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Answer

Now, the total mass of the craft at any instant is:

$$m_t = m_{bm} + m_f$$

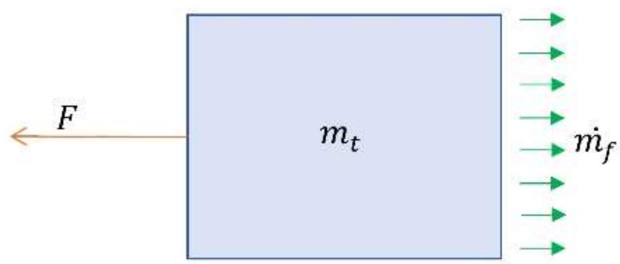
Where,

$m_{bm} = 5000 \text{ lb} \rightarrow$ Burnout mass

$m_f \rightarrow$ Mass of fuel on-board

$$\therefore m_t = 5000 + m_f$$

Now, consider the FBD:



The mass of fuel on-board is continuously varying since the fuel is being ejected from the craft for propulsion. The force of propulsion is:

$$F = \dot{m}_f I_{sp} g$$

Where,

$\dot{m}_f = -\dfrac{dm_f}{dt} \rightarrow$ Rate of mass flow rate of fuel ejection

$I_{sp} = 1000 \text{ s} \rightarrow$ Specific impulse

$g = 32.18 \text{ ft/s}^2 \rightarrow$ Gravitational acceleration

$$\therefore F = \left(-\frac{dm_f}{dt}\right) \times 1000 \times 32.18$$

Next, apply Newton's second law on the craft:

$$F = m_t a$$

Where,

$a = 9g \rightarrow$ Constant acceleration of craft

$$\therefore \left(-\frac{dm_f}{dt}\right) \times 1000 \times 32.18 = \left(5000 + m_f\right) \times 9 \times 32.18$$

$$\therefore -\frac{1000}{9}\left(\frac{dm_f}{5000 + m_f}\right) = dt$$

Integrating both sides: (assuming the entire fuel m_{f_0} runs out in $t = 11\ \text{min} = 660\ \text{s}$;

$$-\frac{1000}{9}\int_{m_{f_0}}^{0} \frac{dm_f}{5000 + m_f} = \int_0^{660} dt$$

$$\therefore -\frac{1000}{9}\left(\ln\left(5000 + m_f\right)\right)_{m_{f_0}}^{0} = (t)_0^{660}$$

$$\therefore -\frac{1000}{9}\left(\ln(5000 + 0) - \ln\left(5000 + m_{f_0}\right)\right) = (660 - 0)$$

$$\therefore -\ln\frac{5000}{5000 + m_{f_0}} = 5.94$$

$$\therefore \frac{5000 + m_{f_0}}{5000} = 379.93$$

$$\boxed{\therefore m_{f_0} = 1894650\ \text{lb}}$$

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