AE 330/708

AEROSPACE PROPULSION

Instructor

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Definitions and Fundamentals of Rocket Performance

Propulsion is achieved by applying a force to a vehicle, that is, accelerating the vehicle or, alternatively, maintaining a given velocity against a resisting force.

This propulsive force is obtained by ejecting propellant at high velocity.

We are going to cover the definitions and the basic relations of this propulsive force, the exhaust velocity, and the efficiencies of creating and converting the energy and other basic parameters.

Specific impulse: Most important performance parameter in rocket propulsion (alternate term – specific thrust)

Calculated as the ratio of the thrust delivered to the weight flow rate of the propellant

$$I_s = \frac{F}{\dot{w}}$$

Interpretation – thrust that can be obtained from an equivalent rocket which has a propellant weight flow rate of unity.

In above equation, *Is* is specific impulse, *F* is the thrust and *w(dot)* is the weight flow rate of the propellant

Weight flow rate of the propellant can also be expressed as the propellant mass flow rate times the gravitational acceleration

$$I_s = \frac{F}{\dot{w}} = \frac{F}{\dot{m}g}$$

The impulse (or total impulse) is the integral of the thrust (F) over the operating duration (t_b).

In general, the thrust (F) is not constant; but a function of time (t).

$$\boldsymbol{I_t = \int_0^{t_b} F.d_t = \int_0^{t_b} I_s.\dot{w}.d_t}$$

For constant thrust, $\boldsymbol{I_t = F.t_b = I_s.w}$

w = Total weight of the propellant loaded in the rocket

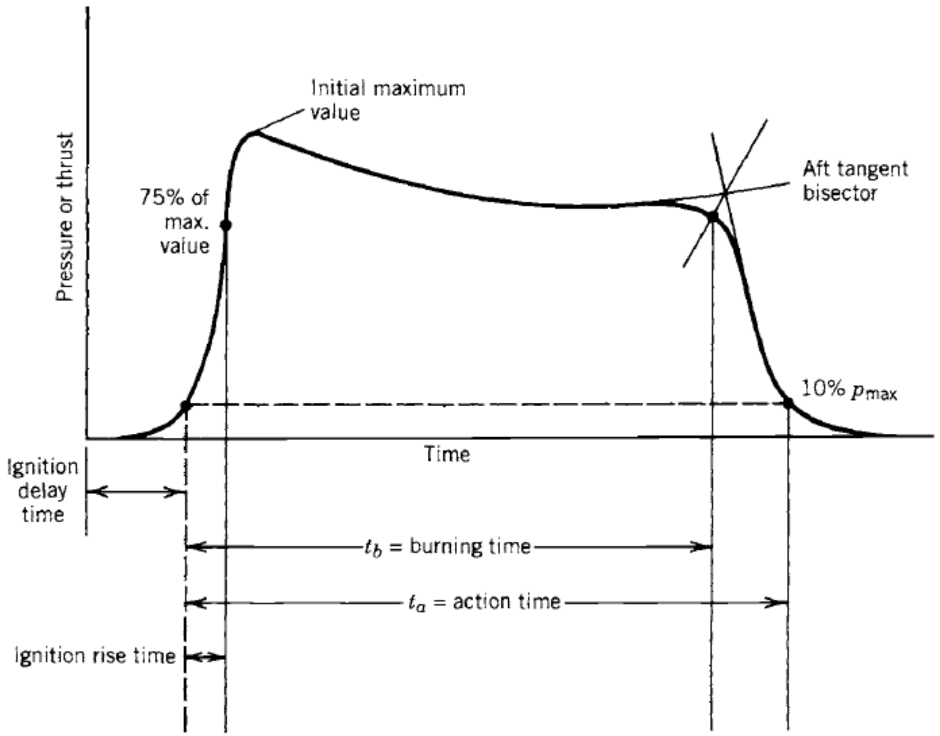
Another definition for specific impulse: $\boldsymbol{I_s = \frac{I_t}{w}}$

In many rockets, for example solid propellant rockets, it is difficult to measure propellant flow rate accurately

Hence, average specific impulse is calculated from thrust-time data over the operation of the rocket and the initial propellant weight loaded in the rocket

In Liquid Propellant Rockets (LPR), it is possible to control and measure the propellant flow rate during engine operation and hence the specific impulse can be directly evaluated

A typical thrust-time curve for a rocket engine looks like



There are three major masses of the rocket propulsion system which are of prime importance in deciding the quality of the design

m_0 = Initial mass of the rocket vehicle before firing the engine
m_b = Burn-out mass of the vehicle (mass of the vehicle after the whole propellant is exhausted)
m_p = Mass of the propellant loaded in the vehicle

Propellant mass fraction (important parameter)

$$\eta = \frac{m_p}{m_0} = \frac{m_0 - m_b}{m_0} = \frac{m_p}{m_p + m_b}$$

For rocket engine system, the value of propellant mass fraction indicates the quality of design.

η = 0.93 indicates that only 7% of the mass is inert rocket hardware and this small fraction is used to burn effectively much larger mass of propellant.

A high value of η is desirable.

The impulse-to-weight ratio is defined as the total impulse divided by the initial vehicle weight.

$$\frac{I_t}{w_0} = \frac{I_t}{(m_b + m_p)g} = \frac{I_s}{(m_b + m_p)g/w}$$
$$= \frac{I_s}{\frac{m_b}{m_p} + 1}$$

In ideal weightless design (i.e. mb = 0),

$$\frac{I_t}{w_0} \rightarrow I_s$$

Impulse-to-weight ratio value cannot exceed the value of specific impulse.

Specific power : This parameter indicates the utilization of mass in the propulsion system in producing kinetic gas power of the ejected matter.

$$\boldsymbol{P_s = \frac{\frac{1}{2}\dot{m}c^2}{w_0} = \frac{F I_s g}{2w_0}}$$

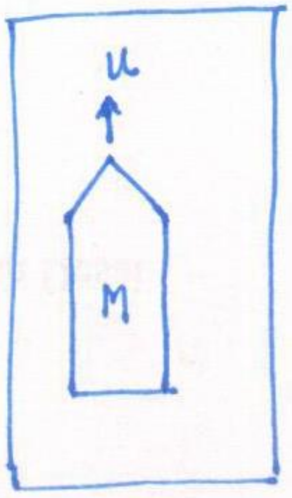
$$\boldsymbol{P_s \propto F, I_s}$$

Thrust to weight ratio : Expresses the acceleration (in multiples of g) that the engine is capable of giving to its own loaded propulsion system mass

Acceleration plays a very critical role in the performance and design of the propulsion system and payload.

Thrust from a Rocket Engine

Thrust of a rocket engine:-

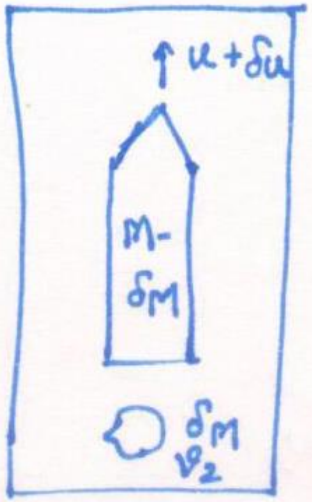


Time = t

After time interval Δt

δM

goes out with velocity v_2 w.r.t. the rocket



Drag-free & gravity free environment

$\Rightarrow$ No external force.

Inertial frame observer.

$\vec{u}$ = Velocity of rocket

$\vec{v_2}$ = Velocity of ejected mass (relative)

In the absence of external forces, the momentum in the inertial frame is conserved.

At $t \Rightarrow$ Mu

At $t + \Delta t \Rightarrow (M - \delta_M)(u + \delta u) + \delta_M (u + \delta u + v_2)$

$$= M \cdot u + M \delta u - \cancel{\delta_M u} - \cancel{\delta_M \cdot \delta u}$$

$$+ \cancel{\delta_M \cdot u} + \cancel{\delta_M \cdot \delta u} + \delta_M v_2$$

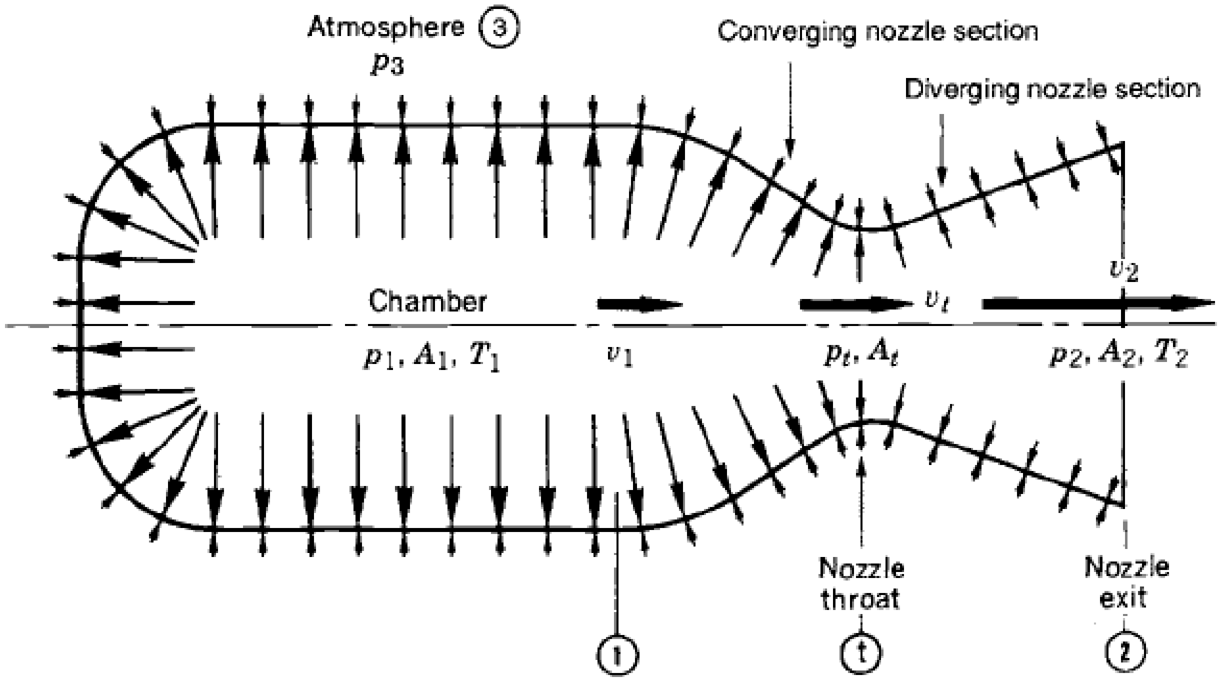
Equating the momentum, we get $M \cdot \delta u + \delta_m \cdot v_2 = 0$

$$M \cdot \delta u = -\delta_M \cdot v_2 \Rightarrow M \cdot \frac{\delta u}{\delta_t} = -\frac{\delta_M}{\delta_t} \cdot v_2$$

In the limit, $\delta_t \rightarrow 0$, $\quad M \dfrac{du}{dt} = -\dfrac{dM}{dt} \cdot v_2$

$\dfrac{dM}{dt}$ = Rate of change of vehicle mass $= -\underline{\dot{m}}$

$$M \frac{du}{dt} = \dot{m} v_2 = F \text{ (Thrust)}$$



Effect of surrounding atmospheric fluid

Axial thrust due to pressure forces

Imbalance of pressure at the open end (nozzle exit) creates additional force in axial direction

Integrate all pressures acting on areas perpendicular to the axis

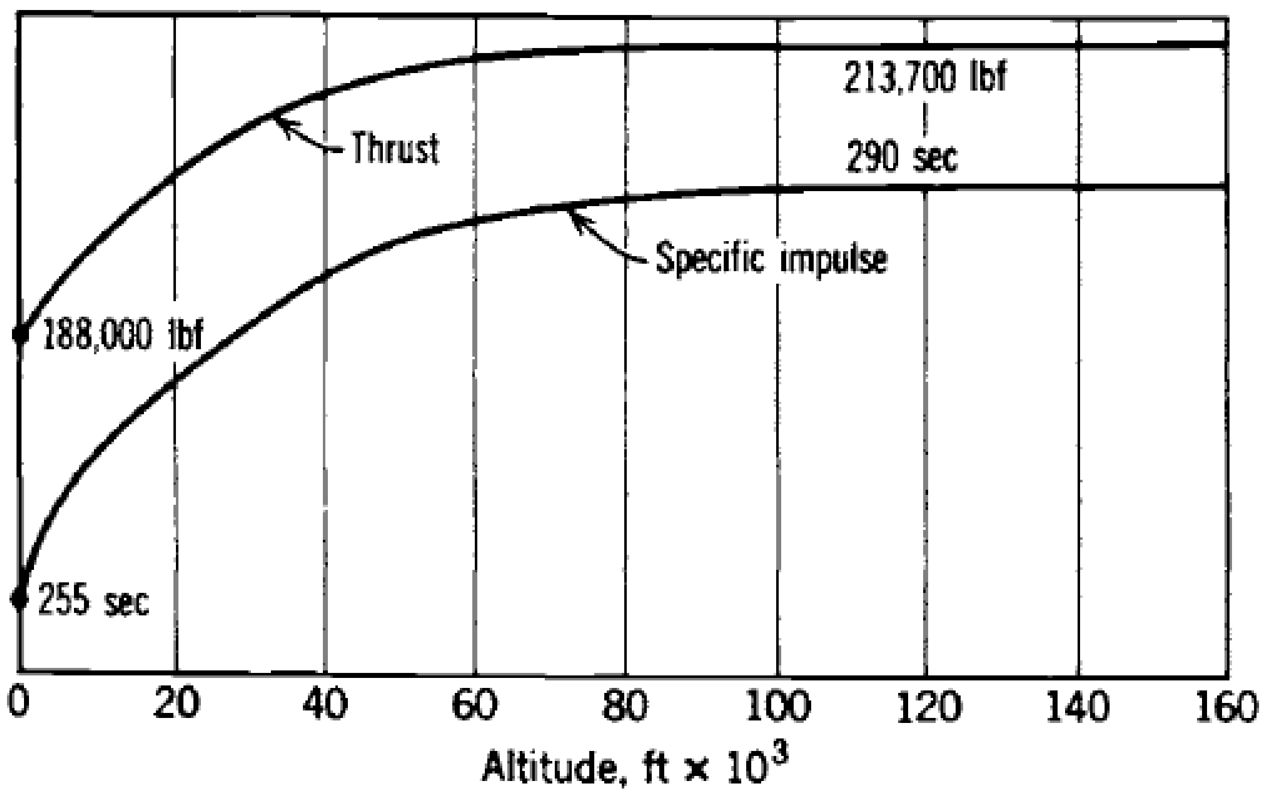
The net thrust from a rocket nozzle may be written as,

$$F = \underbrace{\dot{m}v_2}_{\text{Momentum thrust}} + \underbrace{(P_2 - P_3)A_2}_{\text{Pressure thrust}}$$

P2 = Pressure of the hot gases just at the exit plane of the nozzle

P3 = Ambient pressure

A2 = Exit area of the nozzle



$$F = \dot{m}v_2 + (P_2 - P_3)A_2$$

P2 > P3 : Positive contribution from the pressure thrust

P2 < P3 : Negative contribution from the pressure thrust

P2 = P3 : No pressure thrust. Optimum expansion condition

$$\frac{F}{\dot{m}} = v_2 + \frac{(P_2 - P_3)A_2}{\dot{m}} = c$$

Effect of altitude on the net thrust from a given rocket engine (due to reduction in ambient pressure with altitude)

c = Effective exhaust velocity

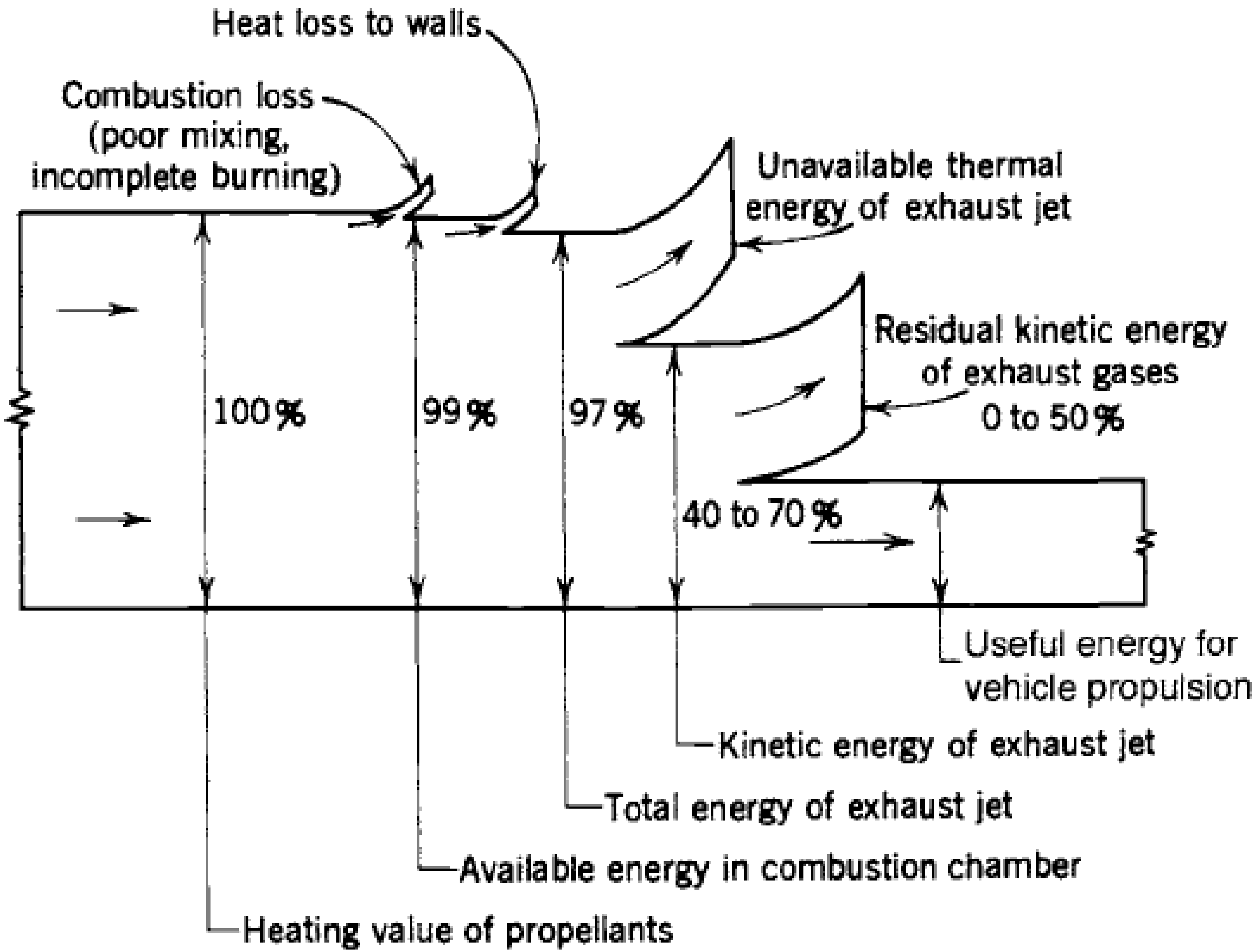
Typically 10-30% rise in thrust due to decrease in P3 with altitude

For optimum expansion condition, c = v2

Another definition of the Specific Impulse

$$I_s = \frac{F}{\dot{w}} = \frac{\dot{m}c}{\dot{m}g} = \frac{c}{g}$$

Typical Energy Balance for a Rocket Engine



Energies and Efficiencies

Kinetic power of the jet = $(1/2)\,\dot{m}v_2^2$

Input power : Chemical energy in the form of combustion of the propellants → generation of thermal energy i.e. high pressure and high temperature gases → conversion to kinetic energy of the exhaust gases

Input chemical energy, $P_{chem} = \dot{m}Q$ (where Q is the heating value of the propellant combination)

Internal efficiency, $\eta_{int} = [(1/2)\,\dot{m}v_2^2]/[\eta_c \dot{m}Q]$

Indication of the effectiveness of converting system's energy input into the kinetic energy of the ejected matter.

Here, η_c = Combustion efficiency

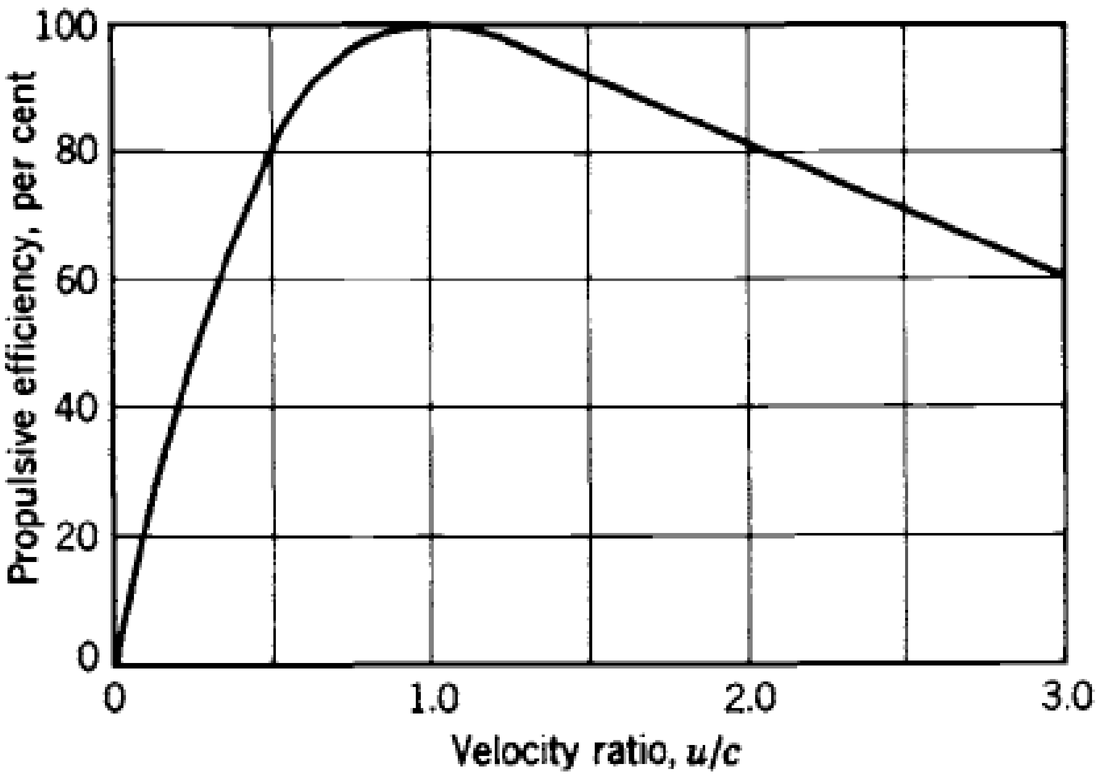
Propulsive Efficiency

Consider a rocket vehicle flying with a velocity u under the action of thrust F.

Propulsive power of the vehicle, $P_{vehicle} = F.u$

$$\eta_P = \frac{\text{vehicle power}}{\text{vehicle power} + \text{residual kinetic jet power}}$$

$$= \frac{Fu}{Fu + \frac{1}{2}(\dot{w}/g_0)(c-u)^2} = \frac{2u/c}{1+(u/c)^2}$$



Characteristic Velocity

Frequently used performance parameter in rocket propulsion

$$c^* = [P_1 . A_t]/\dot{m}$$

Here, P_1 is the combustion chamber pressure, A_t is the throat area of the nozzle.

Comparing relative performance of different chemical propulsion systems and their propellants

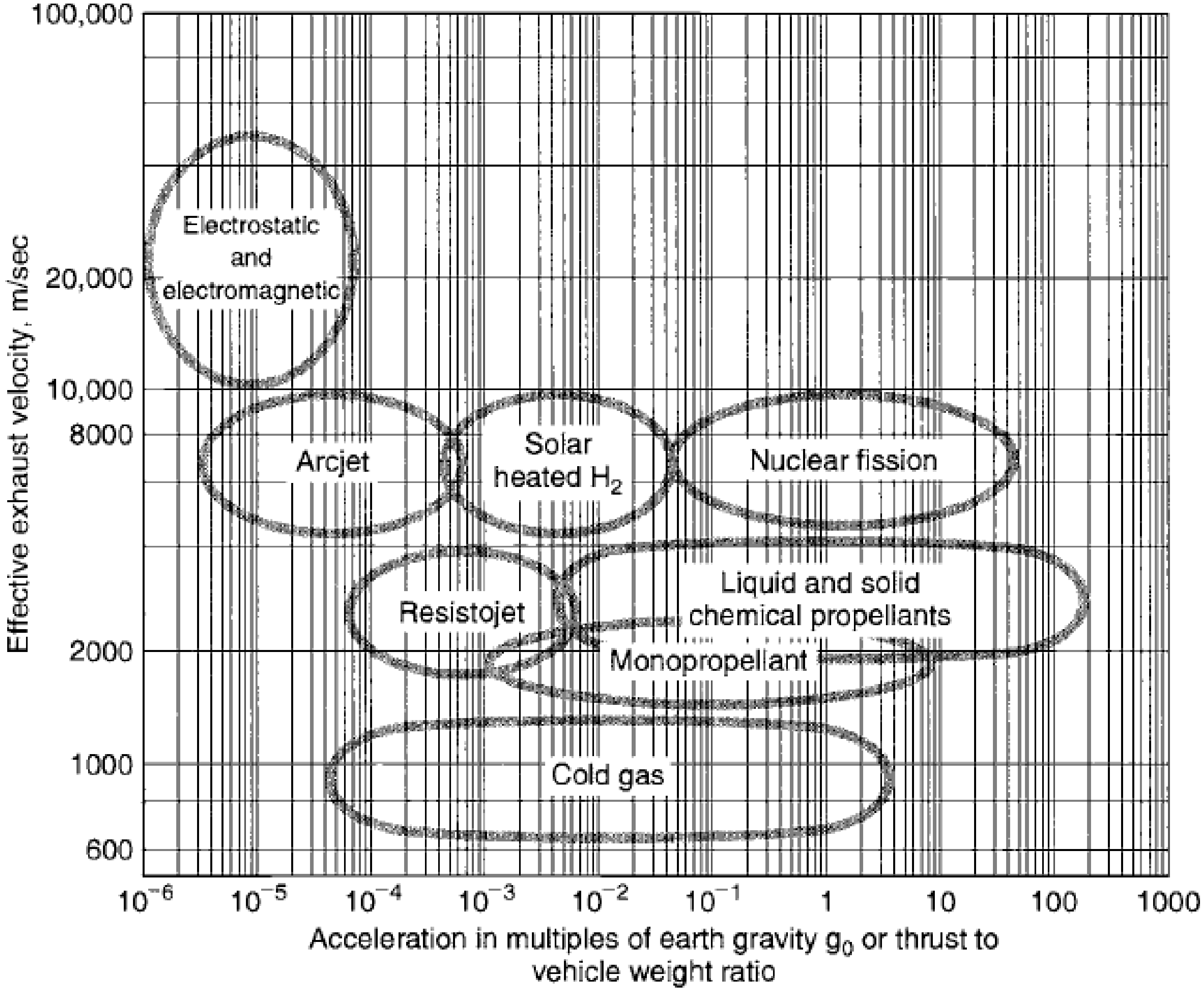
Indicative of the efficiency of combustion and independent of nozzle characteristics

We shall revisit this concept in more detail in Nozzle Theory

TABLE 2–1. Ranges of Typical Performance Parameters for Various Rocket Propulsion Systems

Engine Type	Specific Impulse[a] (sec)	Maximum Temperature (°C)	Thrust-to-Weight Ratio[b]	Propulsion Duration	Specific Power[c] (kW/kg)	Typical Working Fluid	Status of Technology
Chemical—solid or liquid bipropellant	200–410	2500–4100	10^{-2}–100	Seconds to a few minutes	10^{-1}–10^{3}	Liquid or solid propellants	Flight proven
Liquid monopropellant	180–223	600–800	10^{-1}–10^{-2}	Seconds to minutes	0.02–200	N_2H_4	Flight proven
Nuclear fission	500–860	2700	10^{-2}–30	Seconds to minutes	10^{-1}–10^{3}	H_2	Development was stopped
Resistojet	150–300	2900	10^{-2}–10^{-4}	Days	10^{-3}–10^{-1}	H_2, N_2H_4	Flight proven
Arc heating—electrothermal	280–1200	20,000	10^{-4}–10^{-2}	Days	10^{-3}–1	N_2H_4,H_2,NH_3	Flight proven
Electromagnetic including Pulsed Plasma (PP)	700–2500	—	10^{-6}–10^{-4}	Weeks	10^{-3}–1	H_2 Solid for PP	Flight proven
Hall effect	1000–1700	—	10^{-4}	Weeks	10^{-1}–5×10^{-1}	Xe	Flight proven
Ion—electrostatic	1200–5000	—	10^{-6}–10^{-4}	Months	10^{-3}–1	Xe	Several have flown
Solar heating	400–700	1300	10^{-3}–10^{-2}	Days	10^{-2}–1	H_2	In development

[a] At $p_1 = 1000$ psia and optimum gas expansion at sea level ($p_2 = p_3 = 14.7$ psia).
[b] Ratio of thrust force to full propulsion system sea level weight (with propellants, but without payload).
[c] Kinetic power per unit exhaust mass flow.



100,000
20,000
10,000
8000
2000
1000
600
Effective exhaust velocity, m/sec
Electrostatic
and
electromagnetic
Arcjet
Solar
heated H_2
Nuclear fission
Resistojet
Liquid and solid
chemical propellants
Monopropellant
Cold gas
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
100
1000
Acceleration in multiples of earth gravity g_0 or thrust to
vehicle weight ratio