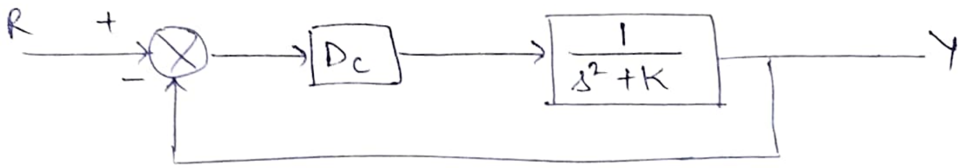
Quiz - 2 Solutions

Ans-1 (a) Consider $W = 0$



To make the system track a ramp input i.e. $R(s) = \frac{1}{s^2}$ with constant steady-state error, it is required that the open-loop transfer function of the system should be Type 1 OR velocity error constant

$K_v = \lim_{s \to 0} s G(s) H(s)$ should be constant. +0.5

Open-loop transfer function of system, $G(s)$ is

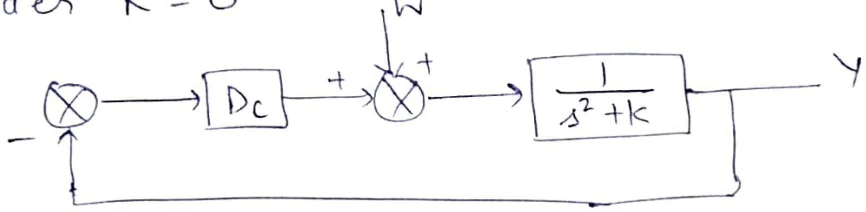
$$G(s) = \frac{D_c}{s^2 + K} \quad ; \quad H(s) = 1$$ +0.5

if $D_c(s) = \frac{1}{s}$ then $G(s) = \frac{1}{s(s^2+K)}$ (Type 1)

$$K_v = \lim_{s \to 0} s \times \frac{1}{s} \times \frac{1}{s^2 + K} = \frac{1}{K}$$

Hence, $D_c(s) = \frac{1}{s}$ (or any other Type 1 transfer function) is suitable. +0.5

(b) Consider $R = 0$



$$\frac{Y(s)}{W(s)} = \frac{\frac{1}{s^2+K}}{1+\frac{D_c}{s^2+K}} = \frac{1}{s^2+K+D_c}$$ $+0.5$

from part (a) $D_c = \frac{1}{s}$

$$\frac{Y(s)}{W(s)} = \frac{s}{s^3+Ks+1}$$

Desired $\lim_{t\to\infty} y(t) = 0 \Rightarrow \lim_{s\to 0} sY(s) = 0$ $+0.5$

$$\lim_{s\to 0} sY(s) = \lim_{s\to 0} s \frac{s}{s^3+Ks+1} W(s)$$

$$= \lim_{s\to 0} \frac{s^2}{s^3+Ks+1} W(s)$$

$$= \lim_{s\to 0} s^2 W(s)$$

$\lim_{s\to 0} sY(s) = 0$ is possible only if $W(s) = 1$ or $\frac{1}{s}$

~~[illegible]~~

$\Rightarrow$ system can reject step-disturbances. $+0.5$

$w(t) = c\,u(t)$, c is any real value

Ans-2 Open loop : $G(s) = \frac{K}{s(s+7)(s+11)}$

Closed loop (unity feedback), $T(s)$

$$T(s) = \frac{K}{s^3+18s^2+77s+K}$$

Routh-Table

s^3	1	77
s^2	18	K
s^1	$\frac{1386-K}{18}$	
s^0	K	

+0.5

(i) for stable

constant terms appearing in first column are positive hence, to ensure stable system terms containing 'K' should also be positive.

$K > 0$ (given)

$\frac{1386-K}{18} > 0 \Rightarrow K < 1386$

System is stable for $0 < K < 1386$ +1

(ii) for unstable

there should be sign change in enteries present in the first column

if $K > 1386$, there are two sign changes hence, two poles in right half s-plane implying system is unstable.

System is unstable for $K > 1386$. +0.5

(iii) for marginal stability

when $K = 1386$, an all zero row is encountered, replacing it with $\frac{d}{ds}(18s^2+1386) = 36s$ and making new Routh-table

s^3	1	77
s^2	18	1386
s^1	36	
s^0	1386	

* no sign changes - no pole in right-half s-plane
* an all zero row is encountered just below $18s^2 + 1386$ polynomial hence, two poles are symmetric about $j\omega$-axis in s-plane. +0.5

Since, there are no-poles in RHP, the symmetric poles are on $j\omega$-axis. These poles are simple hence system is marginally stable for $K = 1386$. +0.5

Ans-3

$$T(s) = \frac{s^3 + 2s^2 + 7s + 21}{s^5 + 6s^4 + 15s^3 + 30s^2 + 44s + 24}$$

To check the location of zeros in s-plane, we will use numerator polynomial of $T(s)$

s^3	1	7
s^2	2	21
s^1	-3.5	
s^0	21	

+0.5

2 - sign changes hence, two zeros are in RHP. +0.5

So, system is non-minimum phase. +0.5