

Assignment - 5 (AE-330)

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Ans 1) Given, $F_{sea} = 8896.4 \text{ N}$, $P_1 = 6.89 \text{ MPa}$, $t_b = 10 \text{ sec}$, $T_{amb} = 294 \text{ K}$
 $T_1 = 1755.3 \text{ K}$, $\gamma = 2.54 \times 10^{-3} \text{ m/s}$, $k = 1.22$, $MW = 22$, $\rho_p = 1550 \text{ kg/m}^3$

$$C_F = 1.57, \quad \varepsilon = 7.8,$$

We know that $C_F = \frac{F}{P_1 A_t}$

$$A_t = \frac{F}{C_F P_1} = \frac{8896.4}{6.89 \times 10^6 \times 1.57}$$

$$A_t = 8.224 \times 10^{-4} \text{ m}^2$$

Now, $\dot{m} = \frac{A_t P_1 k}{\sqrt{k R T_1}} \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$

$$\dot{m} = \frac{8.224 \times 10^{-4} \times 6.89 \times 10^6 \times 1.22}{\sqrt{1.22 \times \frac{8314}{22} \times 1755.3}} \times \sqrt{\left(\frac{2}{2.22}\right)^{\frac{2.22}{0.22}}} = 1550 \times A_b \times 2.54 \times 10^{-3}$$

$$\dot{m} = \frac{6.913 \times 10^3}{8.996 \times 10^2} \times (0.59)$$

$$\dot{m} = 4.534 \text{ kg/sec} = 3.937 \times A_b$$

$$\Rightarrow \begin{cases} \dot{m} = 4.534 \text{ kg/sec} \\ A_b = 1.1516 \text{ m}^2 \end{cases}$$

Now, Specific Impulse $\Rightarrow I_{sp} = \frac{F}{\dot{m} g} = 200 \text{ sec}$

$$m_{\text{propellant}} = \dot{m} \times t_b$$

$$m_p = 4.534 \times 10$$

$$m_p = 45.34 \text{ kg}$$

$$\text{Total Impulse (I}_t\text{)} = I_{sp} \times m_p \times g$$

$$I_t = 88957.08 \text{ kg-m/s}$$

Hence the answers are,

$$I_{sp} = 200 \text{ sec}$$

$$A_t = 8.224 \times 10^{-4} \text{ m}^2$$

$$\dot{m} = 4.534 \text{ kg/s}$$

$$m_p = 45.34 \text{ kg}$$

$$I_t = 88957.08$$

$$A_b = 1.1516 \text{ m}^2$$

For $A_{\text{exit}} (A_2)$, we use,

$$A_2 = \epsilon A_t$$

$$A_2 = 6.414 \times 10^{-3} \text{ m}^2$$

Answers

Ans 2) Given, $\rho_b = 1710 \text{ kg/m}^3$, $P_1 = 14 \text{ MPa}$, $a = 17.21 \times 10^{-3}$, $k = 1.27$,

$T_1 = 2220 \text{ K}$, $MW = 23 \Rightarrow R = \frac{8314}{23} \Rightarrow R = 361.48$, $n = 0.3$

$F_{\text{req}} = 5000 \text{ N}$, \Rightarrow Optimum expansion condition at 0.1 MPa , $t_b = 15 \text{ sec}$

Firstly, we evaluate A_t ,

$$F = P_1 A_t \sqrt{\frac{2k}{k-1} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{k}{k-1}} \right]} = 0$$

$$5000 = 14 \times 10^6 \times A_t \times 1.6328$$

$$A_t = 2.187 \times 10^{-4} \text{ m}^2$$

$$\text{Now } \dot{m} = \frac{A_t P_1 k}{\sqrt{kRT_1}} \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

$$\dot{m} = \frac{2.187 \times 10^{-4} \times 14 \times 10^6 \times 0.587}{1009.53}$$

$$\dot{m} = 1.78 \text{ kg/s}$$

$$\text{Also } \dot{m} = \rho_b A_b (a P_1^n) \times 10^{-3}$$

$$(1.78) = 1710 \times A_b (17.21 \times (14)^{0.3}) \times 10^{-3}$$

$$\frac{(1.78)}{1.71} = A_b \times 37.986$$

$$A_b = 2.74 \times 10^{-2} \text{ m}^2$$

$$\text{Now, } \frac{A_t}{A_2} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}} \sqrt{\frac{k+1}{k-1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$\frac{1}{\epsilon} = 0.0762 \Rightarrow \epsilon = 13.123 \Rightarrow A_2 = 2.87 \times 10^{-3} \text{ m}^2$$

$$\text{Also, } m_{\text{propellant}} = \dot{m} t_b$$

$$\Rightarrow m_p = 26.7 \text{ kg}$$

Hence, the designed SRM has the following characteristics

mass of the propellant stored

Area of burning propellant

Area of Throat section

Area of Exit Section

$$m_p = 26.7 \text{ kg}$$

$$A_b = 2.74 \times 10^{-2} \text{ m}^2$$

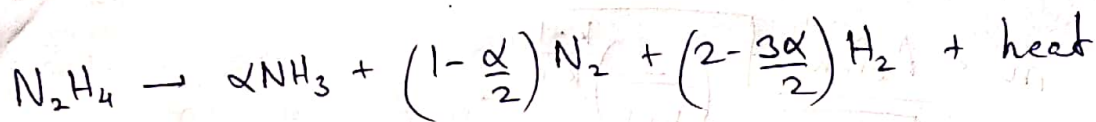
$$A_t = 2.187 \times 10^{-4} \text{ m}^2$$

$$A_2 = 2.87 \times 10^{-3} \text{ m}^2$$

And

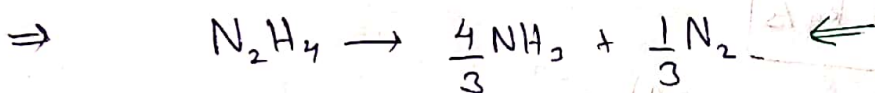
$$\epsilon = 13.123$$

Ans 3) The reaction of Hydrazine is,



As it is assumed that no NH_3 decomposes,

$$2 - \frac{3\alpha}{2} = 0 \Rightarrow \boxed{\alpha = \frac{4}{3}}$$



Now, $\Delta H_{\text{reactants}} = \Delta H_{\text{products}}$

$$1 \times 50.3 \times 10^3 = \frac{4}{3} (-45.9) \times 10^3 + \frac{4}{3} (45.06) (T_{\text{ad}} - 298) + \frac{1}{3} (0) + \frac{1}{3} (38) (T_{\text{ad}} - 298)$$

$$\Rightarrow 111.5 \times 10^3 = 72.747 \times (T_{\text{ad}} - 298)$$

$$\Rightarrow \boxed{T_{\text{ad}} = 1830.71 \text{ K}}$$

Now, given, $T_{\text{vacuum}} = 1000 \text{ N}$, $P_1 = 1 \text{ MPa}$, $P_2 = 12 \text{ kPa}$,

$k = 1.25$.

$$\text{MW}_{\text{products}} = \frac{\frac{4}{3}(17)}{\frac{5}{3}} + \frac{\frac{1}{3}(28)}{\frac{5}{3}} = \underline{\underline{19.2}}$$

$$\Rightarrow R = \frac{8314}{19.2}$$

$$\boxed{R = 433 \text{ J/kg-mol}}$$

Hence, $C^* = \frac{\sqrt{kRT_1}}{k \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}}$

$$C^* = \frac{995.42}{1.25 \times 0.5886}$$

$$C^* = 1352.93 \text{ m/s}$$

Nozzle expansion ratio can be found as,

$$\frac{A_2}{A_1} = \frac{1}{\epsilon} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}} \sqrt{\frac{k+1}{k-1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$\frac{1}{\epsilon} = 1.6018 \times 6.931 \times 10^{-3} \times 2.53$$

$$\frac{1}{\epsilon} = 2.809 \times 10^{-2} \Rightarrow \epsilon = 35.596$$

$$V_2 = \sqrt{\frac{2kRT_1}{k-1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$V_2 = 2374.8 \text{ m/sec}$$

First we will find A_1 & A_2 .

$$F = P_1 A_1 \sqrt{\frac{2k^2}{k-1} \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} + (P_2 - 0) \epsilon A_1$$

$$1000 = A_1 \left[1.7553 \times 10^6 + 2 \times 10^3 \times 35.596 \right]$$

$$A_1 = \frac{1000}{1.8265 \times 10^6} \Rightarrow A_1 = 5.475 \times 10^{-4} \text{ m}^2$$

$$A_2 = \epsilon A_1$$

$$= 35.596 \times 5.475 \times 10^{-4}$$

$$A_2 = 1.9379 \times 10^{-2} \text{ m}^2$$

$$\text{Now, as } \frac{\pi d^2}{4} = A, \quad d_1 = \sqrt{\frac{4A_1}{\pi}} \quad \& \quad d_2 = \sqrt{\frac{4A_2}{\pi}}$$

$$d_1 = 2.64 \text{ cm}$$

$$d_2 = 15.7 \text{ cm}$$

Now let's calculate \dot{m} as,

$$\dot{m} = \frac{A_1 P_1 k}{\sqrt{kRT_1}} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}$$

$$\dot{m} = \frac{5.475 \times 1 \times (0.5886) \times 10^{-4} \times 10^6}{995.425}$$

$$\dot{m} = 0.319 \text{ kg/s}$$

$$I_{sp, vacuum} = \frac{F_{vacuum}}{\dot{m}g} = \frac{1000}{0.319 \times 9.81} \Rightarrow I_{sp, v} = 319.55 \text{ s}$$

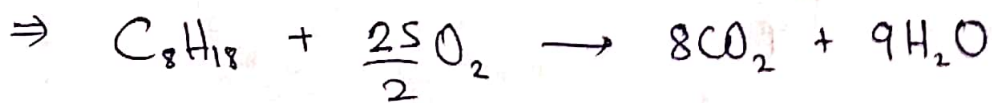
Hence, the answers are,

$$MW = 19.2 \text{ g}, \quad C^* = 1352.9 \text{ m/s}, \quad \epsilon = 35.596,$$

$$V_2 = 2374.8 \text{ m/s}, \quad \dot{m} = 0.319 \text{ kg/s}, \quad d_1 = 2.64 \text{ cm}, \quad d_2 = 15.7 \text{ cm}$$

$$I_{sp, vacuum} = 319.55 \text{ s}$$

Ans 4) Reaction \Rightarrow Between gaseous kerosene ($C_{12}H_{26}$)
and oxygen at $\phi = 1$ (complete combustion)



$$MR = \frac{\frac{25}{2} \times 32}{12 \times 44 + 13 \times 18} = \frac{400}{114} = \frac{200}{57}$$

Hence, $\frac{\dot{m}_{O_2}}{\dot{m}_{fuel}} = \frac{200}{57} \Rightarrow$ We will use this later.

For the combustion, $\Delta H_{reactants} = \Delta H_{products}$, Hence

$$1 \times (-249.95) = 12(-393.978) + 12(T_{ad} - 298) 60.43 \times 10^3 \\ + 13(-241.997) + 13(T_{ad} - 298) 53.9 \times 10^3$$

$$5079.874 \times 10^3 = 968.54 (T_{ad} - 298)$$

$$T_{ad} - 298 = 5244.88$$

$$T_{ad} = 5542.88 \text{ K}$$

Due to cooling processes, T_{flame} drops to 80% of T_{ad}

$$\Rightarrow T_{flame} = 0.8 T_{ad}$$

$$T_{flame} = 4434.3 \text{ K}$$

Now we will use this value as T_i for further calculations

Given, $F_{\text{vacuum}} = 100 \times 10^3 \text{ N}$, $P_1 = 6 \text{ MPa}$, $P_2 = 30 \text{ KPa}$

and $k = 1.22$

Now before all these, let's calculate MW

$$\text{MW} = \frac{8(44) + 9(18)}{17}$$

$$\text{MW} = 30.235 \text{ g} \Rightarrow R = \frac{8314}{30.235}$$

$$\Rightarrow R = 274.977 \text{ J/kg-K}$$

$$R \approx 275 \text{ J/kg-K}$$

$$C^* = \frac{\sqrt{kRT_1}}{k \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} = \frac{1219.71}{1.22 \times 0.5906}$$

$$C^* = 1692.8 \text{ m/s}$$

ϵ can be found as,

$$\frac{A_1}{A_2} \cdot \frac{1}{\epsilon} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}} \sqrt{\frac{k+1}{k-1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$\frac{1}{\epsilon} = (1.607)(0.013)(2.492)$$

$$\frac{1}{\epsilon} = 0.052 \Rightarrow \epsilon = 19.21$$

Now considering Thrust (vacuum),

$$F = P_1 A_t \sqrt{\frac{2k^2}{k-1} \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} + P_2 A_t$$

$$10^5 = A_t \left[6 \times 10^6 (1.704) + 30 \times 10^3 (19.21) \right]$$

$$10^5 = A_t \times 10.8 \times 10^6$$

$$A_t = 9.259 \times 10^{-3} \text{ m}^2$$

$$\dot{m} = \frac{P_1 A_t k}{\sqrt{k R T_1}} \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} = \frac{P_1 A_t}{C^*}$$

$$\dot{m} = \frac{6 \times 10^6 \times 9.259 \times 10^{-3}}{1692.8}$$

$$\dot{m} = 32.818 \text{ kg/s}$$

Now, $\dot{m}_{ox} + \dot{m}_{fuel} = \dot{m}$

$$\& \frac{\dot{m}_{ox}}{\dot{m}_{fuel}} = \frac{200}{57}$$

$$\Rightarrow \dot{m}_{ox} = \frac{200}{257} \dot{m} ; \dot{m}_{fuel} = \frac{57}{257} \dot{m}$$

$$\dot{m}_{ox} = 25.539 \text{ kg/s} ; \dot{m}_{fuel} = 7.2787 \text{ kg/s}$$

$$I_{sp, vacuum} = \frac{F_{vacuum}}{\dot{m}g}$$

$$= \frac{100 \times 10^3}{32818 \times 9.81}$$

$$I_{sp} = 310.612 \text{ sec}$$

Answers:-

$$C^* = 1692.8 \text{ m/s}$$

$$\epsilon = 19.12$$

$$\dot{m}_{ox} = 25.539 \text{ kg/s} ; \dot{m}_{fuel} = 7.2787$$

$$I_{sp} = 310.612 \text{ sec}$$