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Answer

We will be using three main formulae

$$\Delta V = I_{sp} g_o \log \frac{M_i}{M_o} = I_{sp} g_o \log MR \quad \text{--- B}$$

$$\xi = \frac{M_p}{M_i} = \frac{MR - 1}{MR} \quad \text{-----C}$$

$$\frac{M_{pl}}{M_i} = 1 - \frac{\xi}{\lambda} \quad \text{-----D}$$

where

$$\lambda = \frac{M_p}{M_s + M_p}$$

$$M_i = M_{pl} + M_s + M_p$$

Now

$$\Delta V_T = \Delta V_1 + \Delta V_2 \quad \text{and both are equal}$$

$$\text{so } \Delta V_1 = \Delta V_2 = \Delta V_T / 2 = 5000 \quad ft/s$$

Now using relation B

$$MR = e^{\frac{\Delta V}{I_{sp} g_o}}$$

$$\text{using } g_o = 32.152 \text{ ft/s}^2$$

so

$$\mathbf{MR_1 = 1.77885}$$

$$\mathbf{MR_2 = 1.55943}$$

Now using relation C

$$\xi_1 = 0.437839$$

$$\xi_2 = 0.35874$$

Using relation D

$$\frac{M_{pl1}}{M_{i1}} = 0.513512$$

$$\frac{M_{pl2}}{M_{i2}} = 0.551575$$

Now we know $M_{pl2} = M_{pl} = 500 \text{ lb}$

so we get

$$M_{i2} = 500 / 0.551575 \rightarrow \mathbf{M_{i2} = 906.495 \text{ lb}}$$

Now

$$M_{pl1} = M_{i2} + M_{inter_stage} = 906.495 + 200 = 1106.495 \text{ lb}$$

so

$$M_{i1} = M_{pl1} / 0.513512 \rightarrow \mathbf{M_{i1} = 2154.76 \text{ lb}}$$

Now using propellant fraction we have

$$\begin{aligned} M_{p1} &= \xi_1 M_{i1} \\ &= 0.437839 \times 2154.76 = 943.438 \text{ lb} \end{aligned}$$

$$\begin{aligned} M_{p2} &= \xi_2 M_{i2} \\ &= 0.35874 \times 906.495 = 325.196 \text{ lb} \end{aligned}$$

using definition of lambda we have

$$\lambda = \frac{M_p}{M_s + M_p}$$

$$\frac{1}{\lambda} = \frac{M_s + M_p}{M_p}$$

$$\frac{1}{\lambda} = 1 + \frac{M_s}{M_p}$$

$$\frac{M_s}{M_p} = \frac{1}{\lambda} - 1$$

so we have

$$M_s = M_p \left(\frac{1}{\lambda} - 1 \right)$$

$$\mathbf{M_{s1} = 104.826 \text{ lb}}$$

$$\mathbf{M_{s2} = 81.299 \text{ lb}}$$

Please note small difference in values is due to difference in value of g , kindly use your g in above method

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