AE 308: Control Theory
AE 775: System Modelling, Dynamics & Control

Lecture 11: Non Minimum Phase System

Dr. Arnab Maity
Department of Aerospace Engineering
Indian Institute of Technology Bombay
Powai, Mumbai 400076, India

Introduction - Minimum Phase System

Minimum phase system

- **Definition** - Systems which have all their zeros and poles in the LH s-plane are termed minimum phase system due to their specific phase characteristics.
- For example,

$$G(s) = \frac{(s+1)}{(s+2)(s+3)(s+4)}$$

Introduction - Non Minimum Phase System

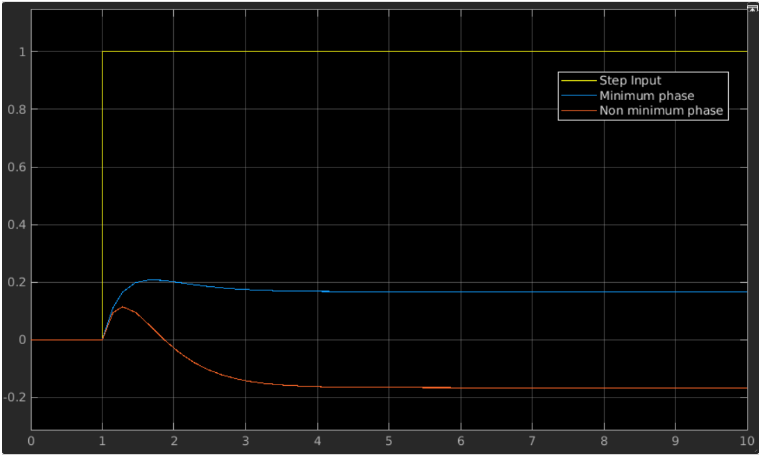
Non minimum phase system

- There are quite a few systems which violate the criteria of minimum phase systems and have either a pole or a zero in RH s-plane.
- In all such systems an additional phase of 180^0 (per pole/zero) gets added to the phase of the basic TF.
- All such systems are called non-minimum phase systems as their phase characteristics are significantly different.
- For example,

$$G(s) = \frac{(s-1)}{(s+2)(s+3)(s+4)}$$

Introduction - Non Minimum Phase System

Response - minimum phase system vs non minimum phase system



Introduction - Non Minimum Phase System

Non minimum phase system (cont...)

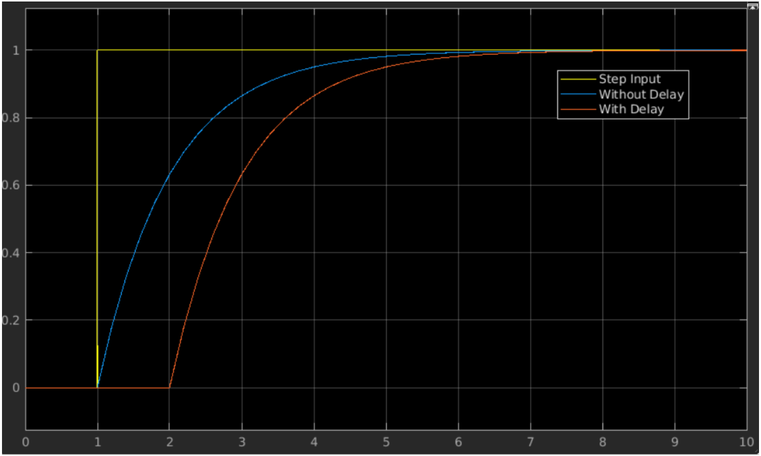
- A common source of such non-minimum phase is the presence of transport lag and computational time delays, which add an exponential factor to the TF.

Non minimum phase system situations

- Fluids and gases flowing through pipes take a finite time to reach the destination.
- Heat flux takes finite time to reach the object to be heated / cooled.
- In communication systems, signals take small, but finite, time to generate triggers.

Non Minimum Phase System - Impact

Transportation delay example



Non Minimum Phase System - Impact

Transportation delay

- Transfer function of time delayed system gets modified, as shown below.

$$G'(s) = e^{-\tau_d s}.G(s)$$

- $e^{-\tau_d s}$ is **irrational**. So we need to convert it into rational form, as required by the transfer function.
- On of the method is to use Pade's approximation

Approximation of Exponential Term

Pade's Approximation (cont...)

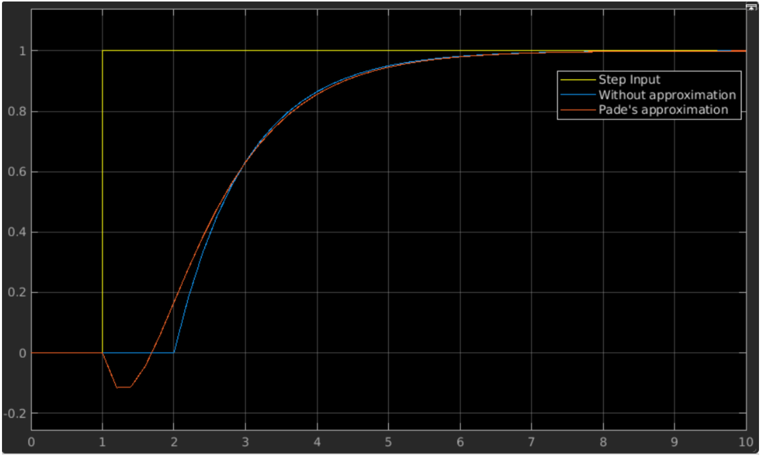
- Pade's series is used to derive an appropriate rational representation of irrational function
- The Pade's series is a ratio of two infinite series, which is normally truncated to a specific order depending on the accuracy requirements
- The first order approximation, for the transportation delay is

$$e^{-\tau_d s} \approx \frac{1 - \left(\dfrac{\tau_d}{2}\right) s}{1 + \left(\dfrac{\tau_d}{2}\right) s}$$

- It is seen that the representation results in a zero in right half of s-plane.

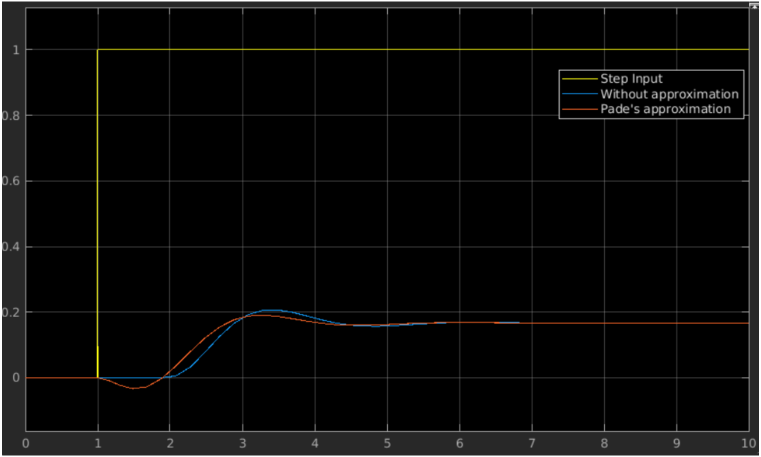
Comparison - Pade's Approximation

1^{st}order example



Comparison - Pade's Approximation

2^{nd}**order example**



References I

- Gene F. Franklin, J. David Powell, and Abbas Emami-Naeini: "*Feedback Control of Dynamic Systems*", Pearson Education, Inc., Upper Saddle River, New Jersey, Seventh Edition, 2015.
- Katsuhiko Ogata: "*Modern Control Engineering*", Pearson Education, Inc., Upper Saddle River, New Jersey, Fifth Edition, 2010.
- Farid Golnaraghi and Benjamin C. Kuo: "*Automatic Control Systems*", John Wiley & Sons, Inc., New Jersey, Ninth Edition, 2010.
- Norman S. Nise: "*Control Systems Engineering*", John Wiley & Sons, Inc., New Jersey, Sixth Edition, 2011.
- Ashok Joshi: "*System Modeling Dynamics and Control*", Lecture Notes, IIT Bombay, Mumbai, 2019.