Assignment I

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Group 1 (Members: 190050031, 200100115, 210050027)

Problem 1. Let X be a random variable with probability density function given by

$$f(x) = \begin{cases} \theta e^{-\theta x} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{0.1}$$

Find the probability density function of

1. $Y_1 = \cos X$
2. $Y_2 = (X - 1/\theta)^2$.

Group 2 (Members: 210010035, 210100097, 21B090003)

Problem 2. Let X be a random variable with probability density function given by

$$f(x) = \begin{cases} \theta e^{-\theta x} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{0.2}$$

Find the probability density function of $Y = \tan X$.

Group 3 (Members: 210050011, 210260036)

Problem 3. Suppose that a projectile is fired at an angle Θ above the earth with a velocity v (deterministic). Assuming that Θ is a random variable with probability density function given by

$$f_\Theta(\theta) = \begin{cases} 12/\pi & \text{if } \pi/6 < \theta < \pi/4 \\ 0 & \text{otherwise.} \end{cases} \tag{0.3}$$

Find the probability density function of the range R of the projectile motion where $R = v^2 \sin\theta/g$ with gravitational constant g.

Problem 4. Let $\mathbb{F} : \mathbb{R} \to [0, 1]$ be distribution function and continuous. Then for some fixed $h > 0$, show that the following functions are distribution functions

$$\mathbb{F}_1(x) = \frac{1}{h}\int_x^{x+h} \mathbb{F}(u)du \text{ for all } x \in \mathbb{R} \tag{0.4}$$

$$\text{and } \mathbb{F}_2(x) = \frac{1}{2h}\int_{x-h}^{x+h} \mathbb{F}(u)du \text{ for all } x \in \mathbb{R}. \tag{0.5}$$

Group 4 (Members: 210050083, 210050076, 20D170022)

Problem 5. Suppose that X is a standard normal random variable that is, X has p.d.f.

$$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2} \text{ for all } x \in \mathbb{R}. \tag{0.6}$$

Then find the probability density function of the following random variables

1. $Y_1 = e^X$
2. $Y_2 = 2X^2 + 1$ and
3. $$Y_3 = g(X) = \begin{cases} 1 & \text{if } X > 0 \\ \frac{1}{2} & \text{if } X = 0 \\ -1 & \text{if } X < 0 \end{cases} \tag{0.7}$$

Group 5 (Members: 210260037, 210050055)

Problem 6. Do the following functions define distribution function? Sketch or plot each of the functions.

1. $\mathbb{F}(x) = 1/2 + (1/\pi)\tan^{-1} x$ for all $x \in \mathbb{R}$.
2. $$\mathbb{F}(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ \frac{x}{2} & \text{if } 0 \leq x < 1 \\ \frac{1}{2} & \text{if } 1 \leq x < 2 \\ \frac{x}{4} & \text{if } 2 \leq x < 3 \\ 1 & \text{if } x \geq 3. \end{cases} \tag{0.8}$$

If $\mathbb{F}$ is the distribution function, then compute the density and mass function associated to it.

Problem 7. Check whether the following function is a probability density function

$$f(x) = \begin{cases} 0 & \text{if } X < 0 \\ \frac{1}{9}(x+1) & \text{if } 0 \le x < 1 \\ \frac{2}{9}(2x-1) & \text{if } 1 \le x < 3/2 \\ \frac{2}{9}(5-2x) & \text{if } 3/2 \le x < 2 \\ \frac{4}{27} & \text{if } 2 \le x < 5 \\ 0 & \text{otherwise.} \end{cases} \tag{0.9}$$

Group 6 (Members: 210050018, 210050073)

Problem 8. Suppose that a data analyst predict that duration in minutes of a long-distance telephone calls made from a city is found to be a random variable with a probability distribution $\mathbb{F}$ given by

$$\mathbb{F}(x) = \begin{cases} 0 & \text{for } x \le 0 \\ 1 - \frac{2}{3}e^{-x/3} - \frac{1}{3}e^{-\lfloor x/3 \rfloor} & \text{for } x > 0 \end{cases} \tag{0.10}$$

where $\lfloor x \rfloor$ is the largest integer less than or equal to x.

1. Check that the prediction of the data analyst is correct that is, $\mathbb{F}$ is indeed a distribution function.
2. Plot/sketch the function $\mathbb{F}$.
3. Find out the associated probability mass/density function.