Assignment - 3 (AE330)

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Ans 1) Given, MW = 12 $\Rightarrow R = R_u/_{MW} \Rightarrow R = \frac{8.314 \times 10^3}{12} = \underline{692.83 \; J/_{kg\text{-}K}}$

$T_1 = T_{01} = 2950 \; K$, $T = 55 \; kN$, $I_{s(opt)} = 295 \; s$

$P_3 = 0.101 \; MPa$, $k = 1.3$

Assuming optimum expansion condition, $P_2 = P_3 = 0.101 \; MPa$

$$F = \dot{m} v_2$$

$$F = \dot{m} g_0 I_{sp(opt)} \quad — ①$$

Where $\dot{m} = \frac{A_t P_1 k}{\sqrt{kRT_1}} \sqrt{\left[\frac{2}{k+1}\right]^{\frac{k+1}{k-1}}}$, Putting values of P_1, k & R, T_1,

$$\dot{m} = A_t P_1 \frac{(1.3)}{1.63 \times 10^3} (0.585)$$

$$\dot{m} = A_t P_1 (4.667 \times 10^{-4}) \quad — ②$$

P_1 can be found by solving $v_2 = g_0 I_{sp(opt)}$

$$g_0 I_{sp} = \sqrt{\frac{2k}{k-1} RT_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$\Rightarrow \boxed{P_1 = 1.618 \; MPa}$$

Putting ② in ①,

$$F = A_t (1.618 \times 10^6) \, 9.81 \times 295 \left(4.667 \times 10^{-4} \right)$$

$$A_t = \frac{55 \times 10^3}{1.618 \times 9.81 \times 295 \times 4.667 \times 10^2}$$

$$\boxed{A_t = 0.02517 \; m^2}$$

Now, $\frac{A_t}{A_2} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\left(\frac{P_2}{P_1}\right)^{1/k}\sqrt{\frac{k+1}{k-1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k}{k-1}}\right]}$

$\frac{A_t}{A_2} = (1.59)(0.1184)(2.769)$

$\Rightarrow \boxed{A_2 = 0.048286\ m^2}$

Now, P_3 is changed to 0.03 MPa,

$F = F_{opt} + (P_2 - P_3)A_2$

$= 55\times10^3 + 3.428\times10^3$

$\boxed{F = 58.428\ kN}$

Now if A_2 is kept constant & P_2 is changed to 0.03 MPa, for optimum expansion at P_3, First we should find A_t

$\frac{A_t}{A_2} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\left(\frac{P_2}{P_1}\right)^{1/k}\sqrt{\frac{k+1}{k-1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k}{k-1}}\right]}$, for $P_2 = 0.03$, $P_1 = 1.618$ & $A_2 = 0.04828 m^2$

$\frac{A_t}{A_2} = (1.59)(0.0465)(2.769) \Rightarrow A_t = 9.885\times10^{-3} m^2$

Now, $F = P_1 A_t \sqrt{\frac{2k^2}{(k-1)}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$

Putting in the values

$F = (1.6\times10^4)(1.5236)$

$\boxed{F_{new} = 24.377\ kN}$

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Ans 2) Given,

$\dot{m} = 3.7\ kg/s$, $P_1 = 2.1\ MPa$, $T_1 = 2585\ K$

$MW = 18 \Rightarrow R = \frac{8.314 \times 10^3}{18} \Rightarrow \boxed{R = 461.89\ J/kg\text{-}K}$

$k = 1.3$, Also, at sea level $\rightarrow P_3 = 0.1013\ MPa$

As it is an optimum expansion $P_2 = 0.1013\ MPa$

Now,

$$v_2 = \sqrt{\frac{2kRT_1}{k-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$\boxed{v_2 = 2.282\ km/s} \qquad v_2^2 = 5.207 \times 10^6$$

Assuming chamber velocity $v_1 \approx 0$,

$$T_2 = T_1 - \frac{v_2^2}{2C_p}$$

Here $C_p = \frac{1.3 \times 461.89}{0.3} = 2001.52$

$$T_2 = 2585 - \frac{5.207 \times 10^6}{2 \times 2001.52}$$

$$T_2 = 2585 - 1300.76$$

$$\boxed{T_2 = 1284.24\ K}$$

$$C_F = \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} + 0\ (A_s\ P_2 - P_3)$$

$$\boxed{C_F = 1.393}$$

Ans 3) Given, $P_3 = 0.002549$ MPa (at 25 km altitude)

$T = 5.5$ kN, $P_1 = 2.068$ MPa, $T_1 = 2900$ K

$k = 1.3$, $R = 355.4$ J/kg-K

We have to find A_t, A_2, V_t, T_2.

Assuming the nozzle is designed for optimum expansion at 25 km,

$P_2 = P_3 = 0.002549$ MPa

$$F = A_t P_1 \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

Putting in the values,

$$5.5\times10^3 = A_t\,(2.068\times10^6)(1.744)$$

$$\boxed{A_t = 0.001512\ m^2}$$

$$\frac{A_t}{A_2} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\left(\frac{P_2}{P_1}\right)^{1/k}\sqrt{\frac{k+1}{k-1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k}{k-1}}\right]}$$

$$\frac{A_t}{A_2} = (0.009215)(2.768)$$

$$\Rightarrow \boxed{A_2 = 0.05926\ m^2} \Rightarrow \varepsilon = 39.2$$

$$V_t = \sqrt{\frac{2k}{k+1}RT_1}$$

$$\boxed{V_t = 1.079\ km/s}$$

$$\frac{V_2}{V_t} = \sqrt{\frac{k+1}{k-1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$V_2 = V_1 \times 2.456$

$$\boxed{V_2 = 2.65 \text{ km/s}}$$

Now, $T_2 = T_1 - \dfrac{V_2^2}{2C_p}$

where $C_p = \dfrac{kR}{k-1} = \dfrac{1.3}{0.3} \times 355.4 = 1540.07 \text{ J/kg-K}$

$$T_2 = 2900 - \frac{(2.65)^2 \times 10^6}{2 \times 1540.07}$$

$$= 2900 - 2280$$

$$\boxed{T_2 = 620 \text{ K}}$$

Ans 4) Given, $\varepsilon = \dfrac{A_2}{A_t} = 8$, $T_1 = 3000 \text{ K}$, $R = 378 \text{ J/kg-K}$, $k = 1.3$

$\boxed{P_3 = 265 \text{ millibars} = 0.0268 \text{ MPa}}$ (at 10 km altitude)

We need to find M_2, V_2, T_2, P_1

Using the relation $\left(\dfrac{A}{A_t}\right)^2 = \dfrac{1}{M^2}\left[\dfrac{2}{k+1}\left(1 + \dfrac{k-1}{2}M^2\right)\right]^{\frac{k+1}{k-1}}$

and calculating the value for M at $A/A_t = 8$,

we get,

$$\boxed{M_2 = 3.386}$$

Now, $T_2 = \dfrac{T_1}{1 + \dfrac{k-1}{2}M_2^2} = \dfrac{3000}{1 + 0.15(3.386)^2}$

$$\boxed{T_2 = 1103.04 \text{ K}}$$

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$$V_2 = M_2 \sqrt{kRT_2}$$

$$\boxed{\cancel{V_2 = 4.111 \text{ km/s}}}$$

$$\boxed{V_2 = 2.49 \text{ km/s}}$$

Now V_2 is also equal to,

$$V_2 = \sqrt{\frac{2k}{k-1} RT_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$V_2^2 = \frac{26}{3} \times RT_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]$$

$$\Rightarrow \quad \frac{P_2}{P_1} = 0.1332 \times 10^{7}$$

$$P_1 = \frac{P_2}{0.01332}$$

As rocket is designed to operate at 10 km altitude,

$$P_2 = P_3 \quad \Rightarrow \quad P_2 = 0.0268 \text{ MPa}$$

$$\boxed{P_1 = 2.012 \text{ MPa}}$$

Now if P_1 is doubled keeping rest of the parameters same, (T_1).

Now, By A/A_t - Mach number relation, it is clear that,

$$\boxed{\frac{A_2}{A_t} = f(M_2)}$$

So if A/A_t is not changing, M_2 must also remain the same

From the relation

$$T_1 = T_2\left(1 + \frac{k-1}{2}M_2^2\right)$$

If M is constant, and T_1 is assumed to be constant, then T_2 will also be constant.

As $V_2 = M_2\sqrt{kRT_2}$, V_2 will also remain the same.

Hence, $\boxed{V_{2\,(new)} = V_{2\,(old)} = 2.49 \text{ km/s}}$

Also, $$V_2 = \sqrt{\frac{2kRT_1}{k-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

As V_2, R, T_1, k are the same, the ratio P_2/P_1 should also remain the same

Hence $P_{2\,(new)} = 2P_{2\,(old)}$

$P_2' = 0.0536$ MPa , $P_1' = 4.024$ MPa

Considering the expression of Thrust,

$F_{old} = F_{opt} = \dot{m}V_2$

here $\dot{m} \propto P_1$

So, $\dot{m}' = 2\dot{m}$, Hence,

$$F_{new} = 2F_{opt} + (P_2' - P_3)A_2$$

As $2F_{opt} > F_{opt}$ & $(P_2' - P_3)A_2 > 0$,

$\boxed{F_{new} > F_{old}}$

Hence when pressure is doubled in the combustion chamber, the exit velocity remains the same but thrust increases

If we try to compute the deviation from the optimum expansion condition in terms of C_F, we can write

$$\frac{\Delta C_F}{C_{F,opt}} = \frac{S-(S+C\varepsilon)}{S} = \frac{C_{F,opt} - C_{F,new}}{C_{F,opt}}$$

Where $S = \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$, $C = \left(\frac{P_2' - P_3}{P_1'}\right)$

$S = 1.56$, $C\varepsilon = 0.1065$

$$\Rightarrow \left|\frac{\Delta C_F}{C_{F,opt}}\right| = \frac{C\varepsilon}{S}$$

$$= \frac{0.1065}{1.56} \times 100$$

$$\boxed{\left|\frac{\Delta C_F}{C_{F,opt}}\right| = 6.83\%}$$

The nozzle in this case is <u>underexpanded</u>

So, statiscally, there is a ~~def~~ deviation 6.83 % from the optimum expansion condition in the terms of C_F.

Ans 5) Given, $T = 2000\,K$, $P_1 = 15\,MPa$, $k = 1.32$,

$MW = 22 \Rightarrow R = \frac{8314}{22} \Rightarrow \boxed{R = 377.91}$, $A_t = 0.1\,m^2$

$\Rightarrow$ Optimum Expansion $\Rightarrow P_2 = P_3 = 0.1\,MPa$

We have to find V_2, c^*, F, C_F, I_{sp}

$$V_2 = \sqrt{\frac{2kRT_1}{k-1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$\boxed{V_2 = 2.094\ km/s}$$

$$C^* = \frac{\sqrt{kRT_1}}{k\sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} = \frac{9.9884 \times 10^2}{(0.77075)}$$

$$\boxed{C^* = 1.296 \text{ km/s}}$$

$$F = (P_1 A_t)\sqrt{\frac{2k^2}{(k-1)}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$F = (1.5 \times 10^6) \times 1.6158$$

$$\boxed{F = 2.4237 \text{ MN}} \Rightarrow \boxed{C_F = 1.6158}$$

Now, $I_{sp} = \frac{c}{g_0} = \frac{V_2}{g_0}$ (Optimum expansion)

$$I_{sp} = \frac{2.094}{9.81} \Rightarrow \boxed{I_{sp} = 213.455}$$

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