Q.1. A linear time-invariant system, initially at rest, when subjected to a unit step input gave a response

$$c(t) = te^{-t} \quad (t \geq 0)$$

Find the transfer function of the system.

Solution:

The input to the system is

$$r(t) = u(t) \Rightarrow R(s) = \frac{1}{s}$$

The output to the system is

$$c(t) = te^{-t} \Rightarrow C(s) = \frac{1}{(s+1)^2}$$

The transfer function of the system is then given by

$$T(s) = \frac{C(s)}{R(s)} = \frac{s}{(s+1)^2}$$

Q.2. The unit impulse response of a second-order system is

$$c(t) = \frac{1}{6}\, e^{-0.8t}\, sin(0.6t)$$

Find the natural frequency and damping ratio of the system.

Solution:

The transfer function of the system is given by

$$T(s) = \frac{C(s)}{R(s)} = \frac{1}{6}\frac{0.6}{(s+0.8)^2 + (0.6^2)} = \frac{1}{10}\left(\frac{1}{s^2 + 1.6s + 1}\right)$$

Comparing the characteristic polynomial with $s^2 + 2\zeta\omega_n s + {\omega_n}^2$, we get

$${\omega_n}^2 = 1 \Rightarrow \omega_n = 1\ rad/s \quad \text{and} \quad 2\zeta\omega_n = 1.6 \Rightarrow \zeta = 0.8$$

Q.3. The transfer function of a system, initially at rest, is given by

$$T(s) = \frac{V(s)}{I(s)} = \frac{s+3}{4s+5}$$

If the excitation $i(t)$ is a unit step signal, then find the initial and steady-state values of $v(t)$.

Solution:

The initial value of the response is given by

$$\begin{aligned} v(0) &= \lim_{s\to\infty} \; sV(s) \\ &= \lim_{s\to\infty} \; sT(s)I(s) \\ &= \lim_{s\to\infty} \; s\left(\frac{s+3}{4s+5}\right)\frac{1}{s} \\ &= \frac{1}{4} = 0.25 \end{aligned}$$

The steady-state value of the response is given by

$$\begin{aligned} \lim_{t\to\infty} v(t) &= \lim_{s\to 0} \; sV(s) \\ &= \lim_{s\to 0} \; sT(s)I(s) \\ &= \lim_{s\to 0} \; s\left(\frac{s+3}{4s+5}\right)\frac{1}{s} \\ &= \frac{3}{5} = 0.6 \end{aligned}$$

Q.4. Find the equivalent transfer function $T(s) = \dfrac{C(s)}{R(s)}$ for the system shown in Figure 1.

Answer:

$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_3(s)[1+G_2(s)]}{[1+H_3(s)G_3(s)][1+H_2(s)G_2(s)+H_1(s)G_1(s)G_2(s)]}$$

Q.5. For a unity feedback control system with a forward-path transfer function $G(s) = \dfrac{16}{s(s+a)}$, find the value of a to yield a closed-loop step response that has 5% overshoot.

Answer: $a = 5.52$

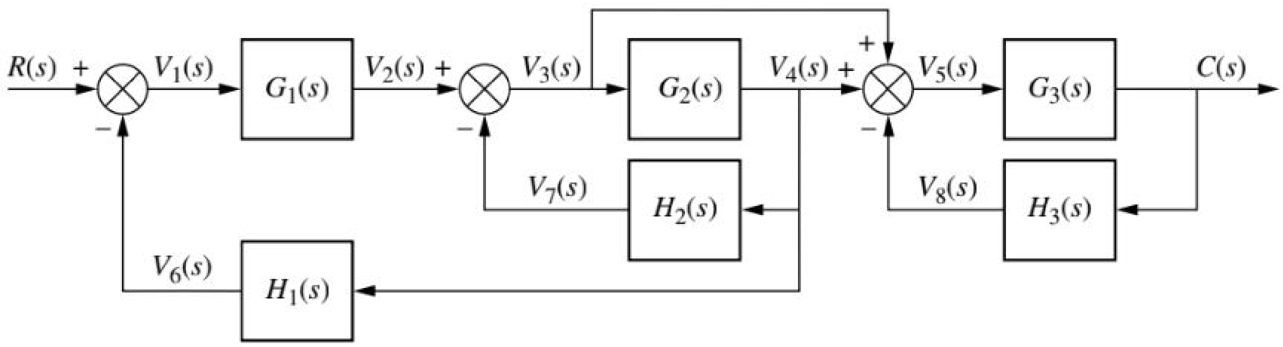


Figure 1: Block Diagram

Q.6. For the system of Figure 2, find the values of K_1 and K_2 to yield a peak time of 1.5 second and a settling time of 3.2 seconds for the closed-loop system response, when $R(s)$ is a step input.

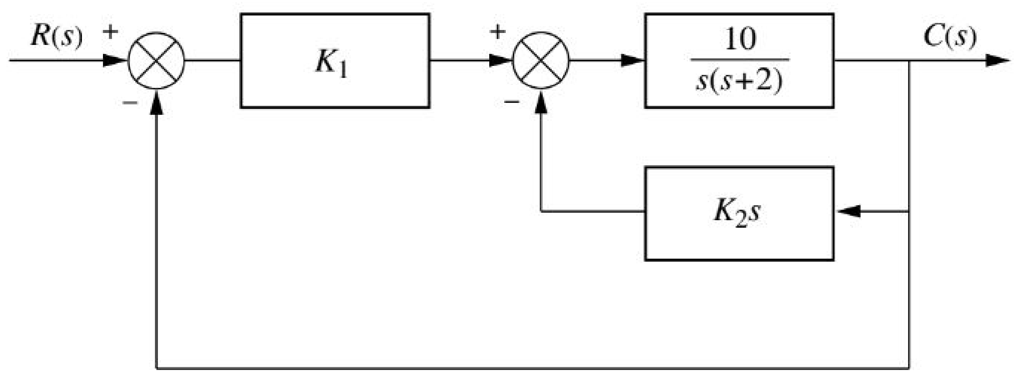
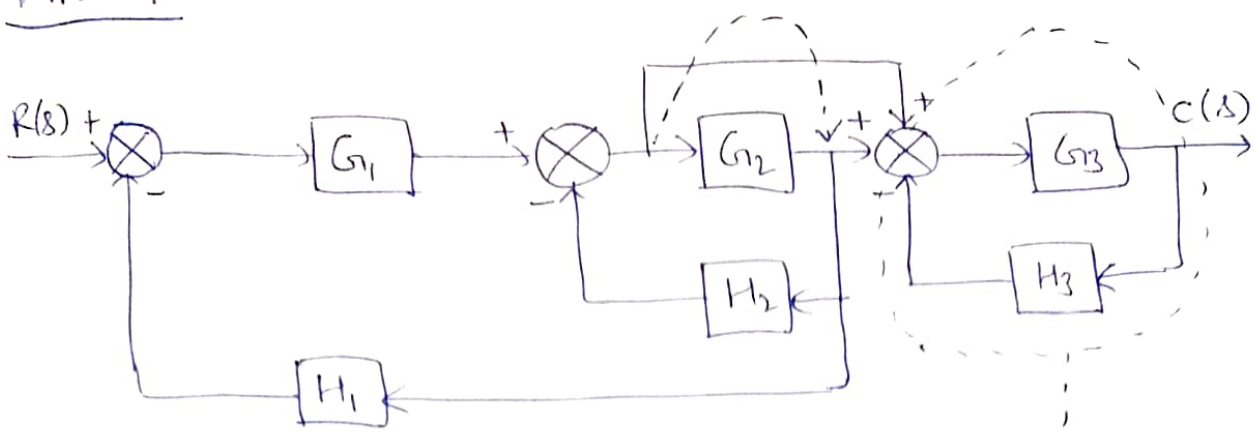
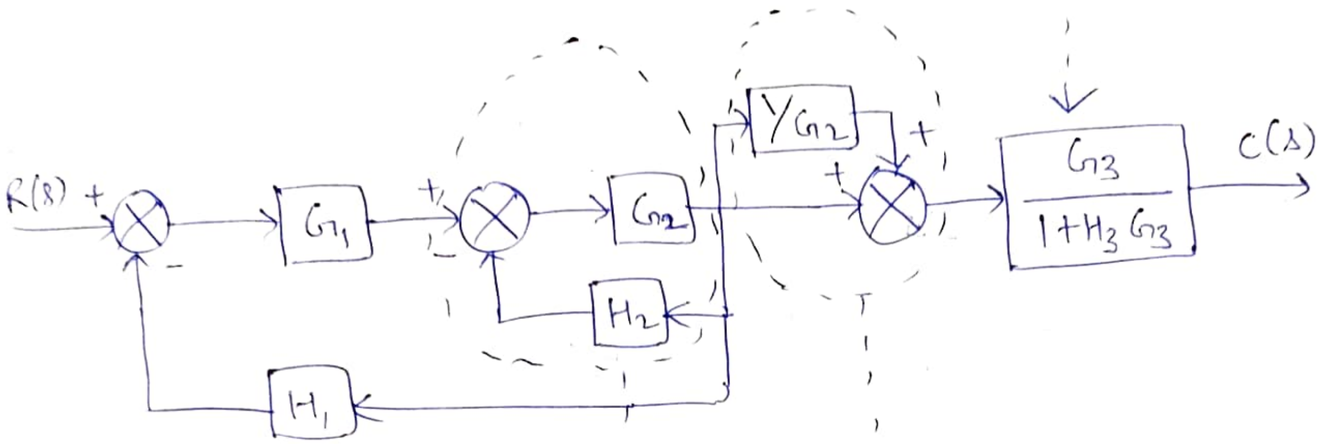


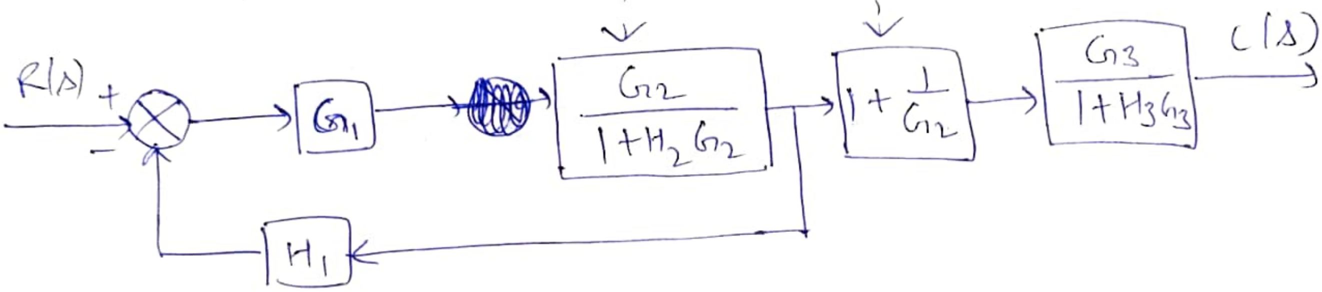
Figure 2: Block Diagram

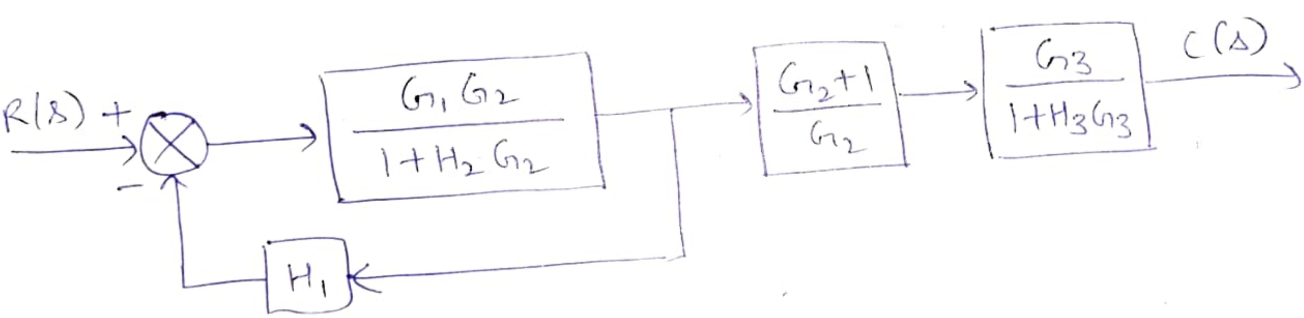
Answer: $K_1 = 0.5951$, $K_2 = 0.05$.

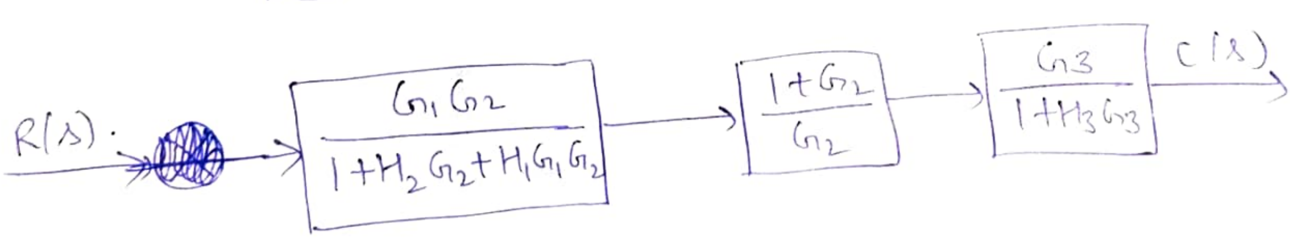
Ans-4


R(s)
G1
G2
G3
H2
H3
H1
C(s)

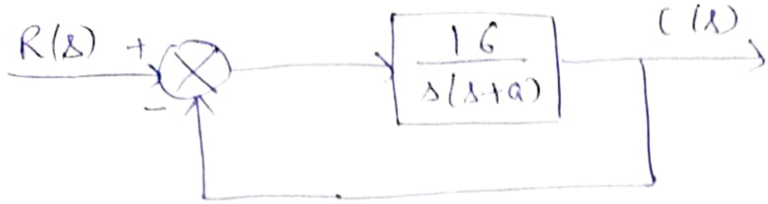

R(s)
G1
G2
1/G2
H2
H1
$\frac{G_3}{1+H_3G_3}$
C(s)


R(s)
G1
$\frac{G_2}{1+H_2G_2}$
$1+\frac{1}{G_2}$
$\frac{G_3}{1+H_3G_3}$
H1
C(s)


R(s)
$\frac{G_1G_2}{1+H_2G_2}$
$\frac{G_2+1}{G_2}$
$\frac{G_3}{1+H_3G_3}$
H1
C(s)


R(s)
$\frac{G_1G_2}{1+H_2G_2+H_1G_1G_2}$
$\frac{1+G_2}{G_2}$
$\frac{G_3}{1+H_3G_3}$
C(s)

Ans - 5



$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + as + 16}$$

Comparing with $\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$,

we get $\omega_n = 4$ and $\boxed{a = 8\xi}$

Given overshoot desired is 5%.

$$e^{-\pi \cot\phi} = 0.05$$

where $\cos\phi = \xi$

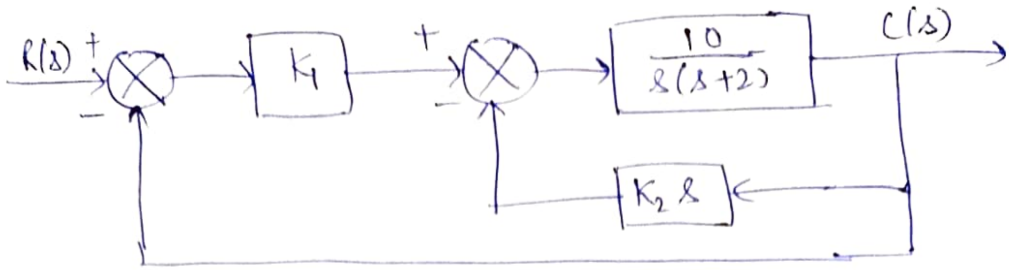
$e^{-\pi \cot\phi} = 0.05 \Rightarrow -\pi \cot\phi = -2.9957$

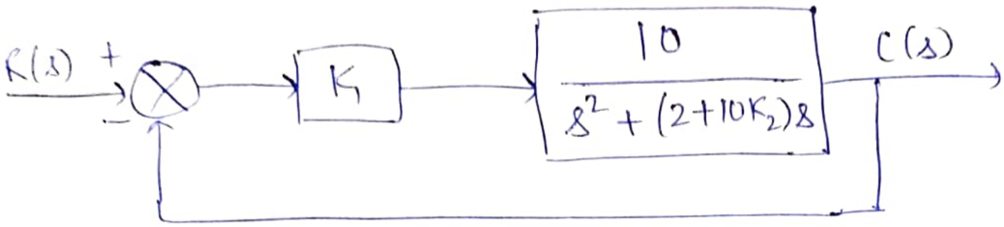
$\Rightarrow \tan\phi = \dfrac{\pi}{2.9957} \Rightarrow \phi = 46.3617^\circ$

$\Rightarrow \xi = \cos\phi = 0.6901$

$a = 8\xi = \underline{\underline{5.52}}$

Ans- 6





$$\frac{C(s)}{R(s)} = \frac{10K_1}{s^2+(2+10K_2)s+10K_1}$$

Comparing with $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$$2+10K_2 = 2\xi\omega_n \Rightarrow K_2 = \frac{2(\xi\omega_n-1)}{10}$$

$$10K_1 = \omega_n^2 \Rightarrow K_1 = \frac{\omega_n^2}{10}$$

given settling time, $T_s = 3.2$; $T_s = \frac{4}{\xi\omega_n}$

$$\Rightarrow \xi\omega_n = \frac{4}{3.2} = 1.25$$

we, get $K_2 = \frac{2(1.25-1)}{10} = \underline{0.05}$

also, peak time, $T_p = 1.5$; $T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$$\frac{T_s}{T_p} = \frac{4}{\xi\cancel{\omega_n}} \times \frac{\cancel{\omega_n}\sqrt{1-\xi^2}}{\pi} = \frac{3.2}{1.5} \Rightarrow \frac{1-\xi^2}{\xi^2} = \frac{64\pi^2}{225}$$

$$\Rightarrow \xi = 0.5124 \Rightarrow \omega_n = \frac{1.25}{0.5124} = 2.4395$$

$$K_1 = \frac{\omega_n^2}{10} = \underline{0.5951}$$