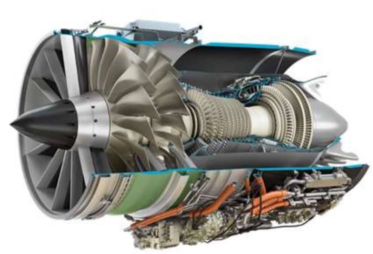
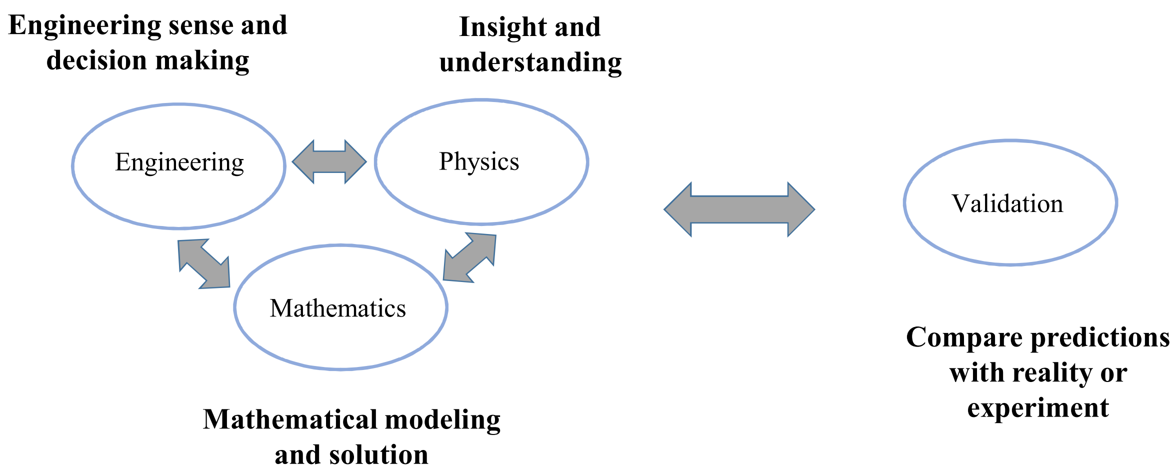
Analysis

Analysis of real life systems

- Real-life structures are complex multi-component systems with distributed mass and stiffness properties
- Key questions in analysis:
  - What is the goal of the analysis?
  - What essential features do we need to include?



What does analysis entail ?



Key steps in analysis

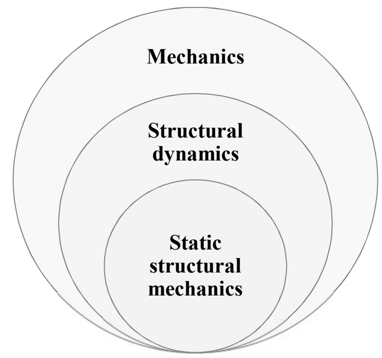
1) Identify the physical system of interest as an object of analysis or design. Physical system may be a complete system, sub-system, or component. (**Engineering judgement and decision making**)
2) Conceive and sketch an idealized representation of the real life system. This is version of the real life system we use for all subsequent purposes. (**Abstraction**)
3) Apply mathematical reasoning to convert the idealized representation to a mathematical model. (**Mathematical modeling**)
   a) Formulation of governing equations,
   b) Identification of boundary and initial conditions
   c) Application of selected solution procedures
4) Compare with physical observations from a laboratory model or actual system. (**Validation**)
5) Interpret comparisons physically in context of the original object of the analysis of design (**Insight & understanding**)
6) Return to step 1 or step 2 based on nature of discrepancy or improvement sought. (**Iteration / Improvement**)

Engineering quantities that we are likely to encounter in the analysis

Motion induced
- Position, velocity, acceleration
- Inertia force (due to mass)
- Dissipation (due to damping mechanisms)

Formulation
- Applied loads
- Equilibrium equations
- Compatibility conditions
- Strain-displacement relations
- Constitutive relations
- Boundary conditions
- Initial conditions

Output
- Internal stress
- Strain
- Deformation (displacement, rotation, shear, twist)



Independent variables: Space & Time

Design of vibratory systems

- Identifying and / or anticipating the predominant features of motion and motion-induced quantities over the entire operational envelope is the most important challenge.
- Simple models that capture the predominant features of motion and motion-induced quantities with reasonable accuracy are extremely useful.
- Relevant concepts:

  (a) Concepts of force, momentum, and energy

  (b) Kinematics and degrees of freedom

  (c) Discretization

  (d) Abstraction

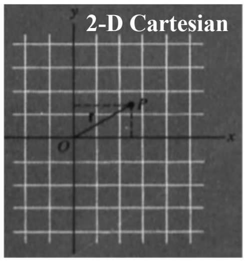
  (e) Formulation and solution

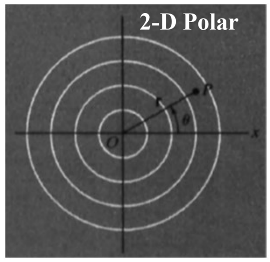
Kinematics, coordinates and degrees of Freedom

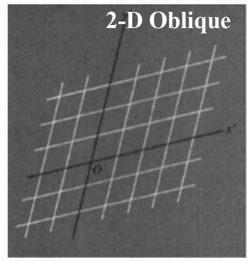
What are kinematics?

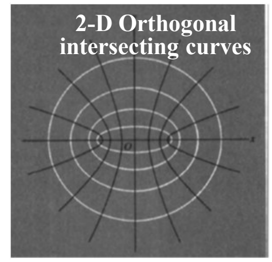
- Kinematics is the branch of mechanics that deals with the geometry of motion.
- Kinematics includes a complete description of the motion, i.e., the positions, velocities, and accelerations, of all points within a system at all times.
- The description of motion is intimately dependent on the reference coordinate system in which the motion of observed.
- Coordinate system can be inertial (fixed or moving with constant velocity) or non-inertial (accelerating).
- It is advantageous to choose a reference coordinate system in which the description of kinematics is the simplest.
- However, one may describe the kinematics uniquely and effectively in several types of coordinate systems.

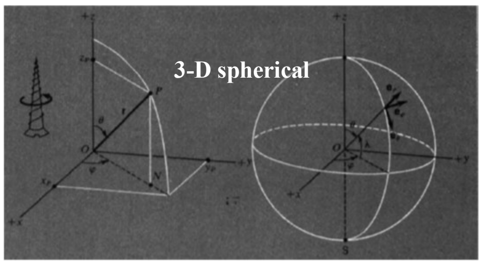
How to choose a reference coordinate system?











Oblique: Non-orthogonal intersecting linear axes.

Orthogonal intersecting curves: Confocal ellipses and hyperbolas

3-D spherical:
- $R - \theta - \phi$ for solid
- $\theta - \lambda$ (latitude-longitude) for shell

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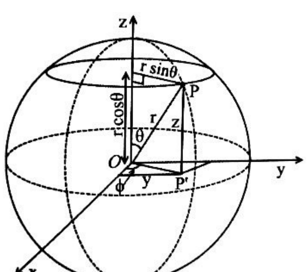
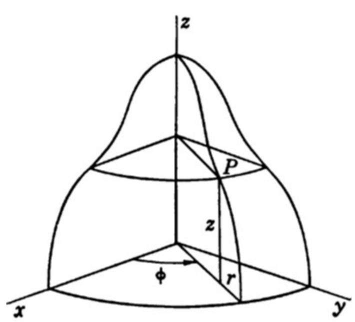
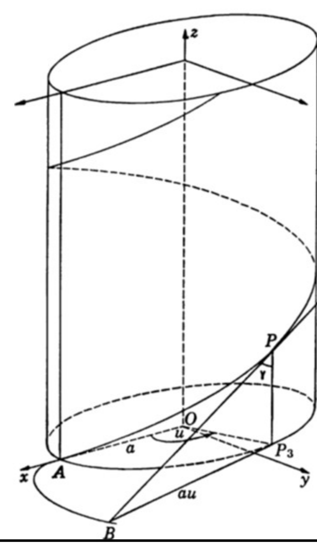
What are coordinates?

Coordinates provide a unique and complete measure of the position of a particle along each of the selected coordinate axes.

- Coordinates are mutually-independent functions of time.
- Coordinates and their temporal derivates are intimately tied to the specific coordinate frame.
- More than one physical coordinate system can be used to describe any given motion.







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What are degrees of freedom ?

- Consider a mechanical system of N free particles. Here, ‘free’ implies that they are not restricted in any manner.
- In a rectangular Cartesian coordinate system, we require a minimum of 3N coordinates (*x, y, z* values for each particle) to uniquely and completely describe the motion of each particle at any given time.
- Note that require a minimum of 3N coordinates are required even if any other coordinate system was used to describe the motion.
- Thus, n = 3N is a characteristic constant of the system, irrespective of the coordinate system used.
- This characteristic constant identifies the <u>number of degrees of freedom</u> or simply the degrees of freedom (DOF) of the system.

What happens when some directions of motion are constrained ?

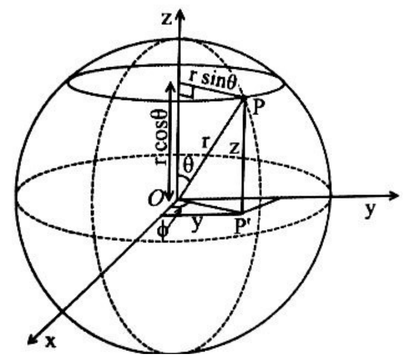
- Now what happens if we impose m independent kinematic constraints on the particles?
- In this case, the total number of ‘free’ or ‘unrestricted’ coordinates is reduced to (3N - m).
- Thus, in this case n = (3N – m) is the characteristic constant, or the DOFs, of the system.
- Note that the DOFs of a given system are fixed (or constant) whether we are able to identify a set of n generalized coordinates or not.

Example: Unconstrained and constrained motion

Consider a particle moving <u>inside</u> a sphere

We may use (x, y, z) or (r, θ, ϕ) coordinates to describe the motion. In this case, the relations are

$$x = r \sin\theta \cos\phi \quad y = r \sin\theta \sin\phi \quad \text{and} \quad z = r\cos\theta$$



Consider a particle moving <u>on</u> a sphere

Now suppose we confine the particle to move only along the surface of the sphere, or the spherical shell. Then, r = R = constant.

We still require (x,y,z) to fully describe the motion. We have to introduce a constraint that $x^2 + y^2 + z^2 = R^2$ along with the description.

Handling the constraint is very straightforward if (r, θ, ϕ) coordinates are used to describe the motion.

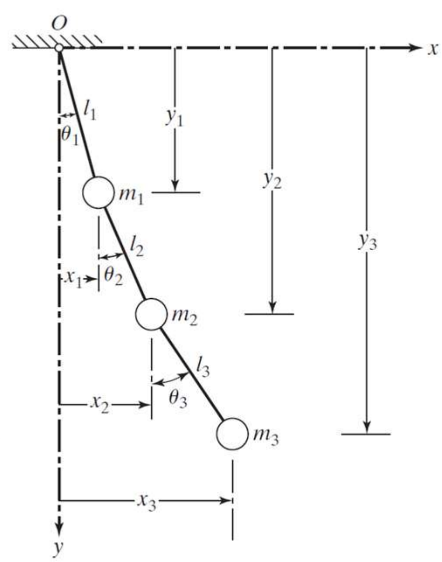
Another example: Constrained and generalized coordinates

Consider a triple pendulum

The positions of the pendulum bobs can be uniquely described using 6 constrained coordinates: (x_i, y_i), where i = 1, 2, 3. The constraints are

$$x_1^2 + y_2^2 = l_1^2$$
$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = l_2^2$$
$$(x_3 - x_2)^2 + (y_3 - y_2)^2 = l_3^2$$

Alternately, one may choose the angular positions of the bobs, for example with respect to the vertical - θ_i, where i = 1, 2, 3 - as coordinate. In this case, only 3 coordinates are adequate to fully describe the positions of the bob.



Discretization

All matter as a collection of particles

- One can attempt to describe the motion of any object theoretically by describing the motion of each particle that is present in the system.
- Problem - we end up with a large number of particles when dealing with realistic structures !

Example:

The molar mass of iron is 55.845 grams per mole.

Each mole contains an Avagadro number of atoms, i.e. 6.023×10^{23} atoms.

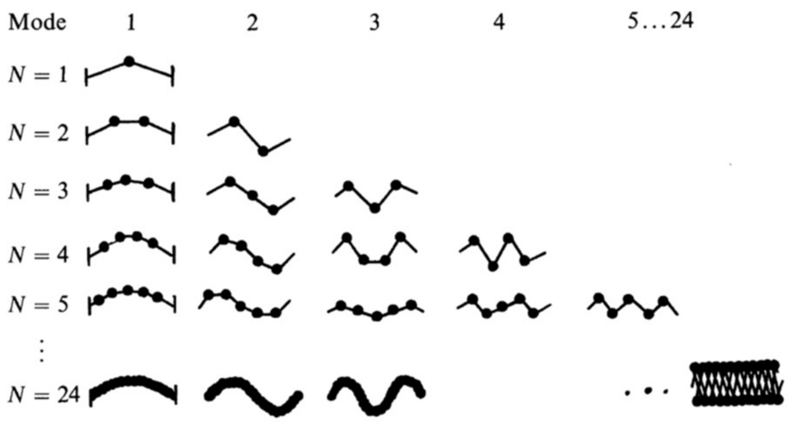
We end up with 6.023×10^{23} descriptions !

Most practical structures composed of iron weigh in the order of kilograms or more.

- To avoid impractical scenarios, we rely on the assumption of continuum and produce '*continuous*' structures that have distributed thermo-mechanical properties.

How do we visualize a continuous structure ?

Example: Transverse vibration of beads on a string.



- Each arrangement shown in the picture is independent and cannot be obtained as a weighted combination of the other arrangements.

As the number of beads increases

- Geometry approaches a continuous shape
- Number of basic arrangements approaches infinity

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Can we analyze continuous structures ?

The answer is 'No' except for very simple geometries.

Discretization is a necessity !

What is discretization?
It is a systematic procedure to convert an theoretically infinite DOF system to one that has finite DOFs.

Why does it work ?
- Only a few degrees of freedom are prominent in any given situation.
- Adequate accuracy of representation can be achieved using only a limited number of DOFs.

Useful discretization procedures
- Distributed coordinates
- Lumping
- Weighted residuals

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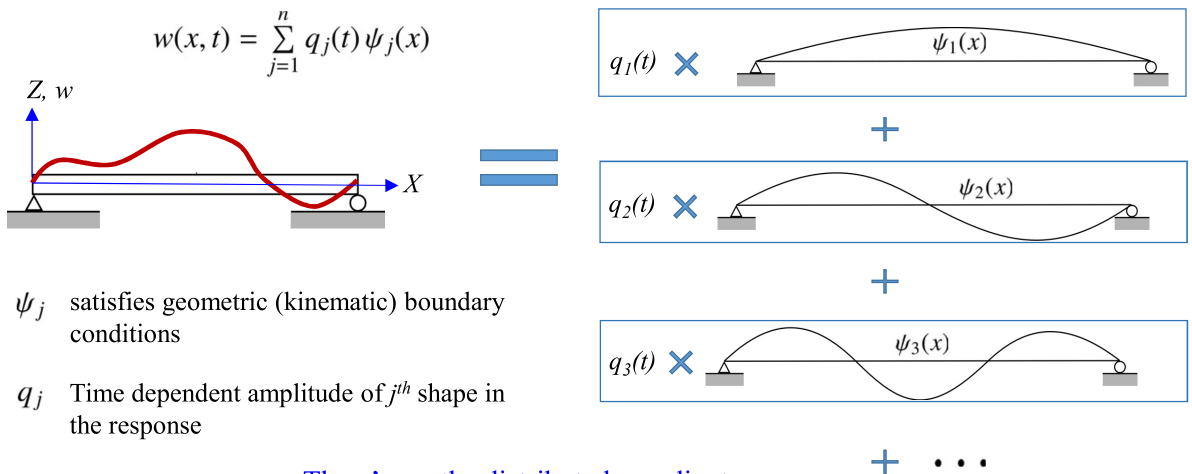
Distributed coordinates

Response variables are expressed as a weighted sum of predetermined shapes

$$w(x,t) = \sum_{j=1}^{n} q_j(t)\psi_j(x)$$



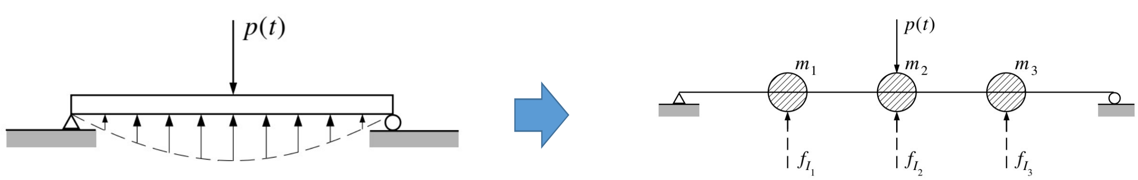
ψ_j satisfies geometric (kinematic) boundary conditions

q_j Time dependent amplitude of j^{th} shape in the response

The q's are the distributed coordinates

Lumping procedures

Mass / stiffness of a continuous system is assumed to be concentrated or *lumped* at specific locations.



Distributed inertia loads

Discrete inertia loads

- **Mass is lumped** by selecting points on the structure to represent inertia forces to desired level of accuracy.
- **Stiffness is lumped** using either stiffness or flexibility approach.

Finite element procedures are based on weighted residual methods

Model a continuum (fluid or solid) as an assembly of interconnected finite number of discrete elements

Beams (1D)

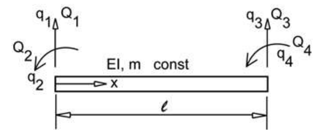
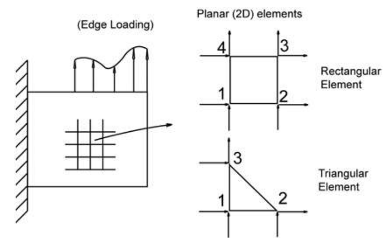


Plate or shell (2D)



- Divide structure into a number of interconnected segments (called *elements*) whose ends are connected at '*nodes*'.
- Nodal displacements are the generalized coordinates.
- Deflection of the entire element is obtained by interpolating nodal displacements using user defined *shape functions*.
- Procedure involves formulation of the element (i.e., the repeating unit) and assembly based on connectivity.

Solid (3D)