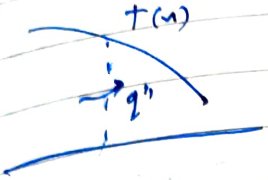
1/10/22

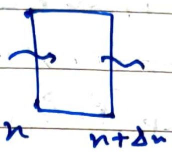
→ Heat Transfer

→ conduction



$$q''_x = -k\frac{dT}{dx}$$ thermal conductivity

$$\vec{q}'' = -k\vec{\nabla}TA$$



$$\dot{q}''_{in} = -\left(kA\frac{dT}{dx}\right)_x \qquad \dot{q}''_{out} = -\left(kA\frac{dT}{dx}\right)_{x+\Delta x}$$

$$\frac{dE}{dt} = \dot{q}''_{in} - \dot{q}''_{out}$$

$$E = \rho CT(A\Delta x\rho)$$

$$\frac{\partial}{\partial t}(\rho CT) = \left(Ak\frac{\partial T}{\partial x}\right)_{x+\Delta x} - \left(Ak\frac{\partial T}{\partial x}\right)_x$$

$$A\Delta x\frac{\partial}{\partial t}[\rho CT] = \Delta x\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right)$$

$$\frac{\partial}{\partial t}[\rho CT] = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)$$

If ρ C k are const

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2} \qquad \alpha = \frac{k}{\rho C_p} \qquad [\alpha] = [LT^{-1}]$$

thermal diffusivity

prandtl No. - $Pr = \frac{\nu}{\alpha}$

most gases $Pr = 1$

air $Pr = 0.7$

low conductivity — $Pr > 1$
(eg: water)

high conductivity — $Pr << 1$
(eg: liq metal)

if $c = c(T)$, $k = k(T)$

c_p, c_v → c_p, c_v are const — calorically perfect gas.
→ $c_p(T), c_v(T)$ ← thermally perfect
→ $c_p(P, \rho), c_v(P, \rho)$ — for eqlbm only at

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$ parabolic PDE

variable seperable.

$$T(t, x) = \Theta_t(t)\, \Theta_x(x)$$

$$\frac{\dot{\Theta}_t}{\Theta_t} = \alpha \frac{\Theta_x''}{\Theta_x} = a$$

$$\Theta_x'' = \frac{a}{\alpha} \Theta_x$$

$$\Theta_x = c_1 e^{a/\alpha\, x} + c_2 e^{-a/\alpha\, x}$$

$$T(t, x) = \sum_j T_j(t)\, e^{i k_j x}$$

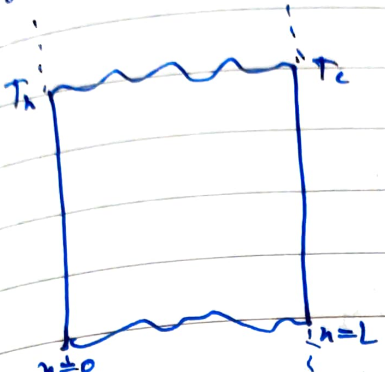
↓

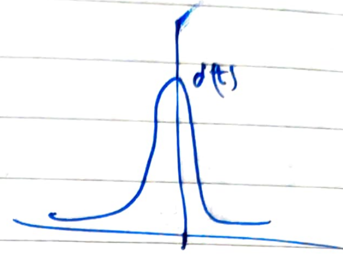
$$= \frac{1}{2\pi} \int T_k(t, t)\, e^{ikx}\, dk$$

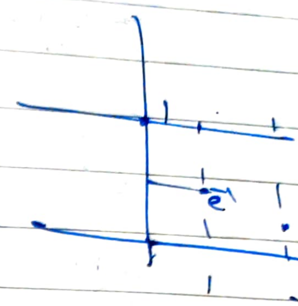
$$\frac{dT_j}{dt} = -k^2 T_j \rightarrow T_j(t) = e^{-k_j^2 t}\, T_j(0)$$

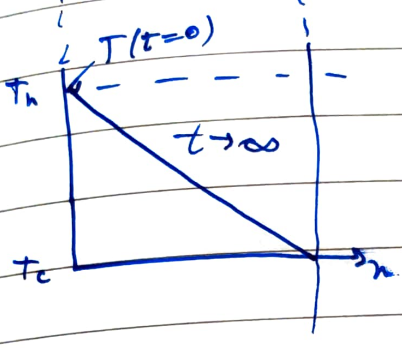
$$T_j(x,0) = \sum_j T_{j0}\, e^{ik_j x}$$

$$T_j(x,t) = \sum_j T_{j0}\, e^{-k_j^2 t}\, e^{ik_j x}$$









$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

assume some solⁿ as $t \to \infty$ (steady state)

$$0 = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$T(x) = ax + b$$

$$T = T_h + \frac{(T_c - T_h)}{L} x$$

$$\Theta(x) = \frac{T - T_c}{T_h - T_c} = 1 - \frac{x}{L}$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\eta = a x^{k} t^{d}$$

$T(\eta) = ?$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial \eta}\frac{\partial \eta}{\partial t} = \frac{d\eta}{t} T_\eta$$

$$\frac{\partial T}{\partial x} = \quad = \frac{k\eta}{x} T_\eta$$

let $k = 1$

$$\eta = a x t^{d}$$

$$\frac{\partial T}{\partial x} = a t^{d} T_\eta$$

$$\frac{\partial^2 T}{\partial x^2} = a^2 t^{2d} T_{\eta\eta}$$

$$d\frac{\eta}{t} T_\eta = \alpha a^2 t^{2d} T_{\eta\eta}$$

$$\eta T_\eta = \frac{a^2}{d} t^{2d+1} T_{\eta\eta}$$

$2d+1 = 0$ for no dependence on t

$d = -1/2$

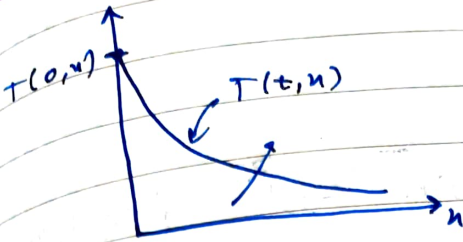
$$\therefore \eta = \frac{x}{2\sqrt{\alpha t}}$$ semi infinite block

$$\eta_{inf} = \frac{L}{2\sqrt{\alpha t_c}} \qquad t_c \sim \frac{L^2}{4\alpha \eta_{inf}^2}$$

$$T_{\eta\eta} = -2\eta T_\eta$$

$$T(\eta) = C_0 e^{-\eta^2}$$

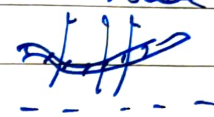
$$= \mathrm{erf}(\eta)$$



steady

$$T(x) = T_h + (T_c - T_h)\frac{x}{L}$$

losses



T → safety of material

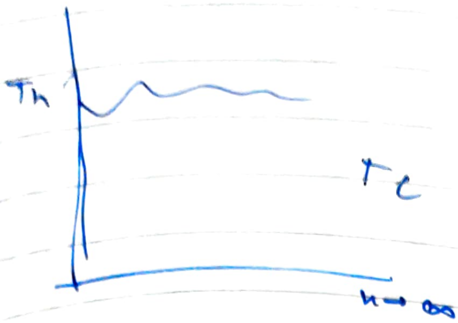
q'' → efficiency (losses)

$$q'' = -k\frac{\partial T}{\partial x} = \frac{k(T_h - T_c)}{L} \quad \text{steady}$$

T_h, T_c, L, k

AE330

7/10/21



$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\eta = \frac{x}{2\sqrt{\alpha t}}$$

$$\theta_{\eta\eta} = -2\eta\,\theta_\eta$$

I.C. $t=0,\ x>0,\ \theta=0$

B.C. $t \geq 0,\ x=0,\ \theta=1$

$x \to \infty \quad \theta = 0$

$$\theta = \frac{T - T_c}{T_h - T_c}$$

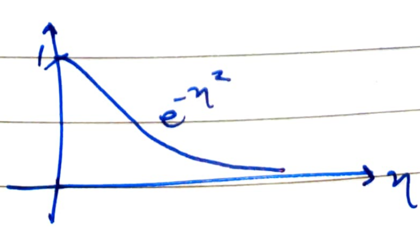
$t \to \infty \ \to \eta = 0$

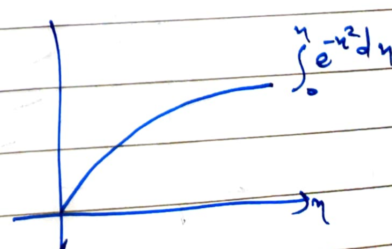
$$\left.\begin{matrix} t=0 \\ \text{or} \\ x \to \infty \end{matrix}\right\} \to \eta \to \infty$$

$$\theta_\eta = e^{-\eta^2}$$

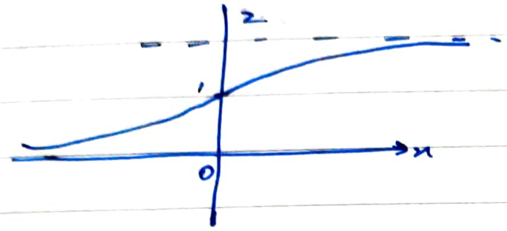
$$\theta = \operatorname{erfc}(\eta) = 1 - \operatorname{erf}(\eta)$$

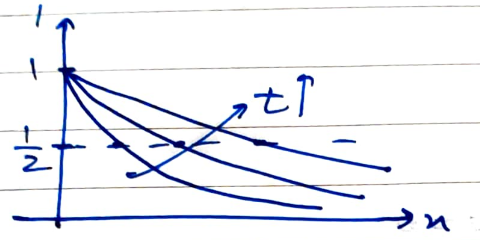
$$\operatorname{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-\eta^2}\, d\eta$$



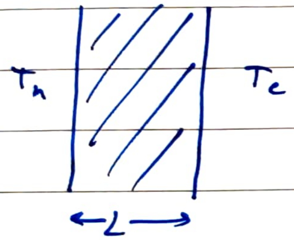


$$\mathrm{erf}(x) = \int_{-\infty}^{\infty} e^{-x^2}\,dx$$





$x = \sqrt{\alpha t}$ conduc^n depth

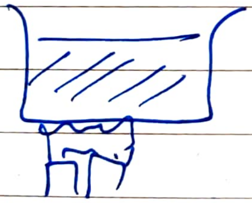


$$\frac{L}{2\sqrt{\alpha t_c}} = 1$$

$$t_c = \frac{L^2}{4\alpha}$$

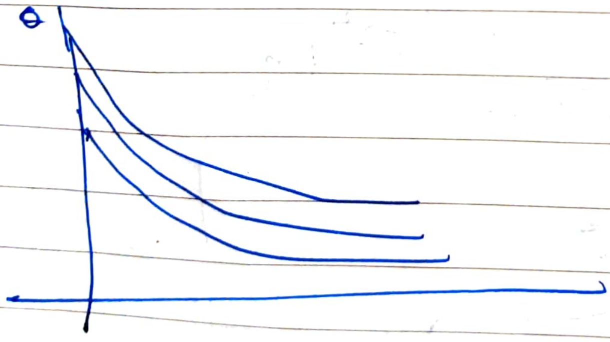
$t > t_c$ steady state

high cond lower t_c



$$q'' = -k\frac{\partial T}{\partial x}$$

$$\frac{\partial T}{\partial x} = \text{const} \leftarrow \text{BC at } x=0$$



$$\sqrt{\alpha t} = \frac{x}{2\eta}$$

$$\frac{h\sqrt{\alpha t}}{k} = \left[\frac{hx}{k} \times \frac{1}{2\eta}\right]$$

$$\frac{h^2 \alpha t}{k^2} = \left(\frac{hx}{k}\right)^2 \frac{1}{4\eta^2}$$

$$h(T_\infty - T_s) = q''$$

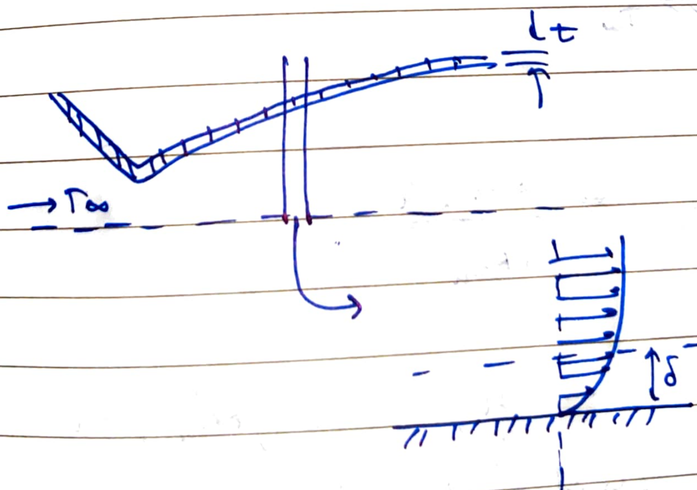
$$q'' = -k\frac{dT}{dx}$$

$$q' \sim k\frac{\delta T}{L}$$

$$\frac{hL}{k} \sim 1$$

$$\frac{hx}{k} = \frac{x}{L}$$

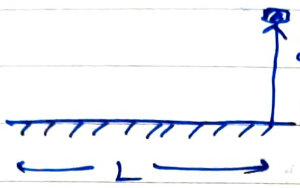
$\frac{hx}{k} \rightarrow$ Biot no.



$$C_f = \frac{\mu\, dv/dy}{\frac{1}{2}\rho V_\infty^2}$$

$$t_c \sim \frac{L}{V_\infty}$$

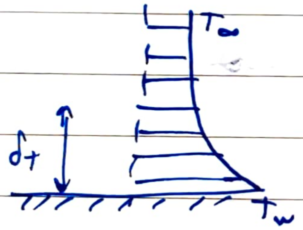
$$t_{cond} \sim \frac{\delta_T^2}{\alpha}$$



$$\frac{\delta_T}{x} \sim \frac{L}{U_\infty} \Rightarrow \left(\frac{\delta_T}{L}\right)^2 \sim \frac{\alpha}{V_\infty L}$$

$$\frac{\delta_T}{L} \sim \sqrt{\frac{\alpha}{V_\infty L}} = \sqrt{\frac{\nu}{U_\infty L}}\sqrt{\frac{\alpha}{\nu}}$$

$$\frac{\delta_T}{L} \sim \underbrace{\frac{1}{\sqrt{Re_L}}}_{\frac{\delta}{L} \longrightarrow \text{momentum BL}} \frac{1}{\sqrt{Pr}} \quad \text{thermal BL}$$



$$q'' = \frac{k\Delta T}{\delta_T} = h\Delta T$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} = \frac{0.664}{\sqrt{Re_x}}$$

$$Nu = \frac{hx}{k} = \frac{x}{\delta_T} \sim \sqrt{Re_x}\sqrt{Pr}$$

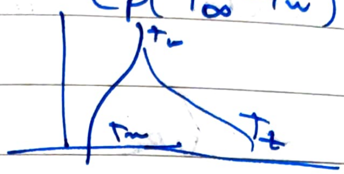
k — fluid conductivity

(Nusselt no.)

Stanton No. $= \dfrac{h(T_\infty - T_w)}{\rho U C_p (T_\infty - T_w)} = \dfrac{h}{\rho U C_p}$

$$St = \frac{Nu}{Re \cdot Pr}$$

Eckert no. $E_c = \dfrac{U^2}{C_p(T_\infty - T_w)}$ also people's worry



$$h_t = h + \frac{1}{2}U^2$$
$$= C_p T_\infty \left[1 + \frac{U_\infty^2}{2 C_p T_\infty}\right]$$

enthalpy

$T_t - T_\infty$ is ~~duty~~ driving force. if isent heat

$T_{ad} - T_\infty$ in real as it is not isent deceleration

$$T_{ad} = T_\infty \left[1 + r\left(\frac{\gamma - 1}{2}\right)M^2\right]$$

recovery factor $r \leq 1$

$r =$ $Pr^{1/2}$ for laminar
$Pr^{1/3}$ for turbulent

$$q'' = h(T_{ad} - T_w)$$

reentry $T \sim 200K$

$T_w = 1500 - 3000K$

$T_t = 6000 - 10000K$

back to incomp flow

$$St = \frac{Nu}{Re\,Pr} \sim \frac{1}{\sqrt{Re}} \frac{1}{\sqrt{Pr}}$$

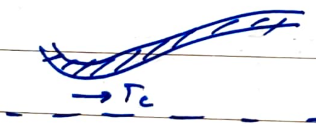
$$C_f \sim 1/\sqrt{Re}$$

$St = C_f \, 1/\sqrt{Pr} \; \frac{1}{2}$ Reynold's analogy

↳ true for $Pr > 0.5$

turb flat plate

$\cancel{St\,Pr}$ $St \cdot Pr^{0.67} = \frac{1}{2} C_f$ → Colburn analogy



duct flow

↳ hydraulic dia - d

$$St \cdot Pr^{0.67} = C_f/2$$

$St \cdot \frac{h_f}{\rho C_p \bar{U}}$ — mean vel in duct

Chilton-colburn analogy

1/11/22

GH_2 fuel as enter cc semicryo

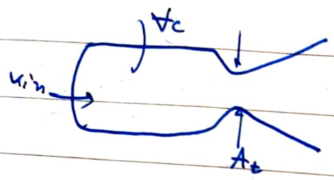
LO_2 — LH_2 cryo

hydrazine variants UDMH/MMH

\+

N_2O_4 ~~or~~ RFNA

→ characteristic length



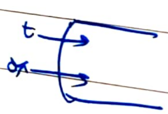
$$L^* = \frac{V_c}{A_t} = \text{char length}$$

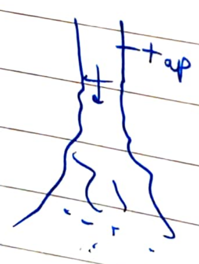
└ in practice ~~for~~ complete combustion

$$T_{res} = \frac{L}{U_{in}} \qquad \frac{A_c}{A_t}$$

residence

not single injector, many! called injector manifold

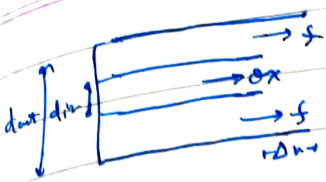




breakdown
↓
droplets
↓
evaporate
↓
mix
↓
heat

$q'' = h(A)\Delta T$

$= \rho C_p V A \Delta T$ create droplets to inc surf area.



$\dot{m} \propto \frac{\Delta p}{\Delta n}$

din/dout ∝ Δn/din

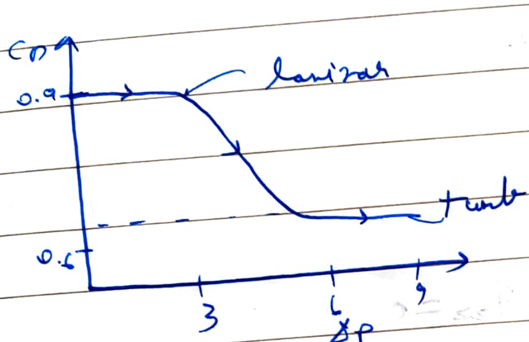
$\dot{m} = C_D A_{inj} \sqrt{2 \rho \Delta p}$

↳ discharg coeff

$\Delta p = \frac{1}{2} \rho u^2$ bernoulli

$\dot{m} = \rho A u$

$= A\sqrt{2 \rho \Delta p}$



$C_D = \begin{cases} 0.6-0.7 & \text{straight hole injector} \\ 0.3-0.5 & \text{swirl injector} \end{cases}$

$u_{inj} \approx 35-45$ m/s

$\sim 100-120$ m/s for GH_2