Aerospace Engineering Department, IIT Bombay
AE 308 & AE 775 - Control Theory
Tutorial 1 Solution

Q1

Classify the given below systems as open-loop or closed-loop systems

1. Ceiling fan with speed control.
2. Automatic electric iron.
3. Air conditioners.
4. Timer based toasters.
5. Cruise control in a car.

Ans1

1. Open-loop.
2. Closed-loop.
3. Closed-loop.
4. Open-loop.
5. Closed-loop.

Q2

A temperature control system operates by sensing the difference between the thermostat setting and the actual temperature and then opening a fuel valve an amount proportional to this difference. Draw a closed-loop block diagram identifying the input and output transducers, the controller, and the plant. Further, identify the input and output signals of all subsystems previously described.

Ans2

Block diagram explained in tutorial class.

Q3

Write state-space model for the spring mass damper system shown in Fig.1, where

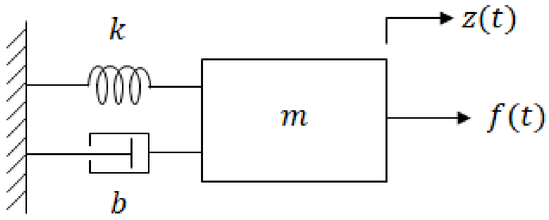

Figure 1: Spring Mass Damper System

$z(t)$ is displacement of the mass, k is spring constant, b is damping coefficient, and $f(t)$ is the external force applied on the mass.

Ans3

The differential equation representing the given spring mass damper system is

$$m\ddot{z}(t) + b\dot{z}(t) + kz(t) = f(t). \tag{1}$$

The order of the system is 2. To split the given system equation (1) in two first order differential equations we choose $x_1 = z$ and $x_2 = \dot{z}$ as the state variables. We get,

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \frac{1}{m}f. \end{aligned} \tag{2}$$

Output y is the displacement $z(t)$ of mass $'m'$.
The state space model is

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f, \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \end{aligned} \tag{3}$$

Q4

Determine the state-space equations for the circuit shown in Fig.2.

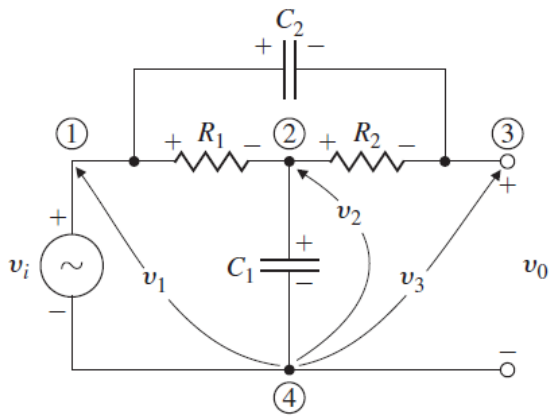

Figure 2: Bridged Tee Circuit

Ans4

To solve the given electrical circuit given in Fig.2, it is required to get the solution of nodal voltages. Node 4 is considered as the reference node.
At node 1:

$$v_1 = v_i. \tag{4}$$

At node 2:

$$\frac{v_1 - v_2}{R_1} + C_1 \frac{d}{dt}(0 - v_2) + \frac{v_3 - v_2}{R_2} = 0. \tag{5}$$

At node 3:

$$\frac{v_2 - v_3}{R_2} + C_2 \frac{d}{dt}(v_1 - v_3) = 0. \tag{6}$$

The differential equations (5),(6) obtained are already of the first order. So, these equations represent the state equations if nodal voltages are considered state variables.

However, nodal voltages are not considered state variables of the system. In place

of nodal voltages, voltages across the capacitors present in the circuit are considered state variables of the system.
In electrical circuits, the current through an inductor and the voltage across a capacitor is considered state variables as they represent the physical quantity of a component in the system.

Let v_{c1} and v_{c2} are the voltages across the capacitors C_1 and C_2 respectively.

$$\begin{aligned} v_{c1} &= v_2, \\ v_{c2} &= v_i - v_3. \end{aligned} \tag{7}$$

Using (5), (6), (7) we get,
Node 2:

$$\frac{v_i - v_{c1}}{R_1} - C_1 \frac{d}{dt} v_{c1} + \frac{v_i - v_{c2} - v_{c1}}{R_2} = 0$$

$$\dot{v}_{c1} = -\frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_{c1} - \frac{1}{R_2 C_1} v_{c2} + \frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_i \tag{8}$$

Node 3:

$$\frac{v_{c1} - v_i + v_{c2}}{R_2} + C_2 \frac{d}{dt}(v_i - v_i + v_{c2}) = 0$$

$$\dot{v}_{c2} = -\frac{1}{R_2 C_2} v_{c1} - \frac{1}{R_2 C_2} v_{c2} + \frac{1}{R_2 C_2} v_i \tag{9}$$

Output:

$$y = v_0 = v_3 = -v_{c2} + v_i \tag{10}$$

The state space model is

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} -\frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) & -\frac{1}{R_2 C_1} \\ -\frac{1}{R_2 C_2} & -\frac{1}{R_2 C_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \\ \frac{1}{R_2 C_2} \end{bmatrix} v_i, \\ y &= \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} v_{c1} \\ v_{c2} \end{bmatrix} + v_i. \end{aligned} \tag{11}$$

Note: All the questions except *Q4* are discussed thoroughly in tutorial class held on Aug 8, 2022. So answers are not in detail while solution with explanation is provided for *Q4*.
In case of any queries/concern, please contact any of the TA's.