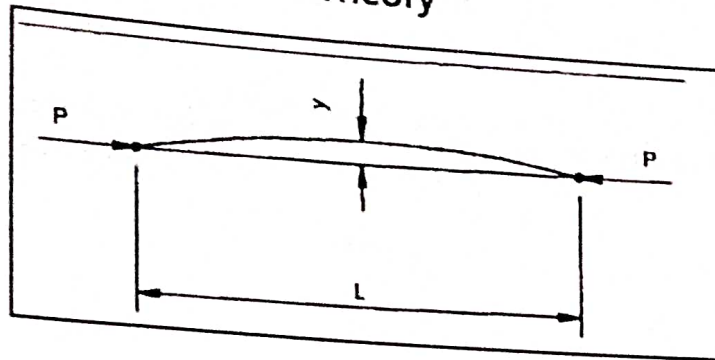


## Theory



Euler's basic strut was a pin ended column length  $L$  with perfect axial loading  $P$ . If such a strut should deflect transversely then a bending moment  $P_y$  is imposed where  $y$  (+ve in the diagram) is the deflection due to that bending moment. With the sign convention adopted it can be seen that the moment is -ve because the strut is hogging. Then by substitution in

we get

$$EI \frac{d^2y}{dx^2} =$$

$$\frac{d^2y}{dx^2} = \frac{-Py}{EI}$$

and writing  $a^2$  for

$$\frac{P}{EI}$$

The solution of this differential equation is

$$y = A \cos ax + B \sin ax$$

Since

$$y = 0$$

When

$$x = 0$$

$$L, A = 0$$

And

$$0 = B \sin aL$$

This equality is satisfied if

$$B = 0$$

or if

$$\sin aL = 0$$

If  $B = 0$  there is no deflection anywhere and hence no bending moment. Suppose that for any reason there is some deflection. Then from

$$\sin aL = 0$$

it follows that

$$aL = 0 \text{ or } \pi \text{ or higher multiples}$$

If

$$aL = 0$$

Then

$$P = 0$$

Which is again not real. Hence the real solution is

$$aL = \pi$$

From which

$$P = \frac{\pi^2 EI}{L^2}$$

For Pinned/Pinned

## Experiment

### Objectives

The purpose of the work is to compare the buckling loads of the four struts, with Euler's mathematical critical loads namely using Southwell Plots

A: Pinned

$$P = \frac{\pi^2 EI}{L^2}$$

B: Fixed/Pinned)

$$P = 2.045 \frac{\pi^2 EI}{L^2}$$

C: Fixed

$$P = \frac{4\pi^2 EI}{L^2}$$

### Procedure

There are four lengths of specimen and three different end conditions, so there are twelve possible combinations that can be tested. For the purpose of this experiment we will demonstrate the procedure using the 500mm Specimen. The procedure is exactly the same if using the shorter Specimens.

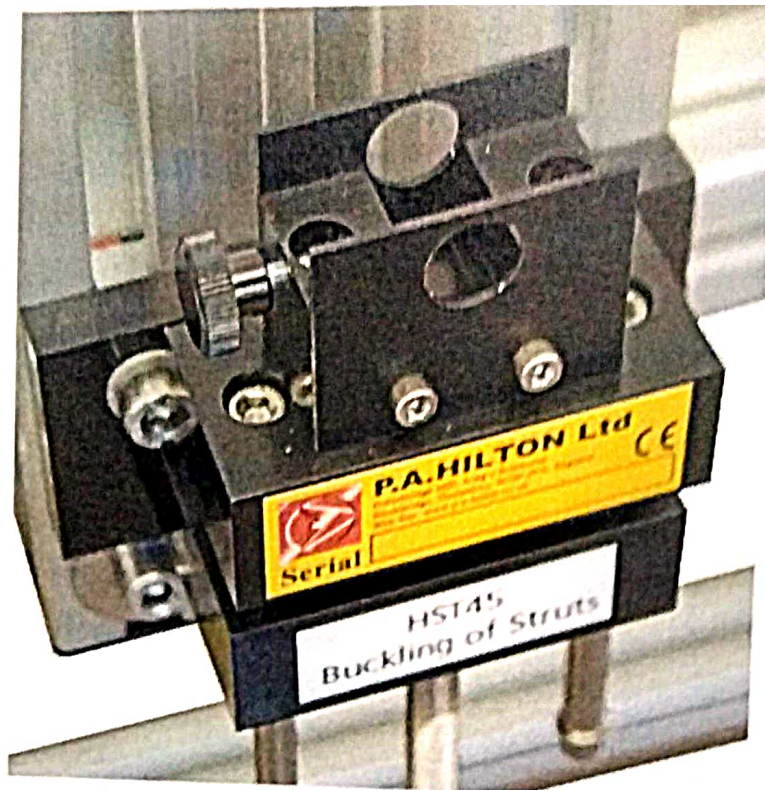
Ensure that the Dial Gauge and Load Cell are connected to the HDA200.

Select the 500mm Specimen and measure and record its length, width (b) and depth (d), in mm in Table 1.

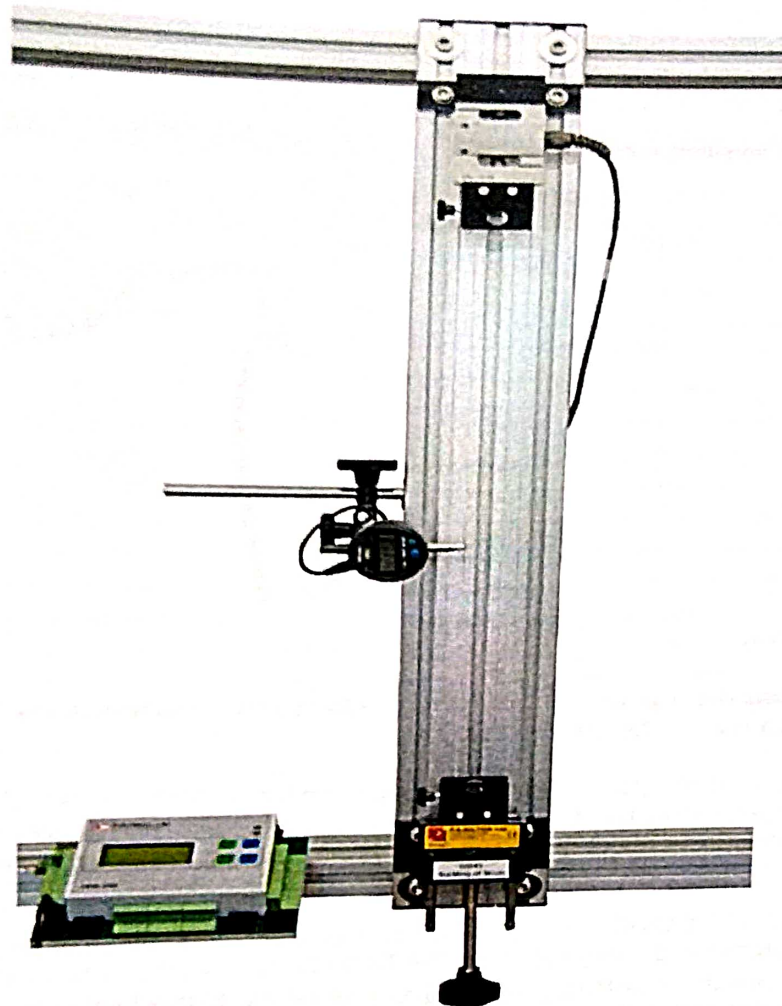
Ensure that the Specimen Ends have been fitted to the Specimen.

**Set up the Fixed/ Fixed end condition.**

Place the Strut Assembly into the Bottom Chuck Holder of the Loading Mechanism. Ensure that the Specimen Clamp is facing the side of the Chuck where the fixing Screw is located. Lock the specimen in position using the Locking Screw.

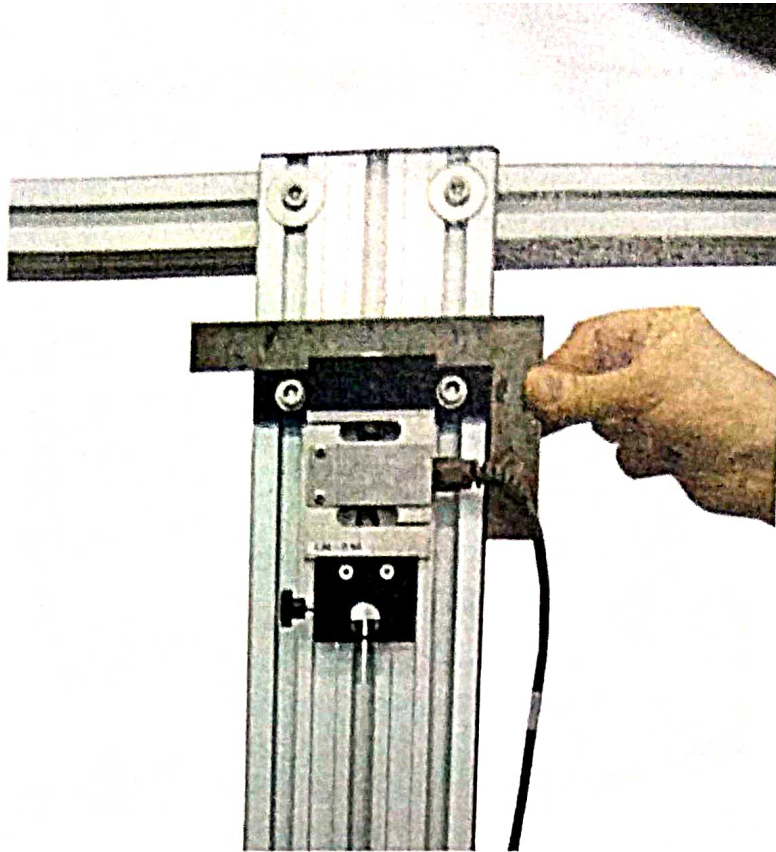


Turn the Knob of the Loading Mechanism fully counter clockwise and then turn it 2 turns clockwise.  
Gently drop the Load Cell Head down so that the Specimen end locates into the Top Chuck Holder.



Using the Square ensure that the Load Cell Head is perpendicular to the side of the Back Plate. Tighten the M8 Cap Head Screws and recheck. Once you are satisfied that the Load Cell Head is in the correct position back the Loading Mechanism back by half a turn.





Tare the Load Cell reading.

Raise the Loading Mechanism up slowly until the reading just registers a very small load value. Lock the specimen in position using the Locking Screw.

Position the Dial Gauge at mid-span of the Specimen, ensuring the Ball Anvil of the Dial Gauge is touching the Specimen and that the Dial Gauge has at least 12mm of travel after it is zeroed. To help establish the centre of the strut is permissible to draw a line through the centre of the Strut using a pencil.

Rotate the Grip Knob of the Loading Mechanism to get a feel for the buckling of the Specimen and which direction it will deform during testing. The Dial Gauge needs to be on the side which will PUSH the Dial Gauge Spindle into the Dial Gauge main body. If the Dial Gauge is on the wrong side then swap the Dial Gauge around.

Return the Loading Mechanism to its initial rest position, whereby the specimen is very lightly loaded. Tare the Dial Gauge reading by pressing the light grey button on the Dial Gauge front face. Tare the Load Cell reading using the button on the front of the HDA200 Interface.

Record the start values of the Dial Gauge and Load Cell in Table 1 below.

In increments of initially 0.1mm up to 0.6mm then in 0.2mm up to 1mm then 0.5mm (approximately), compress the strut specimen, recording the applied force reading from the HDA200 at each interval. Continuing recording and compressing until the deflection reaches 12.00mm for the strut.

Table 1		
End Condition	Pinned / Pinned	
Member Length, L, mm		
End Condition		
Member Width, b, mm		
Member depth, d, mm		
Young's Modulus of elasticity, E, N/mm <sup>2</sup>	207000	
Second Moment of Area, I, mm <sup>4</sup>		
Theoretical Load, PE, N		
Load P, (N)	Deflection S, mm	Southwell Number
	0.00	
	0.10	
	0.20	
	0.30	
	0.40	
	0.50	
	0.60	
	0.80	
	1.00	
	1.50	
	2.00	
	2.50	
	3.00	
	3.50	
	4.00	
	4.50	
	5.00	
	5.50	
	6.00	

Once the testing has been completed remove the load from the Strut.

**Setup for the Fixed/Pinned end condition.**

Disengage the Locking Screw from the Top Chuck Holder and gently lift the Load Cell head Assembly away from the Specimen.

Fit the Knife Edge Block into the Top Chuck Holder.





Turn the Knob of the Loading Mechanism fully counter clockwise and then turn it 2 turns clockwise.  
Gently drop the Load Cell Head down so that the Specimen Holder Vee locates onto Knife Edge of the Knife Edge Holder.





Using the Square ensure that the Load Cell Head is perpendicular to the side of the Back Plate. Tighten the M8 Cap Head Screws and recheck. Once you are satisfied that the Load Cell Head is in the correct position back the Loading Mechanism back by half a turn.

Tare the Load Cell reading.

Raise the Loading Mechanism up slowly until the reading just registers a very small load value.

Position the Dial Gauge at mid-span of the Specimen, ensuring the Ball Anvil of the Dial Gauge is touching the Specimen and that the Dial Gauge has at least 12mm of travel after it is zeroed

Rotate the Grip Knob of the Loading Mechanism to get a feel for the buckling of the Specimen and which direction it will deform during testing. The Dial Gauge needs to be on the side which will PUSH the Dial Gauge Spindle into the Dial Gauge main body. If the Dial Gauge is on the wrong side then swap the Dial Gauge around.

Return the Loading Mechanism to its initial rest position, whereby the specimen is very lightly loaded. Tare the Dial Gauge reading by pressing the light grey button on the Dial Gauge front face. Tare the Load Cell reading using the button on the front of the HDA200 Interface.

Record the start values of the Dial Gauge and Load Cell in Table 1.

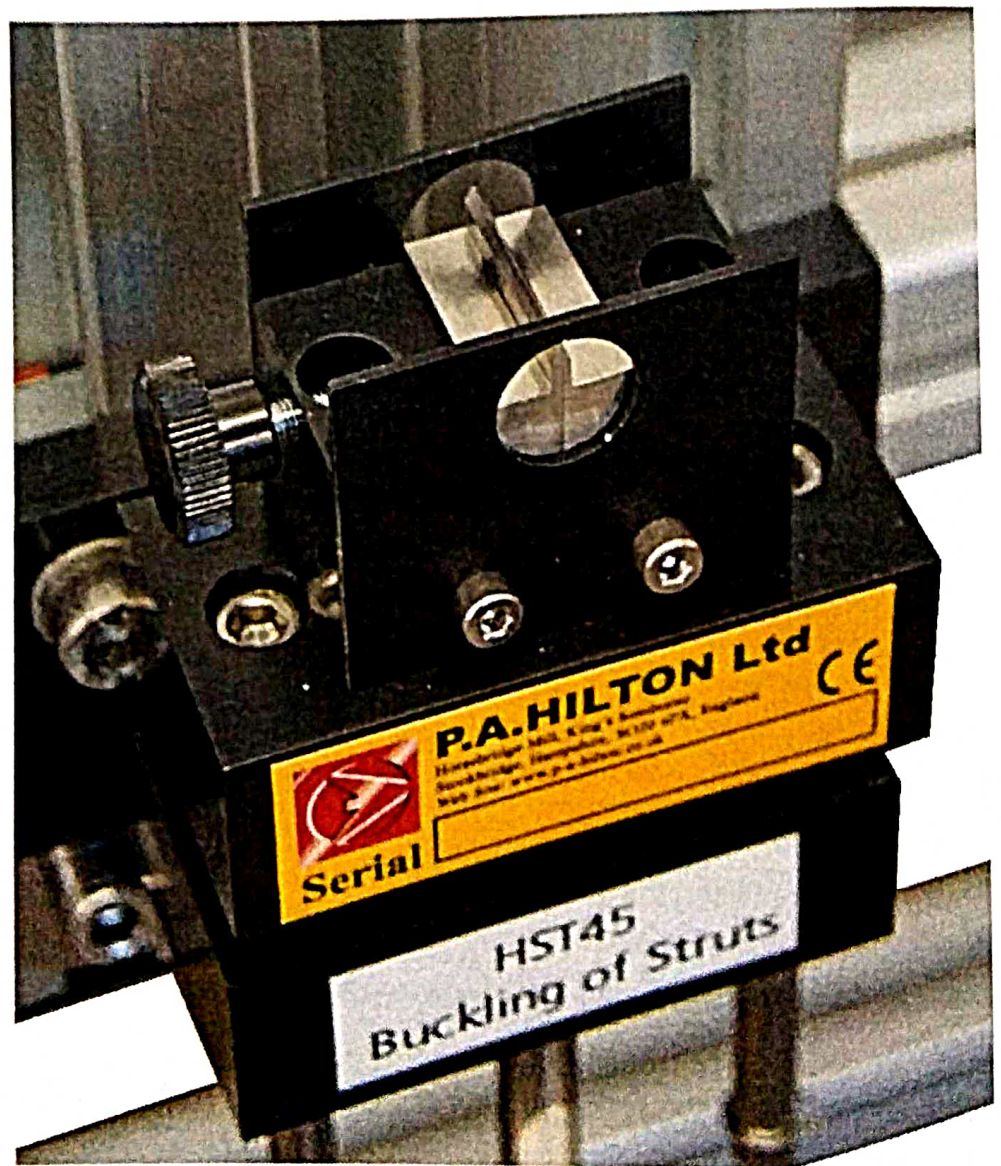
In increments of initially 0.1mm up to 0.6mm then in 0.2mm up to 1mm then 0.5mm (approximately), compress the strut specimen, recording the applied force reading from the HDA200 at each interval. Continuing recording and compressing until the deflection reaches 12.00mm for the strut.



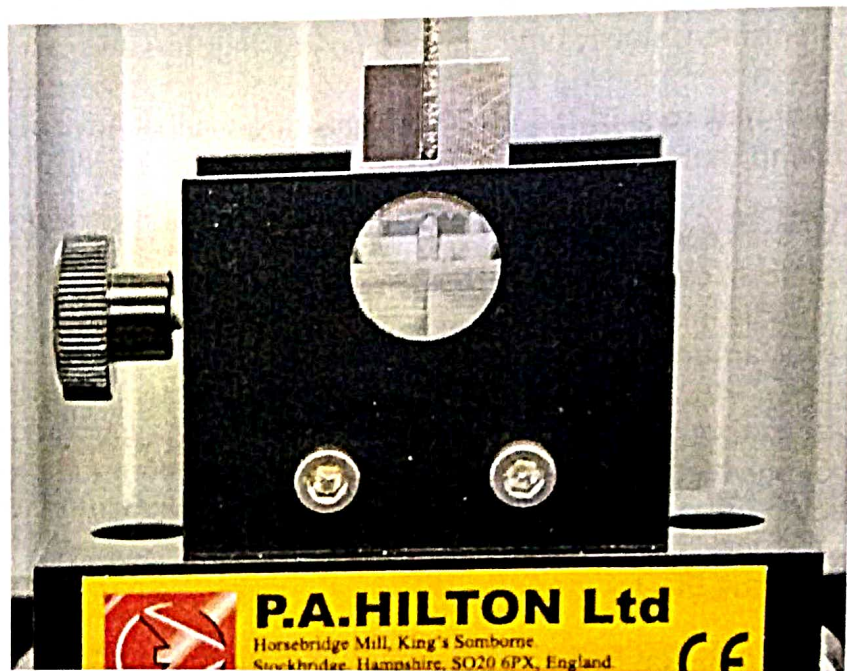
Setup for the Pinned/Pinned end condition.

Disengage the Locking Screw from the Top Chuck Holder and gently lift the Load Cell holder Assembly away from the Specimen.

Fit the Knife Edge Block into the Bottom Chuck Holder.



Turn the Knob of the Loading Mechanism fully counter clockwise and then turn it 2 turns clockwise.  
Place the Vee Groove of the Strut Assembly on the Knife Edge of the Knife Edge Block so that it runs through the centre of the Specimen.



Gently drop the Load Cell Head down so that the Specimen Holder Vee locates onto Knife Edge of the Knife Edge Holder.



Using the Square ensure that the Load Cell Head is perpendicular to the side of the Back Plate. Tighten the M8 Cap Head Screws and recheck. Once you are satisfied that the Load Cell Head is in the correct position back the Loading Mechanism back by half a turn.

Tare the Load Cell reading.

Raise the Loading Mechanism up slowly until the reading just registers a very small load value.

Position the Dial Gauge at mid-span of the Specimen, ensuring the Ball Anvil of the Dial Gauge is touching the Specimen and that the Dial Gauge has at least 12mm of travel after it is zeroed

Rotate the Grip Knob of the Loading Mechanism to get a feel for the buckling of the Specimen and which direction it will deform during testing. The Dial Gauge needs to be on the side which will PUSH the Dial Gauge Spindle into the Dial Gauge main body. If the Dial Gauge is on the wrong side then swap the Dial Gauge around.

Return the Loading Mechanism to its initial rest position, whereby the specimen is very lightly loaded. Tare the Dial Gauge reading by pressing the light grey button on the Dial Gauge front face. Tare the Load Cell reading using the button on the front of the HDA200 Interface.

Record the start values of the Dial Gauge and Load Cell in Table 1.

In increments of initially 0.1mm up to 0.6mm then in 0.2mm up to 1mm then 0.5mm (approximately), compress the strut specimen, recording the applied force reading from the HDA200 at each interval. Continuing recording and compressing until the deflection reaches 12.00mm for the strut.

## Example Results

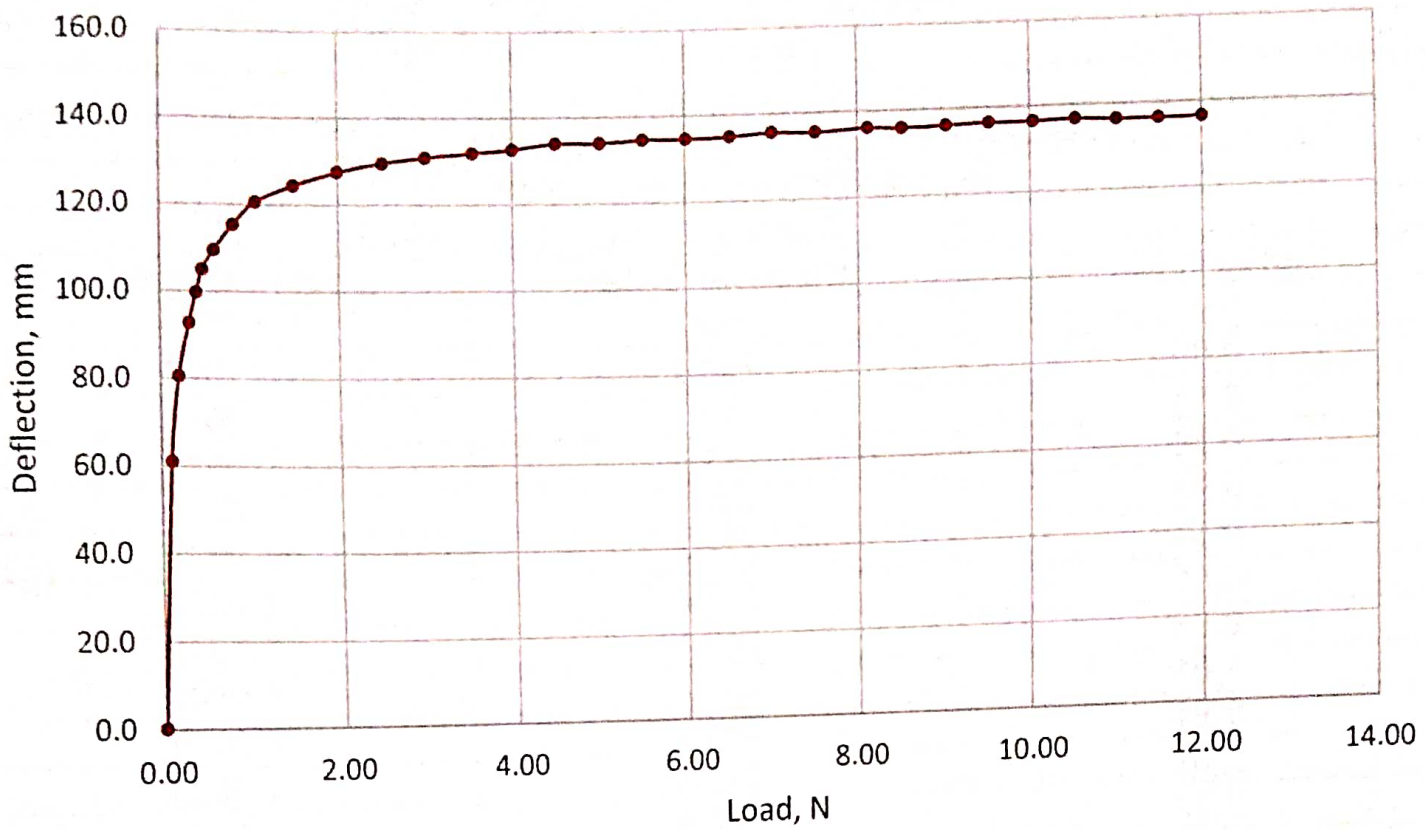
The following results are an illustration of the capabilities of the HST45 Buckling of Struts. These Observations should not be considered correct. Each Machine is different and ambient conditions vary. They are to be used as a guide for the Calculations only.

### 350mm Strut

End Condition		Pinned/Pinned
Effective Strut Length, $L_e$ , mm		360.60
Member Width, $b$ , mm		25.03
Member depth, $d$ , mm		1.64
Young's Modulus of elasticity, $E$ , N/mm <sup>2</sup>		207000
Second Moment of Area, $I$ , mm <sup>4</sup>		9.20
Theoretical Load, $P_E$ , N		144.55
Load $P$ , (N)	Deflection $S$ , mm	Southwell Number
0.00	0.0	0.0000
0.11	60.9	0.0018
0.20	80.5	0.0025
0.32	92.7	0.0035
0.40	99.7	0.0040
0.47	104.9	0.0045
0.60	109.4	0.0055
0.82	115.3	0.0071
1.07	120.5	0.0089
1.50	124.3	0.0121
2.00	127.4	0.0157
2.51	129.5	0.0194
3.00	130.8	0.0229
3.54	131.8	0.0269
4.00	132.6	0.0302
4.50	133.9	0.0336
5.01	133.9	0.0374
5.51	134.5	0.0410
6.01	134.5	0.0447
6.52	134.8	0.0484
7.01	135.5	0.0517
7.51	135.3	0.0555
8.12	135.9	0.0597
8.52	135.7	0.0628
9.03	136.0	0.0664
9.53	136.3	0.0699
10.04	136.3	0.0737
10.53	136.6	0.0771
11.01	136.3	0.0808
11.50	136.3	0.0844
12.00	136.4	0.0880
Gradient =		138.34



### Deflection against Load, 350mm Pinned/Pinned



### Southwell Number against Deflection, 350mm Pinned/Pinned

