AE 330/708

AEROSPACE PROPULSION

Instructor

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Mission objectives, performance and multistaging

Basic orbital mechanics

Mission objectives

Rocket equation

Effects of drag and gravity

Single stage rocket performance

Multistage rocket analysis

Basics of space and flight mechanics - revisited

Newton's law of gravitation : Gravitational force is directly proportional to the product of the masses and inversely proportional to the square of the distance between the masses

Force of gravitational attraction, (for satellites around earth)

$$F = \frac{G \cdot M \cdot m}{R^2} = \frac{\mu \cdot m}{R^2}$$

G = Universal gravitational constant = 6.67×10^{-11} $m^3/kg \cdot s^2$

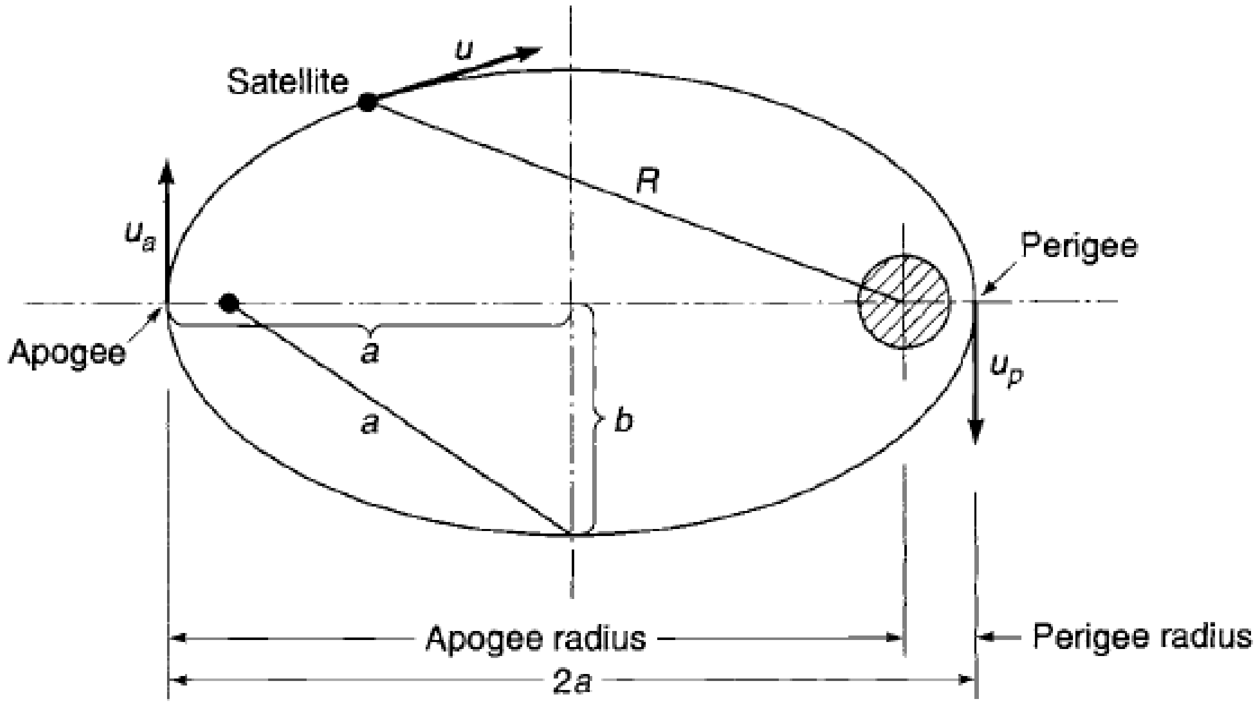
M = Mass of earth = 5.974×10^{24} kg

$\mu = G \cdot M = 3.986 \times 10^{14}$ m^3/s^2

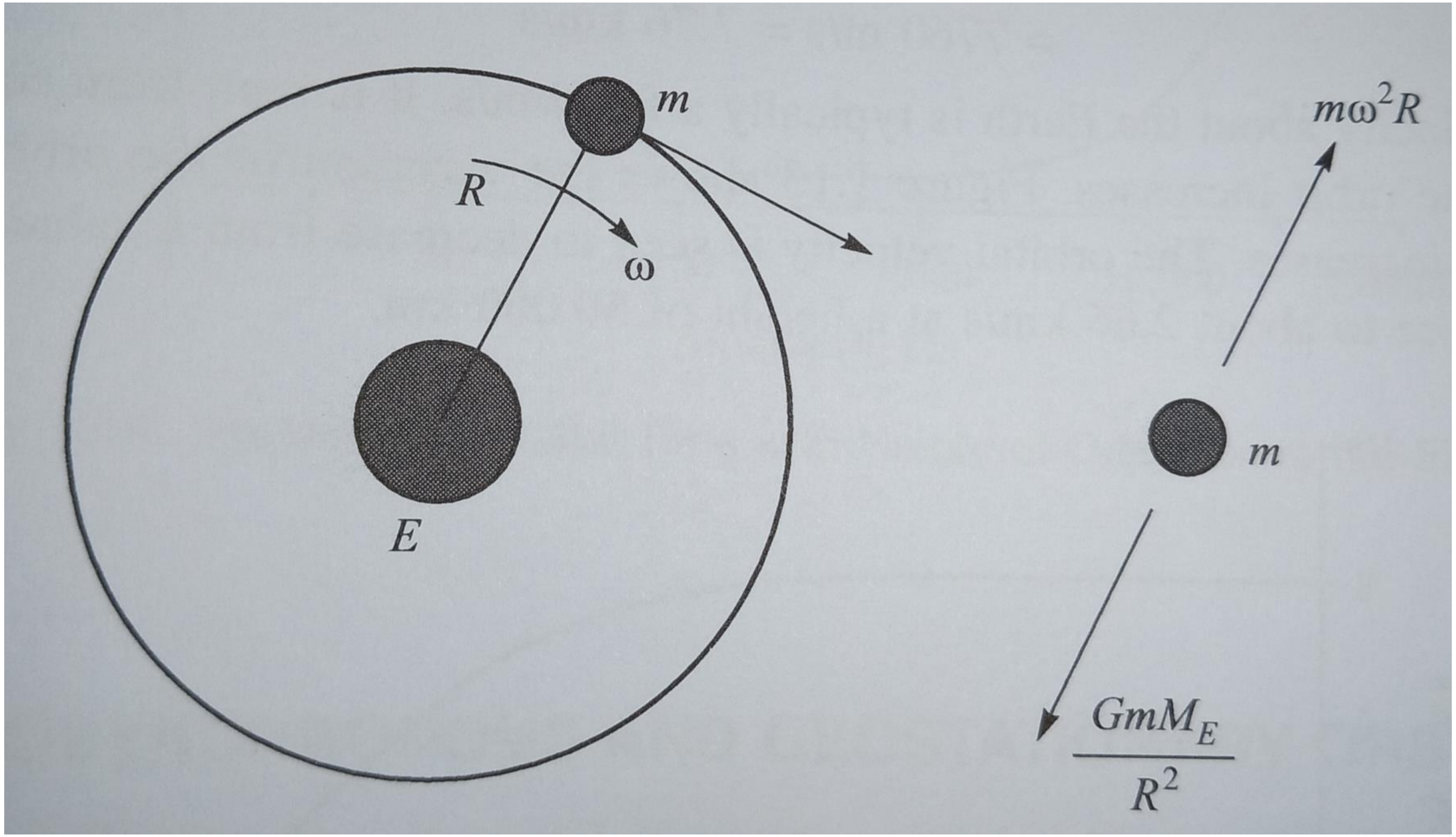
m = Mass of orbiting body

Kepler's laws of planetary motion:

1. The orbit of the planet is an ellipse with the sun at one of the foci.
2. The line segment joining a planet and the sun sweeps out equal areas during equal intervals of time.
3. The square of the orbital period of a planet is proportional to the cube of the semi major axis of its orbit.



Velocity in circular orbit



Velocity in circular orbit

Velocity of satellite in circular orbit,

Force of gravitation = Centrifugal force

$$m \cdot g = \frac{m \cdot u^2}{R}$$

Since gravitational force, $= \frac{\mu \cdot m}{R^2} \Rightarrow g = \frac{\mu}{R^2}$

$g =$ Gravitational acceleration

$$g \sim \frac{1}{R^2} \Rightarrow \cancel{g(r)} \quad g(R) \cdot R^2 = g_0 \cdot R_0^2$$

$g_0 =$ Gravitational acceleration at the earth's surface

$R_0 =$ Radius of earth $= 6374 \cdot 2$ km

$R = R_0 + h$ ($h =$ altitude from earth surface)

$$\therefore \quad g(R) = g_0 \cdot \left(\frac{R_0}{R}\right)^2 = g_0 \left(\frac{R_0}{R_0 + h}\right)^2$$

Velocity in circular orbit

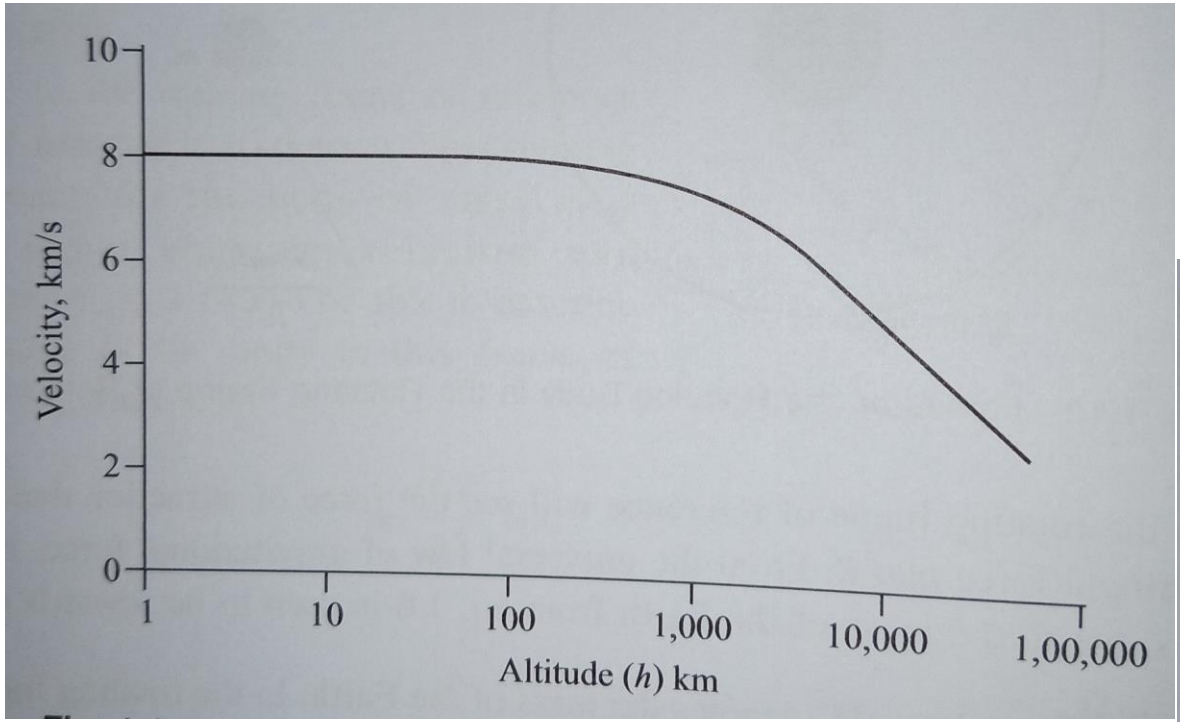
$\therefore$ Velocity of orbiting satellite in circular trajectory,

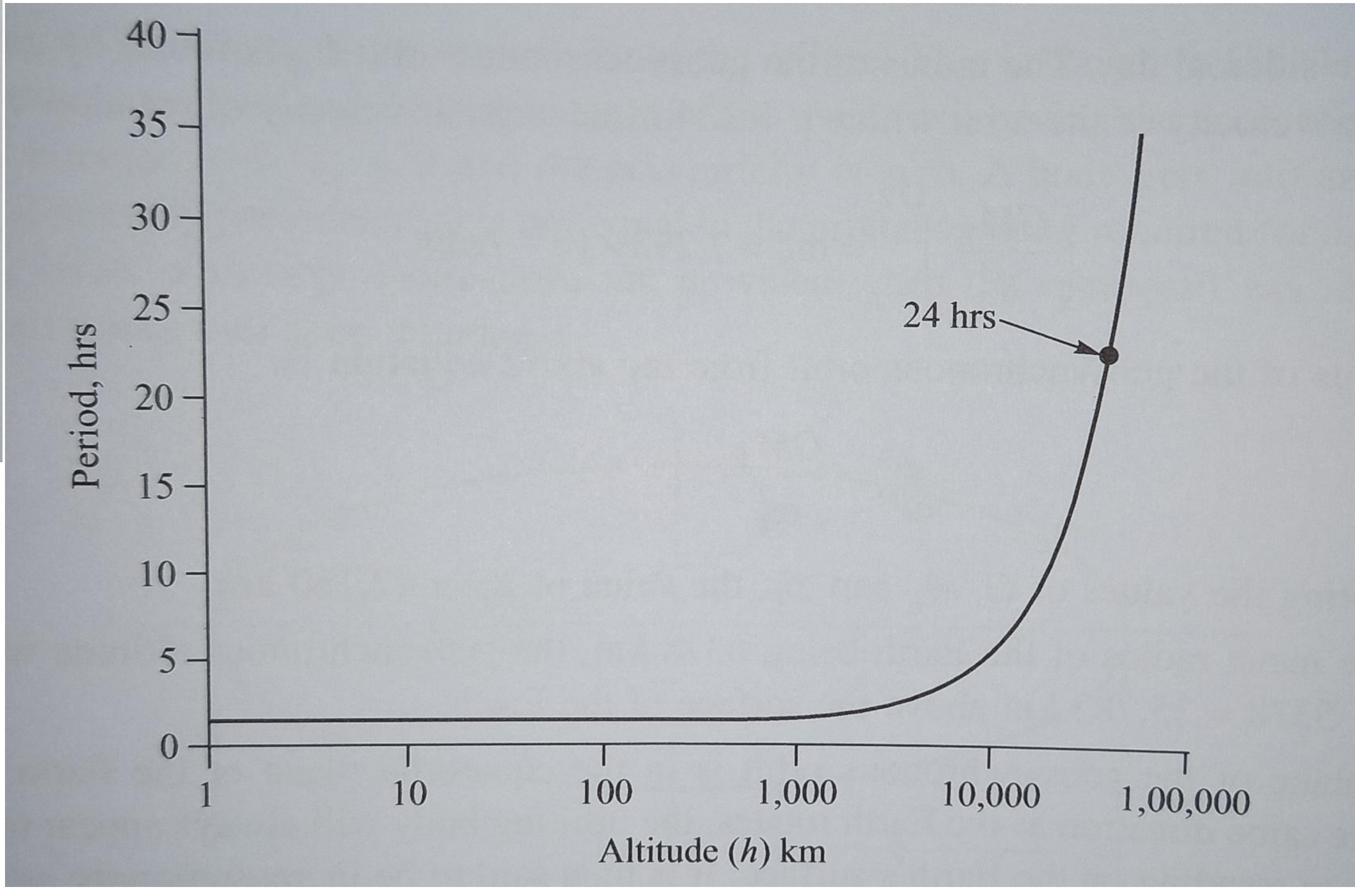
$$u = \sqrt{\frac{\mu}{R}} = R_o \sqrt{\frac{g_o}{R_o + h}}$$

Period of rotation, $T = \dfrac{2\pi (R_o + h)}{u} = \dfrac{2\pi \cdot (R_o + h)^{3/2}}{R_o \sqrt{g_o}}$

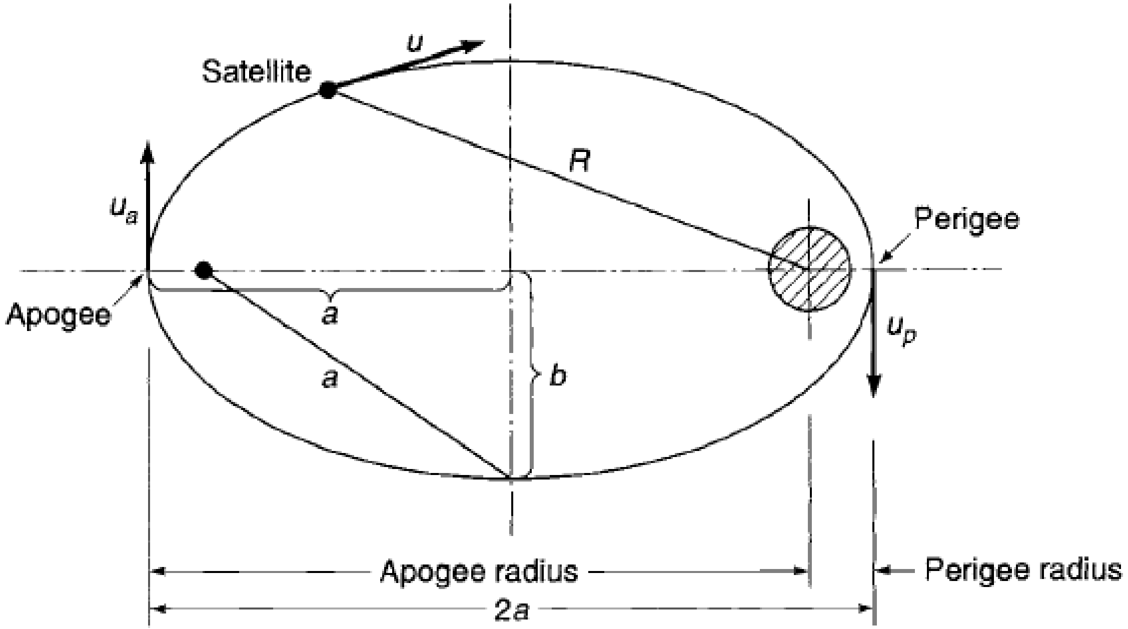
$$\therefore \quad T \sim (R_o + h)^{3/2}$$

Velocity in circular orbit





Velocity in elliptic orbit



a – Semi major axis
b – Semi minor axis

Eccentricity, $$e = \frac{\sqrt{a^2 - b^2}}{a}$$

Orbital velocity, $$u = \left[\mu\left(\frac{2}{R} - \frac{1}{a}\right)\right]^{1/2}$$

Ideal energy requirements for putting a payload in circular orbit around earth

Work done to raise the body to a distance R from the earth.

$$= \int_{R_0}^{R} \frac{GMm}{r^2}\, dr = \text{Energy provided to the body (Potential Energy)}$$

$$= GMm \left(\frac{1}{R_0} - \frac{1}{R} \right)$$

Ideal energy requirements for putting a payload in circular orbit around earth

Total energy provided to satellite in orbit with radius R,

$$E_T = P.E. + K.E. = GMm\left(\frac{1}{R_0} - \frac{1}{R}\right) + \frac{mu^2}{2}$$

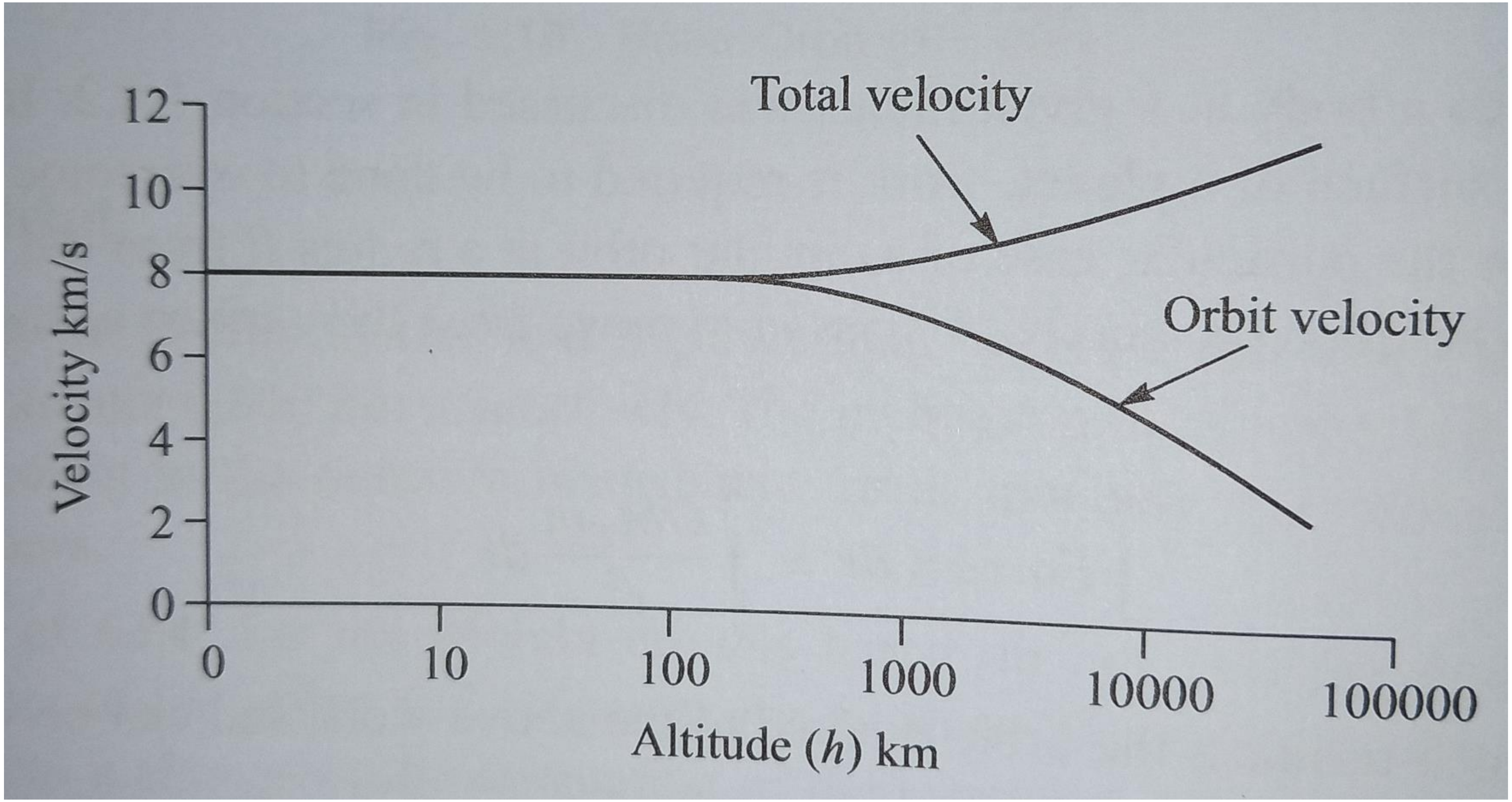
For circular orbit, $u = \sqrt{\frac{GM}{R}}$

$$\therefore \quad E_T = GMm\left[\frac{1}{R_0} - \frac{1}{R} + \frac{1}{2R}\right] = \boxed{GMm\left[\frac{1}{R_0} - \frac{1}{2R}\right]}$$

Velocity requirement, $E_T = \frac{1}{2} m u_T^2$

$$\therefore \quad \boxed{u_T = \sqrt{\frac{GM(R_0+2h)}{R_0(R_0+h)}}}$$

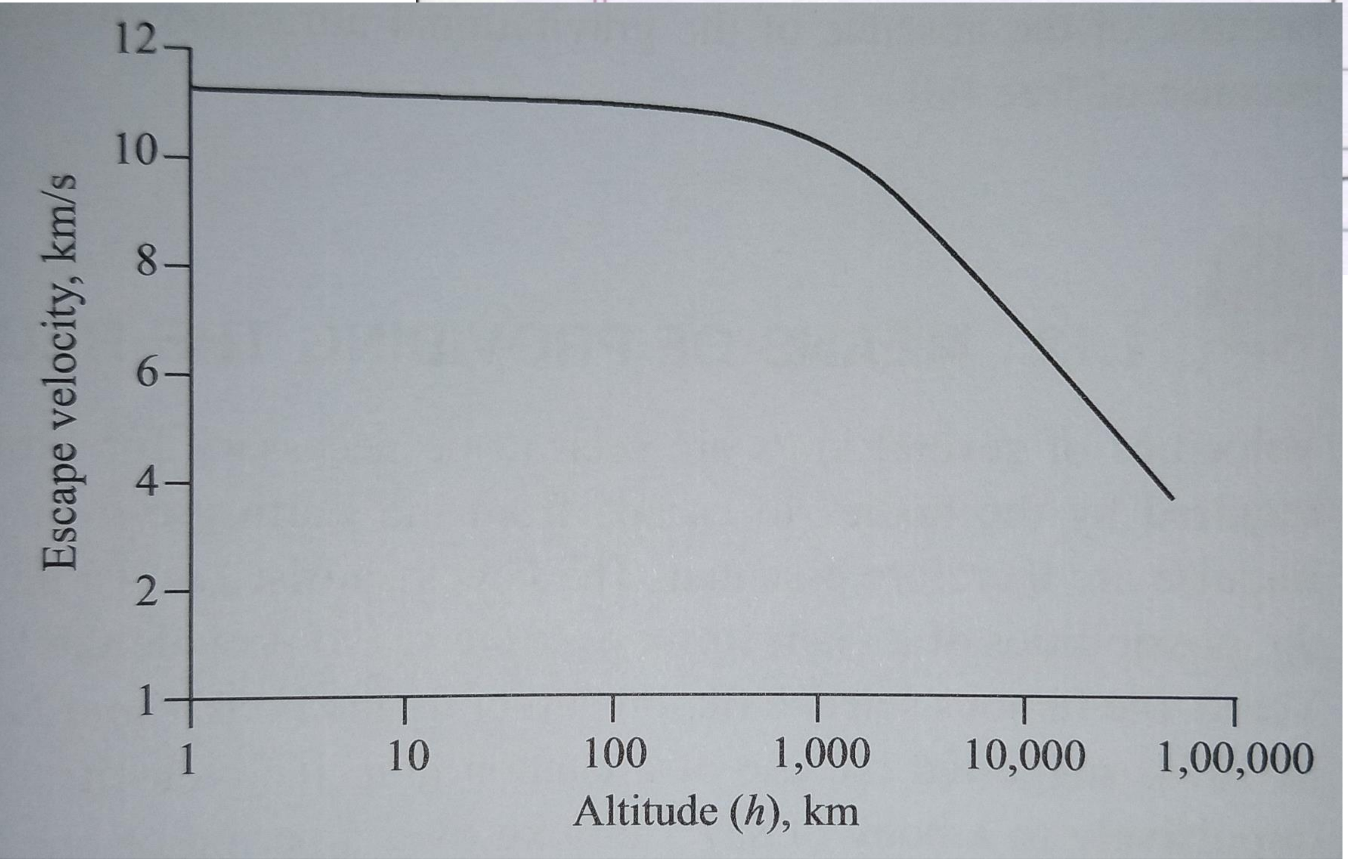
Ideal energy requirements for putting a payload in circular orbit around earth

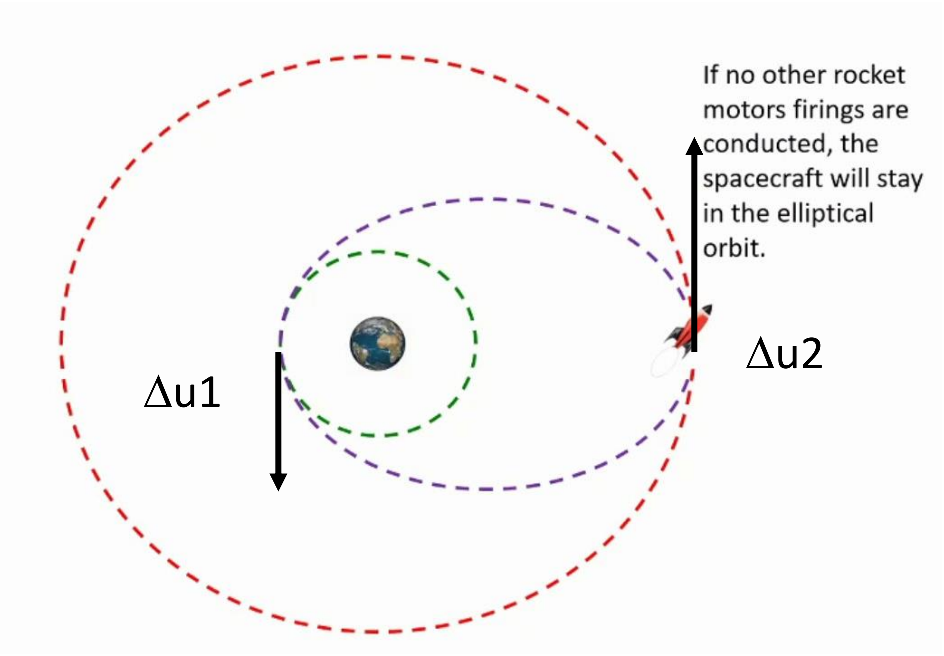


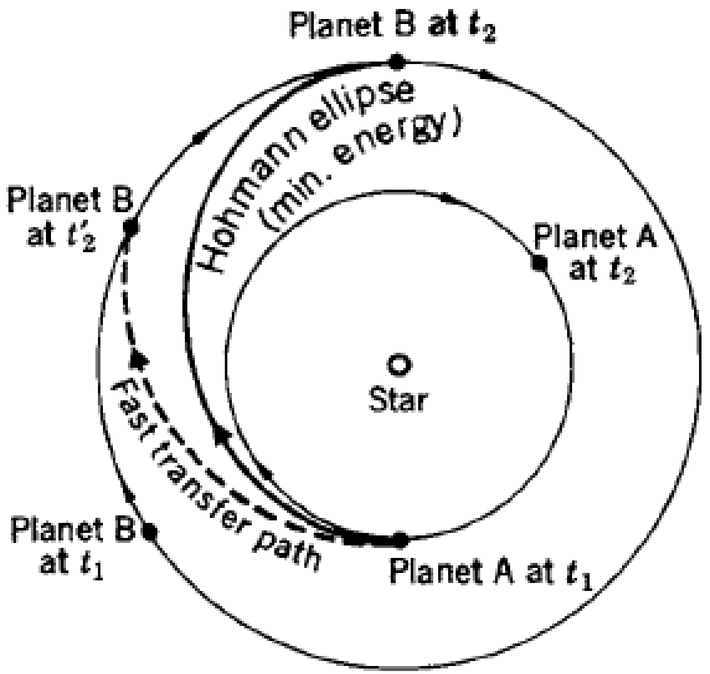
Escape velocity from earth's surface

Escape velocity, $\Rightarrow \frac{1}{2} m u_e^2 = \int_{R_o}^{\infty} \frac{GMm}{r^2}\, dr$

$$\boxed{u_e = \sqrt{\frac{2GM}{R_o}}} \approx 11.2 \text{ km/s}$$







Different orbits and orbital mechanics

Hohmann transfer : Transfer of payload from low earth circular orbit to high earth circular orbit

This transfer orbit is elliptic and the transfer is realized by giving velocity impulses at apogee and perigee

Minimum energy transfer

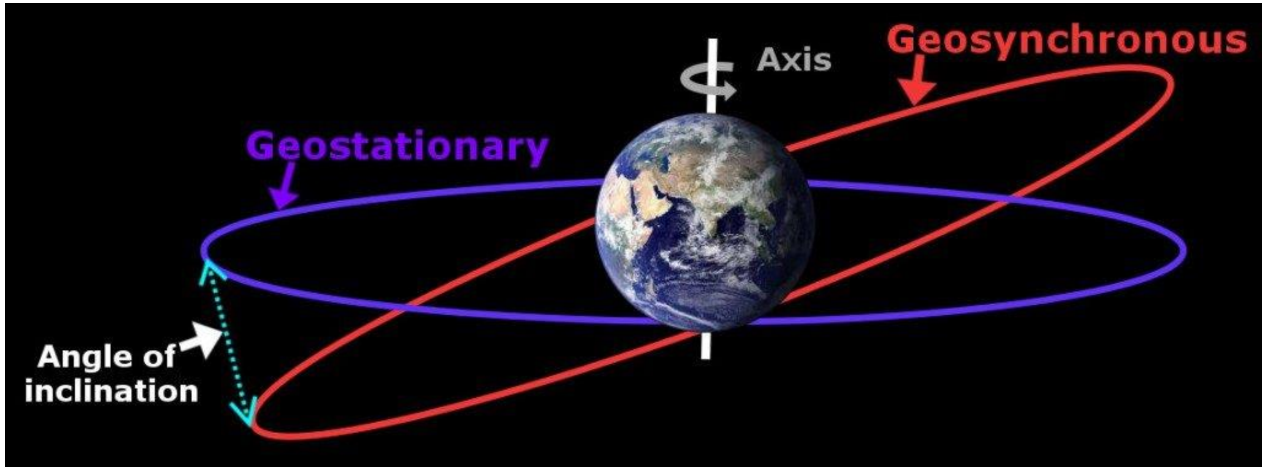
Single transfer orbit is used when the ratio of altitudes of final and initial orbits is less than 16

Geosynchronous/Geostationary orbit:

Period of the orbit = 24 hrs

Typical altitude ~ 36000 km

Communication satellites

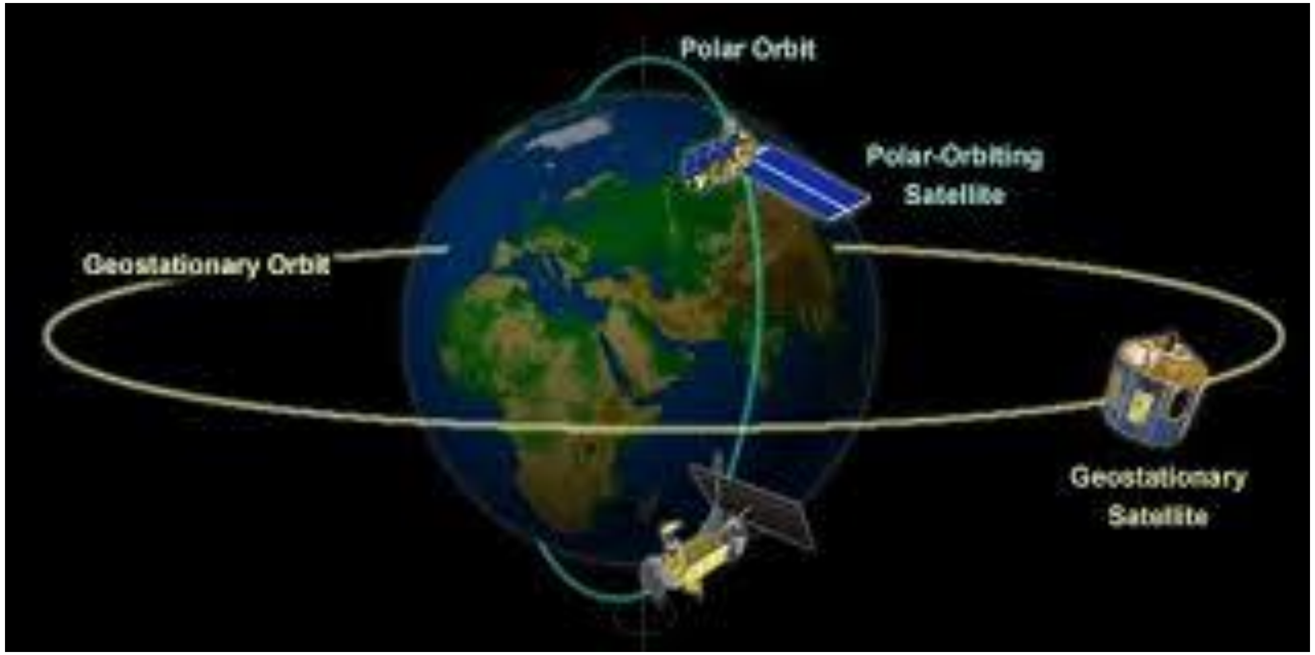


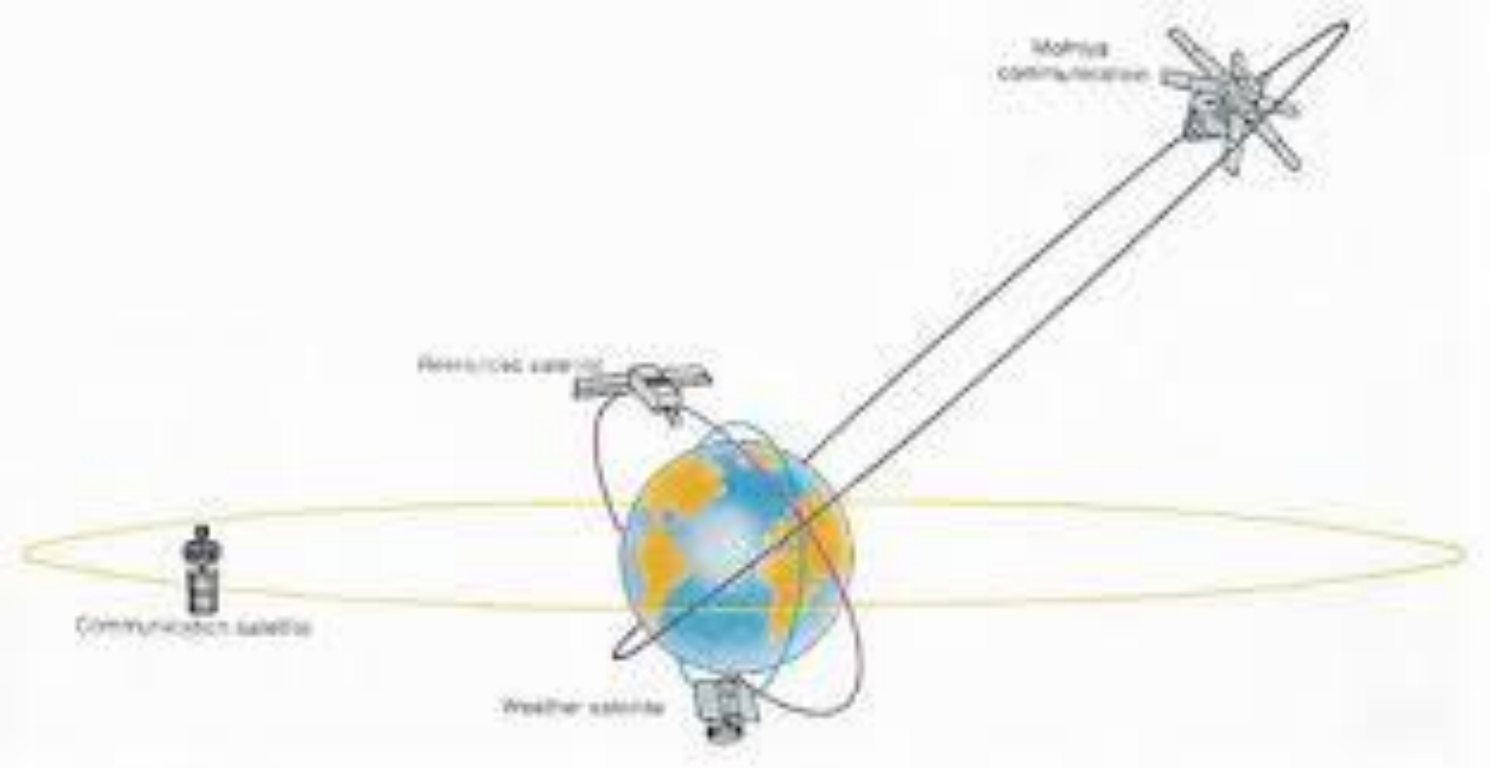
Polar orbit:

Altitude: a few hundred kilometers

Inclination ~ 90-95 degrees

Remote sensing, surveillance satellites





Molniya orbit

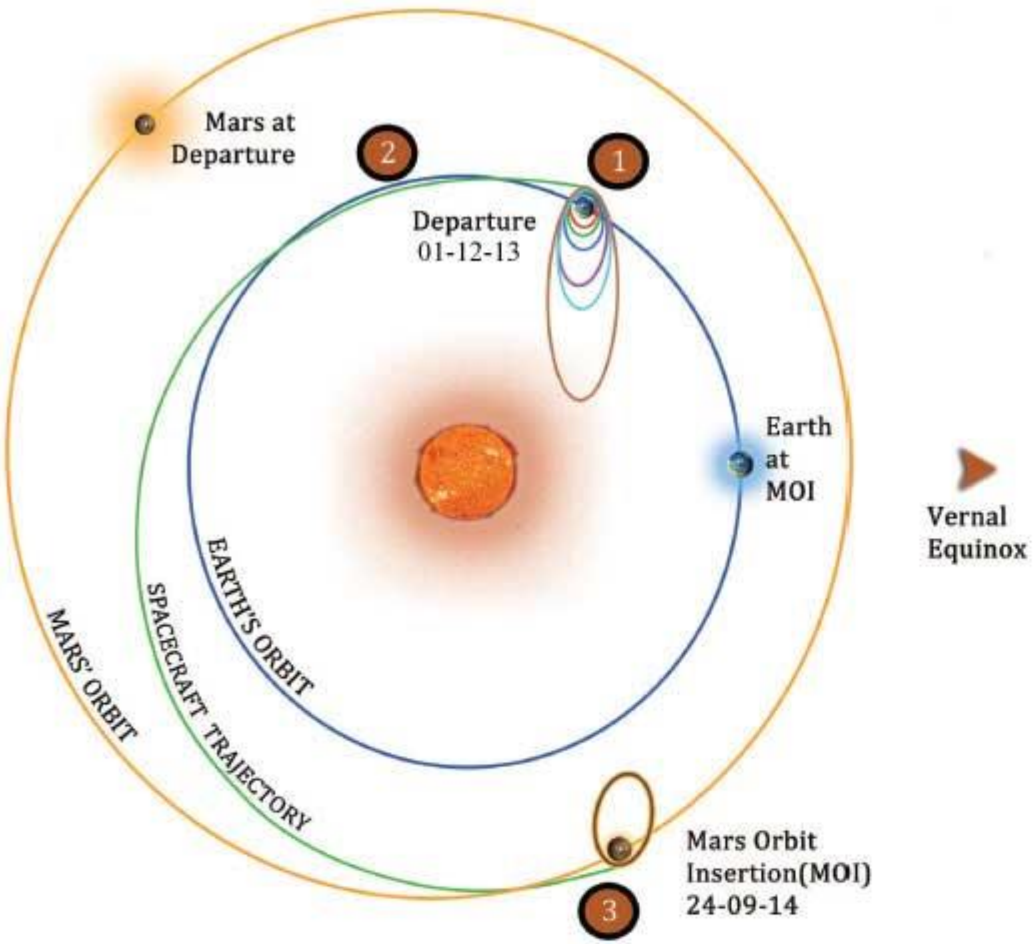
Useful for countries with high latitudes

Inclination – 63.4 degress

Elliptic orbit with perigee ~ 300-400 km and apogee ~ 40000-50000 km

Orbital period – 12 hrs

A set of 3 satellites will give a coverage of 24 hrs



Trajectory Design

Typical orbit for Mars mission

Parking orbit around earth ~ 248 x 23000 km

Escape from earth and transfer orbit in the sphere of influence of sun

Departure in hyperbolic orbit and insertion in elliptic coasting orbit around sun

Capture in the orbit around Mars

Mission Velocity

Important parameter to be decided for selecting/designing the propulsion system

Indicative of the magnitude of energy requirement for any space mission

Summation of all the velocity increments needed to attain mission objective

Some velocity increments are obtained by retro-action (i.e. negative Δu); these require energy and hence their magnitudes are also counted in mission velocity

Test case:

Estimation of mission velocity for launching at Cape Kennedy, bringing a space vehicle in an orbit of 110 km and then entering a de-orbit manoeuvre.

Mission velocity for launching a space vehicle in an circular orbit of 110 km and then entering in a deorbit maneuver

Ideal satellite velocity	7790 m/s
Velocity requirement against gravity losses	1220 m/s
Velocity requirement for turning the vehicle	360 m/s
Velocity requirement against drag	118 m/s
Orbit injection	145 m/s
Deorbit to reenter	60 m/s
Correction and velocity adjustment	62 m/s
Initial earth’s velocity	-408 m/s
Total mission velocity	**9347 m/s**

Mission	Ideal velocity, km/s	Actual velocity, km/s
Satellite (no return)	7-10	9-12.5
Escape	11.2	12.9
Escape from moon	2.3	2.6
Earth to moon (no return)	13.1	15.2
Earth-moon-earth	15.9	17.7
Earth to Mars	17.5	20

How to correlate the mission objective (i.e. payload velocity or the payload energy) with the parameters of the propulsion system?

Gravity-free and drag-free space flight

Mainly applicable for outer deep space where there is no drag and very minimal gravitational influence

Flight direction is in line with the thrust direction i.e. one dimensional straight line acceleration

Thrust, $F = M \dfrac{du}{dt}$

m = instantaneous vehicle mass

u = instantaneous vehicle velocity

$\therefore$ Change in vehicle velocity

$$du = \frac{F}{M} dt \qquad \text{——— (1)}$$

Let $\dot{m}$ = Mass flow rate of propellant

c = Effective exhaust velocity

$$\Rightarrow F = \dot{m} \cdot c$$

Let $M_0 =$ Initial mass of the vehicle

$M_p =$ Mass of the propellant loaded in the vehicle

$M_b =$ Final mass of vehicle after all the propellant is burnt.

Let $t_b =$ Burning duration

Using this nomenclature, $\dot{m} = \dfrac{M_p}{t_b}$

From Eq^n ①, $du = \dfrac{\dot{m} c}{M} dt$ —— ②

If we consider a time instant 't' after the start of the propellant burning, instantaneous mass M can be written as,

$$M = M_0 - \dot{m} \cdot t = M_0 - \frac{M_p}{t_b} \cdot t$$

From Eq^n (2), $du = \dfrac{\dot{m}c}{M_0 - \dot{m}t} \, dt$

Integrate with limits '0' to 't_b' and we can get the net change in velocity of the vehicle 'Δu'.

$$\therefore \quad \Delta u = c \int_0^{t_b} \frac{\dot{m} \, dt}{M_0 - \dot{m}t}$$

$$\therefore \quad \Delta u = -c \Big[\ln (M_0 - \dot{m}t) \Big]_0^{t_b}$$

$$\therefore \quad \Delta u = -c \Big[\ln (M_b) - \ln (M_0) \Big]$$

At $t = t_b$

$$M_0 - \dot{m} \cdot t_b = M_0 - M_p$$
$$= M_b$$

At $t = 0$,

$$M_0 - \dot{m}(0) = M_0$$

$$\therefore \quad \Delta u = -c \ln \left[\frac{M_b}{M_o}\right] \qquad \text{or}$$

$$\Delta u = c \ln \left[\frac{M_o}{M_b}\right]$$

$$\frac{M_o}{M_b} = \text{Mass Ratio} = R$$

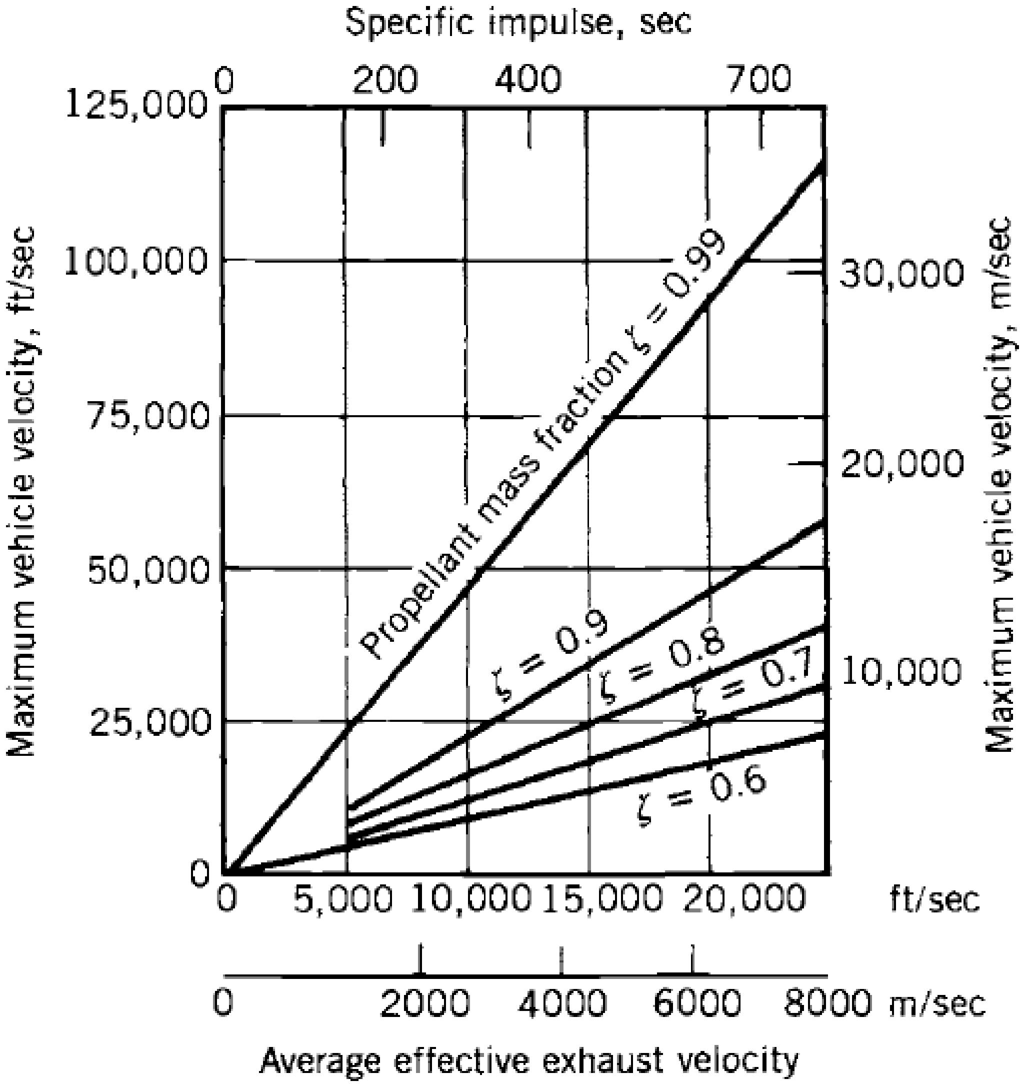
$$\therefore \quad \boxed{\Delta u = c \ln R}$$

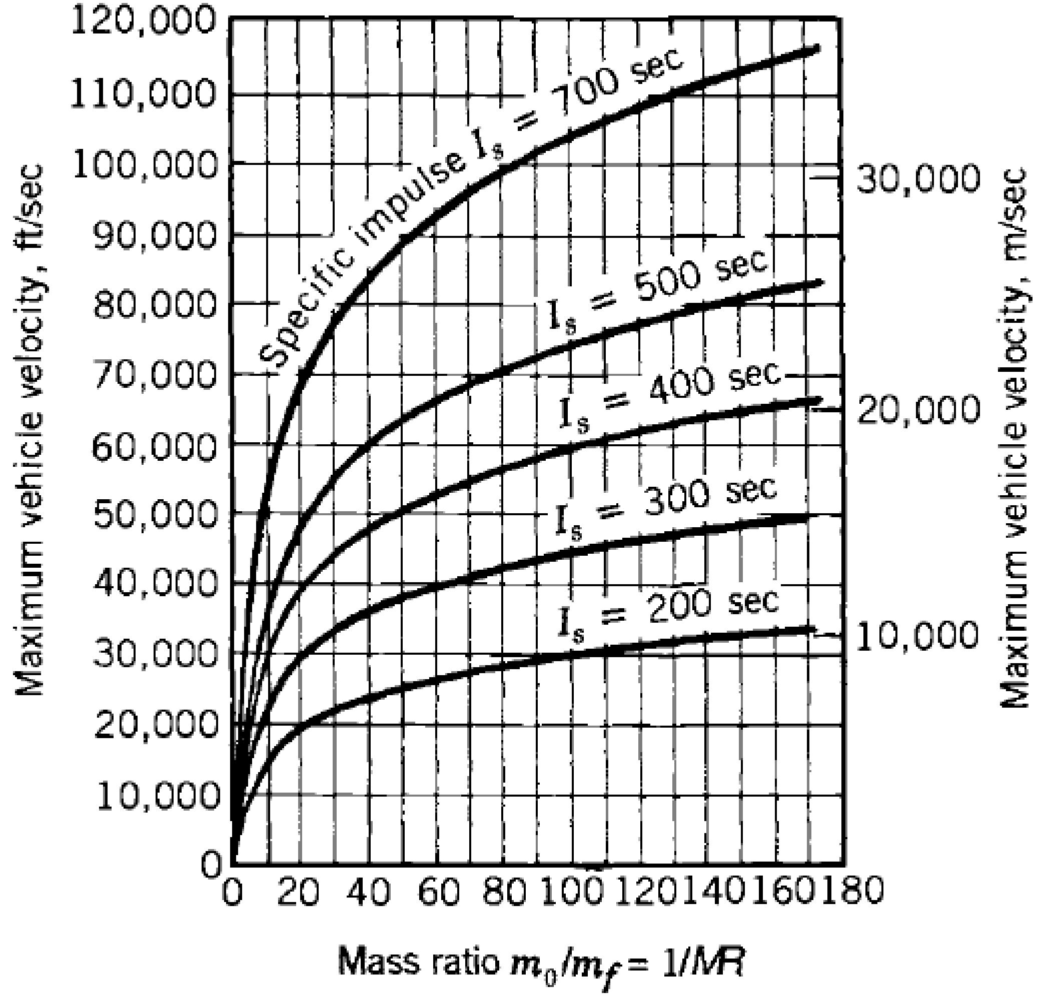
In terms of specific impulse,

$$\boxed{\Delta u = I_s \cdot g_o \cdot \ln(R)}$$

$$\boxed{R = \frac{M_o}{M_b} = e^{\Delta u / c}}$$

Important parameter in the design of a rocket





Change in the propellant mass fraction from 0.8 to 0.9 increases the vehicle velocity by 43%.

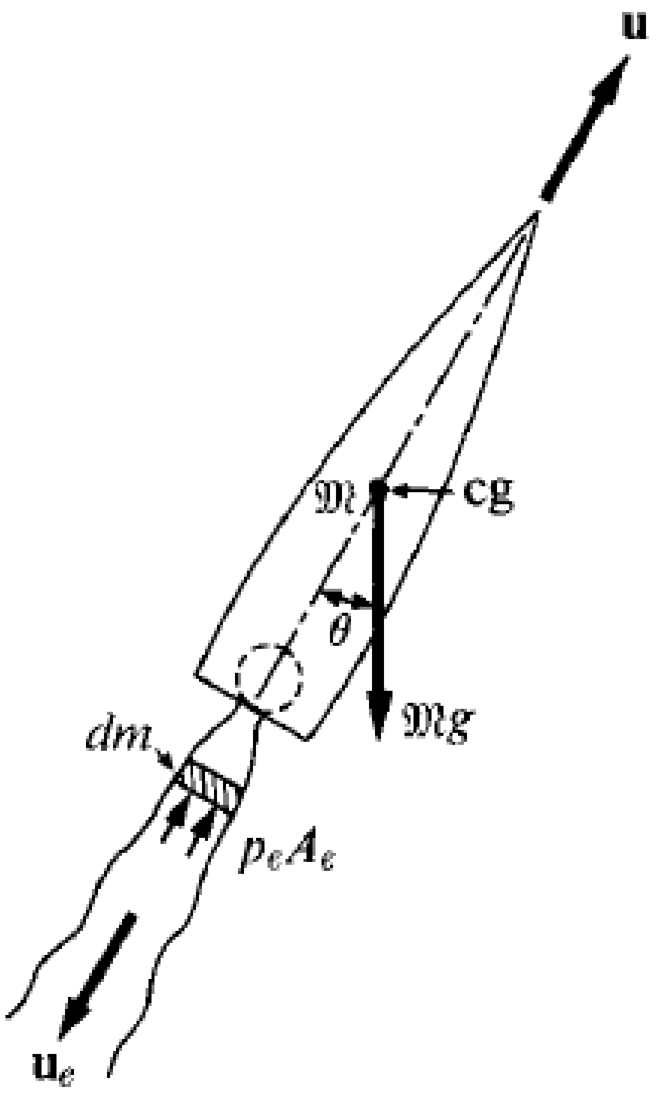
From mission velocity of payload → vehicle mass ratio is determined (a starting point for the vehicle design)

Example: Consider a cryogenic engine with specific impulse of 450 sec and assume gravitational acceleration to be 10 m/s^2. Determine the propellant loading for such a rocket vehicle if the mission objective is 9 km/s.

$$\boxed{R = \frac{M_o}{M_b} = e^{\Delta u/c}}$$

Important parameter in the design of a rocket

Propellant loading ($M_p/M_o = 1 - 1/R$) is 87%.



$$M \cdot \frac{du}{dt} = \sum \text{All forces acting on the vehicle}$$

$$\therefore \quad M\frac{du}{dt} = \underset{\text{Thrust}}{F} - \underset{\text{Drag}}{D} - \underset{\text{Component of vehicle weight}}{M \cdot g \cdot \cos\theta}$$

$$\therefore \quad du = \frac{\dot{m}c}{M}\,dt - \frac{D}{M}\,dt - g \cdot \cos\theta\,dt$$

On integrating,

$$\Delta u = c\ln(R) - \int_0^{t_b} \frac{D}{M}\,dt - \int_0^{t_b} g \cdot \cos\theta\,dt$$

Gravity

Gravity:

$$g = g_0 \left(\frac{R_0}{R_0 + h} \right)^2$$

When $h \ll R_0$

$$g \approx g_0$$

$$\boxed{\Delta u = c \ln(R) - g_0 \overline{\cos\theta}\, t_b}$$

Gravitational acceleration does not change much for altitudes small compared to earth's surface

Absolute thrust as important as specific impulse for achieving reasonable acceleration near earth's surface

Thrust of the vehicle ~ 1.5 – 2 times the initial weight of the vehicle

Drag

Drag:

$$D = C_D \cdot \frac{1}{2}\rho u^2 \cdot A$$

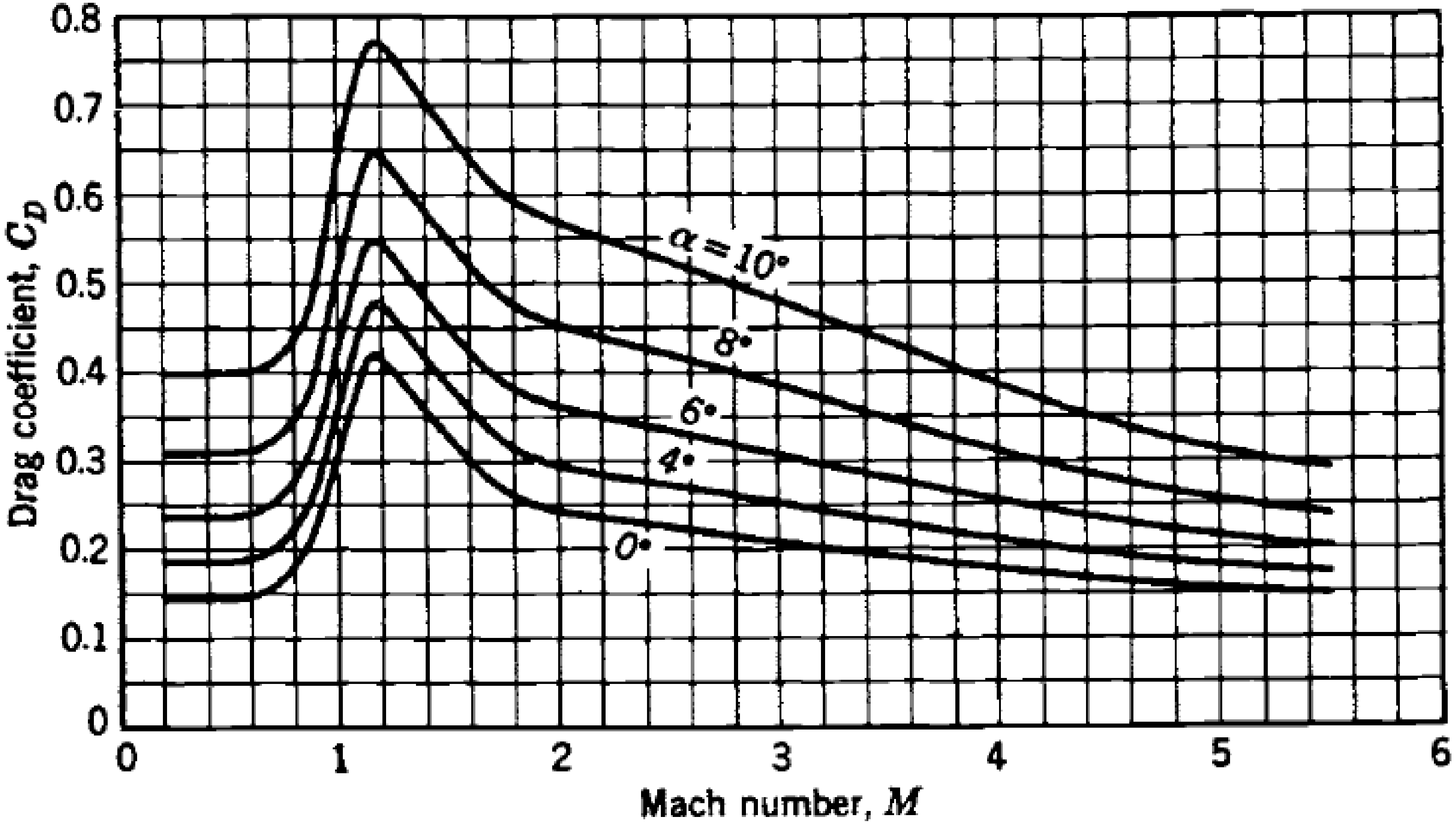
$C_D = f(M)$

(Drag coefficient)

$$\rho(h) = a \cdot \exp\left(-b \cdot h^{1.15}\right)$$

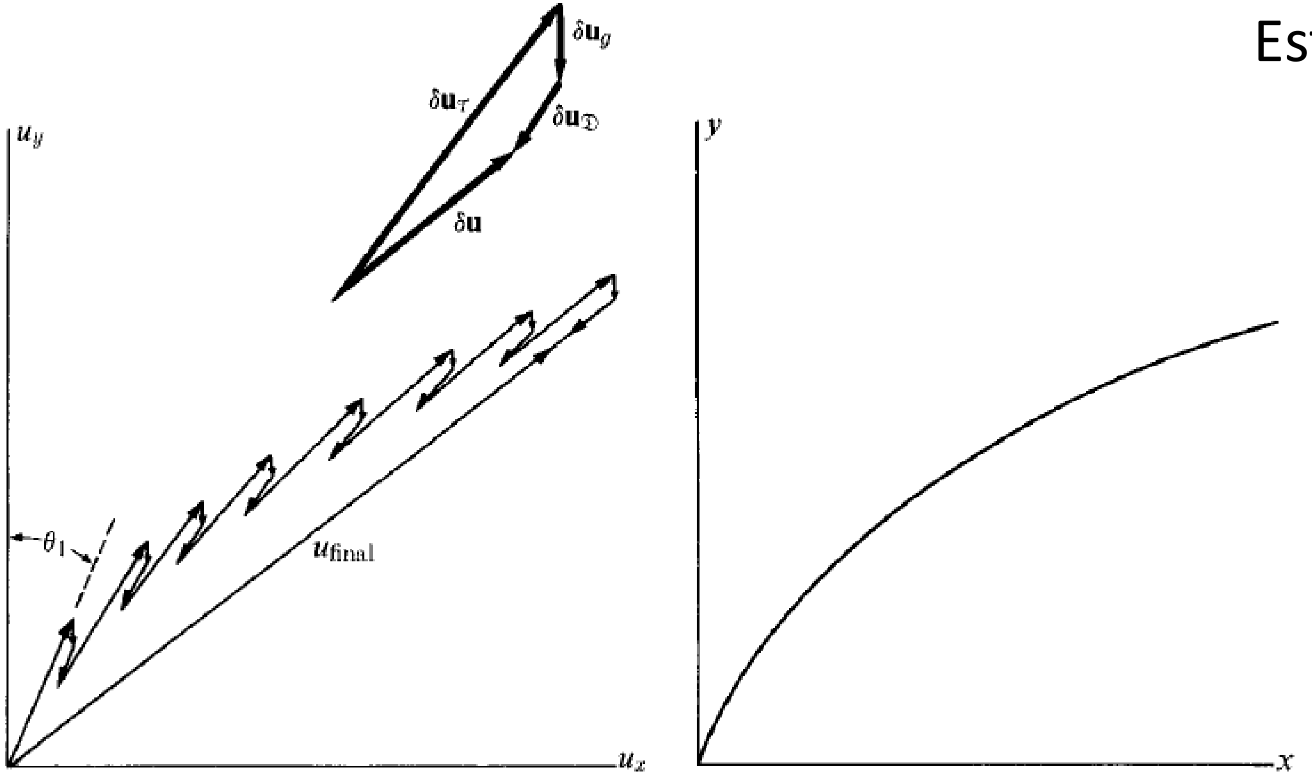
$a = 1.2$, $b = 2.9 \times 10^{-5}$

(for SI units)



At h ~ 30 km, density becomes 1 % of the sea level value

With adequate drag date, it is possible to calculate the actual performance of the engine



(a) Velocity coordinates (b) Position coordinates

Estimation of the trajectory of the rocket

Gravity turn: Turning the vehicle under the presence of gravity

Stepwise changes in the velocity components under various forces

Accuracy of the technique depends upon relative magnitude of Δt with respect to tb.

Net δu in time $\Delta t = \vec{\delta u_F} + \vec{\delta u_D} + \vec{\delta u_g}$

~~Pa~~ Parallel to u

Parallel & oppusite to 'u'

Parallel to $\vec{g}$

$\delta y = \Delta t \cdot u_y$ & $\delta x = \Delta t \cdot u_x$

Single stage sounding rocket – Determination of height at burn-out and maximum height reached

Neglect drag and assume constant effective exhaust velocity

For vertical flight, the altitude attained at burn-out (i.e. at time instant tb):

$$h_b = \int_0^{t_b} u \cdot dt \qquad \text{where} \qquad u = -c \ln \frac{M}{M_o} - g \cdot t$$

Mass `M' (instantaneous) varies with time $\Rightarrow$ $M = M_o - \frac{(M_o - M_b)}{t_b} \cdot t$

$$\therefore \quad u = -c \ln \left[1 - \left(1 - \frac{1}{R} \right) \frac{t}{t_b} \right] - gt$$

$$\Rightarrow \quad h_b = -c \cdot t_b \frac{\ln R}{R-1} + c \cdot t_b - \frac{1}{2} g \cdot t_b^2$$

Equating K.E. of the mass at burn-out with its P.E. between that point and maximum height (h_{max}) $\Rightarrow$ $M_b \cdot \frac{u_b^2}{2} = M_b \cdot g \cdot (h_{max} - h_b)$

$\therefore$ $h_{max} = h_b + \frac{u_b^2}{2g}$ $\Rightarrow$ $h_{max} = \frac{c^2 (\ln R)^2}{2g} - c \cdot t_b \left[\frac{R}{R-1} \ln R - 1 \right]$

Importance of burning time

Short burning times are desirable because the burn-out velocity as well as the altitude reached decrease with increase in burning time when rest of the parameters are same.

Very short burning times are not desirable → due to structural integrity issue as well as max. acceleration constraints from instrumentation. Further, high vehicle velocities would increase drag in near earth region.

Burning time → of not much importance in the absence of gravity and drag