AE 330/708

AEROSPACE PROPULSION

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Multistage rockets - terminologies

Total mass of rocket stage = Summation of various contributing masses

Primary distinction of systems of importance are:

M_L = Mass of payload

M_P = Mass of propellant

M_S = Structural mass (includes engine hardware, guidance instrument, supporting structure, vehicle frame, tanks, etc)

Initial vehicle mass, $M_O = M_L + M_P + M_S$

Similarly burnout mass, $M_b = M_L + M_S$

Mass ratio $= R = \dfrac{M_o}{M_b} = \dfrac{M_o}{M_L + M_s}$

Payload ratio $= \lambda = \dfrac{M_L}{M_o - M_L} = \dfrac{M_L}{M_p + M_s}$

Structural coefficient, $= \varepsilon = \dfrac{M_s}{M_p + M_s} = \dfrac{M_b - M_L}{M_o - M_L}$

$$\Rightarrow \boxed{R = \frac{1+\lambda}{\varepsilon + \lambda}}$$

For stage $i \Rightarrow \quad \lambda_i = \dfrac{M_{o(i+1)}}{M_{o(i)} - M_{o(i+1)}} \quad ; \quad \varepsilon_i = \dfrac{M_{s_i}}{M_{o(i)} - M_{o(i+1)}}$

$$R_i = \frac{M_{o_i}}{M_{b_i}} \quad \text{and} \quad R_i = \frac{1+\lambda_i}{\varepsilon_i + \lambda_i}$$

Multistaging of rocket vehicles

Propellant mass – significantly higher than payload mass. So discarding propellant tank and structures is necessary, otherwise lot of energy will be wasted in accelerating dead mass.

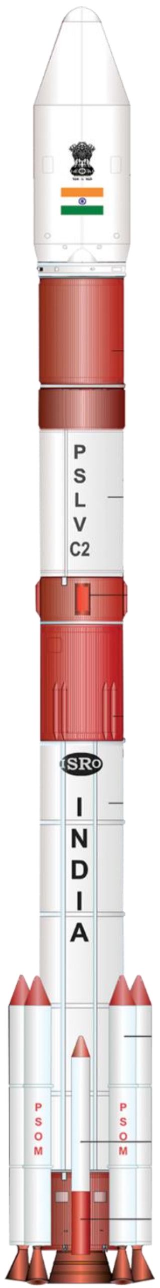
Engines large enough to accelerate initial vehicle mass produces heavy stress once the propellant is near consumption. It is not a good idea to operate the engine at reduced thrust.

Hence, mulistaging of vehicles is adopted.

Series of individual vehicles stacked over one another. Each separate stage is a complete rocket in itself having its on propellant, structure and payload mass.

Stages are numbered in the order of their firing

Polar Satellite Launch Vehicle (PSLV)



Payload

Forth stage

Third stage

Second stage

Strap on boosters + first stage

Case study – single stage to multistage vehicle

Consider a single stage vehicle with following details:
Effective exhaust velocity, c = 3048 m/s
Total initial mass, Mo = 15000 kg
Payload mass, Ml = 1000 kg
Structural mass, Ms = 2000 kg

Configure a two stage vehicle with same initial mass and structural mass that is capable of launching same payload with similar engines.

With given data for single stage vehicle, $\varepsilon = 0.143$ and $\lambda = 0.0714$

In drag-free and gravity-free environment, the payload velocity, $u = c \ln (R) = c \ln [(1+ \lambda)/(\varepsilon + \lambda)]$

We get, the final payload velocity, $u = 4904$ m/s

Now consider a two-stage vehicle with same initial mass, total structural mass, payload and same engines.

Further assume that the payload ratios and the structural coefficients are same for both the stages (for simplicity and gaining the insight into the problem)

For $\lambda 1 = \lambda 2$, we get Mo2 = 3873 kg and $\lambda = 0.348$

Similarly by imposing $\varepsilon 1 = \varepsilon 2$ and Ms1 + Ms2 = 2000 kg, we get Ms1 = 1589 kg, Ms2 = 411 kg, and $\varepsilon = 0.143$

Now the final payload velocity for two-stage vehicle will become, $u = 2\, c \ln [(1+ \lambda)/(\varepsilon + \lambda)] = 6160$ m/s

	Single stage	Stage 1	Stage 2
Mo (kg)	15000	15000	3873
Payload (kg)	1000	3873	1000
Structure (kg)	2000	1589	411
ε	0.143	0.143	0.143
λ	0.0714	0.348	0.348
R	5	2.75	2.75
u (m/s)	4904	3080	3080
Total u (m/s)	4904	6160	

Velocity increment provide by n-stage rocket vehicle

n- stage rocket : $u_n = \sum c_i \ln R_i$

If exhaust velocities are all same $(c_1, c_2, \dots = c)$

$$\Rightarrow \quad u_n = c \sum \ln R_i \quad = \quad c \ln \left[\prod R_i \right]$$

If structural coefficients and payload ratios of all stages are identical (i.e. identical stages assuption):

$$u_n = n\, c \ln R \quad = \quad n\, c \ln \left(\frac{1+\lambda}{\varepsilon+\lambda} \right)$$

Velocity increment provide by n-stage rocket vehicle

$$\frac{M_{o1}}{M_{o2}} = \frac{1+\lambda_1}{\lambda_1} \; ; \quad \frac{M_{o2}}{M_{o3}} = \frac{1+\lambda_2}{\lambda_2} \; ; \quad \ldots\ldots \quad \frac{M_{on}}{M_L} = \frac{1+\lambda_n}{\lambda_n}$$

Multiplying, $\displaystyle \frac{M_{o1}}{M_L} = \prod_{i=1}^{n} \frac{1+\lambda_i}{\lambda_i}$

for identical payload ratios (i.e. $\lambda_i = \lambda$) $\Rightarrow$ $\displaystyle \frac{M_{o1}}{M_L} = \left[\frac{1+\lambda}{\lambda}\right]^n$

Taking the value of λ in terms of $(M_{o1}/M_L) \Rightarrow$

$$\frac{u_n}{c} = n \ln \left\{ \frac{(M_{o1}/M_L)^{1/n}}{\varepsilon\left[(M_{o1}/M_L)^{1/n} - 1\right] + 1} \right\}$$

Mo1/ML ==> Total mass of vehicle per unit mass of payload

Velocity increment provide by n-stage rocket vehicle

As $n \to \infty$ (infinite number of stages) $\Rightarrow$

$$\lim_{n\to\infty} u_n = c \lim_{n\to\infty} \left\{ -n \ln \left[\varepsilon + \frac{1-\varepsilon}{(M_{01}/M_L)^{1/n}} \right] \right\}$$

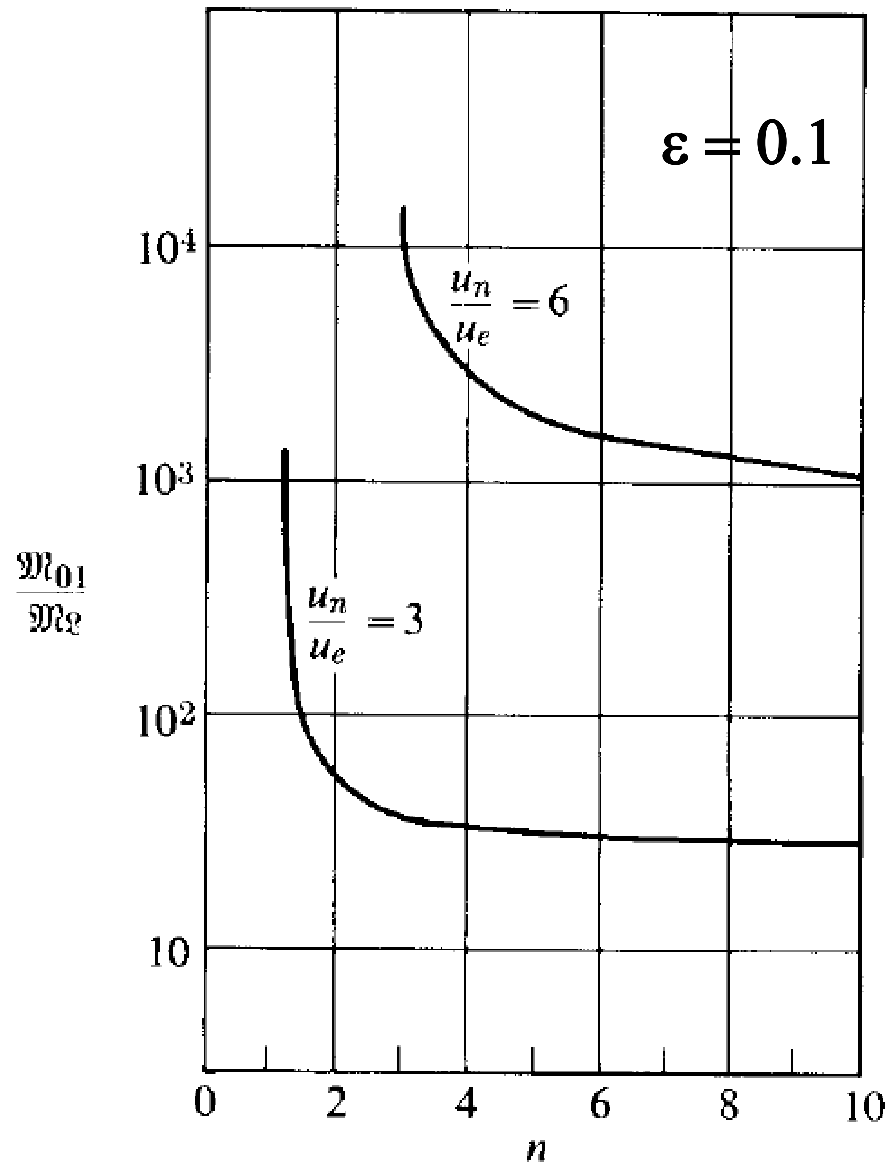
$$\boxed{\lim_{n\to\infty} u_n = c\,(1-\varepsilon) \ln \left(\frac{M_{01}}{M_L} \right)}$$

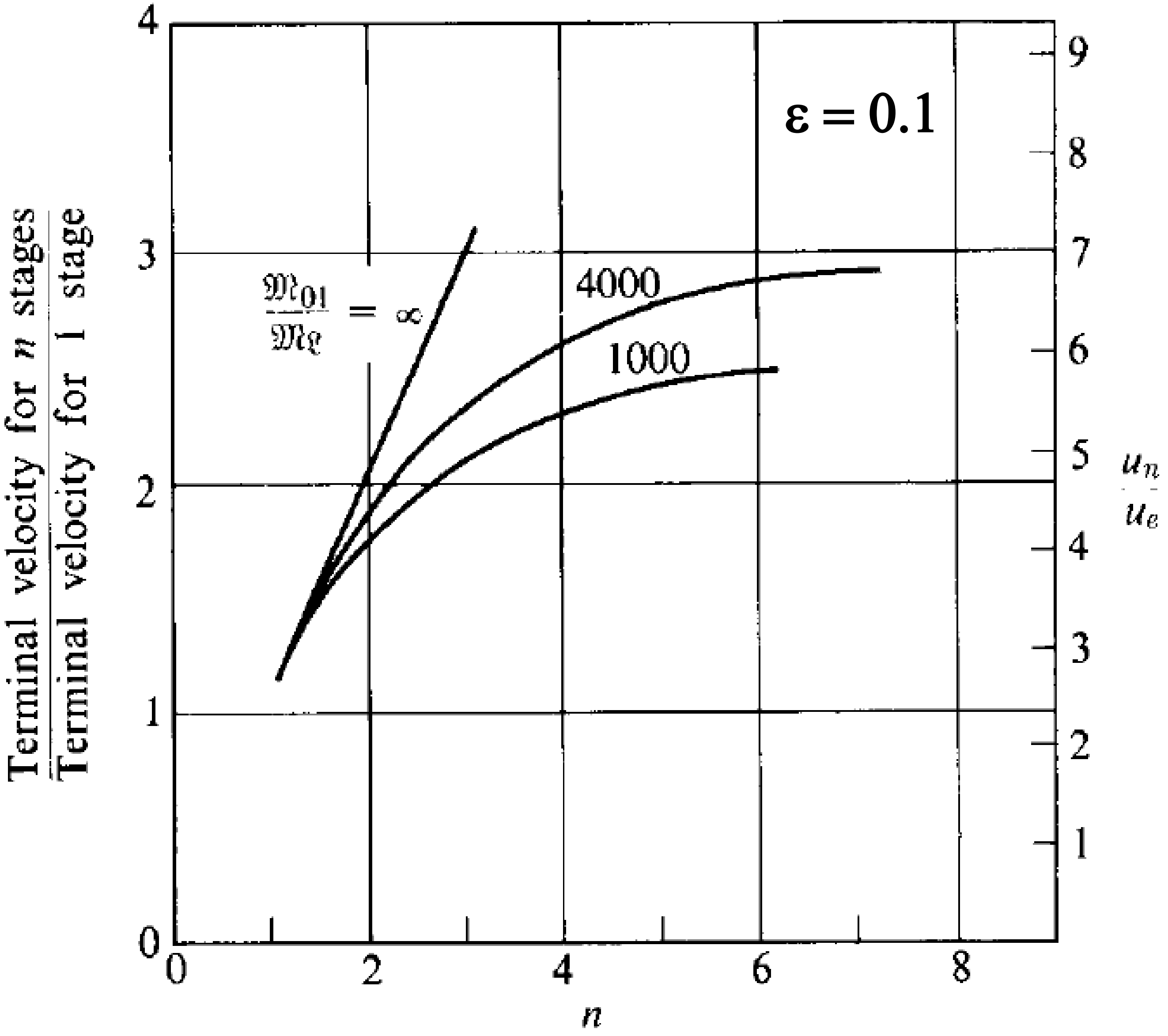
Max. possible u_n when $\varepsilon \to 0$ i.e. ideal massless structure.

But for $\varepsilon = 0$ (or mass-less structure), there is no advantage of multistaging.

If we consider the case study from previous module, it is easy to verify that as n becomes infinity, the payload velocity will be 7074 m/s.

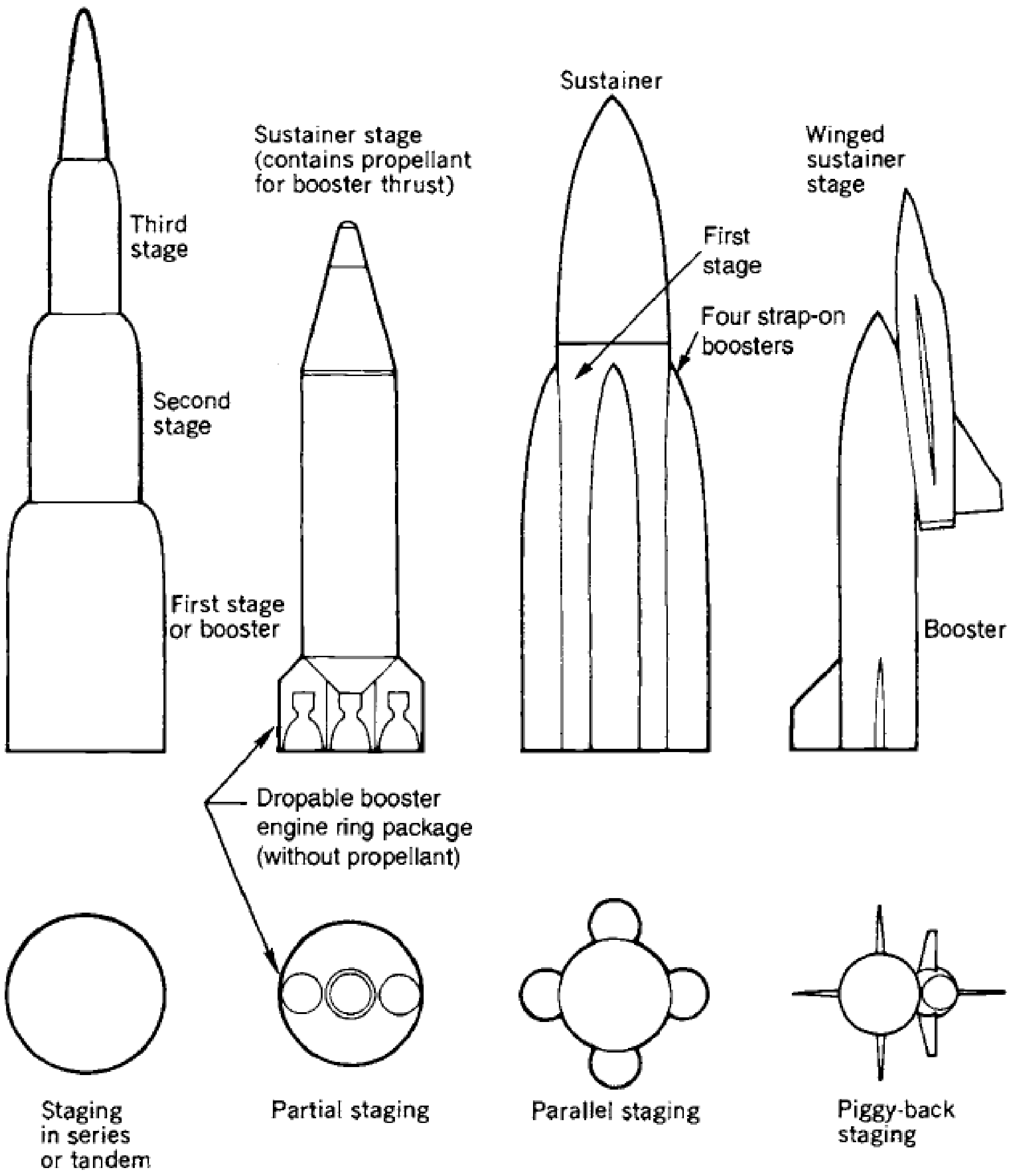
Velocity increment provide by n-stage rocket vehicle





Details of stages used in Apollo mission

Mass and thrust features	Stage I	Stage II	Stage III
Engine	F-1	J-2	J-2
Propellants	RP1-LOx	LH2-LOx	LH2-LOx
Number of engines	5	5	1
Total thrust, kN	33,400	4,450	890
M_0, kg	2,780,000	677,000	215,000
Mp, kg	1,997,000	429,000	109,000
ε	0.05	0.071	0.191
λ	0.321	0.446	0.603



Various multistaging configurations

Staging in series or tandem

Partial staging

Parallel staging

Piggy-back staging