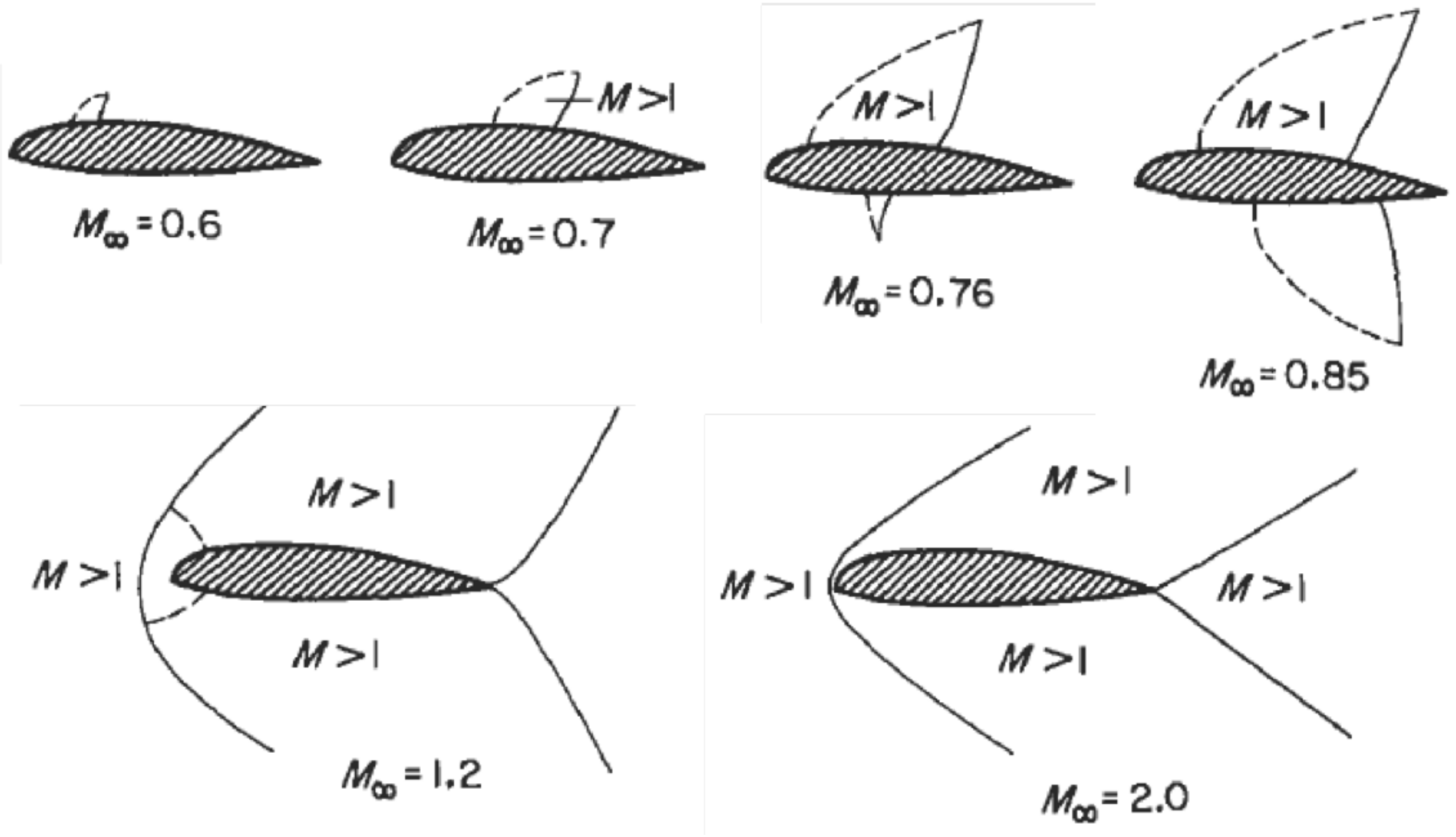
Transonic Aerodynamics – Mitigating Compressibility Effects

Aniruddha Sinha

Progression of flow characteristics with M_∞

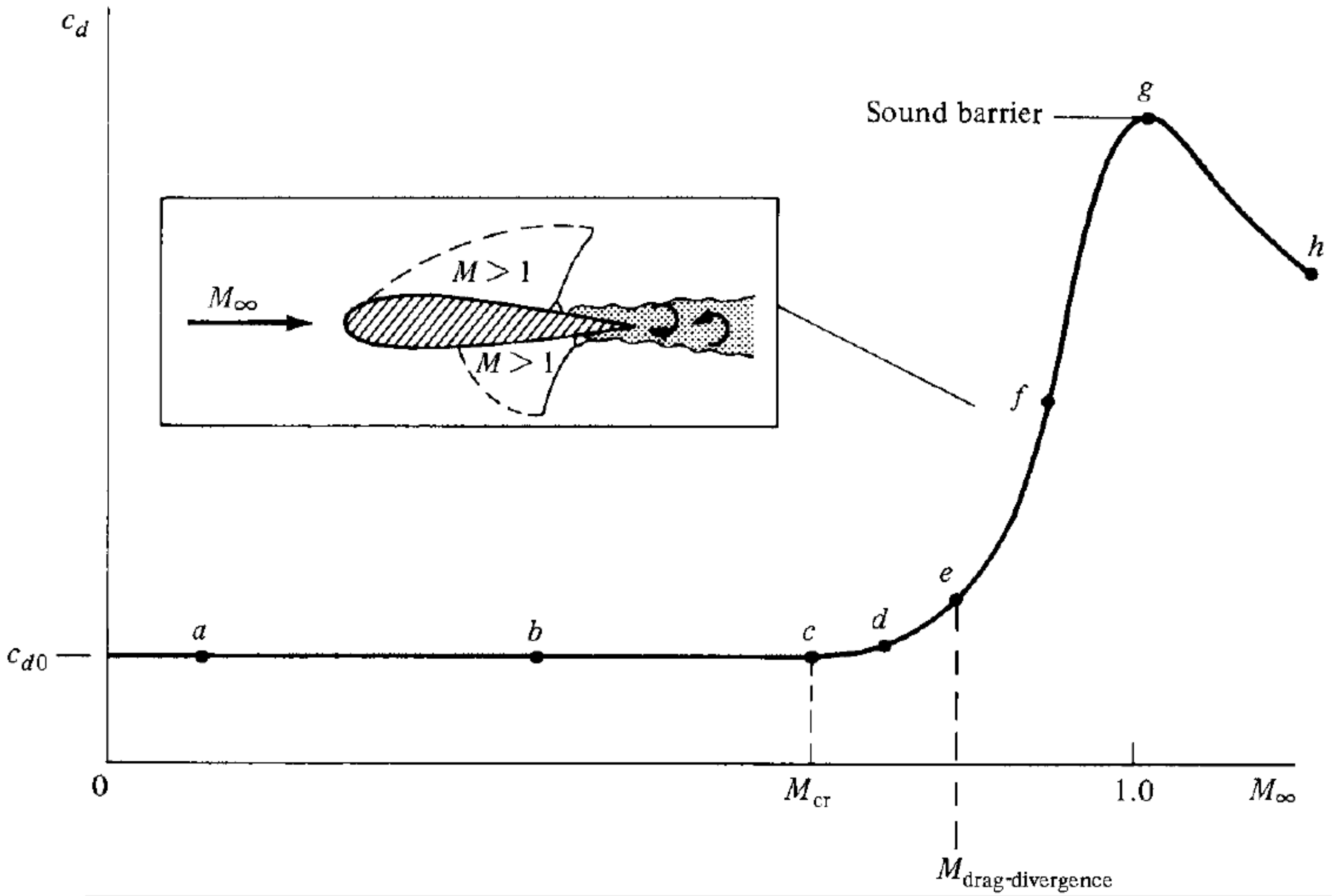


Houghton et al., 2013

Drag-divergence Mach number

Mach number at which drag increases significantly

- Depends on geometry
- Depends on AoA
- Static pressure increases behind the shock
- Slows down flow
  - Causes flow separation
- Called ‘wave drag’



Anderson, 2011

Drag-divergence Mach number – Quantitative

Drag-divergence Mach number

- Decreases with AoA
- Decreases with thickness

Initially, researchers extrapolated that drag will be infinite at sonic speed

- Of course this is not true
- Requires careful engineering
    - Sweep
    - Supercritical airfoil
- Requires powerful propulsion

Abbott & Doenhoff, 1959: NACA 2315

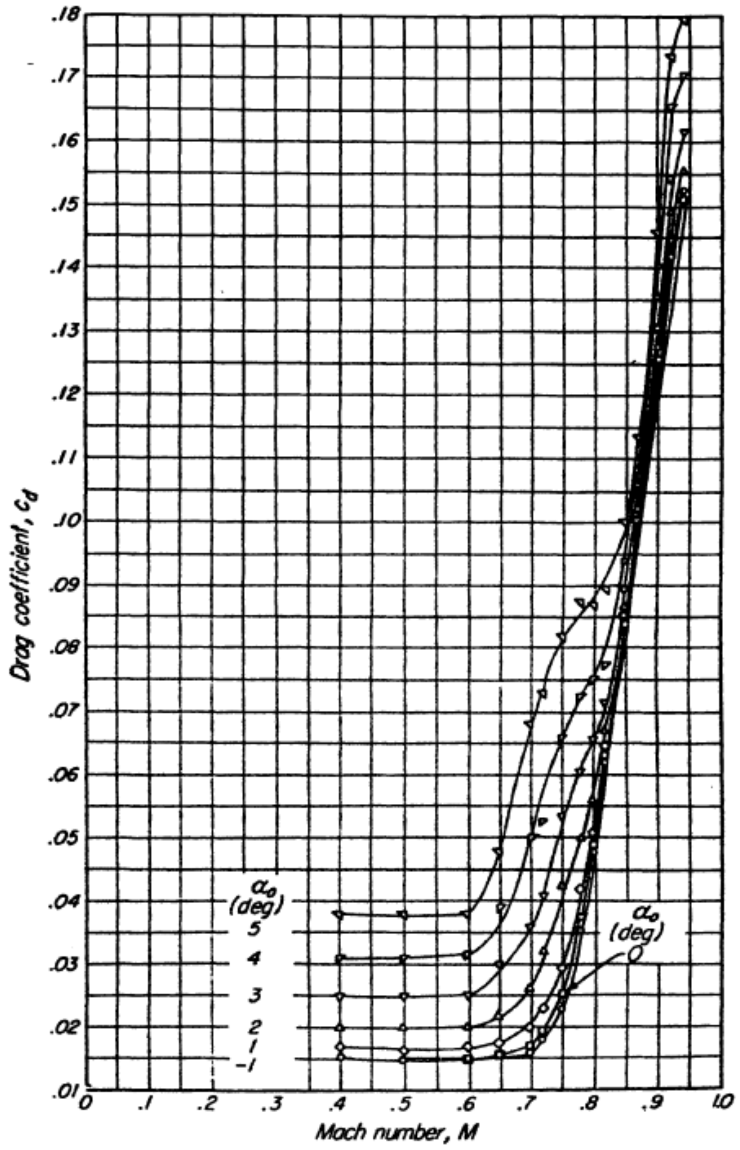
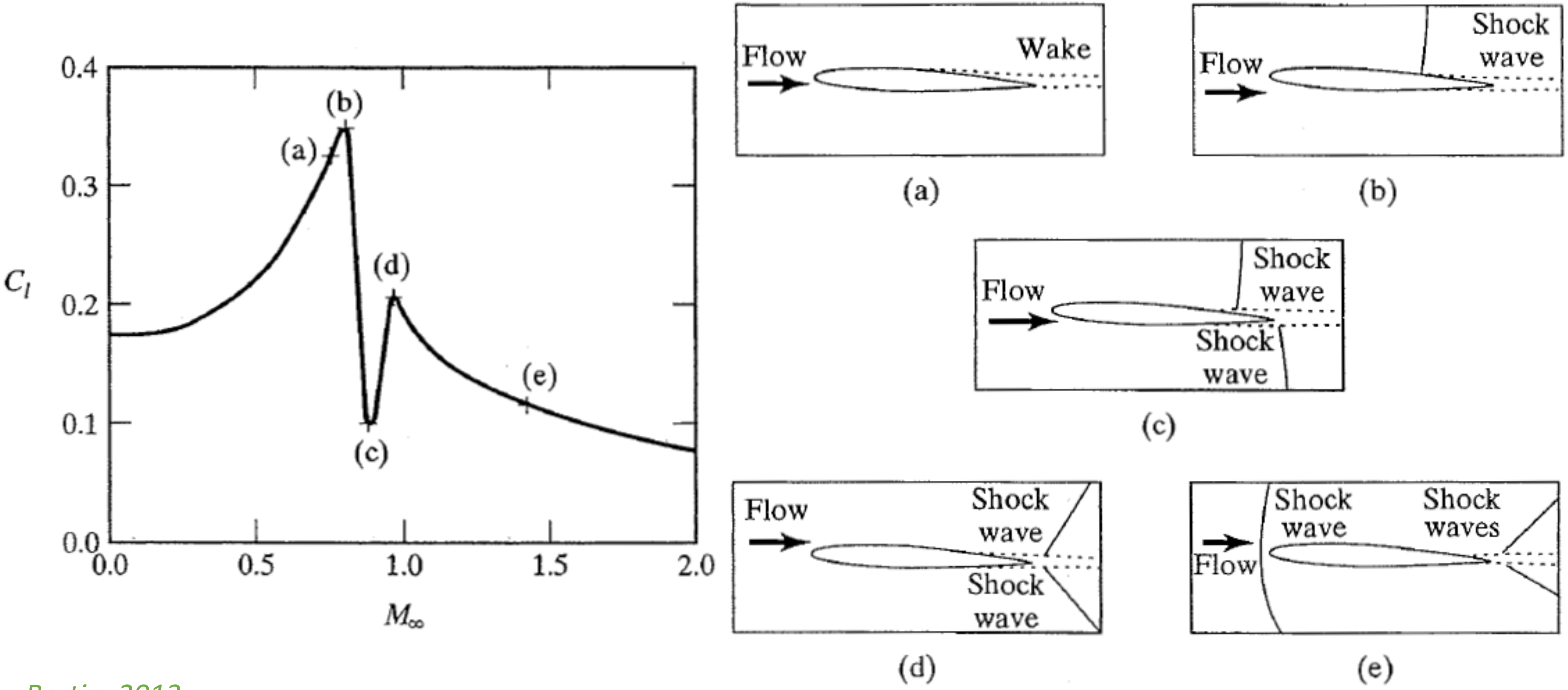


FIG. 179. Effect of compressibility on the drag of the NACA 2315 airfoil.

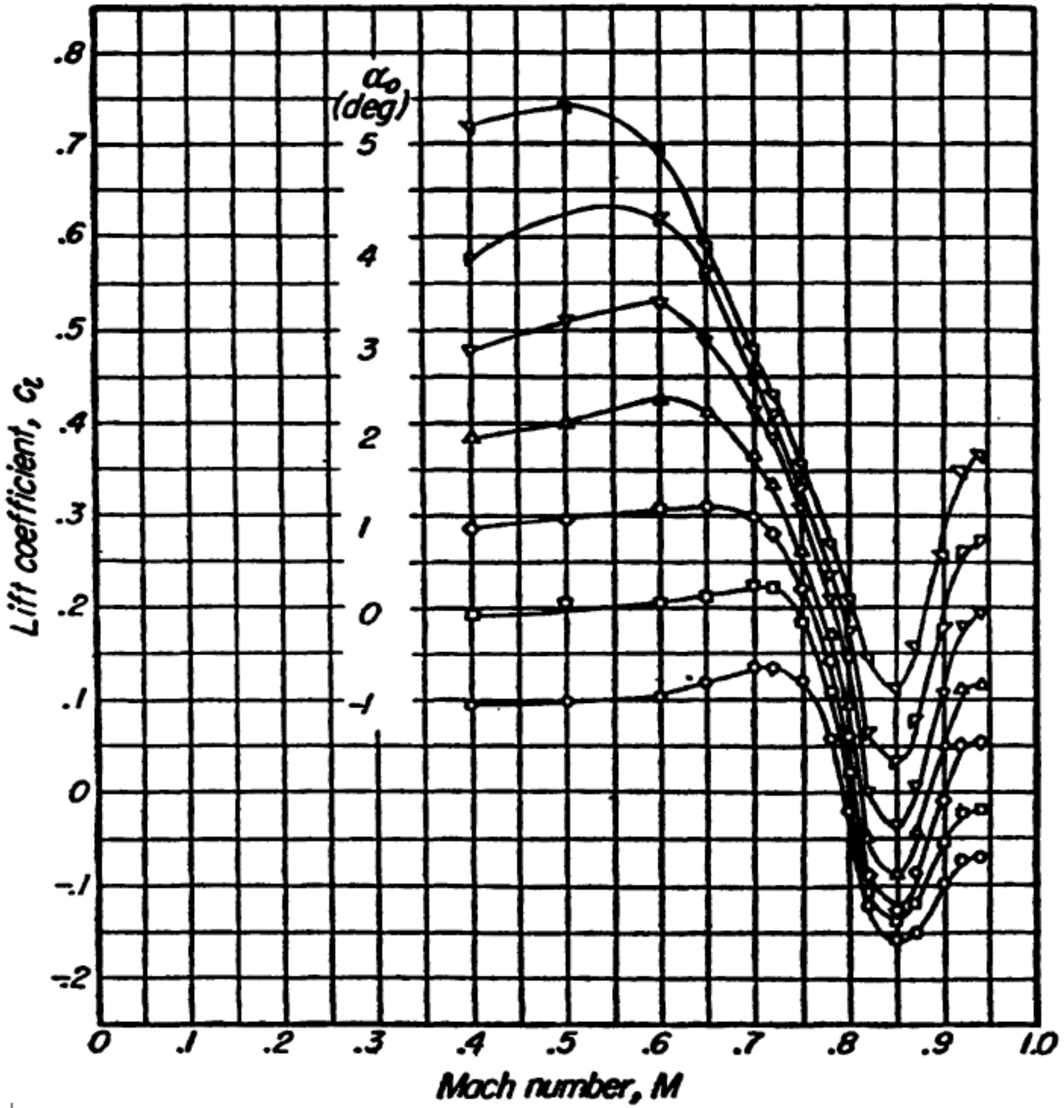
Lift vs. Mach number



Bertin, 2013

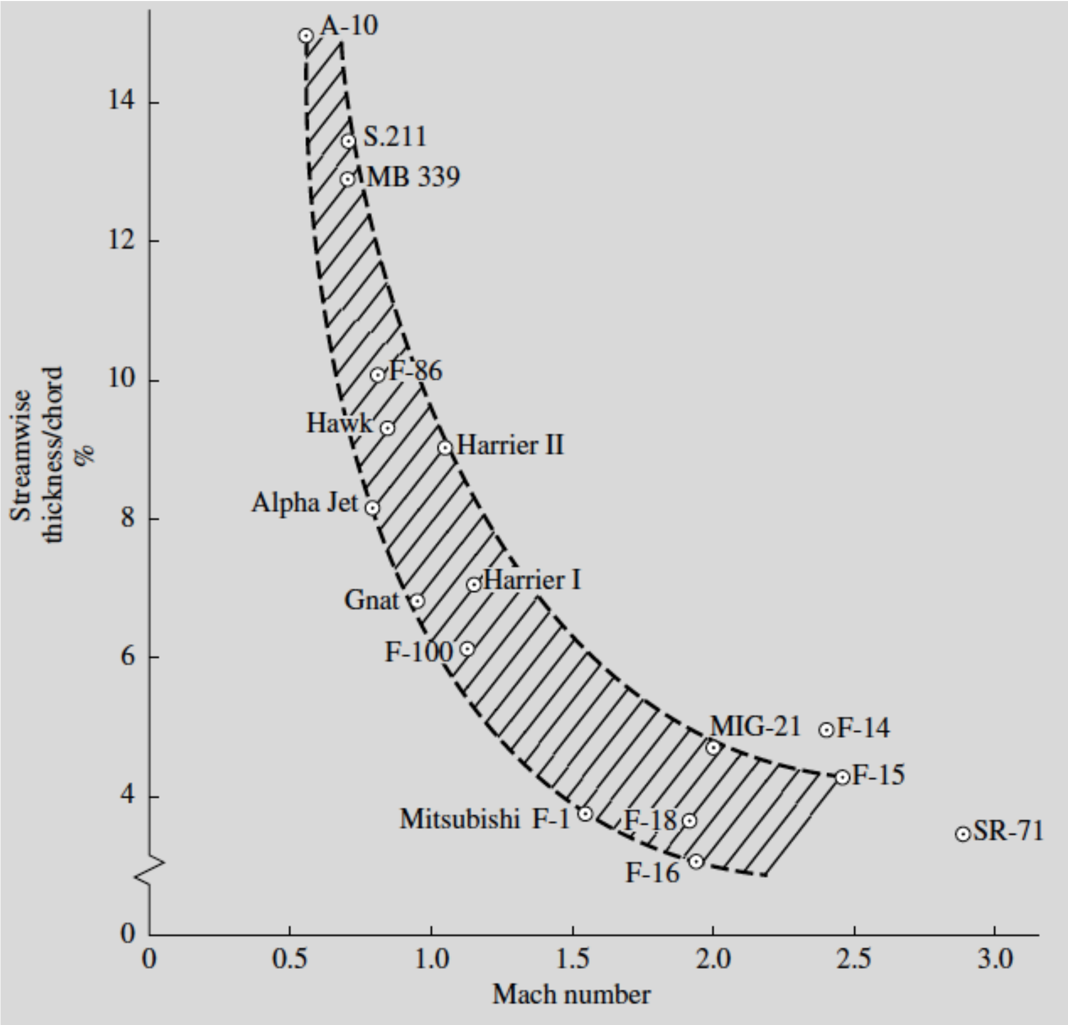
Lift vs. Mach number – Quantitative

- At low Mach nos. lift increases per Prandtl-Glauert rule
- Lift decreases precipitously as Mach number approaches drag-divergence Mach number
- Lift decrease is higher at higher AoA
- Note that lift recovers before sonic condition



Abbott & Doenhoff, 1959: NACA 2315

Variation of thickness ratio vs. design Mach no.



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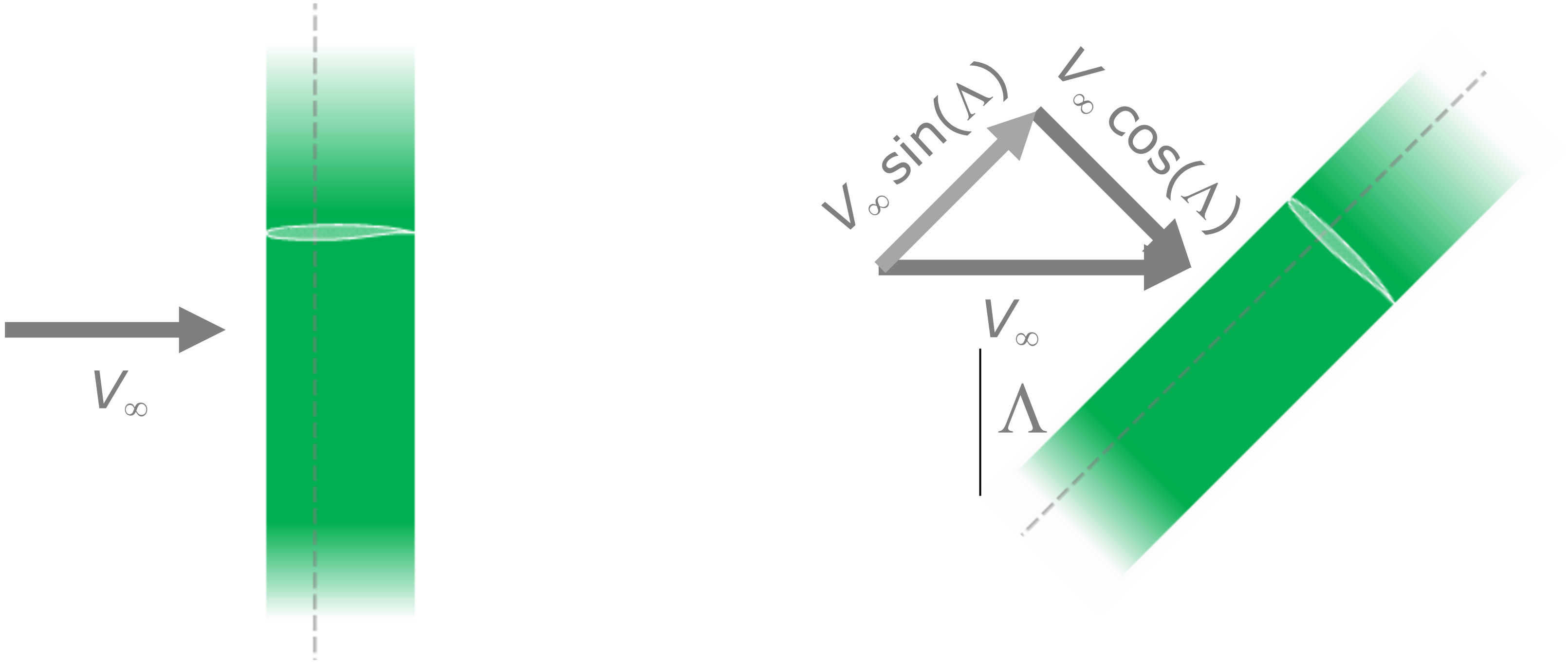
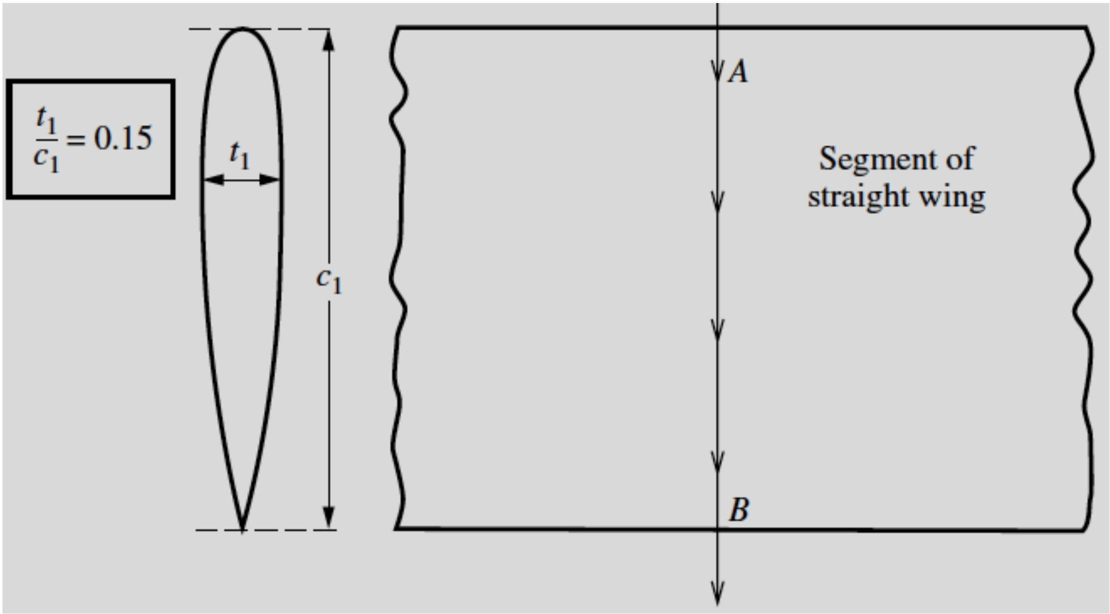
Remedy: Sweep (Adolf Busemann & R. T. Jones)

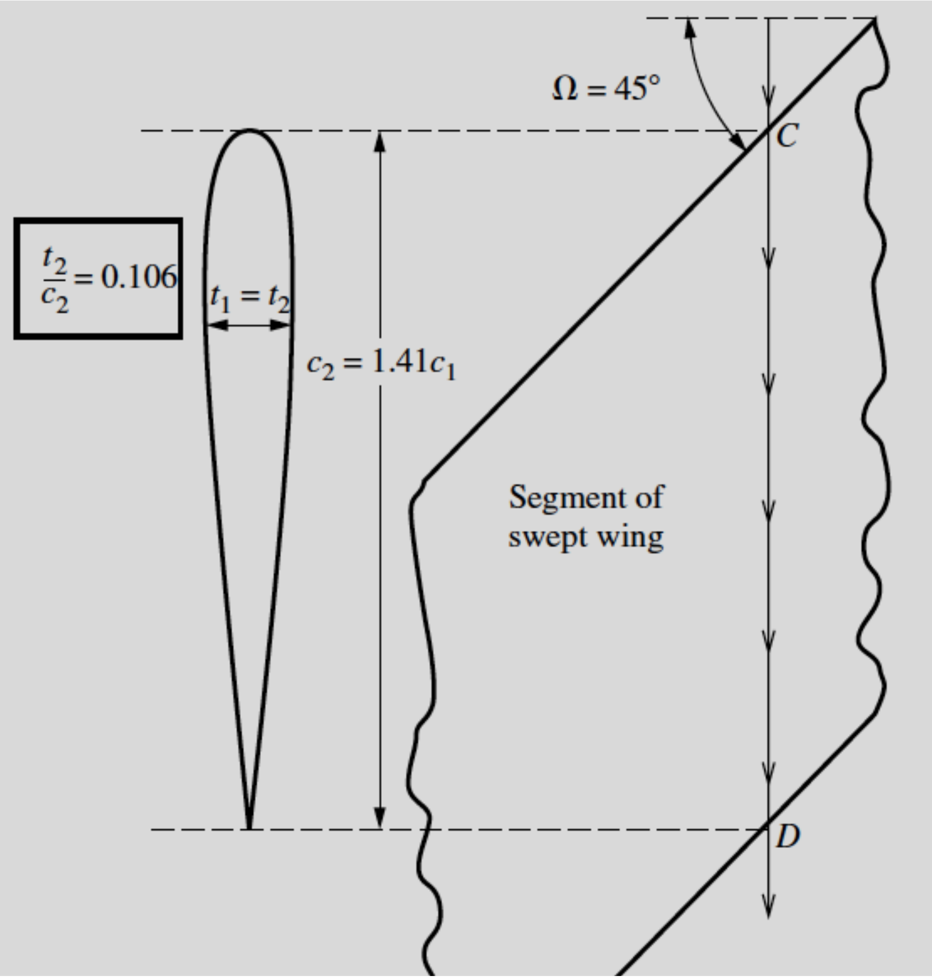
Component of freestream velocity (or Mach no.) in span-wise direction doesn't affect the pressure field



Remedy: Sweep (Adolf Busemann & R. T. Jones)

Alternatively, apparent thickness ratio can be thought to have been decreased by $\cos\Lambda$, Λ being sweep angle

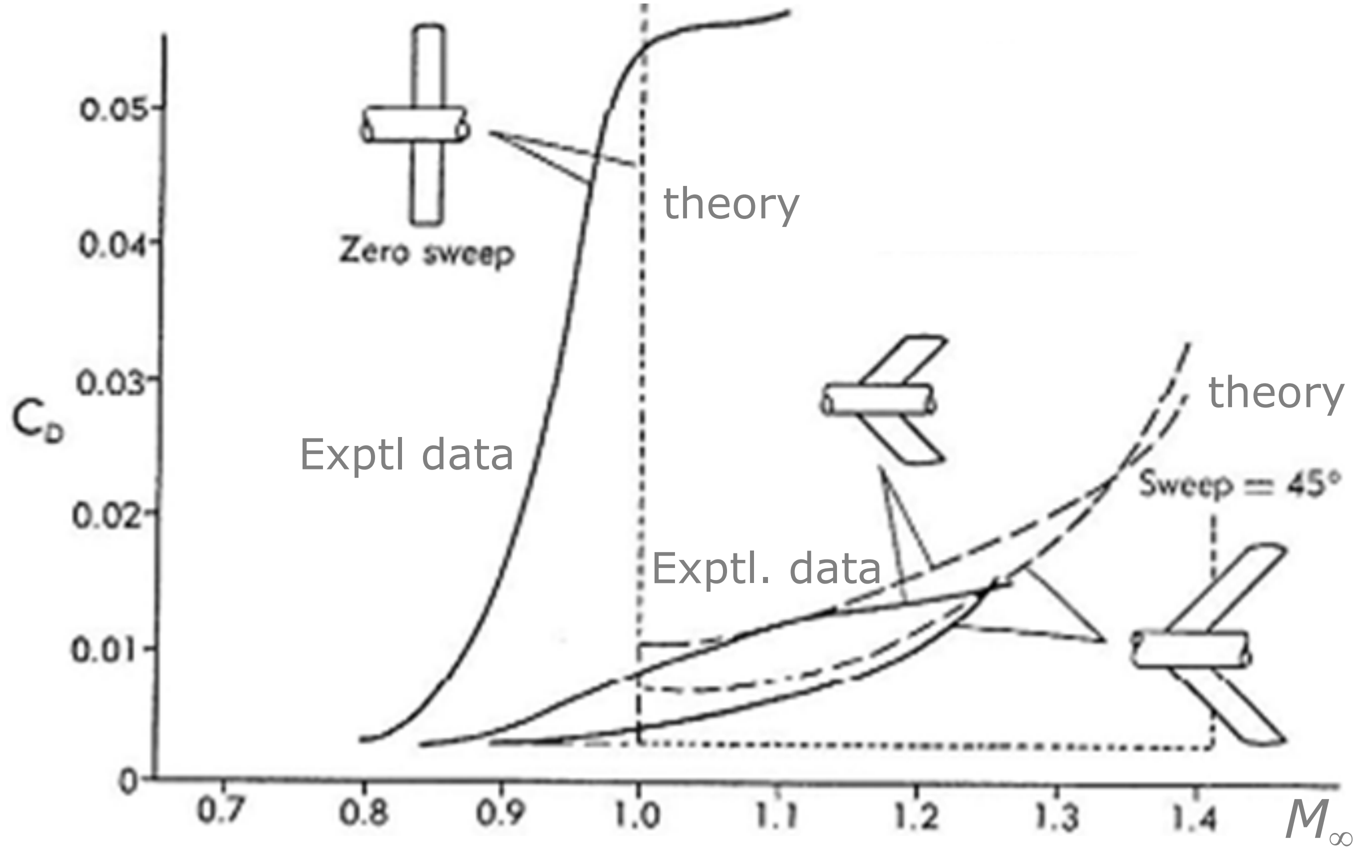




Anderson, 2011

Sweep – Effect on drag divergence

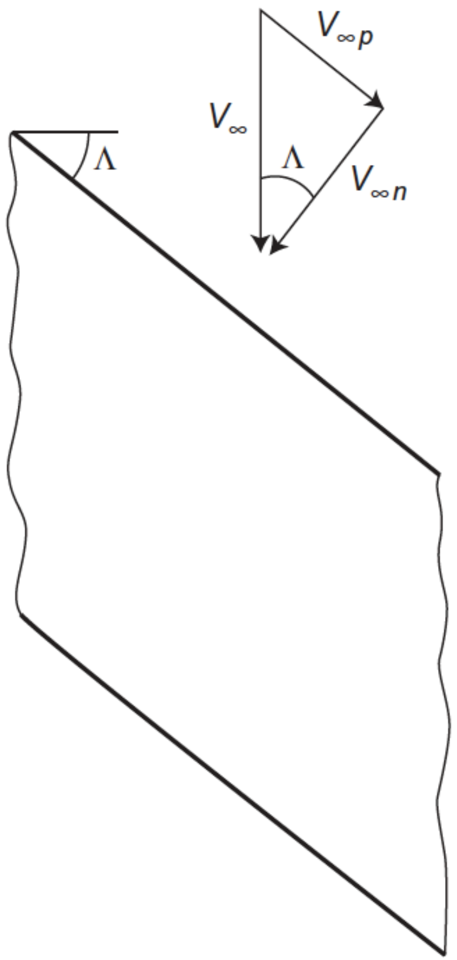
- Wave drag is drastically reduced for swept wings
  - As apparent thickness ratio is decreased
  - Or, as effective Mach no. is decreased
- Both result in smaller perturbations
- Larger aspect ratio gives greater reduction



Lift of swept wings of infinite span

Lift of swept wing of very large span can be predicted as

$$L' = \frac{1}{2}\rho(U_\infty \cos\Lambda)^2 c \left(\frac{dc_l}{d\alpha}\right)_{\text{unswept}} (\alpha_n - \alpha_{0n})$$



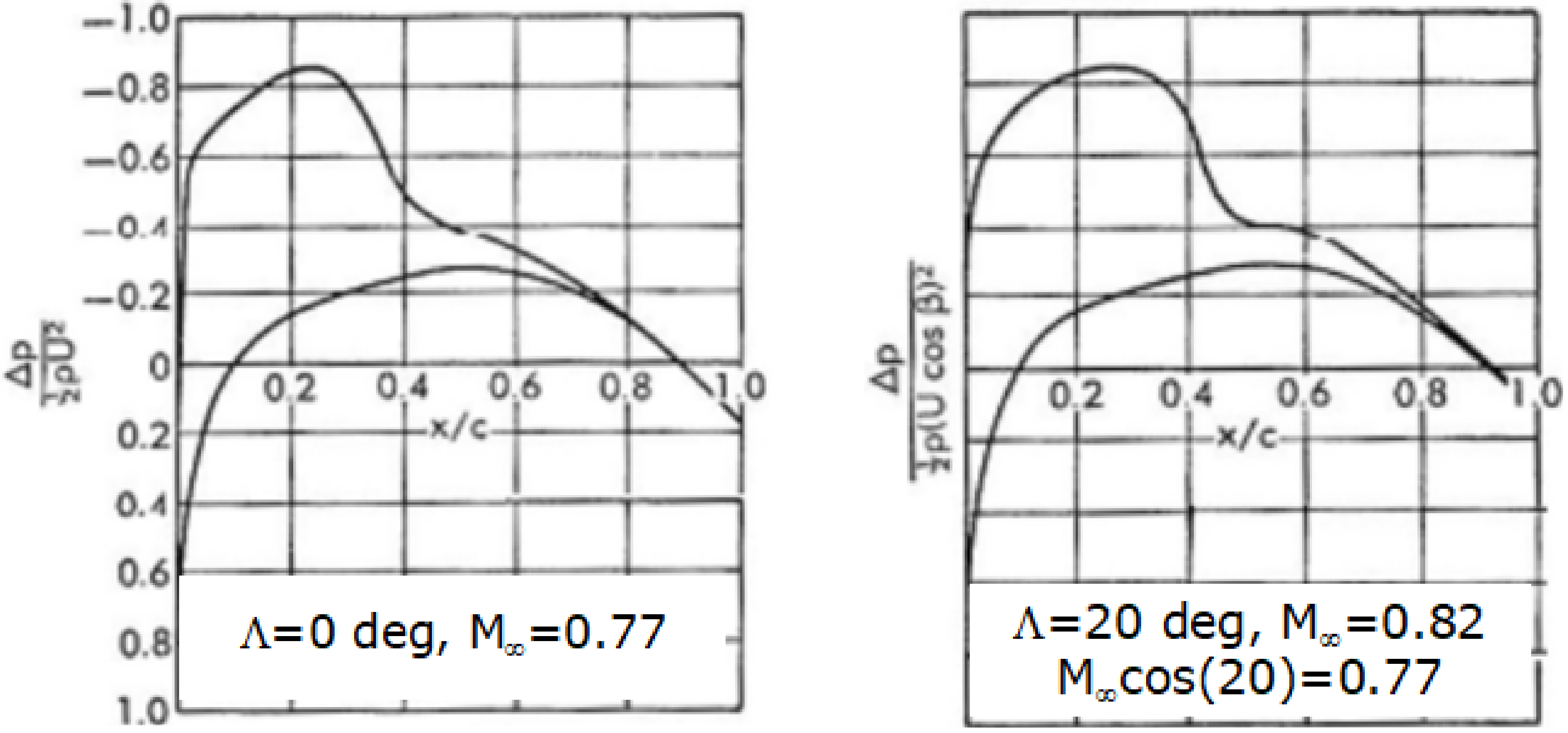
But $\alpha_n = \alpha/\cos\Lambda$ since, w.r.t. chord, vertical component of velocity remains same ($U_\infty \sin\alpha$) whereas horizontal component of normal velocity becomes ($U_\infty \cos\Lambda \cos\alpha$)

Now, $c_l = \dfrac{L}{0.5\rho U_\infty^2 S} = \left(\dfrac{dc_l}{d\alpha}\right)_{\text{unswept}} \cos^2\Lambda \left(\dfrac{\alpha}{\cos\Lambda} - \alpha_{0n}\right)$

So, $$\boxed{\frac{dc_l}{d\alpha} = \left(\frac{dc_l}{d\alpha}\right)_{\text{unswept}} \cos\Lambda}$$

Sweep – Effect on lift

Experimental evidence that swept airfoil (not wing!) provides same amount of lift at higher Mach no. as component normal to span



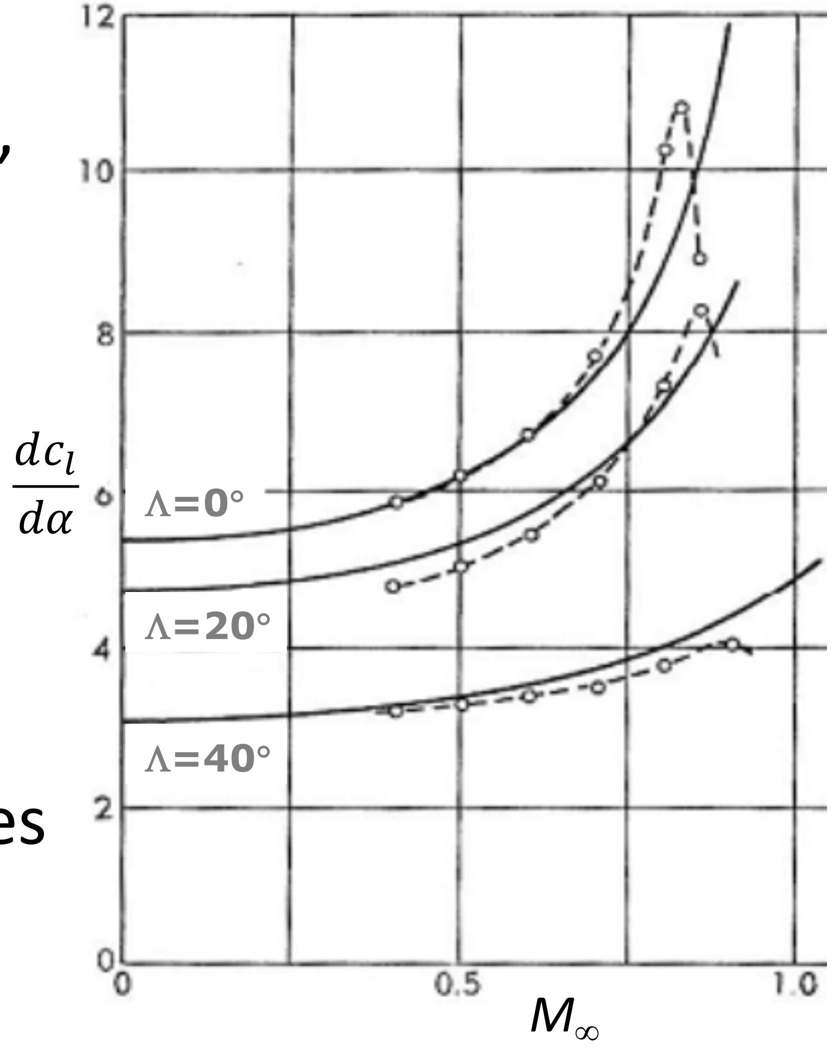
Compressibility effects in swept airfoils/wings

Compressibility corrections remain applicable in the 'normal' direction. W/ Prandtl-Glauert's rule,

$$c_l = \frac{c_l^0}{\sqrt{1 - M_{\infty n}^2}} = \frac{c_l^0}{\sqrt{1 - M_\infty^2 \cos^2 \Lambda}}$$

$$\frac{dc_l}{d\alpha} = \frac{\cos \Lambda}{\sqrt{1 - M_\infty^2 \cos^2 \Lambda}} \left(\frac{dc_l^0}{d\alpha} \right)_{\text{unswept}}$$

- Lift slope decreases as sweep increases
- But, range of operation in terms of M_∞ increases
- Behaviour complicated for wings, but similar



Sweep – Forward or backward

Foregoing discussion would suggest that benefit of sweep will be same if it is forward or backward

However, structural load, stability and handling characteristics are overall worse for forward swept wings

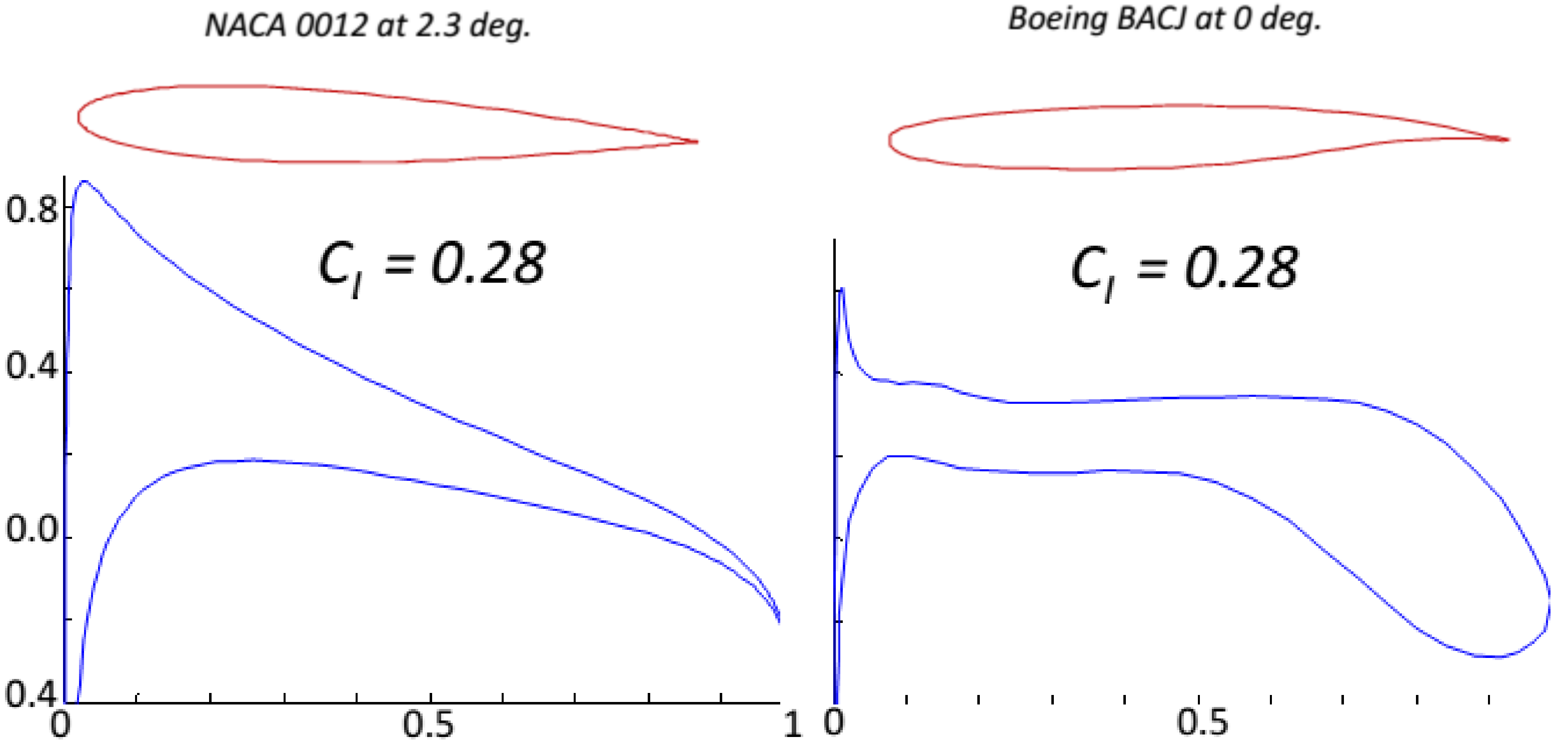
Boeing B-52 Stratofortress: Backward sweep

Grunman X-29: Forward sweep

Remedy: Supercritical airfoil (R. T. Whitcombe)

Aircraft should have high L/D, *AND* high drag-divergence Mach no.

Limit $C_{p,min}$; avoid peaky C_p distribution



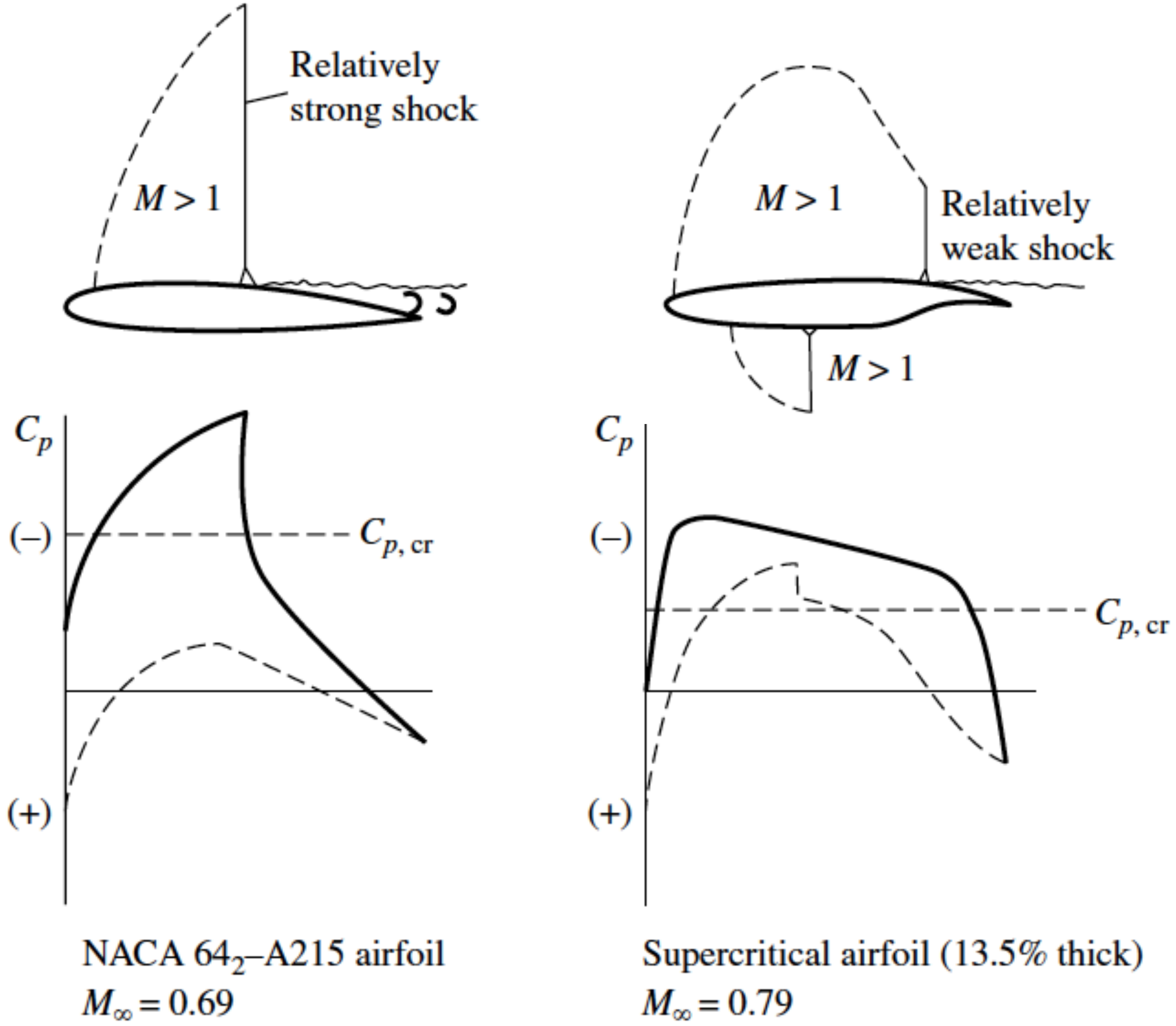
Area under c_p plot is the same

Supercritical airfoil – How it works

Idea: increase Mach no. range between M_{cr} & $M_{\text{drag-divergence}}$

Supercritical airfoil is designed with flat suction surface so that

- Region of supersonic flow is smaller
- Local supersonic Mach numbers are less
- Weaker terminating shock



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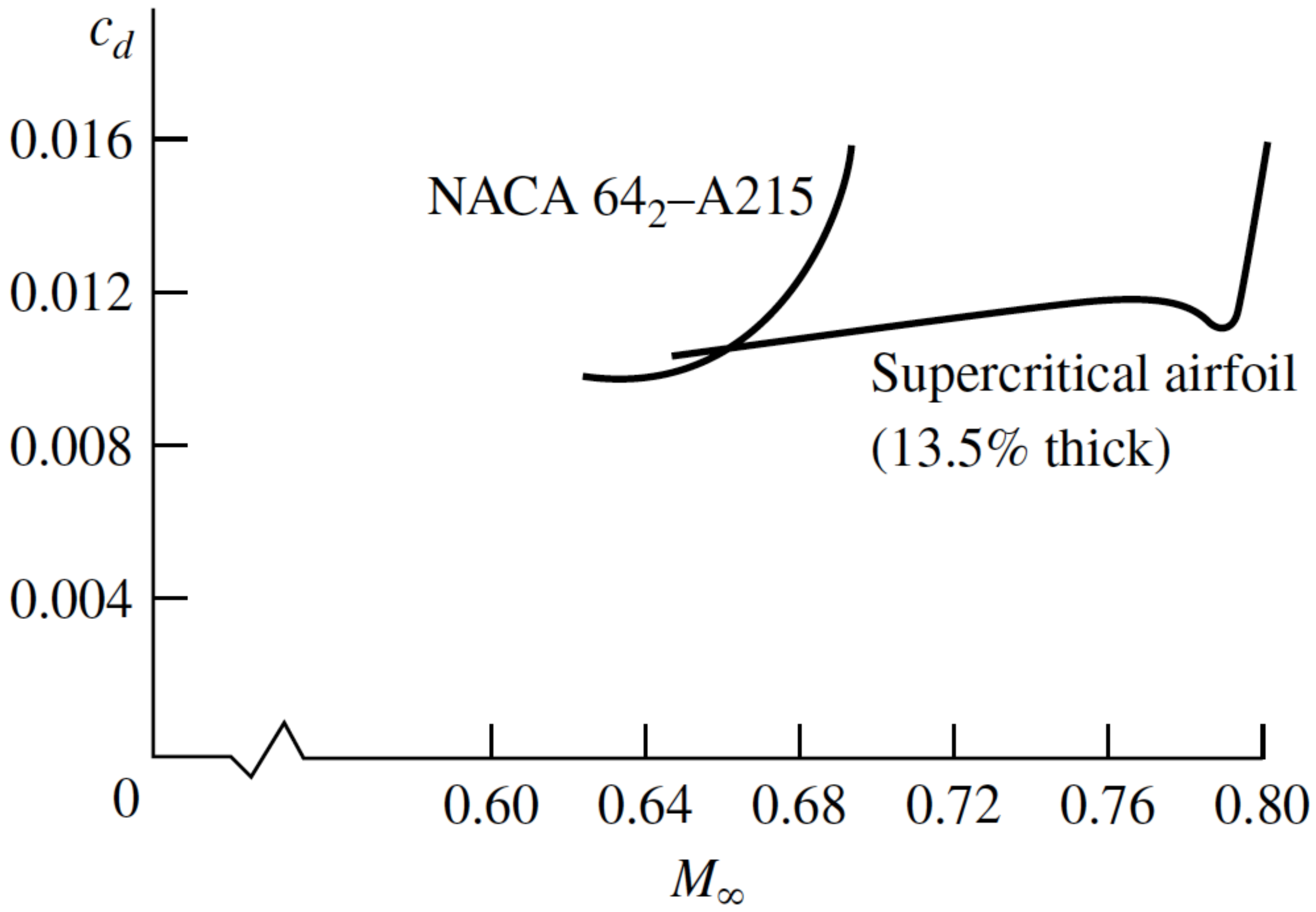
Drag divergence Mach no. of supercritical airfoil



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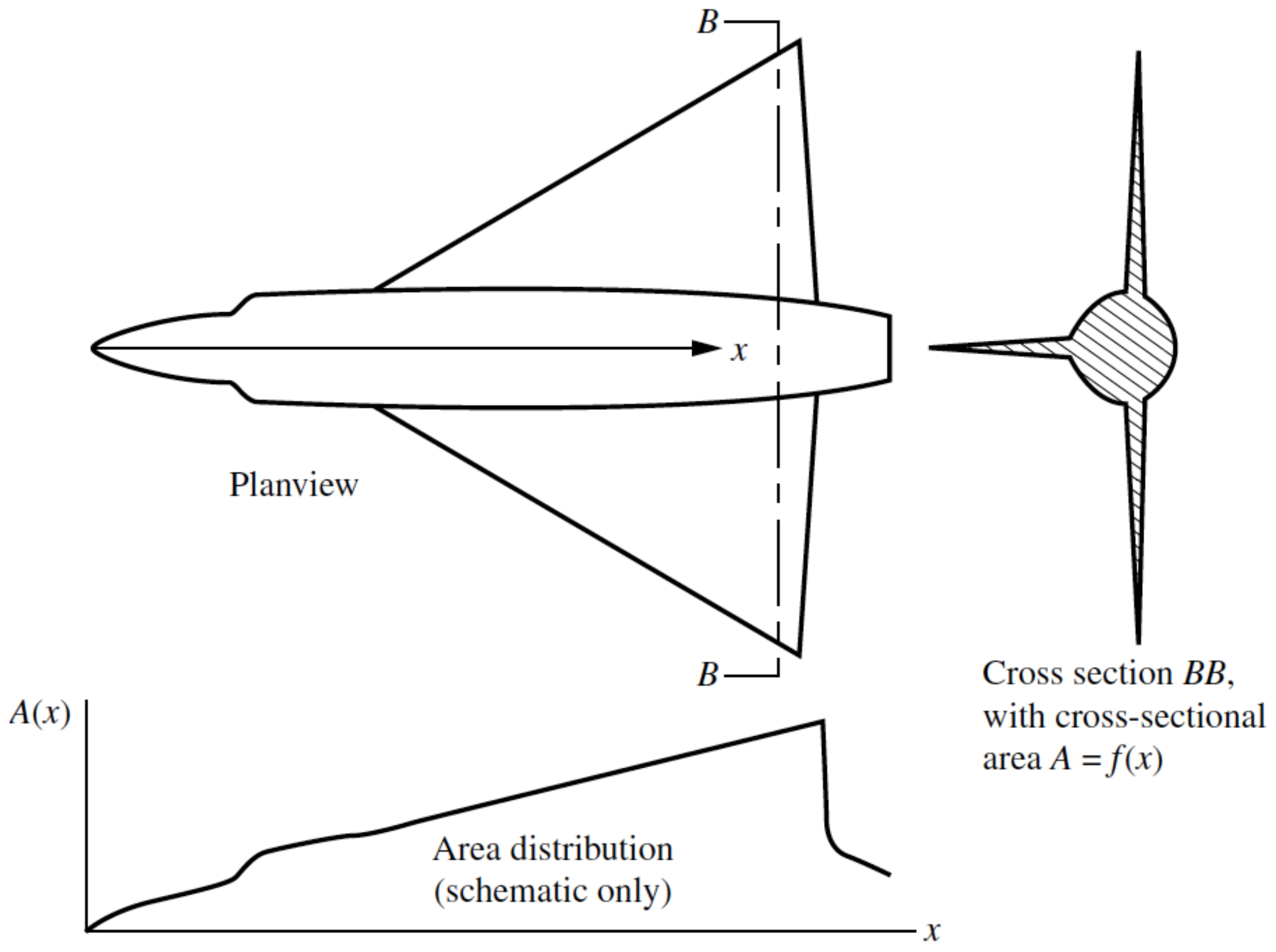
Remedy: Area rule (Richard T. Whitcombe)

Considering full aircraft, area distribution of typical early 1950s displayed discontinuities



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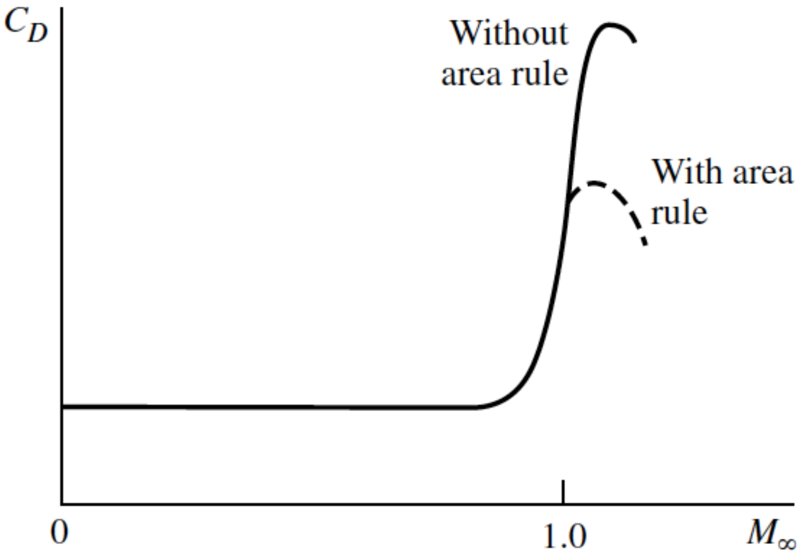
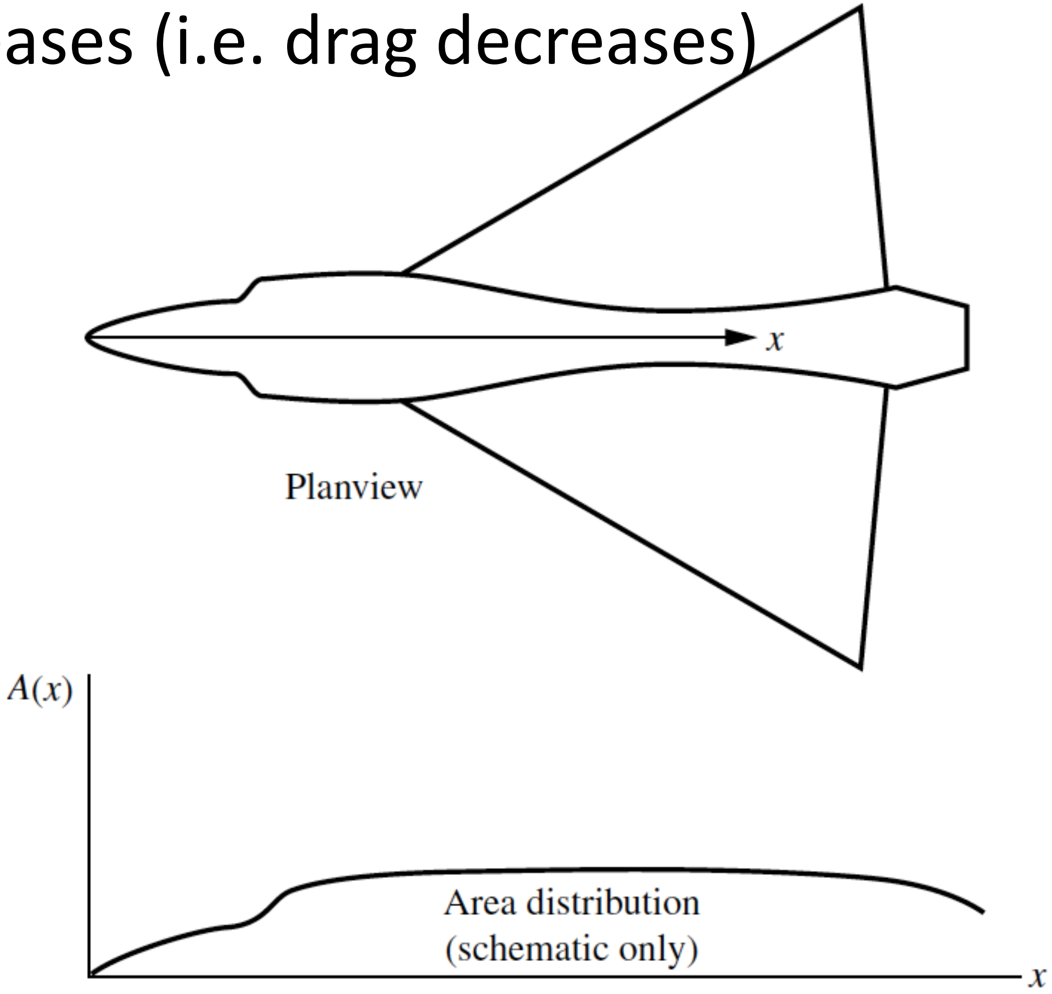
Area rule (contd.)

- For bullets, it was known that speed increases (i.e. drag decreases) with smooth area variation
- Whitcombe applied to whole aircraft
- Resulted in “coke-bottle” fuselage
- Max drag reduced by factor of 2





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End of Topic