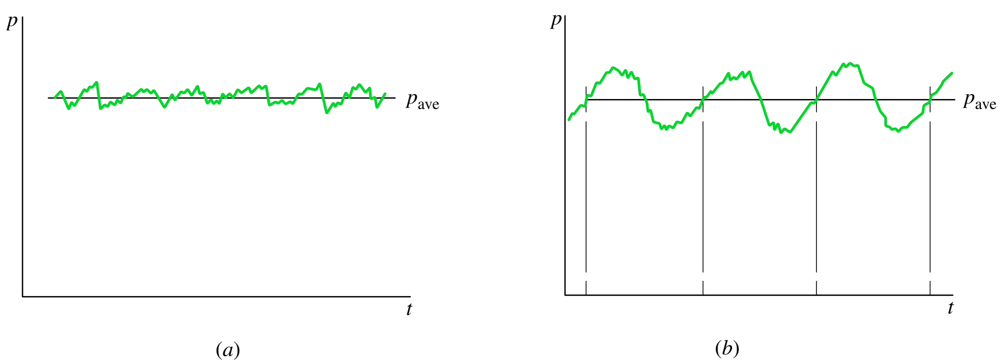
When the mass and energy rate balances are applied to a control volume, simplifications are normally needed to make the analysis manageable. That is, the control volume of interest is *modeled* by making assumptions. The *careful* and *conscious* step of listing assumptions is necessary in every engineering analysis. Therefore, an important part of this section is devoted to considering various assumptions that are commonly made when applying the conservation principles to different types of devices. As you study the examples presented in Sec. 4.3.3, it is important to recognize the role played by careful assumption making in arriving at solutions. In each case considered, steady-state operation is assumed. The flow is regarded as one-dimensional at places where mass enters and exits the control volume. Also, at each of these locations equilibrium property relations are assumed to apply.

In several of the examples to follow, the heat transfer term $\dot{Q}_{cv}$ is set to zero in the energy rate balance because it is small relative to other energy transfers across the boundary. This may be the result of one or more of the following factors:

- ▶ The outer surface of the control volume is well insulated.
- ▶ The outer surface area is too small for there to be effective heat transfer.
- ▶ The temperature difference between the control volume and its surroundings is so small that the heat transfer can be ignored.
- ▶ The gas or liquid passes through the control volume so quickly that there is not enough time for significant heat transfer to occur.

The work term $\dot{W}_{cv}$ drops out of the energy rate balance when there are no rotating shafts, displacements of the boundary, electrical effects, or other work mechanisms associated with the control volume being considered. The kinetic and potential energies of the matter entering and exiting the control volume are neglected when they are small relative to other energy transfers.

In practice, the properties of control volumes considered to be at steady state do vary with time. The steady-state assumption would still apply, however, when properties fluctuate only slightly about their averages, as for pressure in Fig. 4.5*a*. Steady state also might be assumed in cases where *periodic* time variations are observed, as in Fig. 4.5*b*. For example, in



▲ **Figure 4.5** Pressure variations about an average. (*a*) Fluctuation. (*b*) Periodic.

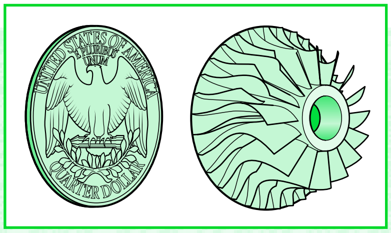
Thermodynamics in the News...

Smaller *Can* Be Better

Engineers are developing miniature systems for use where weight, portability, and/or compactness are critically important. Some of these applications involve tiny *micro systems* with dimensions in the micrometer to millimeter range. Other somewhat larger *meso-scale* systems can measure up to a few centimeters.

Microelectromechanical systems (*MEMS*) combining electrical and mechanical features are now widely used for sensing and control. Medical applications of MEMS include minute pressure sensors that monitor pressure within the balloon inserted into a blood vessel during angioplasty. Air bags are triggered in an automobile crash by tiny acceleration sensors. MEMS are also found in computer hard drives and printers.

Miniature versions of other technologies are being investigated. One study aims at developing an entire gas turbine power plant the size of a shirt button. Another involves micromotors with shafts the diameter of a human hair. Emergency workers wearing fire-, chemical-, or biological-protection suits might in the future be kept cool by tiny heat pumps imbedded in the suit material.



Engineers report that size reductions cannot go on indefinitely. As designers aim at smaller sizes, frictional effects and uncontrolled heat transfers pose significant challenges. Fabrication of miniature systems is also demanding. Taking a design from the concept stage to high-volume production can be both expensive and risky, industry representatives say.

reciprocating engines and compressors, the entering and exiting flows pulsate as valves open and close. Other parameters also might be time varying. However, the steady-state assumption can apply to control volumes enclosing these devices if the following are satisfied for each successive period of operation: (1) There is no *net* change in the total energy and the total mass within the control volume. (2) The *time-averaged* mass flow rates, heat transfer rates, work rates, and properties of the substances crossing the control surface all remain constant.

▶ 4.3.3 Illustrations

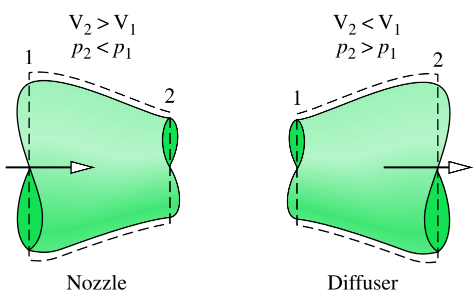
In this section, we present brief discussions and examples illustrating the analysis of several devices of interest in engineering, including nozzles and diffusers, turbines, compressors and pumps, heat exchangers, and throttling devices. The discussions highlight some common applications of each device and the important modeling assumptions used in thermodynamic analysis. The section also considers system integration, in which devices are combined to form an overall system serving a particular purpose.

NOZZLES AND DIFFUSERS

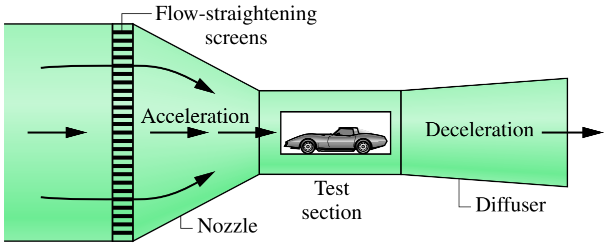
nozzle

diffuser

A ***nozzle*** is a flow passage of varying cross-sectional area in which the velocity of a gas or liquid increases in the direction of flow. In a ***diffuser,*** the gas or liquid decelerates in the direction of flow. Figure 4.6 shows a nozzle in which the cross-sectional area decreases



◀ **Figure 4.6** Illustration of a nozzle and a diffuser.



◀ **Figure 4.7** Wind-tunnel test facility.

in the direction of flow and a diffuser in which the walls of the flow passage diverge. In Fig. 4.7, a nozzle and diffuser are combined in a wind-tunnel test facility. Nozzles and diffusers for high-speed gas flows formed from a converging section followed by diverging section are studied in Sec. 9.13.

For nozzles and diffusers, the only work is *flow work* at locations where mass enters and exits the control volume, so the term $\dot{W}_{cv}$ drops out of the energy rate equation for these devices. The change in potential energy from inlet to exit is negligible under most conditions. At steady state the mass and energy rate balances reduce, respectively, to

$$\cancelto{0}{\frac{dm_{cv}}{dt}} = \dot{m}_1 - \dot{m}_2$$

$$\cancelto{0}{\frac{dE_{cv}}{dt}} = \dot{Q}_{cv} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) - \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right)$$

where 1 denotes the inlet and 2 the exit. By combining these into a single expression and dropping the potential energy change from inlet to exit

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}} + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) \tag{4.21}$$

where $\dot{m}$ is the mass flow rate. The term $\dot{Q}_{cv}/\dot{m}$ representing heat transfer with the surroundings per unit of mass flowing through the nozzle or diffuser is often small enough relative to the enthalpy and kinetic energy changes that it can be dropped, as in the next example.

EXAMPLE 4.3 Calculating Exit Area of a Steam Nozzle

Steam enters a converging–diverging nozzle operating at steady state with $p_1 = 40$ bar, $T_1 = 400°C$, and a velocity of 10 m/s. The steam flows through the nozzle with negligible heat transfer and no significant change in potential energy. At the exit, $p_2 = 15$ bar, and the velocity is 665 m/s. The mass flow rate is 2 kg/s. Determine the exit area of the nozzle, in m^2.

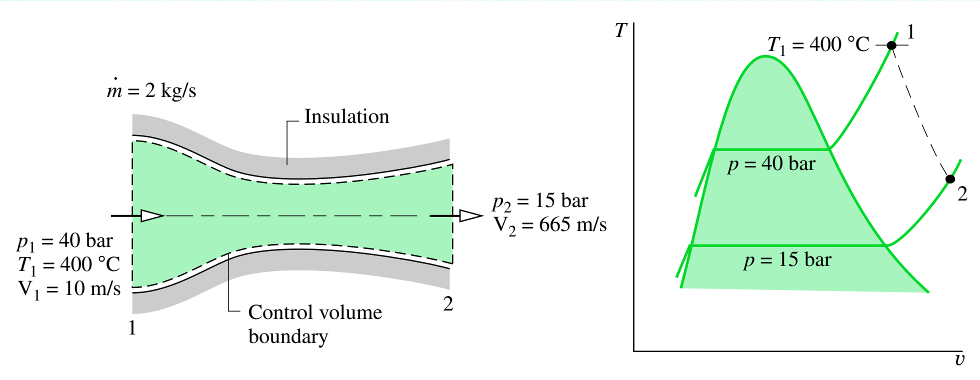
SOLUTION

Known: Steam flows at steady state through a nozzle with known properties at the inlet and exit, a known mass flow rate, and negligible effects of heat transfer and potential energy.

Find: Determine the exit area.

Schematic and Given Data:

❶



◀ **Figure E4.3**

Assumptions:

1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer is negligible and $\dot{W}_{cv} = 0$.
3. The change in potential energy from inlet to exit can be neglected.

Analysis: The exit area can be determined from the mass flow rate $\dot{m}$ and Eq. 4.4b, which can be arranged to read

$$A_2 = \frac{\dot{m}v_2}{V_2}$$

To evaluate A_2 from this equation requires the specific volume v_2 at the exit, and this requires that the exit state be fixed.

The state at the exit is fixed by the values of two independent intensive properties. One is the pressure p_2, which is known. The other is the specific enthalpy h_2, determined from the steady-state energy rate balance

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) - \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right)$$

where $\dot{Q}_{cv}$ and $\dot{W}_{cv}$ are deleted by assumption 2. The change in specific potential energy drops out in accordance with assumption 3 and $\dot{m}$ cancels, leaving

$$0 = (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

Solving for h_2

$$h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

From Table A-4, $h_1 = 3213.6$ kJ/kg. The velocities V_1 and V_2 are given. Inserting values and converting the units of the kinetic energy terms to kJ/kg results in

❷

$$\begin{aligned} h_2 &= 3213.6 \text{ kJ/kg} + \left[\frac{(10)^2 - (665)^2}{2}\right]\left(\frac{\text{m}^2}{\text{s}^2}\right)\left|\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}}\right| \\ &= 3213.6 - 221.1 = 2992.5 \text{ kJ/kg} \end{aligned}$$

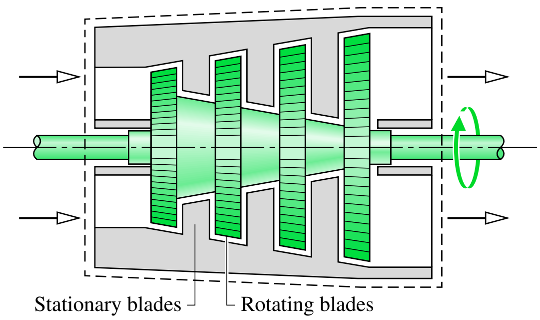
Finally, referring to Table A-4 at $p_2 = 15$ bar with $h_2 = 2992.5$ kJ/kg, the specific volume at the exit is $v_2 = 0.1627$ m^3/kg. The exit area is then

❸
$$A_2 = \frac{(2 \text{ kg/s})(0.1627 \text{ m}^3\text{/kg})}{665 \text{ m/s}} = 4.89 \times 10^{-4} \text{ m}^2$$

❶ Although equilibrium property relations apply at the inlet and exit of the control volume, the intervening states of the steam are not necessarily equilibrium states. Accordingly, the expansion through the nozzle is represented on the T–v diagram as a dashed line.

❷ Care must be taken in converting the units for specific kinetic energy to kJ/kg.

❸ The area at the nozzle inlet can be found similarly, using $A_1 = \dot{m}v_1/V_1$.

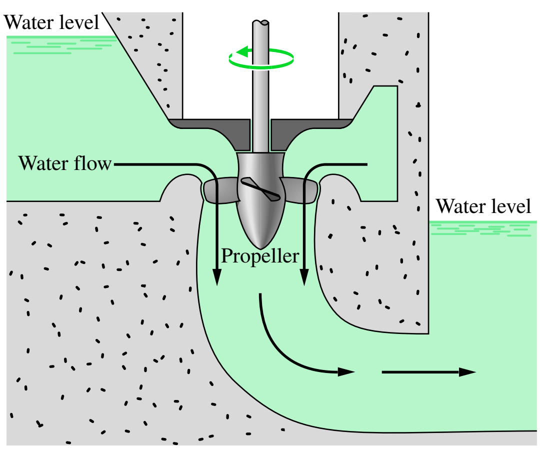


◀ **Figure 4.8** Schematic of an axial-flow turbine.

TURBINES

A ***turbine*** is a device in which work is developed as a result of a gas or liquid passing through a set of blades attached to a shaft free to rotate. A schematic of an axial-flow steam or gas turbine is shown in Fig. 4.8. Turbines are widely used in vapor power plants, gas turbine power plants, and aircraft engines (Chaps. 8 and 9). In these applications, superheated steam or a gas enters the turbine and expands to a lower exit pressure as work is developed. A hydraulic turbine installed in a dam is shown in Fig. 4.9. In this application, water falling through the propeller causes the shaft to rotate and work is developed.

turbine



◀ **Figure 4.9** Hydraulic turbine installed in a dam.

For a turbine at steady state the mass and energy rate balances reduce to give Eq. 4.20b. When gases are under consideration, the potential energy change is typically negligible. With a proper selection of the boundary of the control volume enclosing the turbine, the kinetic energy change is usually small enough to be neglected. The only heat transfer between the turbine and surroundings would be unavoidable heat transfer, and as illustrated in the next example this is often small relative to the work and enthalpy terms.

EXAMPLE 4.4 Calculating Heat Transfer from a Steam Turbine

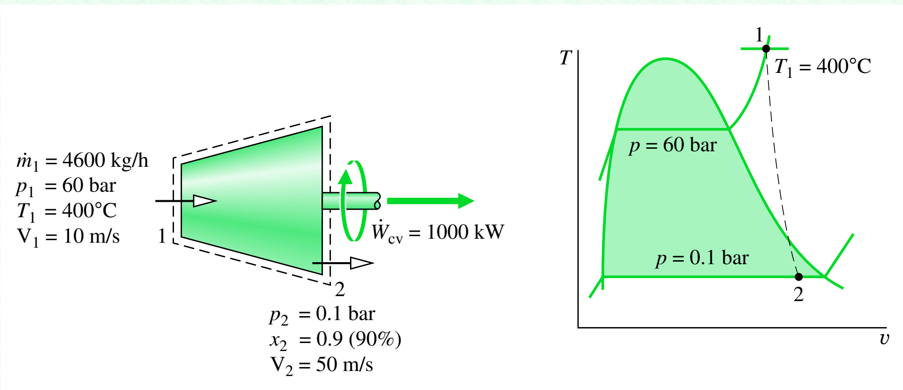
Steam enters a turbine operating at steady state with a mass flow rate of 4600 kg/h. The turbine develops a power output of 1000 kW. At the inlet, the pressure is 60 bar, the temperature is 400°C, and the velocity is 10 m/s. At the exit, the pressure is 0.1 bar, the quality is 0.9 (90%), and the velocity is 50 m/s. Calculate the rate of heat transfer between the turbine and surroundings, in kW.

SOLUTION

Known: A steam turbine operates at steady state. The mass flow rate, power output, and states of the steam at the inlet and exit are known.

Find: Calculate the rate of heat transfer.

Schematic and Given Data:



◀ **Figure E4.4**

Assumptions:

1. The control volume shown on the accompanying figure is at steady state.
2. The change in potential energy from inlet to exit can be neglected.

Analysis: To calculate the heat transfer rate, begin with the one-inlet, one-exit form of the energy rate balance for a control volume at steady state

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) - \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right)$$

where $\dot{m}$ is the mass flow rate. Solving for $\dot{Q}_{cv}$ and dropping the potential energy change from inlet to exit

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}\left[(h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)\right]$$

To compare the magnitudes of the enthalpy and kinetic energy terms, and stress the unit conversions needed, each of these terms is evaluated separately.

First, the specific *enthalpy difference* $h_2 - h_1$ is found. Using Table A-4, $h_1 = 3177.2$ kJ/kg. State 2 is a two-phase liquid–vapor mixture, so with data from Table A-3 and the given quality

$$h_2 = h_{f2} + x_2(h_{g2} - h_{f2})$$
$$= 191.83 + (0.9)(2392.8) = 2345.4 \text{ kJ/kg}$$

Hence

$$h_2 - h_1 = 2345.4 - 3177.2 = -831.8 \text{ kJ/kg}$$

Consider next the specific *kinetic energy difference*. Using the given values for the velocities

❶ $$\left(\frac{V_2^2 - V_1^2}{2}\right) = \left[\frac{(50)^2 - (10)^2}{2}\right]\left(\frac{\text{m}^2}{\text{s}^2}\right)\left|\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}}\right|$$
$$= 1.2 \text{ kJ/kg}$$

Calculating $\dot{Q}_{cv}$ from the above expression

❷ $$\dot{Q}_{cv} = (1000 \text{ kW}) + \left(4600 \frac{\text{kg}}{\text{h}}\right)(-831.8 + 1.2)\left(\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1 \text{ h}}{3600 \text{ s}}\right|\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$
$$= -61.3 \text{ kW}$$

❶ The magnitude of the change in specific kinetic energy from inlet to exit is much smaller than the specific enthalpy change.

❷ The negative value of $\dot{Q}_{cv}$ means that there is heat transfer from the turbine to its surroundings, as would be expected. The magnitude of $\dot{Q}_{cv}$ is small relative to the power developed.

COMPRESSORS AND PUMPS

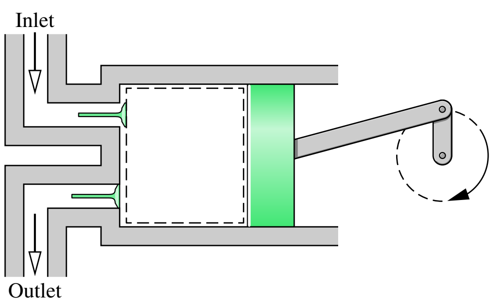
Compressors are devices in which work is done on a *gas* passing through them in order to raise the pressure. In ***pumps,*** the work input is used to change the state of a *liquid* passing through. A reciprocating compressor is shown in Fig. 4.10. Figure 4.11 gives schematic diagrams of three different rotating compressors: an axial-flow compressor, a centrifugal compressor, and a Roots type.

compressor

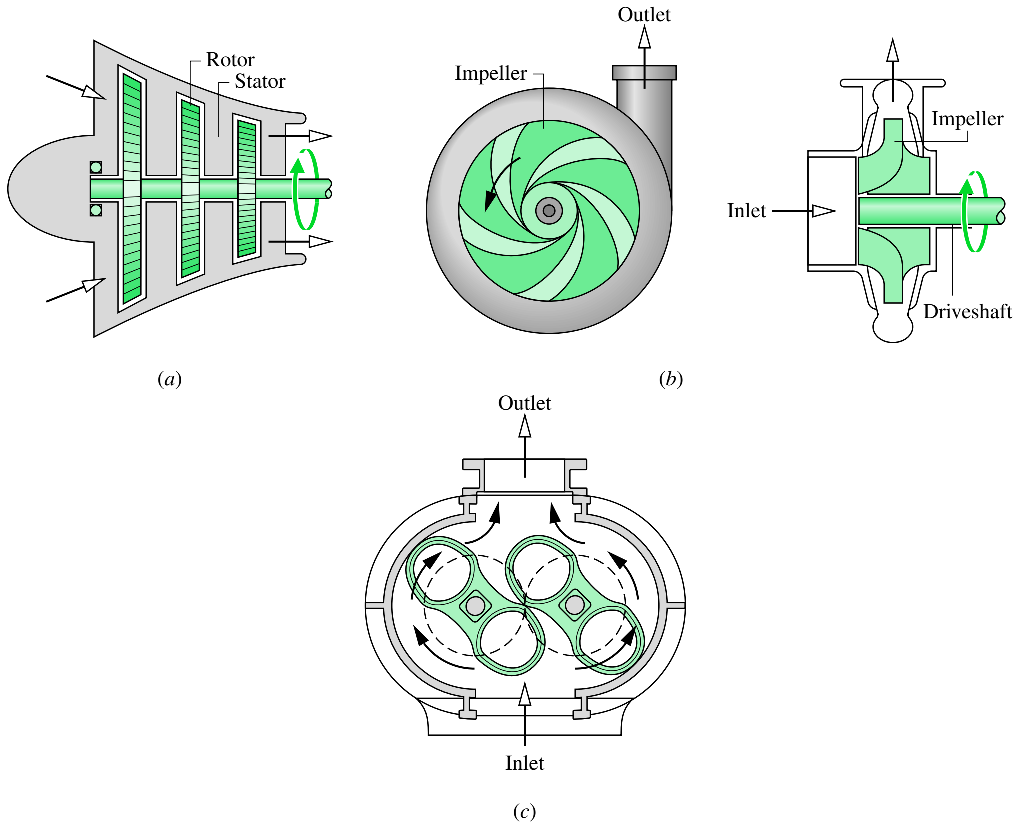
pump

The mass and energy rate balances reduce for compressors and pumps at steady state, as for the case of turbines considered previously. For compressors, the changes in specific kinetic and potential energies from inlet to exit are often small relative to the work done per unit of mass passing through the device. Heat transfer with the surroundings is frequently a secondary effect in both compressors and pumps.

The next two examples illustrate, respectively, the analysis of an air compressor and a power washer. In each case the objective is to determine the power required to operate the device.



◀ **Figure 4.10** Reciprocating compressor.



▲ **Figure 4.11** Rotating compressors. (*a*) Axial flow. (*b*) Centrifugal. (*c*) Roots type.

EXAMPLE 4.5 Calculating Compressor Power

Air enters a compressor operating at steady state at a pressure of 1 bar, a temperature of 290 K, and a velocity of 6 m/s through an inlet with an area of 0.1 m^2. At the exit, the pressure is 7 bar, the temperature is 450 K, and the velocity is 2 m/s. Heat transfer from the compressor to its surroundings occurs at a rate of 180 kJ/min. Employing the ideal gas model, calculate the power input to the compressor, in kW.

SOLUTION

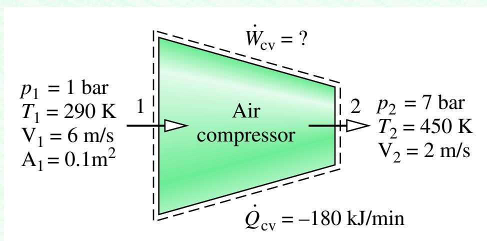
Known: An air compressor operates at steady state with known inlet and exit states and a known heat transfer rate.

Find: Calculate the power required by the compressor.

Schematic and Given Data:

Assumptions:

1. The control volume shown on the accompanying figure is at steady state.
2. The change in potential energy from inlet to exit can be neglected.
3. The ideal gas model applies for the air.



◄ **Figure E4.5**

Analysis: To calculate the power input to the compressor, begin with the energy rate balance for the one-inlet, one-exit control volume at steady state:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) - \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right)$$

Solving

$$\dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\right]$$

The change in potential energy from inlet to exit drops out by assumption 2.

The mass flow rate $\dot{m}$ can be evaluated with given data at the inlet and the ideal gas equation of state.

$$\dot{m} = \frac{A_1V_1}{v_1} = \frac{A_1V_1p_1}{(\bar{R}/M)T_1} = \frac{(0.1\text{ m}^2)(6\text{ m/s})(10^5\text{ N/m}^2)}{\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(290\text{ K})} = 0.72\text{ kg/s}$$

The specific enthalpies h_1 and h_2 can be found from Table A-22. At 290 K, $h_1 = 290.16$ kJ/kg. At 450 K, $h_2 = 451.8$ kJ/kg. Substituting values into the expression for $\dot{W}_{cv}$

$$\dot{W}_{cv} = \left(-180\frac{\text{kJ}}{\text{min}}\right)\left|\frac{1\text{ min}}{60\text{ s}}\right| + 0.72\frac{\text{kg}}{\text{s}}\left[(290.16 - 451.8)\frac{\text{kJ}}{\text{kg}}\right.$$
$$\left. + \left(\frac{(6)^2 - (2)^2}{2}\right)\left(\frac{\text{m}^2}{\text{s}^2}\right)\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\right]$$

2 $$= -3\frac{\text{kJ}}{\text{s}} + 0.72\frac{\text{kg}}{\text{s}}(-161.64 + 0.02)\frac{\text{kJ}}{\text{kg}}$$

3 $$= -119.4\frac{\text{kJ}}{\text{s}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = -119.4\text{ kW}$$

1 The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart.

2 The contribution of the kinetic energy is negligible in this case. Also, the heat transfer rate is seen to be small relative to the power input.

3 In this example $\dot{Q}_{cv}$ and $\dot{W}_{cv}$ have negative values, indicating that the direction of the heat transfer is *from* the compressor and work is done *on* the air passing through the compressor. The magnitude of the power *input* to the compressor is 119.4 kW.

EXAMPLE 4.6 Power Washer

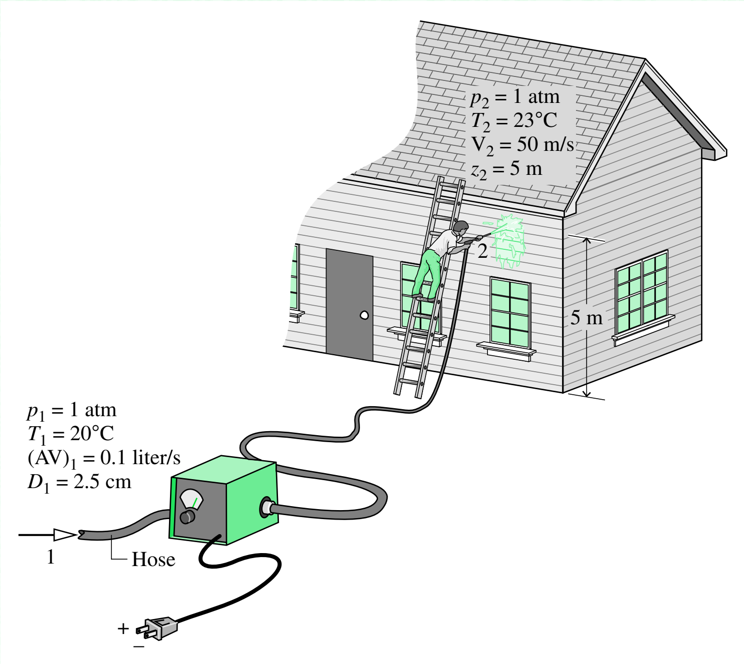
A power washer is being used to clean the siding of a house. Water enters at 20°C, 1 atm, with a volumetric flow rate of 0.1 liter/s through a 2.5-cm-diameter hose. A jet of water exits at 23°C, 1 atm, with a velocity of 50 m/s at an elevation of 5 m. At steady state, the magnitude of the heat transfer rate *from* the power unit *to* the surroundings is 10% of the power input. The water can be considered incompressible, and $g = 9.81$ m/s^2. Determine the power input to the motor, in kW.

SOLUTION

Known: A power washer operates at steady state with known inlet and exit conditions. The heat transfer rate is known as a percentage of the power input.

Find: Determine the power input.

Schematic and Given Data:



Assumptions:

1. A control volume enclosing the power unit and the delivery hose is at steady state.
2. The water is modeled as incompressible.

◄ **Figure E4.6**

Analysis: To calculate the power input, begin with the one-inlet, one-exit form of the energy balance for a control volume at steady state

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

❶ Introducing $\dot{Q}_{cv} = (0.1)\dot{W}_{cv}$, and solving for $\dot{W}_{cv}$

$$\dot{W}_{cv} = \frac{\dot{m}}{0.9}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

The mass flow rate $\dot{m}$ can be evaluated using the given volumetric flow rate and $v \approx v_f(20°C) = 1.0018 \times 10^{-3}\ m^3/kg$ from Table A-2, as follows

$$\begin{aligned}\dot{m} &= (AV)_1/v \\ &= (0.1\ L/s)/(1.0018 \times 10^{-3}\ m^3/kg)\left|\frac{10^{-3}\ m^3}{1\ L}\right| \\ &= 0.1\ kg/s\end{aligned}$$

❷ Dividing the given volumetric flow rate by the inlet area, the inlet velocity is $V_1 = 0.2$ m/s.

The specific enthalpy term is evaluated using Eq. 3.20b, with $p_1 = p_2 = 1$ atm and $c = 4.18$ kJ/kg · K from Table A-19

$$\begin{aligned}h_1 - h_2 &= c(T_1 - T_2) + v\cancel{(p_1 - p_2)}^{0} \\ &= (4.18\ kJ/kg \cdot K)(-3\ K) = -12.54\ kJ/kg\end{aligned}$$

Evaluating the specific kinetic energy term

$$\frac{V_1^2 - V_2^2}{2} = \frac{[(0.2)^2 - (50)^2]\left(\frac{m}{s}\right)^2}{2}\left|\frac{1\ N}{1\ kg \cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N \cdot m}\right| = -1.25\ kJ/kg$$

Finally, the specific potential energy term is

$$g(z_1 - z_2) = (9.81\ \text{m/s}^2)(0 - 5)\text{m}\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| = -0.05\ \text{kJ/kg}$$

Inserting values

3 $$\dot{W}_{cv} = \frac{(0.1\ \text{kg/s})}{0.9}[(-12.54) + (-1.25) + (-0.05)]\left(\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

Thus

$$\dot{W}_{cv} = -1.54\ \text{kW}$$

where the minus sign indicates that power is provided to the washer.

1 Since power is required to operate the washer, $\dot{W}_{cv}$ is negative in accord with our sign convention. The energy transfer by heat is from the control volume to the surroundings, and thus $\dot{Q}_{cv}$ is negative as well. Using the value of $\dot{W}_{cv}$ found below, $\dot{Q}_{cv} = (0.1)\dot{W}_{cv} = -0.154$ kW.

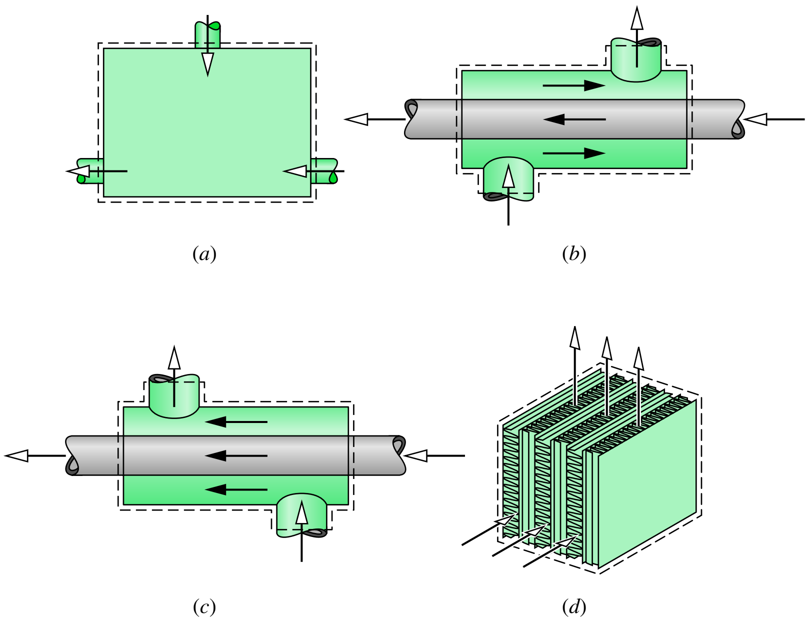
2 The power washer develops a high-velocity jet of water at the exit. The inlet velocity is small by comparison.

3 The power input to the washer is accounted for by heat transfer from the washer to the surroundings and the increases in specific enthalpy, kinetic energy, and potential energy of the water as it is pumped through the power washer.

HEAT EXCHANGERS

Devices that transfer energy between fluids at different temperatures by heat transfer modes such as discussed in Sec. 2.4.2 are called ***heat exchangers***. One common type of heat exchanger is a vessel in which hot and cold streams are mixed directly as shown in Fig. 4.12*a*. An open feedwater heater is an example of this type of device. Another common type of heat

heat exchanger



▲ **Figure 4.12** Common heat exchanger types. (*a*) Direct contact heat exchanger. (*b*) Tube-within-a-tube counterflow heat exchanger. (*c*) Tube-within-a-tube parallel flow heat exchanger. (*d*) Cross-flow heat exchanger.

exchanger is one in which a gas or liquid is *separated* from another gas or liquid by a wall through which energy is conducted. These heat exchangers, known as recuperators, take many different forms. Counterflow and parallel tube-within-a-tube configurations are shown in Figs. 4.12*b* and 4.12*c*, respectively. Other configurations include cross-flow, as in automobile radiators, and multiple-pass shell-and-tube condensers and evaporators. Figure 4.12*d* illustrates a cross-flow heat exchanger.

The only work interaction at the boundary of a control volume enclosing a heat exchanger is flow work at the places where matter enters and exits, so the term $\dot{W}_{cv}$ of the energy rate balance can be set to zero. Although high rates of energy transfer may be achieved from stream to stream, the heat transfer from the outer surface of the heat exchanger to the surroundings is often small enough to be neglected. In addition, the kinetic and potential energies of the flowing streams can often be ignored at the inlets and exits.

The next example illustrates how the mass and energy rate balances are applied to a condenser at steady state. Condensers are commonly found in power plants and refrigeration systems.

EXAMPLE 4.7 Power Plant Condenser

Steam enters the condenser of a vapor power plant at 0.1 bar with a quality of 0.95 and condensate exits at 0.1 bar and 45°C. Cooling water enters the condenser in a separate stream as a liquid at 20°C and exits as a liquid at 35°C with no change in pressure. Heat transfer from the outside of the condenser and changes in the kinetic and potential energies of the flowing streams can be ignored. For steady-state operation, determine

(a) the ratio of the mass flow rate of the cooling water to the mass flow rate of the condensing stream.

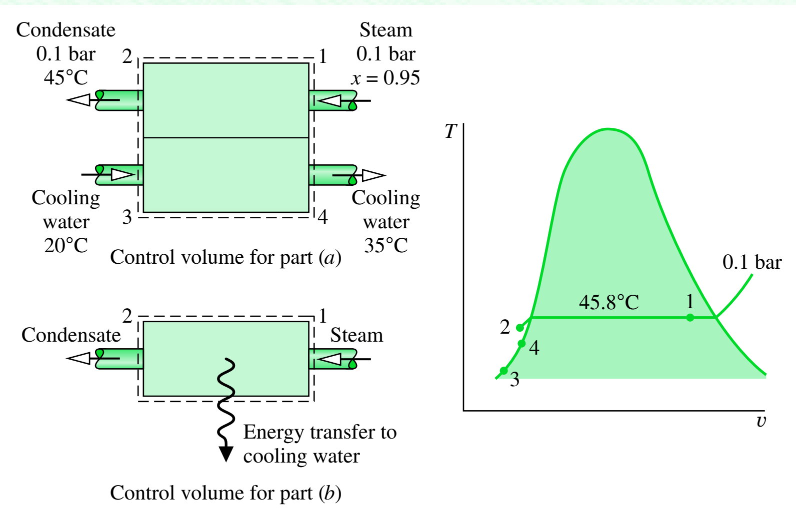
(b) the rate of energy transfer from the condensing steam to the cooling water, in kJ per kg of steam passing through the condenser.

SOLUTION

Known: Steam is condensed at steady state by interacting with a separate liquid water stream.

Find: Determine the ratio of the mass flow rate of the cooling water to the mass flow rate of the steam and the rate of energy transfer from the steam to the cooling water.

Schematic and Given Data:



◀ **Figure E4.7**

Assumptions:

1. Each of the two control volumes shown on the accompanying sketch is at steady-state.
2. There is no significant heat transfer between the overall condenser and its surroundings, and $\dot{W}_{cv} = 0$.
3. Changes in the kinetic and potential energies of the flowing streams from inlet to exit can be ignored.
4. At states 2, 3, and 4, $h \approx h_f(T)$ (see Eq. 3.14).

Analysis: The steam and the cooling water streams do not mix. Thus, the mass rate balances for each of the two streams reduce at steady state to give

$$\dot{m}_1 = \dot{m}_2 \qquad \text{and} \qquad \dot{m}_3 = \dot{m}_4$$

(a) The ratio of the mass flow rate of the cooling water to the mass flow rate of the condensing steam, $\dot{m}_3/\dot{m}_1$, can be found from the steady-state form of the energy rate balance applied to the overall condenser as follows:

$$0 = \underline{\dot{Q}_{cv}} - \underline{\dot{W}_{cv}} + \dot{m}_1\left(h_1 + \underline{\frac{V_1^2}{2} + gz_1}\right) + \dot{m}_3\left(h_3 + \underline{\frac{V_3^2}{2} + gz_3}\right)$$
$$- \dot{m}_2\left(h_2 + \underline{\frac{V_2^2}{2} + gz_2}\right) - \dot{m}_4\left(h_4 + \underline{\frac{V_4^2}{2} + gz_4}\right)$$

The underlined terms drop out by assumptions 2 and 3. With these simplifications, together with the above mass flow rate relations, the energy rate balance becomes simply

$$0 = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$$

Solving, we get

$$\frac{\dot{m}_3}{\dot{m}_1} = \frac{h_1 - h_2}{h_4 - h_3}$$

The specific enthalpy h_1 can be determined using the given quality and data from Table A-3. From Table A-3 at 0.1 bar, h_f = 191.83 kJ/kg and h_g = 2584.7 kJ/kg, so

$$h_1 = 191.83 + 0.95(2584.7 - 191.83) = 2465.1 \text{ kJ/kg}$$

1 Using assumption 4, the specific enthalpy at 2 is given by $h_2 \approx h_f(T_2) = 188.45$ kJ/kg. Similarly, $h_3 \approx h_f(T_3)$ and $h_4 \approx h_f(T_4)$, giving $h_4 - h_3 = 62.7$ kJ/kg. Thus

$$\frac{\dot{m}_3}{\dot{m}_1} = \frac{2465.1 - 188.45}{62.7} = 36.3$$

(b) For a control volume enclosing the steam side of the condenser only, the steady-state form of energy rate balance is

2 $$0 = \dot{Q}_{cv} - \underline{\dot{W}_{cv}} + \dot{m}_1\left(h_1 + \underline{\frac{V_1^2}{2} + gz_1}\right) - \dot{m}_2\left(h_2 + \underline{\frac{V_2^2}{2} + gz_2}\right)$$

The underlined terms drop out by assumptions 2 and 3. Combining this equation with $\dot{m}_1 = \dot{m}_2$, the following expression for the rate of energy transfer between the condensing steam and the cooling water results:

$$\dot{Q}_{cv} = \dot{m}_1(h_2 - h_1)$$

Dividing by the mass flow rate of the steam, $\dot{m}_1$, and inserting values

$$\frac{\dot{Q}_{cv}}{\dot{m}_1} = h_2 - h_1 = 188.45 - 2465.1 = -2276.7 \text{ kJ/kg}$$

where the minus sign signifies that energy is transferred *from* the condensing steam *to* the cooling water.

1 Alternatively, $(h_4 - h_3)$ can be evaluated using the incompressible liquid model via Eq. 3.20b.

2 Depending on where the boundary of the control volume is located, two different formulations of the energy rate balance are obtained. In part (a), both streams are included in the control volume. Energy transfer between them occurs internally and not across the boundary of the control volume, so the term $\dot{Q}_{cv}$ drops out of the energy rate balance. With the control volume of part (b), however, the term $\dot{Q}_{cv}$ must be included.

Excessive temperatures in electronic components are avoided by providing appropriate cooling, as illustrated in the next example.

EXAMPLE 4.8 Cooling Computer Components

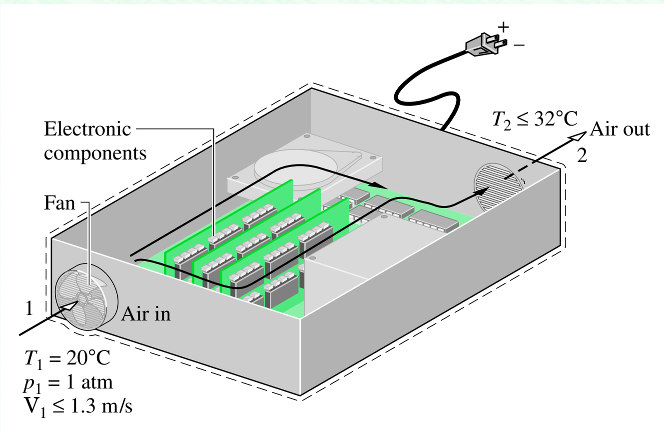
The electronic components of a computer are cooled by air flowing through a fan mounted at the inlet of the electronics enclosure. At steady state, air enters at 20°C, 1 atm. For noise control, the velocity of the entering air cannot exceed 1.3 m/s. For temperature control, the temperature of the air at the exit cannot exceed 32°C. The electronic components and fan receive, respectively, 80 W and 18 W of electric power. Determine the smallest fan inlet diameter, in cm, for which the limits on the entering air velocity and exit air temperature are met.

SOLUTION

Known: The electronic components of a computer are cooled by air flowing through a fan mounted at the inlet of the electronics enclosure. Conditions are specified for the air at the inlet and exit. The power required by the electronics and the fan are also specified.

Find: Determine for these conditions the smallest fan inlet diameter.

Schematic and Given Data:



◀ **Figure E4.8**

Assumptions:

1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer from the *outer* surface of the electronics enclosure to the surroundings is negligible. Thus, $\dot{Q}_{cv} = 0$.
3. ❶ Changes in kinetic and potential energies can be ignored.
4. ❷ Air is modeled as an ideal gas with $c_p = 1.005$ kJ/kg · K.

Analysis: The inlet area A_1 can be determined from the mass flow rate $\dot{m}$ and Eq. 4.4b, which can be rearranged to read

$$A_1 = \frac{\dot{m}v_1}{V_1}$$

The mass flow rate can be evaluated, in turn, from the steady-state energy rate balance

$$0 = \underline{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \underline{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \underline{g(z_1 - z_2)}\right]$$

The underlined terms drop out by assumptions 2 and 3, leaving

$$0 = -\dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

where $\dot{W}_{cv}$ accounts for the *total* electric power provided to the electronic components and the fan: $\dot{W}_{cv} = (-80\ \text{W}) + (-18\ \text{W}) = -98\ \text{W}$. Solving for $\dot{m}$, and using assumption 4 with Eq. 3.51 to evaluate $(h_1 - h_2)$

$$\dot{m} = \frac{(-\dot{W}_{cv})}{c_p(T_2 - T_1)}$$

Introducing this into the expression for A_1 and using the ideal gas model to evaluate the specific volume v_1

$$A_1 = \frac{1}{V_1}\left[\frac{(-\dot{W}_{cv})}{c_p(T_2 - T_1)}\right]\left(\frac{RT_1}{p_1}\right)$$

From this expression we see that A_1 *increases* when V_1 and/or T_2 *decrease*. Accordingly, since $V_1 \leq 1.3$ m/s and $T_2 \leq 305$ K (32°C), the inlet area must satisfy

$$A_1 \geq \frac{1}{1.3\ \text{m/s}}\left[\frac{98\ \text{W}}{\left(1.005\dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(305 - 293)\text{K}}\left|\frac{1\ \text{kJ}}{10^3\ \text{J}}\right|\left|\frac{1\ \text{J/s}}{1\ \text{W}}\right|\right]\left(\frac{\left(\dfrac{8314\ \text{N}\cdot\text{m}}{28.97\ \text{kg}\cdot\text{K}}\right)293\ \text{K}}{1.01325 \times 10^5\ \text{N/m}^2}\right)$$

$$\geq 0.005\ \text{m}^2$$

Then, since $A_1 = \pi D_1^2/4$

$$D_1 \geq \sqrt{\frac{(4)(0.005\ \text{m}^2)}{\pi}} = 0.08\ \text{m}\left|\frac{10^2\ \text{cm}}{1\ \text{m}}\right|$$

$$D_1 \geq 8\ \text{cm}$$

For the specified conditions, the smallest fan inlet diameter is 8 cm.

1 Cooling air typically enters and exits electronic enclosures at low velocities, and thus kinetic energy effects are insignificant.

2 The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart. Since the temperature of the air increases by no more than 12°C, the specific heat c_p is nearly constant (Table A-20).

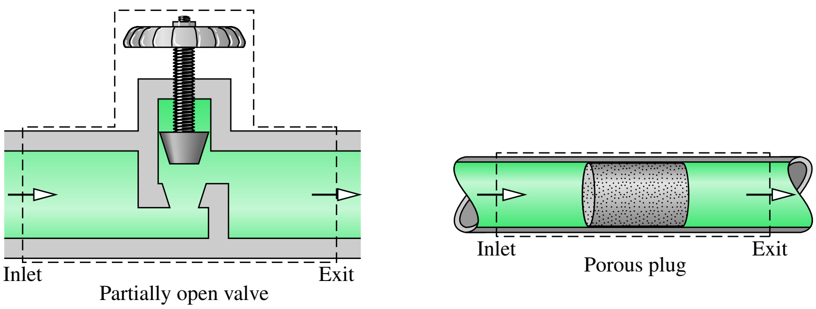
THROTTLING DEVICES

A significant reduction in pressure can be achieved simply by introducing a restriction into a line through which a gas or liquid flows. This is commonly done by means of a partially opened valve or a porous plug, as illustrated in Fig. 4.13.

For a control volume enclosing such a device, the mass and energy rate balances reduce at steady state to

$$0 = \dot{m}_1 - \dot{m}_2$$

$$0 = \dot{Q}_{cv} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) - \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right)$$



▲ **Figure 4.13** Examples of throttling devices.

There is usually no significant heat transfer with the surroundings and the change in potential energy from inlet to exit is negligible. With these idealizations, the mass and energy rate balances combine to give

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

Although velocities may be relatively high in the vicinity of the restriction, measurements made upstream and downstream of the reduced flow area show in most cases that the change in the specific kinetic energy of the gas or liquid between these locations can be neglected. With this further simplification, the last equation reduces to

throttling process

$$h_1 = h_2 \tag{4.22}$$

When the flow through a valve or other restriction is idealized in this way, the process is called a ***throttling process.***

An application of the throttling process occurs in vapor-compression refrigeration systems, where a valve is used to reduce the pressure of the refrigerant from the pressure at the exit of the *condenser* to the lower pressure existing in the *evaporator*. We consider this further in Chap. 10. The throttling process also plays a role in the *Joule–Thomson* expansion considered in Chap. 11. Another application of the throttling process involves the ***throttling calorimeter,*** which is a device for determining the quality of a two-phase liquid–vapor mixture. The throttling calorimeter is considered in the next example.

throttling calorimeter

EXAMPLE 4.9 Measuring Steam Quality

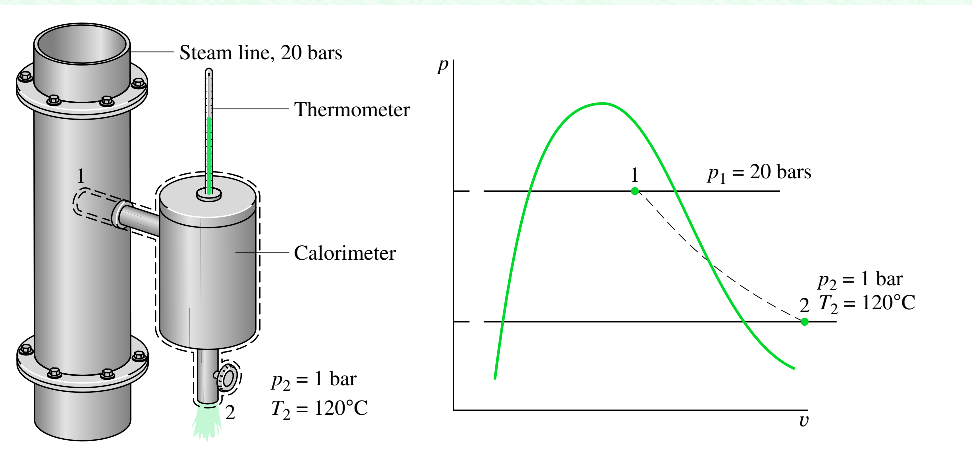
A supply line carries a two-phase liquid–vapor mixture of steam at 20 bars. A small fraction of the flow in the line is diverted through a throttling calorimeter and exhausted to the atmosphere at 1 bar. The temperature of the exhaust steam is measured as 120°C. Determine the quality of the steam in the supply line.

SOLUTION

Known: Steam is diverted from a supply line through a throttling calorimeter and exhausted to the atmosphere.

Find: Determine the quality of the steam in the supply line.

Schematic and Given Data:



◀ **Figure E4.9**

Assumptions:

1. The control volume shown on the accompanying figure is at steady state.
2. The diverted steam undergoes a throttling process.

Analysis: For a throttling process, the energy and mass balances reduce to give $h_2 = h_1$, which agrees with Eq. 4.22. Thus, with state 2 fixed, the specific enthalpy in the supply line is known, and state 1 is fixed by the known values of p_1 and h_1.

1 As shown on the accompanying p–v diagram, state 1 is in the two-phase liquid–vapor region and state 2 is in the superheated vapor region. Thus

$$h_2 = h_1 = h_{f1} + x_1(h_{g1} - h_{f1})$$

Solving for x_1

$$x_1 = \frac{h_2 - h_{f1}}{h_{g1} - h_{f1}}$$

From Table A-3 at 20 bars, $h_{f1} = 908.79$ kJ/kg and $h_{g1} = 2799.5$ kJ/kg. At 1 bar and 120°C, $h_2 = 2766.6$ kJ/kg from Table A-4. Inserting values into the above expression, the quality in the line is $x_1 = 0.956$ (95.6%).

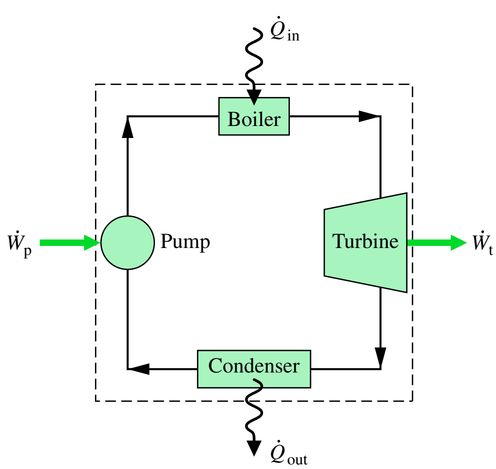
---

1 For throttling calorimeters exhausting to the atmosphere, the quality in the line must be greater than about 94% to ensure that the steam leaving the calorimeter is superheated.

SYSTEM INTEGRATION

Thus far, we have studied several types of components selected from those commonly seen in practice. These components are usually encountered in combination, rather than individually. Engineers often must creatively combine components to achieve some overall objective, subject to constraints such as minimum total cost. This important engineering activity is called *system integration*.

Many readers are already familiar with a particularly successful system integration: the simple power plant shown in Fig. 4.14. This system consists of four components in series, a turbine-generator, condenser, pump, and boiler. We consider such power plants in detail in subsequent sections of the book. The example to follow provides another illustration. Many more are considered in later sections and in end-of-chapter problems.



◀ **Figure 4.14** Simple vapor power plant.

Thermodynamics in the News...

Sensibly Built Homes Cost No More

Healthy, comfortable homes that cut energy and water bills and protect the environment cost no more, builders say. The "I have a Dream House," a highly energy efficient and environmentally responsible house located close to the Atlanta boyhood home of Dr. Martin Luther King Jr., is a prime example.

The house, developed under U.S. Department of Energy auspices, can be heated and cooled for less than a dollar a day, and uses 57% less energy for heating and cooling than a conventional house. Still, construction costs are no more than for a conventional house.

Designers used a whole-house *integrated system* approach whereby components are carefully selected to be complementary in achieving an energy-thrifty, cost-effective outcome.

The walls, roof, and floor of this 1565 square-foot house are factory-built structural insulated panels incorporating foam insulation. This choice allowed designers to reduce the size of the heating and cooling equipment, thereby lowering costs. The house also features energy-efficient windows, tightly sealed ductwork, and a high-efficiency air conditioner that further contribute to energy savings.

EXAMPLE 4.10 Waste Heat Recovery System

An industrial process discharges gaseous combustion products at 478°K, 1 bar with a mass flow rate of 69.78 kg/s. As shown in Fig. E4.10, a proposed system for utilizing the combustion products combines a heat-recovery steam generator with a turbine. At steady state, combustion products exit the steam generator at 400°K, 1 bar and a separate stream of water enters at .275 MPa, 38.9°C with a mass flow rate of 2.079 kg/s. At the exit of the turbine, the pressure is 0.07 bars and the quality is 93%. Heat transfer from the outer surfaces of the steam generator and turbine can be ignored, as can the changes in kinetic and potential energies of the flowing streams. There is no significant pressure drop for the water flowing through the steam generator. The combustion products can be modeled as air as an ideal gas.

(a) Determine the power developed by the turbine, in kJ/s.

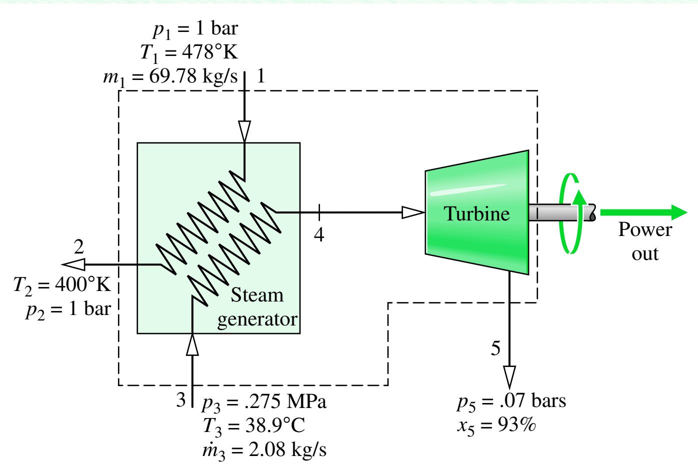
(b) Determine the turbine inlet temperature, in °C.

SOLUTION

Known: Steady-state operating data are provided for a system consisting of a heat-recovery steam generator and a turbine.

Find: Determine the power developed by the turbine and the turbine inlet temperature. Evaluate the annual value of the power developed.

Schematic and Given Data:



Assumptions:

1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer is negligible, and changes in kinetic and potential energy can be ignored.
3. There is no pressure drop for water flowing through the steam generator.
4. The combustion products are modeled as air as an ideal gas.

◄ **Figure E4.10**

Analysis:

(a) The power developed by the turbine is determined from a control volume enclosing both the steam generator and the turbine. Since the gas and water streams do not mix, mass rate balances for each of the streams reduce, respectively, to give

$$\dot{m}_1 = \dot{m}_2, \qquad \dot{m}_3 = \dot{m}_5$$

The steady-state form of the energy rate balance is

$$0 = \underline{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_1\left(h_1 + \underline{\frac{V_1^2}{2} + gz_1}\right) + \dot{m}_3\left(h_3 + \underline{\frac{V_3^2}{2} + gz_3}\right)$$
$$- \dot{m}_2\left(h_2 + \underline{\frac{V_2^2}{2} + gz_2}\right) - \dot{m}_5\left(h_5 + \underline{\frac{V_5^2}{2} + gz_5}\right)$$

The underlined terms drop out by assumption 2. With these simplifications, together with the above mass flow rate relations, the energy rate balance becomes

$$\dot{W}_{cv} = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_5)$$
$$\text{where } \dot{m}_1 = 69.78 \text{ kg/s}, \dot{m}_3 = 2.08 \text{ kg/s}$$

The specific enthalpies h_1 and h_2 can be found from Table A-22: At 478 K, $h_1 = 480.35$ kJ/kg, and at 400 K, $h_2 = 400.98$ kJ/kg. At state 3, water is a liquid. Using Eq. 3.14 and saturated liquid data from Table A-2, $h_3 \approx h_f(T_3) = 162.9$ kJ/kg. State 5 is a two-phase liquid–vapor mixture. With data from Table A-3 and the given quality

$$h_5 = 161 + 0.93(2571.72 - 161) = 2403 \text{ kJ/kg}$$

Substituting values into the expression for $\dot{W}_{cv}$

$$\dot{W}_{cv} = (69.78 \text{ kg/s})(480.3 - 400.98) \text{ kJ/kg}$$
$$+ (2.079 \text{ kg/s})(162.9 - 2403) \text{ kJ/kg}$$
$$= 876.8 \text{ kJ/s} = 876.8 \text{ kW}$$

(b) To determine T_4, it is necessary to fix the state at 4. This requires two independent property values. With assumption 3, one of these properties is pressure, $p_4 = 0.275$ MPa. The other is the specific enthalpy h_4, which can be found from an energy rate balance for a control volume enclosing just the steam generator. Mass rate balances for each of the two streams give $\dot{m}_1 = \dot{m}_2$ and $\dot{m}_3 = \dot{m}_4$. With assumption 2 and these mass flow rate relations, the steady-state form of the energy rate balance reduces to ❶

$$0 = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$$

Solving for h_4

$$h_4 = h_3 + \frac{\dot{m}_1}{\dot{m}_3}(h_1 - h_2)$$
$$= 162.9\frac{\text{kJ}}{\text{kg}} + \left(\frac{69.78}{2.079}\right)(480.3 - 400.98)\frac{\text{kJ}}{\text{kg}}$$
$$= 2825 \text{ kJ/kg}$$ ❷

Interpolating in Table A-4 at $p_4 = .275$ MPa with h_4, $T_4 = 180$°C.

---

❶ Alternatively, to determine h_4 a control volume enclosing just the turbine can be considered. This is left as an exercise.

❷ The decision about implementing this solution to the problem of utilizing the hot combustion products discharged from an industrial process would necessarily rest on the outcome of a detailed economic evaluation, including the cost of purchasing and operating the steam generator, turbine, and auxiliary equipment.

4.4 Transient Analysis

transient

Many devices undergo periods of ***transient*** operation in which the state changes with time. Examples include the startup or shutdown of turbines, compressors, and motors. Additional examples are provided by vessels being filled or emptied, as considered in Example 4.2 and in the discussion of Fig. 1.3. Because property values, work and heat transfer rates, and mass flow rates may vary with time during transient operation, the steady-state assumption is not appropriate when analyzing such cases. Special care must be exercised when applying the mass and energy rate balances, as discussed next.

MASS BALANCE

First, we place the control volume mass balance in a form that is suitable for transient analysis. We begin by integrating the mass rate balance, Eq. 4.2, from time 0 to a final time t. That is

$$\int_0^t \left(\frac{dm_{\text{cv}}}{dt}\right) dt = \int_0^t \left(\sum_i \dot{m}_i\right) dt - \int_0^t \left(\sum_e \dot{m}_e\right) dt$$

This takes the form

$$m_{\text{cv}}(t) - m_{\text{cv}}(0) = \sum_i \left(\underline{\int_0^t \dot{m}_i \, dt}\right) - \sum_e \left(\underline{\int_0^t \dot{m}_e \, dt}\right)$$

Introducing the following symbols for the underlined terms

$$m_i = \int_0^t \dot{m}_i \, dt \quad \begin{cases} \text{amount of mass} \\ \text{entering the control} \\ \text{volume through inlet } i, \\ \text{from time 0 to } t \end{cases}$$

$$m_e = \int_0^t \dot{m}_e \, dt \quad \begin{cases} \text{amount of mass} \\ \text{exiting the control} \\ \text{volume through exit } e, \\ \text{from time 0 to } t \end{cases}$$

the mass balance becomes

$$m_{\text{cv}}(t) - m_{\text{cv}}(0) = \sum_i m_i - \sum_e m_e \tag{4.23}$$

In words, Eq. 4.23 states that the change in the amount of mass contained in the control volume equals the difference between the total incoming and outgoing amounts of mass.

ENERGY BALANCE

Next, we integrate the energy rate balance, Eq. 4.15, ignoring the effects of kinetic and potential energy. The result is

$$U_{\text{cv}}(t) - U_{\text{cv}}(0) = Q_{\text{cv}} - W_{\text{cv}} + \sum_i \left(\underline{\int_0^t \dot{m}_i h_i \, dt}\right) - \sum_e \left(\underline{\int_0^t \dot{m}_e h_e \, dt}\right) \tag{4.24a}$$

where Q_{cv} accounts for the net amount of energy transferred by heat into the control volume and W_{cv} accounts for the net amount of energy transferred by work, except for flow work.

The integrals shown underlined in Eq. 4.24a account for the energy carried in at the inlets and out at the exits.

For the *special case* where the states at the inlets and exits are *constant with time*, the respective specific enthalpies, h_i and h_e, would be constant, and the underlined terms of Eq. 4.24a become

$$\int_0^t \dot{m}_i h_i \, dt = h_i \int_0^t \dot{m}_i \, dt = h_i m_i$$
$$\int_0^t \dot{m}_e h_e \, dt = h_e \int_0^t \dot{m}_e \, dt = h_e m_e$$

Equation 4.24a then takes the following special form

$$U_{\text{cv}}(t) - U_{\text{cv}}(0) = Q_{\text{cv}} - W_{\text{cv}} + \sum_i m_i h_i - \sum_e m_e h_e \tag{4.24b}$$

Whether in the general form, Eq. 4.24a, or the special form, Eq. 4.24b, these equations account for the change in the amount of energy contained within the control volume as the difference between the total incoming and outgoing amounts of energy.

Another special case is when the intensive properties within the control volume are *uniform with position* at each instant. Accordingly, the specific volume and the specific internal energy are uniform throughout and can depend only on time, that is $v(t)$ and $u(t)$. Thus

$$\begin{aligned} m_{\text{cv}}(t) &= V_{\text{cv}}(t)/v(t) \\ U_{\text{cv}}(t) &= m_{\text{cv}}(t)u(t) \end{aligned} \tag{4.25}$$

When the control volume is comprised of different phases, the state of each phase would be assumed uniform throughout.

The following examples provide illustrations of the transient analysis of control volumes using the conservation of mass and energy principles. In each case considered, we begin with the general forms of the mass and energy balances and reduce them to forms suited for the case at hand, invoking the idealizations discussed in this section when warranted.

The first example considers a vessel that is partially emptied as mass exits through a valve.

EXAMPLE 4.11 Withdrawing Steam from a Tank at Constant Pressure

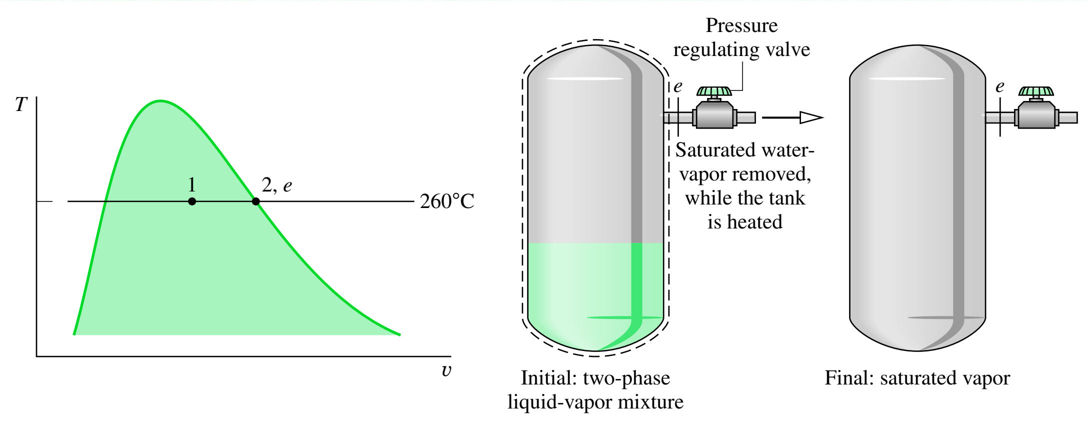
A tank having a volume of 0.85 m^3 initially contains water as a two-phase liquid—vapor mixture at 260°C and a quality of 0.7. Saturated water vapor at 260°C is slowly withdrawn through a pressure-regulating valve at the top of the tank as energy is transferred by heat to maintain the pressure constant in the tank. This continues until the tank is filled with saturated vapor at 260°C. Determine the amount of heat transfer, in kJ. Neglect all kinetic and potential energy effects.

SOLUTION

Known: A tank initially holding a two-phase liquid–vapor mixture is heated while saturated water vapor is slowly removed. This continues at constant pressure until the tank is filled only with saturated vapor.

Find: Determine the amount of heat transfer.

Schematic and Given Data:



▲ **Figure E4.11**

Assumptions:

1. The control volume is defined by the dashed line on the accompanying diagram.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects can be neglected.
3. At the exit the state remains constant.
4. The initial and final states of the mass within the vessel are equilibrium states.

Analysis: Since there is a single exit and no inlet, the mass rate balance takes the form

$$\frac{dm_{cv}}{dt} = -\dot{m}_e$$

With assumption 2, the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Combining the mass and energy rate balances results in

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + h_e \frac{dm_{cv}}{dt}$$

By assumption 3, the specific enthalpy at the exit is constant. Accordingly, integration of the last equation gives

$$\Delta U_{cv} = Q_{cv} + h_e \, \Delta m_{cv}$$

Solving for the heat transfer Q_{cv}

$$Q_{cv} = \Delta U_{cv} - h_e \, \Delta m_{cv}$$

or

$$Q_{cv} = (m_2 u_2 - m_1 u_1) - h_e(m_2 - m_1)$$

where m_1 and m_2 denote, respectively, the initial and final amounts of mass within the tank.

The terms u_1 and m_1 of the foregoing equation can be evaluated with property values from Table A-2 at 260°C and the given value for quality. Thus

$$\begin{aligned} u_1 &= u_f + x_1(u_g - u_f) \\ &= 1128.4 + (0.7)(2599.0 - 1128.4) = 2157.8 \text{ kJ/kg} \end{aligned}$$

Also,

$$v_1 = v_f + x_1(v_g - u_f)$$
$$= 1.2755 \times 10^{-3} + (0.7)(0.04221 - 1.2755 \times 10^{-3}) = 29.93 \times 10^{-3}\ \text{m}^3/\text{kg}$$

Using the specific volume v_1, the mass initially contained in the tank is

$$m_1 = \frac{V}{v_1} = \frac{0.85\ \text{m}^3}{(29.93 \times 10^{-3}\ \text{m}^3/\text{kg})} = 28.4\ \text{kg}$$

The final state of the mass in the tank is saturated vapor at 260°C, so Table A-2 gives

$$u_2 = u_g(260°\text{C}) = 2599.0\ \text{kJ/kg}, \qquad v_2 = v_g(260°\text{C}) = 42.21 \times 10^{-3}\ \text{m}^3/\text{kg}$$

The mass contained within the tank at the end of the process is

$$m_2 = \frac{V}{v_2} = \frac{0.85\ \text{m}^3}{42.21 \times 10^{-3}\ \text{m}^3/\text{kg}} = 20.14\ \text{kg}$$

Table A-2 also gives $h_e = h_g(260°\text{C}) = 2796.6$ kJ/kg.

Substituting values into the expression for the heat transfer yields

$$Q_{cv} = (20.14)(2599.0) - (28.4)(2157.8) - 2796.6(20.14 - 28.4)$$
$$= 14{,}162\ \text{kJ}$$

1. In this case, idealizations are made about the state of the vapor exiting *and* the initial and final states of the mass contained within the tank.
2. This expression for Q_{cv} could be obtained by applying Eq. 4.24b together with Eqs. 4.23 and 4.25

In the next two examples we consider cases where tanks are filled. In Example 4.12, an initially evacuated tank is filled with steam as power is developed. In Example 4.13, a compressor is used to store air in a tank.

EXAMPLE 4.12 Using Steam for Emergency Power Generation

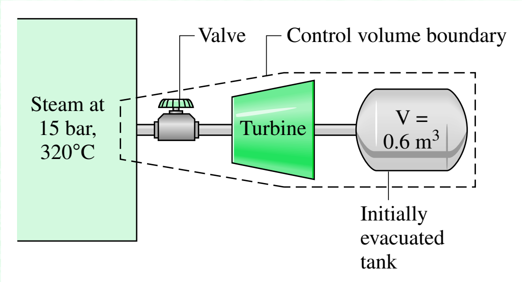
Steam at a pressure of 15 bar and a temperature of 320°C is contained in a large vessel. Connected to the vessel through a valve is a turbine followed by a small initially evacuated tank with a volume of 0.6 m^3. When emergency power is required, the valve is opened and the tank fills with steam until the pressure is 15 bar. The temperature in the tank is then 400°C. The filling process takes place adiabatically and kinetic and potential energy effects are negligible. Determine the amount of work developed by the turbine, in kJ.

SOLUTION

Known: Steam contained in a large vessel at a known state flows from the vessel through a turbine into a small tank of known volume until a specified final condition is attained in the tank.

Find: Determine the work developed by the turbine.

Schematic and Given Data:



◄ **Figure E4.12**

Assumptions:

1. The control volume is defined by the dashed line on the accompanying diagram.
2. For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible.
3. The state of the steam within the large vessel remains constant. The final state of the steam in the smaller tank is an equilibrium state.
4. The amount of mass stored within the turbine and the interconnecting piping at the end of the filling process is negligible.

Analysis: Since the control volume has a single inlet and no exits, the mass rate balance reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

The energy rate balance reduces with assumption 2 to

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + \dot{m}_i h_i$$

Combining the mass and energy rate balances gives

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + h_i \frac{dm_{cv}}{dt}$$

Integrating

$$\Delta U_{cv} = -W_{cv} + h_i \, \Delta m_{cv}$$

In accordance with assumption 3, the specific enthalpy of the steam entering the control volume is constant at the value corresponding to the state in the large vessel.

Solving for W_{cv}

$$W_{cv} = h_i \, \Delta m_{cv} - \Delta U_{cv}$$

ΔU_{cv} and Δm_{cv} denote, respectively, the changes in internal energy and mass of the control volume. With assumption 4, these terms can be identified with the small tank only.

Since the tank is initially evacuated, the terms ΔU_{cv} and Δm_{cv} reduce to the internal energy and mass within the tank at the end of the process. That is

$$\Delta U_{cv} = (m_2 u_2) - (\cancel{m_1 u_1})^0, \qquad \Delta m_{cv} = m_2 - \cancel{m_1}^0$$

where 1 and 2 denote the initial and final states within the tank, respectively.

Collecting results yields

$$W_{cv} = m_2(h_i - u_2) \tag{1}$$

The mass within the tank at the end of the process can be evaluated from the known volume and the specific volume of steam at 15 bar and 400°C from Table A-4

$$m_2 = \frac{V}{v_2} = \frac{0.6 \text{ m}^3}{(0.203 \text{ m}^3/\text{kg})} = 2.96 \text{ kg}$$

The specific internal energy of steam at 15 bar and 400°C from Table A-4 is 2951.3 kJ/kg. Also, at 15 bar and 320°C, $h_1 = 3081.9$ kJ/kg.

Substituting values into Eq. (1)

❸ $$W_{cv} = 2.96 \text{ kg}(3081.9 - 2951.3)\text{kJ/kg} = 386.6 \text{ kJ}$$

❶ In this case idealizations are made about the state of the steam entering the tank *and* the final state of the steam in the tank. These idealizations make the transient analysis manageable.

❷ A significant aspect of this example is the energy transfer into the control volume by flow work, incorporated in the pv term of the specific enthalpy at the inlet.

❸ If the turbine were removed and steam allowed to flow adiabatically into the small tank, the final steam temperature in the tank would be 477°C. This may be verified by setting W_{cv} to zero in Eq. (1) to obtain $u_2 = h_i$, which with $p_2 = 15$ bar fixes the final state.

EXAMPLE 4.13 Storing Compressed Air in a Tank

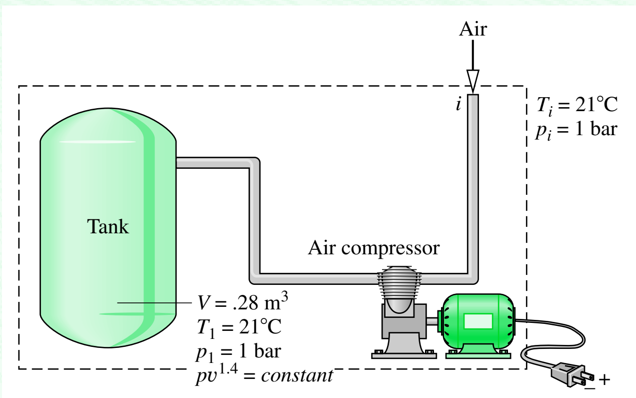
An air compressor rapidly fills a .28m^3 tank, initially containing air at 21°C, 1 bar, with air drawn from the atmosphere at 21°C, 1 bar. During filling, the relationship between the pressure and specific volume of the air in the tank is $pv^{1.4} = constant$. The ideal gas model applies for the air, and kinetic and potential energy effects are negligible. Plot the pressure, in atm, and the temperature, in °F, of the air within the tank, each versus the ratio m/m_1, where m_1 is the initial mass in the tank and m is the mass in the tank at time $t > 0$. Also, plot the compressor work input, in kJ, versus m/m_1. Let m/m_1 vary from 1 to 3.

SOLUTION

Known: An air compressor rapidly fills a tank having a known volume. The initial state of the air in the tank and the state of the entering air are known.

Find: Plot the pressure and temperature of the air within the tank, and plot the air compressor work input, each versus m/m_1 ranging from 1 to 3.

Schematic and Given Data:



◀ **Figure E4.13*a***

Assumptions:

1. The control volume is defined by the dashed line on the accompanying diagram.
2. Because the tank is filled rapidly, $\dot{Q}_{cv}$ is ignored.
3. Kinetic and potential energy effects are negligible.
4. The state of the air entering the control volume remains constant.
5. The air stored within the air compressor and interconnecting pipes can be ignored.
6. The relationship between pressure and specific volume for the air in the tank is $pv^{1.4} = constant$.
7. The ideal gas model applies for the air.

Analysis: The required plots are developed using *Interactive Thermodynamics: IT*. The *IT* program is based on the following analysis. The pressure p in the tank at time $t > 0$ is determined from

$$pv^{1.4} = p_1 v_1^{1.4}$$

where the corresponding specific volume v is obtained using the known tank volume V and the mass m in the tank at that time. That is, $v = V/m$. The specific volume of the air in the tank initially, v_1, is calculated from the ideal gas equation of state and the known initial temperature, T_1, and pressure, p_1. That is

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\dfrac{8314 \text{ N} \cdot \text{m}}{28.97 \text{ kg} \cdot {}^\circ\text{K}}\right)(294^\circ\text{K})}{(1 \text{ bar})} \left| \frac{1 \text{ bar}}{10^5 \text{ N/m}^2} \right| = .8437 \text{ m}^3/\text{kg}$$

Once the pressure p is known, the corresponding temperature T can be found from the ideal gas equation of state, $T = pv/R$.

To determine the work, begin with the mass rate balance for the single-inlet control volume

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

Then, with assumptions 2 and 3, the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + \dot{m}_i h_i$$

Combining the mass and energy rate balances and integrating using assumption 4 gives

$$\Delta U_{cv} = -W_{cv} + h_i \Delta m_{cv}$$

Denoting the work *input* to the compressor by $W_{in} = -W_{cv}$ and using assumption 5, this becomes

$$W_{in} = mu - m_1 u_1 - (m - m_1)h_i \tag{1}$$

where m_1 is the initial amount of air in the tank, determined from

$$m_1 = \frac{V}{v_1} = \frac{.28 \text{ m}^3}{0.8437 \text{ m}^3/\text{kg}} = 0.332 \text{ kg}$$

As a *sample* calculation to validate the *IT* program below, consider the case $m = 0.664$ kg, which corresponds to $m/m_1 = 2$. The specific volume of the air in the tank at that time is

$$v = \frac{V}{m} = \frac{0.28 \text{ m}^3}{0.664 \text{ kg}} = 0.422 \text{ m}^3/\text{kg}$$

The corresponding pressure of the air is

$$p = p_1\left(\frac{v_1}{v}\right)^{1.4} = (1\text{ bar})\left(\frac{0.8437\text{ m}^3/\text{kg}}{0.422\text{ m}^3/\text{kg}}\right)^{1.4}$$
$$= 2.64\text{ bars}$$

and the corresponding temperature of the air is

$$T = \frac{pv}{R} = \left(\frac{(2.64\text{ bars})(.422\text{ m}^3/\text{kg})}{\left(\dfrac{8314}{28.97}\dfrac{\text{J}}{\text{kg}\cdot{}^\circ\text{K}}\right)}\right)\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|$$
$$= 388^\circ\text{K}\ (114.9^\circ\text{C})$$

Evaluating u_1, u, and h_i at the appropriate temperatures from Table A-22, $u_1 = 209.8$ kJ/kg, $u = 277.5$ kJ/kg, $h_i = 294.2$ kJ/kg. Using Eq. (1), the required work input is

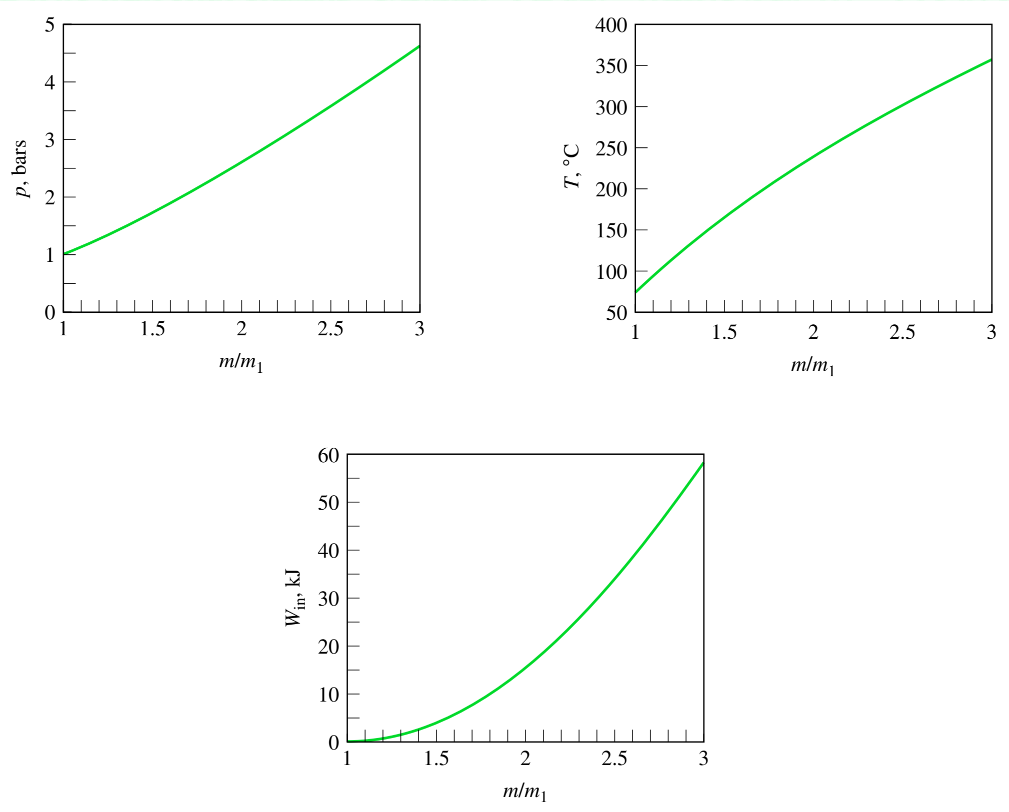
$$W_{\text{in}} = mu - m_1u_1 - (m - m_1)h_i$$
$$= (0.664\text{ kg})\left(277.5\,\frac{\text{kJ}}{\text{kg}}\right) - (0.332\text{ kg})\left(209.8\,\frac{\text{kJ}}{\text{kg}}\right) - (0.332\text{ kg})\left(294.2\,\frac{\text{kJ}}{\text{kg}}\right)$$
$$= 16.9\text{ kJ}$$

IT Program. Choosing SI units from the **Units** menu, and selecting Air from the **Properties** menu, the *IT* program for solving the problem is

```
// Given data
p1 = 1 // bar
T1 = 21 // °C
Ti = 21 // °C
V = .28 // m³
n = 1.4

// Determine the pressure and temperature for t > 0
v1 = v_TP("Air", T1, p1)
v = V/m
p * v ^n = p1 * v1 ^n
v = v_TP("Air", T, p)

// Specify the mass and mass ratio r
v1 = V/m1
r = m/m1
r = 2

// Calculate the work using Eq. (1)
Win = m * u−m1 * u1−hi * (m−m1)
u1 = u_T("Air", T1)
u = u_T("Air", T)
hi = h_T("Air", Ti)
```

Using the **Solve** button, obtain a solution for the sample case $r = m/m_1 = 2$ considered above to validate the program. Good agreement is obtained, as can be verified. Once the program is validated, use the **Explore** button to vary the ratio m/m_1 from

1 to 3 in steps of 0.01. Then, use the **Graph** button to construct the required plots. The results are:



▲ **Figure E4.13*b***

We conclude from the first two plots that the pressure and temperature each increase as the tank fills. The work required to fill the tank increases as well. These results are as expected.

1 This pressure-specific volume relationship is in accord with what might be measured. The relationship is also consistent with the uniform state idealization, embodied by Eqs. 4.25.

The final example of transient analysis is an application with a *well-stirred* tank. Such process equipment is commonly employed in the chemical and food processing industries.

EXAMPLE 4.14 Temperature Variation in a Well-Stirred Tank

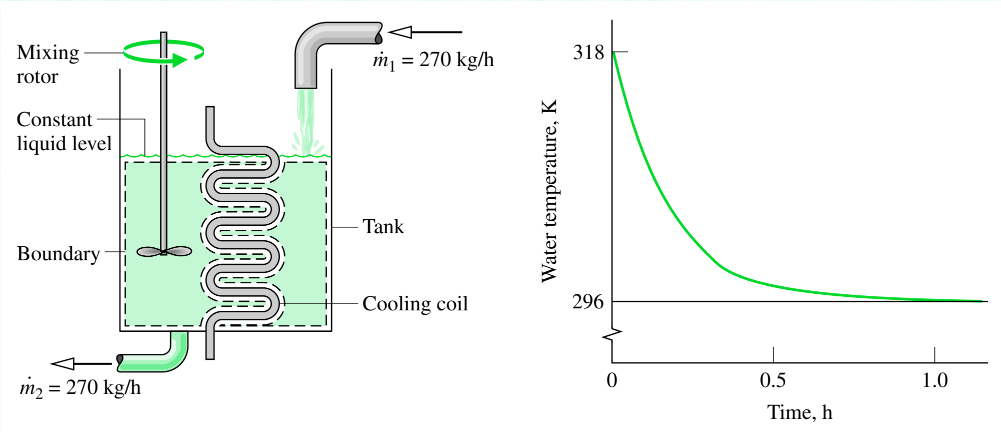
A tank containing 45 kg of liquid water initially at 45°C has one inlet and one exit with equal mass flow rates. Liquid water enters at 45°C and a mass flow rate of 270 kg/h. A cooling coil immersed in the water removes energy at a rate of 7.6 kW. The water is well mixed by a paddle wheel so that the water temperature is uniform throughout. The power input to the water from the paddle wheel is 0.6 kW. The pressures at the inlet and exit are equal and all kinetic and potential energy effects can be ignored. Plot the variation of water temperature with time.

SOLUTION

Known: Liquid water flows into and out of a well-stirred tank with equal mass flow rates as the water in the tank is cooled by a cooling coil.

Find: Plot the variation of water temperature with time.

Schematic and Given Data:



◄ **Figure E4.14**

Assumptions:

1. The control volume is defined by the dashed line on the accompanying diagram.
2. For the control volume, the only significant heat transfer is with the cooling coil. Kinetic and potential energy effects can be neglected.
3. The water temperature is uniform with position throughout: $T = T(t)$.
4. The water in the tank is incompressible, and there is no change in pressure between inlet and exit.

Analysis: The energy rate balance reduces with assumption 2 to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

where $\dot{m}$ denotes the mass flow rate.

The mass contained within the control volume remains constant with time, so the term on the left side of the energy rate balance can be expressed as

$$\frac{dU_{cv}}{dt} = \frac{d(m_{cv}u)}{dt} = m_{cv}\frac{du}{dt}$$

Since the water is assumed incompressible, the specific internal energy depends on temperature only. Hence, the chain rule can be used to write

$$\frac{du}{dt} = \frac{du}{dT}\frac{dT}{dt} = c\frac{dT}{dt}$$

where c is the specific heat. Collecting results

$$\frac{dU_{cv}}{dt} = m_{cv}c\frac{dT}{dt}$$

With Eq. 3.20b the enthalpy term of the energy rate balance can be expressed as

$$h_1 - h_2 = c(T_1 - T_2) + v(\cancelto{0}{p_1 - p_2})$$

where the pressure term is dropped by assumption 4. Since the water is well mixed, the temperature at the exit equals the temperature of the overall quantity of liquid in the tank, so

$$h_1 - h_2 = c(T_1 - T)$$

where T represents the uniform water temperature at time t.

With the foregoing considerations the energy rate balance becomes

$$m_{cv} c \frac{dT}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}c(T_1 - T)$$

As can be verified by direct substitution, the solution of this first-order, ordinary differential equation is

$$T = C_1 \exp\left(-\frac{\dot{m}}{m_{cv}} t\right) + \left(\frac{\dot{Q}_{cv} - \dot{W}_{cv}}{\dot{m}c}\right) + T_1$$

The constant C_1 is evaluated using the initial condition: at $t = 0$, $T = T_1$. Finally

$$T = T_1 + \left(\frac{\dot{Q}_{cv} - \dot{W}_{cv}}{\dot{m}c}\right)\left[1 - \exp\left(-\frac{\dot{m}}{m_{cv}} t\right)\right]$$

Substituting given numerical values together with the specific heat c for liquid water from Table A-19

❷
$$\begin{aligned} T &= 318 \text{ K} + \left[\frac{[-7.6 - (-0.6)] \text{ kJ/s}}{\left(\frac{270}{3600}\frac{\text{kg}}{\text{s}}\right)\left(4.2\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\right]\left[1 - \exp\left(-\frac{270}{45} t\right)\right] \\ &= 318 - 22[1 - \exp(-6t)] \end{aligned}$$

where t is in hours. Using this expression, we can construct the accompanying plot showing the variation of temperature with time.

❶ In this case idealizations are made about the state of the mass contained within the system and the states of the liquid entering and exiting. These idealizations make the transient analysis manageable.

❷ As $t \rightarrow \infty$, $T \rightarrow 296$ K. That is, the water temperature approaches a constant value after sufficient time has elapsed. From the accompanying plot it can be seen that the temperature reaches its constant limiting value in about 1 h.

Chapter Summary and Study Guide

The conservation of mass and energy principles for control volumes are embodied in the mass and energy rate balances developed in this chapter. Although the primary emphasis is on cases in which one-dimensional flow is assumed, mass and energy balances are also presented in integral forms that provide a link to subsequent fluid mechanics and heat transfer courses. Control volumes at steady state are featured, but discussions of transient cases are also provided.

The use of mass and energy balances for control volumes at steady state is illustrated for nozzles and diffusers, turbines, compressors and pumps, heat exchangers, throttling devices, and integrated systems. An essential aspect of all such applications is the careful and explicit listing of appropriate assumptions. Such model-building skills are stressed throughout the chapter.

The following checklist provides a study guide for this chapter. When your study of the text and end-of-chapter exercises has been completed you should be able to

- write out the meanings of the terms listed in the margins throughout the chapter and understand each of the related concepts. The subset of key concepts listed below is particularly important in subsequent chapters.
- list the typical modeling assumptions for nozzles and diffusers, turbines, compressors and pumps, heat exchangers, and throttling devices.
- apply Eqs. 4.18–4.20 to control volumes at steady state, using appropriate assumptions and property data for the case at hand.
- apply mass and energy balances for the transient analysis of control volumes, using appropriate assumptions and property data for the case at hand.

Key Engineering Concepts

mass flow rate *p. 122*
mass rate balance *p. 122*
one-dimensional flow *p. 124*
volumetric flow rate *p. 124*
steady state *p. 125*
flow work *p. 130*
energy rate balance *p. 131*
nozzle *p. 134*
diffuser *p. 134*
turbine *p. 137*
compressor *p. 139*
pump *p. 139*
heat exchanger *p. 143*
throttling process *p. 148*

Exercises: Things Engineers Think About

1. Why does the relative velocity *normal* to the flow boundary, V_n, appear in Eqs. 4.3 and 4.8?

2. Why might a computer cooled by a *constant-speed* fan operate satisfactorily at sea level but overheat at high altitude?

3. Give an example where the inlet and exit mass flow rates for a control volume are equal, yet the control volume is not at steady state.

4. Does $\dot{Q}_{cv}$ accounting for energy transfer by heat include heat transfer across inlets and exits? Under what circumstances might heat transfer across an inlet or exit be significant?

5. By introducing enthalpy h to replace each of the $(u + pv)$ terms of Eq. 4.13, we get Eq. 4.14. An even simpler algebraic form would result by replacing each of the $(u + pv + V^2/2 + gz)$ terms by a single symbol, yet we have not done so. Why not?

6. Simplify the general forms of the mass and energy rate balances to describe the process of blowing up a balloon. List all of your modeling assumptions.

7. How do the general forms of the mass and energy rate balances simplify to describe the exhaust stroke of a cylinder in an automobile engine? List all of your modeling assumptions.

8. Waterwheels have been used since antiquity to develop mechanical power from flowing water. Sketch an appropriate control volume for a waterwheel. What terms in the mass and energy rate balances are important to describe steady-state operation?

9. When air enters a diffuser and decelerates, does its pressure increase or decrease?

10. Even though their outer surfaces would seem hot to the touch, large steam turbines in power plants might not be covered with much insulation. Why not?

11. Would it be desirable for a coolant circulating inside the engine of an automobile to have a large or a small specific heat c_p? Discuss.

12. A hot liquid stream enters a counterflow heat exchanger at $T_{h,in}$, and a cold liquid stream enters at $T_{c,in}$. Sketch the variation of temperature with location of each stream as it passes through the heat exchanger.

13. What are some examples of commonly encountered devices that undergo periods of transient operation? For each example, which type of system, closed system or control volume, would be most appropriate?

14. An insulated rigid tank is initially evacuated. A valve is opened and atmospheric air at 20°C, 1 atm enters until the pressure in the tank becomes 1 bar, at which time the valve is closed. Is the final temperature of the air in the tank equal to, greater than, or less than 20°C?

Problems: Developing Engineering Skills

Applying Conservation of Mass

4.1 The mass flow rate at the inlet of a one-inlet, one-exit control volume varies with time according to $\dot{m}_i = 100(1 - e^{-2t})$, where $\dot{m}_i$ has units of kg/h and t is in h. At the exit, the mass flow rate is constant at 100 kg/h. The initial mass in the control volume is 50 kg.

(a) Plot the inlet and exit mass flow rates, the instantaneous rate of change of mass, and the amount of mass contained in the control volume as functions of time, for t ranging from 0 to 3 h.

(b) *Estimate* the time, in h, when the tank is nearly empty.

4.2 A control volume has one inlet and one exit. The mass flow rates in and out are, respectively, $\dot{m}_i = 1.5$ and $\dot{m}_e = 1.5(1 - e^{-0.002t})$, where t is in seconds and $\dot{m}$ is in kg/s. Plot the time *rate of change* of mass, in kg/s, and the net *change in the amount* of mass, in kg, in the control volume versus time, in s, ranging from 0 to 3600 s.

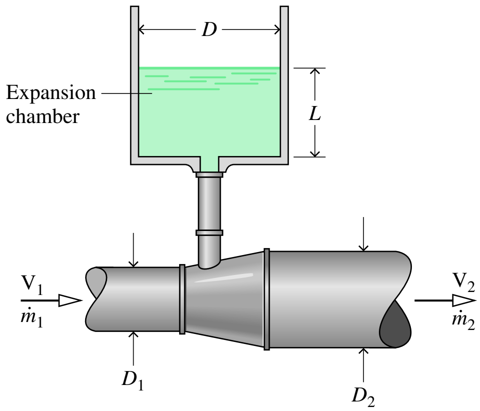
4.3 A 0.5-m^3 tank contains ammonia, initially at 40°C, 8 bar. A leak develops, and refrigerant flows out of the tank at a constant mass flow rate of 0.04 kg/s. The process occurs slowly enough that heat transfer from the surroundings maintains a constant temperature in the tank. Determine the time, in s, at which half of the mass has leaked out, and the pressure in the tank at that time, in bar.

4.4 A water storage tank initially contains 400 m^3 of water. The average daily usage is 40 m^3. If water is added to the tank at an average rate of $20[\exp(-t/20)]$ m^3 per day, where t is time in days, for how many days will the tank contain water?

4.5 A pipe carrying an incompressible liquid contains an expansion chamber as illustrated in Fig. P4.5.

(a) Develop an expression for the time rate of change of liquid level in the chamber, dL/dt, in terms of the diameters D_1, D_2, and D, and the velocities V_1 and V_2.

(b) Compare the relative magnitudes of the mass flow rates $\dot{m}_1$ and $\dot{m}_2$ when $dL/dt > 0$, $dL/dt = 0$, and $dL/dt < 0$, respectively.



▲ **Figure P4.5**

4.6 Velocity distributions for *laminar* and *turbulent* flow in a circular pipe of radius R carrying an incompressible liquid of density ρ are given, respectively, by

$$V/V_0 = [1 - (r/R)^2]$$
$$V/V_0 = [1 - (r/R)]^{1/7}$$

where r is the radial distance from the pipe centerline and V_0 is the centerline velocity. For each velocity distribution

(a) plot V/V_0 versus r/R.

(b) derive expressions for the mass flow rate and the average velocity of the flow, V_{ave}, in terms of V_0, R, and ρ, as required.

(c) derive an expression for the *specific* kinetic energy carried through an area normal to the flow. What is the percent error if the specific kinetic energy is evaluated in terms of the average velocity as $(V_{ave})^2/2$?

Which velocity distribution adheres most closely to the idealizations of one-dimensional flow? Discuss.

4.7 Vegetable oil for cooking is dispensed from a cylindrical can fitted with a spray nozzle. According to the label, the can is able to deliver 560 sprays, each of duration 0.25 s and each having a mass of 0.25 g. Determine

(a) the mass flow rate of each spray, in g/s.

(b) the mass remaining in the can after 560 sprays, in g, if the initial mass in the can is 170 g.

4.8 Air enters a one-inlet, one-exit control volume at 8 bar, 600 K, and 40 m/s through a flow area of 20 cm^2. At the exit, the pressure is 2 bar, the temperature is 400 K, and the velocity is 350 m/s. The air behaves as an ideal gas. For steady-state operation, determine

(a) the mass flow rate, in kg/s.

(b) the exit flow area, in cm^2.

4.9 *Infiltration* of outside air into a building through miscellaneous cracks around doors and windows can represent a significant load on the heating equipment. On a day when the outside temperature is –18°C, 0.042 m^3/s of air enters through the cracks of a particular office building. In addition, door openings account for about .047 m^3/s of outside air infiltration. The internal volume of the building is 566 m^3, and the inside temperature is 22°C. There is negligible pressure difference between the inside and the outside of the building. Assuming ideal gas behavior, determine at steady state the volumetric flow rate of air exiting the building through cracks and other openings, and the number of times per hour that the air within the building is changed due to infiltration.

4.10 Refrigerant 134a enters the condenser of a refrigeration system operating at steady state at 9 bar, 50°C, through a 2.5-cm-diameter pipe. At the exit, the pressure is 9 bar, the temperature is 30°C, and the velocity is 2.5 m/s. The mass flow rate of the entering refrigerant is 6 kg/min. Determine

(a) the velocity at the inlet, in m/s.

(b) the diameter of the exit pipe, in cm.

4.11 Steam at 160 bar, 480°C, enters a turbine operating at steady state with a volumetric flow rate of 800 m^3/min. Eighteen percent of the entering mass flow exits at 5 bar, 240°C, with a velocity of 25 m/s. The rest exits at another location with a pressure of 0.06 bar, a quality of 94%, and a velocity of 400 m/s. Determine the diameters of each exit duct, in m.

4.12 Air enters a compressor operating at steady state with a pressure of 1 bar, a temperature of 20°C, and a volumetric flow rate of 0.25 m^3/s. The air velocity in the exit pipe is 210 m/s and the exit pressure is 1 MPa. If each unit mass of air passing from inlet to exit undergoes a process described by $pv^{1.34}$ = *constant,* determine the exit temperature, in °C.

4.13 Air enters a 0.6-m-diameter fan at 16°C, 101 kPa, and is discharged at 18°C, 105 kPa, with a volumetric flow rate of 0.35 m^3/s. Assuming ideal gas behavior, determine for steady-state operation

(a) the mass flow rate of air, in kg/s.

(b) the volumetric flow rate of air at the inlet, in m^3/s.

(c) the inlet and exit velocities, in m/s.

4.14 Ammonia enters a control volume operating at steady state at p_1 = 14 bar, T_1 = 28°C, with a mass flow rate of 0.5 kg/s. Saturated vapor at 4 bar leaves through one exit, with a volumetric flow rate of 1.036 m^3/min, and saturated liquid at 4 bar leaves through a second exit. Determine

(a) the minimum diameter of the inlet pipe, in cm, so the ammonia velocity does not exceed 20 m/s.

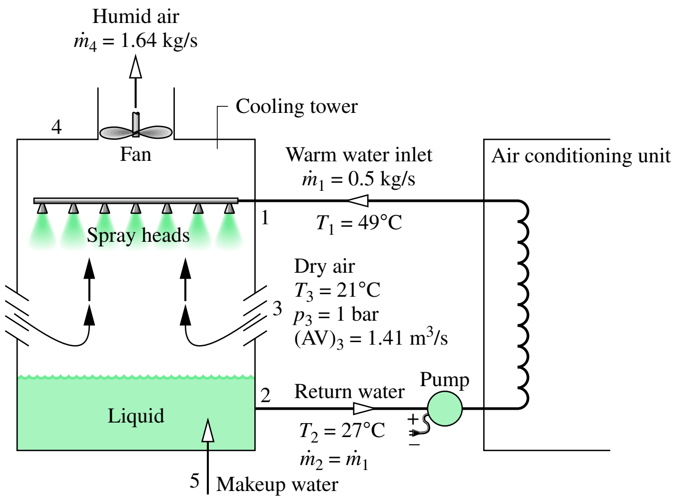
(b) the volumetric flow rate of the second exit stream, in m^3/min.

4.15 At steady state, a stream of liquid water at 20°C, 1 bar is mixed with a stream of ethylene glycol (M = 62.07) to form a refrigerant mixture that is 50% glycol by mass. The water molar flow rate is 4.2 kmol/min. The density of ethylene glycol is 1.115 times that of water. Determine

(a) the molar flow rate, in kmol/min, and volumetric flow rate, in m^3/min, of the entering ethylene glycol.

(b) the diameters, in cm, of each of the supply pipes if the velocity in each is 2.5 m/s.

4.16 Figure P4.16 shows a cooling tower operating at steady state. Warm water from an air conditioning unit enters at 49°C with a mass flow rate of 0.5 kg/s. Dry air enters the tower at 21°C, 1 atm with a volumetric flow rate of 1.41 m^3/s. Because of evaporation within the tower, humid air exits at the top of the tower with a mass flow rate of 1.64 kg/s. Cooled liquid water is collected at the bottom of the tower for return to the air conditioning unit together with makeup water. Determine the mass flow rate of the makeup water, in kg/s.



◀ **Figure P4.16**

Energy Analysis of Control Volumes at Steady State

4.17 Air enters a control volume operating at steady state at 1.05 bar, 300 K, with a volumetric flow rate of 12 m^3/min and exits at 12 bar, 400 K. Heat transfer occurs at a rate of 20 kW from the control volume to the surroundings. Neglecting kinetic and potential energy effects, determine the power, in kW.

4.18 Steam enters a nozzle operating at steady state at 30 bar, 320°C, with a velocity of 100 m/s. The exit pressure and temperature are 10 bar and 200°C, respectively. The mass flow rate is 2 kg/s. Neglecting heat transfer and potential energy, determine

(a) the exit velocity, in m/s.

(b) the inlet and exit flow areas, in cm^2.

4.19 Methane (CH_4) gas enters a horizontal, well-insulated nozzle operating at steady state at 80°C and a velocity of 10 m/s. Assuming ideal gas behavior for the methane, plot the temperature of the gas exiting the nozzle, in °C, versus the exit velocity ranging from 500 to 600 m/s.

4.20 Air enters an uninsulated nozzle operating at steady state at 420°K with negligible velocity and exits the nozzle at 290°K with a velocity of 460 m/s. Assuming ideal gas behavior and neglecting potential energy effects, determine the heat transfer per unit mass of air flowing, in kJ/kg.

4.21 Air enters an insulated diffuser operating at steady state with a pressure of 1 bar, a temperature of 300 K, and a velocity of 250 m/s. At the exit, the pressure is 1.13 bar and the velocity is 140 m/s. Potential energy effects can be neglected. Using the ideal gas model, determine

(a) the ratio of the exit flow area to the inlet flow area.

(b) the exit temperature, in K.

4.22 The inlet ducting to a jet engine forms a diffuser that steadily decelerates the entering air to zero velocity relative to the engine before the air enters the compressor. Consider a jet airplane flying at 1000 km/h where the local atmospheric pressure is 0.6 bar and the air temperature is 8°C. Assuming ideal gas behavior and neglecting heat transfer and potential energy effects, determine the temperature, in °C, of the air entering the compressor.

4.23 Refrigerant 134a enters an insulated diffuser as a saturated vapor at 7 bars with a velocity of 370 m/s. At the exit, the pressure is 16 bars and the velocity is negligible. The diffuser operates at steady state and potential energy effects can be neglected. Determine the exit temperature, in °C.

4.24 Air expands through a turbine from 10 bar, 900 K to 1 bar, 500 K. The inlet velocity is small compared to the exit velocity of 100 m/s. The turbine operates at steady state and develops a power output of 3200 kW. Heat transfer between the turbine and its surroundings and potential energy effects are negligible. Calculate the mass flow rate of air, in kg/s, and the exit area, in m^2.

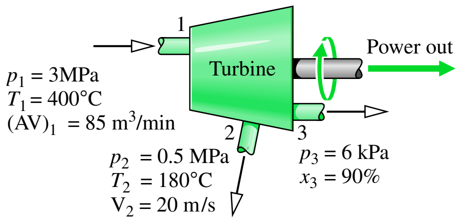
4.25 A well-insulated turbine operating at steady state develops 23 MW of power for a steam flow rate of 40 kg/s. The steam enters at 360°C with a velocity of 35 m/s and exits as saturated vapor at 0.06 bar with a velocity of 120 m/s. Neglecting potential energy effects, determine the inlet pressure, in bar.

4.26 Nitrogen gas enters a turbine operating at steady state with a velocity of 60 m/s, a pressure of 0.345 Mpa, and a temperature of 700 K. At the exit, the velocity is 0.6 m/s, the pressure is 0.14 Mpa, and the temperature is 390 K. Heat transfer from the surface of the turbine to the surroundings occurs at a rate of 36 kJ per kg of nitrogen flowing. Neglecting potential energy effects and using the ideal gas model, determine the power developed by the turbine, in kW.

4.27 Steam enters a well-insulated turbine operating at steady state with negligible velocity at 4 MPa, 320°C. The steam expands to an exit pressure of 0.07 MPa and a velocity of 90 m/s. The diameter of the exit is 0.6 m. Neglecting potential energy effects, plot the power developed by the turbine, in kW, versus the steam quality at the turbine exit ranging from 0.9 to 1.0.

4.28 The intake to a hydraulic turbine installed in a flood control dam is located at an elevation of 10 m above the turbine exit. Water enters at 20°C with negligible velocity and exits from the turbine at 10 m/s. The water passes through the turbine with no significant changes in temperature or pressure between the inlet and exit, and heat transfer is negligible. The acceleration of gravity is constant at $g = 9.81$ m/s^2. If the power output at steady state is 500 kW, what is the mass flow rate of water, in kg/s?

4.29 A well-insulated turbine operating at steady state is sketched in Fig. P4.29. Steam enters at 3 MPa, 400°C, with a volumetric flow rate of 85 m^3/min. Some steam is extracted from the turbine at a pressure of 0.5 MPa and a temperature of 180°C. The rest expands to a pressure of 6 kPa and a quality of 90%. The total power developed by the turbine is 11,400 kW. Kinetic and potential energy effects can be neglected. Determine



▲ **Figure P4.29**

(a) the mass flow rate of the steam at each of the two exits, in kg/h.
(b) the diameter, in m, of the duct through which steam is extracted, if the velocity there is 20 m/s.

4.30 Air is compressed at steady state from 1 bar, 300 K, to 6 bar with a mass flow rate of 4 kg/s. Each unit of mass passing from inlet to exit undergoes a process described by $pv^{1.27} = constant$. Heat transfer occurs at a rate of 46.95 kJ per kg of air flowing to cooling water circulating in a water jacket enclosing the compressor. If kinetic and potential energy changes of the air from inlet to exit are negligible, calculate the compressor power, in kW.

4.31 A compressor operates at steady state with Refrigerant 22 as the working fluid. The refrigerant enters at 5 bar, 10°C, with a volumetric flow rate of 0.8 m^3/min. The diameters of the inlet and exit pipes are 4 and 2 cm, respectively. At the exit, the pressure is 14 bar and the temperature is 90°C. If the magnitude of the heat transfer rate from the compressor to its surroundings is 5% of the compressor power input, determine the power input, in kW.

4.32 Refrigerant 134a enters an air conditioner compressor at 3.2 bar, 10°C, and is compressed at steady state to 10 bar, 70°C. The volumetric flow rate of refrigerant entering is 3.0 m^3/min. The power *input* to the compressor is 55.2 kJ per kg of refrigerant flowing. Neglecting kinetic and potential energy effects, determine the heat transfer rate, in kW.

4.33 A compressor operating at steady state takes in 45 kg/min of methane gas (CH_4) at 1 bar, 25°C, 15 m/s, and compresses it with negligible heat transfer to 2 bar, 90 m/s at the exit. The power input to the compressor is 110 kW. Potential energy effects are negligible. Using the ideal gas model, determine the temperature of the gas at the exit, in K.

4.34 Refrigerant 134a is compressed at steady state from 2.4 bar, 0°C, to 12 bar, 50°C. Refrigerant enters the compressor with a volumetric flow rate of 0.38 m^3/min, and the power input to the compressor is 2.6 kW. Cooling water circulating through a water jacket enclosing the compressor experiences a temperature rise of 4°C from inlet to exit with a negligible change in pressure. Heat transfer from the outside of the water jacket and all kinetic and potential energy effects can be neglected. Determine the mass flow rate of the cooling water, in kg/s.

4.35 Air enters a water-jacketed air compressor operating at steady state with a volumetric flow rate of 37 m^3/min at 136 kPa, 305 K and exits with a pressure of 680 kPa and a temperature of 400 K. The power input to the compressor is 155 kW. Energy transfer by heat from the compressed air to the cooling water circulating in the water jacket results in an increase in the temperature of the cooling water from inlet to exit with no change in pressure. Heat transfer from the outside of the jacket as well as all kinetic and potential energy effects can be neglected.

(a) Determine the temperature increase of the cooling water, in K, if the cooling water mass flow rate is 82 kg/min.

(b) Plot the temperature increase of the cooling water, in K, versus the cooling water mass flow rate ranging from 75 to 90 kg/min.

4.36 A pump steadily delivers water through a hose terminated by a nozzle. The exit of the nozzle has a diameter of 2.5 cm and is located 4 m above the pump inlet pipe, which has a diameter of 5.0 cm. The pressure is equal to 1 bar at both the inlet and the exit, and the temperature is constant at 20°C. The magnitude of the power input required by the pump is 8.6 kW, and the acceleration of gravity is $g = 9.81$ m/s^2. Determine the mass flow rate delivered by the pump, in kg/s.

4.37 An oil pump operating at steady state delivers oil at a rate of 5.5 kg/s and a velocity of 6.8 m/s. The oil, which can be modeled as incompressible, has a density of 1600 kg/m^3 and experiences a pressure rise from inlet to exit of .28 Mpa. There is no significant elevation difference between inlet and exit, and the inlet kinetic energy is negligible. Heat transfer between the pump and its surroundings is negligible, and there is no significant change in temperature as the oil passes through the pump. If pumps are available in 1/4-horsepower increments, determine the horsepower rating of the pump needed for this application.

4.38 Ammonia enters a heat exchanger operating at steady state as a superheated vapor at 14 bar, 60°C, where it is cooled and condensed to saturated liquid at 14 bar. The mass flow rate of the refrigerant is 450 kg/h. A separate stream of air enters the heat exchanger at 17°C, 1 bar and exits at 42°C, 1 bar. Ignoring heat transfer from the outside of the heat exchanger and neglecting kinetic and potential energy effects, determine the mass flow rate of the air, in kg/min.

4.39 A steam boiler tube is designed to produce a stream of saturated vapor at 200 kPa from saturated liquid entering at the same pressure. At steady state, the flow rate is 0.25 kg/min. The boiler is constructed from a well-insulated stainless steel pipe through which the steam flows. Electrodes clamped to the pipe at each end cause a 10-V direct current to pass through the pipe material. Determine the required size of the power supply, in kW, and the expected current draw, in amperes.

4.40 Carbon dioxide gas is heated as it flows steadily through a 2.5-cm-diameter pipe. At the inlet, the pressure is 2 bar, the temperature is 300 K, and the velocity is 100 m/s. At the exit, the pressure and velocity are 0.9413 bar and 400 m/s, respectively. The gas can be treated as an ideal gas with constant specific heat $c_p = 0.94$ kJ/kg · K. Neglecting potential energy effects, determine the rate of heat transfer to the carbon dioxide, in kW.

4.41 A feedwater heater in a vapor power plant operates at steady state with liquid entering at inlet 1 with $T_1 = 45$°C and $p_1 = 3.0$ bar. Water vapor at $T_2 = 320$°C and $p_2 = 3.0$ bar enters at inlet 2. Saturated liquid water exits with a pressure of $p_3 = 3.0$ bar. Ignore heat transfer with the surroundings and all kinetic and potential energy effects. If the mass flow rate of the liquid entering at inlet 1 is $\dot{m}_1 = 3.2 \times 10^5$ kg/h, determine the mass flow rate at inlet 2, $\dot{m}_2$, in kg/h.

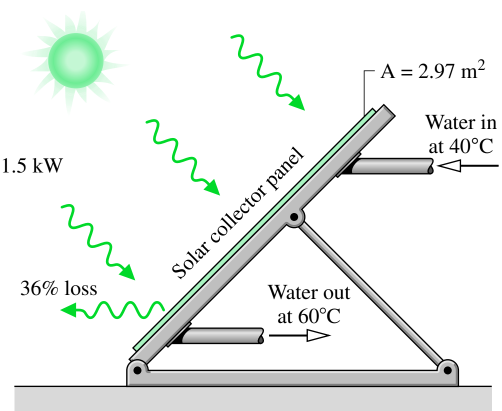
4.42 The cooling coil of an air-conditioning system is a heat exchanger in which air passes over tubes through which Refrigerant 22 flows. Air enters with a volumetric flow rate of 40 m^3/min at 27°C, 1.1 bar, and exits at 15°C, 1 bar. Refrigerant enters the tubes at 7 bar with a quality of 16% and exits at 7 bar, 15°C. Ignoring heat transfer from the outside of the heat exchanger and neglecting kinetic and potential energy effects, determine at steady state

(a) the mass flow rate of refrigerant, in kg/min.

(b) the rate of energy transfer, in kJ/min, from the air to the refrigerant.

4.43 Refrigerant 134a flows at steady state through a long horizontal pipe having an inside diameter of 4 cm, entering as saturated vapor at −8°C with a mass flow rate of 17 kg/min. Refrigerant vapor exits at a pressure of 2 bar. If the heat transfer rate to the refrigerant is 3.41 kW, determine the exit temperature, in °C, and the velocities at the inlet and exit, each in m/s.

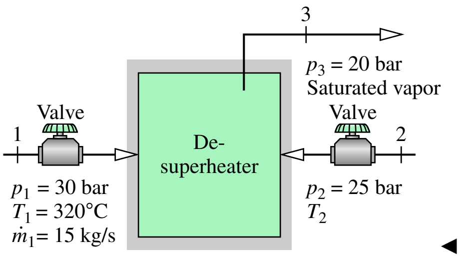
4.44 Figure P4.44 shows a solar collector panel with a surface area of 2.97 m^2. The panel receives energy from the sun at a rate of 1.5 kW. Thirty-six percent of the incoming energy is lost to the surroundings. The remainder is used to heat liquid water from 40°C to 60°C. The water passes through the solar collector with a negligible pressure drop. Neglecting kinetic and potential energy effects, determine at steady state the mass flow rate of water, in kg. How many gallons of water at 60°C can eight collectors provide in a 30-min time period?



▲ **Figure P4.44**

4.45 As shown in Fig. P4.45, 15 kg/s of steam enters a desuperheater operating at steady state at 30 bar, 320°C, where it is mixed with liquid water at 25 bar and temperature T_2 to produce saturated vapor at 20 bar. Heat transfer between the device and its surroundings and kinetic and potential energy effects can be neglected.

(a) If $T_2 = 200°C$, determine the mass flow rate of liquid, $\dot{m}_2$, in kg/s.



◀ **Figure P4.45**

(b) Plot $\dot{m}_2$, in kg/s, versus T_2 ranging from 20 to 220°C.

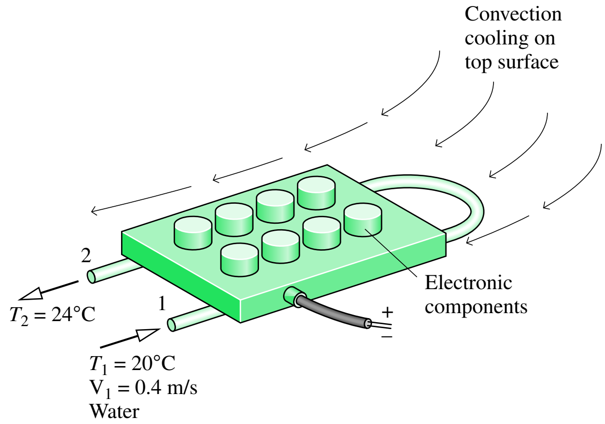
4.46 A feedwater heater operates at steady state with liquid water entering at inlet 1 at 7 bar, 42°C, and a mass flow rate of 70 kg/s. A separate stream of water enters at inlet 2 as a two-phase liquid–vapor mixture at 7 bar with a quality of 98%. Saturated liquid at 7 bar exits the feedwater heater at 3. Ignoring heat transfer with the surroundings and neglecting kinetic and potential energy effects, determine the mass flow rate, in kg/s, at inlet 2.

4.47 The electronic components of Example 4.8 are cooled by air flowing through the electronics enclosure. The rate of energy transfer by forced convection from the electronic components to the air is $hA(T_s - T_a)$, where $hA = 5$ W/K, T_s denotes the average surface temperature of the components, and T_a denotes the average of the inlet and exit air temperatures. Referring to Example 4.8 as required, determine the largest value of T_s, in °C, for which the specified limits are met.

4.47 The electronic components of a computer consume 0.1 kW of electrical power. To prevent overheating, cooling air is supplied by a 25-W fan mounted at the inlet of the electronics enclosure. At steady state, air enters the fan at 20°C, 1 bar and exits the electronics enclosure at 35°C. There is no significant energy transfer by heat from the outer surface of the enclosure to the surroundings and the effects of kinetic and potential energy can be ignored. Determine the volumetric flow rate of the entering air, in m^3/s.

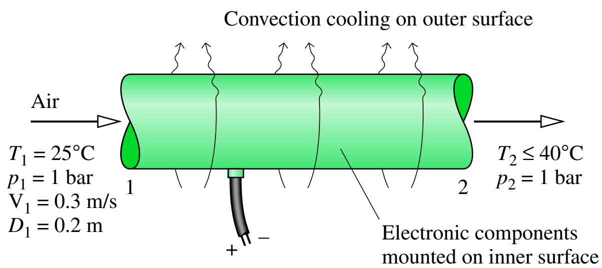
4.49 Ten kg/min of cooling water circulates through a water jacket enclosing a housing filled with electronic components. At steady state, water enters the water jacket at 22°C and exits with a negligible change in pressure at a temperature that cannot exceed 26°C. There is no significant energy transfer by heat from the outer surface of the water jacket to the surroundings, and kinetic and potential energy effects can be ignored. Determine the maximum electric power the electronic components can receive, in kW, for which the limit on the temperature of the exiting water is met.

4.50 As shown in Fig. P4.50, electronic components mounted on a flat plate are cooled by convection to the surroundings and by liquid water circulating through a U-tube bonded to the plate. At steady state, water enters the tube at 20°C and a velocity of 0.4 m/s and exits at 24°C with a negligible change in pressure. The electrical components receive 0.5 kW of electrical power. The rate of energy transfer by convection from the plate-mounted electronics is estimated to be 0.08 kW. Kinetic and potential energy effects can be ignored. Determine the tube diameter, in cm.



▲ **Figure P4.50**

4.51 Electronic components are mounted on the inner surface of a horizontal cylindrical duct whose inner diameter is 0.2 m, as shown in Fig. P4.51. To prevent overheating of the electronics, the cylinder is cooled by a stream of air flowing through it and by convection from its outer surface. Air enters the duct at 25°C, 1 bar and a velocity of 0.3 m/s and exits with negligible changes in kinetic energy and pressure at a temperature that cannot exceed 40°C. If the electronic components require 0.20 kW of electric power at steady state, determine the minimum rate of heat transfer by convection from the cylinder's outer surface, in kW, for which the limit on the temperature of the exiting air is met.



▲ **Figure P4.51**

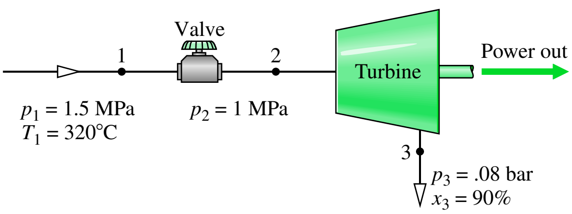
4.52 Ammonia enters the expansion valve of a refrigeration system at a pressure of 1.4 MPa and a temperature of 32°C and exits at 0.08 MPa. If the refrigerant undergoes a throttling process, what is the quality of the refrigerant exiting the expansion valve?

4.53 Propane vapor enters a valve at 1.6 MPa, 70°C, and leaves at 0.5 MPa. If the propane undergoes a throttling process, what is the temperature of the propane leaving the valve, in °C?

4.54 A large pipe carries steam as a two-phase liquid–vapor mixture at 1.0 MPa. A small quantity is withdrawn through a throttling calorimeter, where it undergoes a throttling process to an exit pressure of 0.1 MPa. For what range of exit temperatures, in °C, can the calorimeter be used to determine the quality of the steam in the pipe? What is the corresponding range of steam quality values?

4.55 As shown in Fig. P4.55, a steam turbine at steady state is operated at part load by throttling the steam to a lower pressure before it enters the turbine. Before throttling, the pressure and temperature are, respectively, 1.5 MPa and 320°C. After throttling, the pressure is 1 MPa. At the turbine exit, the steam is at .08 bar and a quality of 90%. Heat transfer with the surroundings and all kinetic and potential energy effects can be ignored. Determine

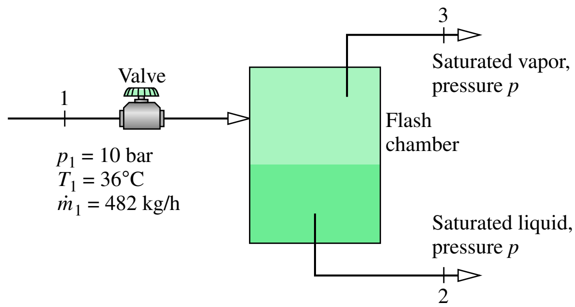
(a) the temperature at the turbine inlet, in °C.
(b) the power developed by the turbine, in kJ/kg of steam flowing.



▲ **Figure P4.55**

4.56 Refrigerant 134a enters the flash chamber operating at steady state shown in Fig. P4.56 at 10 bar, 36°C, with a mass flow rate of 482 kg/h. Saturated liquid and saturated vapor exit as separate streams, each at pressure p. Heat transfer to the surroundings and kinetic and potential energy effects can be ignored.

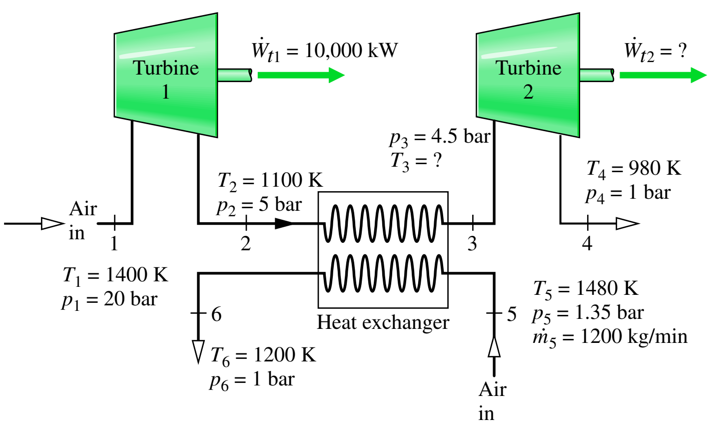
(a) Determine the mass flow rates of the exiting streams, each in kg/h, if $p = 4$ bar.
(b) Plot the mass flow rates of the exiting streams, each in kg/h, versus p ranging from 1 to 9 bar.



▲ **Figure P4.56**

4.57 Air as an ideal gas flows through the turbine and heat exchanger arrangement shown in Fig. P4.57. Data for the two flow streams are shown on the figure. Heat transfer to the surroundings can be neglected, as can all kinetic and potential energy effects. Determine T_3, in K, and the power output of the second turbine, in kW, at steady state.

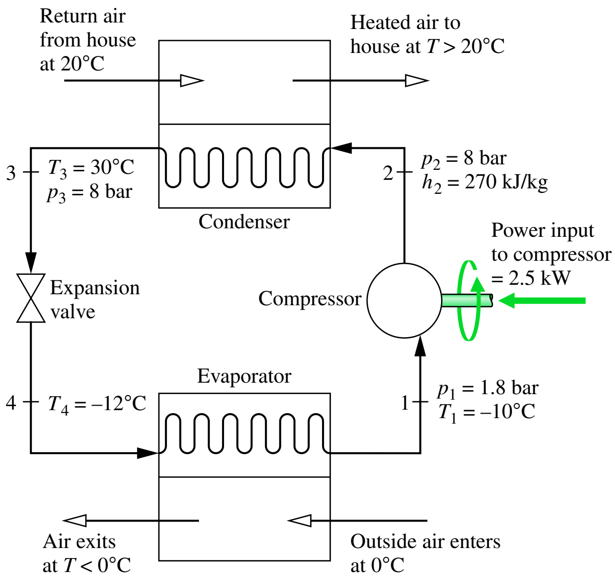
4.58 A residential heat pump system operating at steady state is shown schematically in Fig. P4.58. Refrigerant 134a circulates through the components of the system, and property data at the numbered locations are given on the figure. The mass



◀ **Figure P4.57**

flow rate of the refrigerant is 4.6 kg/min. Kinetic and potential energy effects are negligible. Determine

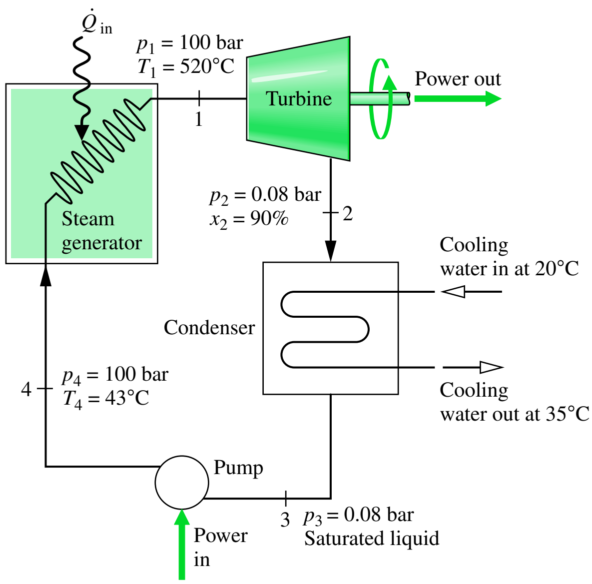
(a) rate of heat transfer between the compressor and the surroundings, in kJ/min.
(b) the coefficient of performance.



▲ **Figure P4.58**

4.59 Figure P4.59 shows a simple vapor power plant operating at steady state with water circulating through the components. Relevant data at key locations are given on the figure. The mass flow rate of the water is 109 kg/s. Kinetic and potential energy effects are negligible as are all stray heat transfers. Determine

(a) the thermal efficiency.
(b) the mass flow rate of the cooling water passing through the condenser, in kg/s.



▲ **Figure P4.59**

Transient Analysis

4.60 A tiny hole develops in the wall of a rigid tank whose volume is 0.75 m^3, and air from the surroundings at 1 bar, 25°C leaks in. Eventually, the pressure in the tank reaches 1 bar. The process occurs slowly enough that heat transfer between the tank and the surroundings keeps the temperature of the air inside the tank constant at 25°C. Determine the amount of heat transfer, in kJ, if initially the tank

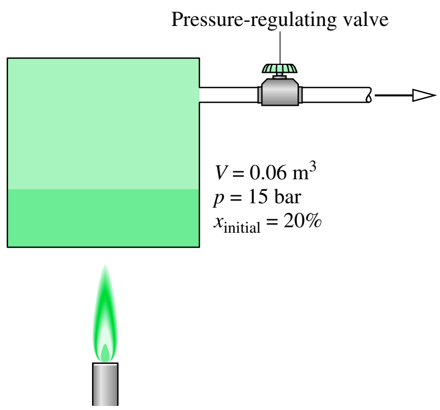
(a) is evacuated.
(b) contains air at 0.7 bar, 25°C.

4.61 A rigid tank of volume 0.75 m^3 is initially evacuated. A hole develops in the wall, and air from the surroundings at 1 bar, 25°C flows in until the pressure in the tank reaches 1 bar. Heat transfer between the contents of the tank and the surroundings is negligible. Determine the final temperature in the tank, in °C.

4.62 A rigid, well-insulated tank of volume 0.5 m^3 is initially evacuated. At time $t = 0$, air from the surroundings at 1 bar, 21°C begins to flow into the tank. An electric resistor transfers energy to the air in the tank at a constant rate of 100 W for 500 s, after which time the pressure in the tank is 1 bar. What is the temperature of the air in the tank, in °C, at the final time?

4.63 The rigid tank illustrated in Fig. P4.63 has a volume of 0.06 m^3 and initially contains a two-phase liquid–vapor mixture of H_2O at a pressure of 15 bar and a quality of 20%. As the tank contents are heated, a pressure-regulating valve keeps the pressure constant in the tank by allowing saturated vapor to escape. Neglecting kinetic and potential energy effects

(a) determine the total mass in the tank, in kg, and the amount of heat transfer, in kJ, if heating continues until the final quality is $x = 0.5$.
(b) plot the total mass in the tank, in kg, and the amount of heat transfer, in kJ, versus the final quality x ranging from 0.2 to 1.0.



◄ **Figure P4.63**

4.64 A well-insulated rigid tank of volume 10 m^3 is connected to a large steam line through which steam flows at 15 bar and 280°C. The tank is initially evacuated. Steam is allowed to flow into the tank until the pressure inside is p.

(a) Determine the amount of mass in the tank, in kg, and the temperature in the tank, in °C, when $p = 15$ bar.

(b) Plot the quantities of part (a) versus p ranging from 0.1 to 15 bar.

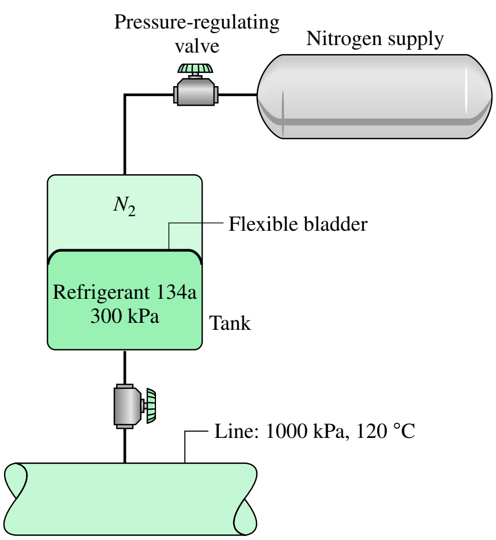
4.65 A tank of volume 1 m^3 initially contains steam at 6 MPa and 320°C. Steam is withdrawn slowly from the tank until the pressure drops to p. Heat transfer to the tank contents maintains the temperature constant at 320°C. Neglecting all kinetic and potential energy effects

(a) determine the heat transfer, in kJ, if $p = 1.5$ MPa.

(b) plot the heat transfer, in kJ, versus p ranging from 0.5 to 6 MPa.

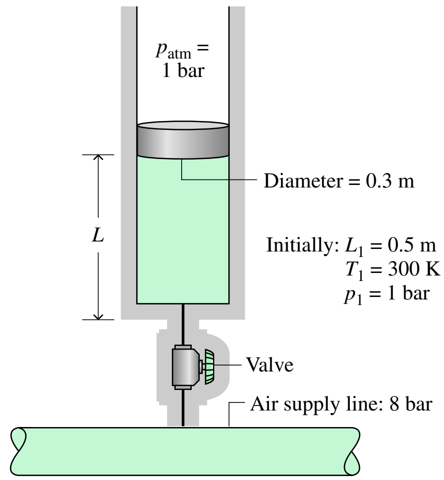
4.66 A 1 m^3 tank initially contains air at 300 kPa, 300 K. Air slowly escapes from the tank until the pressure drops to 100 kPa. The air that remains in the tank undergoes a process described by $pv^{1.2} = constant$. For a control volume enclosing the tank, determine the heat transfer, in kJ. Assume ideal gas behavior with constant specific heats.

4.67 A well-insulated tank contains 25 kg of Refrigerant 134a, initially at 300 kPa with a quality of 0.8 (80%). The pressure is maintained by nitrogen gas acting against a flexible bladder, as shown in Fig. P4.67. The valve is opened between the tank and a supply line carrying Refrigerant 134a at 1.0 MPa, 120°C. The pressure regulator allows the pressure in the tank to remain at 300 kPa as the bladder expands. The valve between the line and the tank is closed at the instant when all the liquid has vaporized. Determine the amount of refrigerant admitted to the tank, in kg.



▲ **Figure P4.67**

4.68 A well-insulated piston–cylinder assembly is connected by a valve to an air supply line at 8 bar, as shown in Fig. P4.68. Initially, the air inside the cylinder is at 1 bar, 300 K, and the piston is located 0.5 m above the bottom of the cylinder. The atmospheric pressure is 1 bar, and the diameter of the piston face is 0.3 m. The valve is opened and air is admitted slowly until the volume of air inside the cylinder has doubled. The weight of the piston and the friction between the piston and the cylinder wall can be ignored. Using the ideal gas model, plot the final temperature, in K, and the final mass, in kg, of the air inside the cylinder for supply temperatures ranging from 300 to 500 K.



▲ **Figure P4.68**

4.69 Nitrogen gas is contained in a rigid 1-m tank, initially at 10 bar, 300 K. Heat transfer to the contents of the tank occurs until the temperature has increased to 400 K. During the process, a pressure-relief valve allows nitrogen to escape, maintaining constant pressure in the tank. Neglecting kinetic and potential energy effects, and using the ideal gas model with constant specific heats evaluated at 350 K, determine the mass of nitrogen that escapes, in kg, and the amount of energy transfer by heat, in kJ.

4.70 The air supply to a 56 m^3 office has been shut off overnight to conserve utilities, and the room temperature has dropped to 4°C. In the morning, a worker resets the thermostat to 21°C, and 6 m^3/min of air at 50°C begins to flow in through a supply duct. The air is well mixed within the room, and an equal mass flow of air at room temperature is withdrawn through a return duct. The air pressure is nearly 1 bar everywhere. Ignoring heat transfer with the surroundings and kinetic and potential energy effects, estimate how long it takes for the room temperature to reach 21°C. Plot the room temperature as a function of time.

Design & Open Ended Problems: Exploring Engineering Practice

4.1D What practical measures can be taken by manufacturers to use energy resources more efficiently? List several specific opportunities, and discuss their potential impact on profitability and productivity.

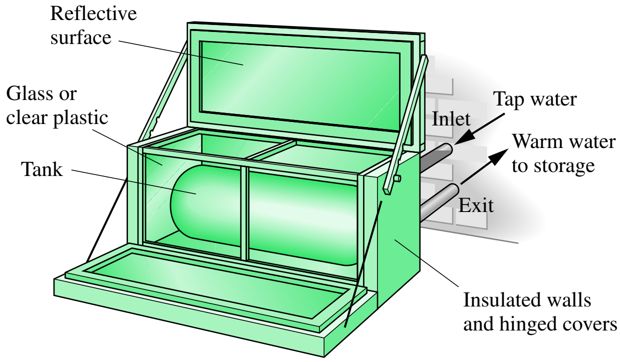
4.2D Methods for measuring mass flow rates of gases and liquids flowing in pipes and ducts include: *rotameters, turbine flowmeters, orifice-type flowmeters, thermal flowmeters, and Coriolis-type flowmeters.* Determine the principles of operation of each of these flow-measuring devices. Consider the suitability of each for measuring liquid or gas flows. Can any be used for two-phase liquid–vapor mixtures? Which measure volumetric flow rate and require separate measurements of pressure and temperature to determine the state of the substance? Summarize your findings in a brief report.

4.3D Wind turbines, or windmills, have been used for generations to develop power from wind. Several alternative wind turbine concepts have been tested, including among others the Mandaras, Darrieus, and propeller types. Write a report in which you describe the operating principles of prominent wind turbine types. Include in your report an assessment of the economic feasibility of each type.

4.4D Prepare a memorandum providing guidelines for selecting fans for cooling electronic components. Consider the advantages and disadvantages of locating the fan at the inlet of the enclosure containing the electronics. Repeat for a fan at the enclosure exit. Consider the relative merits of alternative fan types and of fixed- versus variable-speed fans. Explain how *characteristic curves* assist in fan selection.

4.5D Pumped-hydraulic storage power plants use relatively inexpensive *off-peak baseload* electricity to pump water from a lower reservoir to a higher reservoir. During periods of *peak* demand, electricity is produced by discharging water from the upper to the lower reservoir through a hydraulic turbine-generator. A single device normally plays the role of the pump during upper-reservoir charging and the turbine-generator during discharging. The ratio of the power developed during discharging to the power required for charging is typically much less than 100%. Write a report describing the features of the pump-turbines used for such applications and their size and cost. Include in your report a discussion of the economic feasibility of pumped-hydraulic storage power plants.

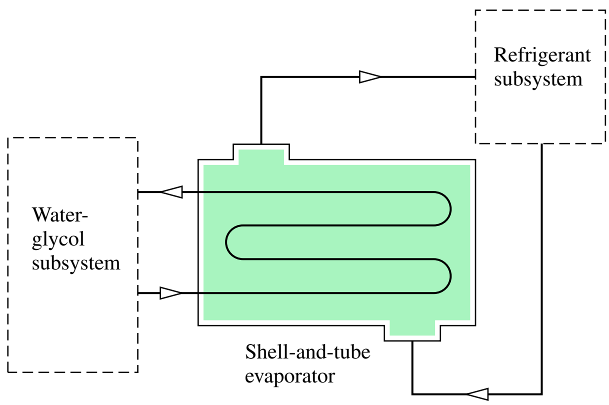
4.6D Figure P4.6D shows a *batch-type* solar water heater. With the exit closed, cold tap water fills the tank, where it is heated by the sun. The batch of heated water is then allowed to flow to an existing conventional gas or electric water heater. If the *batch-type* solar water heater is constructed primarily from salvaged and scrap material, estimate the time for a typical family of four to recover the cost of the water heater from reduced water heating by conventional means.



▲ **Figure P4.6D**

4.7D *Low-head* dams (3 to 10 m), commonly used for flood control on many rivers, provide an opportunity for electric power generation using hydraulic turbine-generators. Estimates of this hydroelectric potential must take into account the available head and the river flow, each of which varies considerably throughout the year. Using U.S. Geological Survey data, determine the typical variations in head and flow for a river in your locale. Based on this information, estimate the total annual electric generation of a hydraulic turbine placed on the river. Does the peak generating capacity occur at the same time of year as peak electrical demand in your area? Would you recommend that your local utility take advantage of this opportunity for electric power generation? Discuss.

4.8D Figure P4.8D illustrates an experimental apparatus for steady-state testing of Refrigerant 134a *shell-and-tube* evaporators having a *capacity* of 100 kW. As shown by the dashed lines on the figure, two subsystems provide refrigerant and a water-glycol mixture to the evaporator. The water-glycol mixture is chilled in passing through the evaporator tubes, so the water-glycol subsystem must reheat and recirculate the mixture to the evaporator. The refrigerant subsystem must remove the energy added to the refrigerant passing through the evaporator, and deliver saturated liquid refrigerant at −20°C. For each subsystem, draw schematics showing layouts of heat exchangers, pumps, interconnecting piping, etc. Also, specify



▲ **Figure P4.8D**

the mass flow rates, heat transfer rates, and power requirements for each component within the subsystems, as appropriate.

4.9D The stack from an industrial paint-drying oven discharges 30 m^3/min of gaseous combustion products at 240°C. Investigate the economic feasibility of installing a heat exchanger in the stack to heat air that would provide for some of the space heating needs of the plant.

4.10D **Smaller *can* be Better** (see box Sec. 4.3). Investigate the scope of current medical applications of MEMS. Write a report including at least three references.

4.11D **Sensibly Built Homes Cost No More** (see box Sec. 4.3). In energy-efficient homes, indoor air quality can be a concern. Research the issue of carbon monoxide and radon buildup in tightly sealed houses. Write a report including at least three references.