Indian Institute of Technology Bombay | Aerospace Engineering
AE 308 **Control Theory** | Assignment 1
AE 775 **System Modelling, Dynamics and Control** | October 30, 2022

Q.1. Find the transfer function of the system whose step response is shown in Figure 1. Assume the system is of second-order. [10]

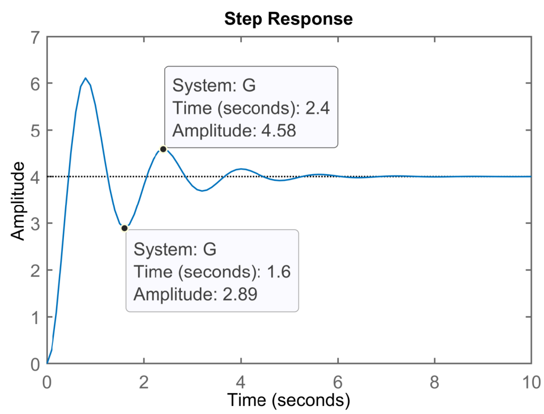

Figure 1: Step Response of the unknown system.

Ans.1.

$$c(t) = 1 - \frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} \cos\left(w_d t - \phi\right), \qquad \phi = \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)$$

for $r(t) = 1$.

For $r(t) = 4$, observed from the plot

$$c(t) = 4 - 4\frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} \cos\left(w_d t - \phi\right), \qquad \phi = \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)$$

Given: $\dfrac{2\pi}{w_d} = 1.6$, $\dfrac{3\pi}{w_d} = 2.4$, $c(1.6) = 2.89$, $c(2.4) = 4.58$.

Gives: $\dfrac{\pi}{w_d} = 0.8 \implies w_d = \dfrac{\pi}{0.8}$.

Also,

$$0.722 = 1 - \frac{e^{-1.6\zeta w_n}}{\sqrt{1-\zeta^2}}\cos\left(1.6w_d - \phi\right) \tag{1}$$

$$1.145 = 1 - \frac{e^{-2.4\zeta w_n}}{\sqrt{1-\zeta^2}}\cos\left(2.4w_d - \phi\right) \tag{2}$$

We have,
$\cos(1.6w_d - \phi) = \cos(2\pi - \phi) = \cos(\phi) = \sqrt{1-\zeta^2}$

$\cos(2.4w_d - \phi) = \cos(3\pi - \phi) = -\cos(\phi) = -\sqrt{1-\zeta^2}$.

Putting in above

$$0.722 = 1 - e^{-1.6\zeta w_n} \tag{3}$$

$$1.145 = 1 + e^{-2.4\zeta w_n} \tag{4}$$

$$\zeta w_n = \frac{\zeta w_d}{\sqrt{1-\zeta^2}} = \frac{\zeta\pi}{0.8\sqrt{1-\zeta^2}} = \frac{\pi}{0.8}\tan\phi$$

$$0.722 = 1 - e^{-2\pi\tan\phi} \tag{5}$$

$$1.145 = 1 + e^{-3\pi\tan\phi} \tag{6}$$

On solving any of them we get $\phi = 0.2$, giving $\zeta = 0.2$, $w_n = 4$.

$$G(s) = \frac{4w_n^2}{s^2 + 2\zeta w_n s + w_n^2} = \frac{64}{s^2 + 1.6s + 16}$$

Q.2. Consider a second-order system transfer function in normalized form

$$G(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}.$$

Let $G_1(s)$, $G_2(s)$ and $G_3(s)$ be three different transfer functions given in the above structure and the location of poles of $G_1(s)$, $G_2(s)$ and $G_3(s)$ be different. The poles are, respectively, (p_{11}, p_{12}), (p_{21}, p_{22}) and (p_{31}, p_{32}). Find (p_{11}, p_{12}), (p_{21}, p_{22}) and (p_{31}, p_{32}) in the complex plane such that $G_1(s)$, $G_2(s)$ and $G_3(s)$ have the same settling time (2%). Give justification. [10]

Ans.2. In all three $G(s)$'s, the coefficient of $'s'$ should be the same and w_n^2 terms should be different.

Q.3. Find the transfer function of the system whose step response is shown in Figure 2. Assume the system is of second-order. Let the transfer function so obtained is $G(s)$, now introducing a pure integrator before the input to the system obtaining modified system $M(s) = \frac{1}{s}G(s)$,

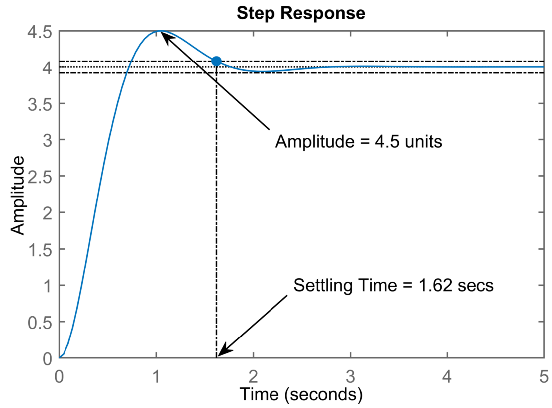

Figure 2: Step Response of the unknown system.

calculate the steady state error of the modified system $M(s)$ when input fed to the system is (a) impulse (b) step. [10]

Ans.3. OS% $= \dfrac{4.5-4}{4} = 12.5\%$, gives $\zeta = 0.5519$.

Settling time $\dfrac{4}{\zeta w_n} = 1.62$, gives $w_n = 4.4739$.

$$G(s) = \frac{4w_n^2}{s^2 + 2\zeta w_n s + w_n^2} = \frac{80}{s^2 + 4.94s + 20}$$

$$M(s) = \frac{80}{s(s^2 + 4.94s + 20)}$$

(a) For $r(t) = \delta(t), R(s) = 1, \lim_{s\to 0} sC(s) = sM(s)R(s) = s\dfrac{80}{s(s^2+4.94s+20)} = 4$, error$= \pm 4$.

(b) For $r(t) = 1, R(s) = \dfrac{1}{s}, \lim_{s\to 0} sC(s) = sM(s)R(s) = s\dfrac{80}{s(s^2+4.94s+20)}\dfrac{1}{s} = \infty$, error$= \pm\infty$.

Q.4. Consider the control system described by the signal flow graph given below. Obtain the closed–loop transfer function using Mason's gain formula. [10]

Solution :- There are FIVE forward path gains which are:

$$\mathrm{P}_1 = \mathrm{G}_1\mathrm{G}_2\mathrm{G}_3\mathrm{G}_4\mathrm{G}_5\mathrm{G}_6\mathrm{G}_7 \quad \mathrm{P}_2 = \mathrm{G}_1\mathrm{G}_2\mathrm{G}_3\mathrm{G}_{11}\mathrm{G}_7$$
$$\mathrm{P}_3 = \mathrm{G}_1\mathrm{G}_2\mathrm{G}_8\mathrm{G}_9\mathrm{G}_5\mathrm{G}_6\mathrm{G}_7 \quad \mathrm{P}_4 = \mathrm{G}_1\mathrm{G}_2\mathrm{G}_8\mathrm{G}_9\mathrm{G}_5\mathrm{H}_2\mathrm{G}_{11}\mathrm{G}_7$$

$$\mathrm{P}_5 = \mathrm{G}_1\mathrm{G}_2\mathrm{G}_8\mathrm{G}_{10}\mathrm{G}_7$$

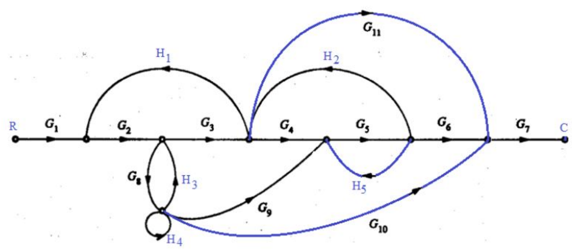

Figure 3: SFG

There are SIX individual loops, the gains of these loops are

$$\begin{aligned}
L_1 &= G_8H_3 \\
L_2 &= H_4 \\
L_3 &= G_2G_3H_1 \\
L_4 &= G_4G_5H_2 \\
L_5 &= G_2G_8G_9G_5H_2H_1 \\
L_6 &= G_5H_5
\end{aligned}$$

There are SIX pairs of non-touching loops, the gains of these loops are

$$\begin{aligned}
L_1\ L_4 &= G_8H_3G_4G_5H_2 \\
L_1\ L_6 &= G_8H_3G_5H_5 \\
L_2\ L_3 &= H_4G_2G_3H_1 \\
L_2\ L_4 &= H_4G_3G_5H_2 \\
L_2\ L_6 &= H_4G_5H_5 \\
L_3\ L_6 &= G_2G_3H_1G_5H_5
\end{aligned}$$

There is only <u>ONE</u> three non-touching loops, the gains of this loops are $L_2\ L_3\ L_6 = H_4G_2G_3H_1G_5H_5$

$$\Delta_1 = 1 - L_2$$
$$\Delta_2 = 1 - \{L_2 + L_6\} + \{L_2L_6\}$$
$$\Delta_3 = \Delta_4 = 1$$
$$\Delta_5 = 1 - L_6 - L_4$$
$$\Delta = 1 - \{L_1 + \cdots + L_6\} + \{L_1L_4 + L_1L_6 + \cdots + L_36\} - \{L_2L_3L_6\}$$
$$\frac{C(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5}{\Delta}$$

Q.5. For the signal flow graph (SFG) of a control system shown below, using Mason's formula, find the system transfer function and the system characteristic equation. [10]

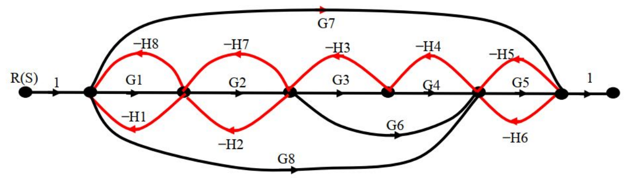

Figure 4: SFG

Solution- There are FOUR Forward Paths:

$$\mathrm{P}_1 = \mathrm{G}_7 \qquad \mathrm{P}_2 = \mathrm{G1G2G3G4}$$
$$\mathrm{P}_3 = \mathrm{G}_1\mathrm{G}_2\mathrm{G}_6\mathrm{G}_5 \qquad \mathrm{P}_4 = \mathrm{G}_8\mathrm{G}_5$$

* There are TWENTY ONE feedback loops:
$\mathrm{L}_1 = \mathrm{G}_1\mathrm{H}_1$; $\mathrm{L}_2 = \mathrm{G}_2\mathrm{H}_2$; $\mathrm{L}_3 = \mathrm{G}_3\mathrm{H}_3$; $\mathrm{L}_4 = \mathrm{G}_4\mathrm{H}_4$; $\mathrm{L}_5 = \mathrm{G}_5\mathrm{H}_5$; $\mathrm{L}_6 = \mathrm{G}_5\mathrm{H}_6$; $\mathrm{L}_7 = \mathrm{G}_2\mathrm{H}_7$; $\mathrm{L}_8 = \mathrm{G}_1\mathrm{H}_8$; $\mathrm{L}_9 = \mathrm{G}_6\mathrm{H}_4\mathrm{H}_3$; $\mathrm{L}_{10} = \mathrm{G}_8\mathrm{H}_4\mathrm{H}_3\mathrm{H}_7\mathrm{H}_8$; $\mathrm{L}_{11} = \mathrm{G}_8\mathrm{H}_4\mathrm{H}_3\mathrm{H}_7\mathrm{H}_1$; $\mathrm{L}_{12} = \mathrm{G}_8\mathrm{H}_4\mathrm{H}_3\mathrm{H}_2\mathrm{H}_1$; $\mathrm{L}_{13} = \mathrm{G}_8\mathrm{H}_4\mathrm{H}_3\mathrm{H}_2\mathrm{H}_8$; $\mathrm{L}_{14} = \mathrm{G}_7\mathrm{H}_5\mathrm{H}_4\mathrm{H}_3\mathrm{H}_7\mathrm{H}_8$; $\mathrm{L}_{15} = \mathrm{G}_7\mathrm{H}_5\mathrm{H}_4\mathrm{H}_3\mathrm{H}_7\mathrm{H}_1$; $\mathrm{L}_{16} = \mathrm{G}_7\mathrm{H}_5\mathrm{H}_4\mathrm{H}_3\mathrm{H}_2\mathrm{H}_1$; $\mathrm{L}_{17} = \mathrm{G}_7\mathrm{H}_5\mathrm{H}_4\mathrm{H}_3\mathrm{H}_2\mathrm{H}_8$; $\mathrm{L}_{18} = \mathrm{G}_7\mathrm{H}_6\mathrm{H}_4\mathrm{H}_3\mathrm{H}_7\mathrm{H}_8$; $\mathrm{L}_{19} = \mathrm{G}_7\mathrm{H}_6\mathrm{H}_4\mathrm{H}_3\mathrm{H}_7\mathrm{H}_1$; $\mathrm{L}_{20} = \mathrm{G}_7\mathrm{H}_6\mathrm{H}_4\mathrm{H}_3\mathrm{H}_2\mathrm{H}_1$; $\mathrm{L}_{21} = \mathrm{G}_7\mathrm{H}_6\mathrm{H}_4\mathrm{H}_3\mathrm{H}_2\mathrm{H}_8$;
* There are EIGHTEEN combination of two-non-touching feedback loops:
$\mathrm{L}_1\ \mathrm{L}_3 = \mathrm{G}_1\mathrm{H}_1\mathrm{G}_3\mathrm{H}_3$ $\mathrm{L}_1\ \mathrm{L}_4 = \mathrm{G}_1\mathrm{H}_1\mathrm{G}_4\mathrm{H}_4$ $\mathrm{L}_1\ \mathrm{L}_5 = \mathrm{G}_1\mathrm{H}_1\mathrm{G}_5\mathrm{H}_5$ $\mathrm{L}_1\ \mathrm{L}_6 = \mathrm{G}_1\mathrm{H}_1\mathrm{G}_5\mathrm{H}_6$ $\mathrm{L}_1\ \mathrm{L}_9 = \mathrm{G}_1\mathrm{H}_1\mathrm{G}_6\mathrm{H}_4\mathrm{H}_3$ $\mathrm{L}_2\ \mathrm{L}_4 = \mathrm{G}_2\mathrm{H}_2\mathrm{G}_4\mathrm{H}_4$

$$\mathrm{L}_2\ \mathrm{L}_5 = \mathrm{G}_2\mathrm{H}_2\mathrm{G}_5\mathrm{H}_5 \qquad \mathrm{L}_2\ \mathrm{L}_6 = \mathrm{G}_2\mathrm{H}_2\mathrm{G}_5\mathrm{H}_6$$
$$\mathrm{L}_3\ \mathrm{L}_5 = \mathrm{G}_3\mathrm{H}_3\mathrm{G}_5\mathrm{H}_5\ \mathrm{L}_3\ \mathrm{L}_6 = \mathrm{G}_3\mathrm{H}_3\mathrm{G}_5\mathrm{H}_6$$
$$\mathrm{L}_3\ \mathrm{L}_8 = \mathrm{G}_3\mathrm{H}_3\mathrm{G}_1\mathrm{H}_8\ \mathrm{L}_4\ \mathrm{L}_7 = \mathrm{G}_4\mathrm{H}_4\mathrm{G}_2\mathrm{H}_7$$
$$\mathrm{L}_4\ \mathrm{L}_8 = \mathrm{G}_4\mathrm{H}_4\mathrm{G}_1\mathrm{H}_8\ \mathrm{L}_5\ \mathrm{L}_7 = \mathrm{G}_5\mathrm{H}_5\mathrm{G}_2\mathrm{H}_7$$
$$\mathrm{L}_5\ \mathrm{L}_8 = \mathrm{G}_5\mathrm{H}_5\mathrm{G}_1\mathrm{H}_8\ \mathrm{L}_6\ \mathrm{L}_7 = \mathrm{G}_5\mathrm{H}_6\mathrm{G}_2\mathrm{H}_7$$
$$\mathrm{L}_6\ \mathrm{L}_8 = \mathrm{G}_5\mathrm{H}_6\mathrm{G}_1\mathrm{H}_8\ \mathrm{L}_8\ \mathrm{L}_9 = \mathrm{G}_1\mathrm{H}_8\mathrm{G}_6\mathrm{H}_4\mathrm{H}_3$$

* There are FOUR combination of three-non-touching feedback loops
$\mathrm{L}_1\ \mathrm{L}_3\ \mathrm{L}_5 = \mathrm{G}_1\mathrm{H}_1\mathrm{G}_3\mathrm{H}_3\mathrm{G}_5\mathrm{H}_5$ $\mathrm{L}_1\ \mathrm{L}_3\ \mathrm{L}_6 = \mathrm{G}_1\mathrm{H}_1\mathrm{G}_3\mathrm{H}_3\mathrm{G}_5\mathrm{H}_6$ $\mathrm{L}_3\ \mathrm{L}_5\ \mathrm{L}_8 = \mathrm{G}_3\mathrm{H}_3\mathrm{G}_5\mathrm{H}_5\mathrm{G}_1\mathrm{H}_8$ $\mathrm{L}_3\ \mathrm{L}_6\ \mathrm{L}_8 = \mathrm{G}_3\mathrm{H}_3\mathrm{G}_5\mathrm{H}_6\mathrm{G}_1\mathrm{H}_8$
$\Delta = 1 - [\mathrm{L}_1 + \mathrm{L}_2 + \ldots + \mathrm{L}_{15}] - [\mathrm{L}_1\ \mathrm{L}_3 + \mathrm{L}_1\ \mathrm{L}_4 + \ldots + \mathrm{L}_8\ \mathrm{L}_9] + [\mathrm{L}_1\ \mathrm{L}_3\ \mathrm{L}_5 + \ldots + \mathrm{L}_3\ \mathrm{L}_6\ \mathrm{L}_8]$ $\Delta 1 = 1 - [\mathrm{L}_2 + \mathrm{L}_3 + \mathrm{L}_4 + \mathrm{L}_7 + \mathrm{L}_9] + [\mathrm{L}_2\ \mathrm{L}_4 + \mathrm{L}_4\ \mathrm{L}_7]$ $\Delta 2 = \Delta 3 = 1$ $\Delta 4 = 1 - [\mathrm{L}_2 + \mathrm{L}_3 + \mathrm{L}_7]$ $\Delta = 1 - \{\mathrm{L}_1 + \mathrm{L}_2 + \mathrm{L}_3 + \ldots + \mathrm{L}_{21}\} + \{\mathrm{L}_1\ \mathrm{L}_3 + \mathrm{L}_1\ \mathrm{L}_4 + \mathrm{L}_1\ \mathrm{L}_5 + \ldots + \mathrm{L}_4\ \mathrm{L}_6\}$ Using Mason's Formula, the system Transfer Function is:

$$\frac{Y(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

The characteristic equation is:

$$\Delta = 1 - [\mathrm{L}_1 + \mathrm{L}_2 + \ldots + \mathrm{L}_{15}] - [\mathrm{L}_1\ \mathrm{L}_3 + \mathrm{L}_1\ \mathrm{L}_4 + \ldots + \mathrm{L}_8\ \mathrm{L}_9] + [\mathrm{L}_1\ \mathrm{L}_3\ \mathrm{L}_5 + \ldots + \mathrm{L}_3\ \mathrm{L}_6\ \mathrm{L}_8] = 0$$

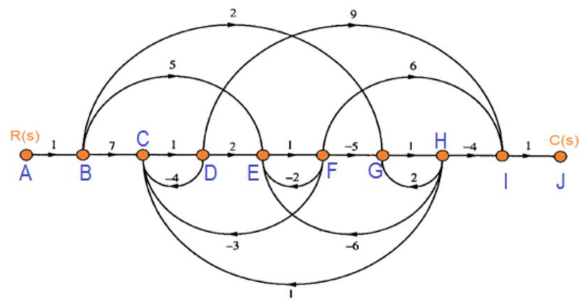

Figure 5: SFG

Q.6. For the signal flow graph (SFG) of a control system shown below in figure 5, find the system transfer function. [10]

Solution- Forward paths;

$$
\begin{aligned}
P_1 &= ABCDEFGHIJ = 7 \times 2 \times -5 \times -4 = 280 \\
P_2 &= ABCDEFIJ = 7 \times 2 \times 6 = 84 \\
P_3 &= ABEFGHIJ = 5 \times -5 \times -4 = 100 \\
P_4 &= ABEFIJ = 5 \times 6 = 30 \\
P_5 &= ABGHIJ = 2 \times -4 = -8 \\
P_6 &= ABCDIJ = 7 \times 9 = 63 \\
P_7 &= ABGHCDIJ = 2 \times 9 = 18 \\
P_8 &= ABGHCDEFIJ = 2 \times 2 \times 6 = 24 \\
P_9 &= ABGHEFIJ = 2 \times -6 \times 6 = -72 \\
P_{10} &= ABEFCDIJ = 5 \times -3 \times 9 = -135 \\
P_{11} &= ABEFGHCDIJ = 5 \times -5 \times 9 = -225 \\
P_{12} &= ABGHEFCDIJ = 2 \times -6 \times -3 \times 9 = 324
\end{aligned}
$$

Loops:

$L_1 = CDC = -4 \quad L_2 = EFE = -2$

$L_3 = GHG = 2 \quad L_4 = CDEFC = 2 \times -3 = -6$

$L_5 = EFGHE = -5 \times -6 = 30 \quad L_6 = CDEFGHC = 2 \times -5 = -10$

Two Non-touching Loops: $L_1\ L_2 = -4 \times -2 = 8$ $\quad L_1\ L_3 = -4 \times 2 = -8$ $L_1\ L_5 = -4 \times 30 = -120$ $L_2\ L_3 = -2 \times 2 = -4$ $L_3\ L_4 = 2 \times -6 = -12$ Three Non-touching Loops: $L_1\ L_2\ L_3 = -4 \times -2 \times 2 = 16$

$$\Delta = 1 - \{-4 - 2 + 2 - 6 + 30 - 10\} + \{8 - 8 - 120 - 4 - 12\} - \{16\} = -161$$

$$
\begin{aligned}
\Delta 1 &= 1 - \{0\} = 1 \\
\Delta 2 &= 1 - \{L_3\} = 1 - 2 = -1
\end{aligned}
$$

$\Delta 3 = 1 - \{\mathrm{L}_1\} = 1 + 4 = 5$

$$\Delta 4 = 1 - \{\mathrm{L}_1 + \mathrm{L}_3\} + \{\mathrm{L}_1\ \mathrm{L}_3\} = 1 - \{-4 + 2\} - 8 = -5$$
$$\Delta 5 = 1 - \{\mathrm{L}_1 + \mathrm{L}_2 + \mathrm{L}_4\} + \{\mathrm{L}_1\ \mathrm{L}_2\} = 1 - \{-4 - 2 - 6\} + 8 = 21$$
$$\Delta 6 = 1 - \{\mathrm{L}_2 + \mathrm{L}_3 + \mathrm{L}_5\} + \{\mathrm{L}_2\ \mathrm{L}_3\} = 1 - \{-2 + 2 + 30\} - 4 = -33$$
$$\Delta 7 = 1 - \{\mathrm{L}_2\} = 1 + 2 = 3$$
$$\Delta 8 = 1 - \{0\} = 1$$
$$\Delta 9 = 1 - \{\mathrm{L}_1\} = 1 - \{-4\} = 5$$
$$\Delta 10 = 1 - \{\mathrm{L}_3\} = 1 - 2 = -1$$
$$\Delta 11 = 1 - \{0\} = 1 \quad \Delta 12 = 1 - \{0\} = 1$$
$$\frac{C(s)}{R(s)} = \frac{\{280 \times 1 + 84 \times (-1) + 100 \times 5 + 30 \times (-5) + (-8) \times 21 + 63 \times (-33) + 18 \times 3 + 24 \times 1 - 72 \times 5 - 135 \times (-1) - 225 \times 1 + 324 \times 1\}}{-161}$$
$$\frac{C(s)}{R(s)} = \frac{-1749}{-161} = 10.8634$$

Q.7. Consider the control system shown below in figure 6, draw the corresponding signal flow graph, and obtain the closed–loop transfer function using Mason's gain formula. [10]

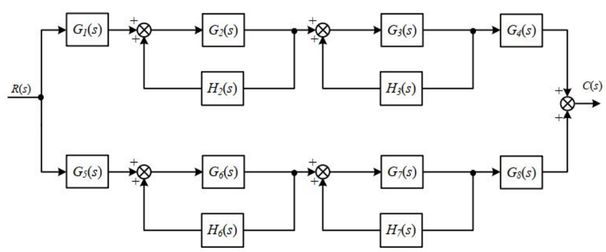

Figure 6: Block Diagram

Solution-

Forward path (2 paths)

$$P_1 = G_1 G_2 G_3 G_4$$
$$P_2 = G_5 G_6 G_7 G_8$$

Loops (4 loops)	three nontouching loops
$L_1 = G_2H_2$	$L_1L_2L_3 = G_2G_3G_6H_2H_3H_6$
$L_2 = G_3H_3$	$L_1L_2L_4 = G_2G_3G_7H_2H_3H_7$
$L_3 = G_6H_6$	$L_1L_3L_4 = G_2G_6G_7H_2H_6H_7$
$L_4 = G_7H_7$	$L_2L_3L_4 = G_3G_6G_7H_3H_6H_7$
two nontonching Loops	four non-tonching Loops
$L_1L_2 = G_2G_3H_2H_3$	$L_1L_2L_3L_4 = G_2G_3G_6G_7H_2H_3H_6H_7$
$L_1L_3 = G_2G_6H_2H_6$	
$L_1L_4 = G_2G_7H_2H_7$	
$L_2L_3 = G_3G_6H_3H_6$	
$L_2L_4 = G_3G_7H_3H_7$	
$L_3L_4 = G_6G_7H_6H_7$	

$\Delta_1 = 1 - [G_6H_6 + G_7H_7] + [G_6G_7H_6H_7]$
$\Delta_2 = 1 - [G_2H_2 + G_3H_3] + [G_2G_3H_2H_3]$
$\Delta = 1-[G_2H_2 + G_3H_3 + G_6H_6 + G_7H_7]+[G_2G_3H_2H_3 + G_2G_6H_2H_6+ \ G_2G_7H_2H_7 + G_3G_6H_3H_6 + G_3G_7H_3H$
$[G_2G_3G_6H_2H_3H_6 \ +G_2G_3G_7H_2H_3H_7 + G_2G_6G_7H_2H_6H_7 + G_3G_6G_7H_3H_6H_7 + P_1G_3G_6G_7H_2H_3H_6H_7]$

$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4\{1-(G_6H_6+G_7H_7)+G_6G_7H_6H_7\}+G_5G_6G_7G_8\{1-(G_2H_2+G_3H_3)}{\Delta}$

Q.8. Use Laplace transform to solve the initial value problem [10]

$$y' + 2y = 26sin3t, y(0) = 3$$

Q.9. Given the standard second-order transfer function

$$G(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}.$$

A) Describe how the addition of a pole on the real axis (left half plane) affect the system step response [5]

Ans.A.

Consider a three-pole systme with compled poles and a third pole on the real axis. Assuming the complex poles are at $-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ and the real pole is at $-\alpha_r$, the step response of the system can be determined from a partial-fraction expansion. Thus, the output transform is
$c(s) = \frac{A}{s} + \frac{B(s+\zeta\omega_n)+C\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2} + \frac{D}{s+\alpha_r}$
or in the time domain,
$c(t) = Au(t) + e^{-\zeta\omega_n t}(Bcos\omega_d t + Csin\omega_d t) + De^{-\alpha_r t}$

Case 1 $\alpha_r >> \zeta\omega_n$(real pole far from the pair), the pure exponential will die out much more rapidly than the second-order underdamped step response. If the pure exponential term decays to an insignificant value at the time of the first overshoot, such parameters as percent overshoot, settling time, and peak time will be generated by the second-order underdamped step response component. Thus, the total response will approach that of a pure second-order system

Case 2 if α_r is not much greater than $\zeta\omega_n$, the real pole's transient response will not decay to insignificance at the peak time or settling time generated by the second-order pair. In this case, the exponential decay is significant, and the system cannot be represented as a second-order system. The system step response become slower in this case.

B) Describe how the addition of a zero on the real axis (left half plane) affect the system step response [5]

Ans .B. Let C(s) be the step resonse of a system, T(s), with unity in the numerator. If we add a zero to the transfer function, yielding (s+1)T(s), the Laplace transform of the response will be $(s+1)C(s) = sC(s) + aC(s)$

Thus the response of a system with zero consists of two parts: the derivative of the original response and a scaled version of the original response. If a, the negative of the zero is very large, the Laplace transform of the response is approximately aC(s), or a scaled version of the original response. If a is not very large, the response has an additional component consisting of the derivative of the original response. As a becomes smaller, the derivative term contributes more to the response and has a greater effect. For step responses, the derivative is typically positive at the start of the response. Thus, for small values of a, we can expect more overshoot and also a faster response in second-order system because the derivative term will be additive around the first overshoot.

Q.10. A) Consider an LTI system with input and output related through the equation

$$y(t) = \int_{-\infty}^{t} e^{-(t-\lambda)} x(\lambda - 2)\, d\lambda$$

What is the impulse response h(t) of the system? [5]

Ans.A. $y(t) = \int_{-\infty}^{t} e^{-(t-\lambda)} x(\lambda - 2) d\lambda = \int_{-\infty}^{t-2} e^{-(t-2-\lambda')} x(\lambda') d\lambda'$
Therefore the impulse response, $h(t) = e^{-(t-2)} u(t-2)$

B) Determine the response of the system when the input x(t) is as shown in the figure below. [5]

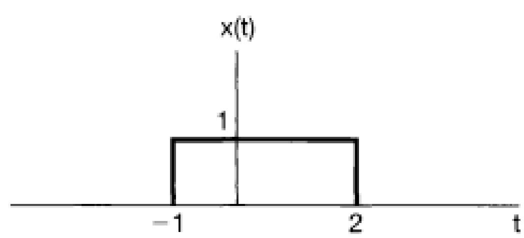

Figure 7: Input x(t)

Ans.B. We have
$y(t) = \int_{-\infty}^{\infty} h(t) x(t-\tau) d\tau = \int_{2}^{\infty} e^{-(\tau-2)} [u(t-\tau+1) - u(t-\tau-2)] d\tau$

From the above we get,

$$y(t) = \begin{cases} 0, t < 1 \\ \int_{2}^{t+1} e^{-(\tau-2)} d\tau = 1 - e^{-(t-1)}, 1 < t < 4 \\ \int_{t-2}^{t+1} e^{-(\tau-2)} d\tau = e^{-(t-4)}[1 - e^{-3}], t > 4 \end{cases}$$