AE 330 Rocket Propulsion Vehicle Trajectory

Kowsik Bodi

Aerospace Engineering, IIT Bombay

Vertically Launched Rocket

$$M\frac{dV}{dt} = \dot{m}u_{eq} - Mg - D$$

$$dV = -u_{eq}\frac{dM}{M} - gdt - \frac{D}{M}dt$$

$$V_b \equiv \Delta V = \underbrace{u_{eq}\log\left(\frac{M_o}{M_f}\right)}_{\Delta V_{id}} - \underbrace{g_o t_b}_{\Delta V_g} - \underbrace{\int\limits_0^{t_b}\frac{D}{M}dt}_{\Delta V_D}$$

Tsiolkovsky Equation

Acceleration

Lift-off: $\eta_o = \dfrac{\mathcal{T}}{M_o g_o}$

Burn-out: $\eta_{max} = \dfrac{\mathcal{T}}{M_f g_o} \equiv \dfrac{1}{1-K} \eta_o$
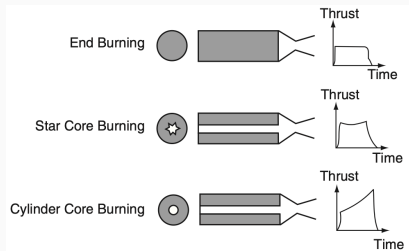
where, $K = M_p/M_o$

image from Rogers

Acceleration

Lift-off: $\eta_o = \dfrac{\mathcal{T}}{M_o g_o}$

Burn-out: $\eta_{max} = \dfrac{\mathcal{T}}{M_f g_o} \equiv \dfrac{1}{1-K}\eta_o$
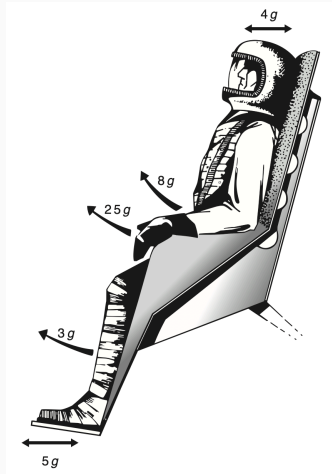
where, $K = M_p/M_o$

image from Rogers

Accelerations and Burn-times

	t_b	η_{max}
Large SLV (Liquid Rocket Engine)	2 - 8 min	1.2 - 6
Strap-on Booster	0.5 - 2 min	1.2 - 3
SAM or Anti-Aircraft or Anti-Missile Missile	2 - 75 sec	5 - 20 (can go upto 100)
Spacecraft Orbit Maneuvres (or Maintenance)	<10 min (cumulative)	0.2 - 6 (upto 0.1 for large
Air-launched guided missile	2 - 5 sec (booster) 10 - 30 sec (sustainer)	upto 25
Rocket-assisted projectiles (gun launched)	a few seconds	upto 20,000 (in the gun barrel)

ΔV vs Propellant fraction
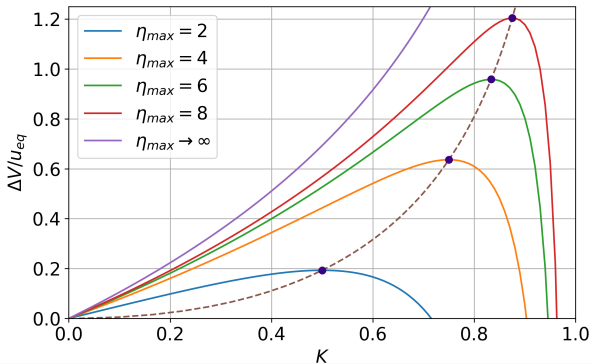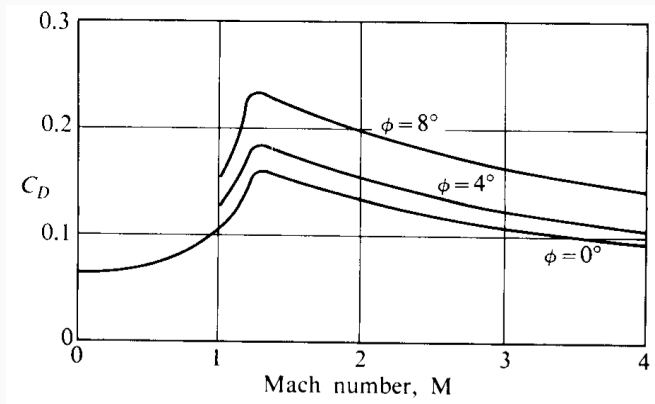
Drag

Drag Coefficient C_D

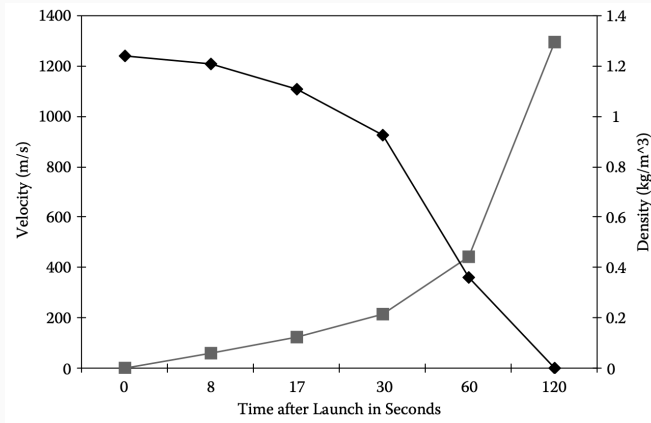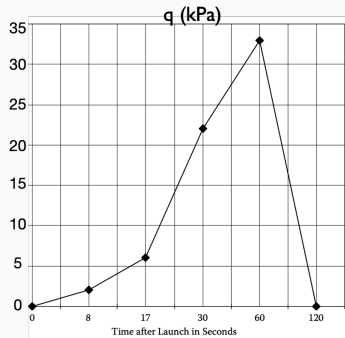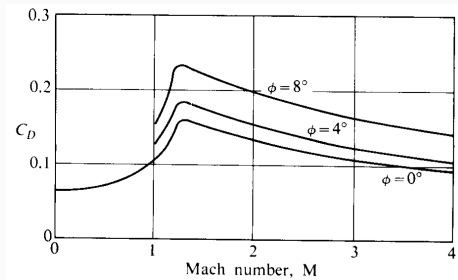image from Hill & Peterson

Dynamic Pressure (q)

image from Taylor
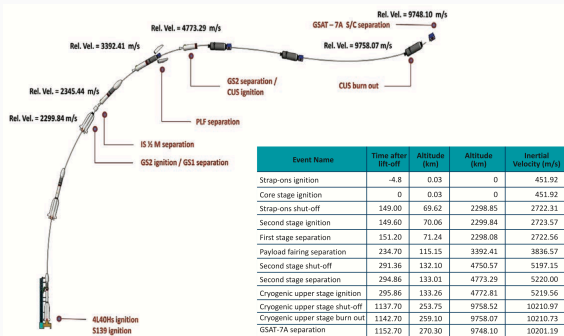
C_D **and** q

images from Hill & Peterson andTaylor

Launch Vehicle Trajectory

Payload destination

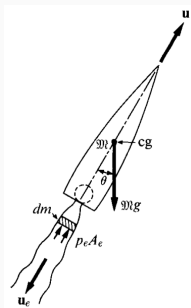| Event Name	Time after lift-off	Altitude (km)	Altitude (km)	Inertial Velocity (m/s)
Strap-ons ignition	-4.8	0.03	0	451.92
Core stage ignition	0	0.03	0	451.92
Strap-ons shut-off	149.00	69.62	2298.85	2722.31
Second stage ignition	149.60	70.06	2299.84	2723.57
First stage separation	151.20	71.24	2298.08	2722.56
Payload fairing separation	234.70	115.15	3392.41	3836.57
Second stage shut-off	291.36	132.10	4750.57	5197.15
Second stage separation	294.86	133.01	4773.29	5220.00
Cryogenic upper stage ignition	295.86	133.26	4772.81	5219.56
Cryogenic upper stage shut-off	1137.70	253.75	9758.52	10210.97
Cryogenic upper stage burn out	1142.70	259.10	9758.07	10210.73
GSAT-7A separation	1152.70	270.30	9748.10	10201.19

images from Heister and ISRO

(a) Velocity coordinates

(b) Position coordinates
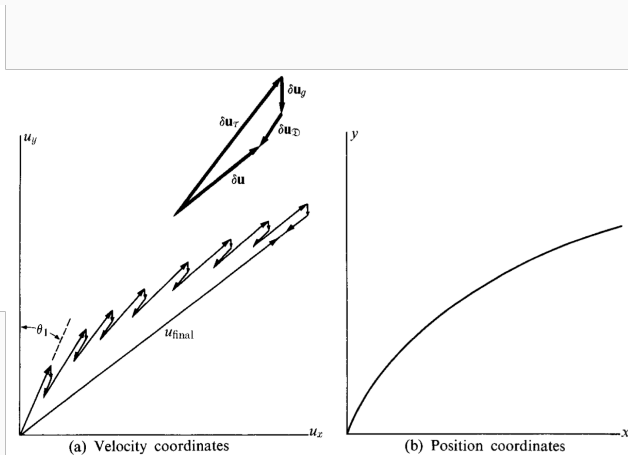
images from Hill & Peterson

Stability

Movable fins

Gimbaled thrust

Vernier rockets

Thrust vane

image from Taylor

12