Introduction to Aerodynamics

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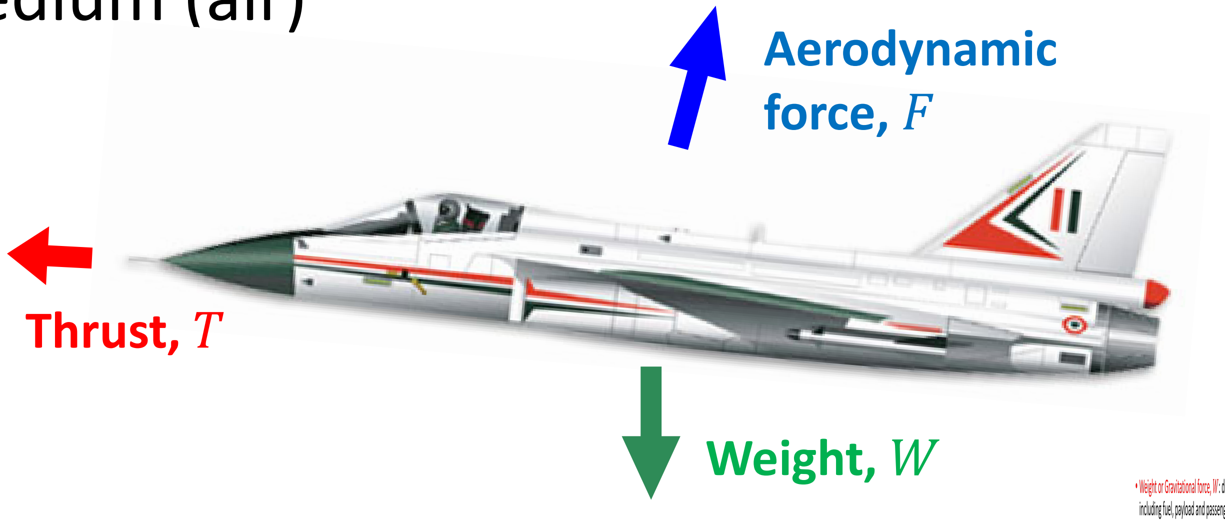
Course objectives

- Appreciate the role of aerodynamics in flight of aerospace vehicles
- Understand which parts of the problem are easy to solve and which are more difficult, and why
- Learn analytical techniques for solving the easy parts of the problem
- Develop qualitative understanding of the approaches for solving the more difficult parts of the problem
- Acquire the background to take up research and development challenges in aerodynamics

Forces on an aerospace vehicle

- Weight or Gravitational force, W: due to vehicle, all its contents, including fuel, payload and passengers
- Thrust or Propulsive force, T: force on vehicle generated by its propulsion system (think propellers, jet engines, rocket engines)
- Aerodynamic force, F: force on ***surface*** of vehicle generated by interaction with surrounding medium (air)
- Thrust and aerodynamic forces are often not easy to decouple
  - Propulsive force is affected by aerodynamics, and vice versa

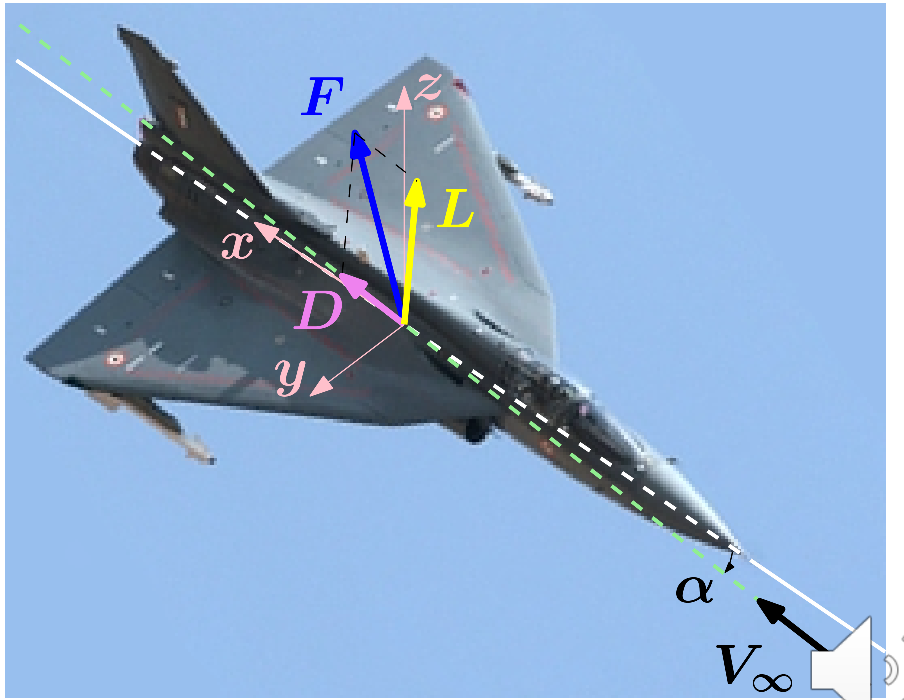


Resolving aerodynamic force into lift and drag

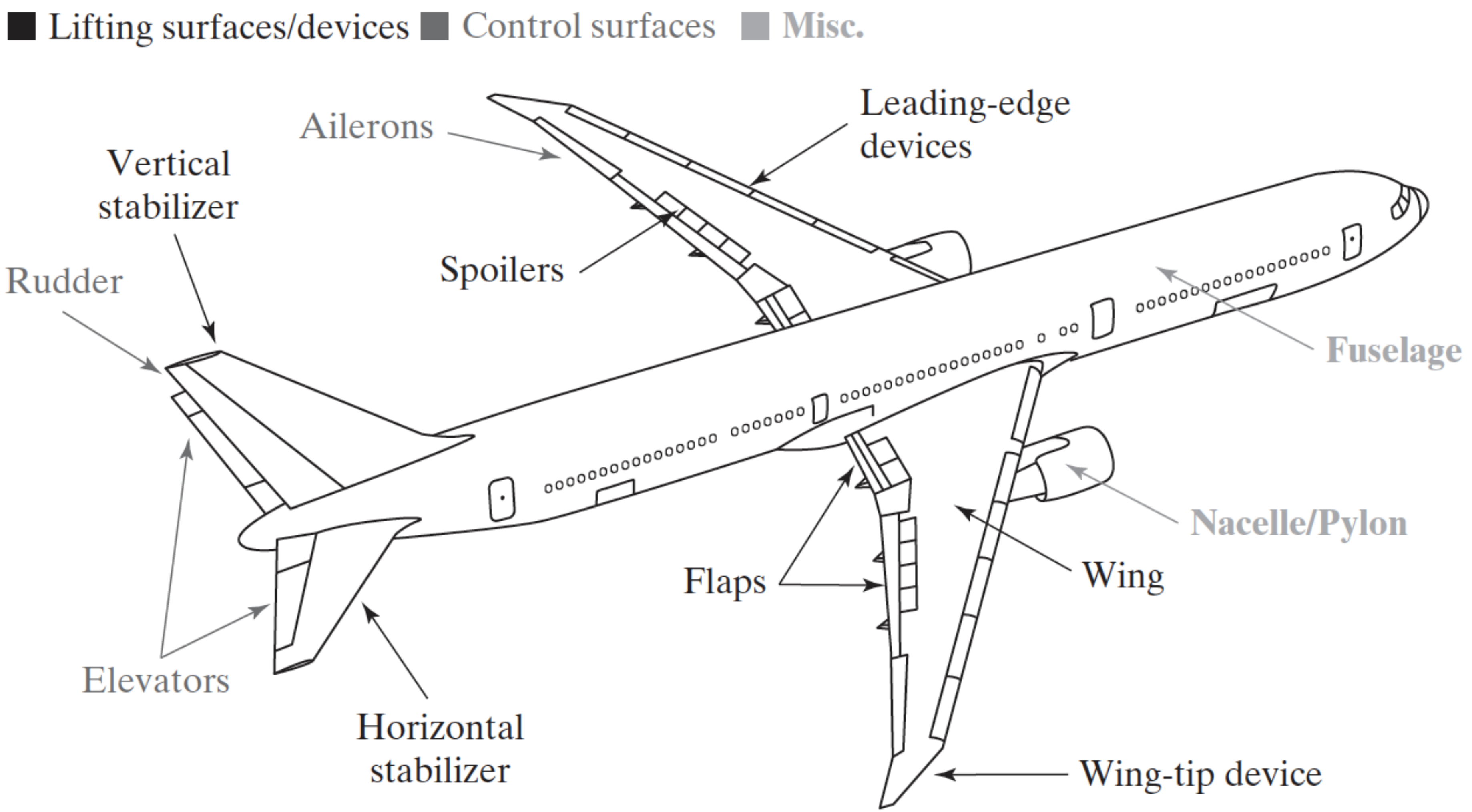
Relative wind vector, $\vec{V}_\infty$: resultant of vehicle motion and breeze

- This also defines the **freestream** direction

- Longitudinal axis of vehicle: x-axis; positive toward rear
- $y = 0$ is plane of symmetry of vehicle; y is +ve toward right wing tip
- 3rd axis in right-handed system: z
- Generally, $\vec{V}_\infty$ is not directed along x
- Assuming no motion along y (no side slip), aerodynamic force, F, is resolved into
  - Component parallel to relative wind, **drag**, D
  - Component normal to relative wind, **lift**, L

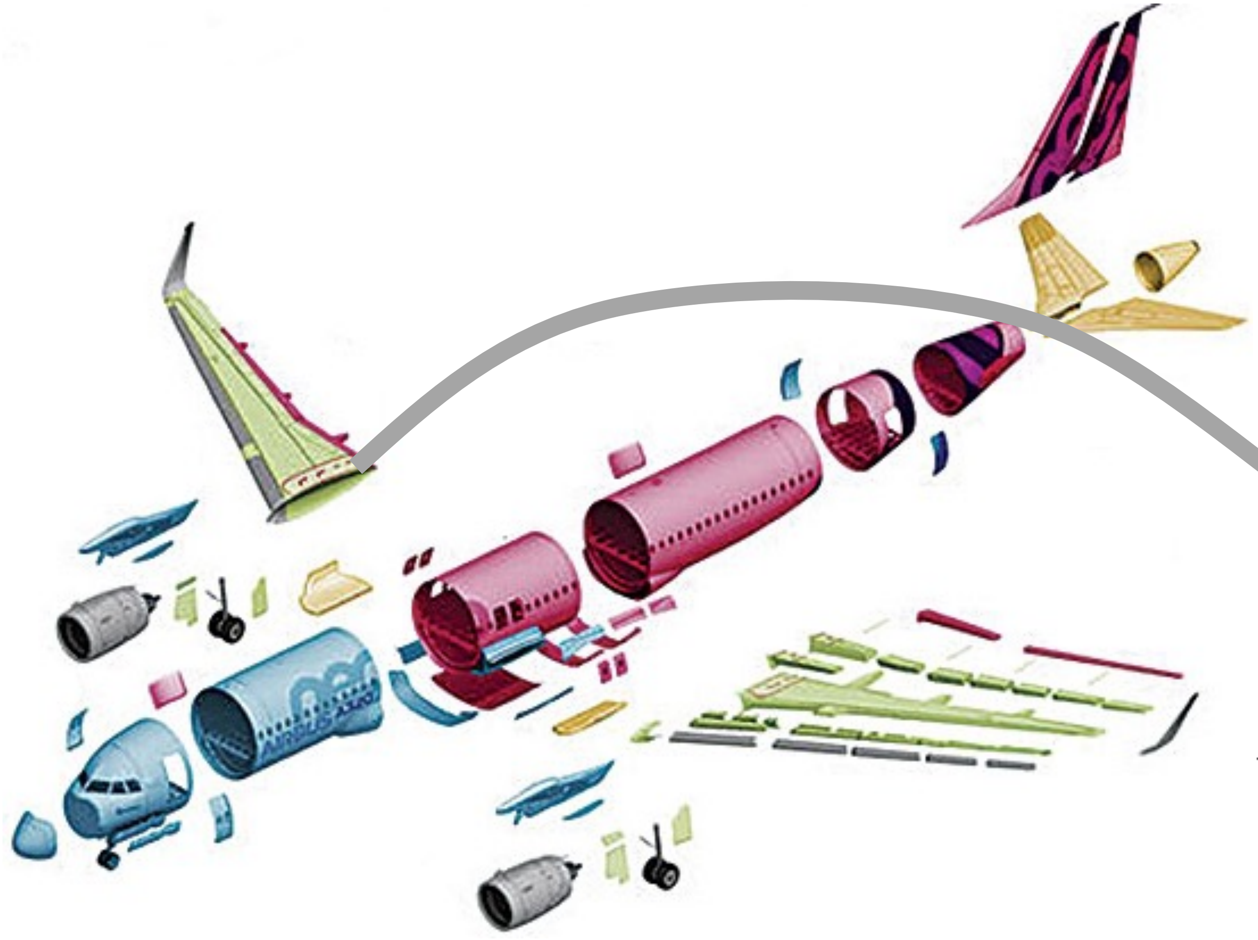


Components of a commercial aircraft



Bertin and Cummings, Aerodynamics for Engineers, 2013

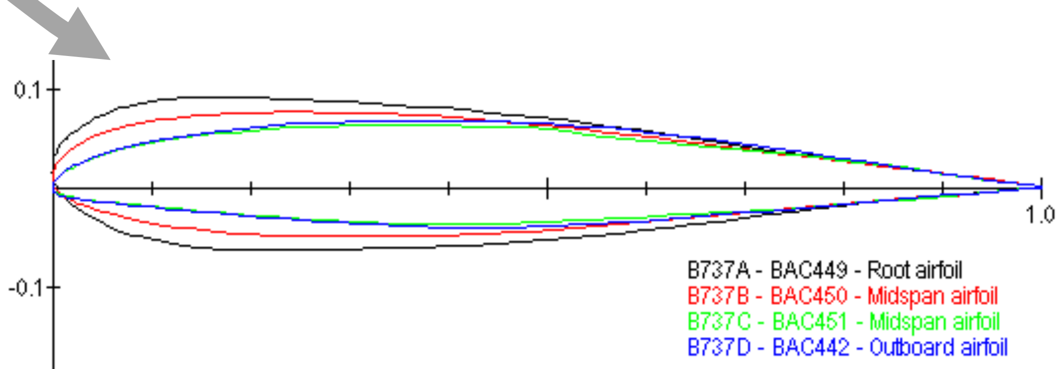
Wing section shape is called airfoil



Cut apart view of Airbus A320

Airfoils have streamlined shape

- Resemble streamlines of flow
- Little or no flow separation

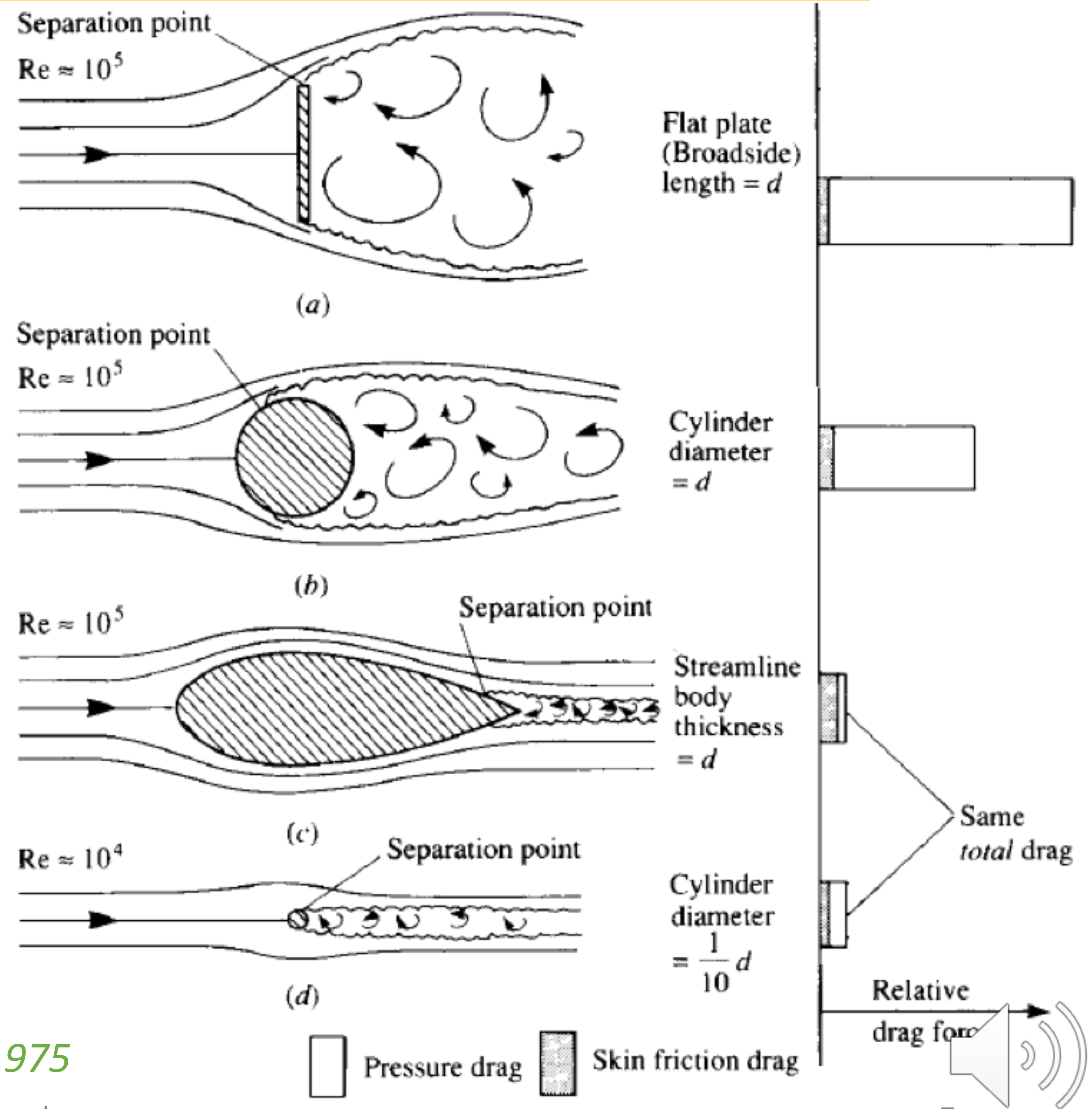


Boeing 737 Airfoil Sections

Drag forces on bluff and streamlined bodies

3 bodies of same size (but different shape), 1 much smaller; all operating at same speed

- Streamlining reduces pressure drag – even in cylinder vs. normal plate, but much more so in teardrop shape
- Skin friction drag increases slightly due to increase in exposed surface
- Much smaller cylinder has same drag at same speed (1/10th *Re*) than airfoil
  - Think of wire bracing in early biplanes



Talay, Introduction to the aerodynamics of flight, NASA SP-367, 1975

Review question

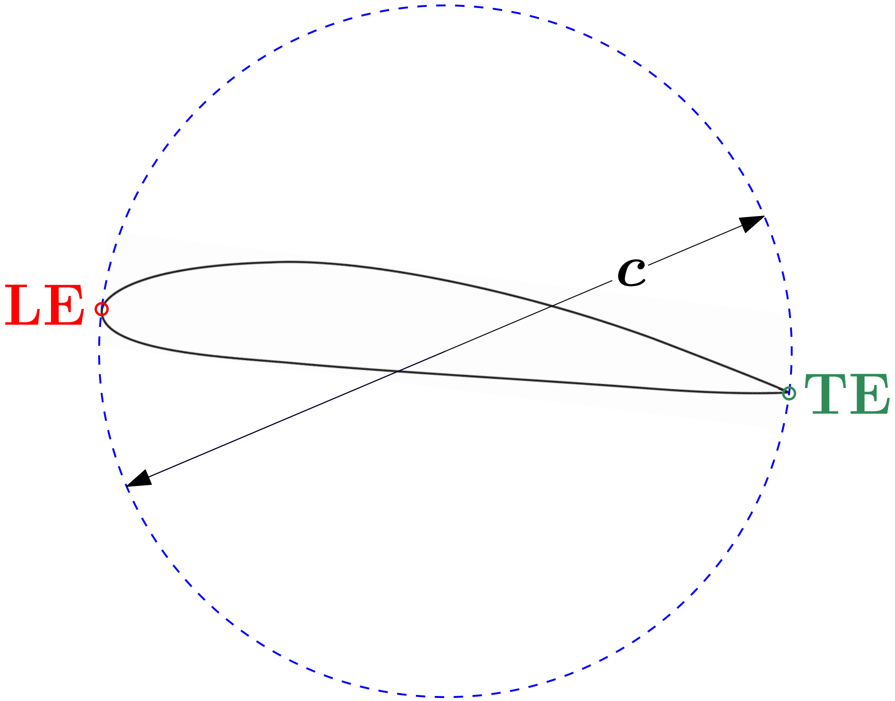
Refresh your prior knowledge of the drag force vs. Reynolds number behaviour of flow over a circular cylinder

Airfoil Geometry

Introduction to Aerodynamics

Airfoil geometry: Chord

- Airfoil section: usual section obtained when horizontal wing is cut by vertical plane parallel to center-plane
- Usually has *rounded leading edge* (LE) and (almost) *sharp trailing edge (TE)*
- Chord, c: diameter of smallest circle circumscribing the airfoil
  - Most unambiguous for sharp TE
- 2 contact points w/ airfoil are LE & TE
- Airfoil 'L' dimensions normalized by c

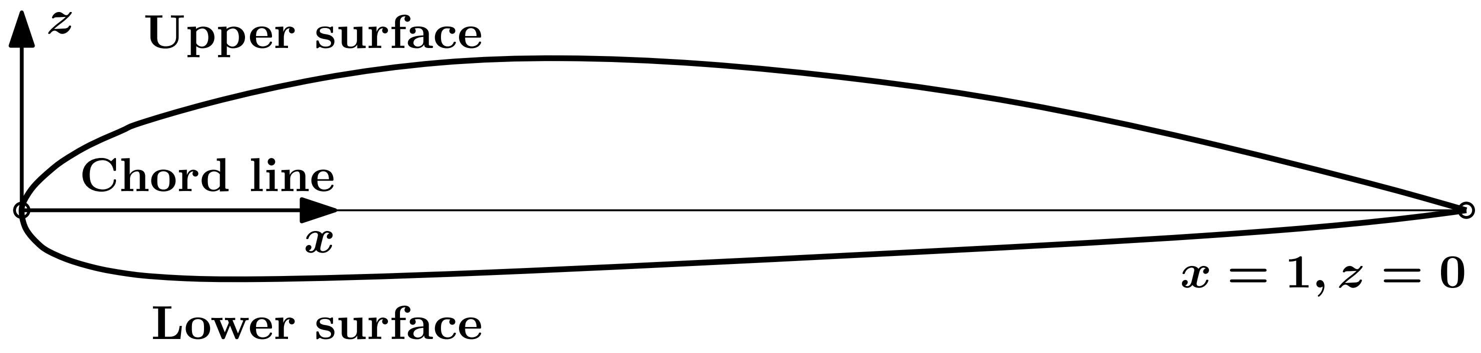


Smallest circumscribing circle

Houghton et al., Aerodynamics for Engineering Students, 2013

Airfoil geometry: Axes convention

- Chord line is x-axis, from LE to TE
  - N.B.: Reused notation! This may differ from longitudinal axis of vehicle!
- y-axis points into the plane of the diagram
- z-axis goes thru LE; completes right-handed system; points "up"
- Flow is assumed to go from left to right (LE to TE)
- Both x- and z-axes are normalized by the chord length c



Airfoil geometry: Camber

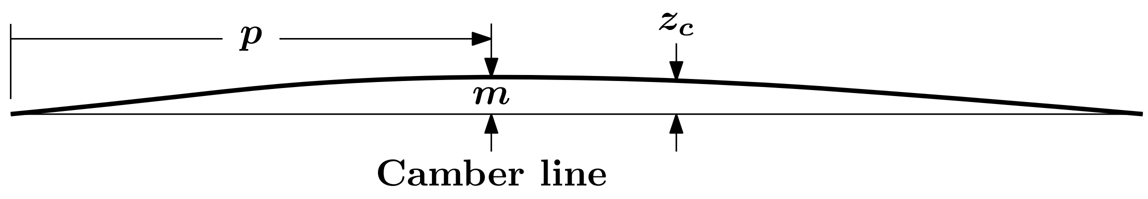
At any point along the chord, a point may be marked midway between the upper and lower surfaces

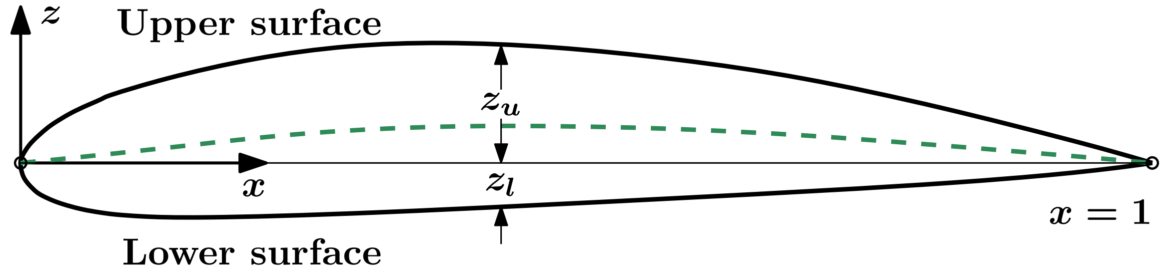
- Camber line: locus of all such points

Camber function, $z_c(x)$, is expressed as a fraction of chord

It is parameterized by:

- Max camber, m (max deviation of camber line from chord line)
  - Typical values up to 5%
- Location of max camber, p





Houghton et al., Aerodynamics for Engineering Students, 2013

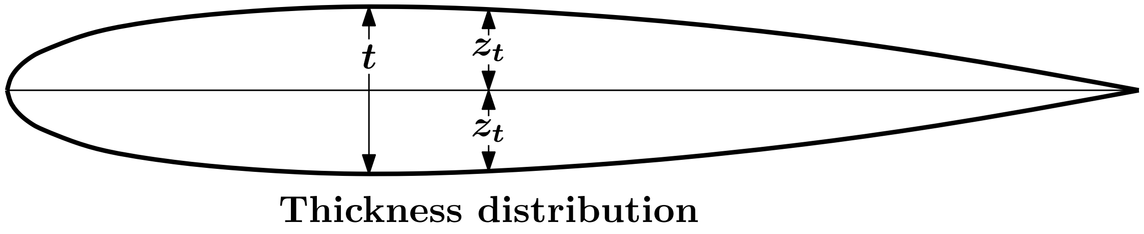
Airfoil geometry: Thickness distribution

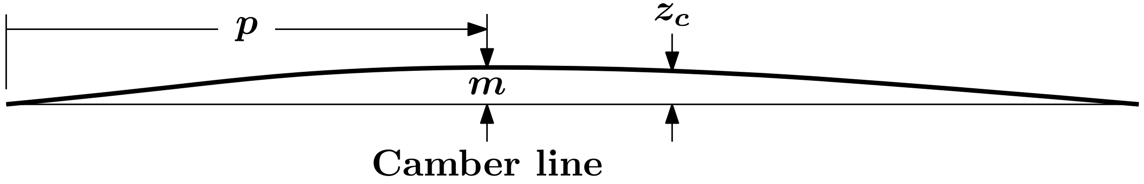
Distances from camber line to upper & lower surfaces at any point along chord are same by definition

This is thickness distribution, $z_t(x)$, expressed as a fraction of chord

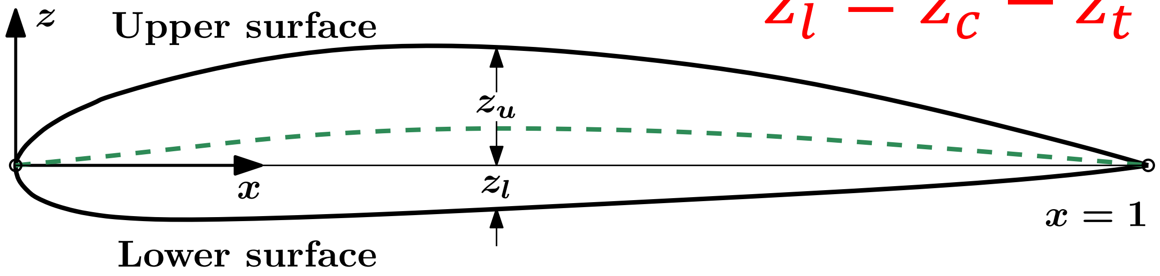
It is parameterized by:

- Max thickness, t (total, not half)
  - Typically limited to 18%
- Location of max thickness
  - Typical values between 25% & 60%



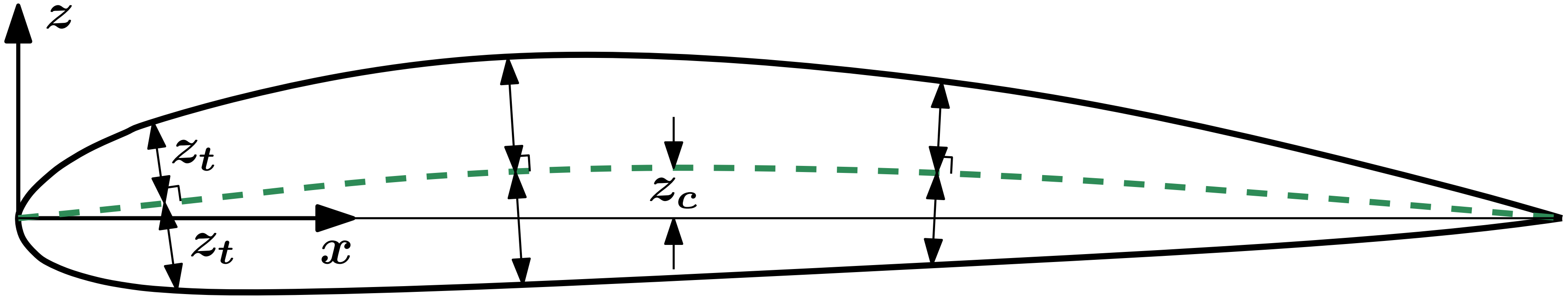


$$z_u = z_c + z_t$$
$$z_l = z_c - z_t$$



True (almost equivalent) airfoil definition

The actual (NACA) definition of the camber line is such that the thickness is distributed *normal* to it, instead of normal to the chord



Parametric equation:

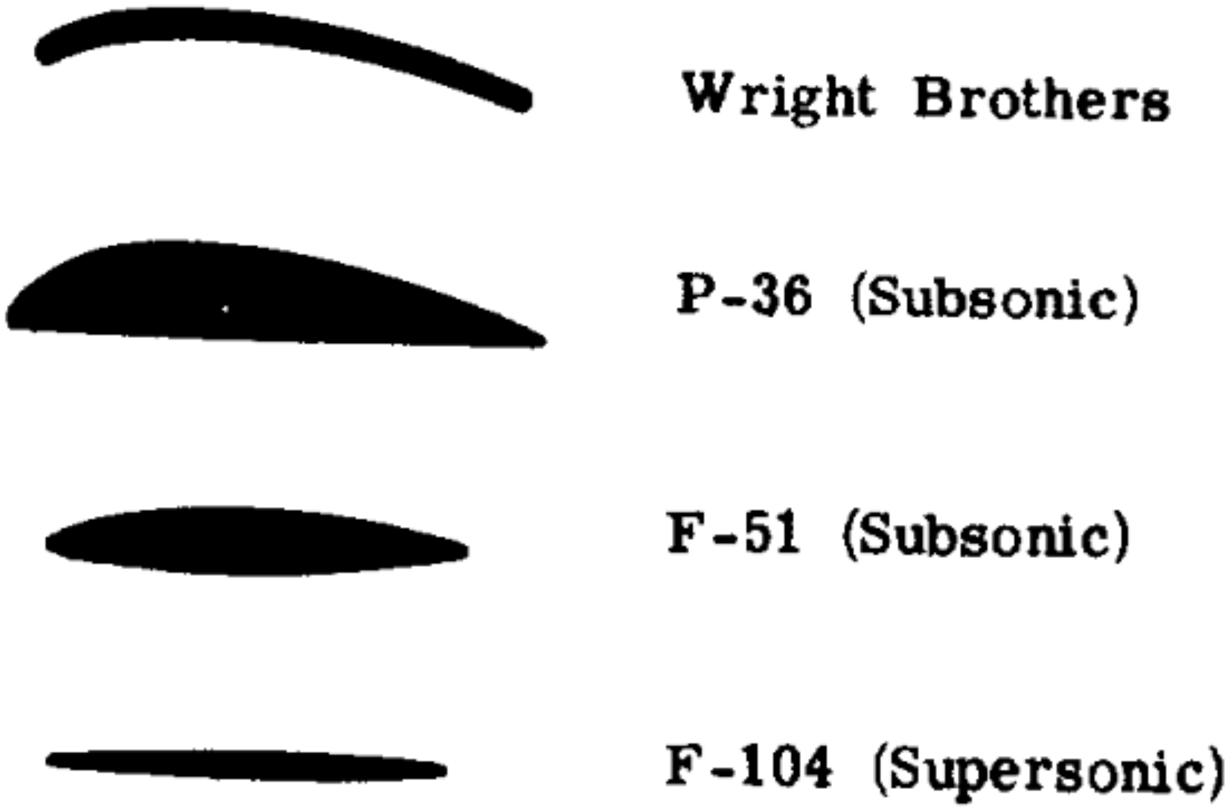
$$x_U = x - z_t \sin\theta\,, \qquad z_U = z_c + z_t \cos\theta$$
$$x_L = x + z_t \sin\theta\,, \qquad z_L = z_c - z_t \cos\theta$$
$$\theta = \tan^{-1}(dz_c/dx)$$

As long as the camber is small, the actual difference is negligible

Jacobs et al., NACA Report No. 460, 1932

Examples of airfoil shapes

Airfoil shapes are carefully tailored to the specific use



Talay, Introduction to the aerodynamics of flight, NASA SP-367, 1975

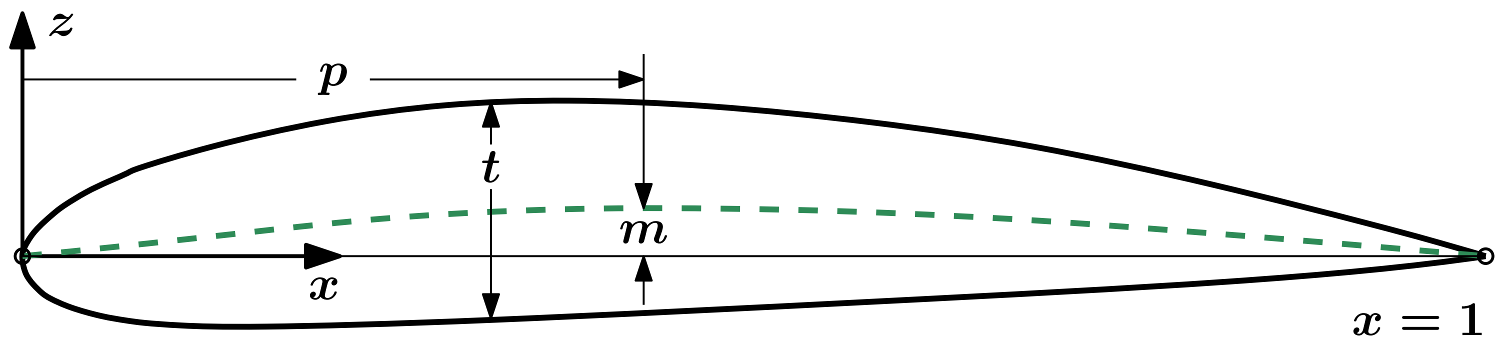
Airfoil nomenclature: NACA 4-digit series

NACA 4-digit series (eg. NACA XXXX) airfoil is easiest to generate

- 1st digit: Max camber (m) in 100ths of chord
- 2nd digit: Location of max camber point (p) in 10ths of chord
- Last 2 digits: Max thickness (t) in 100ths of chord

❑ NACA 2410 => 2% max. camber @ 40% chord; 10% max. thickness

❑ NACA 6508 => 6% max. camber @ 50% chord; 8% max. thickness

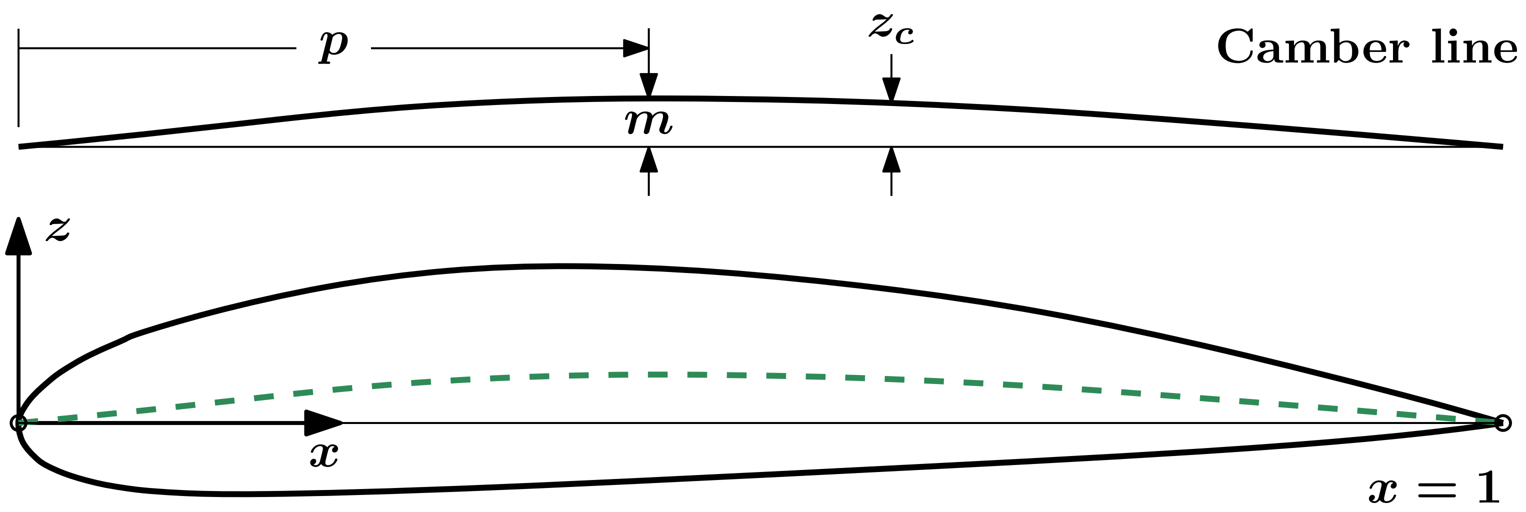


NACA 4-digit series definition: camber

Camber function w/ maximum of m at $x = p$ (all normalized by c):

$$z_c = \frac{m}{p^2}(2px - x^2), \qquad x \in [0, p]$$

$$z_c = \frac{m}{(1-p)^2}(1 - 2p + 2px - x^2), \qquad x \in [p, 1]$$



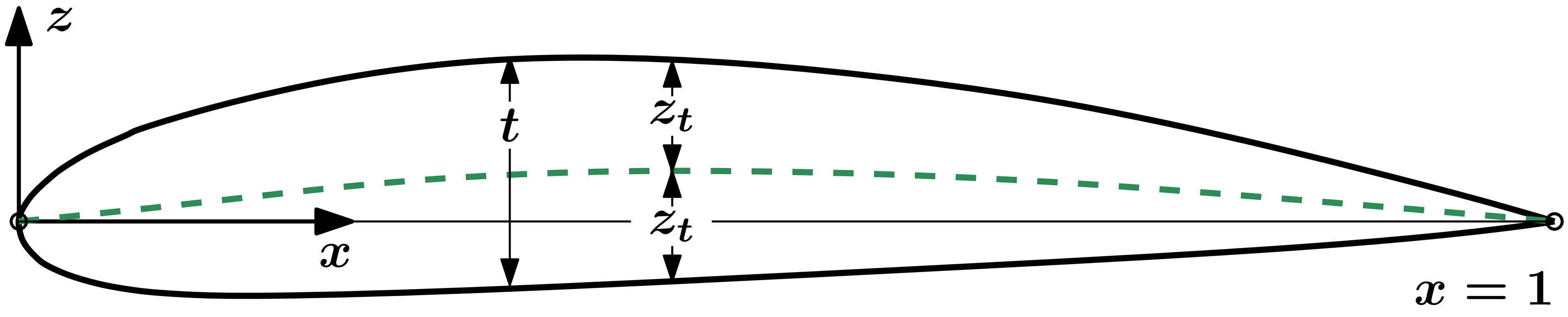
NACA 4-digit series definition

- Thickness distribution having maximum of t (total, not half):

$$z_t = 5t(0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4)$$

- This gives a slightly blunt TE; a *sharp TE* is needed in computations

$$z_t = 5t(0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - \color{red}{0.1036}x^4)$$



Problems

Show for yourself that

1. The camber function is smooth and differentiable
2. The thickness function results in a smoothly rounded LE
3. The thickness function peaks at about 30% of chord
4. The maximum thickness of the airfoil is indeed approximately t
5. The trailing edge is indeed sharp with the last equation

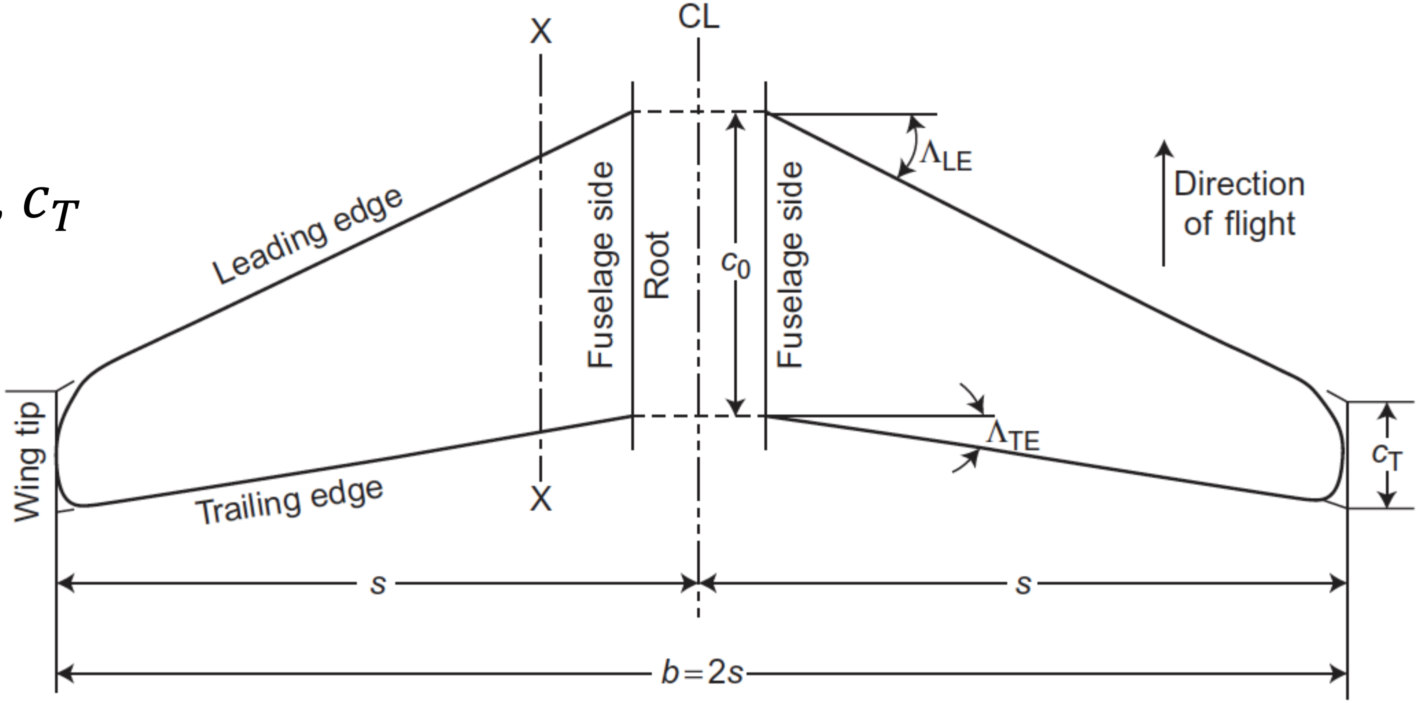
Wing Geometry

Introduction to Aerodynamics

Wing geometry: Terminology for wing planform

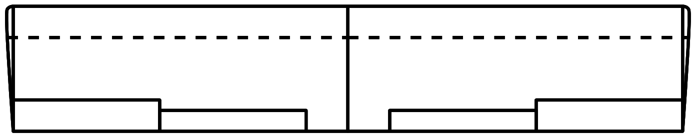
Wing planform is its shape as seen on a plan (top) view of aircraft

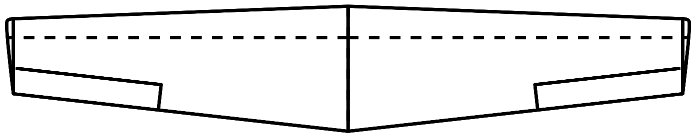
- Wing span, b
- Wing area, S
- Chords
  - Root chord, $c_{0,}$ and tip chord, c_T
  - Taper ratio (c_T/c_0)
  - Mean chord, $\bar{c}$ $(:= S/b)$
- Aspect ratio, $AR = b/\bar{c}$
  - Alternatively, $AR = b^2/S$
- Sweep back, Λ_{LE} and Λ_{TE}

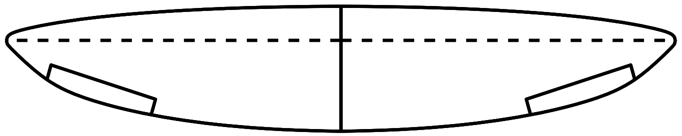


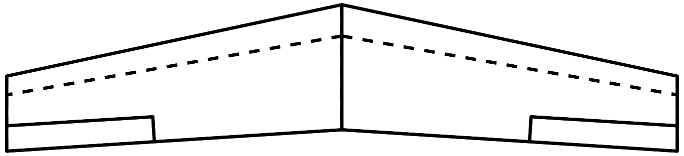
Houghton et al., Aerodynamics for Engineering Students, 2013

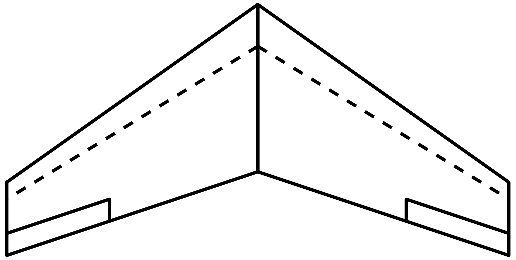
Examples of wing planform

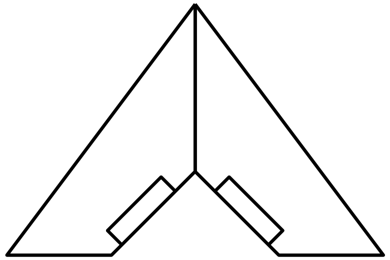

Rectangular straight wing

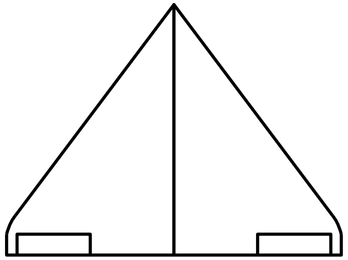

Tapered straight wing

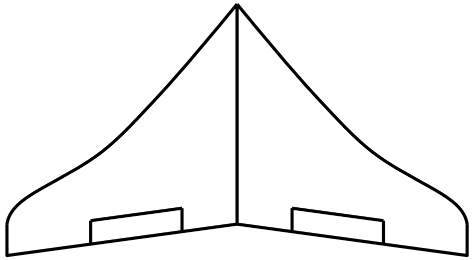

Rounded/elliptical straight wing


Slightly swept wing


Moderately swept wing


Highly swept wing

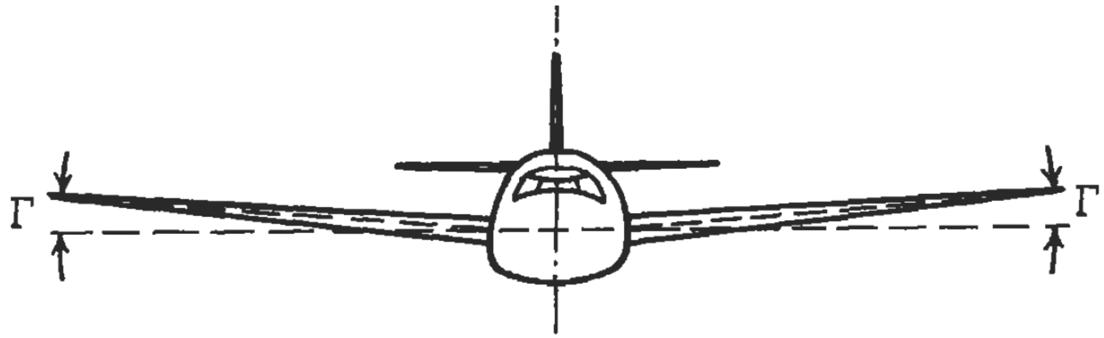

Delta wing

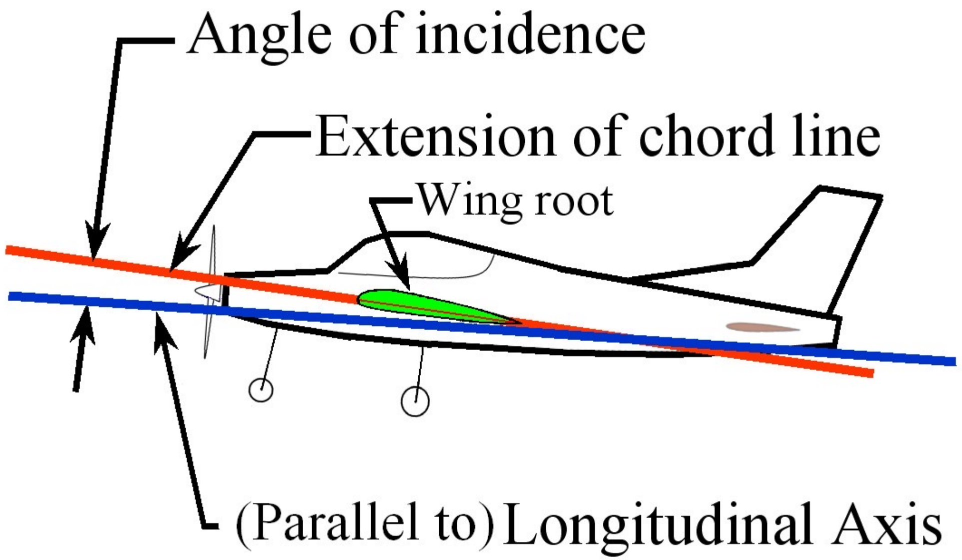

Complex delta wing

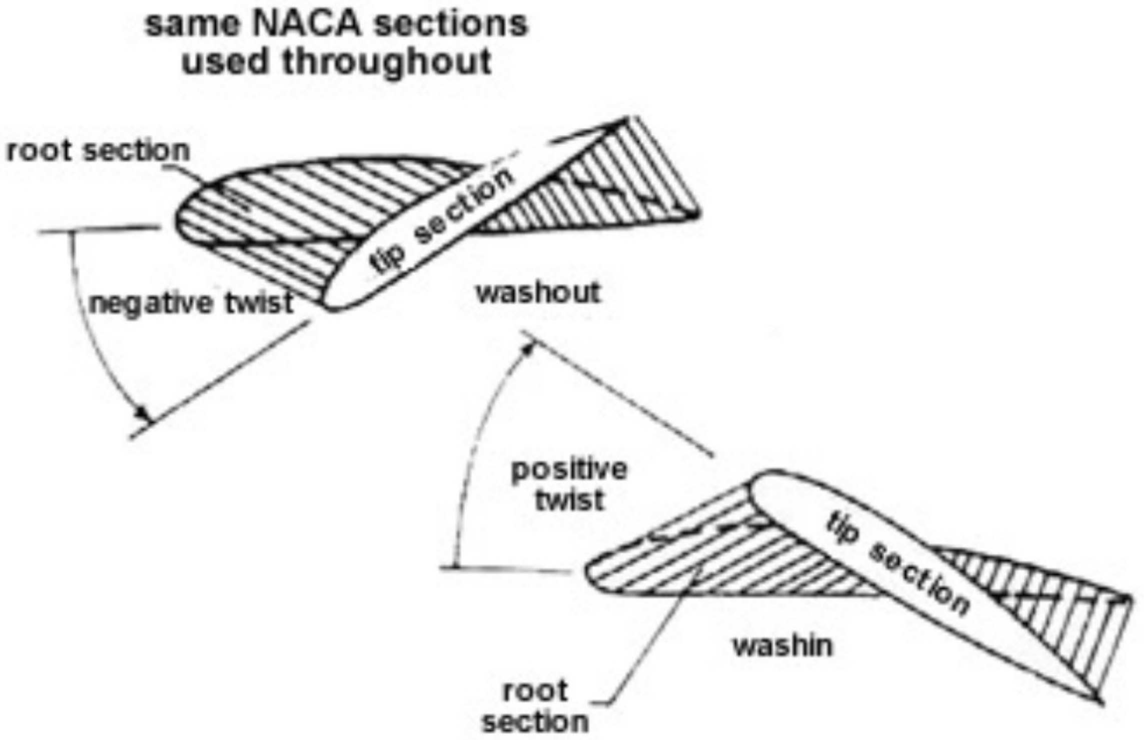
Talay, Introduction to the aerodynamics of flight, NASA SP-367, 1975

The third dimension

- Dihedral angle, Γ
- Geometric angle of incidence
- Twist: wash-in, wash-out



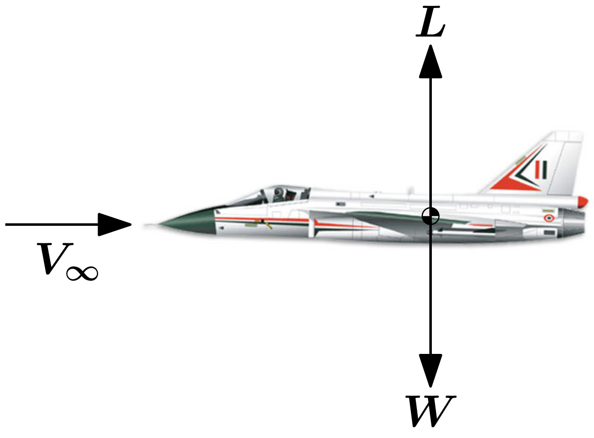




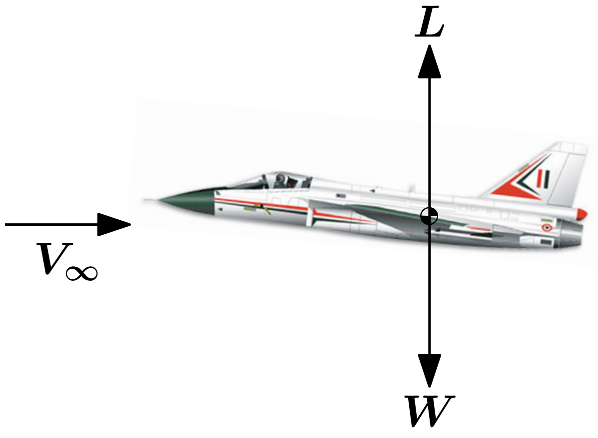
Aerodynamic Forces on an Airfoil

Introduction to Aerodynamics

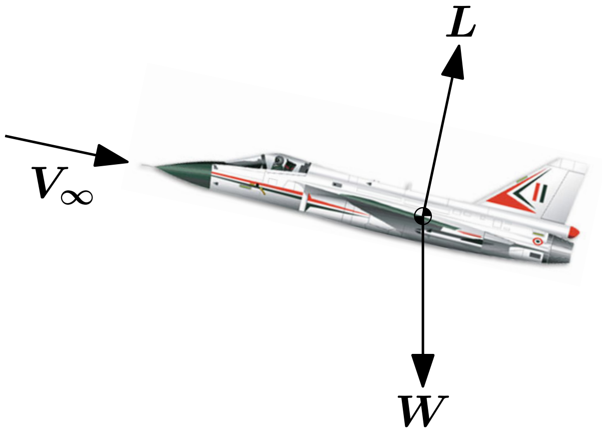
Review: Flight condition and direction of lift



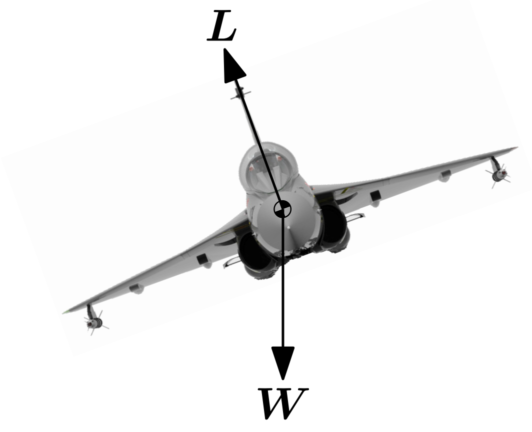
High-speed level flight



Low-speed level flight



Climbing flight



Banked circling flight

Houghton et al., Aerodynamics for Engineering Students, 2013

Aerodynamic forces: Contributions

Aerodynamic forces due to fluid on a surface can be resolved into

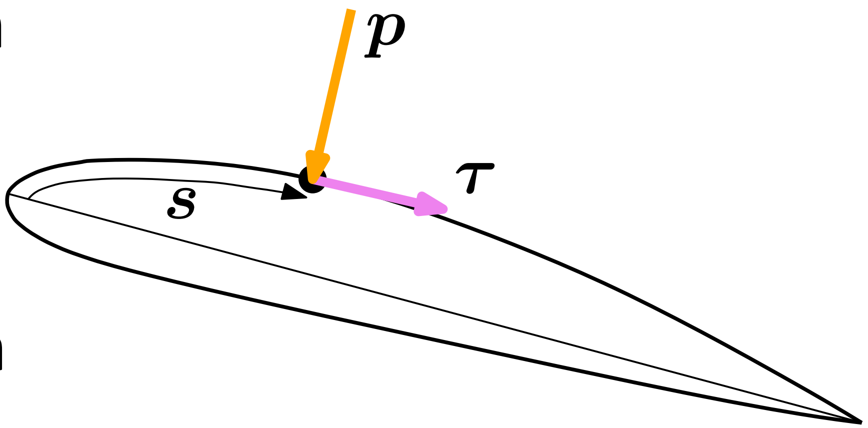
- Contribution from pressure (normal force per unit area)
- Contribution from shear stress (tangential force per unit area)

For 2D (airfoil) section, only one component of shear stress relevant

$s =$ (Body-fitted) coordinate along surface, measured from LE

$p = p(s) =$ Surface pressure distribution on airfoil section

$\tau = \tau(s) =$ Surface shear stress distribution on airfoil section



Anderson, Fundamentals of Aerodynamics, 2011

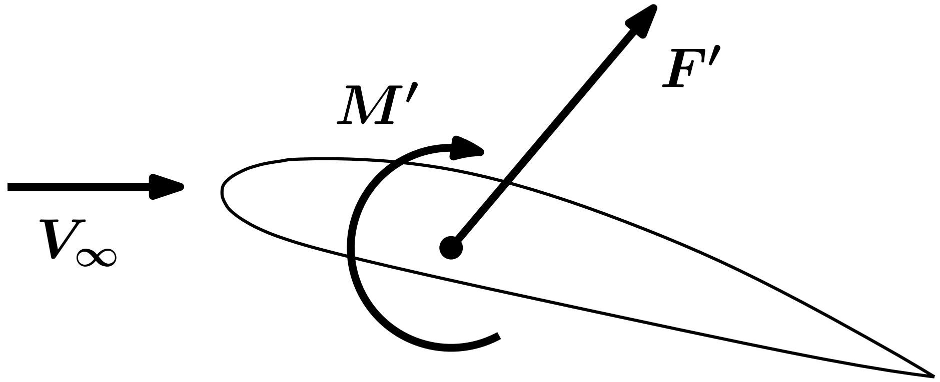
Aerodynamic forces: Resultant

Net effect of surface stresses p & τ *integrated* over entire section:

- A resultant (sectional) aerodynamic force per unit span, F'
- A resultant (sectional) aerodynamic moment per unit span, M' (pitching moment in 2D), about a chosen point

Primes on symbols (F' & M') signify forces/moments per unit span

These are also called *sectional* force & *sectional* pitching moment

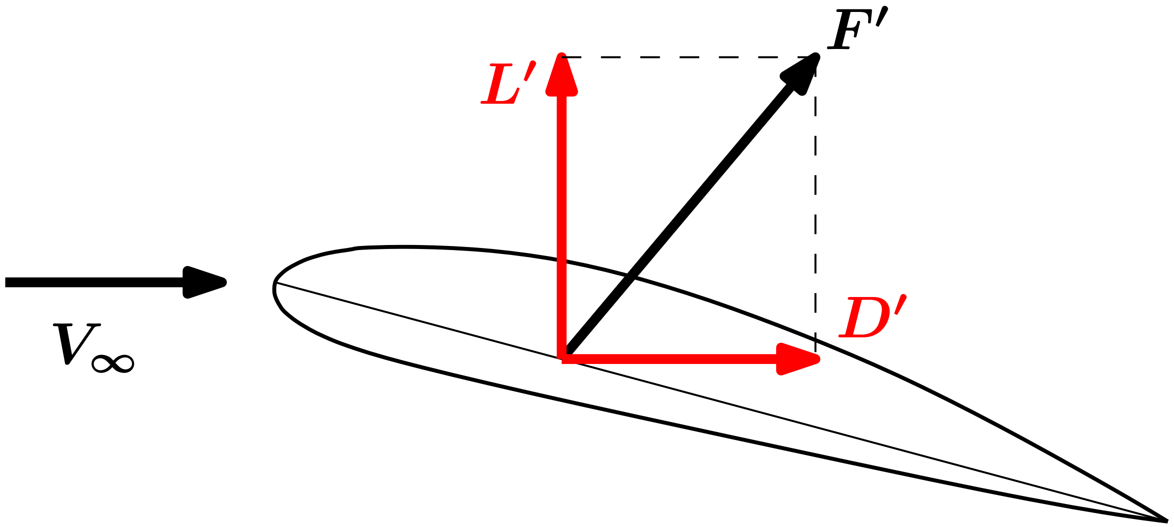


Aerodynamic forces: Components

Resultant force F' can be split into either of 2 sets of components

One resolution is w.r.t. the direction of relative wind ($\vec{V}_\infty$)

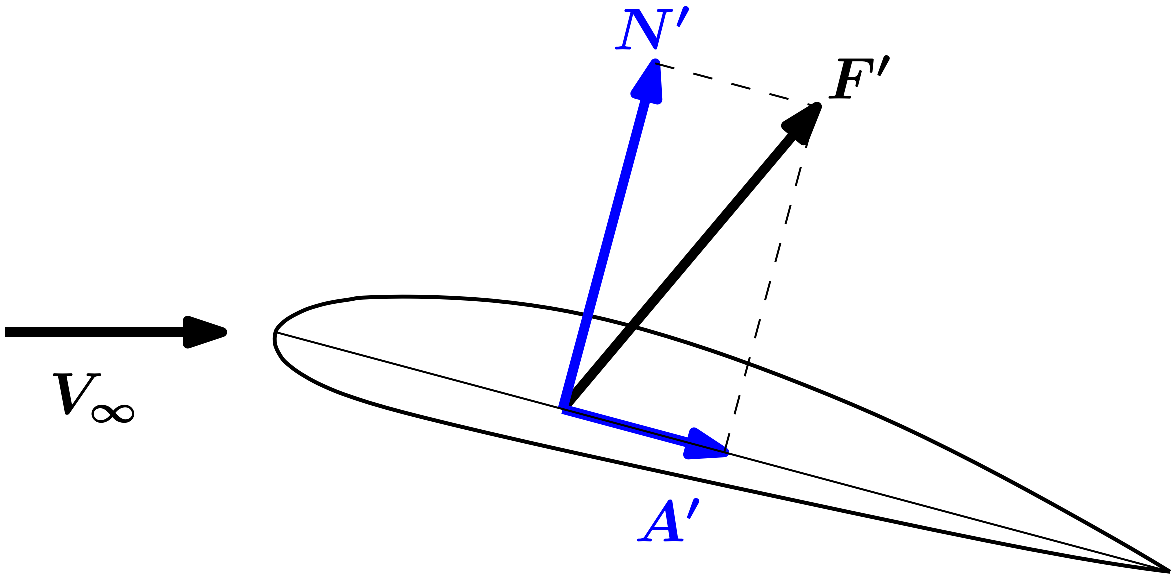
- L' := (Sectional) lift force = component of $F' \perp \vec{V}_\infty$
- D' := Drag force = component of $F' \parallel \vec{V}_\infty$



Aerodynamic forces: Components (alternative)

Another resolution is w.r.t. chord line

- N' := Normal force = component of F' $\perp$ chord
- A' := Axial force = component of F' $\parallel$ chord



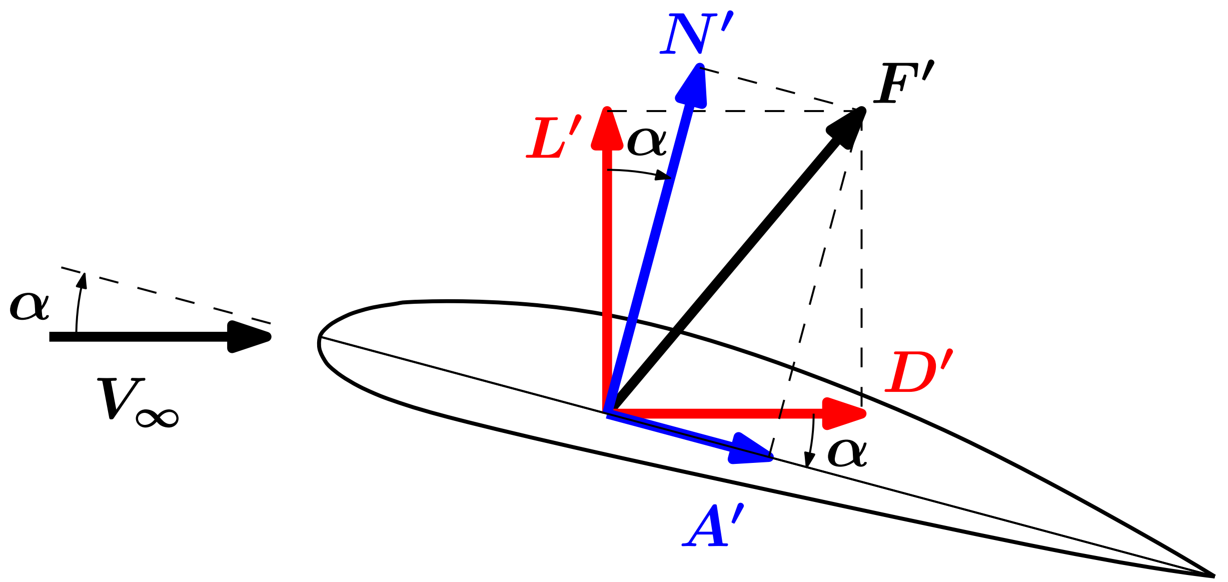
Aerodynamic forces: Components' relation

Angle-of-attack, $\alpha :=$ angle between chord line & $\vec{V}_\infty$

- N.B.: α is measured +ve clockwise (i.e., $\alpha > 0$ means airfoil is pitched up)!

Thus, $L' = N' \cos\alpha - A' \sin\alpha\,, \quad D' = N' \sin\alpha + A' \cos\alpha$

Alt., $N' = L' \cos\alpha + D' \sin\alpha\,, \quad A' = -L' \sin\alpha + D' \cos\alpha$



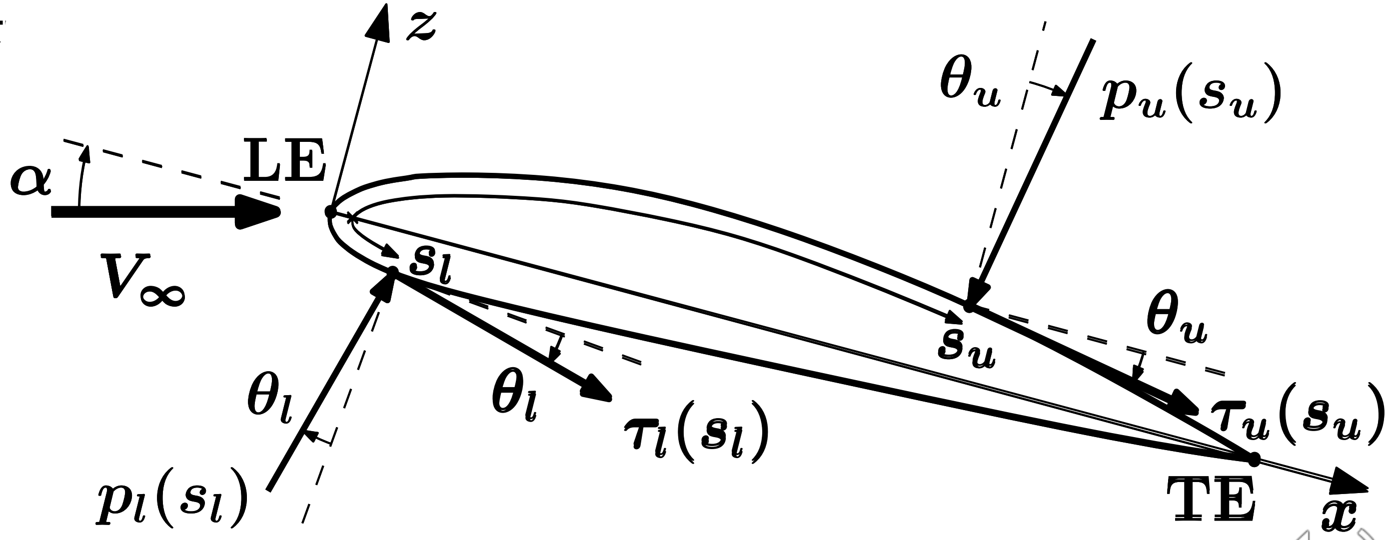
Pressure & shear contributions to axial force

Shear part, $A'_\tau = \int_{\mathrm{LE}}^{\mathrm{TE}} \tau_u \cos\theta_u \, ds_u + \int_{\mathrm{LE}}^{\mathrm{TE}} \tau_l \cos\theta_l \, ds_l$

Pressure part, $A'_p = -\int_{\mathrm{LE}}^{\mathrm{TE}} p_u \sin\theta_u \, ds_u + \int_{\mathrm{LE}}^{\mathrm{TE}} p_l \sin\theta_l \, ds_l$

Total axial force, $A' = A'_p + A'_\tau$

N.B.: θ_u & θ_l are measured positive clockwise!

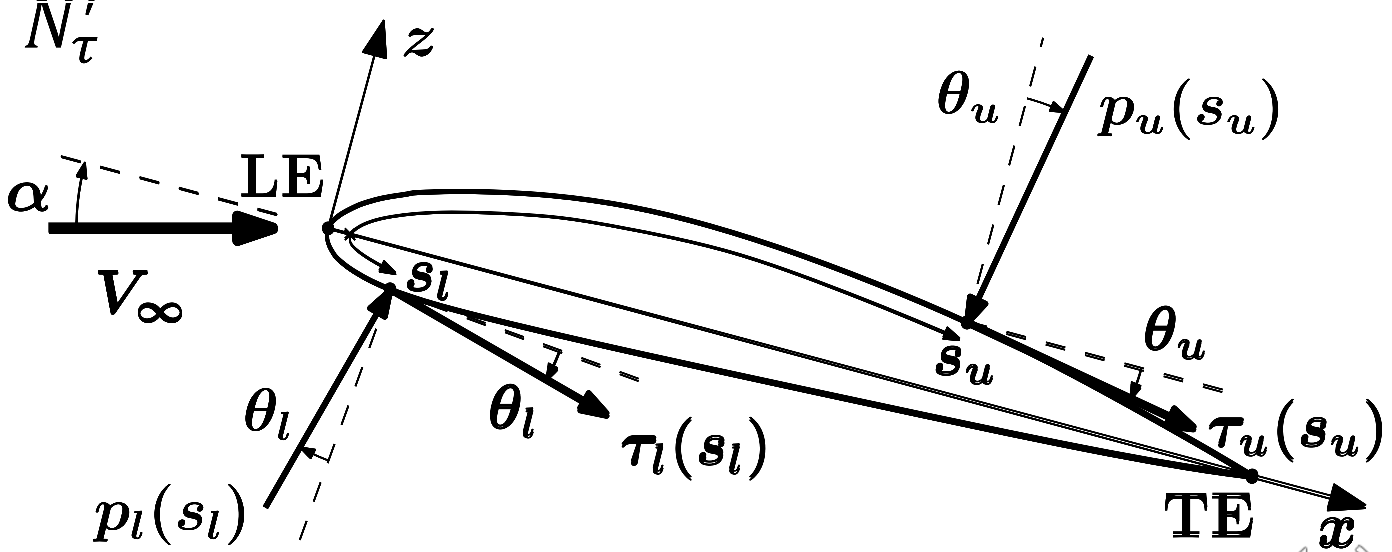


Pressure & shear contributions to normal force

Shear part, $N'_\tau = -\int_{\mathrm{LE}}^{\mathrm{TE}} \tau_u \sin\theta_u \, ds_u - \int_{\mathrm{LE}}^{\mathrm{TE}} \tau_l \sin\theta_l \, ds_l$

Pressure part, $N'_p = -\int_{\mathrm{LE}}^{\mathrm{TE}} p_u \cos\theta_u \, ds_u + \int_{\mathrm{LE}}^{\mathrm{TE}} p_l \cos\theta_l \, ds_l$

Total normal force, $N' = N'_p + N'_\tau$



Approximations for slender airfoils

For slender airfoils, surface's slope (θ_u or θ_l) is small everywhere

I.e., $\cos\theta_u \approx 1$, $\sin\theta_u \approx -dz_u/dx$, $\cos\theta_l \approx 1$, $\sin\theta_l \approx -dz_l/dx$

$$N'_p = -\int_{\mathrm{LE}}^{\mathrm{TE}} p_u \cos\theta_u \, ds_u + \int_{\mathrm{LE}}^{\mathrm{TE}} p_l \cos\theta_l \, ds_l \approx \int_0^c (-p_u + p_l) dx$$

$$N'_\tau = -\int_{\mathrm{LE}}^{\mathrm{TE}} \tau_u \sin\theta_u \, ds_u - \int_{\mathrm{LE}}^{\mathrm{TE}} \tau_l \sin\theta_l \, ds_l \approx \int_0^c \left(\tau_u \frac{dz_u}{dx} + \tau_l \frac{dz_l}{dx}\right) dx$$

$$A'_\tau = \int_{\mathrm{LE}}^{\mathrm{TE}} \tau_u \cos\theta_u \, ds_u + \int_{\mathrm{LE}}^{\mathrm{TE}} \tau_l \cos\theta_l \, ds_l \approx \int_0^c (\tau_u + \tau_l) dx$$

$$A'_p = -\int_{\mathrm{LE}}^{\mathrm{TE}} p_u \sin\theta_u \, ds_u + \int_{\mathrm{LE}}^{\mathrm{TE}} p_l \sin\theta_l \, ds_l \approx \int_0^c \left(p_u \frac{dz_u}{dx} - p_l \frac{dz_l}{dx}\right) dx$$

Approximations for slender airfoils at low AoA

If the AoA, α, is small, then $\cos\alpha \approx 1$, $\sin\alpha \approx \alpha \ll 1$

For *streamlined* airfoils, A' and N' are of similar order

Along with the preceding simplifications for slender airfoils, one has

$$L' = N' \cos\alpha - A' \sin\alpha \approx N' - A'\alpha \approx N'_p \approx \int_0^c (p_l - p_u)dx$$

Sectional lift force is almost solely determined by pressure distribution, not by shear stress

- In particular, by the pressure differential on the upper (suction) and lower (pressure) surfaces

Pitching Moment on an Airfoil

Introduction to Aerodynamics

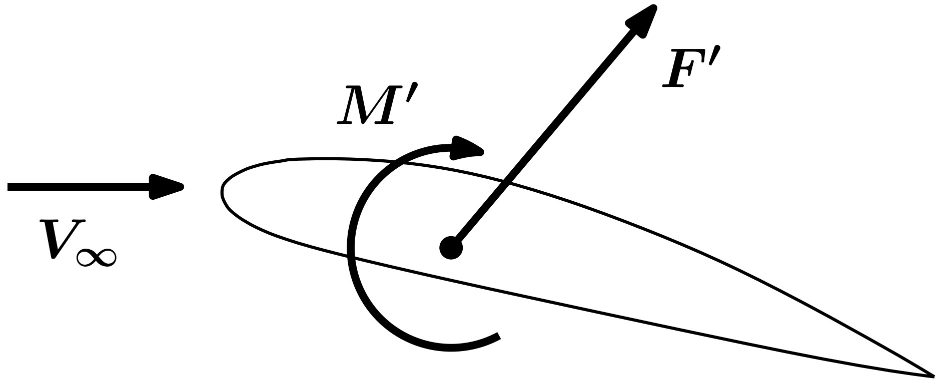
Resultant aerodynamic pitching moment

Net *rotational* effect of surface pressure p and surface shear stress τ *integrated* over entire section:

- A resultant aerodynamic moment M' (pitching moment in 2D), about a chosen point

Prime reflects the fact that the moment is “per unit span”

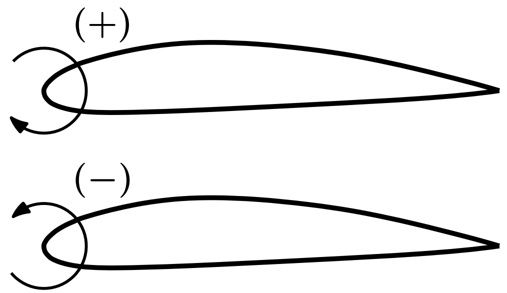
It is thus also called *sectional* pitching moment



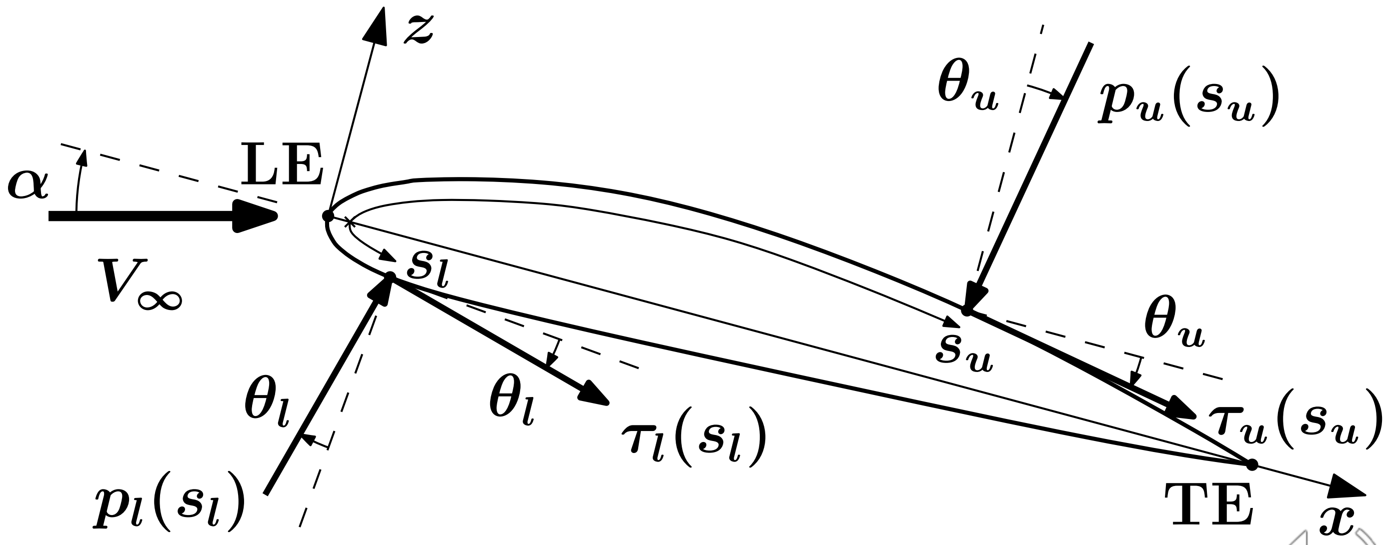
Pitching moment from pressure and shear

About LE, sectional pitching moment is (see sign convention):

$$M'_{LE} = \int_{\mathrm{LE}}^{\mathrm{TE}} [(p_u \cos\theta_u + \tau_u \sin\theta_u)x + (-p_u \sin\theta_u + \tau_u \cos\theta_u)z]ds_u$$
$$+ \int_{\mathrm{LE}}^{\mathrm{TE}} [(-p_l \cos\theta_l + \tau_l \sin\theta_l)x + (p_l \sin\theta_l + \tau_l \cos\theta_l)z]ds_l$$



Positive if tending to increase AoA



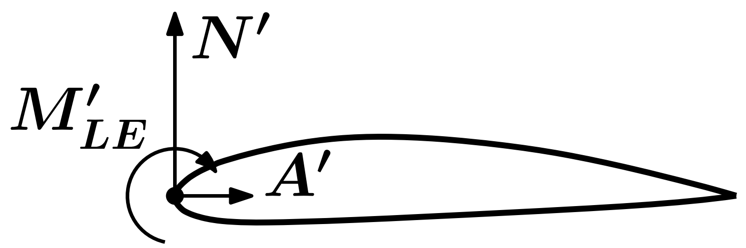
Review of Forces and Moment Systems

Moment on a system can be reported about any point

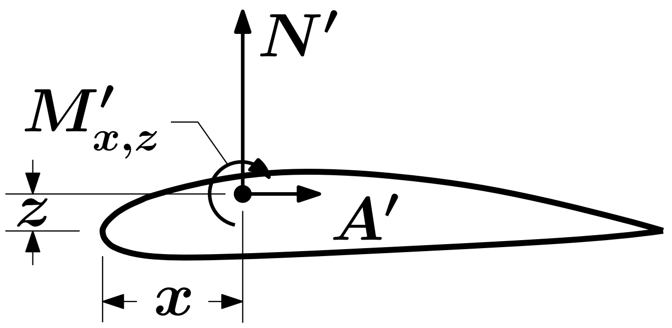
- Given the moment about one point (and forces on the system), moment about another point can be calculated from equivalence

Setting origin of coordinate system at LE,

$$M'_{LE} = M'_{x,z} - xN' + zA'$$



Reference point @ LE



Reference point @ (x, z)

Center of pressure

It's the point thru which resultant of surface-distributed forces act

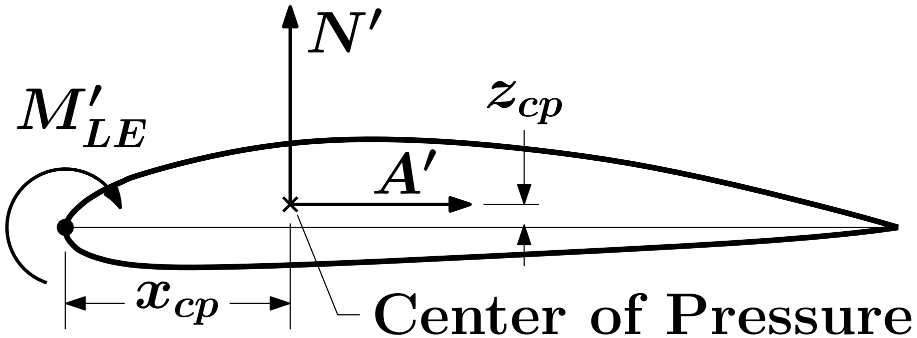
Alternatively, it's the point about which (pitching) moment vanishes

- Skin friction contribution to pitching moment is usually negligible owing to slender shape of airfoils (hence the terminology)

Setting origin at LE, $M'_{LE} = -x_{CP} N' + z_{CP} A'$ (since $M'_{CP} = 0$)

- As airfoils are slender, z_{CP} is usually negligible $\Longrightarrow x_{CP} \approx -M'_{LE}/N'$
- If AoA is small, then $L' \approx N'$

$\Rightarrow x_{CP} \approx -M'_{LE}/L'$



Equivalent specs of airfoil forces & moment

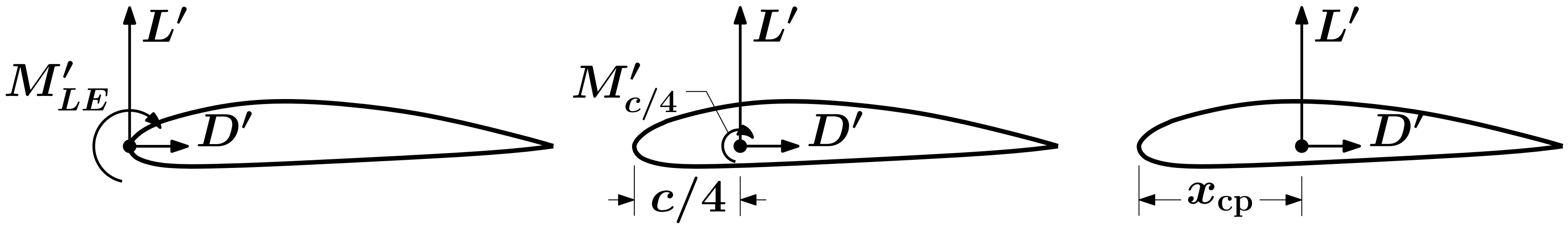
As L' decreases, $x_{CP} \approx -M'_{LE}/L'$ becomes difficult to measure

- Thus, center of pressure is not always a convenient concept

However, this is no problem since L' and D' can be placed at any point as long as M' is also specified about that point

- Later we will find that quarter-chord point is particularly suitable

Equivalent descriptions: $M'_{LE} = -xL' + M'_x = -\frac{c}{4}L' + M'_{c/4} = -x_{CP}L'$



Resultant force at: Leading edge Quarter-chord point Center of pressure

Aerodynamic center

Sectional pitching moment vanishes at center of pressure

- But location of center of pressure may shift with AoA

Aerodynamic center (x_{ac}, z_{ac}) is defined as a point about which sectional pitching moment is independent of AoA

- Usually, $z_{ac} \approx 0$ for slender airfoils

Although this may seem miraculous, existence of such a point is:

- Due to the slenderness of airfoils,
- Due to the limitation of their operation to small AoAs, and
- Limited focus on operation at high Reynolds nos.

Knowledge of A/D center is crucial for design of aerospace vehicles

Dimensional Analysis in Aerodynamics

Introduction to Aerodynamics

Dimensional analysis of airfoil forces

Let F' be any (sectional) A/D force per unit span on a 2D (airfoil) section

$$F' = f(\rho_\infty, V_\infty, c, \alpha, \mu_\infty, a_\infty, \text{geometry})$$

This is a dimensional equation; all terms have dimensions of force/length

Reformulate as an equivalent dimensionless equation; one option is

$$\frac{F'}{0.5\rho_\infty V_\infty^2 c} = \hat{f}\left(\alpha, \frac{\rho_\infty V_\infty c}{\mu_\infty}, \frac{V_\infty}{a_\infty}, \text{shape}\right)$$

Fundamental: Shows that *dimensionless* sectional force (called sectional force *coefficient*) depends *only* on AoA, Reynolds no., Mach no. & shape

N.B.: Reynolds no. and Mach no. refer to freestream values

Sectional force/moment coefficients on 2D sections

$q_\infty \coloneqq 0.5\rho_\infty V_\infty^2$ is "dynamic head" of flow (0.5 is from Bernoulli's eqn.)

Relevant airfoil *sectional* force coefficients (coeff.) are:

Lift coeff., $c_l := \dfrac{L'}{q_\infty c}$; Drag coeff., $c_d := \dfrac{D'}{q_\infty c}$

Normal force coeff., $c_n := \dfrac{N'}{q_\infty c}$; Axial force coeff., $c_a := \dfrac{A'}{q_\infty c}$

By analogy, pitching moment coeff., $c_m := \dfrac{M'}{q_\infty c^2}$

N.B.: *sectional* coefficients' symbols have lower case 'c' and subscripts

All are functions of AoA, Reynolds no. & Mach no. (& shape of course)

Dimensional analysis of 3D aerodynamic forces

Let F be any aerodynamic force on a 3D (wing) body

$$F = f(\rho_\infty, V_\infty, \bar{c}, b, \alpha, \mu_\infty, a_\infty, \text{geometry})$$

Reformulate as an equivalent dimensionless equation; one option is

$$\frac{F}{0.5\rho_\infty V_\infty^2 S} = \hat{f}\left(\alpha, \frac{\rho_\infty V_\infty \bar{c}}{\mu_\infty}, \frac{V_\infty}{a_\infty}, AR, \text{shape}\right)$$

I.e., dimensionless force (called force coefficient) depends only on AoA, Reynolds no., Mach no., aspect ratio and shape

$$C_L = \frac{L}{q_\infty S}; \quad C_D = \frac{D}{q_\infty S}; \quad C_N = \frac{N}{q_\infty S}; \quad C_A = \frac{A}{q_\infty S}; \quad C_M = \frac{M}{q_\infty S \bar{c}}$$

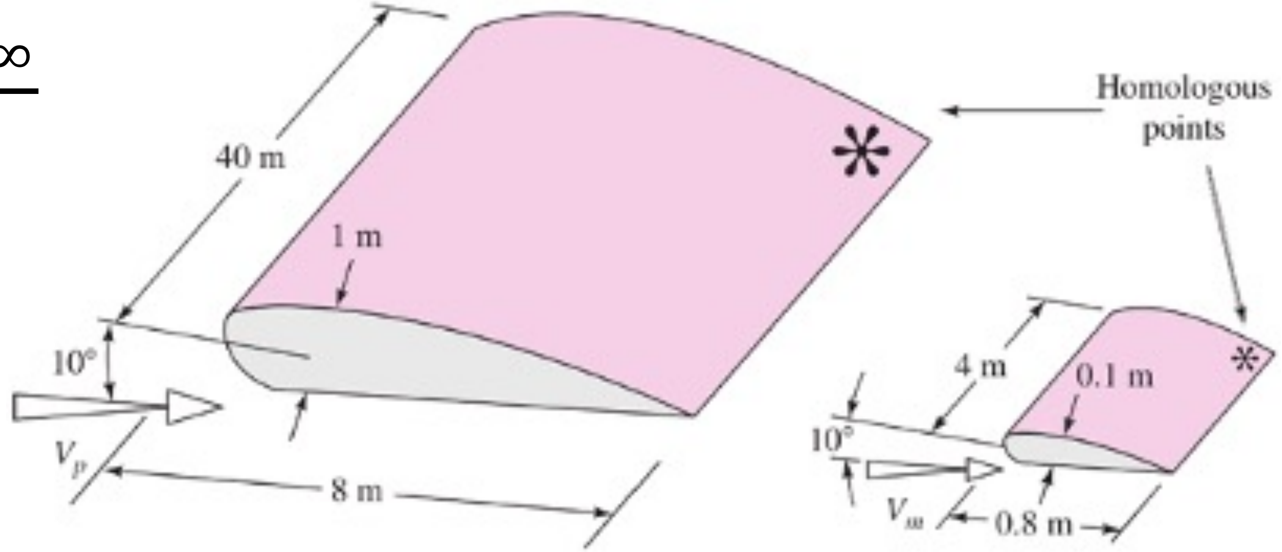
N.B.: coefficients' symbols have upper case 'C' and subscripts

Pressure coefficient and skin friction coefficient

Pressure & shear stresses can also be reduced to non-dimensional coefficients, following the same principle of dimensional analysis

Pressure coeff.: $C_p = \dfrac{p - p_\infty}{q_\infty}$

Skin friction coeff.: $c_f = \dfrac{\tau}{q_\infty}$



Both are functions of AoA, Reynolds no., Mach no. & shape

Also, function of non-dimensional position on body surface (following similarity)

White, Fluid Mechanics, 2009

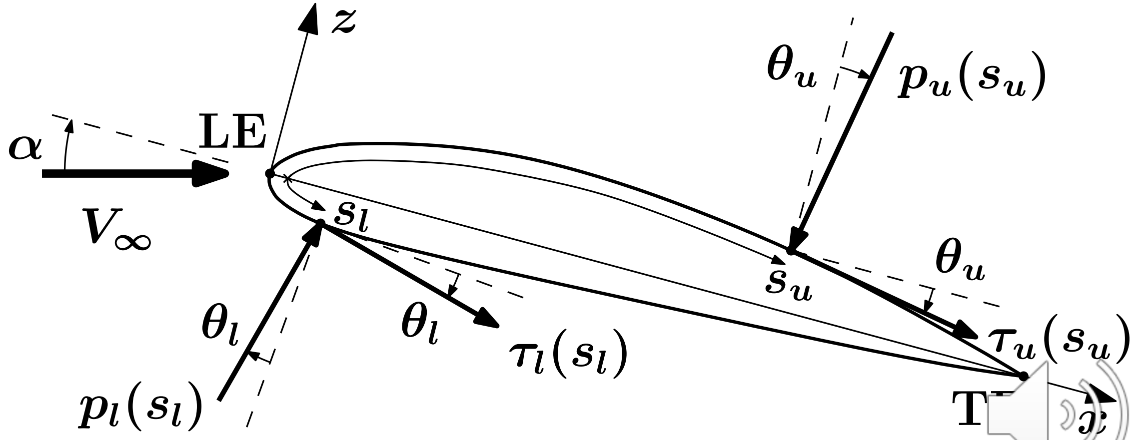
Expressing c_n, c_a, c_l & c_d in terms of C_p & c_f

All aerodynamic force coefficients can be found from the pressure and skin friction coefficients

$$c_n = \int_{\mathrm{LE}}^{\mathrm{TE}} \left(-C_{p,u}\cos\theta_u - c_{f,u}\sin\theta_u\right)\frac{ds_u}{c} + \int_{\mathrm{LE}}^{\mathrm{TE}} \left(C_{p,l}\cos\theta_l - c_{f,l}\sin\theta_l\right)\frac{ds_l}{c}$$

$$c_a = \int_{\mathrm{LE}}^{\mathrm{TE}} \left(-C_{p,u}\sin\theta_u + c_{f,u}\cos\theta_u\right)\frac{ds_u}{c} + \int_{\mathrm{LE}}^{\mathrm{TE}} \left(C_{p,l}\sin\theta_l + c_{f,l}\cos\theta_l\right)\frac{ds_l}{c}$$

$$c_l = c_n\cos\alpha - c_a\sin\alpha$$
$$c_d = c_n\sin\alpha + c_a\cos\alpha$$



Goal of study of aerodynamics

Clearly, the foregoing demonstrates the principle stated earlier, viz.

- Aerodynamic lift, drag & moments on a body are the surface-integrated pressure and shear stress distributions over the body

Goal of aerodynamics is to determine $p(s)$ and $\tau(s)$ for a given body shape and freestream condition, thus yielding the coefficients of aerodynamic forces and moments

Streamlining of airfoils results in pressure dominating over shear for the all-important lift force & pitching moment

A simplified endeavour is to calculate $p(s)$ only on airfoils and wings for *estimating* the coefficients of lift and pitching moment

End of Topic

Introduction to Aerodynamics