Q.1. $G(s) = \dfrac{K}{s(s+1)(s^2+4s+13)}$

$\therefore$ the poles are at $s = 0, -1, -2 \pm j3$

No. of asymptotes $= |P-Z| = |4-0| = 4$

Angle of asymptotes $= \dfrac{180^\circ(2l+1)}{P-Z}$; $l = 0, 1, \ldots, |P-Z|-1$
$= 0, 1, 2, 3$

$= 45^\circ, 135^\circ, 225^\circ, 315^\circ$

Centroid, $\sigma = \dfrac{\Sigma \text{Poles} - \Sigma \text{Zeroes}}{P-Z}$

$= \dfrac{0-1-2-2}{4} = \dfrac{-5}{4} = -1.25$

Breakaway points $\rightarrow$

$\dfrac{dK}{ds} = 0 \Rightarrow \dfrac{d}{ds}[G(s)] = 0$

$\Rightarrow \dfrac{d}{ds}\left[\dfrac{1}{s(s+1)(s^2+4s+13)}\right] = 0$

$\Rightarrow \dfrac{d}{ds}\left[s(s+1)(s^2+4s+13)\right] = 0$

$\Rightarrow \dfrac{d}{ds}\left[s^4 + 4s^3 + 13s^2 + s^3 + 4s^2 + 13s\right] = 0$

$\Rightarrow 4s^3 + 15s^2 + 34s + 13 = 0$

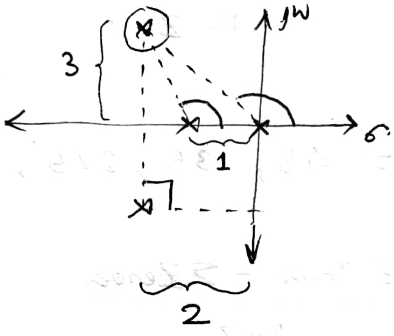
$\Rightarrow s = -0.467, -1.642 \pm j2.067$

$\therefore$ the actual breakaway point is $s = -0.467$ as it lies on the root locus.

Angle of departure $\rightarrow$

$\theta_D = 180^\circ - \phi$, where $\phi = \angle Poles - \angle zeroes$

Considering the pole at $s = -2 + j3$



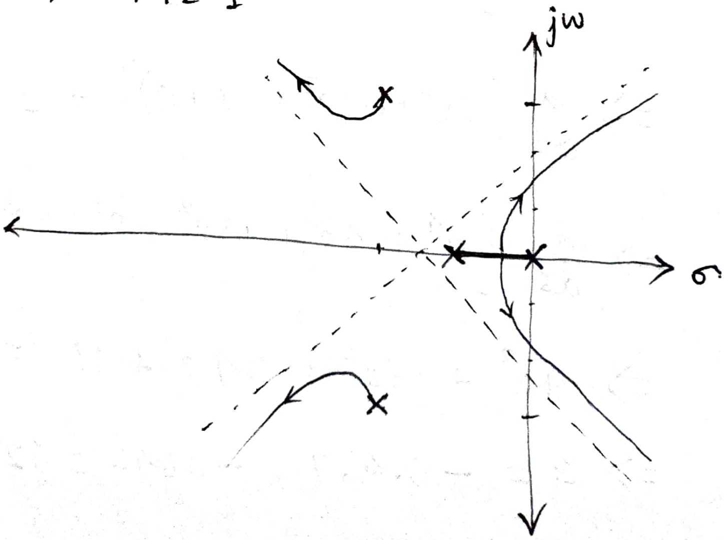
$\angle Poles = \left(90^\circ + \tan^{-1}\frac{2}{3}\right) + \left(90^\circ + \tan^{-1}\frac{1}{3}\right) + 90^\circ$

$= 322.1^\circ$

$\therefore \quad \theta_D = 180^\circ - 322.1^\circ = -142.1^\circ$

Similarly, for the pole at $s = -2 - j3$,

$\theta_D = 142.1^\circ$



Points of intersection with the imaginary axis $\rightarrow$

Characteristic equation is $s^4 + 5s^3 + 17s^2 + 13s + k = 0$

$$
\begin{array}{c|ccc}
s^4 & 1 & 17 & k \\
s^3 & 5 & 13 & \\
s^2 & \frac{72}{5} & k & \\
s^1 & \dfrac{\frac{72}{5}\times 13 - 5k}{\frac{72}{5}} & &
\end{array}
$$

Considering s^1 to be the row of zeroes,

$$5k = \frac{72}{5} \times 13$$

$$\Rightarrow k = 37.44$$

Auxillary equ^n $\rightarrow$ $\frac{72}{5}s^2 + k = 0$

Substituting $k = 37.44$ in the above equ^n,

$$\frac{72}{5}s^2 + 37.44 = 0$$

$$\Rightarrow s = \pm j1.612$$

Q.2. $G(s) = \dfrac{K(s+2)(s+3)}{s(s+1)}$

Since $|P-Z| = |2-2| = 0$, there are no asymptotes.

Break-in & breakaway points →

$$\frac{dK}{ds} = 0 \Rightarrow \frac{d}{ds}\left[G(s)\right] = 0$$

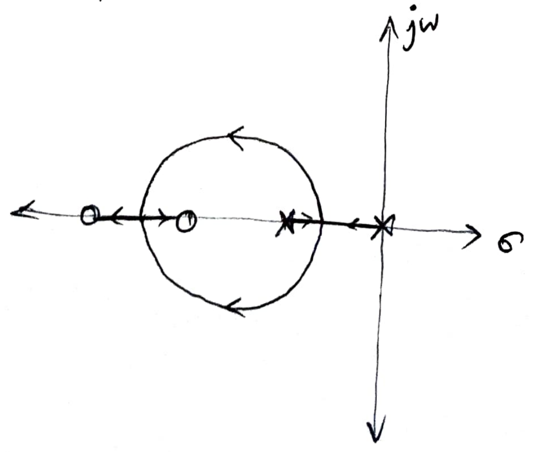
$$\Rightarrow s(s+1)\frac{d}{ds}\left[(s+2)(s+3)\right] - (s+2)(s+3)\frac{d}{ds}\left[s(s+1)\right] = 0$$

$$\Rightarrow s(s+1)(2s+5) - (s+2)(s+3)(2s+1) = 0$$

$$\Rightarrow 4s^2 + 12s + 6 = 0$$

$$\Rightarrow s = -0.634, -2.366$$

Since $s = -0.634$ lies between two poles, it is a breakaway point. And, since $s = -2.366$ lies between two zeroes, it is a break-in point.



Q. 3. $$G(s) = \frac{ks^2}{(s+5)(s+50)}$$

$$= \frac{ks^2}{5\times 50\left(1+\frac{s}{5}\right)\left(1+\frac{s}{50}\right)}$$

$$= \left(\frac{k}{250}\right)\frac{s^2}{\left(1+\frac{s}{5}\right)\left(1+\frac{s}{50}\right)}$$

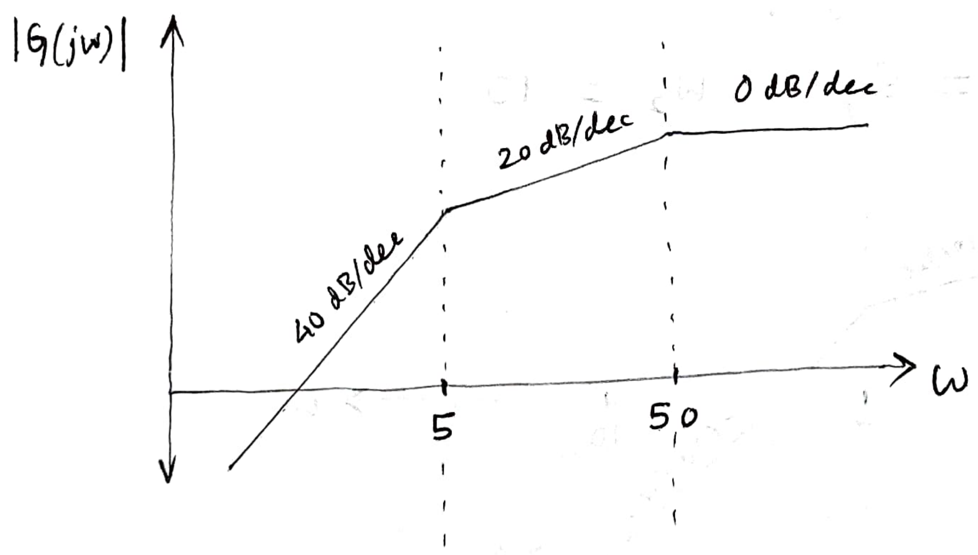
$\therefore$ the corner frequencies are

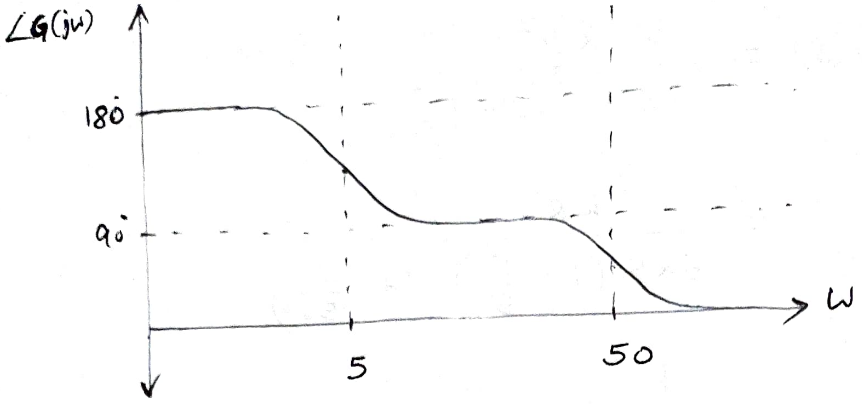
$w_1 = 5$ rad/s

$w_2 = 50$ rad/s

The gain crossover frequency $\rightarrow$

$$|G(jw)| = k' w_{gc}^2 = 1 \Rightarrow w_{gc} = \sqrt{\frac{1}{k'}}$$





Q.4.

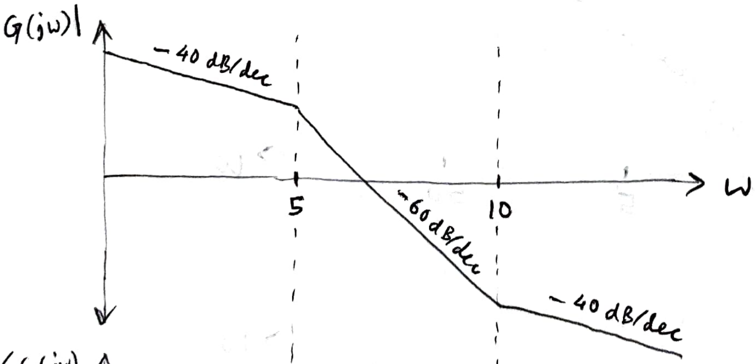
$$G(s) = \frac{K(s+10)}{s^2(s+5)}$$

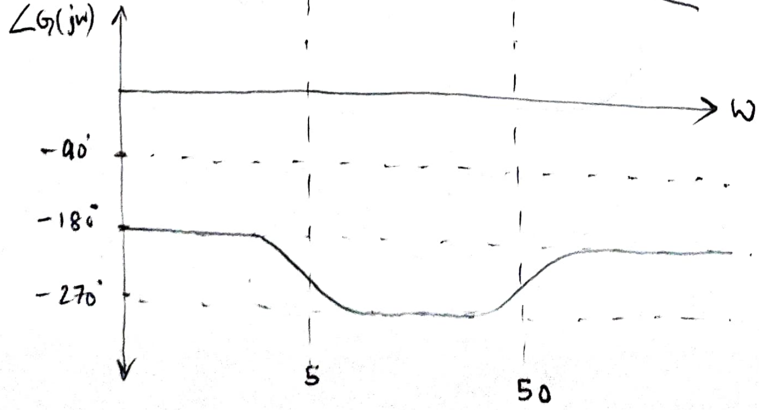
$$= \frac{K \cdot 10\left(1+\frac{s}{10}\right)}{5 \cdot s^2\left(1+\frac{s}{5}\right)}$$

$$= (2K) \cdot \frac{\left(1+\frac{s}{10}\right)}{s^2\left(1+\frac{s}{5}\right)}$$

The corner frequencies are →

$\omega_1 = 5$, $\omega_2 = 10$

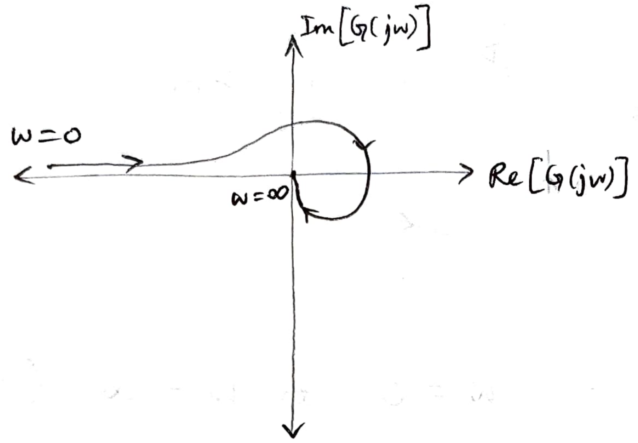




Q. 5. $G(s) = \dfrac{K}{s^2(1+sT_1)(1+sT_2)(1+sT_3)}$

Type = 2

order = 5



$$G(j\omega) = \frac{K}{(j\omega)^2(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}$$

$$\angle G(j\omega) = -180^\circ - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 - \tan^{-1}\omega T_3$$

At $\omega = 0$,

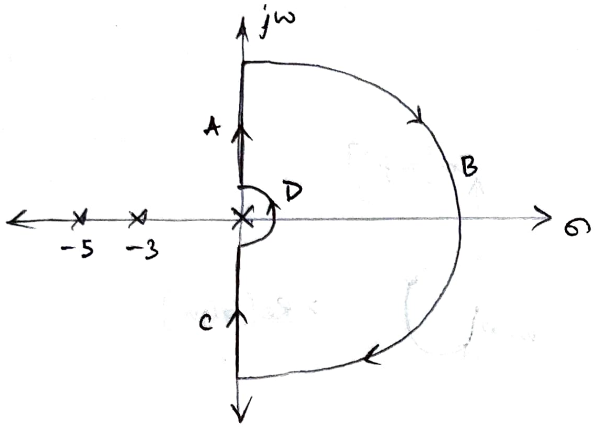
$$\angle G(j\omega) = -180^\circ$$

At $\omega = \infty$,

$$\angle G(j\omega) = -180^\circ - 90^\circ - 90^\circ - 90^\circ$$
$$= -450^\circ$$

Q:6. $$G(s) = \frac{K}{s(s+3)(s+5)}$$

Nyquist contour →



Section A → $w = 0$ to $w = \infty$ (Polar plot)

Section B → Radius 'R' semicircle of the N-contour

Substitute $s = \lim_{R \to \infty} R e^{j\theta}$, $90^\circ \geq \theta \geq -90^\circ$

$$G(jw) = \frac{1}{jw(jw+3)(jw+5)}$$

$$= \lim_{R \to \infty} \frac{1}{Re^{j\theta}(Re^{j\theta}+3)(Re^{j\theta}+5)}$$

$$= 0$$

∴ section B maps to a single point at the origin in the N-plot.

Section C → $w = -\infty$ to $w = 0$

(Reverse of the Polar plot)

Section D $\rightarrow$ Radius 'ϵ' semicircle of the N-contour.

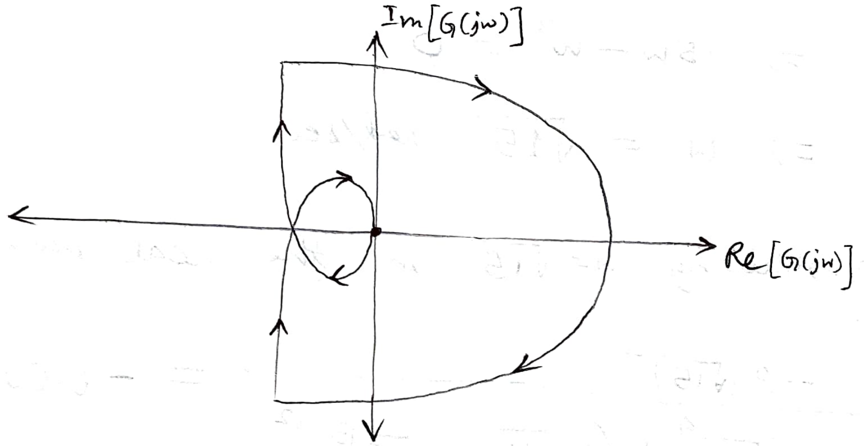
substitute $s = \lim_{\epsilon \to 0} \epsilon e^{j\theta}$, $-90^\circ \leqslant \theta \leqslant 90^\circ$

$$G(s) = \frac{1}{s(s+3)(s+5)}$$

$$= \lim_{\epsilon \to 0} \frac{1}{\epsilon e^{j\theta}(\epsilon e^{j\theta}+3)(\epsilon e^{j\theta}+5)}$$

$$= \infty \angle -\theta$$

$\therefore$ Section D maps to an infinite radius semicircle in the N-plot.



Point of intersection with the real axis $\rightarrow$

$$G(jw) = \frac{1}{jw(jw+3)(jw+5)}$$

$$= \frac{1}{-jw^3 - 5w^2 - 3w^2 + 15jw}$$

$$= \frac{1}{-8w^2 + j(15w - w^3)}$$

$$= \frac{-8w^2 - j(15w - w^3)}{[-8w^2 + j(15w-w^3)][-8w^2 - j(15w-w^3)]}$$

$$= \frac{-8w^2 - j(15w - w^3)}{64w^4 + (15w-w^3)^2}$$

$$= \frac{-8w^2}{64w^4+(15w-w^3)^2} - j\frac{(15w-w^3)}{64w^4+(15w-w^3)^2}$$

For points on the ~~imag~~ real axis, the imaginary part is zero, hence

$$\frac{15w - w^3}{64w^4 + (15w-w^3)^2} = 0$$

$$\Rightarrow 15w - w^3 = 0$$

$$\Rightarrow w = \sqrt{15} \text{ rad/sec}$$

Substituting $w = \sqrt{15}$ in the real part,

$$\frac{-8(\sqrt{15})^2}{64(\sqrt{15})^4 + (15\sqrt{15} - (\sqrt{15})^3)^2} = -0.0083$$

$\therefore$ the point of intersection is $s = -0.0083$

Now, $P = 0$ and $N = 0$ $\Rightarrow Z = P - N = 0$

But as we keep increasing K, the point of intersection will keep on shifting to the left in the real axis. When it would reach $s=-1$, our system would become marginally stable. Beyond that, it would become unstable.

$$\text{Gain margin} = \frac{1}{0.0083} = 120.5$$

Hence, for stability, the range of K should be

$$0 < K < 120.5$$

(7) $G(s) = \dfrac{6}{(s+2)(s^2+2s+2)}$

$$G(j\omega) = \frac{6}{(j\omega+2)(-\omega^2+2j\omega+2)} \times \frac{2-j\omega}{2-j\omega}$$

$$= \frac{6(2-j\omega)}{(4+\omega^2)\{(2-\omega^2)+2j\omega\}} \times \frac{\{(2-\omega^2)-2j\omega\}}{\{(2-\omega^2)-2j\omega\}}$$

$$= \frac{6\{2(2-\omega^2)-4j\omega-j\omega(2-\omega^2)-2\omega^2\}}{(4+\omega^2)\{(2-\omega^2)^2+4\omega^2\}}$$

$$= \frac{6(4-2\omega^2-2\omega^2-4j\omega-2j\omega+j\omega^3)}{(4+\omega^2)((2-\omega^2)^2+4\omega^2)}$$

$$G(j\omega) = \frac{24(1-\omega^2)-6j\omega(6-\omega^2)}{(4+\omega^2)((2-\omega^2)^2+4\omega^2)}$$

Gain margin :- $\omega = ?$ $G(j\omega)$ crosses real axis.

$Im\{G(j\omega)\} = 0$

$6-\omega^2 = 0 \Rightarrow \omega = \sqrt{6}$

$$|G(j\omega)|_{\omega=\sqrt{6}} = \left|\frac{24(1-6)}{(4+6)(16+24)}\right|$$

$$= \left|\frac{-\cancel{24}^{3} \times \cancel{5}}{\cancel{10}_{2} \times \cancel{40}_{10}}\right| = 0.3$$

Gain margin $= 20\log\frac{1}{0.3} = 10.45\,dB$

Phase margin : $\omega = ?$ $|G(j\omega)| = 1$

using solver $\omega = 1.253$

$$\angle G(j\omega)\Big|_{\omega=1.253} = -\tan^{-1}\left(1.253/2\right)$$

$$- \tan^{-1}\left(2.5/0.43\right)$$

$$= -32.07^\circ - 80.24^\circ$$
$$= -112.31^\circ$$

Phase Margin $= 180^\circ - 112.31^\circ$
$= 67.69^\circ$

(8) $$G(s) = \frac{1}{(s+1)(s+2)(s+10)}$$

PI controller $D(s) = K_p + \dfrac{K_I}{s}$

Step input, $e_{ss} = 0$.

$$G(s)\,D(s) = \frac{K_p s + K_I}{s(s+1)(s+2)(s+10)}$$

Compensated System is "Type 1".

Date
Page

So, any K_p, K_I value will result in zero steady-state error to step input.

For same transient-behaviour:
Closed loop dominant poles = ?
Characteristic equation of closed-loop system :-

$$(s+1)(s+2)(s+10)+1 = 0$$
$$s^3 + 13s^2 + 32s + 21 = 0$$
$$s = -1.1295, -1.8567, -10.0138$$

$$G(s)\,D(s) = \frac{K_p s + K_I}{s(s+1)(s+2)(s+10)}$$

Characteristic equation of closed-loop system:(compensated):-

$$s(s+1)(s+2)(s+10) + K_p s + K_I = 0$$
$$s^4 + 13s^3 + 32s^2 + 21s + K_p s + K_I = 0$$
$$s^4 + 13s^3 + 32s^2 + (21+K_p)s + K_I = 0$$

Main idea is to choose K_p and K_I such that zero $(K_p s + K_I)$ cancels out with closed loop pole generating negligible effect on transient behaviour.

Let $K_p = 1$, $K_I = 0.5$

$$s = -0.02, -1.24, -1.71, -10.03.$$

Pole 0.02 is near zero 0.5, cancelling it's effect.