AE 330 Rocket Propulsion

Boundary Layers
&
Turbulence

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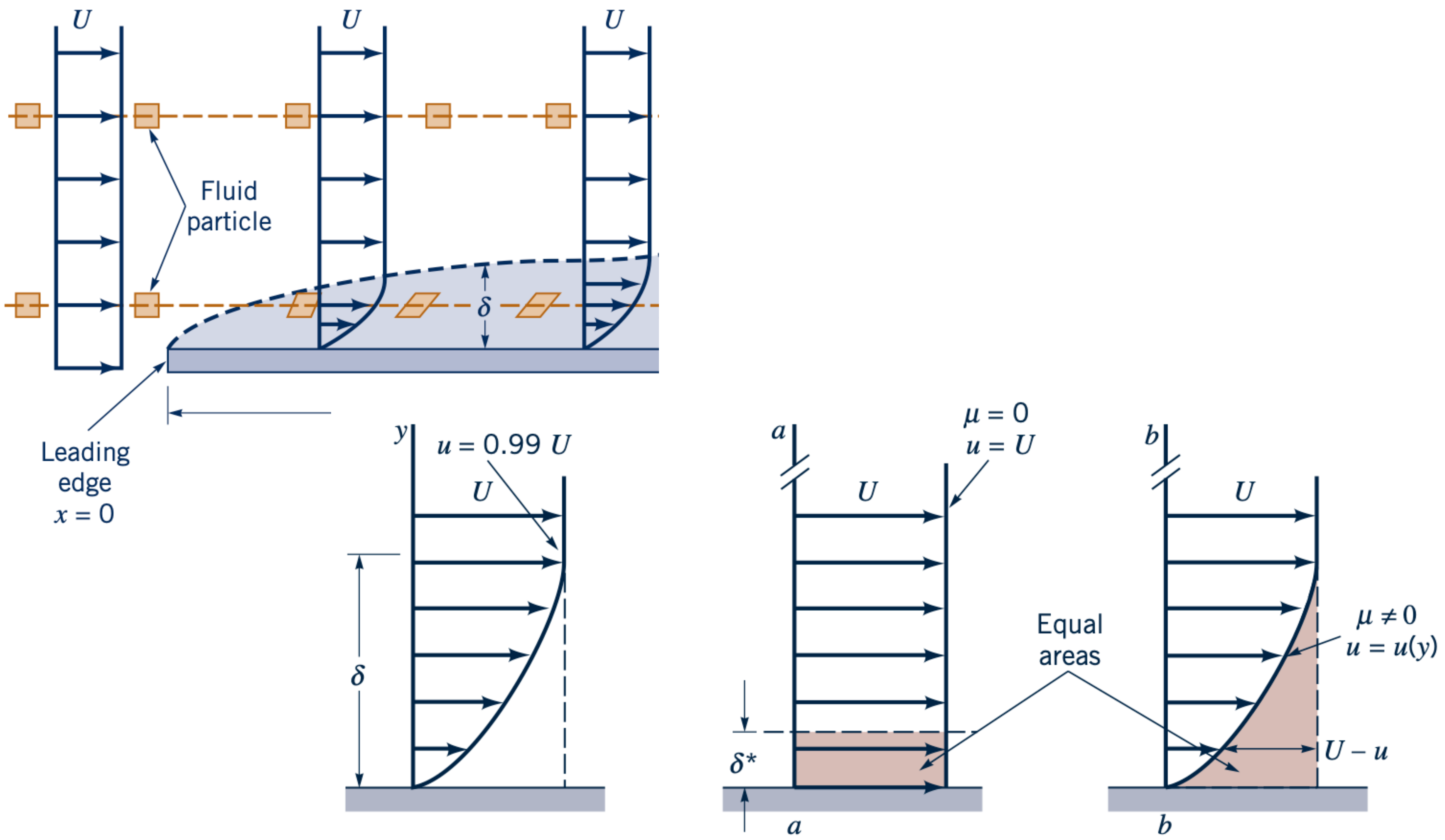
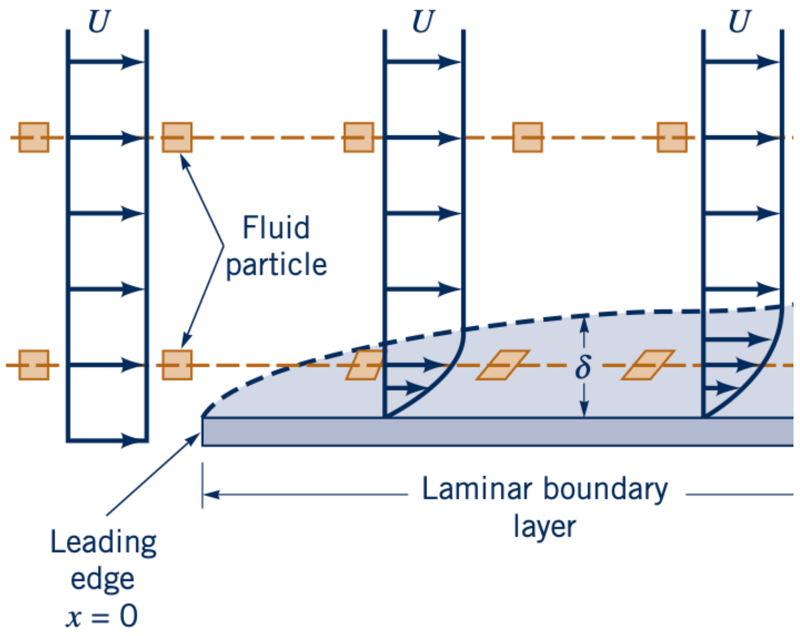
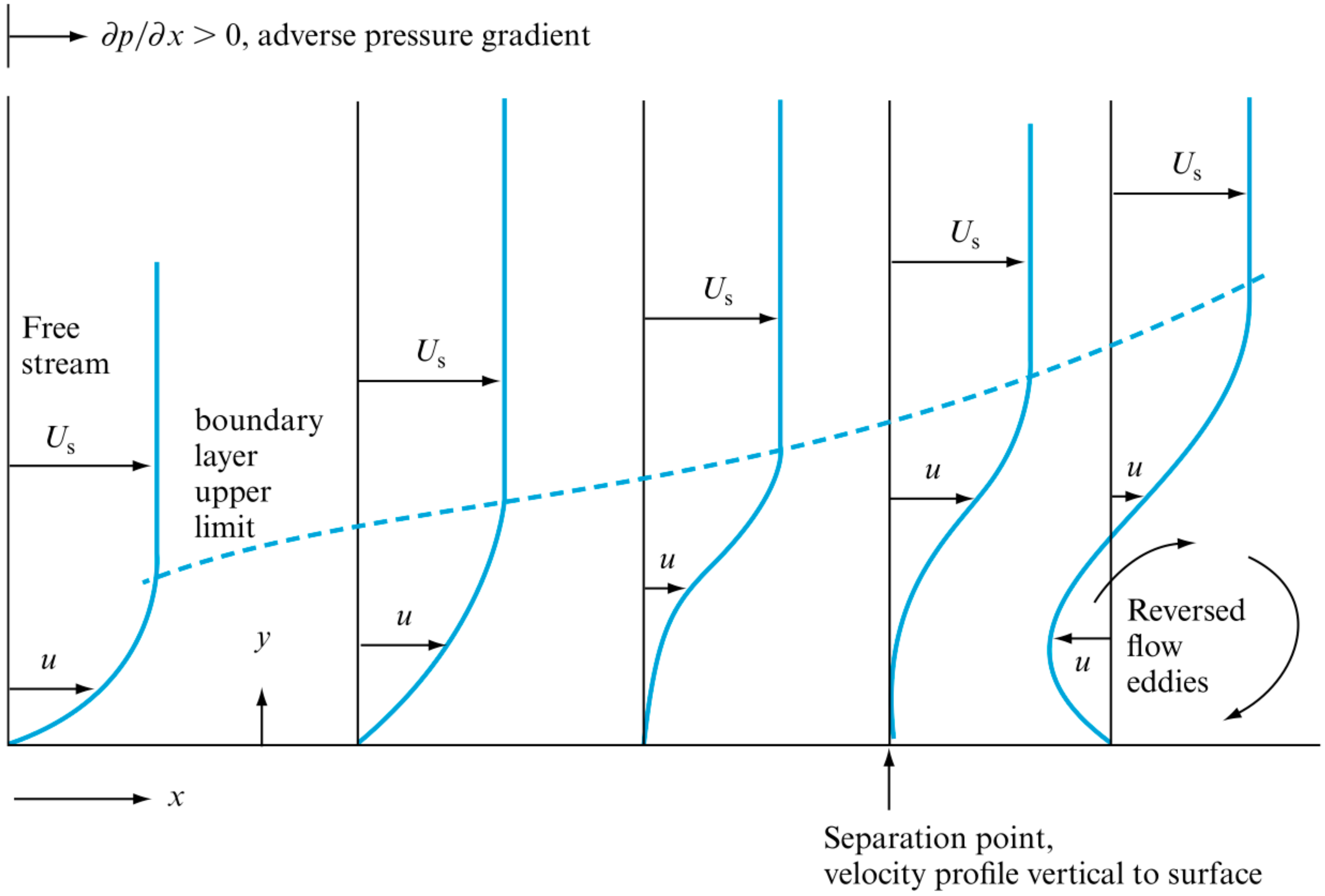
Displacement Thickness

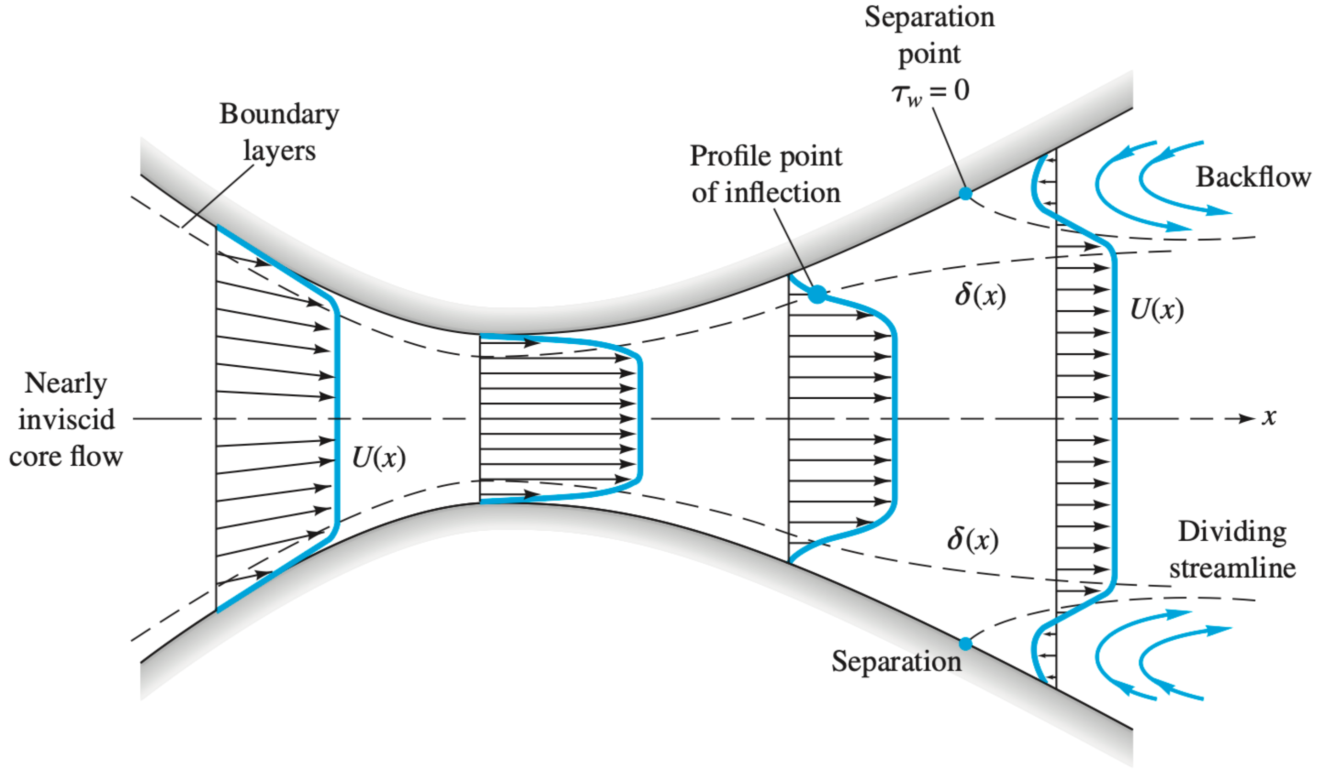


Flow Separation



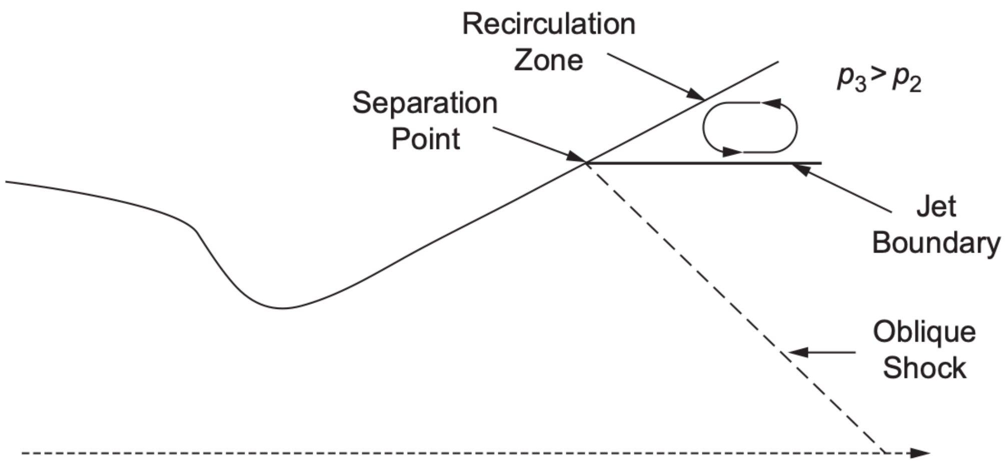


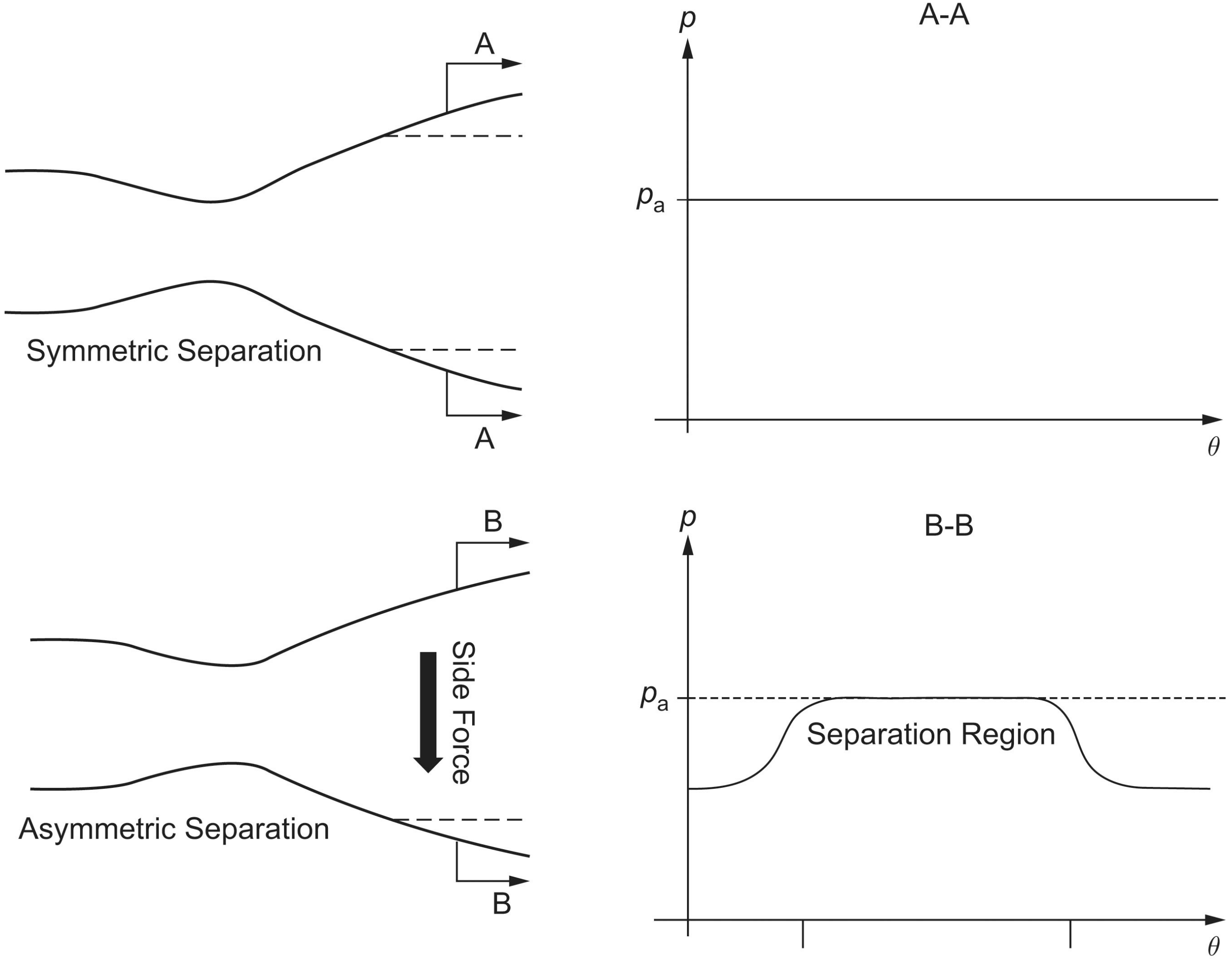
Flow Separation



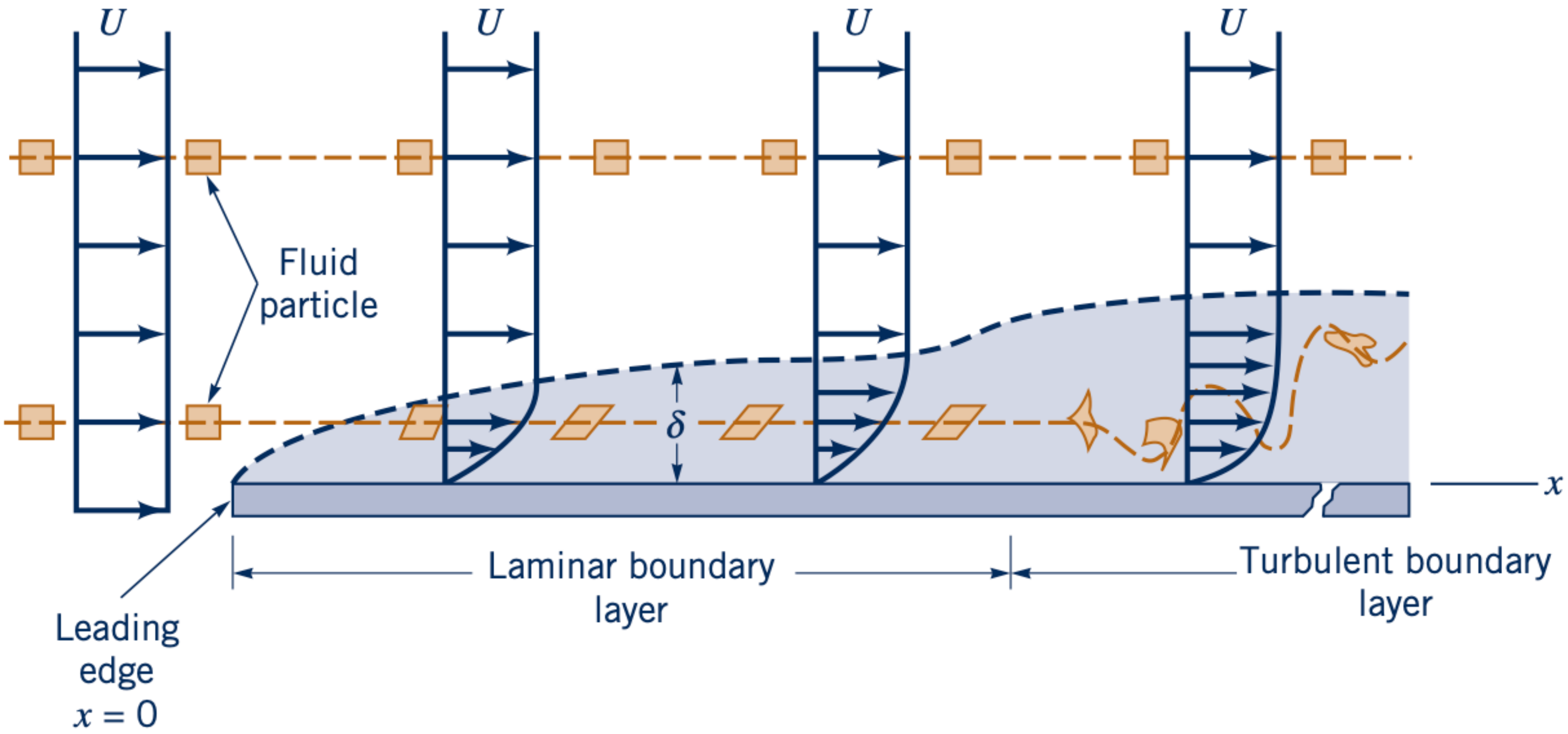
Nozzle:	*Throat*:	*Diffuser*:
Decreasing pressure and area	Constant pressure and area	Increasing pressure and area
Increasing velocity	Velocity constant	Decreasing velocity
Favorable gradient	Zero gradient	Adverse gradient (boundary layer thickens)

Flow Separation

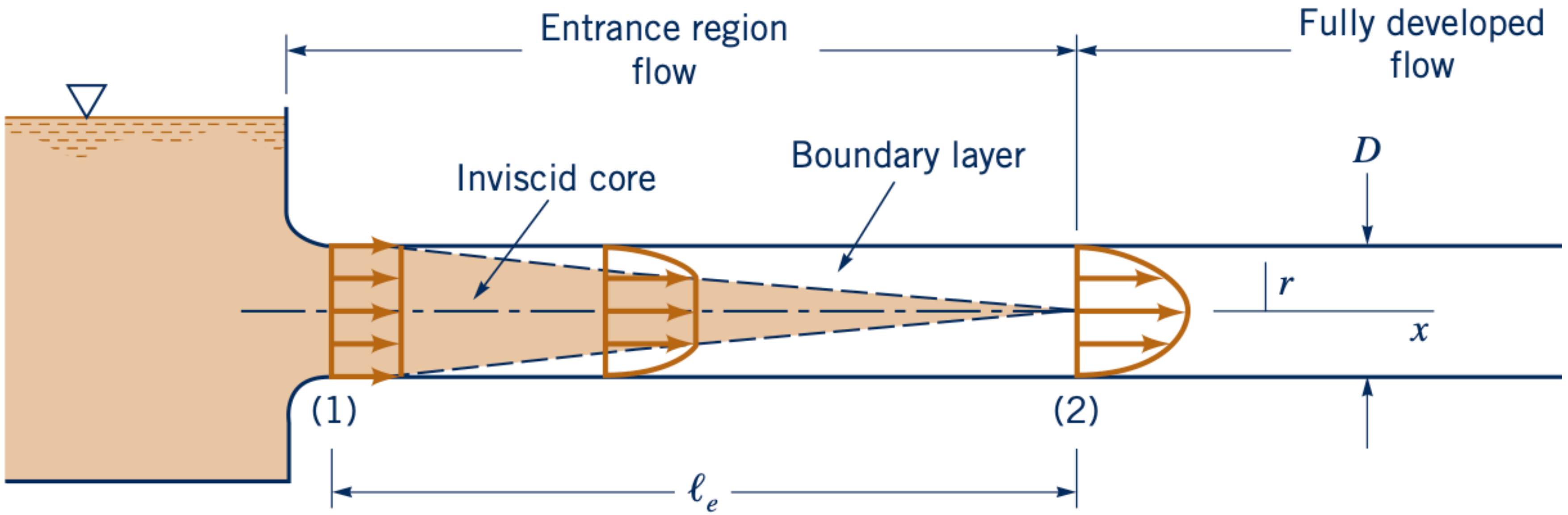




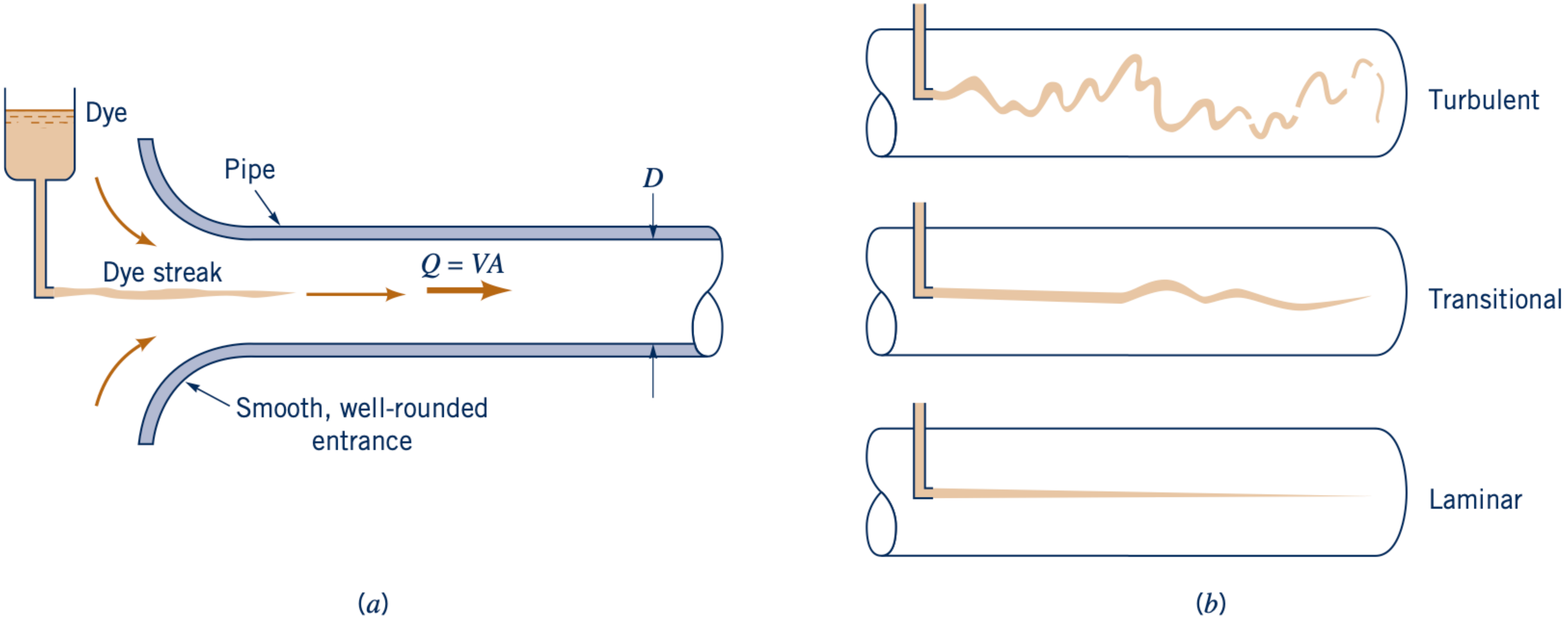
Flat-plate Boundary Layer



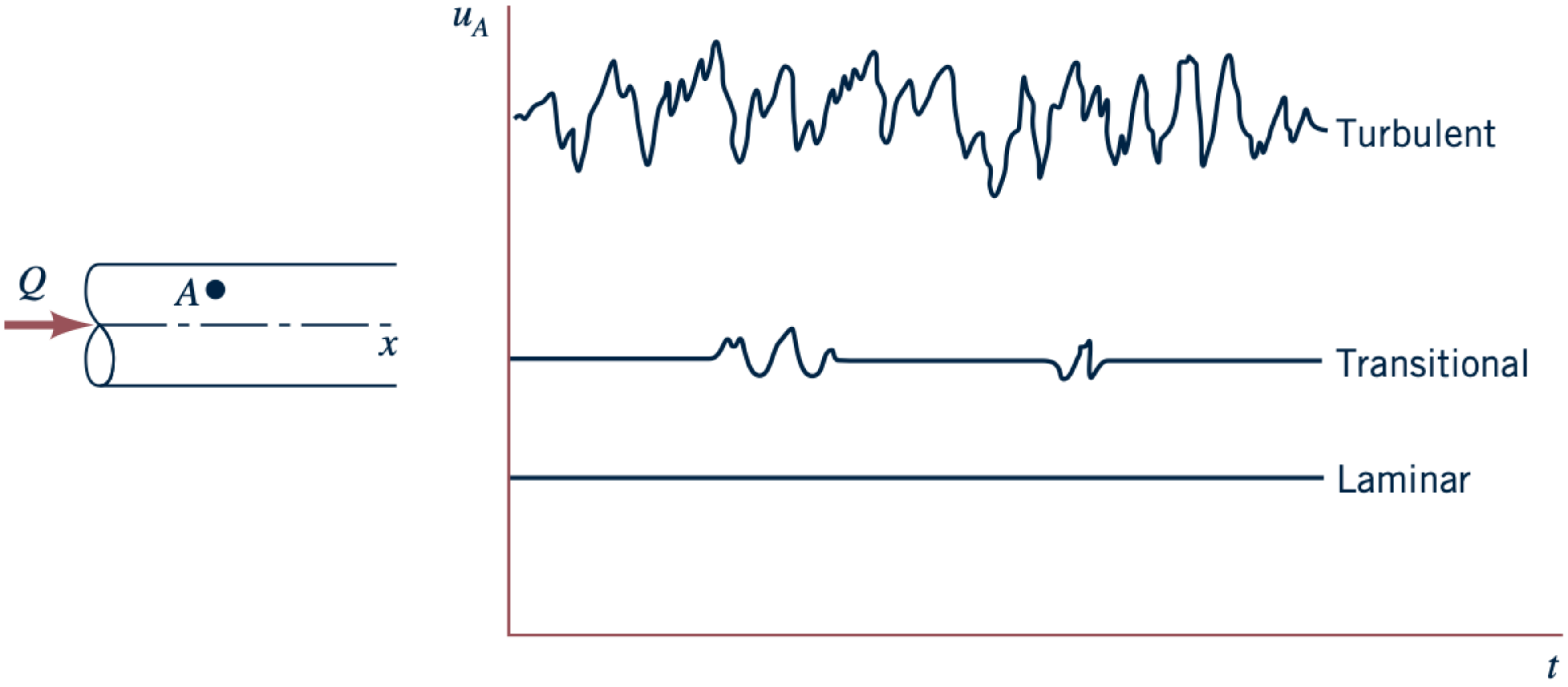
Pipe Flow


Entrance region
flow
Fully developed
flow
Inviscid core
Boundary layer
D
r
x
(1)
(2)
ℓ_e

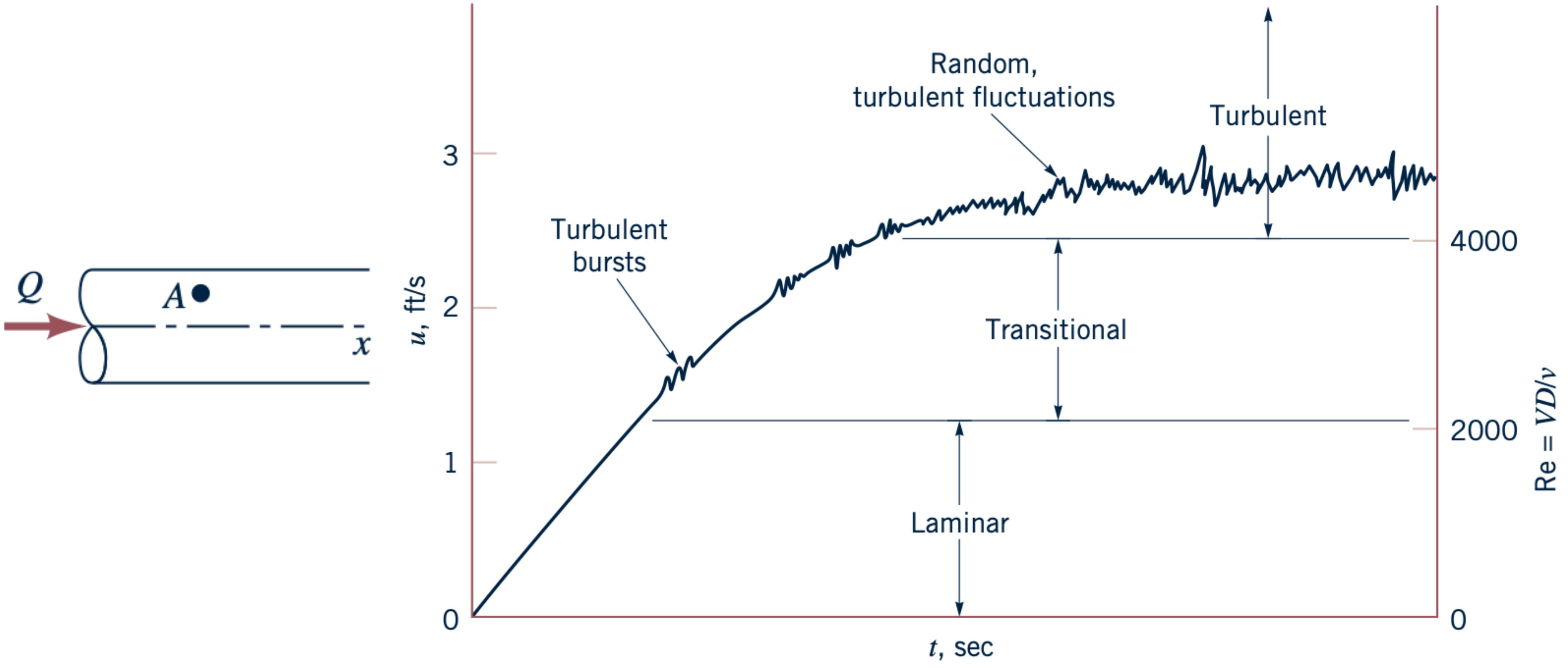
Pipe Flow – Reynolds Experiment



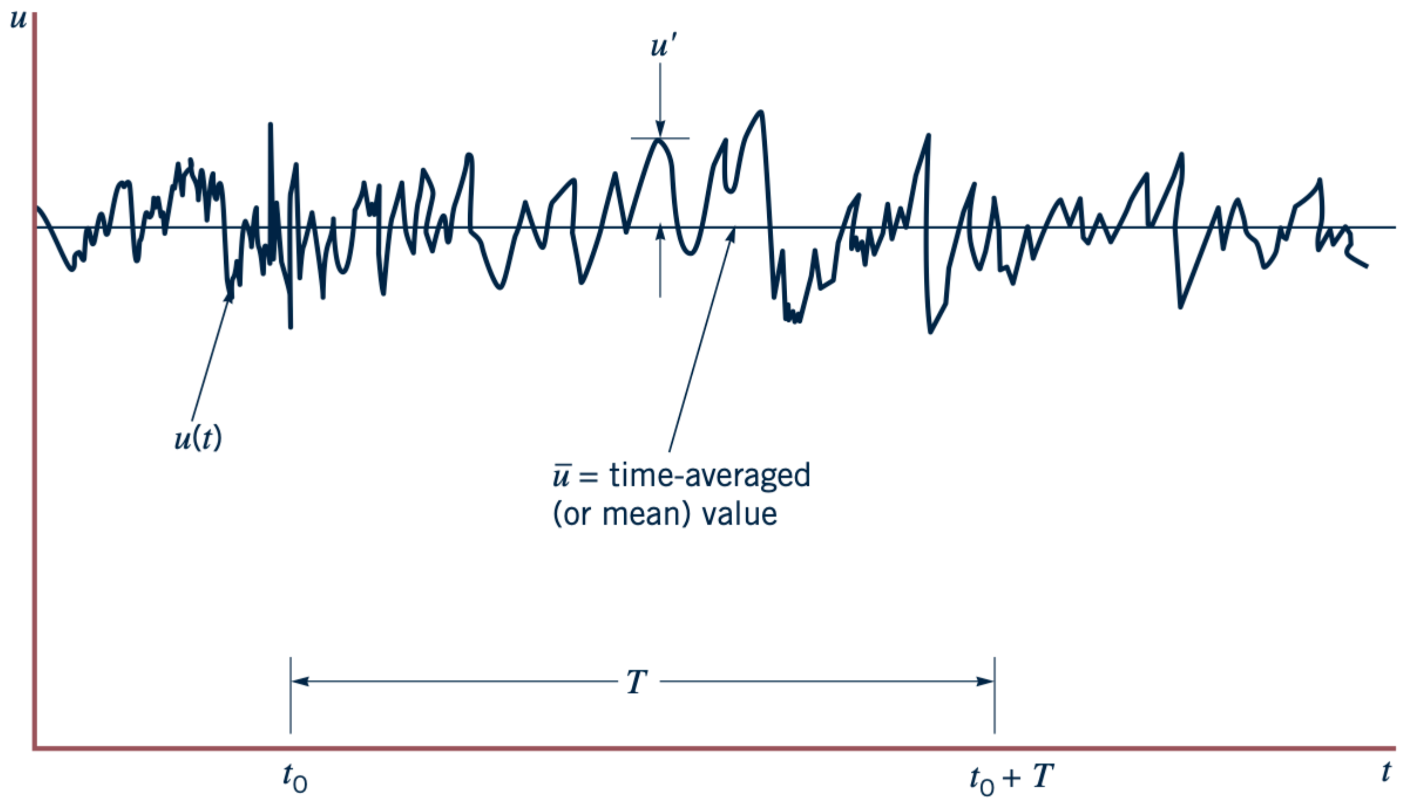
Pipe Flow – Reynolds Experiment



Pipe Flow – Reynolds Experiment

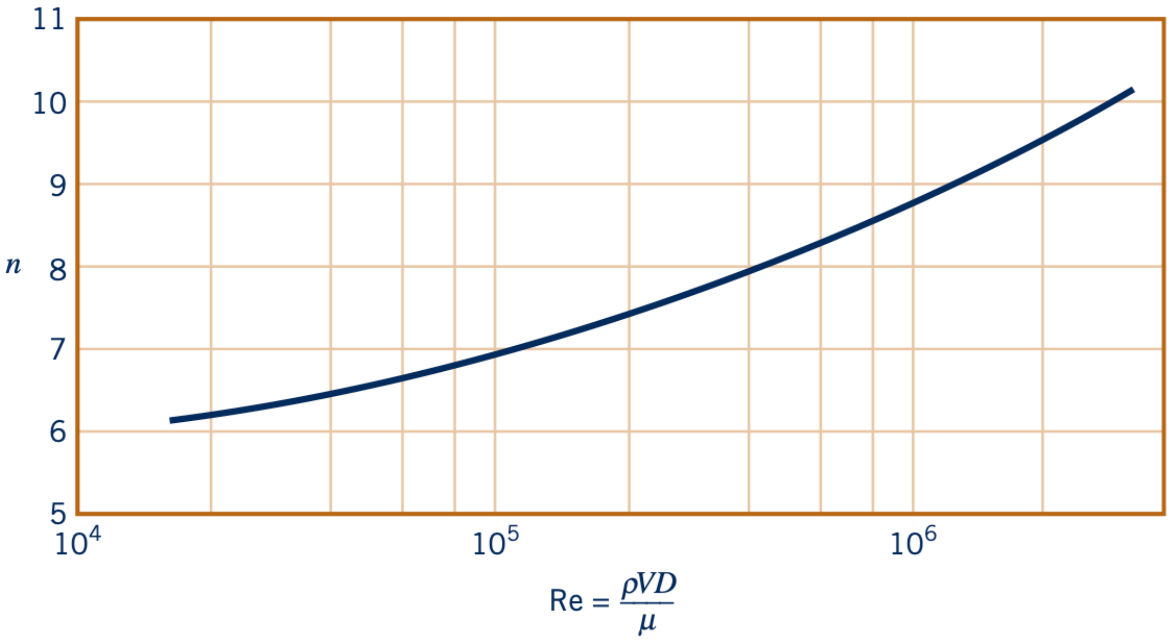


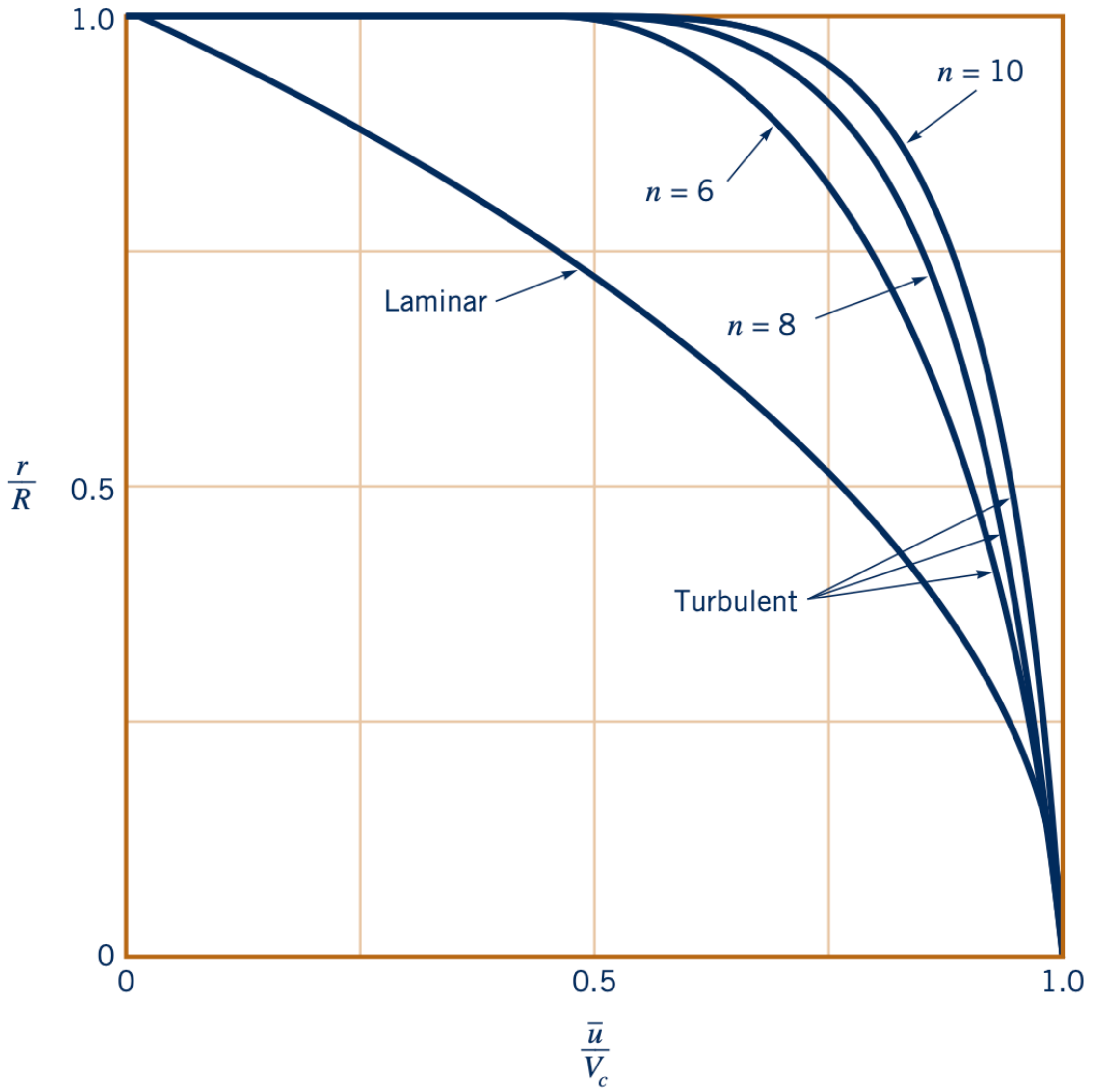
Turbulent Fluctuations



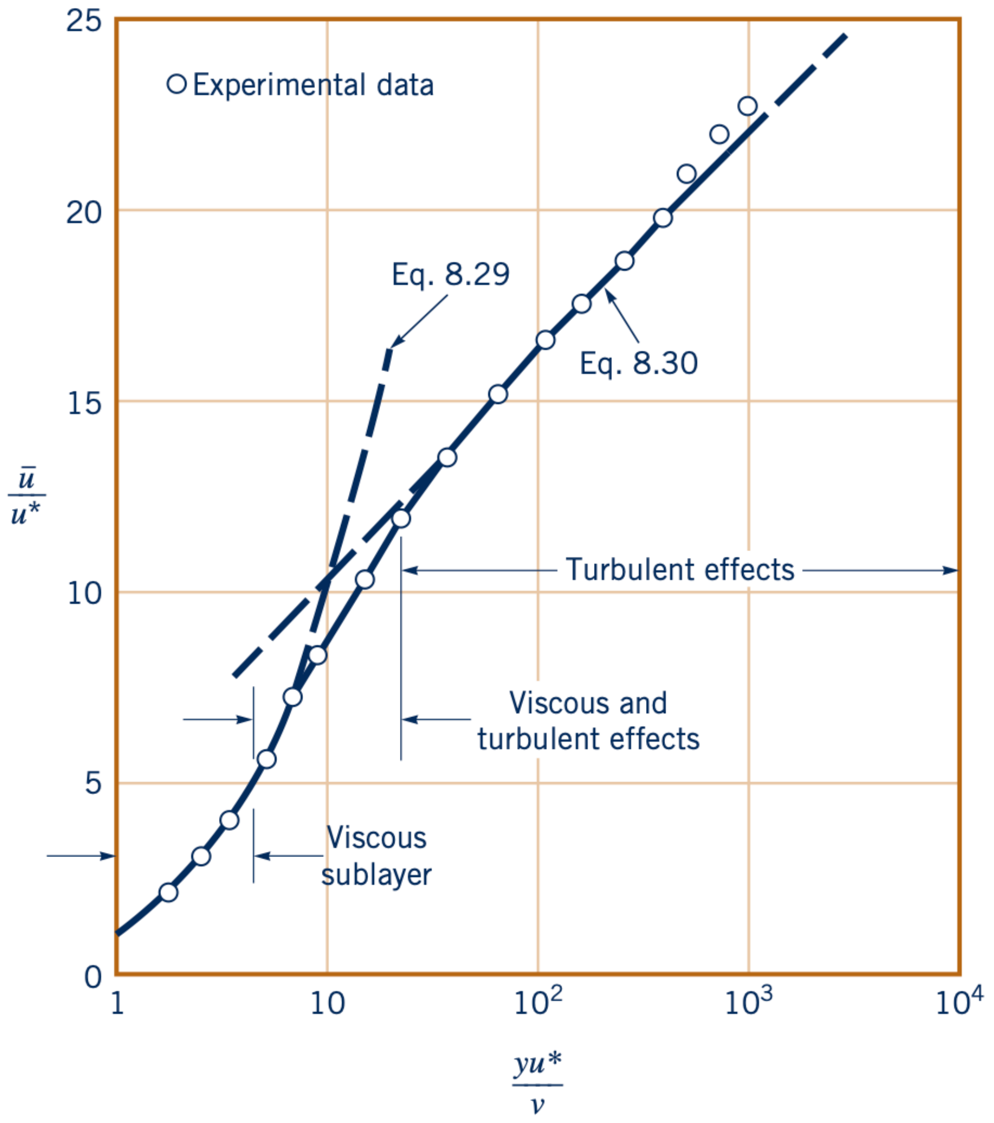
Pipe Flow – Mean Flow Profile

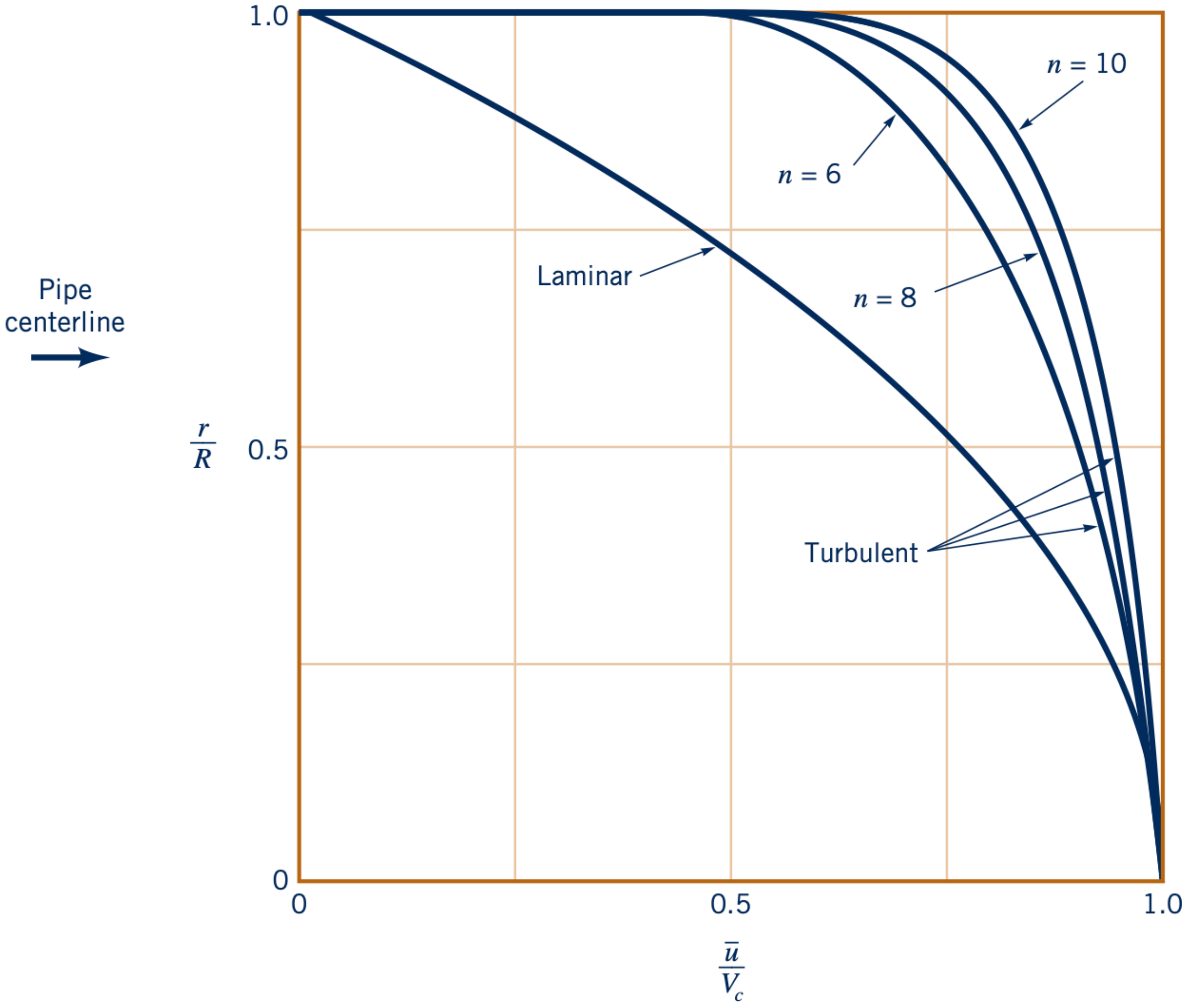
$$\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$



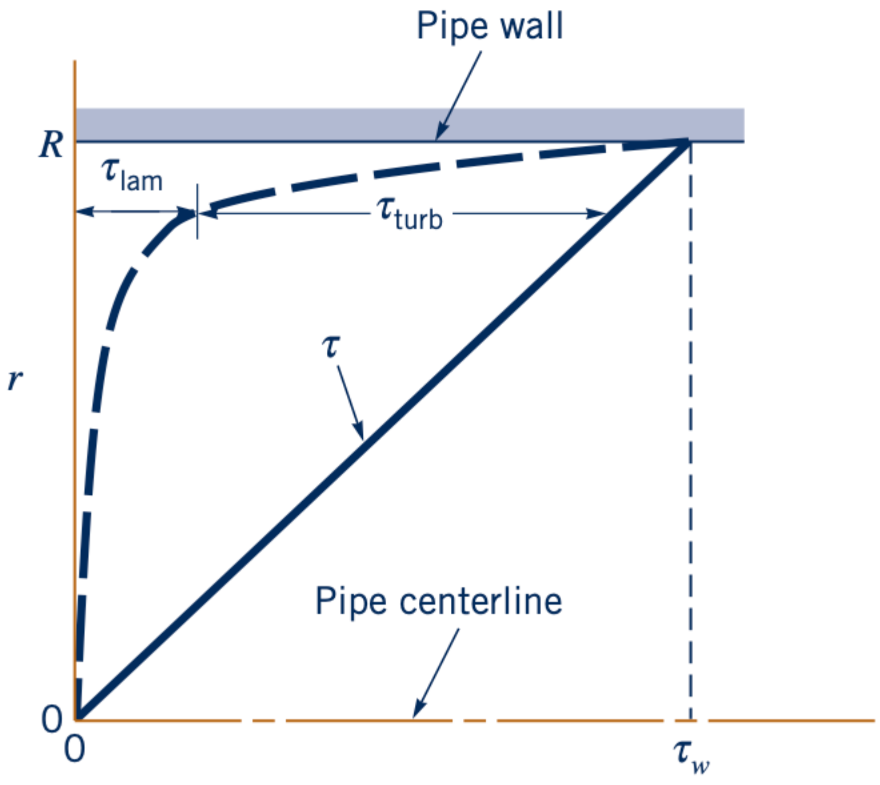


Pipe Flow – Mean Flow Profile

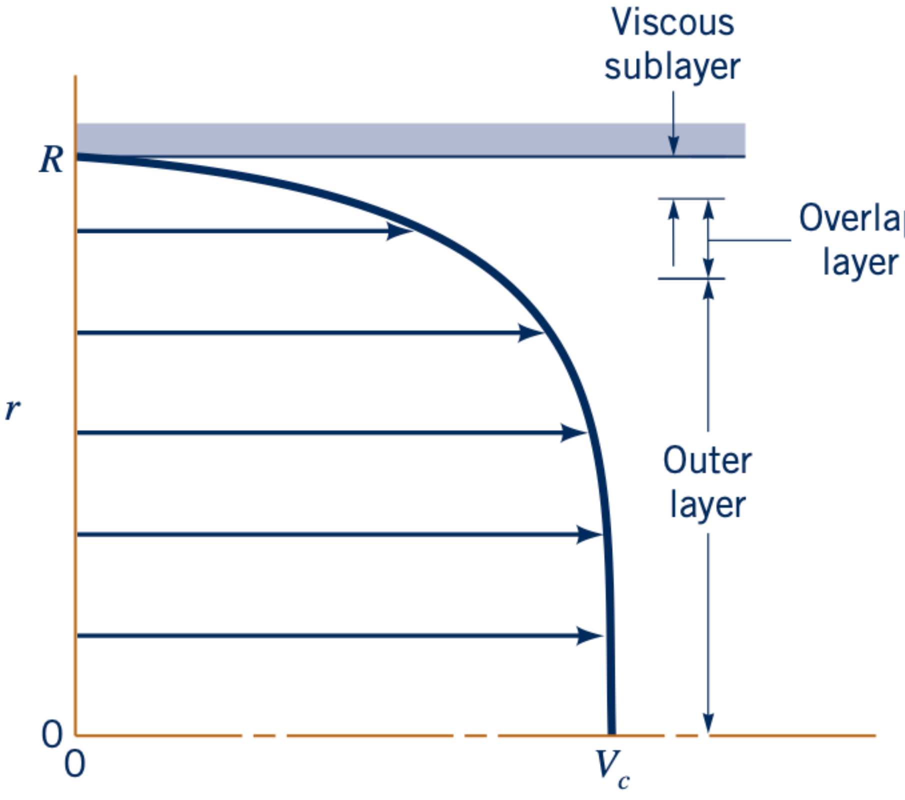




Pipe Flow – Mean Flow

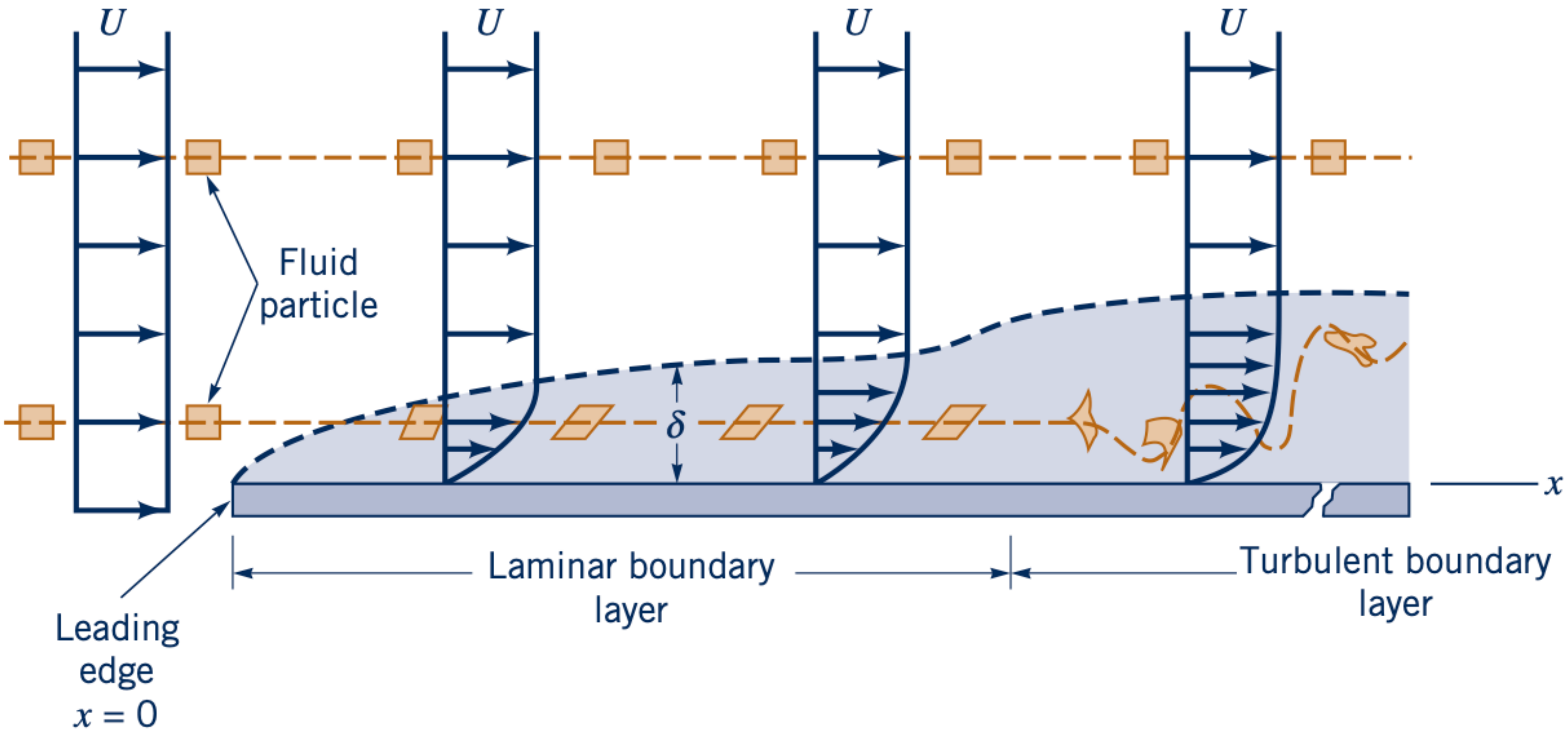


(*a*)

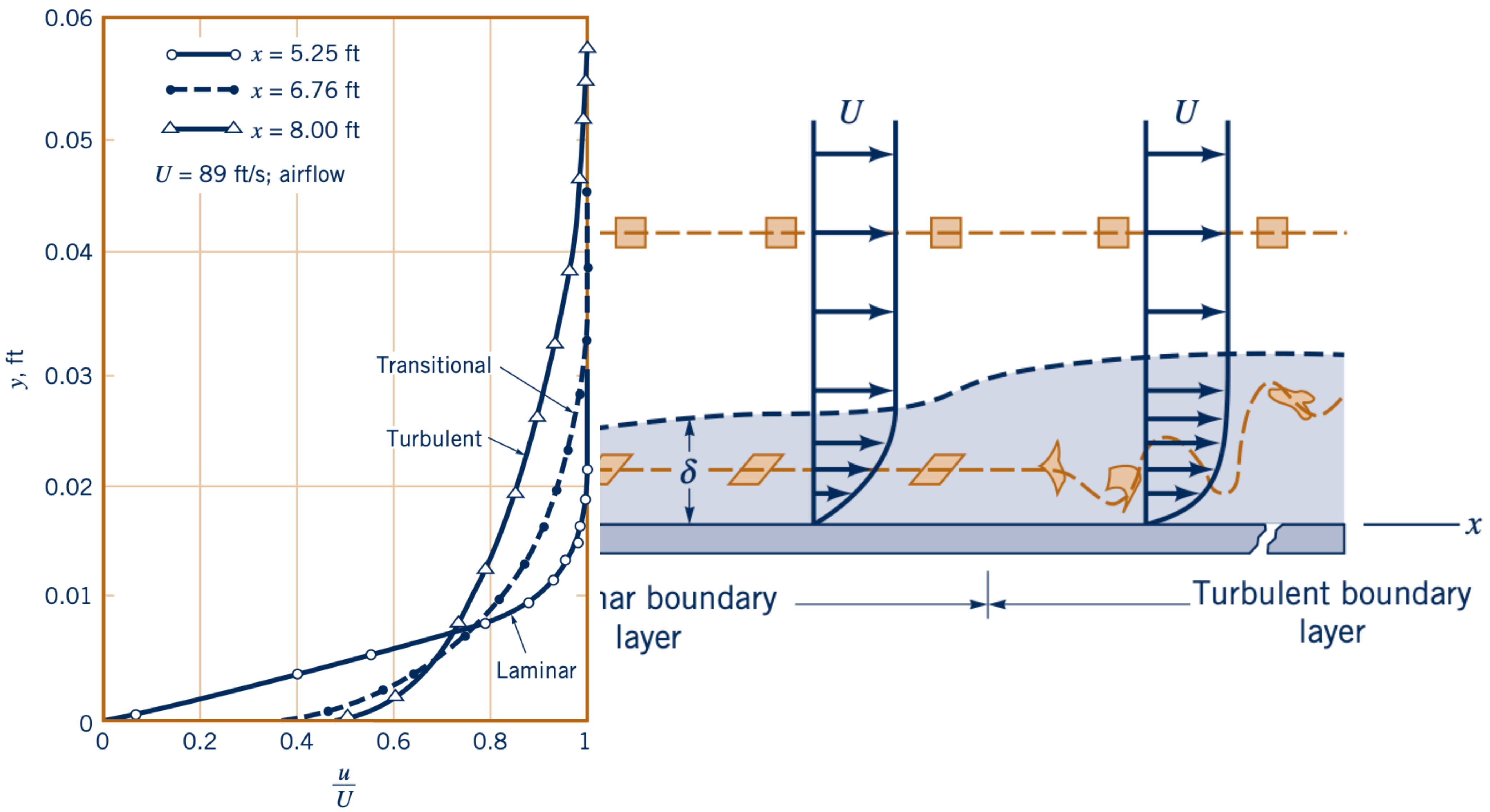


(*b*)

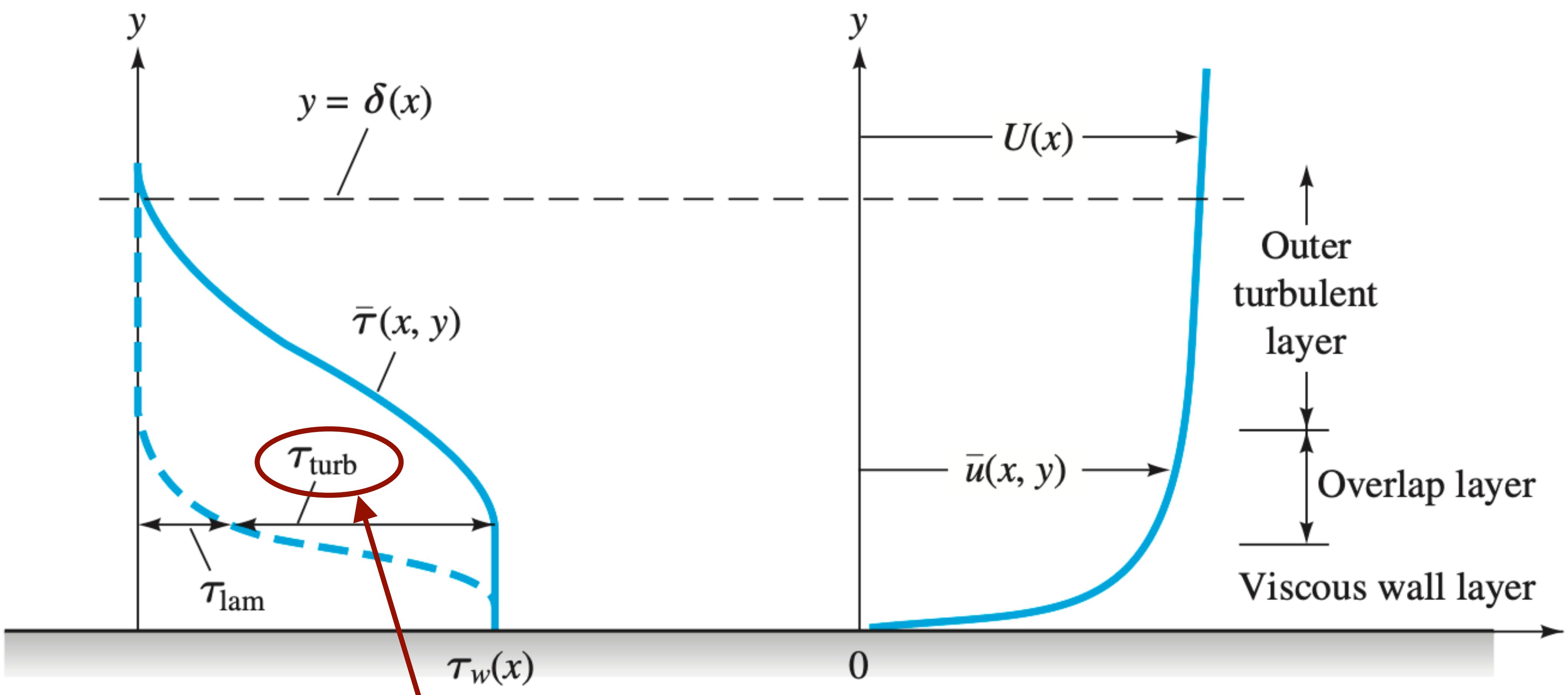
Flat-plate Boundary Layer



Flat-plate Boundary Layer

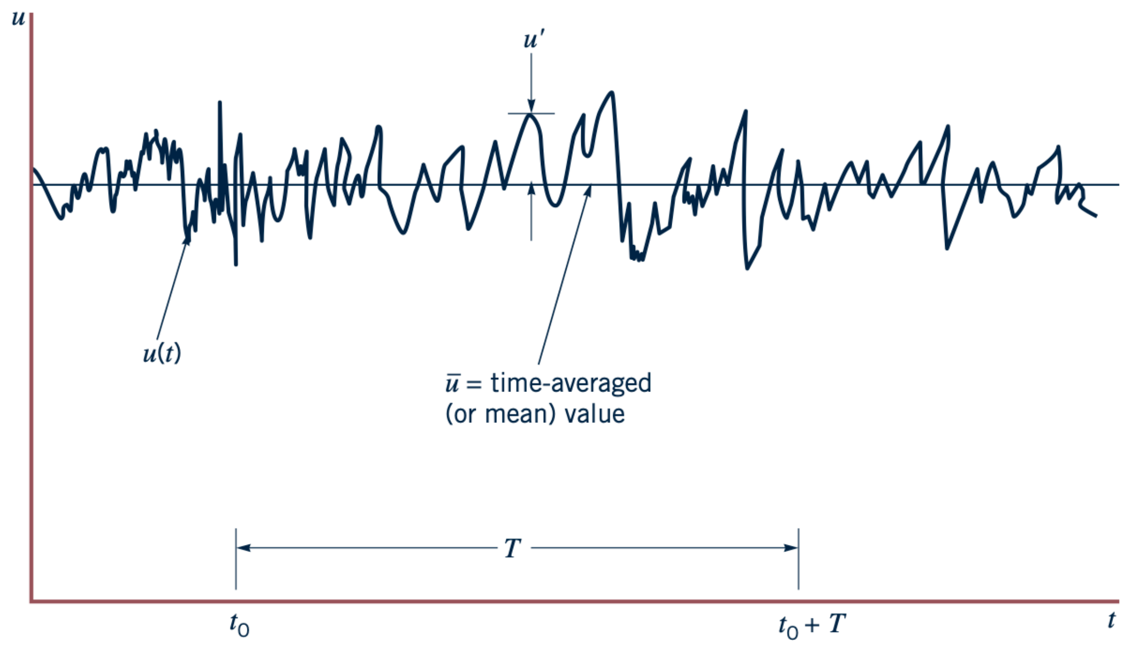


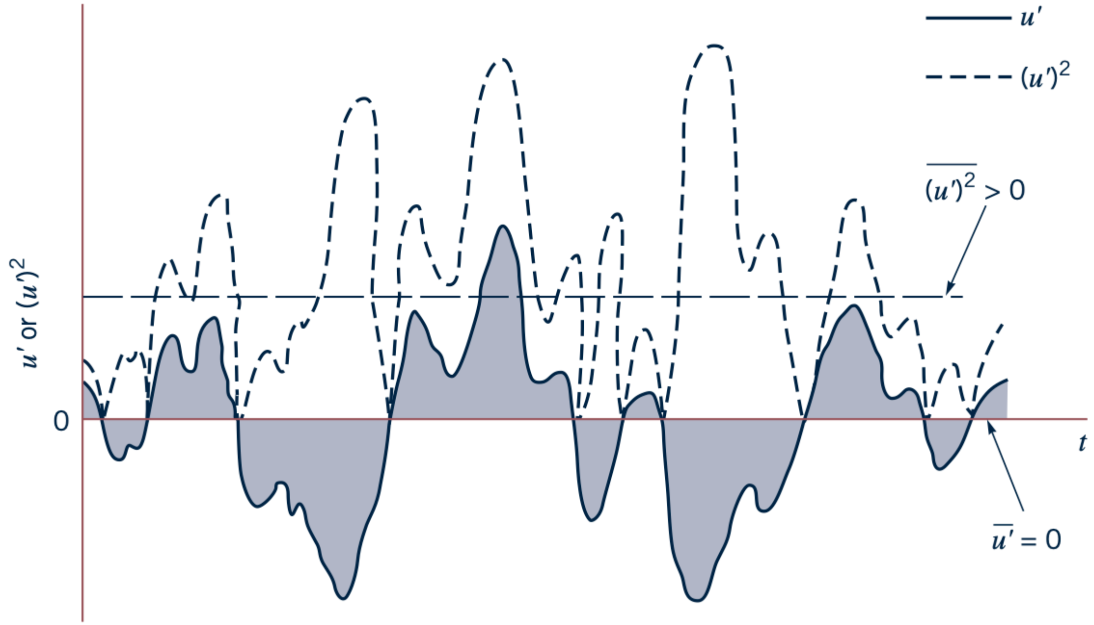
Flat Plate Flow – Mean Flow



How to model?

Turbulent Fluctuations





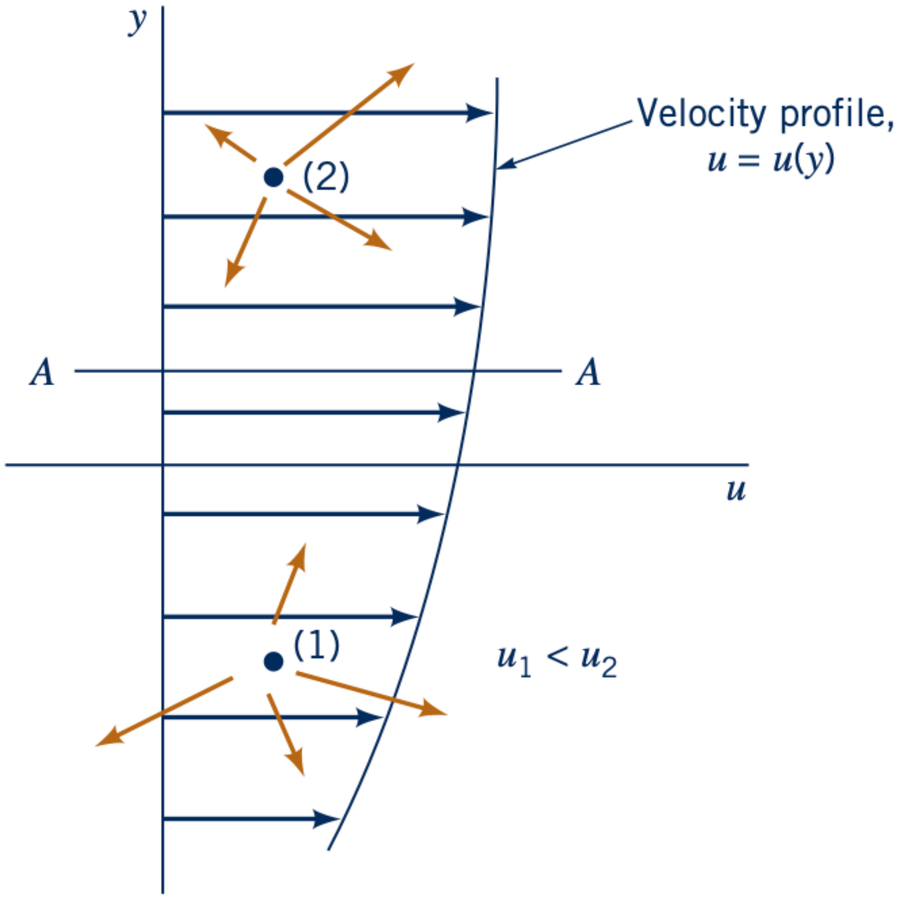
Turbulence Intensity $\mathscr{I} = \frac{\sqrt{\overline{(u')^2}}}{\bar{u}}$

Reynolds Averaged Navier-Stokes (RANS)

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{u}_i \bar{u}_j\right) = -\frac{1}{\rho_o}\frac{\partial \bar{p}}{\partial x_i} - \frac{\partial}{\partial x_j}\left(\overline{u_i' u_j'}\right) + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j}$$

Reynolds
Stress tensor $\tau_{t_{ij}} = -\overline{u_i' u_j'} = \nu_t \frac{\partial \bar{u}_i}{\partial x_j}$

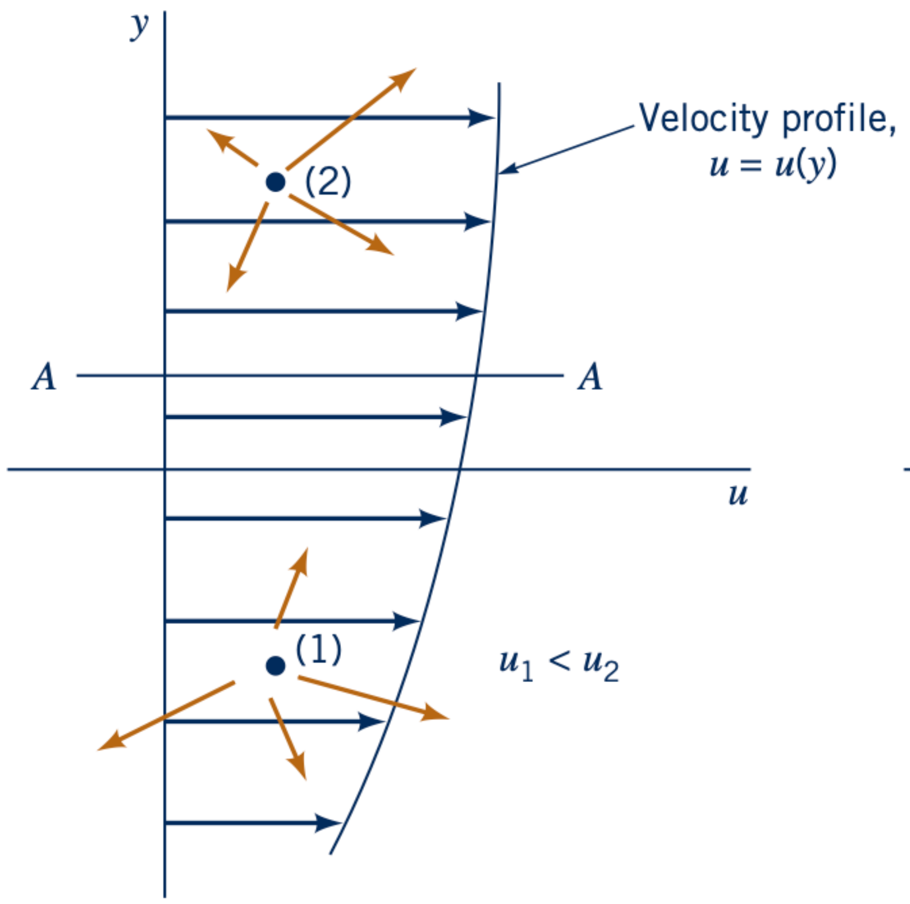
Concept of Eddy viscosity (ν_t)



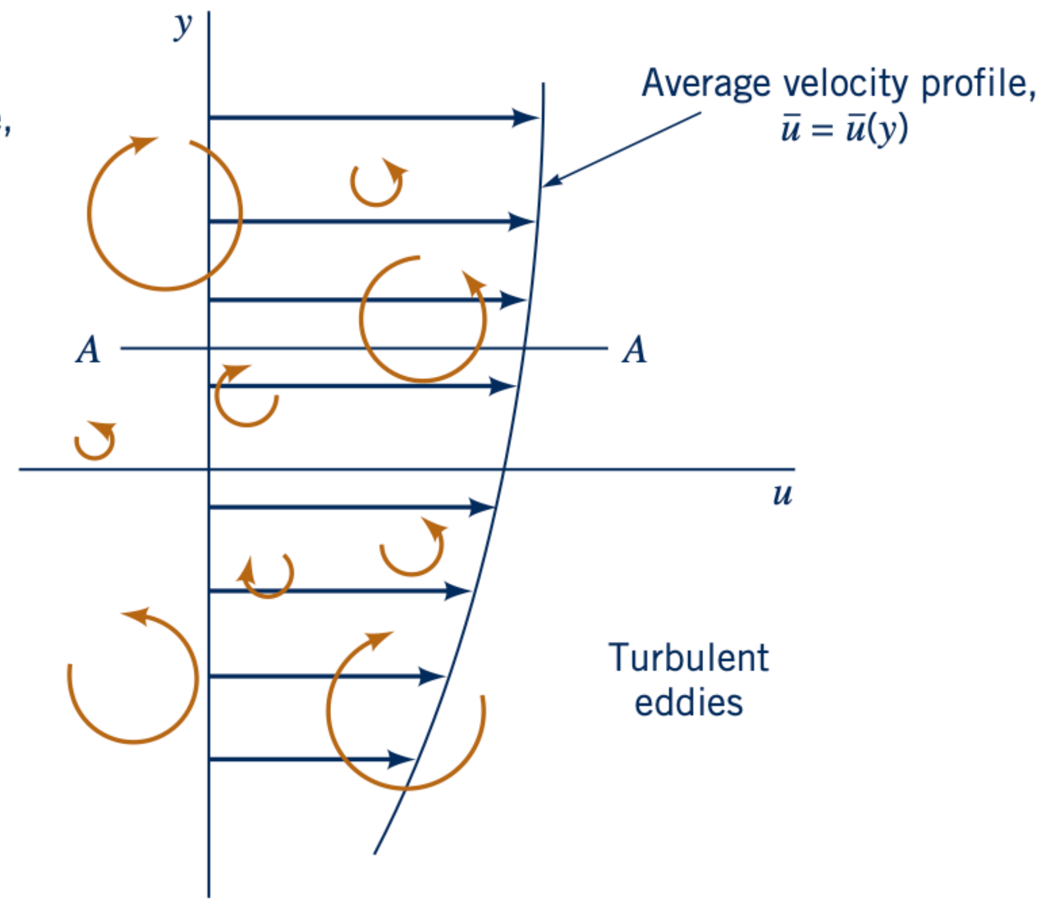
Classical Collisional transport

$$\nu \sim v_{th}\lambda_{mfp}$$

Concept of Eddy viscosity (ν_t)



Classical Collisional transport



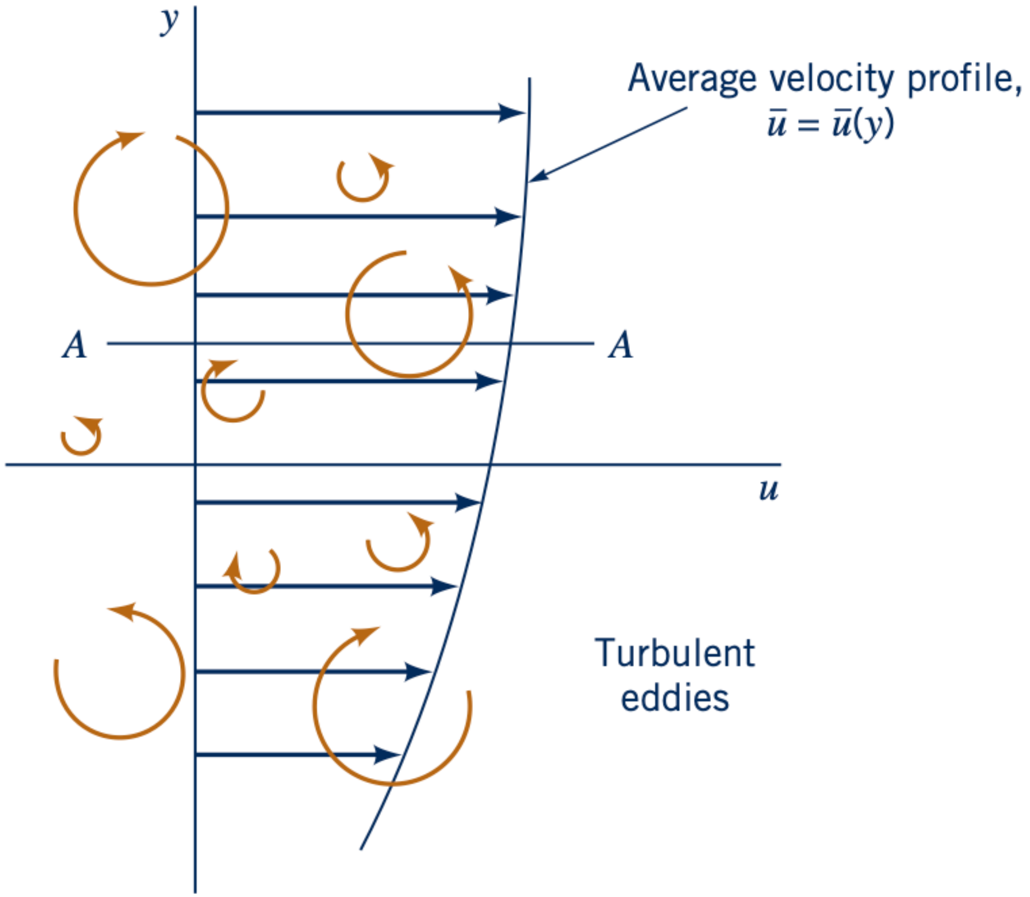
Turbulent transport

Concept of Eddy viscosity (ν_t)

l_m is "mixing length"

$$v_t \sim v_t \lambda_t \sim \frac{l_m^2}{\tau_m} = l_m^2 \frac{\partial \bar{u}}{\partial y}$$

Prandtl



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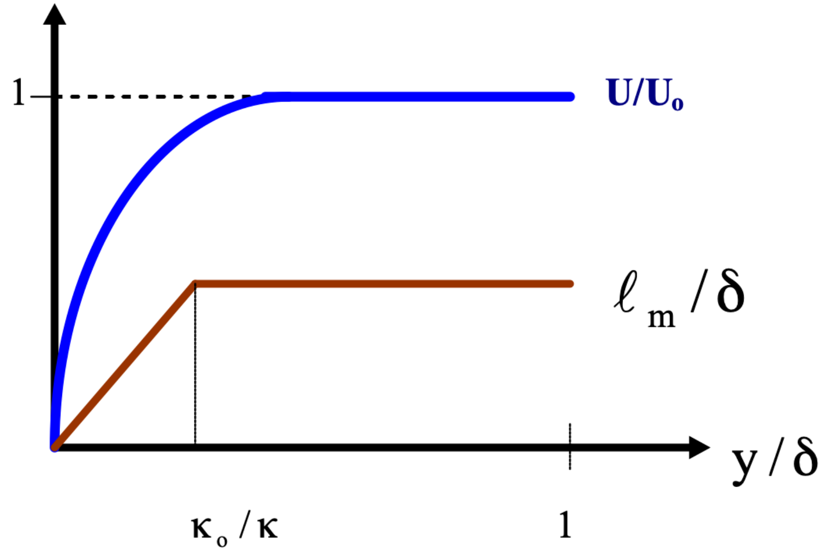
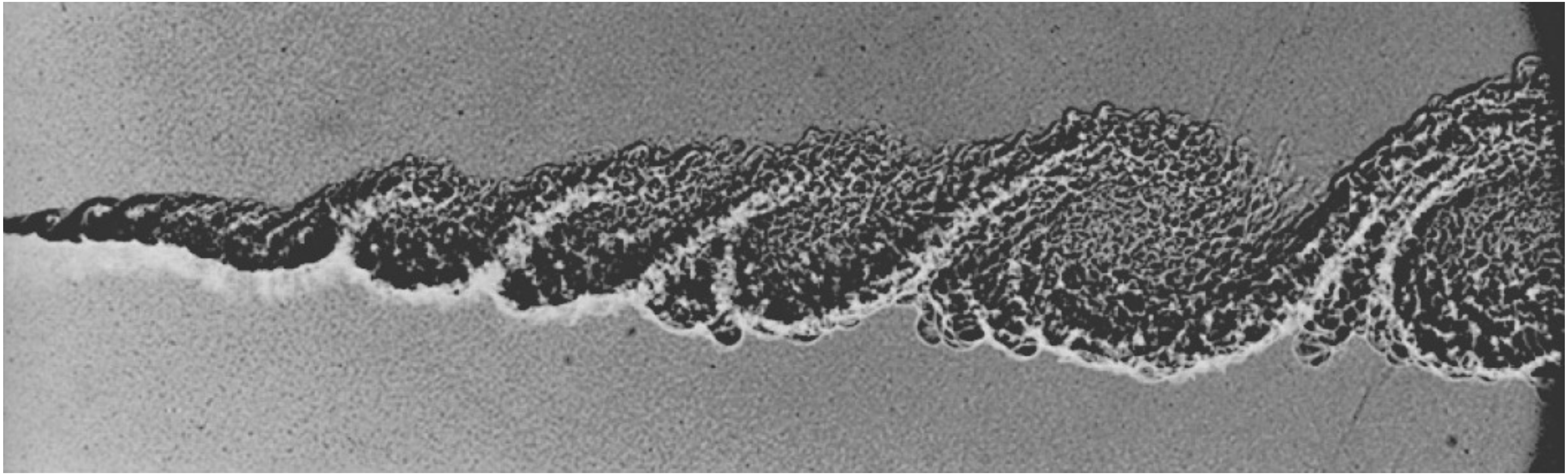


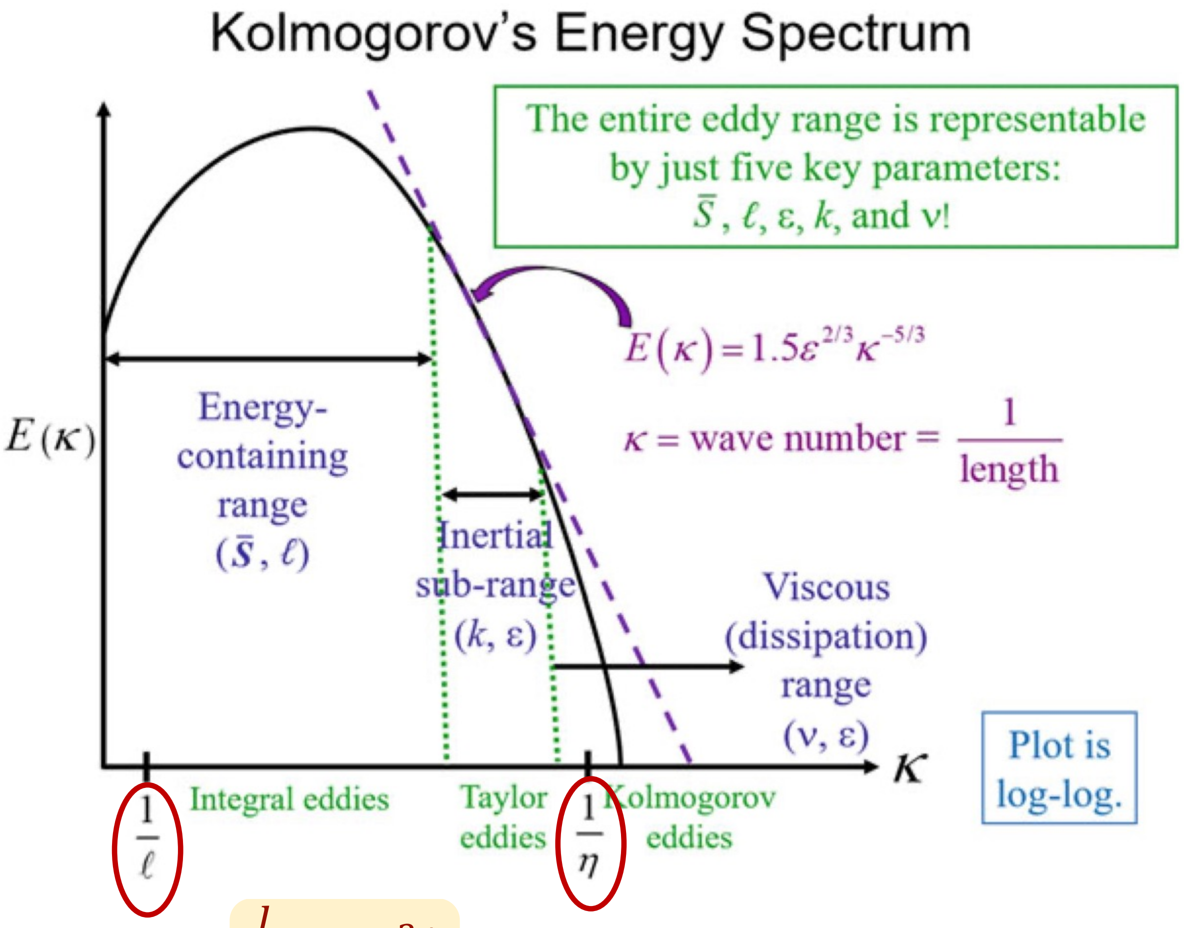
	l_m
plane jet	$0.09\ l$
circular jet	$0.075\ l$
mixing layer	$0.07\ l$
wall boundary layer	$\kappa y, \quad \frac{y}{\delta} \leq \frac{\kappa_o}{\kappa}$ $\kappa\delta, \quad \frac{y}{\delta} \geq \frac{\kappa_o}{\kappa}$ $\kappa = 0.41, \kappa_o = 0.09$

Turbulent Mixing Layer

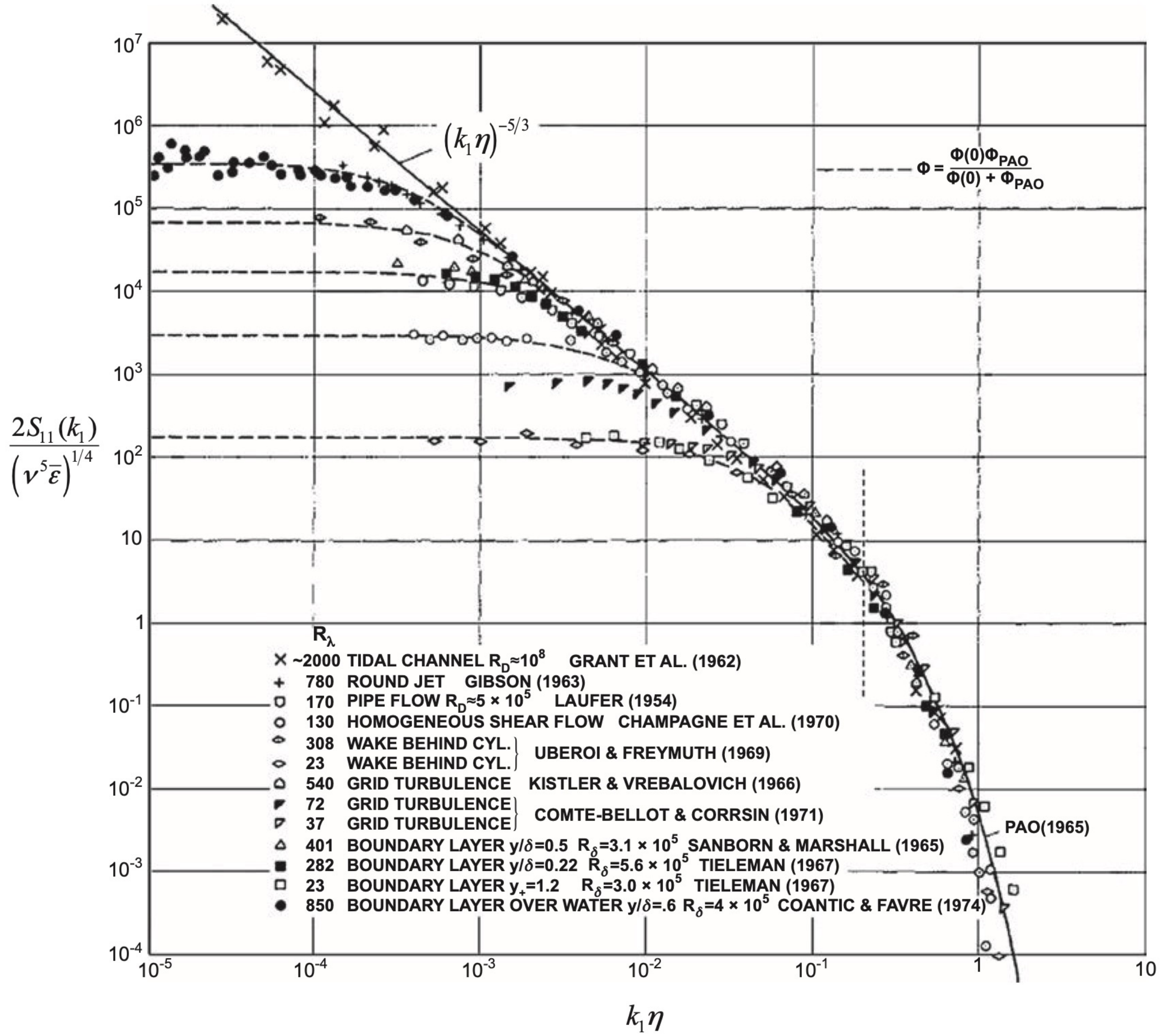


Mixing Layer between Helium (upper) and Nitrogen (lower) streams

Turbulence

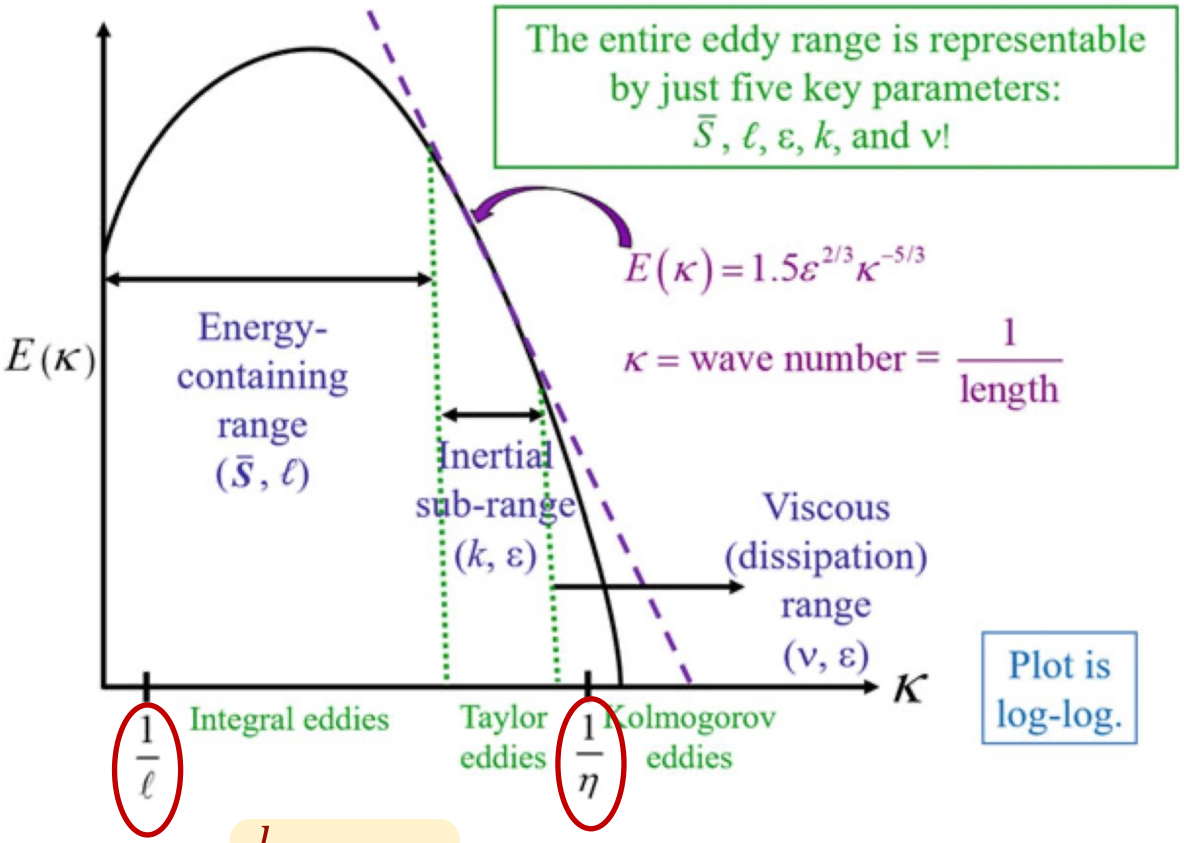


$$\frac{l}{\eta} = Re^{3/4}$$



Turbulence

Kolmogorov's Energy Spectrum



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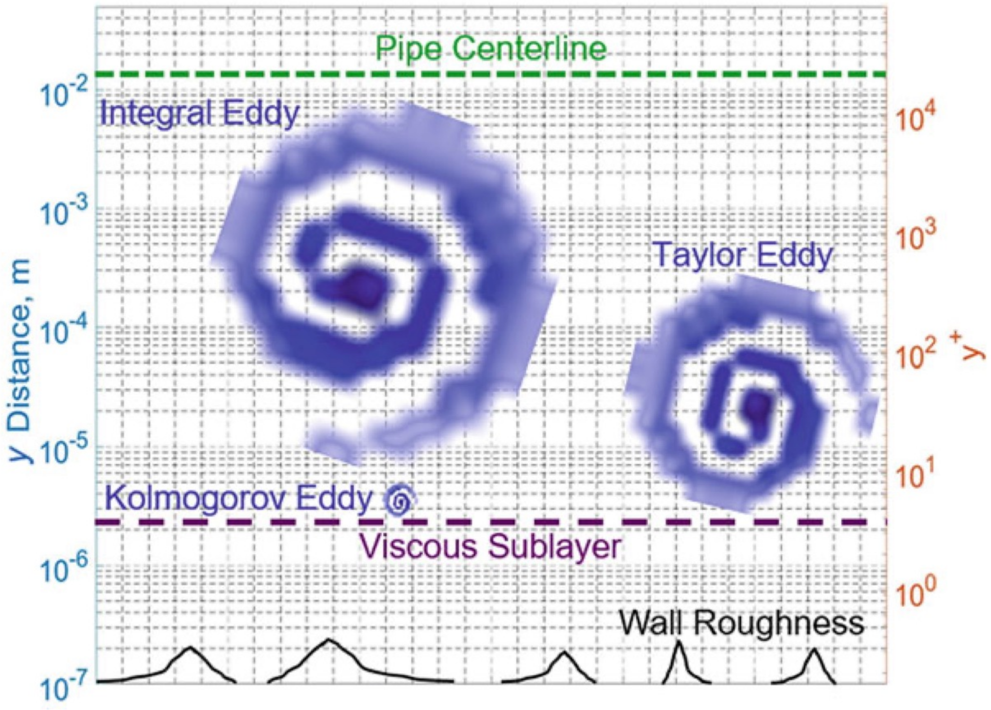
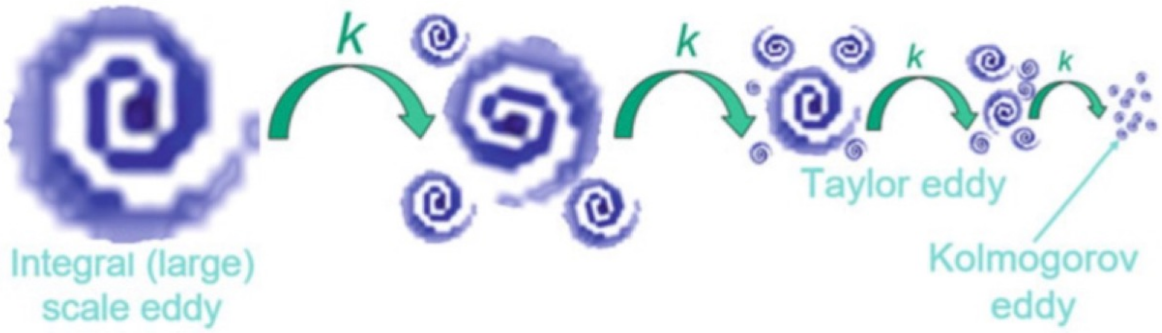
Plot is log-log.





Turbulence

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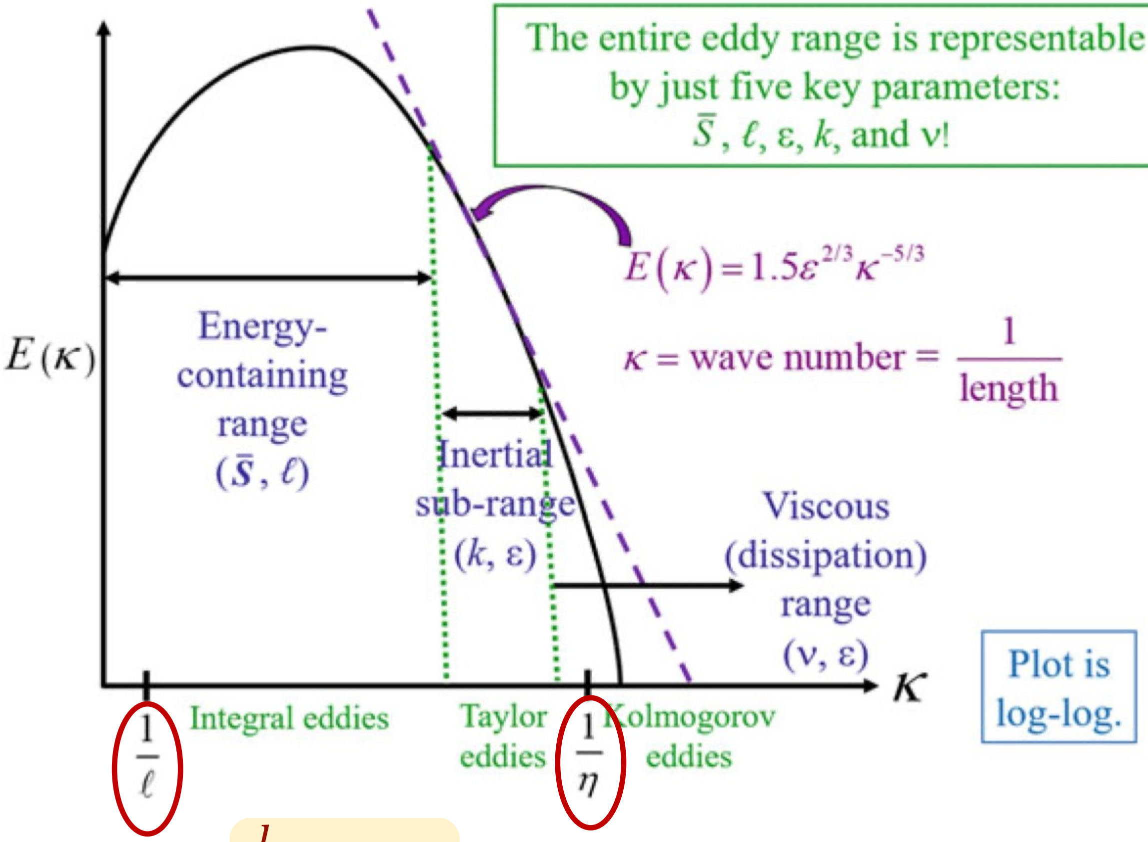

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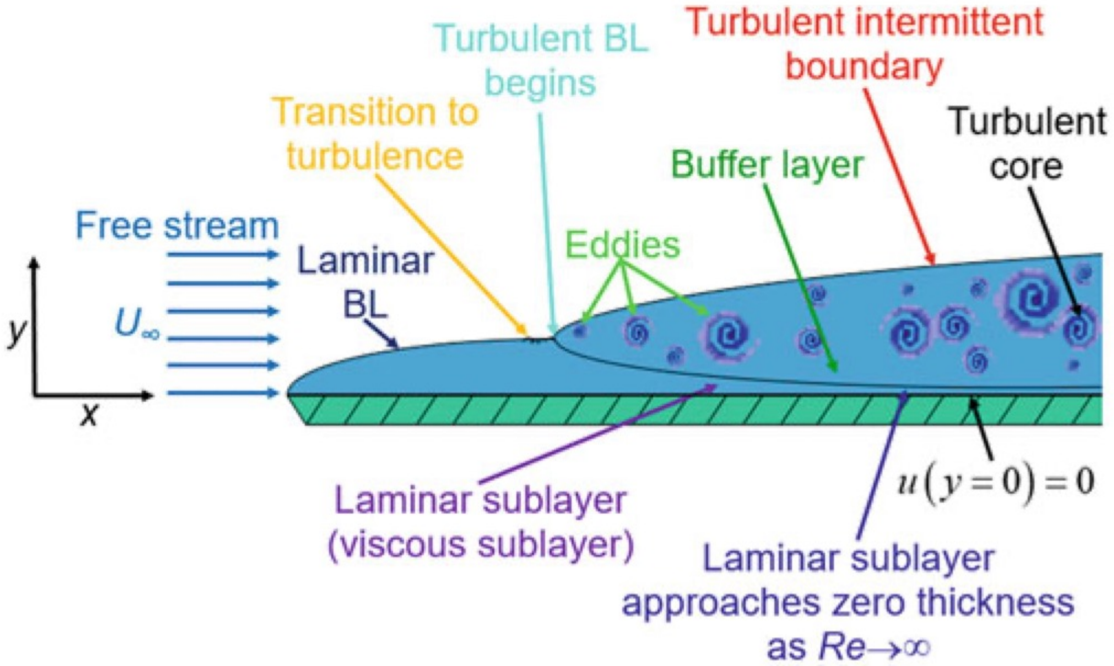

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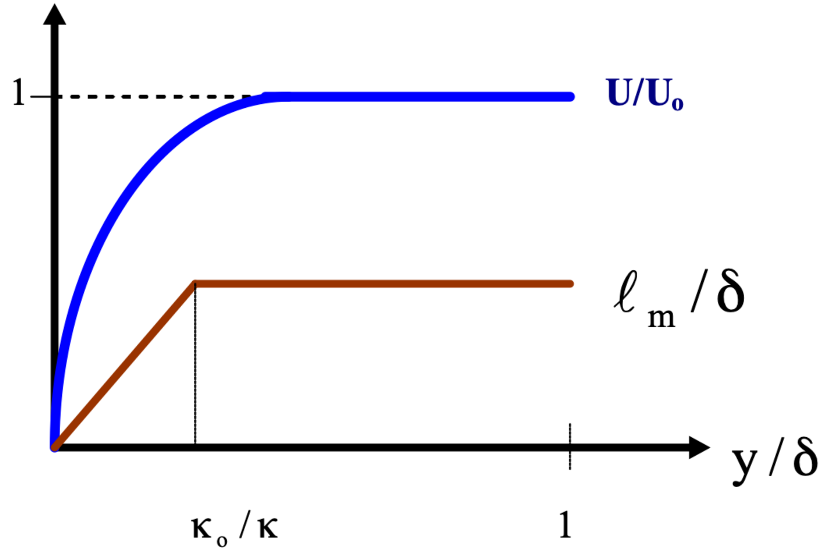

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References

- Images from the books of White, Heister et al., Munson et al., Kundu et al., and Rodriguez