`Experiment 7

VIBRATIONS

Aim: Estimation of elastic modulus of a given specimen by observing the vibration behaviour in first bending mode.

Objectives:

1. To enhance understanding of natural vibrations of a system and to give insight to experimental measurement of natural frequencies.

2. To introduce the concept of making approximate engineering models.

3. To calculate the desired material properties (e.g. E) from the experiment.

4. To perform error analysis and to identify the reasons for the errors in the results obtained.

Equipment Used:

1. Steel Rule
2. Weights
3. Bench vice
4. Watch
5. Rubber bands

Theory:

Any given structure, when subjected to a certain specific initial conditions and allowed to vibrate freely, produces harmonic response.
This particular initial condition is called the mode shape and the corresponding frequency of the harmonic response is called natural frequency of the structure.
Theoretically, a beam like structure has infinite degrees of freedom and thus will have infinite mode shapes and corresponding natural frequencies.
However, only first few natural frequencies are of practical interest because amount of energy required to excite the higher modes is very high and such energy levels may not be achieved during the operating conditions.
To illustrate these facts, consider a simple cantilever beam. The first and second mode shape is as shown in the figure below.
If the initial condition is same as that of a particular mode shape the system will vibrate at its corresponding natural frequency. The first mode shape can be achieved by simply pushing the tip of the cantilever and suddenly releasing it. During this, a few other modes may also get excited; however, only the first mode will be dominant.

Higher modes in a structure mean more number of nodal points in the structure. For a simple cantilever beam, the portion of the beam between each nodal point can be considered as a separate simply supported beam. It is known that the lateral stiffness of the beam is inversely proportional to L3; as a result more number of nodal points implies lesser L which implies higher stiffness. As a result, large force amplitudes will be required to excite these modes.

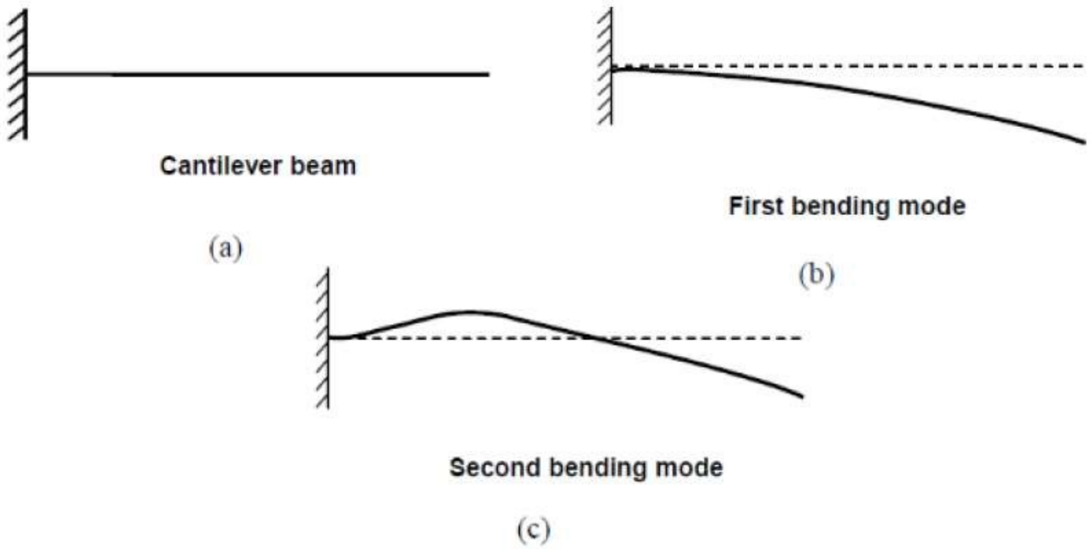


Fig.1: Bending modes of a cantilever beam

The given specimen is a slender beam with high frequency (because of low mass). The vibrations of such a system are difficult to measure directly. To reduce the natural frequency, we add mass sufficient mass at the tip of the beam so that the time period of the oscillations can be measured at ease.

Once the natural frequency is known, material property like Young's Modulus of the structure can be obtained. A cantilever beam with mass at the end can be approximately modelled as a massless beam with whole mass concentrated at the end.

It is assumed that the beam only contributes towards stiffness of the system. It should be noted that this assumption will give us only approximate value of fundamental natural frequency given by

$$\omega_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

For a cantilever beam, using definition of k we have

$$\omega_n = \frac{1}{2\pi}\sqrt{\frac{3EI}{L^3 m}}$$

Where,

$$I = \frac{b * t^3}{12}$$

If T is the time recorded for 20 oscillations, then we have,

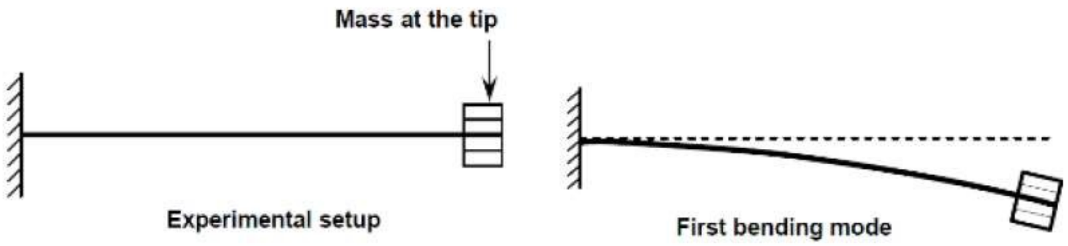
$$\omega_n = \frac{20}{T}$$

From above it can be shown that,

$$E = \frac{400 * 16 * L^3 * m * \pi^2}{b * t^3 * T^2}$$

Experimental Setup:

The figure below shows the experimental setup. It consists of a cantilever specimen (steel ruler fixed at one end using bench vice). Masses are tied at the free end of the beam.
A simple push on the mass produces initial condition sufficient to excite the first mode of the system. The time required for twenty oscillations is recorded for a given length of the beam.



Observation:

Specimen width = 24.32 mm
Specimen Thickness = 1.40 mm
Effective Length = 31.5 cm

Exercise:

1. Do the calculations of E without the mass of the beam and perform the error analysis?

Mass	Time for 20 oscillations	Young's Modulus(GPa)
273.1 g	11.29 s	
	11.30 s	
Mass	Time for 20 oscillations	Young's Modulus(GPa)
546.2 g	16.20	
	16.19	
Mass	Time for 20 oscillations	Young's Modulus(GPa)
819.3 g	21.31	
	21.49	

2. Discuss about the errors which are associated with the current problem, find the possible reasons and make some conclusions from the same. Also suggest possible improvements in the experiment to minimize the errors:
3. What is the effect of mass of the beam on system natural frequencies? How can we incorporate this in the model?
4. How can you simulate the free-free boundary condition for the specimen used?
5. What is the significance of this experiment?