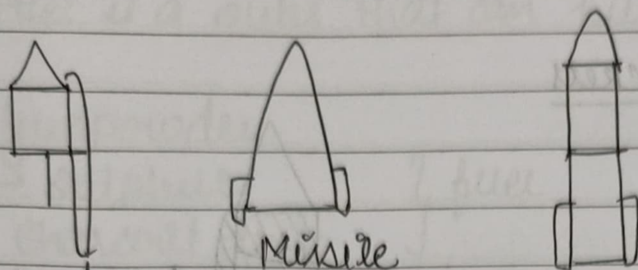
28/7/22

LECTURE-1



Good rockets   Missile   Launch vehicle

1) History classification → Ch 1
2) External Ballistics — Launch vehicles, Missiles — Chap 2, 3, 4
3) Rocket performance — Thermodynamics & comp flows
4) Heat transfer — conduction, convection } Chapter-6
5) Solid Rocket Motors - Ch 7
6) Liquid rocket engines - Ch 8 & 9

* Textbook → Rocket propulsion
Heister, Anderson
Pourpoint & Cassaday

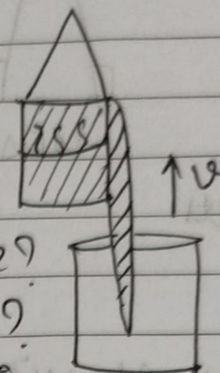
3 Quizzes → 3X20 = 60
Endsem → 40

* Grading statistics:-
Total → 70
AA → 11
AB → 26
BB → 16
BC → 9
CC → 5
CD → 1
DD → 2

Lecture-2

* **Evolution of Rockets**

1) Diwali Rocket

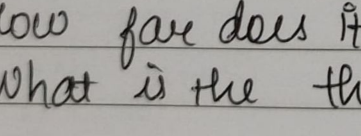


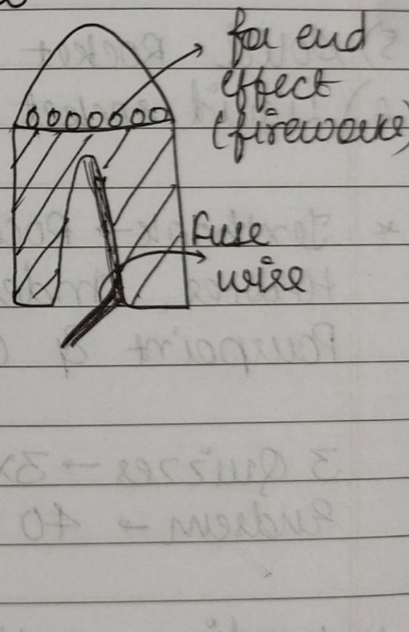
for pressure accumulation (To gain velocity) ← why bottle?

why stick?
- ↳ Direction
- → Produces enough drag to keep it straight
- → Just like

What is inside? gun powder

Gun powder block :-





How far does it go? ~100 m

What is the thrust?

- Mass of components
  - Rocket → 41 gm
  - Propellant − 108 gm
  - Payload → 9 gm
  - Burntime → 5.5s
  - Thrust → 1470 gm

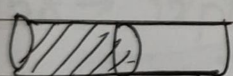
Propellant is a combination of fuel & oxidiser.
Fuel is a subs that can burn in presence of oxygen.

- Gunpowder
  → Sulphur } fuel
  Charcoal dust } fuel
  → Saltpeter } oxidiser
  ↳ Potassium Nitrate (KNO_3) } oxidiser

- Early Rockets ~900 CE (Common Era)

- Diwali Rocket ~1272 CE

- Roger Bacon (Better gunpowder) (13th cent)
- Foissart (14th cent)
  ↳ Fire from inside a tube
  better accuracy.
- Idea of cannon



- Galileo & Newton
  ↳ Laws of motion
  ↳ Trajectory

Brahe
Kepler
$\frac{GMm}{r^2}$

- Feymann's (lost lecture)
- Tipu Sultan used Mysorean rockets (cast iron casing) → Chamber pressure
  ↳ Go far but go
  Range ~ 2km
- Congreve (England)
  Used them in American wars → used them on Mexico.
- Hale (Use spin to stabilise)
  (no stick)

- Tsiolkovskii
  Rocket equation

- Goddard → American

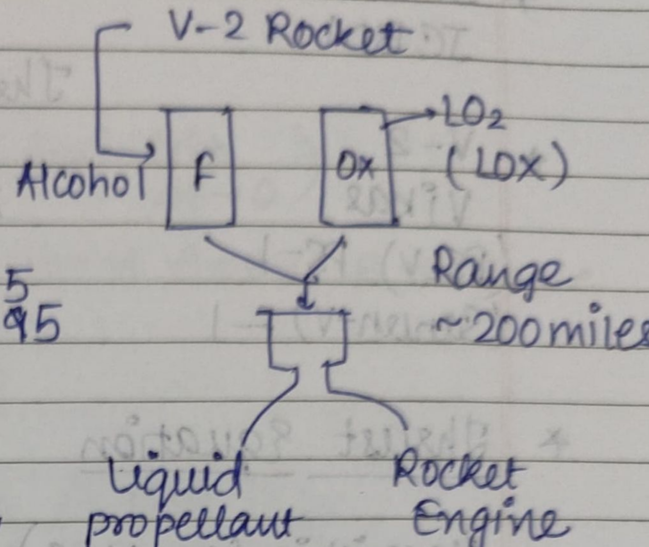
- Oberth → Orbital mechanics
- von Braun → present rocket FINALLY!!
  V-2 Rocket → LOX + Ethanol
  ↳ Liquid oxygen

LOX → Ethanol
&
LH_2

MOVIE → Apollo 13   Date 2/8/22

Lecture - 3

> WW-2 Missiles



Range ~ 7000km by 1995

Rocket → Missile
Rocket → Launch vehicle, Spacecrafts

Space
- → Space Station
  - → Skylab, Mir
  - → ISS

1957 → Sputnik
1958 → Explorer
↓
Communication satellite
↓
Dog
↓
Humans

Indian Space Program:-
Sarabhai
↓
SLV
↓
ASLV
↓
PSLV → LEO
↓
GSLV → GTO
GSLV → GEO

Ballistic missile:-
IGMDP
Prithvi → 150-300 km
Agni → 2000-5000 km

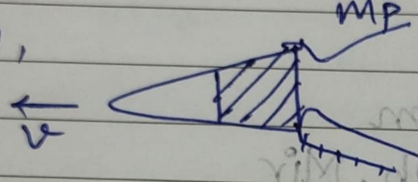
* IRBM (Inter Range Ballistic missile)
ICBM

	Thrust (kN)
* V-2	265
Vikas	735
(PSLV) PS-1	4500
(Saturn-V) F-1	6860

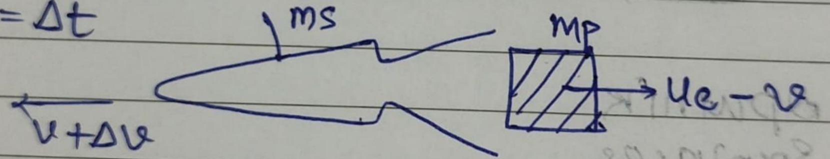
* Thrust Equation

Heuristic (or N-S)

At $t=0$,



At $t=\Delta t$



Mom^n Conservation

$$(m_s + m_p) v = m_s (v + \Delta v) - m_p (u_e - v)$$

u_e → speed at which propellant is ejected from nozzle.

$$(m_s + m_p) v = (m_s + m_p) v + m_s \Delta v - m_p u_e$$

$$m_s \Delta v = m_p u_e$$

$$m_p = \dot{m} \Delta t$$

$\dot{m}$ → rate of consumption of propellant

$$m_s \frac{\Delta u}{\Delta t} = \dot{m} u_e$$

$$T = m_s \frac{dv}{dt} = \dot{m} u_e \quad \left(\text{for } \lim_{\Delta t \to 0}\right)$$

for long duration, Total impulse $\rightarrow m_p u_e$

- Propulsive efficiency

$$\eta_p = \frac{\text{Vehicle power}}{(\text{Vehicle power}) + (\text{Residual jet K.E. power})}$$

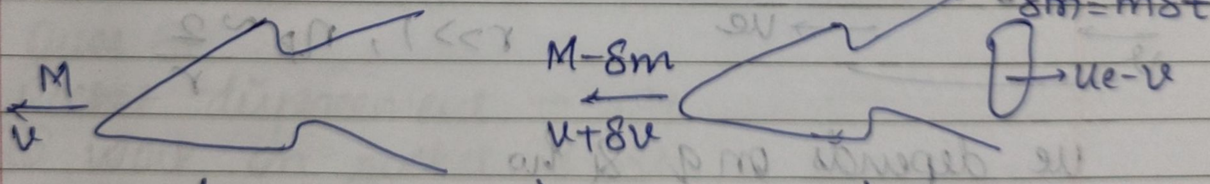
$$= \frac{TV}{TV + \frac{1}{2}\dot{m}(u_e - v)^2}$$

$$\eta_p = \frac{2u_e v}{u_e^2 + v^2}$$

Work done on vehicle = Force x disp
$T \times v\Delta t$

Jet ejected propellant

$$K.E = \frac{1}{2} m_p (u_e - v)^2, \quad m_p = \dot{m}\Delta t$$



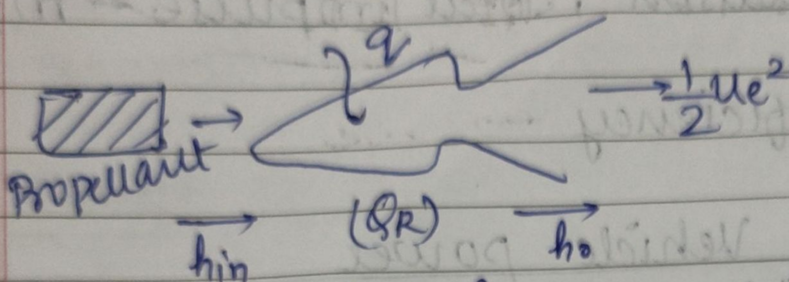
$$Mv = (M - \delta m)(v + \delta v) + \delta m (v - u_e)$$
$$0 = M\delta v - \delta m v + \delta m (v - u_e)$$
$$M\delta v = \delta m u_e$$
$$\delta m = \dot{m}\delta t$$
$$M\frac{dv}{dt} = \dot{m} u_e \qquad \lim_{\delta t \to 0} \quad M\frac{dv}{dt} = \dot{m} u_e$$

Smallness parameter

$\delta m << M$

$(M - \delta m) << M$

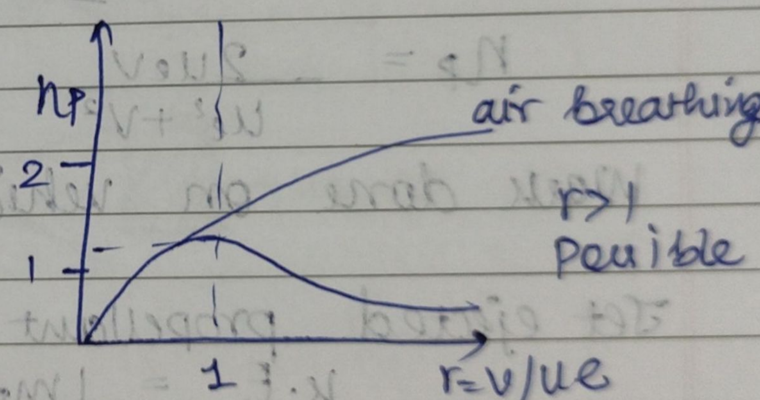
$\delta m \delta V << M \delta V$



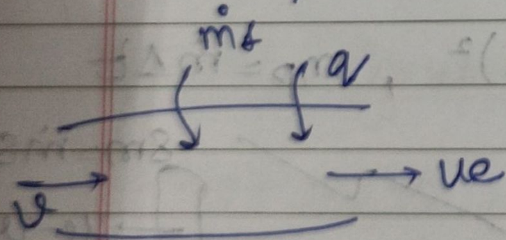
$h_o = h_{in} + q$

$r = \frac{v}{u_e}$

$\eta_P = \frac{2r}{1 + r^2}$



$T = \dot{m}(u_e - v)$



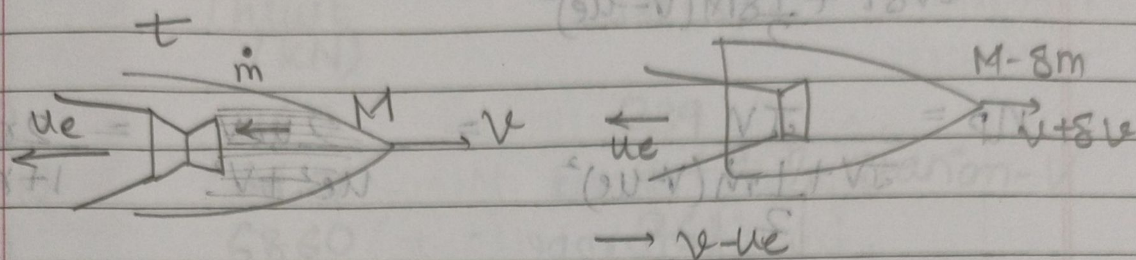
$r << 1, \ \eta_P \sim 2r$

$r >> 1, \ \eta_P \sim \frac{2}{r}$

ue depends on q & ho

LECTURE-4

Thrust Equation



$v < u_e$
or $v > u_e$

$\delta m = \dot{m}\delta t$
No external force

$$MV = (M-\delta m)(v+\delta v) + (\dot{m}\delta t)(v-u_e)$$

$$M\delta V = \dot{m}\delta t\, u_e \xrightarrow{\lim_{\delta t\to 0}} M\frac{dV}{dt} = \dot{m}u_e$$

$$M\delta V = \dot{m}u_e$$

$$\cancel{MV} + M\delta V - \cancel{\delta m v} - \cancel{\delta m \delta v} + \cancel{\dot{m}v\delta t} - \dot{m}u_e$$

Small δt

$$\delta m v = \dot{m}v\delta t$$

Over δt
Rocket displacement → $V\delta t$
ⓐ Work on rocket $TV\delta t$

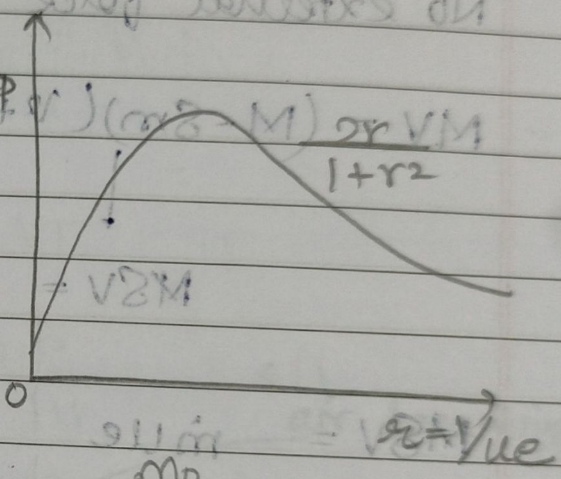
Exhaust gases → $\frac{1}{2}\delta M(v-u_e)^2$ ← ⓑ
(in absolute frame)

Propulsive efficiency → $\eta_p = \frac{(a)}{(a)+(b)}$

$$= \frac{JV\delta t}{JV\delta t + \frac{1}{2}\delta M (V-u_e)^2}$$

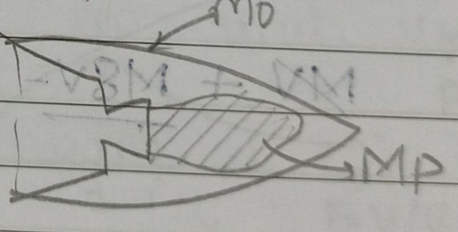
$$\eta_p = \frac{JV}{JV + \frac{1}{2}\dot{m}(V-u_e)^2} = \frac{2u_eV}{u_e^2+V^2} = \frac{2r}{1+r^2}$$

$$J = \dot{m}u_e + (p_e - p_a)A_e = \dot{m}u_{eff}$$



Total propellant mass is M_p.
Rocket mass at start $= M_0$

After all propellant is consumed, Rocket mass → M_e (empty mass)



~~M_0 / M_p~~ $M_0 = M_p + M_e$

Total impulse that the propellant can provide

$$\int J\,dt = M_p u_e$$

$$\int \dot{m}u_e\,dt = u_e \int \dot{m}\,dt$$

Specific Impulse

$$I_{sp} = \frac{\text{Total Impulse}}{\text{Total propellant } \cancel{\text{weight}} \text{ weight at sea level}}$$

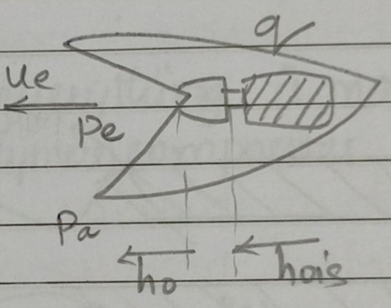
$$I_{sp} = \frac{u_e}{g_0}$$

	Thrust (kN)	I_{sp} (s)	
V-2	265	239	
RL-10	66.7	444	} Saturn-V
F-1	6860	261	
VIKAS	735	295	} PSLV
PS-1	4520	250	

$g_0 \rightarrow$ acc due to gravity at sea level

$$\dot{m} h_0 = \dot{m} h_{0in} + \dot{q}$$

$$\dot{q} = \dot{m} Q_R$$



$$\cancel{h_0 = h_{0in} + Q_R}$$

~~negligible~~

Temp of the gas that is coming in is negligible.

$h_0 \sim Q_R$

$h_0 \rightarrow$ Total enthalpy

h_{0in} - before combustion

$h_0, T_0 \rightarrow$ after combustion

$$Q_R \rightarrow \frac{1}{2} u_e^2$$

$$h_0 \sim Q_R$$

$$u_e \sim \sqrt{2 C_p T_0} \quad , \quad C_p T_0 \sim \frac{1}{2} u_e^2$$

$$u_e \sim \sqrt{2\left(\frac{\gamma}{\gamma - 1}\right)\frac{R}{M_w} T_0}$$

↳ highest for low gamma.

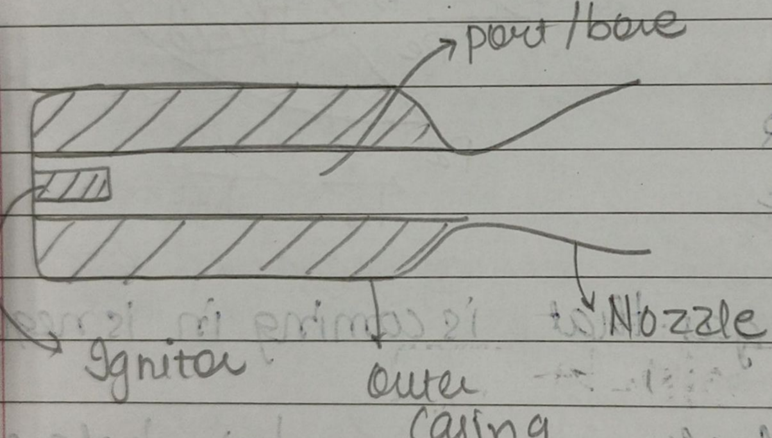
LECTURE-5

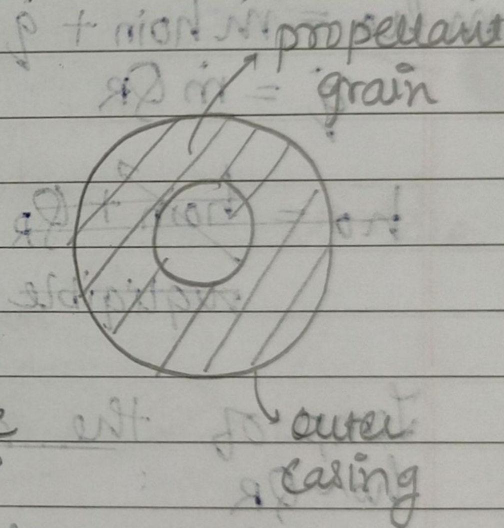
SCHEMATICS

Chemical (Energy) :
Rocket

- → Solid (Propellant)
- → Solid Rocket $\boxed{\text{Motor}}$ (SRM)
- → Liquid (propellant)
- → Liquid rocket $\boxed{\text{engines}}$ (LRE)

SRM

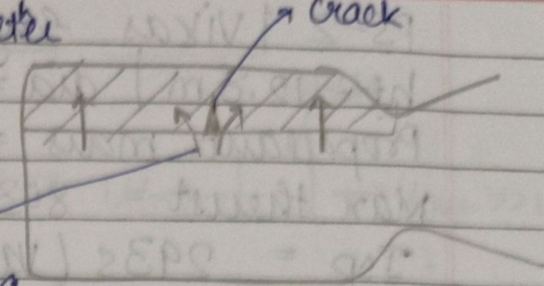




Propellant is fuel + oxidiser
→ Can carry its own weight

PSLV Stage 1
PS-1 - S139 Motor
ht - 20m | dia - 2.8m |
Propellant mass - 138.2 tons
Max. thrust - 4,847.4 KN
Isp ~ 237-269s | Burn time - 110s till alt ~ 55km
Propellant - HTPB hydroxil turmoil polybutadiene

Cannot check whether crack is there or not!



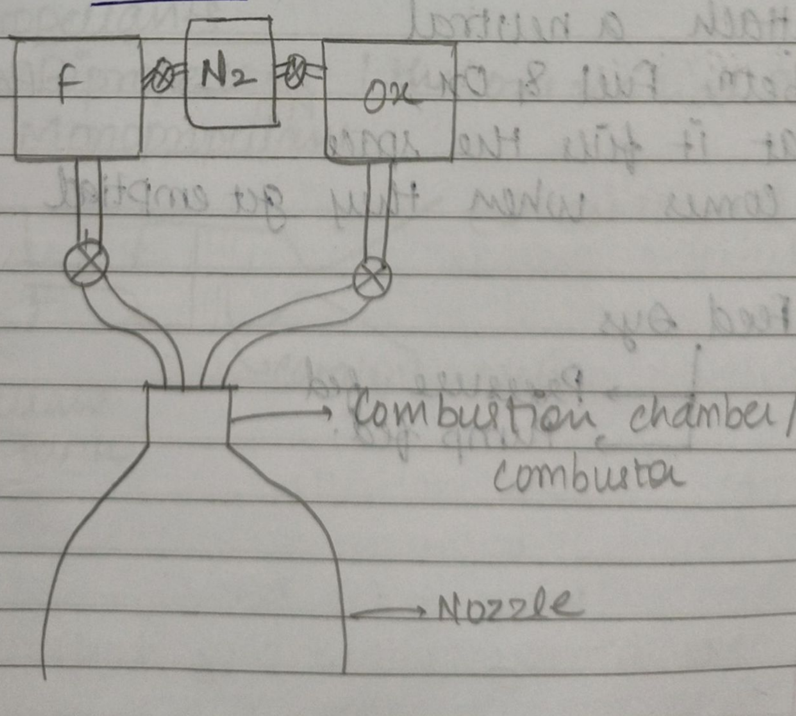
in all directions & reaches wall earlier & can cause explosion.

Once agreed DRDO gives its samples to private companies to manufacture further.

If there is a crack, In beam vibration, the frequency changes & we can detect it → NDT

The core & casing & everything is made of (multiple polymers) composites.

<u>LRE</u> (Fuel & oxidiser are separate)



PS-2 | Vikas Engine
ht = 12.8 m | dia = 2.8 m
Propellant mass = 42 tons
Max thrust = 803.7 kN
Isp = 293 s (vacuum)
Burn time - 133 s (till 130 km)
Propellant - N_2O_4 | UDMH

Again all its parts are made of composites.

① Ignition
- Hypergolic (Ignite as soon as fuel & ox are in contact) → N_2O_4 & UDMH
- Non-hypergolic (need an igniter) → $LO_2 + GH_2$

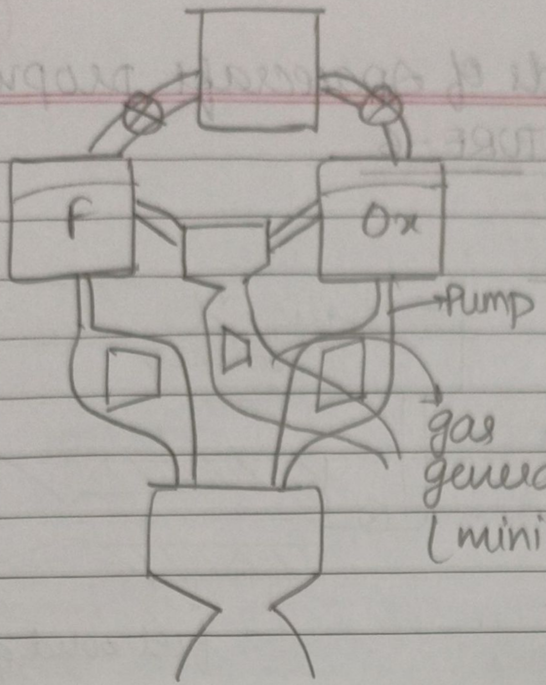
② Feed System
Chamber pressure ⟷ Tank pr

We attach a neutral gas betn fuel & ox, so that it fills the space which comes when they get emptied

$$\frac{\Delta P}{L} \propto \dot{m}$$

$$P_{in} \rightarrow \dot{m} \quad P_{out}$$

Feed sys
- Pressure fed
- Pump fed.



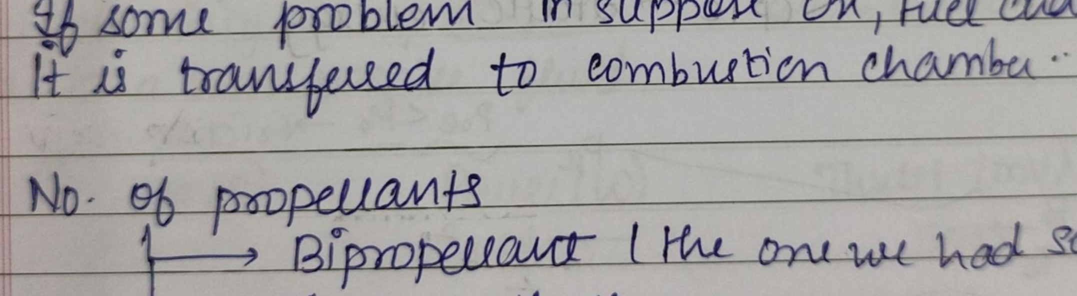
work
$u_{in} \rightarrow \boxed{} \rightarrow u_{out} \gg u_{in}$

$$p + \frac{1}{2}\rho u^2$$

Gas generator gives high speed flow that helps in running turbine of pump also to maintain the pres of three chambers above condenser pres → gas generator

Host of hydrodynamic instabilities-
If some problem in suppose Ox, Fuel chamber It is transferred to combustion chamber..

③ No. of propellants
- → Bipropellant (the one we had seen)
- → Monopropellant



(Methods of Spacecraft propulsion)

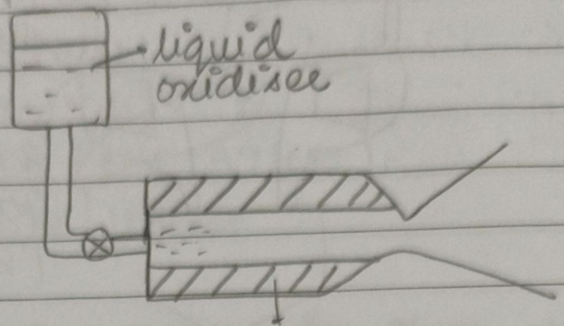
LECTURE - 6

* Chemical reaction

Solid rockets

Liquid rockets

Hybrid rockets



Thermal

$C_pT_0 \equiv h_0 \sim \frac{1}{2}u_e^2$

→ Nuclear

→ Electric

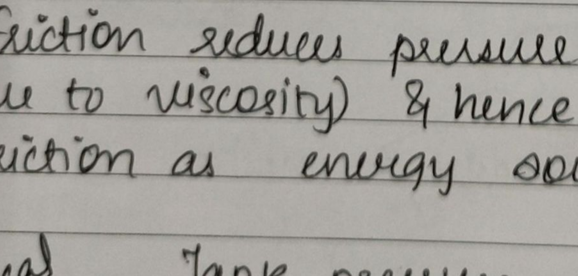
→ Friction

→ Mechanical

· Friction

$$C_pT_0 \begin{cases} \frac{1}{2}u_e^2 \rightarrow \text{Kinetic energy} \\ C_pT_e \rightarrow \text{Static energy} \end{cases}$$

$P_{0e} < P_0 \rightarrow$ Friction



Friction reduces pressure (ΔP pressure reduces due to viscosity) & hence we never use friction as energy source.

· Mechanical

Tank pressure

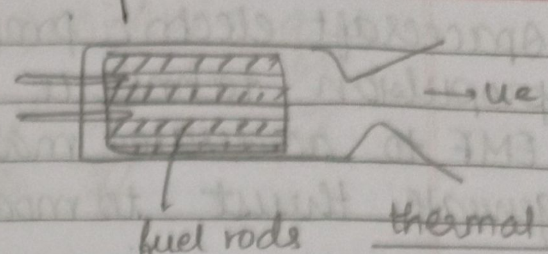
Turbopump

(pump-fed) LRE

Not directly contributing to thrust

• **Nuclear**

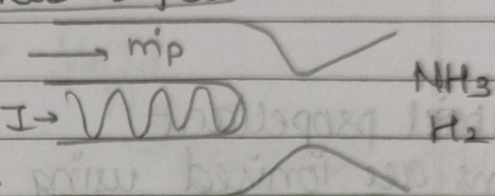
Using nuclear reaction instead of chemical reaction to burn fuel & oxidiser to generate thrust.



NERVA

$T_0 < 4000K$

Resistojet

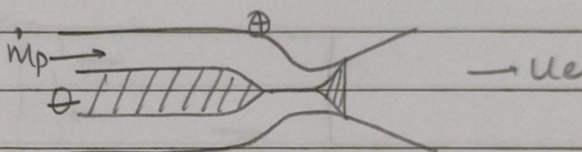


NH_3
H_2

• **electrical**

Arcjet → electric discharge is created in flow of propellant. Additional energy to propellant.

Resistojet → Provides heating by sending electricity through resistor to a non reactive fluid.



N_2H_4

Arcjet

arc ~ 10,000K

	Isp (s)	Power (W)	Thrust (mN)
Res.	300	340	500
Arc	865	365	22500

• **electrostatic** Flow does not change M.F.

$\vec{E}$ & $\vec{B}$ are imposed

$q \sim C_p T_0 \sim \frac{V_e^2}{2}$

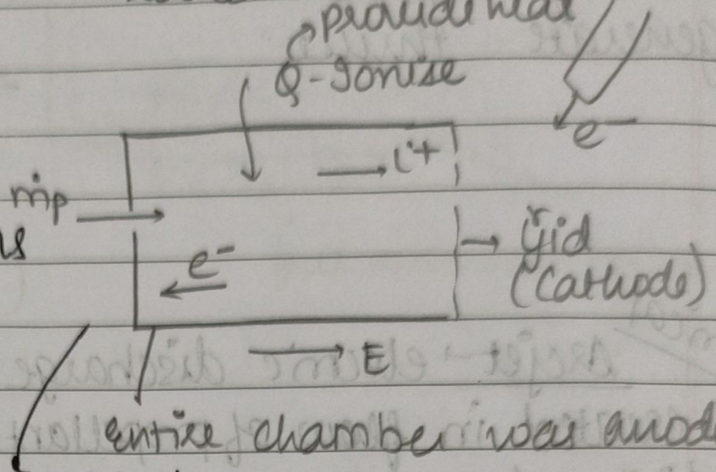
Ionise → accelerate using $\vec{E}$

Spacecraft electric propulsion is a type of spacecraft propulsion technique that uses electrostatic or EMF to accelerate mass to high speed & thus generate thrust to modify velocity of spacecraft in rocket.

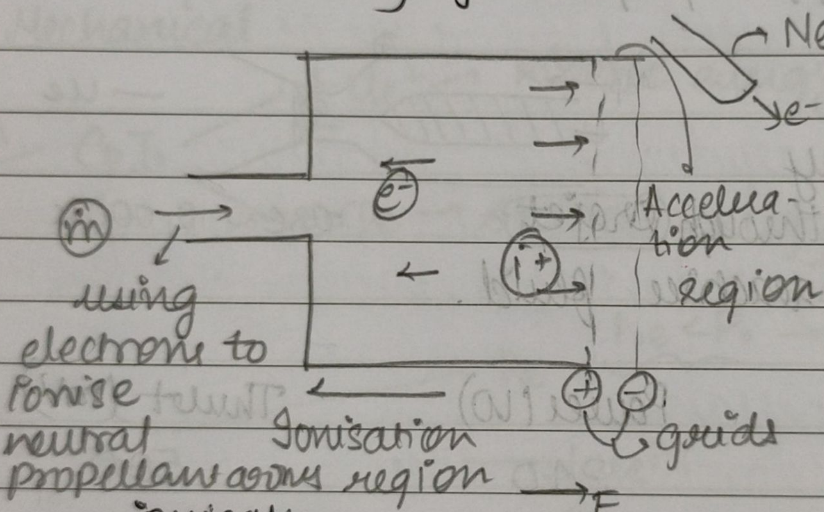
H mass = 1836 e^- mass

ions → thrust

electrons → neutralise ions



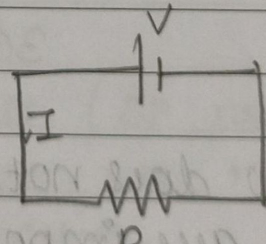
Neutral propellant atoms are ionised using gridded ion thruster



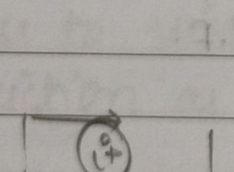
Isp ~ 3000s

$E = \frac{V}{d}$

Power → ionisation, losses, thrust



$I = I_i + I_e$
$= q_i n_i u_i A + q_e n_e u_e A$



$T = \underbrace{\rho_i A_i u_i}_{\dot{m}} u_i$

Potential drop → V

$\frac{1}{2} M u_i^2 = q_i V$

At STP, $\rho \sim 1.225$

1 mole = 22.4

$u_i = \sqrt{\frac{2qV}{M}}$

Ion beam current, $I_b = q_i n_i A u_i = \frac{q}{M}\dot{m}$

$$T = I_b \frac{M}{q}\sqrt{\frac{2qV}{M}} = \sqrt{\frac{2M}{q}}\, \cancel{\frac{I_b}{I_b}}\, I_b \sqrt{V}$$

$$I_{sp} = \frac{u_i}{g_0} = \frac{1}{g_0}\sqrt{\frac{2qV}{M}}$$

$M \rightarrow$ Mass of ions

$$T = \sqrt{\frac{2M}{q}}\, I_b \sqrt{V}$$

$P = V_d I_d$ $= \frac{V_b I_d}{\eta_d}$

V_d → Discharge voltage

η_d → Discharge efficiency

$$\left[\frac{T}{P} = \sqrt{\frac{2M}{q}}\,\frac{\eta_d}{\sqrt{V}}\right]$$

P → Power

For a given power, high M is better

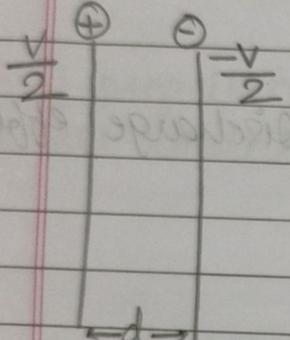
<u>Typical propellants:-</u>

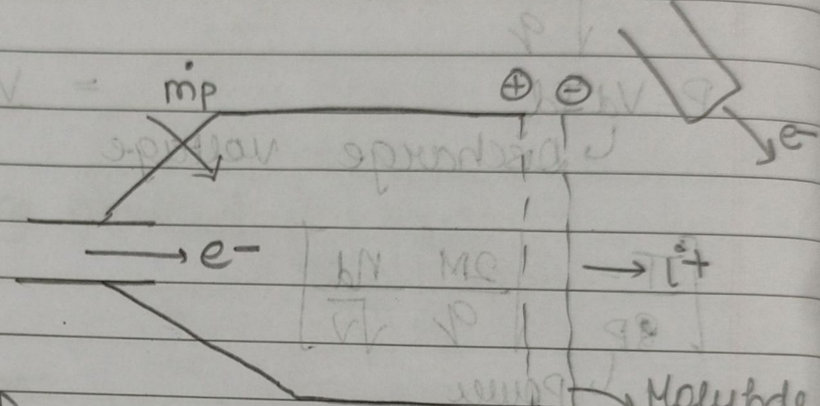
$^{132}_{54}Xe$ — Expensive

$^{133}_{35}Cs$ $\quad$ $^{200}_{80}Hg$ — Corrosive

Lecture-7

6) Raptor semi cryo engine (SpaceX) → LOx + CH4 } Liquid propulsion sys
1) V-2 Rocket → LOx + Ethanol
2) Hybrid propellant rocket engine → Hybrid prop sys
3) Arcjet thruster → Electric prop sys
4) Electrothermal thrusters
5) Gridded Ion thrusters → Electrostatic

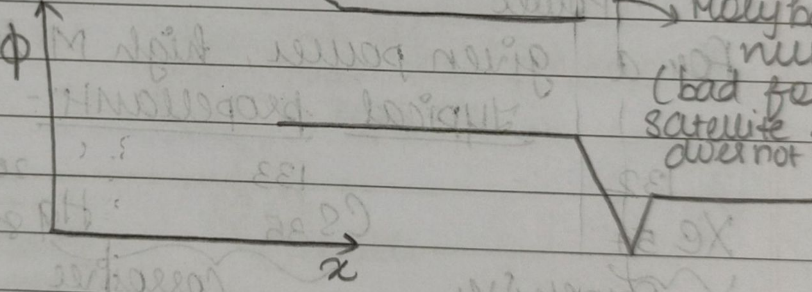


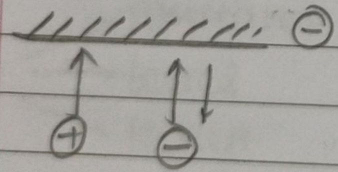


→ Molybdenum (bad for satellite so doesnot use)

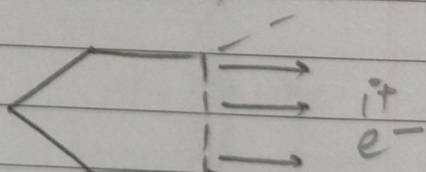
$j = j(V, d)$

$j \propto V^{3/2}$





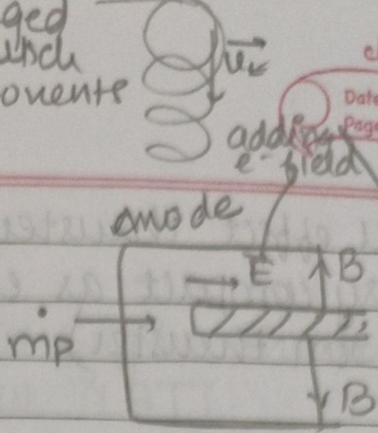
$\Gamma_+ = \Gamma_-$



How to replace molybdenum grid?
That sustains a large pot drop.
HALL THRUSTERS (To inc the area for e-)
If the idea is the reduction of e- flow inwards we can achieve this using a magnetic field
Only electrons are 'magnetised'.
(NASA's evolutionary XENON thruster (NEXT))

of any charged particle

What are $\vec{V_c}$ & $\vec{V_E}$? components of vel of e- along $\vec{E}$ & $\vec{B}$



If we add E.F there are Lorentz forces.

Ions should obey E.F,
e- should obey M.f.

$$m\frac{d\vec{V}}{dt} = q\vec{E} + q\vec{V}\times\vec{B}$$

$$\vec{V} = \vec{V_c} + \vec{V_E}$$

$$\cancel{m\frac{d\vec{V_c}}{dt}} \neq \cancel{q\vec{V_c}\times\vec{B}}$$ c → cyclotron

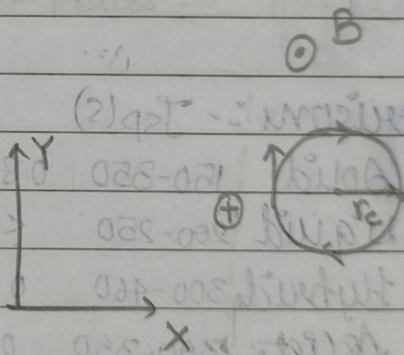
$$m\frac{d}{dt}(\vec{V_c}+\vec{V_E}) = q\vec{E} + q\vec{V_c}\times\vec{B} + q\vec{V_E}\times\vec{B}$$

Define v_c :- $m\frac{d\vec{V_c}}{dt} = q\vec{V_c}\times\vec{B}$

$$m\frac{d\vec{V_E}}{dt} = q\vec{E} + q\vec{V_E}\times\vec{B} = 0$$

$$\vec{V_E} = \frac{\vec{E}\times\vec{B}}{B^2}$$

$$\vec{E} = E\hat{e}_z$$
$$\vec{B} = B\hat{e}_r$$
$$\vec{V_E} = \frac{-E}{B}\hat{e}_\theta$$



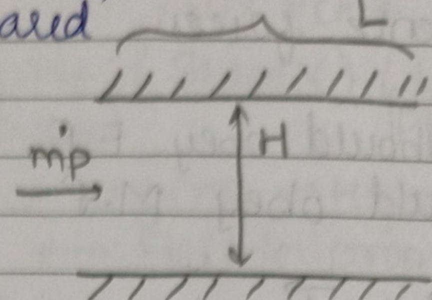
$$m\frac{dv_x}{dt} = qv_yB \rightarrow \frac{dv_x}{dt} = \frac{qB}{m}v_y$$ ($\frac{qB}{m} = \omega_c$)

$$m\frac{dv_y}{dt} = -qv_xB$$

$$\frac{mV^2}{r_c} = qVB$$

$$r_c = \frac{V}{\omega_c} = \frac{mV}{qB}. \quad r_c \propto m$$

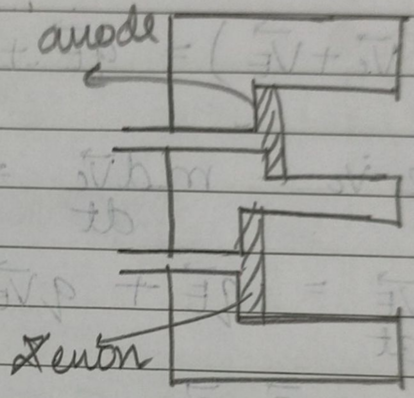
- Hall effect thrusters (HET)
(Very compact as compared to Ion thruster)



Propellants → Xenon, Krypton

Uses an electrostatic potential to accelerate ions upto high speeds.

If $r_{ce} < L, H < r_{ci}$



★ Comparison :-

	Isp(s)	Λ
Solid	150-350	0.8-0.95
Liquid (mono prop, bi prop)	200-250	<0.5
Hybrid	300-460	0.7-0.9
Arcjet	300-350	0.85-0.9
Ion	400-1500	<0.5
Hall Thruster	1000-10000	<0.5.

$m_p \leftrightarrow m_i$ Inert (Tank, pump, pipes etc)

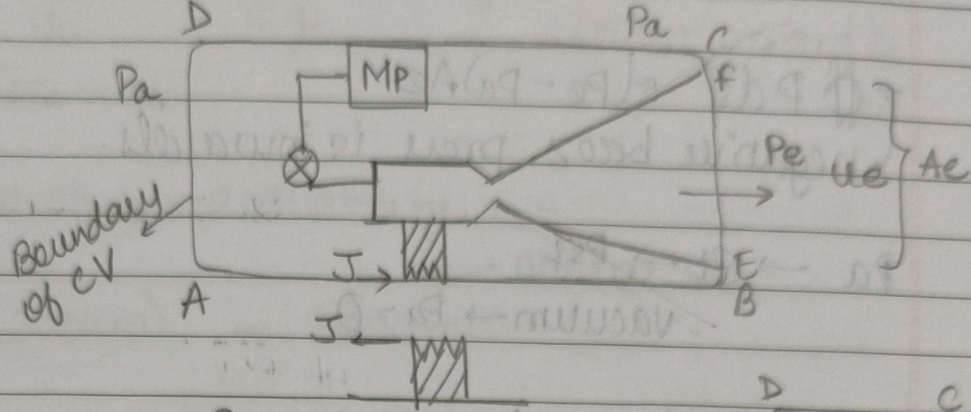
m_p → propellant

$$\Lambda = \frac{m_p}{m_p + m_i}$$ (Propellant mass fraction)

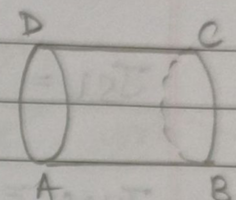
Product gases → CO_2, NO_x, H_2O, SO_x, Al_2O_3.

LECTURE-8

* Thrust equation:- (a jet Engine thruster)
Using Newton's 2^{nd} law & RTT.



$J = ?$ $\dot{m}$



Force ↔ Acceleration

$$\frac{d}{dt}\Big(\underset{1}{\oiiint \rho \vec{V} d\forall}\Big) + \underset{S}{\oiint} \underset{2}{\rho\vec{V}\cdot\vec{V}d\vec{S}} = \text{Forces} \equiv -\underset{3}{\oiint p\, d\vec{S}} + \underset{4}{J\hat{e}_x}$$

Body force exerted on it

Time interval Δt
Mom^n of inflow → 0
Mom^n of outflow → $\dot{m}\Delta t\, u_e$

AD → Ae

Pressure force:-
(AD) @ inflow → $p_a A_o$
(BC) @ outflow → $p_e A_e + p_a(A_o - A_e)$

$J = (p_e - p_a)A_e$
↳ Total force

Force ↔ Acceleration $\left[\dfrac{\dot{m}\Delta t\, u_e - 0}{\Delta t}\right)$

$J = \dot{m}u_e + [p_e - p_a]A_e$

CLASSMATE

Date
Page

$dm = \rho V_n ds$

For here we assume steady state

$\oiint \rho \vec{v} \cdot \vec{v} \cdot d\vec{s} = \dot{m} u_e$

$-\oiint P d\vec{s} \sim (P_e - P_a) A_e$

negative becoz pres is inwards.

$P_a \rightarrow SL \rightarrow P_{SL}$

vacuum $\rightarrow P_a = 0$

$J_{SL} = \dot{m} u_e + (P_e - P_{SL}) A_e$

$J_{vac} = \dot{m} u_e + P_e A_e$

$J_{vac} = J_{SL} + P_{SL} A_e$

$J = \dot{m} u_e + (P_e - P_a) A_e \equiv \dot{m} u_{eq}$

$$u_{eq} = u_e + \frac{(P_e - P_a) A_e}{\dot{m}}$$

$$I_{sp} = \frac{u_{eq}}{g_0} = \frac{u_e}{g_0} + \frac{(P_e - P_a) A_e}{\dot{m} g_0}$$

$J = \dot{m} u_e + (P_e - P_a) A_e \equiv \dot{m} u_{eq}$

$\dot{m} u_e \gg (P_e - P_a) A_e \rightarrow$ Rocket

$\dot{m} u_e \ll (P_e - P_a) A_e$

Rocket :- I_{sp}, λ

$$\updownarrow$$

$$\frac{u_{eq}}{g_0} \text{ or } \frac{J}{\dot{m} g}$$

$\Lambda = \frac{M_p}{M_p + M_i}$

↳ Inert mass like tanks

$\frac{u_{eq}}{g_0}$ or $\frac{J}{\dot{m} g}$ (highest in vacuum lowest on grd).

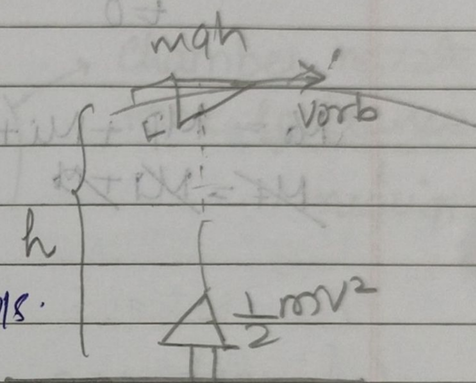
Mission → ΔV

→ ease of use availability

orbital vel

LEO (300km) — 8000 m/s

$\Delta V_{orb} = g_0 \frac{R_0^2}{R_0 + h}$



$\Delta V_{orb} = \Delta V_{alt} = \sqrt{2gh} = \sqrt{6}$ km/s.

$\Delta V = \Delta V_{alt} + \Delta V_{orb} \sim 10$ km/s.

$\frac{mv^2}{r} = \frac{GMm}{r^2}$

$\frac{1}{2}mv^2 \simeq m g_0 h$

$v = \sqrt{2 g_0 h}$

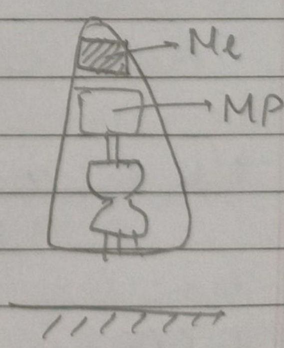
Vikas Engine

$I_{sp} \sim 300$s

Payload ~ 500kg

$\Lambda = \frac{M_p}{M_i + M_p}$

Inert gas $M_i = \frac{1-\Lambda}{\Lambda} M_p$



For the rocket,

Force = (accl) x mass

$J = \dot{m}_p u_{eq} = M \frac{dV}{dt}$

$$\dot{m}_p = \frac{-dM}{dt}$$

assume $u_{eq} = \text{const}$

$$M\frac{dv}{dt} = -\frac{dM}{dt} = -\frac{dM}{dt}u_{eq} \quad \} \quad dV = -u_{eq}\frac{dM}{M}$$

$$dV = -u_{eq}\frac{dM}{M}$$

assume $u_{eq} = \text{const}$.

For the rocket (in absence of gravity)

$$\Delta V = \int_{t=0}^{t=t_f} dV = V_{eq}\ln\left[\frac{M_0}{M_f}\right].$$

$M_0 = M_P + M_i + M_L$

$\cancel{M_f = M_i + M}$ $M_f = M_i + M_L$

LECTURE-9

$$M\frac{dv}{dt} = u_{eq}\dot{m} \quad -① \qquad \left[J = \dot{m}u_e + (P_e - P_a)A_e \equiv \dot{m}u_{eq}\right]$$

$$\frac{dM}{dt} = \dot{m} \equiv \dot{m}_p$$

↳ Total change in rocket mass

$$\int_{t=t_0}^{t_f} ① \, dt \longrightarrow \Delta V = u_{eq} \ln\left[\frac{M_0}{M_p}\right]$$

$t_f = t_b \rightarrow$ final time or burn out time

$$M_0 = M_{\ell} + M_p + M_i$$

Initial mass, Payload, Propellant, (Inert/ structural mass) → Chamber, nozzle, tank, guidance → negligible

$$M_f = M_\ell + M_i$$

Depends on mission, ΔV — LEO (10 km/s) — Orbit Maintainance ~50 m/s /yr

Total thrust over one year that is provided to counter drag,

$$\Delta V = \int J dt$$

$$\Delta V = V_{eq} \ln[MR]$$

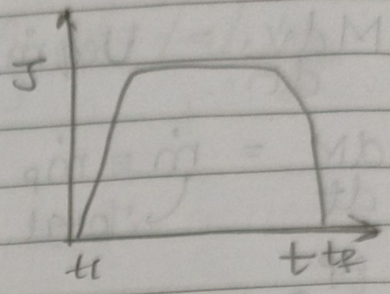
$$MR = \frac{M_0}{M_f} = \frac{M_p + M_i + M_\ell}{M_i + M_\ell}$$

For selected engine → u_{eq} is given.

& $\Lambda = \dfrac{M_p}{M_i + M_p}$ is used to find M_i.

* How to find u_{eq} for any engine?

Measure $\dot{m} = \frac{M_p}{t_f - t_i}$



$MR = \frac{M_p + (\Lambda)M_l}{(1-\Lambda)M_p + \Lambda M_l}$

Using this find M_p.

Measure thrust & mass flow rate, the ratio is u_{eq}.

* Vikas engine:-

$u_{eq} \simeq 3000\,m/s$
$\Delta V = 10\,km/s$
$\Lambda = 0.9$
$M_l = 1\,kg$
Find M_p.

Two cases → $\Delta V = 3\,km/s$
$\Delta V = 10\,km/s$.

$\Delta V = u_{eq} \ln[MR]$

$\frac{10 \times 1000}{3000} = \ln[MR] = \frac{10}{3}$

$MR = e^{100/3}$.

$MR = \frac{x + 0.9}{0.1x + 0.9} = 1.001$

$x + 0.9 = 1.001(0.1x + 0.9) = 0.1001x + 0.9009$

$0.0009 = (1 - 0.1001)x$

$0.0009 = 0.8999\,x$

$x = 1.0001\,kg \rightarrow M_p$

Difference betn u_{exp} & u_{eq}!

$$M_p + \Lambda M_e = MR \{ (1-\Lambda) M_p + \Lambda M_e \}$$

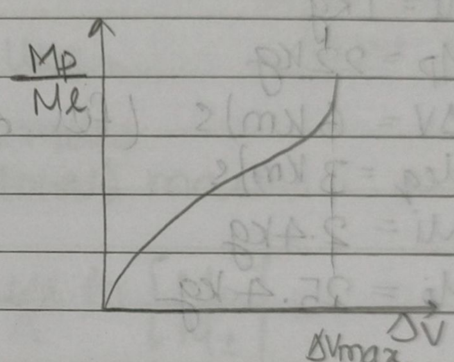
$$M_p [1 - (1-\Lambda) MR] = \Lambda (MR - 1) M_e$$

$$M_p = M_e \left[\frac{(MR-1)\Lambda}{1 - (1-\Lambda)MR} \right]$$

$$\left.\begin{aligned} &1 - (1-\Lambda) MR > 0 \\ &MR < \frac{1}{1-\Lambda} \\ &\Delta V < u_{eq} \ln \left(\frac{1}{1-\Lambda} \right) \end{aligned}\right\} \text{limiting Cases.}$$

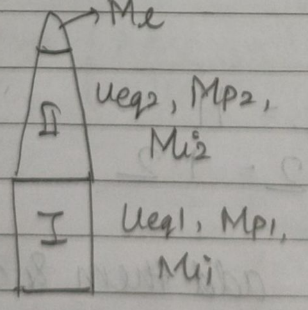
For first case, $\Delta V = 3000\ m/s$, $V_{exp} = 3000\ m/s$

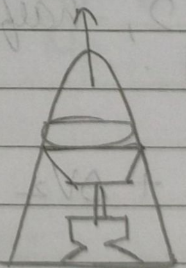
$M_p \sim 2.1\ kg$.



- $\Delta V = 2 u_{eq}$
  $\Lambda = 0.9$
  $\frac{M_p}{M_e} = 22$, $\frac{M_i}{M_e} = 2.4$

- Falcon 9 → Two stages & same engine for both engines.





Tank, chamber, nozzle

* Stage I (M_0 - Total initial mass)

$M_{01} = M_{P1} + M_{i1} + M_{l1}$

$M_{l1} = M_{02}$

$\Delta V_1 = u_{eq1} \ln [MR_1]$

* Stage II

$M_{l2} = M_l$

$M_{02} = M_{P2} + M_{i2} + M_l$

$\Delta V_2 = u_{eq2} \ln [MR_2]$

$\Delta V = \Delta V_1 + \Delta V_2$

* continuing with previous ex:-

$M_l = 1 kg$

$M_P = 22 kg$

$\Delta V = 6 km/s$ (for single stage)

$u_{eq} = 3 km/s$

$M_i = 2.4 kg$

$M_0 = 25.4 kg$

Split into 2, half goes into stage 1 & other to stage II.

$\Delta V = \Delta V_1 + \Delta V_2$

M_{P1} & $M_{P2} = M_P/2 = 11$

M_{i1} & $M_{i2} = M_i/2 = 1.2$

Find ΔV_1 & ΔV_2, add them & compare with 6 ~~10~~ km/s & check wheter it increased or not

1.7 kg 5.6 kg

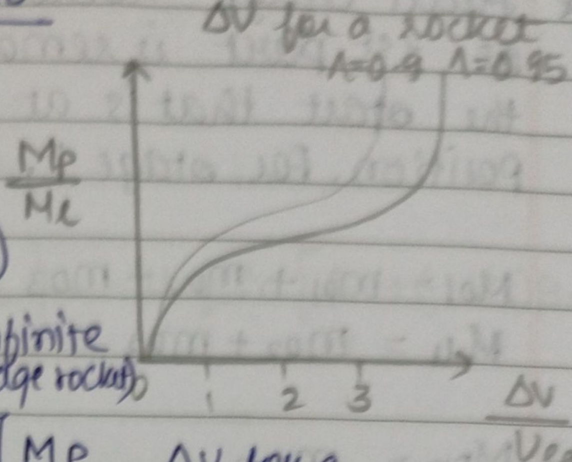
Total 7 kg.

LECTURE-10

Discard unnecessary inert mass

→ discard once (2 stage rocket)

→ discard continuously (infinite stage rocket)



ΔV for a rocket

$\Lambda = \frac{M_p}{M_p + M_i}$, $M_i = \left[\frac{1-\Lambda}{\Lambda}\right] M_p$

$\dot{m}_t = \dot{m}_p + \dot{m}_i = \frac{\dot{m}_p}{\Lambda}$

$\dot{m}_i$ → loss in inert mass

$M\frac{dv}{dt} = \dot{m}\, U_{eq} = \frac{\dot{m}_p}{\Lambda} U_{eq}$

$\dot{m}$ → loss of total mass

M → rocket component (total) mass

$\Delta V = \frac{U_{eq}}{\Lambda} \ln\left[\frac{M_0}{M_f}\right] \equiv \frac{U_{eq}}{\Lambda} \ln\left[\frac{M_0}{M_e}\right]$

$\frac{M_0}{M_e} = \exp\left[\frac{\Lambda \Delta V}{U_{eq}}\right]$, $\frac{M_1}{M_0} = \exp\left[-\frac{\Lambda \Delta V}{U_{eq}}\right]$

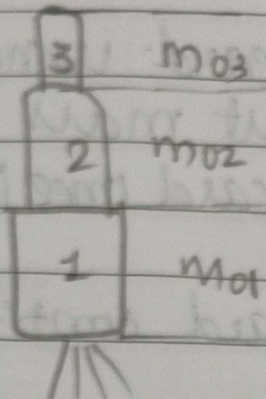
∞-stage rocket

$M_p = \Lambda\left[\exp\left(\frac{\Lambda \Delta V}{U_{eq}}\right) - 1\right] M_e$

$$\left.\begin{aligned} &t \to t + \delta t \\ &\delta m = \dot{m}_p \,\delta t \\ &\delta m = \delta m_p + \delta m_i \end{aligned}\right\} \text{ Infinite stage rocket}$$

Multi stage rockets (N)

As 1st part is removed in the start that is at first position. For stage 1,



$M_{01} = m_{01} + m_{02} + m_{03}$

$M_{f1} = m_{02} + m_{03}$

$M_{pn} = m_{pn}$

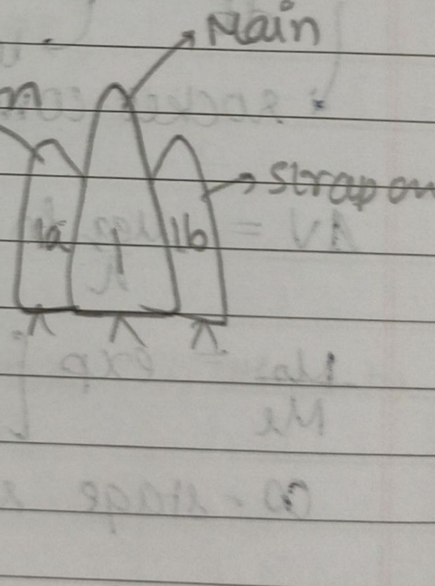
$M_{fn} = m_{fn}$

$$M_{0n} = \sum_{j=n}^{N} m_{0j} \quad \& \quad M_{fn} = \sum_{j=n+1}^{N} m_{0j}$$

1st stage → booster stage.

$J = J_1 + J_{1a} + J_{1b} + \dots + J_{1n}$ strap on

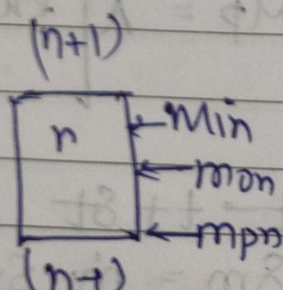
$= \dot{m}_1 u_{eq1} + \dot{m}_{1a} u_{eq1a} + \dots \dot{m}_{1n} u_{eq1n}$



$$u_{eq} = \frac{\dot{m}_1 u_{eq1} + \dots + \dot{m}_{1n} u_{eq1n}}{\dot{m}_1 + \dot{m}_2 + \dots + \dot{m}_{1n}}$$

Stage structural factor

$$\varepsilon_n = \frac{m_{sn}}{m_{0n}} \cong \frac{m_{fn}}{m_{0n}}$$



m_{fn} → final mass of stage n after stage n is completed.

Payload fraction

$$\beta_n = \frac{M_{en}}{M_{on}} = \frac{M_{on+1}}{M_{on}}$$

$$\Delta V_n = u_{eqn} \ln\left(\frac{M_{on}}{M_{fn}}\right) = -u_{eq} \ln\left[\epsilon_n + (1-\epsilon_n)\beta_n\right]$$

$M_L = 1$ ton
$M_0 = 25.5$ ton
$M_p = 22$ -"-
$M_i = 2.4$ ton

$m_{02} = 12.2$ ton
$m_{p2} = 11$ ton
$M_{i2} = 1.2$ ton

$m_{01} = 12.2$ ton
$m_{p1} = 11$ ton
$m_{e1} = 1.2$ ton

$\beta_1 = \frac{M_{02}}{M_0}$, $\beta_2 = \frac{M_L}{M_{02}}$ $\quad M_{02} = M_L + m_{02}$

$\beta_1 = \frac{13.2}{25.4}$ $\quad \beta_2 = \frac{1}{13.2}$, $\epsilon_1 = \epsilon_2 = \frac{1.2}{12.2}$

$$\Delta V = \sum \Delta V_n$$

$$\Delta V = -\sum u_{eqn} \ln\left[\epsilon_n + (1-\epsilon_n)\beta_n\right]$$

given $\frac{M_L}{M_0}$ or ΔV_{max}

$\frac{\partial \Delta V}{\partial \beta_n}$, $\beta = \frac{M_L}{M_0}$ (Payload) (in our control)

$\epsilon_n = \frac{m_{in}}{m_{on}}$

$\epsilon_n = \frac{m_{in}}{m_{on}}$

$$\Delta V = -\sum_{1}^{N} u_{eq} \ln\left[\epsilon + (1-\epsilon)\beta_n\right]$$

Same rocket engine for all stages $\Rightarrow u_{eqn} = u_{eq}$
$\epsilon_n = \epsilon$

$$\frac{M_L}{M_0} = \frac{M_{02}}{M_0} \cdot \frac{M_{03}}{M_{02}} \cdots \frac{M_p}{M_{0N-1}}$$

$$= \prod_{i=1}^{N} \beta_N$$

$u_{eqn} = u_{eq}$
$\epsilon_n = \epsilon$

① $$\Delta V = -\sum_{n=1}^{N} u_{eq} \ln \left[\epsilon + (1-\epsilon)\beta_n\right]$$

$$\frac{M_L}{M_0} = \prod_{n=1}^{N} \beta_N$$

② $$\ln \frac{M_L}{M_0} = \sum_{n=1}^{N}$$

LECTURE-11

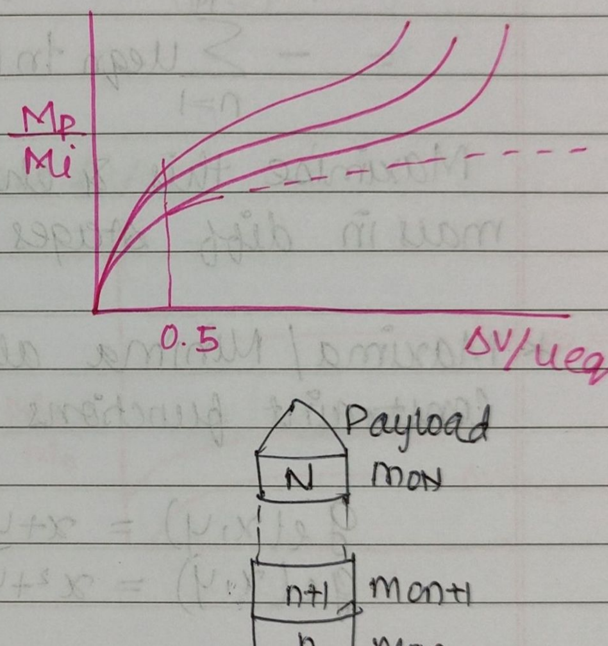
* Single stage rocket

$\Delta V = u_{eq} \ln\left(\frac{M_o}{M_f}\right) \equiv u_{eq} \ln(MR)$ or $MR = \exp\left(\frac{\Delta V}{u_{eq}}\right)$

$\Delta V > u_{eq}/2$

$M\frac{dv}{dt} = \dot{m}_p u_{eq} = \Lambda \dot{m} u_{eq}$

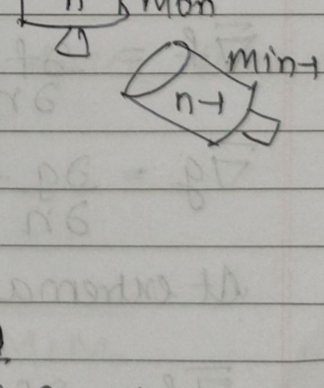
$\Lambda = \frac{M_p}{M_p + M_i} \Rightarrow \dot{m} = \dot{m}_p + \dot{m}_i = \frac{\dot{m}_p}{\Lambda}$



* Multistage rocket

for n'th stage:-

$M_{on} = \sum_{j=n}^{N} m_{oj}$



$B_1 = \frac{M_{o2}}{M_{o1}}$

$M_{e1} = M_{o2}$

$B_2 = \frac{M_{o3}}{M_{o2}}$

$B_n = \frac{M_{en}}{M_{on}} = \frac{M_{on+1}}{M_{on}}$

$M_{on} = \sum_{j=n}^{N} m_{oj}$, $M_{en} - \sum_{j=n+1}^{N} m_{oj}$

$\Delta V_n = u_{eqn} \ln\left[\frac{M_{on}}{M_{fn}}\right]$

Stage mass :- $m_{on} = m_{pn} + m_{in}$

$$\epsilon_n = \frac{m_{in}}{m_{on}}, \quad m_{in} = \epsilon_n m_{on}$$

$M_{fn} = m_{in} + M_{en} \rightarrow$ Final mass

$$m_{in} = \epsilon_n [M_{on} - M_{en}]$$
$$= M_{on}[\epsilon_n + (1-\epsilon_n)\beta_n]$$

$$\Delta V = \sum_{n=1}^{N} \Delta V_n$$
$$= -\sum_{n=1}^{N} u_{eqn} \ln(\epsilon_n + (1-\epsilon_n)\beta_n)$$

Maximise this & check how to distribute mass in diff stages for optimized problem.

* Maxima / Minima along a curve
Constraint functions or constraints

$$g_e(x,y) = x+y-1$$
$$g_c(x,y) = x^2+y^2-4 \quad g_l(x,y)=0, g_c(x,y)=0$$

$$\bar{\nabla} f = \frac{\partial f}{\partial n}\hat{e_n} + \cancel{\frac{\partial f}{\partial j}\hat{e_j}}^{\,0} \text{ at extrema}$$

$$\bar{\nabla} g = \frac{\partial g}{\partial n}\hat{e_n} + \cancel{\frac{\partial g}{\partial j}\hat{e_j}}^{\,0} \text{ everywhere}$$

At extrema

$$\bar{\nabla} f = \frac{\partial f/\partial n}{\partial g/\partial n} \times \bar{\nabla} g = \alpha \bar{\nabla} g$$

For $\partial g/\partial n \neq 0$.

$\nabla(f - \alpha g) = 0$

we minimise $f - \alpha g$ instead of $f(x,y)$

$\alpha \rightarrow$ Lagrange multiplier.

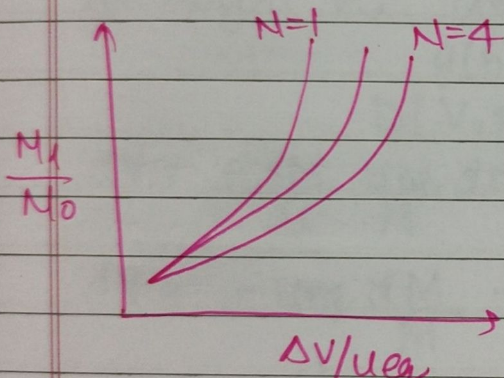
Optimum mass distribution:-

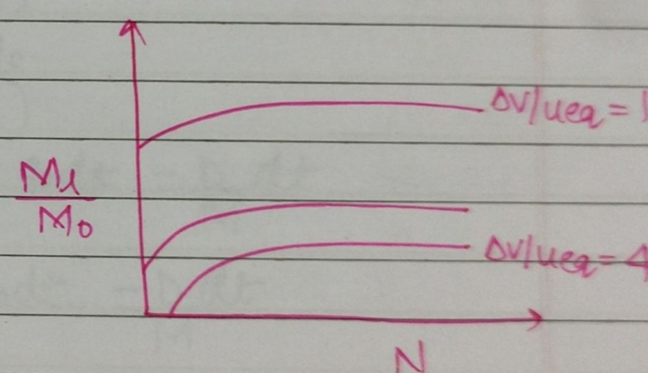
$$\frac{M_L}{M_0} = \prod_{n=1}^{N} \beta_n = \prod_{n=1}^{N} \frac{-\alpha}{(\alpha + u_{eq n})} \frac{\epsilon_n}{(1-\epsilon_n)}$$

Payload fraction:- $\frac{M_L}{M_0} = \frac{(w^{-1/N} - \epsilon)^N}{(1-\epsilon)}$

$w = \exp(\Delta V / u_{eq})$

Payload fraction vs ΔV.





Countries capable of launching space Rockets.

* PSLV, (C53)

  Four stages

  First stage → PS1

  $\beta_1 = \beta_2 \rightarrow \frac{M_{02}}{M_{01}} = \frac{M_L}{M_{02}} \rightarrow M_{02} = \sqrt{M_L M_0}$

* GSLV (Mk-II) F11

  GSLV Mk-III ~~F~~M1

* Launch facilities
Sites → across the world.

Spacex – Falcon 9
Payload (tons), LEO Launch cost, height, diameter, lift-off mass.

Date 30/8/22

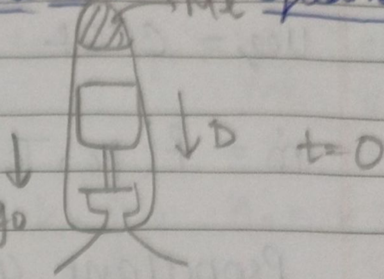
Lecture :- 12

Vertical Trajectories - The Rocket Equation

$M_0 = M_p + M_i + M_l$

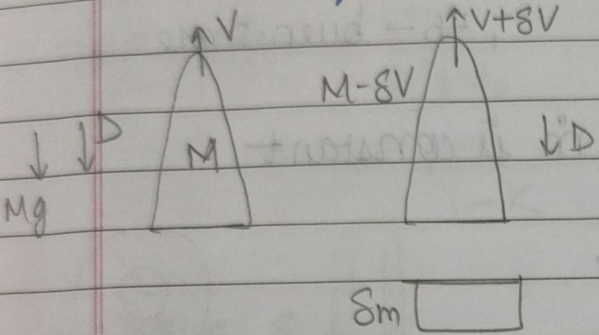
$t = t_b$ - burn-out

$M_f = M_i + M_l$



$$J - Mg - D = M\frac{dV}{dt}$$

$$J = \dot{m}u_{eq} = \dot{m}u_e + (p_e - p_a)A_e$$



$$(t+\delta t)(MV) = -Mg - D$$

$V \rightarrow$ Velocity

At height h, $u_{eq}(h)$

$g(h) = g_0$

$D(V, p, M)$

$$dV = -\frac{dM}{dt}\frac{u_{eq}}{M}dt - g_0 dt - \frac{D}{M}dt$$

$$dV = -u_{eq}\frac{dM}{M} - g_0 dt - \frac{D}{M}dt$$

$$\Delta V = \int_0^2 dV$$

$$= \underbrace{-\int u_{eq}\frac{dM}{M}}_{\Delta V_{ideal}} - \underbrace{g_0 t}_{\substack{\Delta V_g \\ \text{'g-t loss'}}} - \underbrace{\int \frac{D}{M}dt}_{\Delta V_D}$$

$$\Delta V = \Delta V_{id} - \Delta V_g - \Delta V_D$$

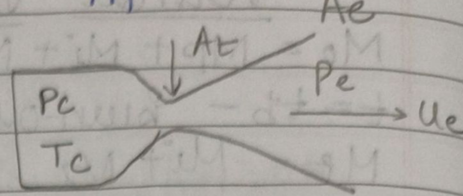
$$J = \dot{m}u_{eq} = \dot{m}u_e + (p_e - p_a)A_e$$

$J = \dot{m}C^* C_f$, $C^* \rightarrow$ characteristic velocity

$C_f \rightarrow$ Thrust coefficient

$\dot{m}c^* = P_c A_t$ or $c^* = \frac{P_c A_t}{\dot{m}}$

$u_{eq} = c^* C_f$



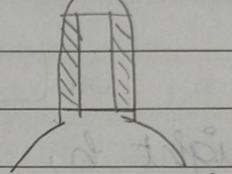
$\dot{m} = f(P_c, T_c, \gamma, A_t)$

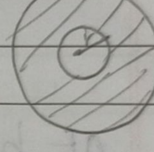
Propellant comb → T_c, γ.

Flow rate $\dot{m} = \frac{M_p}{t_b}$, t_b – burn time

For liquid rocket → $\dot{m}$ is constant

For solid propellant –



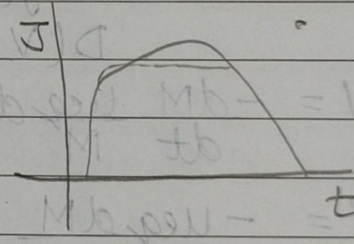


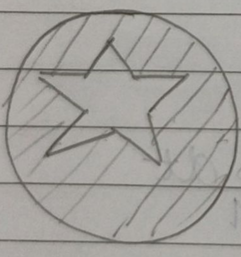
$2\pi r l$

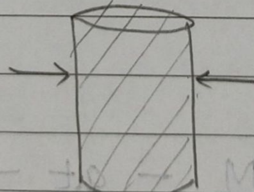
$M\frac{dV}{dt} = J$

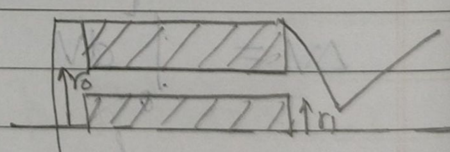
$a = \frac{dV}{dt} = \frac{J}{M}$

$\eta = \frac{a}{g} = \frac{J}{Mg}$

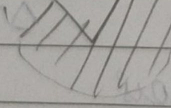




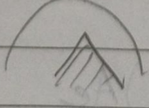




$A = 2\pi(r_i + r_o)L$



area increases



area decreases

$\eta = \frac{J}{Mg}$

$\left(\eta = \frac{a}{g}\right)$

$\pi(r_0^2 - r^2)L - 2\pi r L \dot{r}$

$\eta = \frac{J}{Mg} = \frac{M_p u_{eq}}{M g_0 t_b}$ at $t=0$ $\eta_0 = \frac{M_p}{M_0}\frac{I_{sp}}{t_b}$

$k = \frac{M_p}{M_0}$, $\eta_0 = \frac{k I_{sp}}{t_b}$

At $t = t_b$

$M_f = M_0 - M_p = M_0(1-k)$

$\eta_f \equiv \eta_{max} = \frac{J}{M_f g_0} = \frac{M_0}{M_f}\eta_0$

$\eta_{max} = \frac{\eta_0}{1-k}$

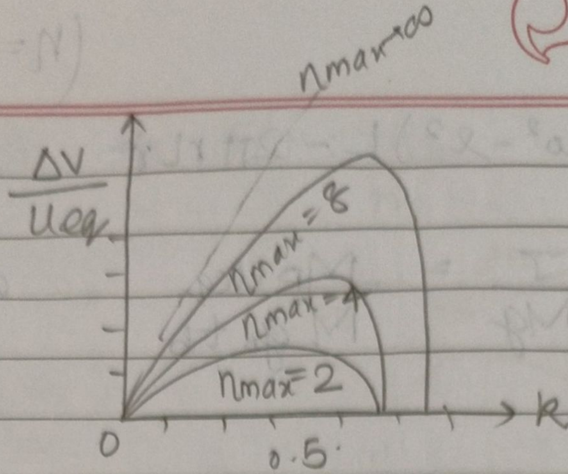
	t_b	η_{max}
* Large SLV (~~Space~~ Launch vehicle) satellite	2-8 min	1.2-6
Strapon Booster	0.5-2 min	1.2-3
s/c orbit maint/ maneuvers	<10 min	0.1-6
SAM (anti a/c missile)	2-75 s	5-20 (<100)
Air launched guided missile	2-5 s (booster) + 10-30 s (sustainer)	<25
Rocket assisted proj (gun launched)	~few seconds	upto 20000

$t_b = \frac{k}{1-k}\frac{I_{sp}}{\eta_{max}}$

$\dot{m} = \frac{M_p}{t_b}$

$\Delta V = u_{eq}\ln\left(\frac{M_0}{M - M_p}\right) - g_0 t_b = u_{eq}\ln\left[\frac{k}{1-k}\right] - \frac{u_{eq}}{\eta_{max}}\left(\frac{K}{1-k}\right)$

$$\frac{d}{dk}(\Delta V) = 0$$



Tsiolkovsky equation

$$V_b \equiv \Delta V = u_{eq} \log\left(\frac{M_0}{M_f}\right) - g_0 t_b$$

classmate

Date 7/9/22

Page

Lecture -13

Drag

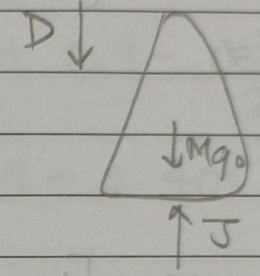
$$\eta_0 = \frac{F}{m_0 g_e}$$ force req for lift off

$$K_{opt} = 1 - \frac{1}{\eta_{max}}$$

$$\Delta V_{opt} = u_{eq}\left[\ln(\eta_{max}) + \frac{1}{\eta_{max}} - 1\right]$$

$$t_{bopt} = \frac{K I_{sp}}{\eta_0} \equiv \frac{K_{opt}}{1-K_{opt}} \frac{I_{sp}}{\eta_{max}} \equiv K_{opt} I_{sp}$$

$$\eta_{0opt} = 1$$



$$M\frac{dV}{dt} = J - D - Mg$$

Neglect drag

$$M\frac{dV}{dt} = \dot{m} u_{eq} - Mg_0$$

$$dV = -\frac{dM}{M} u_{eq} - g_0$$

Beginning, at $t=0, V=0, h=0$

$$V(t) = \int_0^t dV = u_{eq}\ln\left[\frac{M_0}{M(t)}\right] - g_0 t$$

$$M(t) = M_0 - \dot{m}t \equiv M_0 - \frac{Kt}{t_b}$$

$M_p = \dot{m} t_b$ | $t_b \rightarrow$ total burn time

$$V(t) = -u_{eq}\ln\left[1 - \frac{Kt}{t_b}\right] - g_0 t$$

$$h(t) = \int_0^t v(t)\,dt = -u_{eq}\int_0^t \ln\left[1 - \frac{Kt}{t_b}\right]dt - \frac{1}{2}g_0 t^2$$

classmate

Date

Page

$$h(t) = \int_0^t v(t)\,dt = -u_{eq}\int_0^t \ln\left[1 - \frac{kt}{t_b}\right]dt - \frac{1}{2}g_0 t^2$$

$$= u_{eq}\left\{t + \left(\frac{t_b}{k} - t\right)\ln\left(1 - \frac{kt}{t_b}\right)\right\} - \frac{1}{2}g_0 t^2$$

We can use $v(t) = \cancel{g_e} - u_{eq}\ln\left[\left(\frac{m(t)}{m_0}\right)\right] - g_e t$

$$h_{b0} = u_{eq} t_b\left(\frac{m_f}{m_0 - m_f}\ln\left(\frac{m_f}{m_0}\right) + 1\right) - \frac{g_e t_b^2}{2}$$

$$h_{b0} = u_{eq} t_b\left(\frac{1-k}{k}\ln(1-k) + 1\right) - \frac{g_e t_b^2}{2}$$

$$\frac{dh_b}{dk} =$$

$$t_b = \frac{M_p}{\dot{m}} = \frac{M_p u_{eq}}{\eta_{max}[M_0 - M_p]g_0}$$

$$= \frac{k}{1-k}\frac{I_{sp}}{\eta_{max}}$$

$$h_{bom} = \frac{\underbrace{g_e I_{sp}}^2}{\frac{u_{eq}}{g_0}}\left[(1-k)\ln(1-k) + k - \frac{k^2}{2}\right]$$

$$k = k(t_b)$$

$$\frac{dh_b}{dk} = \frac{\partial h_b}{\partial k} + \frac{\partial h_b}{\partial t_b}\cdot\frac{\partial t_b}{\partial k} = 0$$

$$\eta_0 = 1,\ v_{b0} = u_{eq}\left[\ln\left(\frac{1}{1-k}\right) - k\right]$$

$$h_{b0} = \frac{u_{eq}^2}{g_0}\left[(1-k)\ln(1-k) + k - \frac{k^2}{2}\right]$$

$$m g_0 h_{max} = m g_0 h_b + \frac{1}{2} m v_b^2$$

$$h_{max} = h_b + \frac{v_b^2}{2g}$$

Ex:- $u_{eq} = 3000$ m/s

$K = 0.9$

$V_b = 4200$ m/s $\quad h_c = 900$ km

$h_{bo} = 240$ km $\quad h_{max} = 1140$ km

$$V_{bo} = 3000\left[\ln(10) - 0.9\right]$$

$= 3000\,[2.303 - 0.9)$

$= 3000 \times 1.403$

$= 1403 \times 3 = 4209$ m/s

$$h_{bo} = \frac{9\times10^6}{10}\left[0.1\times \ln(0.1) + 0.9 - \frac{0.81}{2}\right]$$

$= 9\times10^5\left[-0.1\times 2.303 + 0.9 - 0.405\right]$

* Sounding Rockets (~20 min above 100 km)

Launch $\angle \rightarrow 85^\circ$

Can reach an apogee upto 1600 km

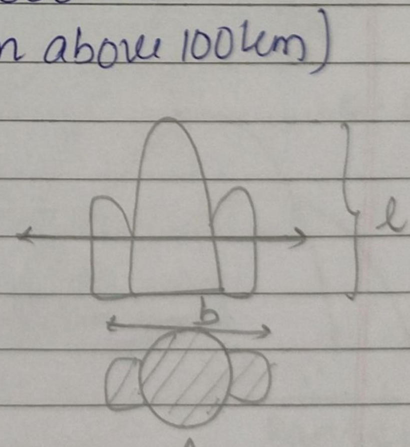
For $M_e \sim 500$ kg, $h_{max} \le 600$ km

(< 10 min above 100 km)

$$D = \frac{1}{2}\rho V^2 A C_D$$

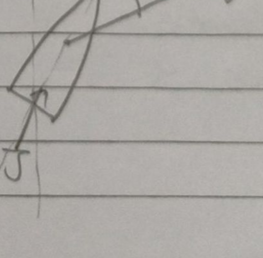
Drag $D \sim L^2$

Diameter $d = \sqrt{\frac{4A}{\pi}}$



$M \sim \forall \rho \sim L^3$

$\frac{D}{M} \sim \frac{1}{L}$



$$dV = \frac{-dM}{u_{eq}} - g\cos\theta\, dt - \frac{D}{M}\, dt$$

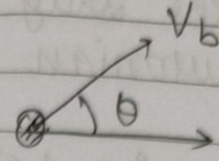
Stability:-
Movable fins
Gimbled Thrust
Vernier rockets
Thrust vane.

Which chapter from book?
No chap from book.

Date 5/9/22

Projectile Basics

$\frac{dV_x}{dt} = 0$, $\frac{dx}{dt} = V_x$



$\frac{dV_y}{dt} = -g_0$ $\frac{dy}{dt} = V_y$

$V_x(t) = V_b \cos\theta$

$V_y(t) = V_b \sin\theta - g_0 t$

$x(t) = V_b \cos\theta\, t$

$h(t) \equiv y(t) = v_b \sin\theta\, t - \frac{1}{2} g_0 t^2$

$h_{max} = \frac{V_b^2 \sin^2\theta}{2g_0} = \frac{R\tan\theta}{4}$

$R = \frac{V_b^2 \sin 2\theta}{g_0}$ (Range)

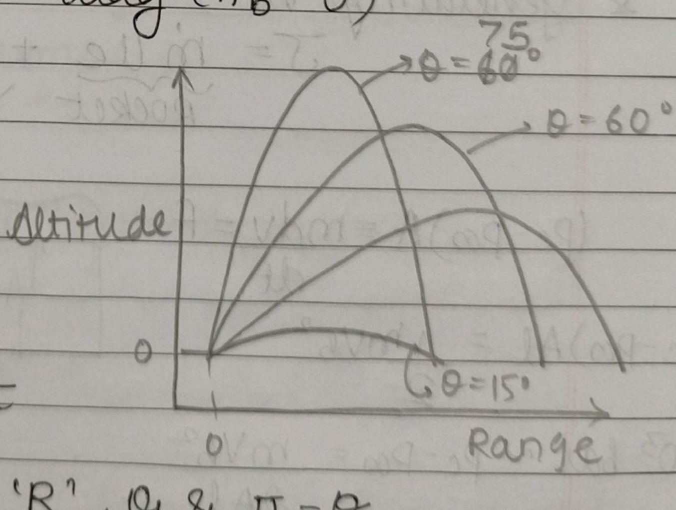
Peak $\rightarrow$ $V_y = 0 \rightarrow t_P = \frac{V_b \sin\theta}{g_0}$

Flight time $\rightarrow t_f = 2t_P$.

* Trajectory without drag ($h_b = 0$)

$\theta_1 < \frac{\pi}{4} \rightarrow \tan\theta_1 < 1$

$\frac{h_{max_1}}{h_{max_2}} = \tan^2\theta_1$



2 angles for max ht

For same range 'R', θ_1 & $\frac{\pi}{2} - \theta$

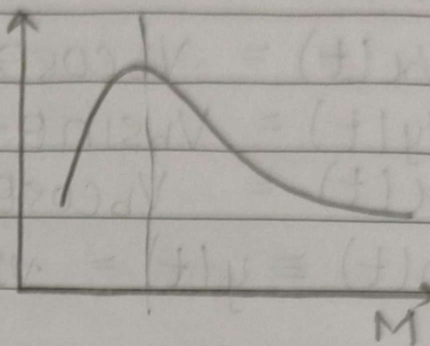
If we consider drag

Stokes Drag $\propto \nu V$

Newtonian drag $\propto \rho V^2$

$$D = AC_D \frac{1}{2}\rho V^2 = AC_D \frac{\gamma}{2} p M^2$$

$M > 2$, $D \propto pM$



$$\frac{d\vec{V}}{dt} = -g_0 \hat{e}_y \mp \frac{D}{\text{Mass}} \hat{e}_u$$

$\uparrow$

ρV^2 or νV (Stokes)
(Newton)

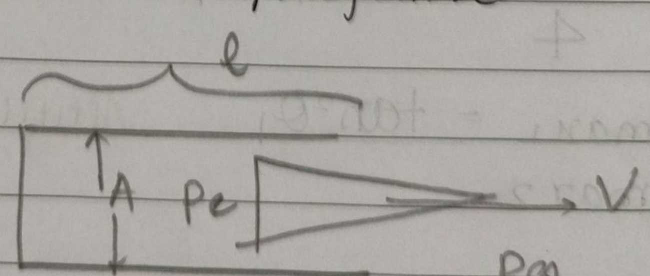
Stokes drag:- Ht decreases & Range increases.

The peak $\angle$ that gives max altitude has reduced.

Generating V_b

$$J = \underbrace{\dot{m} u_e}_{\text{Rocket}} + \underbrace{(p_e - p_a) A_e}_{\text{projectile}}$$

$$(p_c - p_\infty) A \equiv m \frac{dv}{dt} = F$$



$$Al = \frac{1}{2} m V_b^2$$

$$\sim p_c - p_\infty = \frac{m V_b^2}{2Al}$$

* Ballistic Missiles

Without drag, $R' = \frac{V_b^2 \sin 2\theta}{g_0}$

$V_b = \sqrt{\frac{R g_0}{\sin 2\theta}}$ for $\theta = 45°$

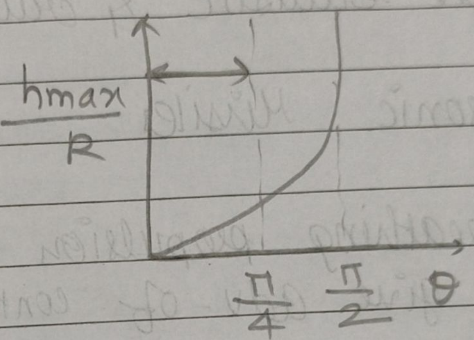
Inter continental Ballistic missile

SRBM, MRBM, IRBM, ICBM

<100 km, 1k–3k km, 3–5k km, >5,500.

>7.3 km/s.

3.13 km/s



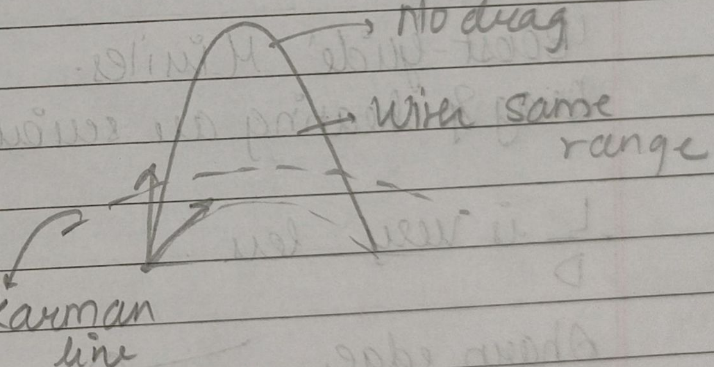
· Avoid Drag

$h_{max} = \frac{R}{4} \tan\theta$

R – 8000 km

$\theta = \pi/4$

$h_{max} = 2000$ km



$h_k = y(t) = V_0 \sin\theta\, t_k - \frac{1}{2} g t_k^2$

→ ht of karman line

tp → time to reach peak altitude

$\frac{2h_k}{2h_{max}} \sim \frac{2t_k}{2t_{ps}} < 1$

* ICBM Trajectories (Depressed angle, lofted ∠)

Atmospheric Re entry
Empirical correlations for stagnation region heat flux.

$$q^n = C\sqrt{\frac{\rho_\infty}{R_C}}\, V_\infty^3\left(1-\frac{h_w}{h_0}\right)$$

Blunt bodies have lower heat flux.

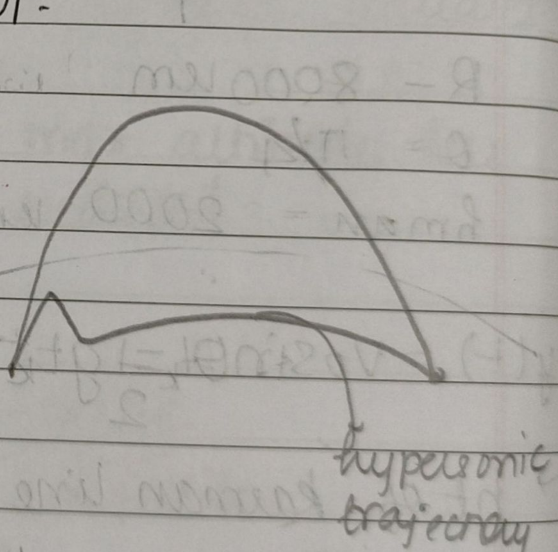
Re entry thermal protection sys (TPS)
Diff materials are used.

Indian Ballistic & cruise missiles

Hypersonic Missiles

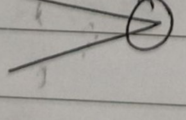
Air breathing propulsion at similar speeds (M>)
would give adv. of control.

'Boost-glide' Missiles.
Drag & heating are serious.



$\frac{L}{D}$ is very less.

Sharp edge



BRAHMOS, At M = 2

$$T_0 = T\left[1+\frac{\gamma-1}{2}M^2\right]$$

$T_0 = 8T \sim 1600k$.
Ramjet is used.

PROJECTILE BASICS FROM SLIDES

$h_b \rightarrow$ burnout ht

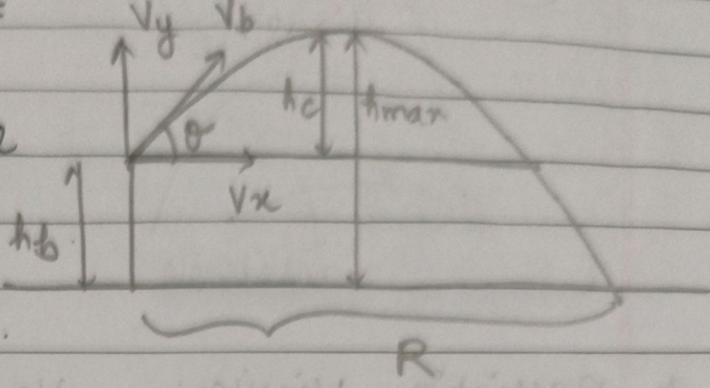
When drag $= 0$, ~~$h_b = 0$~~

h_b can be 0

$\rightarrow$ Range of $0°$ & $75°$ are same.

If $h_b > 0$,

Range of $0° >$ of $75°$.

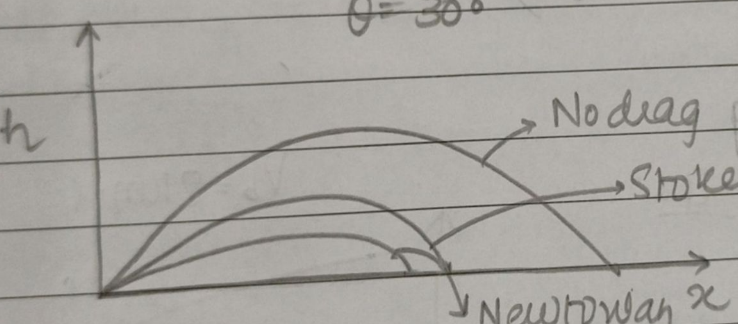


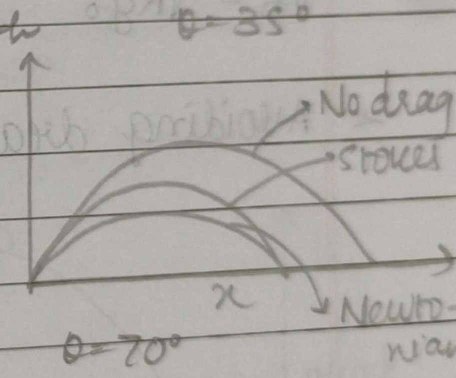
With drag $\rightarrow$

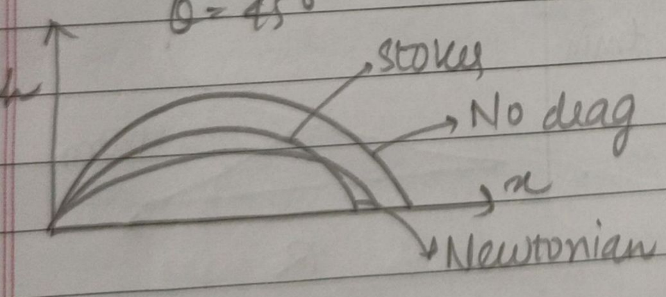
1) Stokes (Viscous) $\propto \nu V$ — coeff of viscosity

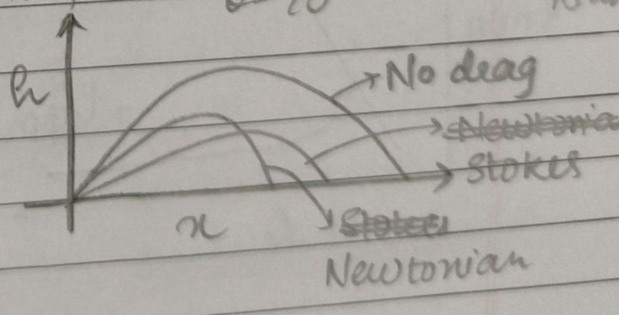
2) Newtonian $\propto \propto V^2$ — $\frac{1}{2}\rho$

Both Range & ht reduces (Stokes, $h_b = 0$)









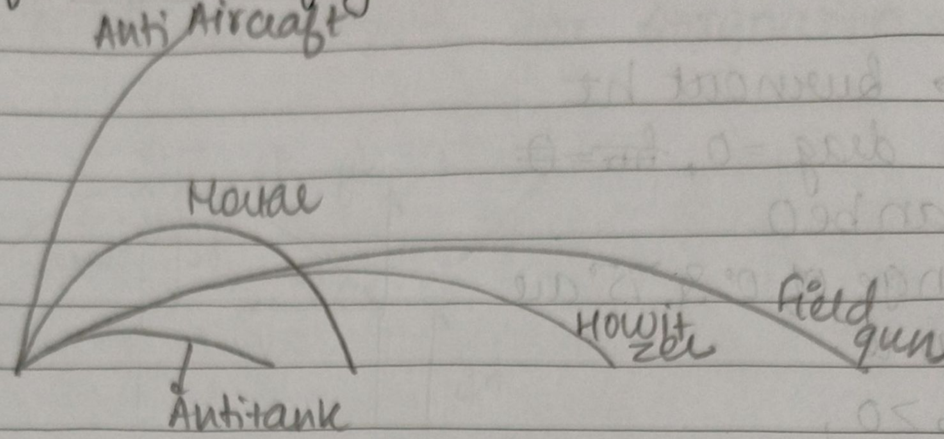
* Generating V_b

$J = \dot{m}u_e + (p_e - p_a)A_e$

rocket — projectile

- Two types $\rightarrow$ gun sys with recoil + Recoilless gun sys (Gases ~~[illegible]~~ Reward).

* Classification of guns



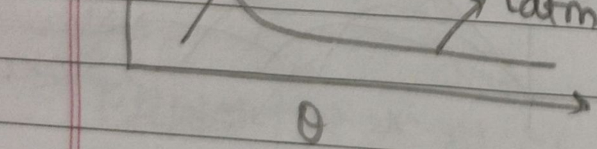
* Ballistic Missiles :-

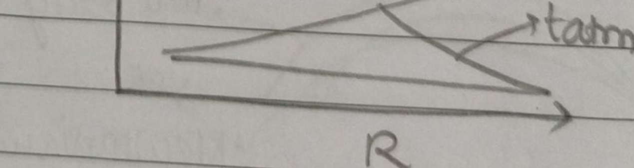
	V_b
1) SRBM	3.13 km/s
2) MRBM	3.13 - 5.4
3) IRBM	5.4 - 7.3
4) ICBM (Intercontinental Ballistic missiles)	> 7.3

$$V_b = \sqrt{R g_0} \quad (\theta = 45°)$$

* Avoiding drag

$V_b = 8$ km/s





BOOK NOTES

CHAPTER-2
Mission analysis fundamentals

* Rocket-propellant types:-

1) Launch vehicles
2) Upper stage or orbital transfer vehicles
3) Interceptors
4) Ballistic missiles

Components
- Inert comp
- Propellant
- Payload
  - Useful
  - Not useful

• Propellant mass fraction →

$\Lambda = \frac{m_p}{m_p + m_i}$, as size inc, thick dec to 'min gauge', propellant inc

$\Lambda \to 1$.

Process:-

1) Select a propulsion sys
2) Derive mission eq.
3) Prelim design & trade studies
4) Detailed design
5) Demonstration & Qualification testing

~~CHAPTER - 3~~

$\Delta V = V_e - V_{surf}$

$F_g = m g_e \left(\frac{r_e}{r}\right)^2 \rightarrow$ at dist r from origin

$$P.E. = -\int \frac{m g_e r_e^2}{(r_e+z)^2} dz = \frac{m g_e r_e z}{(r_e+z)}$$

P.E. on surface $= 0$

$$K.E = \frac{m V_z^2}{2}$$

$$V_z = \sqrt{\frac{2 g_e r_e z}{(r_e+z)}}$$

$V_z \rightarrow$ initial vel of mass at surface that will attain altitude z.

If $z \rightarrow \infty$, $V_\infty = \sqrt{2 g_e r_e} = 11.2 km/s$

* Mission requirements for upper stage or OTV.
Vacuum of space
1) Length & size (Package ability)
2) Propulsion sys weight.
3) Guidance considerations. (Three axis control sys)

Two stage → Interstage in betn
Extendible exit cones (EEC) → skirt to inc size
(Inertial Upper stage) (IUS)

Liquid sys → Centaur upper stage (RL10 engine)

- Placing satellite into GEO.
If $\tau = 1$, $Z = 35800$ km.
Launch vehicle transfers spacecrafts.

Initial GTO (Apogee at GEO & perigee at LEO)

Liquid Apogee engines (LAEs) - provide final apogee impulse.

- First stage → creates an elliptic orbit with A-rad of final orbit.

Perigee kick motors (PKM), apogee kick M.

$$\Delta V = \Delta V_1 + \Delta V_2$$
$$\Delta V_1 = V_{c1}\left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right], \quad \Delta V_2 = V_{c1}\left[\sqrt{\frac{r_1}{r_2}} - \sqrt{\frac{2r_1}{r_2(1+r_2/r_1)}}\right]$$

V_{c1} → vel in initial circular orbit

If solar resources, $t_b >> T$ (No hohmann)
$$\Delta V = \sqrt{V_{c1}^2 - 2V_{c1}V_{c2} + V_{c2}^2}$$
(No plane change)

* With Plane change (Not hohmann)

$$\Delta V = \sqrt{V_{c1}^2 - 2V_{c1}V_{c2}\cos\left(\frac{\pi\alpha}{2}\right) + V_{c2}^2}$$

For Hohmann, $\Delta V = 2V_c \sin(\alpha/2)$

Better to change plane at higher altitude.

Mission requirements for Ballistic missiles.

Deliver a nuclear warhead to fixed location.

Long range - 3 propulsive stages + post boost vehicle

Parameters:-

Burning time of each stage

Short response (i.e. why deploy SRMs)

Inert weights

Guidance sys

Reentry heating (Composite materials)

Zero-lift' trajectory - 0 ∠ of attack.

Elliptical orbit with focal pt at the center.

Free flight range ∠ ψ.

$r_e \psi$ - Range.

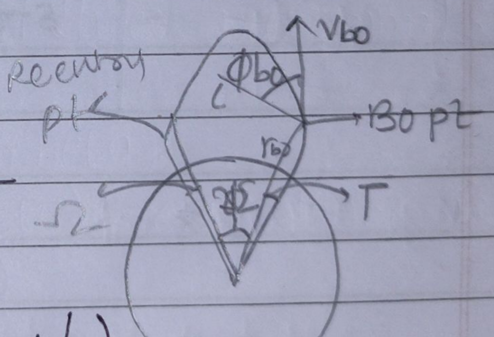
Typical burnout altitudes ~ 250-500 miles

Uses gravity to steer vehicle onto desired trajectory. Adv - uses vehicle's own thrust

Initial ascent phase - even zero ∠ of attack.

Minimizes aerodynamic stress.

$$Q_{bo} = \frac{V_{bo}^2 r_{bo}}{\mu} = \left(\frac{V_{bo}}{V_c}\right)^2$$



$$\cos\left(\frac{\psi}{2}\right) = \frac{1 - Q_{bo}\cos^2\phi_0}{\sqrt{1 + Q_{bo}(Q_{bo}-2)\cos^2\phi_{bo}}}$$

$$\sin(2\phi_{bo} + \psi/2) = \frac{2 - Q_{bo}}{Q_{bo}} \sin(\psi/2)$$ → high trajectory & low trajectory solution

$$Q_{bo} = \frac{2\sin(\psi/2)}{1 + \sin(\psi/2)}$$ - Max Range

$$Alt_{max} = \frac{r_{bo}}{2 - Q_{bo}}\left[1 + \sqrt{1 + Q_{bo}(Q_{bo}-2)\cos^2\phi_{bo}}\right] - r_e$$

* Interceptors

Unique rocket designed to ~~close in~~ defend on a moving target. (Kinetic kill vehicle → KKV)

Boosted by two axial propulsion SRMs.

Two pulses, two coast phases.

1-3 stage

Solid or liq with storable propellants

Liquid → gel propellants.

Considerations →

1) Efficient Packing (carried on the aircraft)
   ↳ low size.

2) Target maneuverability

CHAPTER 3 → Trajectory Analysis & Rocket Design

* Chapter 2 → Δv → mission requirements
  Chapter 4 → Isp
  Chap 3 → Size & Mass

The Rocket eqn → The Tsiolkovsky eqn.

Trajectory optimization

1) Vertical Trajectory
2) t_b & acc levels
3) Why staging
4) general form of trajectory eqns.

* Vertical Trajectory $\quad T = \dot{m} g I_{sp}$

$$-mg - D + \dot{m} g I_{sp} = m\frac{dv}{dt}$$

If ~~the~~ rocket is expelling only propellant gases

$$dv = -g I_{sp}\frac{dm}{m} - \frac{D}{m}dt - g\,dt$$

$$\Delta V = g_e I_{sp} \ln\left(\frac{m_0}{m_f}\right) - \underbrace{\int_0^{t_b} g\,dt}_{\text{g-t loss } (\Delta V_g)} - \underbrace{\int_0^{t_b}\left(\frac{D}{m}\right)dt}_{\text{Drag loss}}$$

(If Isp is const)

$$\Delta V_{ideal} = g_e I_{sp} \ln\left(\frac{m_0}{m_f}\right)$$

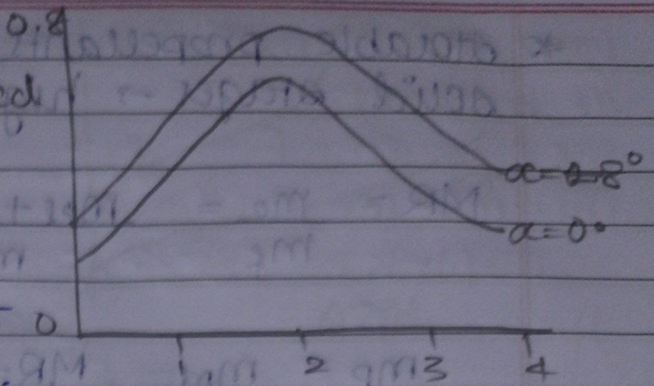
$$D = -C_D A_f \frac{\rho V^2}{2}$$

A_f → frontal area

$q = \frac{\rho V^2}{2}$ → dynamic press

$$\frac{D}{m} \propto \frac{1}{L}$$

$$\Delta V_D = \int_0^{t_b} \left(\frac{D}{m}\right) dt$$



$$M_\infty = \frac{V}{\sqrt{\frac{\gamma\, p(h)}{\rho(h)}}}$$

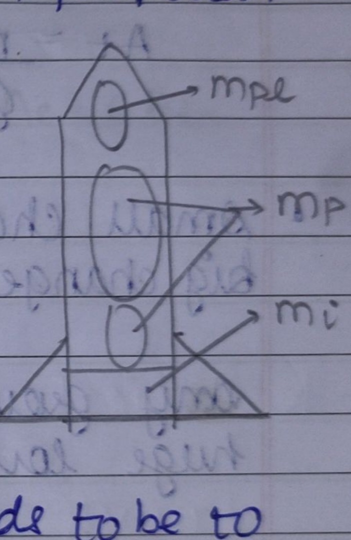
• Steering losses – When thrust is not aligned with vehicle.

$$\Delta V_{ideal} = \Delta v + \Delta v_g + \Delta v_D + \Delta v_s$$

↳ actual vel gain to be imparted to payload

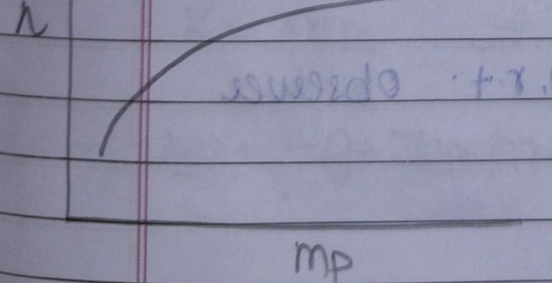
• Rocket comprises of payload, inert, propellant mass

Structural efficiency is determined by $\Lambda = \frac{m_p}{m_p + m_i}$

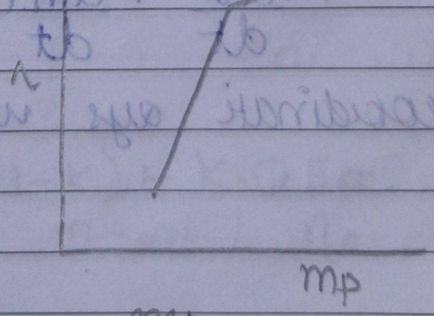


• As sys gets bigger, Λ increases bcoz of 'cubed-squared' law.

We can't make it as thin as it needs to be to retain design loads.



Small motors



mp

* Storable propellants → higher λ than cryogenic
solid stages → higher λ than liq.

$$MR = \frac{m_o}{m_f} = \frac{m_{pl} + m_p + m_i}{m_{pl} + m_i} = \frac{\lambda m_{pl} + m_p}{\lambda m_{pl} + m_p(1-\lambda)}$$

$$m_p = m_{pl} \frac{MR - 1}{MR - (MR-1)/\lambda}$$

$$MR = e^{(\Delta v_{id}/g_e I_{sp})}$$

avg mass flow rate → $\frac{m_p}{t_b}$

$$A_t = \frac{m_p c^*}{g_e t_b P_c}$$, c^* → characteristic vel
P_c → chamber pres.

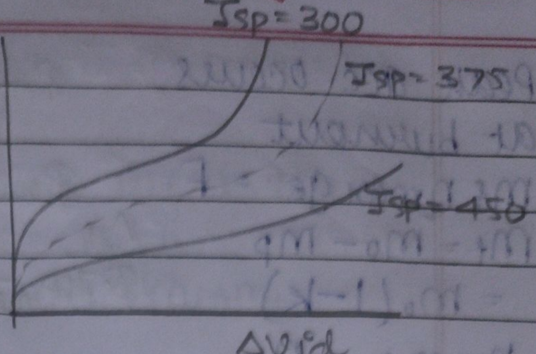
- Small change in performance i.e. Δv can cause big change in payload.

Any growth in structural weight can lead to huge losses in payload. So multiple stages used.

- Considering Newton's 2nd Law
$$\Sigma \text{Forces} = m\frac{du}{dt} + \frac{dm}{dt}(v)$$

Inertial coordinate sys w.r.t. observer

Example



$m_{pl} = 100\ lb$

$\Delta v_{id} = 3000\ ft/s,\ \lambda = 0.8$

$C^* = 4900\ ft/s,\ \gamma = 1.2$

$t_b = 3s,\ p_c = 1000\ psi$

$\frac{p_e}{p_c} = 0.0147,\ p_e = p_a,\ p_a = 14.7\ psi$

$\epsilon = 9\quad C_f = C_{f,opt} = 1.6$

$I_{sp} = \frac{C_{f,opt}\ C^*}{g} = 243\ s$

$\Delta v_{id} = g\ I_{sp} \ln(MR)$

$\text{min}\ \cancel{A} = m_p\left(\frac{1}{\lambda} - 1\right),\ g = \cancel{g}.$

$\text{min} = m_p\left(\frac{1}{\lambda} - 1\right),\ \dot{m} = \frac{m_p}{t_b},\ A_e = \epsilon A_t$

* Burning time & Acc effects:-

$n_0 = \frac{F}{m_0 g_e} > 1,\ n = \frac{a}{g_e}$

$t_b = \frac{m_p}{\dot{m}} = \frac{m_p I_{sp}}{F}$

Trajectory near earth:- $\Delta v = g_e I_{sp} \ln\left(\frac{m_0}{m_f}\right) - g_e t_b$

$K = \frac{m_p}{m_0}$

$\Delta v = -g_e I_{sp} \ln(1-K) - K \frac{g_e I_{sp}}{n_0}$

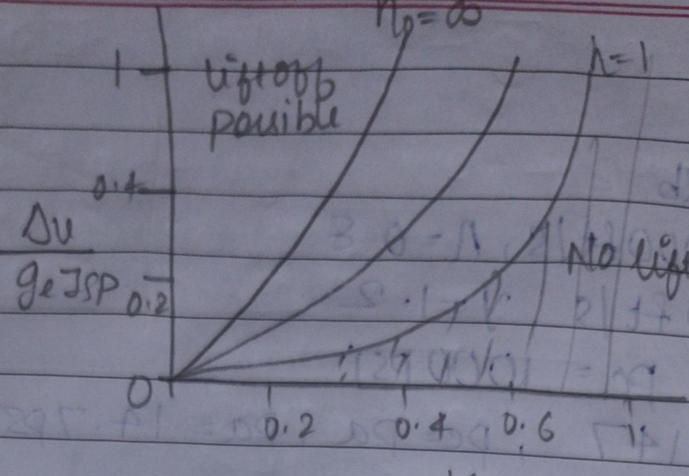
Peak acc occurs at burnout

$m_f \eta_{max} g_e = F$

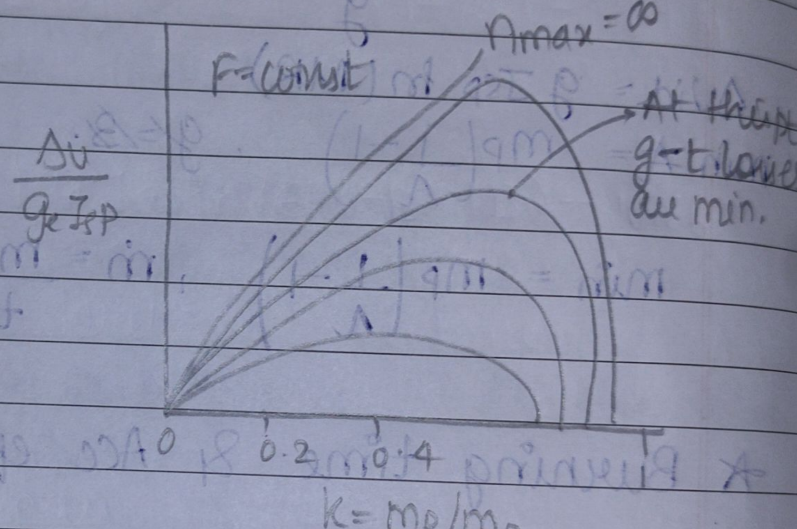
$m_f = m_0 - m_p$

$= m_0(1-k)$

$\eta_{max} = \dfrac{n_0}{1-k}$



$$\Delta V = -g_e I_{sp} \ln(1-k) - \frac{k g_e I_{sp}}{(1-k)\eta_{max}}$$



★ Vertical ht calculations

Launch from earth's surface, $v_0 = h_0 = 0$

$$v(t) = -g_e I_{sp} \ln\left(\frac{m(t)}{m_0}\right) - g_e t$$

$$m(t) = m_0 - \dot{m} \qquad 0 < t < t_b$$

$$h_{b0} = g_e I_{sp} t_b \left(\frac{m_f}{m_0 - m_f} \ln\left(\frac{m_f}{m_0}\right) + 1\right) - \frac{g_e}{2} t_b^2$$

classmate

Date

Page

$h_{bom} = \frac{1}{2} g_e t_b^2$ (max)

consider fixed payload & inert mass

$$K(t_b) = \frac{m_p}{m_p + m_i + m_{pl}} = \frac{\dot{m} t_b}{\dot{m} t_b + m_i + m_{pl}}.$$

For taking K as varying,

$$h_{bom} = g_e I_{sp}^2 \left[(1-K)\ln(1-K) + K - K^2/2 \right]$$

Burnout vel $= V_{bo} = g_e I_{sp} \left[\ln\left(\frac{1}{1-K}\right) - \frac{1}{n_o} \right]$

Additional ht during coasting

K.E at burnout $=$ P.E at apogee.

$$h_c = \frac{1}{2} g_e I_{sp}^2 \left[\ln(1-K) + \frac{K}{n_o} \right]^2$$

* Multistage systems
  (N stage rocket), $n \le N$
- m_{fn} = burnout mass of stage n
- m_{on} = Initial mass of stage n
- ϵ_n = stage structural factor m_{fn}/m_{on}
- β_n = payload factor $= \frac{\sum_{n+1}^{N} m_{on}}{\sum_{n}^{N} m_{on}}$

$$\epsilon_N = \frac{m_{fN} - m_{pl}}{m_{oN} - m_{pl}}, \quad \beta_N = \frac{m_{pl}}{m_{oN}}$$

Small ϵ values.

$$V_{boN} = \sum_{j=1}^{N} \Delta V_j$$

$$\Delta v_j = g_e I_{spj} \ln(MR_j) - \bar{g}_j t_{bj}$$

$\bar{g}_j = \int_0^{t_b} g\,dt / t_b$, $MR_j \rightarrow$ stage mass ratio

$$MR_j = \frac{\sum_{i=j}^{N} m_{oi}}{\sum_{i=j+1}^{N} m_{oi} + m_{PL}}$$

$$MR_j = [\epsilon_j(1-\beta_j) + \beta_j]^{-1}$$

$$\therefore \Delta v_j = -g_e I_{spj} \ln(\epsilon_j(1-\beta_j)+\beta_j) - \bar{g}_j t_{bj}$$

Special case:-

$$V_{boN} = -N g_e I_{sp} \ln(\epsilon(1-\beta)+\beta) - g_e \tau_b$$

$\tau_b = N t_b \rightarrow$ Total power on time

$$\Delta v_j = \frac{V_{boN}}{N}$$

$$G = \frac{\text{liftoff mass}}{\text{payload mass}} = \frac{\sum_1^N m_{oi}}{m_{PL}}$$

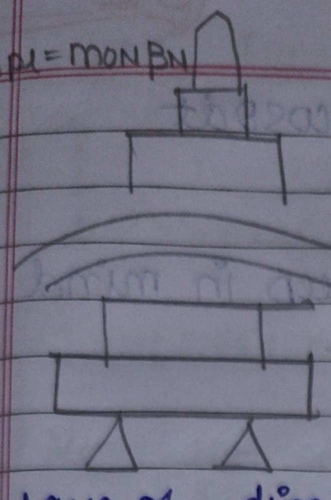
$$S = \frac{V_{boN} + g_e \tau_b}{g_e I_{sp}} = -N \ln(\epsilon(1-\beta)+\beta)$$

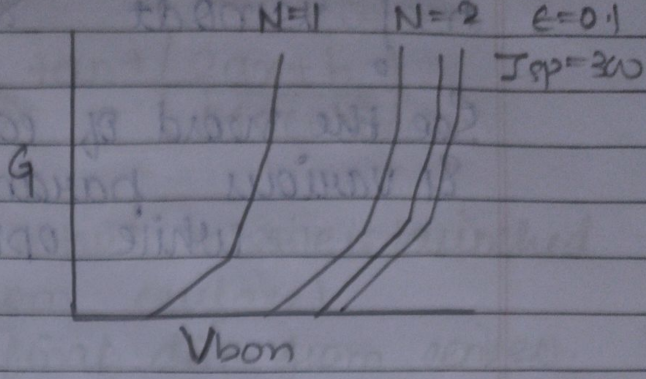
$$G = \prod_{j=1}^{N} \frac{1}{\beta_j} = \beta^{-N}$$

$$G = [(1-\epsilon) / (e^{-S/N} - \epsilon)]^N$$

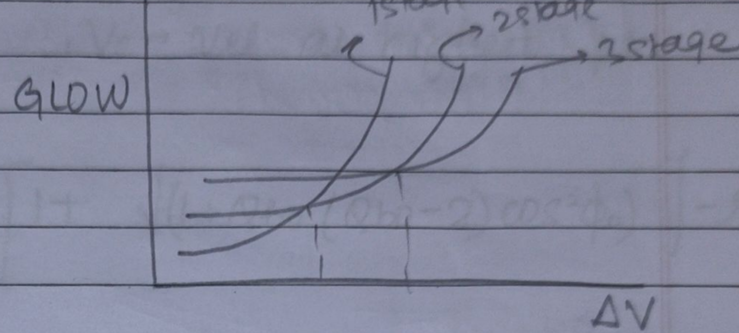
$$\lim_{N\to\infty} \ln G = e^{S(1-\epsilon)}$$

s ratio.

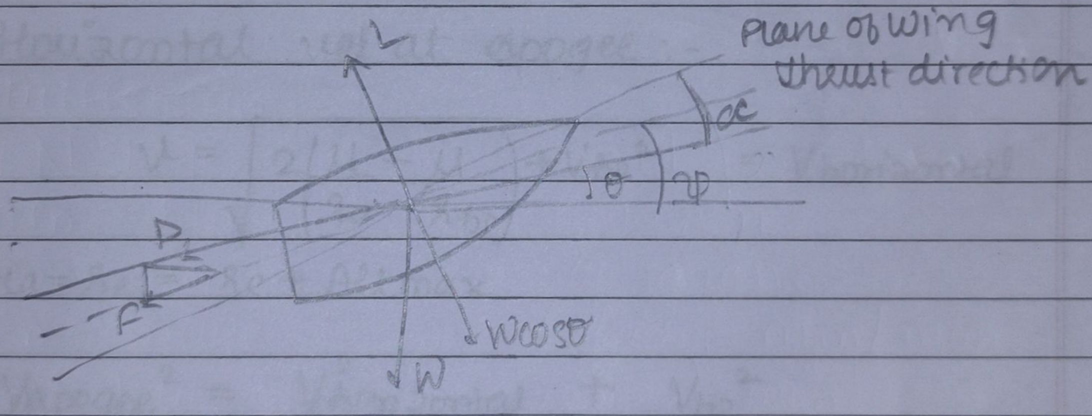




Law of diminishing results for multistage vehicles.



* Generalized Trajectories



$$m\frac{dv}{dt} = F\cos(\psi-\theta) - C_D\rho\frac{V^2A}{2} - mg\sin\theta \quad \text{(horizontal)}$$

$$\frac{V^2}{R} = \frac{V d\theta}{dt}$$

$$mv\frac{d\theta}{dt} = F\sin(\psi-\theta) + C_L\rho\frac{V^2A}{2} - mg\cos\theta$$

(vertical)

Re not included in C_L & C_D as turbulent range

SOME FORMULAE WHILE SOLVING CHAP 2 QUES

1) $\int \frac{1}{ax^2+bx+c}dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)$

2) $V_{surf} = r_e \times \omega$ (Vel at which missile is launched from surface)

$V_c = \sqrt{\frac{\mu_c}{r_c}}$, $r_c \rightarrow$ The final dist from center.

3) $Q_{bo} = \left(\frac{V_{bo}}{V_c}\right)^2$, $V_c \rightarrow$ vel at highest pt.

4) $Alt_{max} = \frac{r_{bo}}{2-Q_{bo}}\left[1 + \sqrt{(1+Q_{bo}(Q_{bo}-2)\cos^2\phi_0)}\right] - r_e$

$r_{bo} \rightarrow$ burnout altitude

5) Horizontal vel at apogee :-

$$v = \sqrt{2\left(\frac{\mu}{r_a} - \frac{\mu}{r_{bo}}\right) + V_{bo}^2} = V_{horizontal}$$

$r_a = r_c = r_e + Alt_{max}$

6) $V_{apogee}^2 = V_{horizontal}^2 + V_{bo}^2$

7) When boost phase ends, it has burnout velocity

8) $\frac{dV_r}{dt} = \frac{-1}{2} \frac{C_d A_r}{m_r} \frac{\rho_g}{m_r} V_r^2$

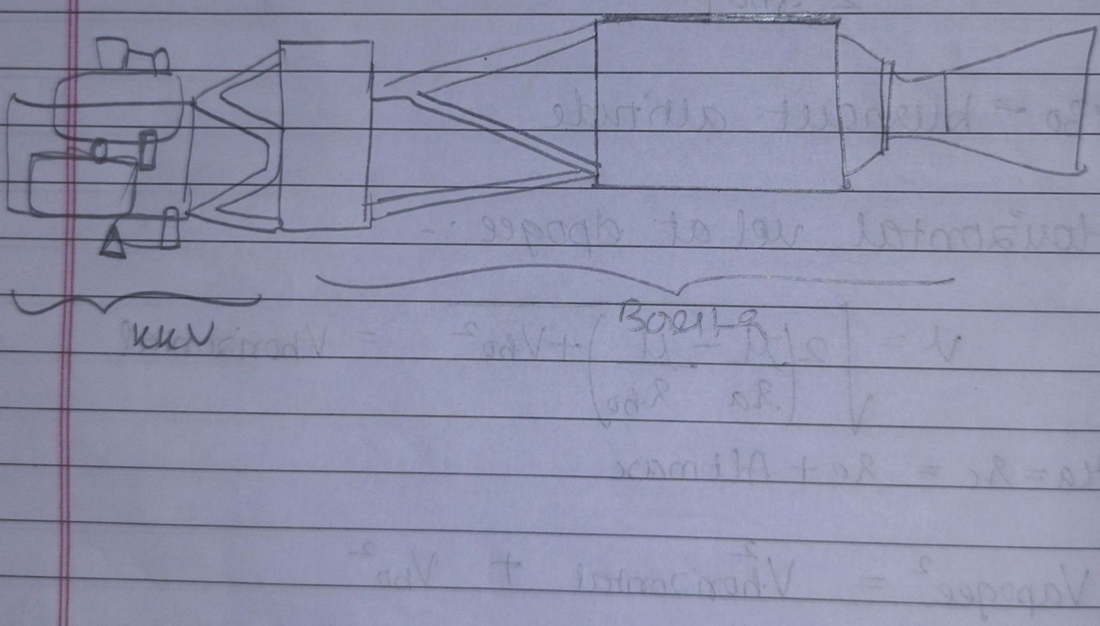
9) Range = $r_e \Psi$

$$\cos(\Psi/2) = \frac{1 - Q_{bo}\cos^2\phi_{bo}}{\sqrt{1 + Q_{bo}(Q_{bo}-2)\cos^2\phi_o}}$$

$$Q_{bo} = \frac{V_{bo}^2 r_{bo}}{\mu}$$

10) For max range, $Q_{bo} = \dfrac{2\sin(\Psi/2)}{1+\sin(\Psi/2)}$

11) $\dfrac{(\text{Dist from center})^3}{(\text{Time period})^2} = \dfrac{\mu}{4\pi^2}$



Confusion about $c^* \, C_f$

$\Delta V = \int_0^t dv$

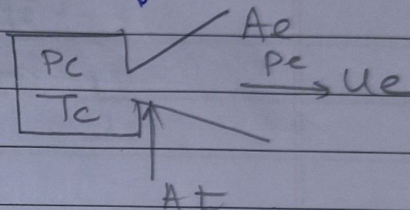
$\Delta V = -\int u_{eq} \frac{dM}{M} - g_0 t - \int \frac{D}{m} dt$

$\Delta V = \Delta V_{id} - \Delta V_g - \Delta V_D$

$J = \dot{m} u_{eq} = \dot{m} u_e + (P_e - P_a) A_e$

$J = \dot{m} c^* C_f$

$\dot{m} c^* = P_c A_t$ or $c^* = \frac{P_c A_t}{\dot{m}}$



$u_{eq} = c^* C_f$

$\dot{m} = f(P_c, T_c, \lambda, A_t)$

$\frac{M dv}{dt} = \dot{m} u_{eq}$

const $\dot{m} = \frac{M_p}{t_b}$, t_b → burn time

$u_{eq} = c^* C_f$

$\frac{(P_e - P_a) A_e}{\dot{m}}$

$c^* = \frac{P_c A_t}{\dot{m}}$

$u_{eq} = u_e +$

CHAPTER-1

Classification of Rocket propulsion systems & Historical perspectives.

* Fathers of Modern Rocketry:-
1) Konstantin Tsiolkovsky
2) Herman Obert
3) Dr Robert Goddard

* V-2 Rocket (Germans used)
  ↳ LOX & alcohol as propellants
  Dr Von Braun (Apollo)

* Intercontinental ballistic missiles (ICBMs)
  (Russia & America in cold war).

Atlas | Titan | Thor or Delta → US unmanned launch vehicle

Jet assisted take off (JATO)

Octob. 4, 1957 (SS-6 ICBM launched 'Sputnik' ~~to~~ into orbit) → Russia

NASA in 1958 (Saturn V → Neil Armstrong on Moon on 20 July, 1969)

• Ratio of payload mass to GLOW - less than 2%
very large devices to place smaller devices in orbit.

* Classification of Rocket prop sys. *

Specific Impulse Isp.

$I_{sp} = \frac{F}{\dot{m}}$ (Propellant efficiency)

Proper measure should be N-s/kg but it's only s.

Total Impulse = $\int_0^{t_b} F\,dt = F t_b$.

$I = I_{sp} \int_0^{t_b} \dot{m}\,dt = I_{sp}\, m_p$

$\Lambda \rightarrow$ propellant mass fraction

$\Lambda = \frac{m_p}{m_0} = \frac{m_p}{m_p + m_i}$

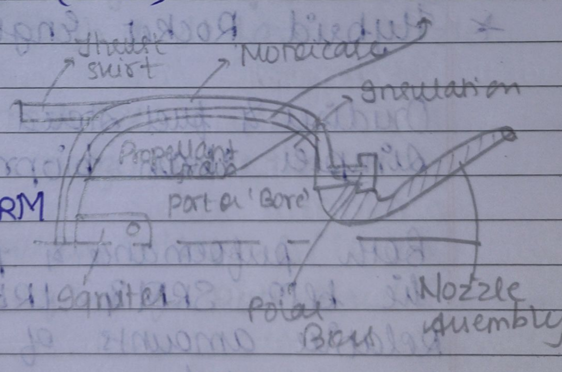
$m_0 \rightarrow$ overall mass of loaded prop sys.

high Isp $\Rightarrow$ high flame temp, high comb press $\rightarrow$ thicker chambers $\rightarrow$ lower mass fraction.

* Solid Rocket Motor (SRM)

Insulation

Propellant grain
$\rightarrow$ main propellant



High mass fraction of SRM
$\rightarrow$ attractive for first stage thrust.

CHAPTER-1

Classification of Rocket propulsion systems & Historical perspectives.

* Fathers of Modern Rocketry:-
1) Konstantin Tsiolkovsky
2) Herman Obert
3) Dr Robert Goddard

* V-2 Rocket (Germans used)
  ↳ LOX & alcohol as propellants
  Dr Von Braun (Apollo)

* Intercontinental ballistic missiles (ICBMs)
  (Russia & America in cold war).

Atlas / Titan / Thor or Delta → US unmanned launch vehicle

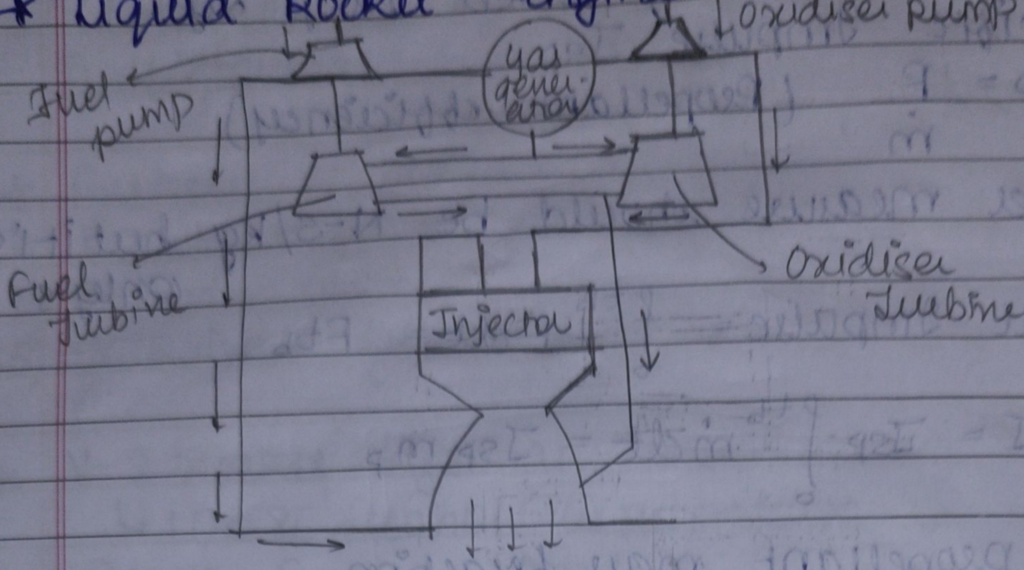
Jet assisted Take off (JATO)

Octob. 4, 1957 (SS-6 ICBM launched 'Sputnik' ~~to~~ into orbit) → Russia

NASA in 1958 (Saturn V → Neil Armstrong on Moon on 20 July, 1969)

• Ratio of payload mass to GLOW – less than 2% very large devices to place smaller devices in orbit.

* Liquid Rocket Engine



Engine power cycle → method used to obtain energy to drive turbine

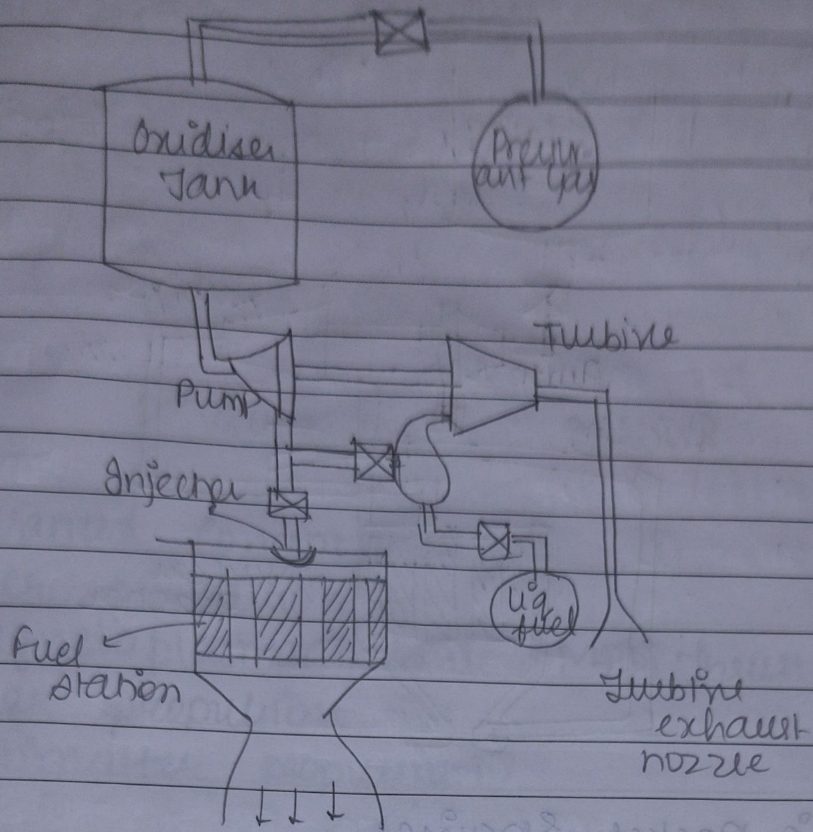
Pressure fed → contains inert gas to force propellants into chamber.

Λ is lower.

* Hybrid Rocket Engines.

Oxidiser & fuel stored separately simpler than biopropellant liq sys.

Both performance & propellant mass fraction lie betn SRM & LRE
Relative amounts of fuel & oxidiser are not controlled.



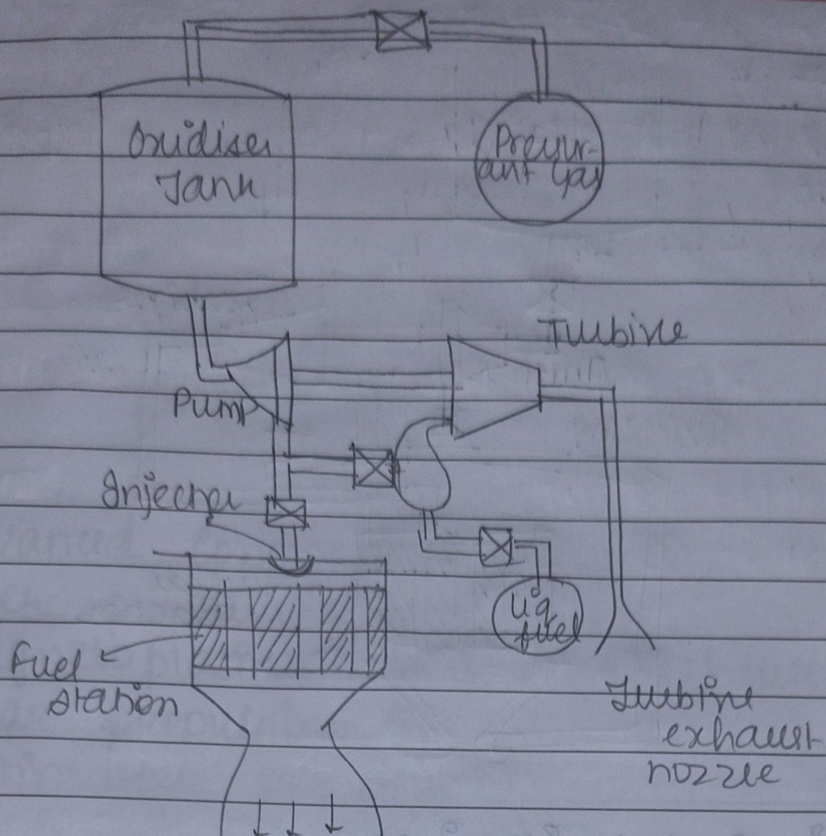
* Nuclear rocket engines

- Nuclear thermal rocket
  Energy from nuclear process is used to heat a working fluid to high temp & hot gases then expanded through a de Laval Nozzle.

- Nuclear electric rocket.
  Utilises electricity to generate thrust.

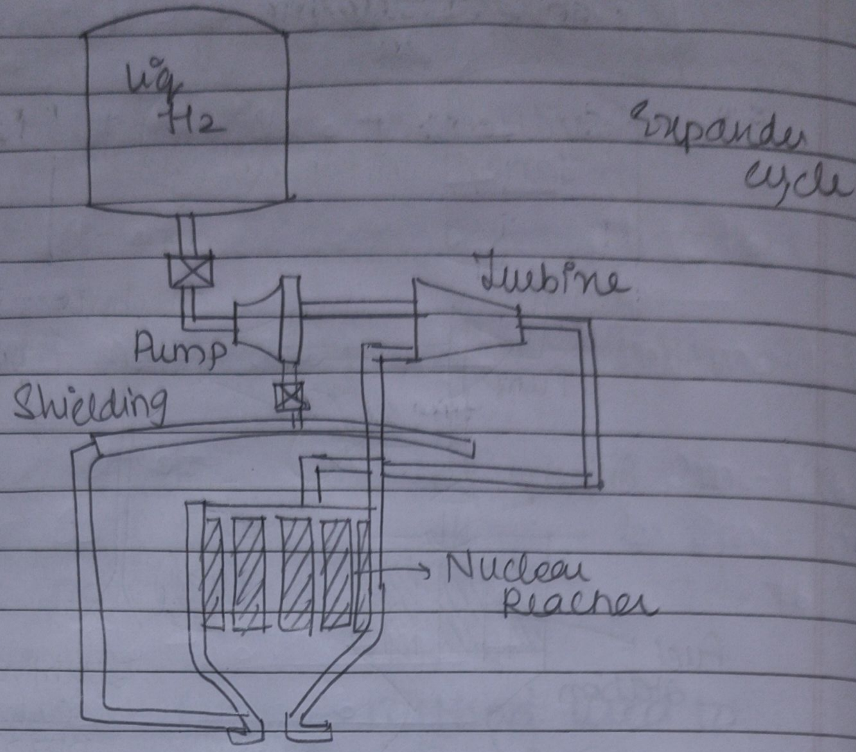
high Isp (Interplanetary missions) (low λ)

NERVA (Nuclear engine for Rocket vehicle app)



* Nuclear rocket engines
- Nuclear thermal rocket
Energy from nuclear process is used to heat a working fluid to high temp & hot gases then expanded through a de Laval Nozzle.

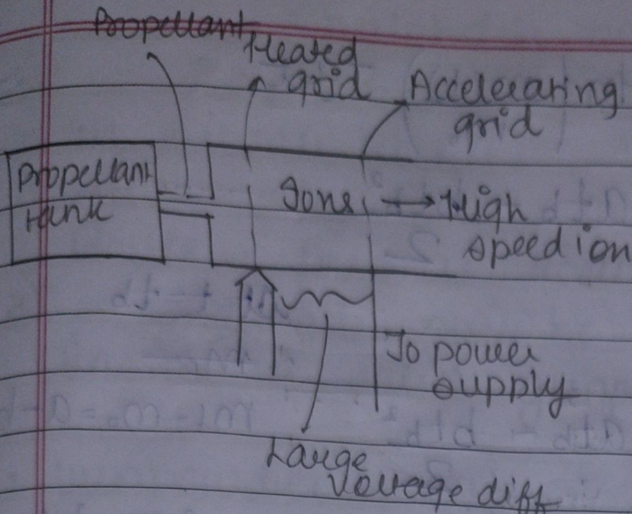
- Nuclear electric rocket
Utilises electricity to generate thrust
high Isp (Interplanetary missions) (low λ)
NERVA (Nuclear engine for Rocket vehicle app)

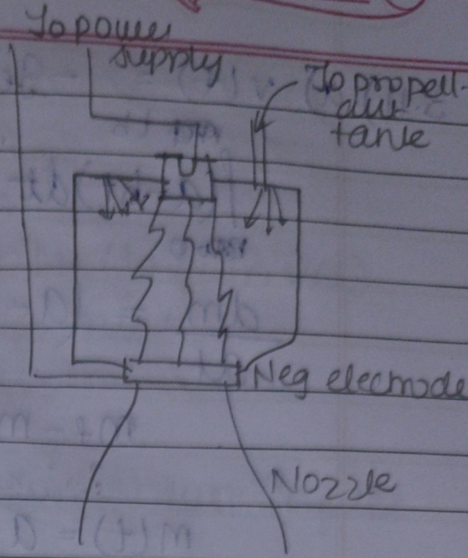


* Electric Rocket Engines
Arcjet/electrothermal thruster
Electric pot is arced betn electrodes
Isp ~ 400-1500 s.
Fluid passing through arc is superheated
H_2, NH_3, He → used succesfully

Electrostatic /Ion thrusters :-
Heavy atoms (Xe) are ionized via electric heating coils, +vely charged ions accelerated using electric pot. to high velocities.

Isp ~ 10000 s
Large amt of electric power
Satellite maneuvering



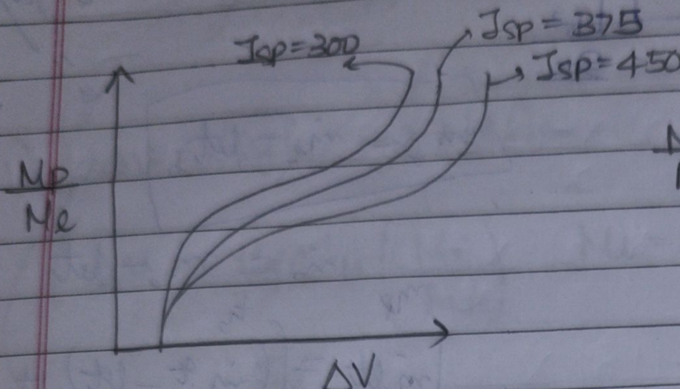


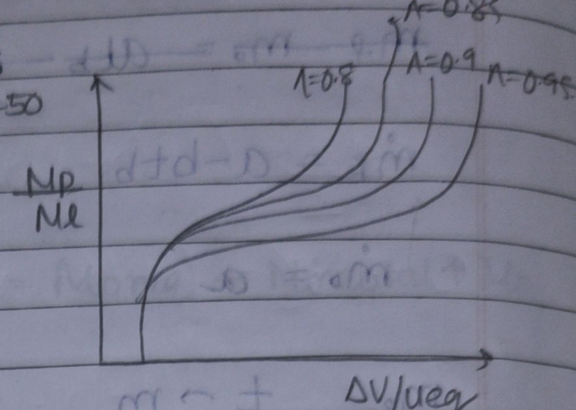
* Advanced Concepts
1) Laser propulsion
2) Magnetoplasmadynamic (MPD Thrusters)
3) Solar propulsion sys.
4) Antimatter propulsion.

Multi stage Rockets notes of sir:

- Rocket eqⁿ for single stage:-

$$\Delta V = u_{eq} \ln\left(\frac{M_0}{M_f}\right) \equiv u_{eq} \ln(MR)$$





- continous ejection

$$\dot{m} = \dot{m}_p + \dot{m}_i = \frac{\dot{m}_p}{\Lambda}$$

$$M\frac{du}{dt} = \dot{m}_p u_{eq} = (\dot{m}_p + \dot{m}_i)\Lambda u_{eq} = -\frac{dM}{dt}\Lambda u_{eq}$$

$$\Delta V_\infty = \Lambda u_{eq} \ln\left(\frac{M_0}{M_e}\right)$$

$$(\text{Mass Ratio})_\infty \equiv \frac{M_0}{M_e} = \exp\left(\frac{\Delta V}{\Lambda u_{eq}}\right)$$

$$1 + \frac{M_p + M_i}{M_e} \equiv 1 + \frac{M_p}{\Lambda M_e} = MR_\infty$$

$$\frac{M_p}{M_L} = \Lambda\left(\exp\left(\frac{\Delta V}{\Lambda u_{eq}}\right) - 1\right), \left(\frac{M_i}{M_L}\right) = (1-\Lambda)\left(\exp\left(\frac{\Delta V}{\Lambda u_{eq}}\right) - 1\right)$$

* Multistage Rocket

Stage mass $m_{0n} = m_{pn} + m_{in}$

structural factor : $\epsilon_n = \frac{m_{fn}}{m_{0n}} \equiv \frac{m_{in}}{m_{in} + m_{pn}}$

Propellant fraction $\Lambda_n = \frac{m_{pn}}{m_{in} + m_{pn}} \equiv 1 - \epsilon_n$

- $M_{on} = \sum_{j=n}^{N} m_{oj}$ $\qquad \beta_n = \frac{M_{ln}}{M_{on}}$, $M_{fn} = m_{in} + M_{ln}$

$M_{ln} = \sum_{n+1}^{N} m_{oj}$

$\Delta V_n = u_{eqn} \ln\left(\frac{M_{on}}{M_{fn}}\right)$

For a given $\frac{M_l}{M_0}$, when is ΔV max?

* Staging

Obj function

$f(u_{eqn}, \epsilon_n, \beta_n) \equiv \Delta V = \sum_{1}^{N} \Delta V_n$

$= -\sum_{1}^{N} u_{eqn} \ln(\epsilon_n + (1-\epsilon_n)\beta_n)$

constraint func

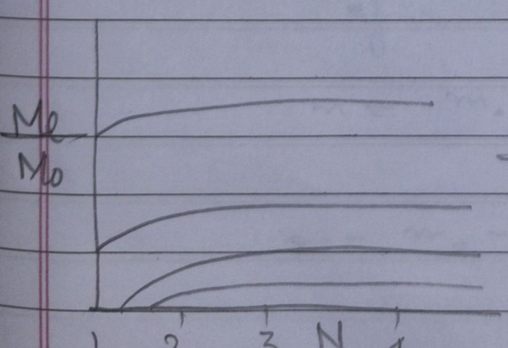
$\frac{M_l}{M_0} = \frac{M_{l1}}{M_0} \cdot \frac{M_{l2}}{M_{02}} \cdot \frac{M_{l3}}{M_{03}} \cdots \frac{M_l}{M_{0N}} = \prod_{n=1}^{N} \beta_n$

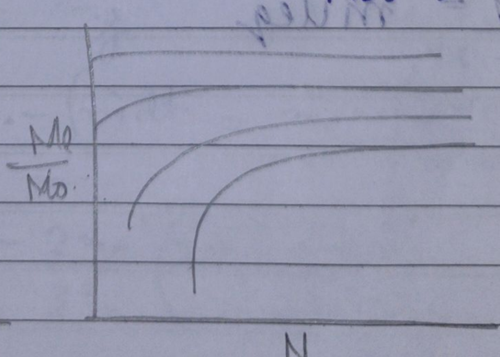
$g(\beta_n) = \ln\left(\frac{M_l}{M_0}\right) = \sum_{1}^{N} \ln \beta_n$

Our constraint

$\frac{M_l}{M_0} = \prod_{1}^{N} \beta_n = \beta^N \Rightarrow \beta_{opt} = \left(\frac{M_l}{M_0}\right)^{1/N}$

$\Delta V_{opt} = -N u_{eq} \ln\left(\epsilon + (1-\epsilon)\left(\frac{M_l}{M_0}\right)^{1/N}\right)$

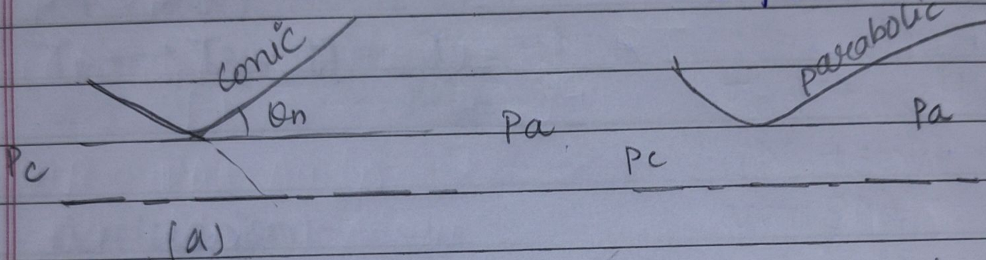


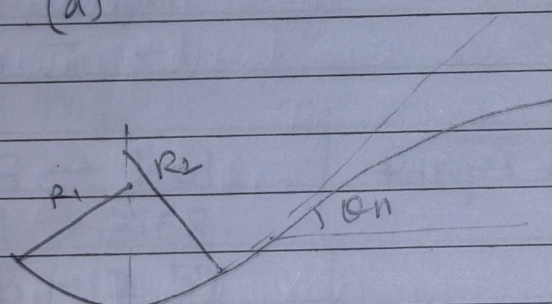


Rocket Nozzle performance - Class 2

* Multiphase flow
  ↳ particles / drops need not follow 'streamlines'.
  ↳ a 'minor effect' for flow modeling
  ↳ 'quasi 1-D' flow $A = A(x)$

* Validity of 'quasi 1-D'
  ↳ no 2D effects — oblique shocks → after throat; near exit $P_a > P_{eis}$
  — expansion ~~shocks~~ fans

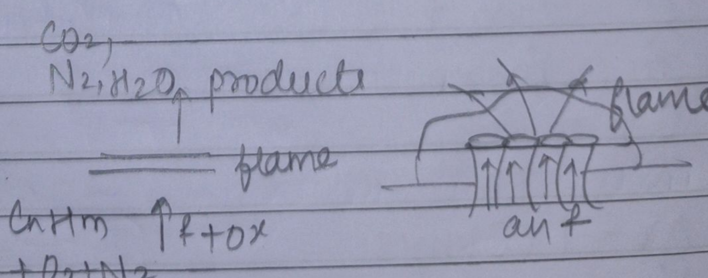




c → chamber
a → ambient

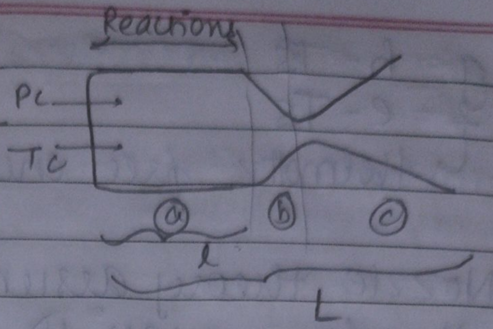
characteristics ↕ waves ↔ direction of info propogation

* Constant properties
  (thermodynamic & transport)



(oo'ly fast reactions)

(a) Combustion chamber
(b) Chamber – throat
(c) Throat – exit



fast-slow (relative)
$t_c = L/V$
convective (or flow) time scale
'residence time'
$t_{ch} \equiv t_r = \frac{Y_f}{\omega'''_f}$ — fuel conc — reaction rate

per length (each ')
per sec per unit vol (''')

Damkohler No.
$Da = \frac{t_c}{t_r}$

equilibrium flow (chemical, composition, shifting)
Infinitely fast reaction, $t_r \to 0$, $Da \to \infty$
Infinitely slow reaction, $t_r \to \infty$, $Da \to 0$
↳ Chemically frozen flow
or
Frozen composition flow / Frozen flow.

Isp → equilibrm
Isp → frozen

finite Da, finite rate chemistry
Isp depends on Da

$$h_t = \sum_{j=1}^{N} \left(h_{fj} + \int_{T_{ref}}^{T} c_{pj}\, dT \right)$$

combinations
↳ STANJAN, CEA, Chemkin, Cantera
finite rate chemistry

↳ Helmoltz free energy

Nozzle theory assumptions
) steady, quasi 1D
) Homogeneous perfect gas
) Isentropic - no reaction, no heat transfer, no friction

Perfect gas
$P = \rho RT$

$C_P = \frac{\gamma}{\gamma - 1} RT$, $C_V = \frac{1}{\gamma - 1} RT$

$R = \frac{R_u}{M_w}$

Dalton partial pressure , $P = \sum p_j$

$p_j = \cancel{p}\, \rho_j R_j T$

$P = \sum \rho_j R_j T$

Thermal equilibrium

$T = T_t = T_r = T_v$

Rocket Nozzles → class 3

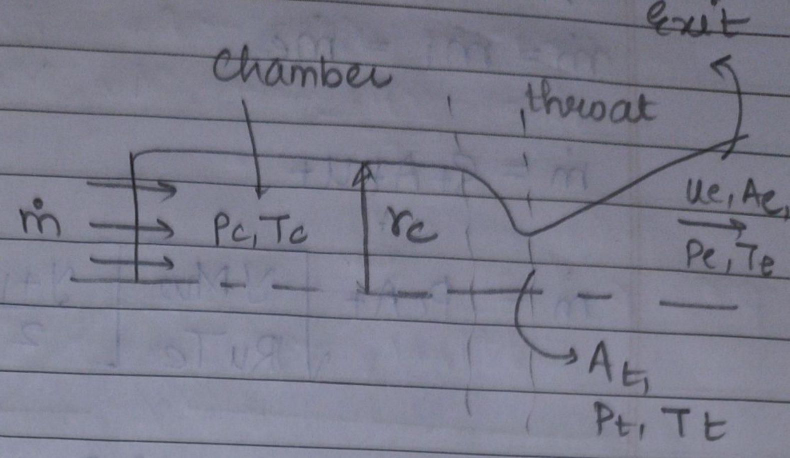
$u \rightarrow$ speed

$v \rightarrow$ specific volume

$v = 1/\rho$

$A_c = \pi r_c^2$, $A_t = \pi r_t^2$

$A_e = \pi r_e^2$



ASSUMPTIONS:-

1) Quasi-1D, steady
2) Homogeneous perfect gas
3) Nozzle flow is isentropic (no friction, heat transfer)
4) No chemical reactions in nozzle → frozen flow

$P_c, T_c \rightarrow$ $\underbrace{\text{stagnation properties}}_{\text{total properties}}$ for nozzle flow

$J = \dot{m} u_e + (p_e - p_a) A_e$ (Express in terms of stagnation properties)

$$\dot{m} = \rho A u = \frac{P}{RT} A M \sqrt{\gamma R T}$$

$$= \frac{P_c}{RT_c} \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{-\gamma}{\gamma-1}+1-\frac{1}{2}} A M \sqrt{\gamma R T}$$

→ any area of CS in nozzle

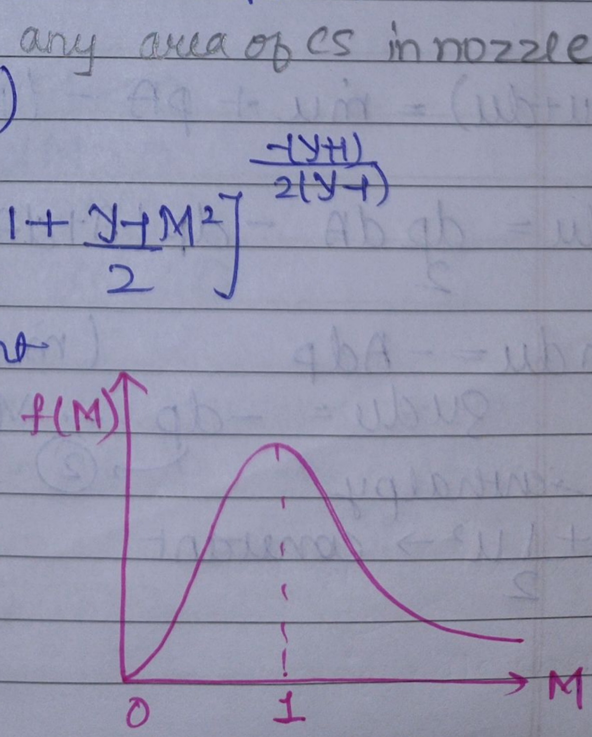
$$\dot{m} = \frac{P_c}{\sqrt{T_c}} A f(\gamma, M_w, M)$$

$$f(\gamma, M_w, M) = \sqrt{\frac{\gamma M_w}{R_u}} \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{-(\gamma+1)}{2(\gamma-1)}}$$

$M_w \rightarrow$ molecular weight

$\frac{\dot{m}}{A} = \rho u \Rightarrow$ Max for $M = 1$

$\dot{m} = \rho_t A_t u_t$



$$\frac{\dot{m}}{A} \equiv \rho u \rightarrow \text{Max for } M=1$$

$$\dot{m} = \dot{m}_t = \dot{m}_e$$

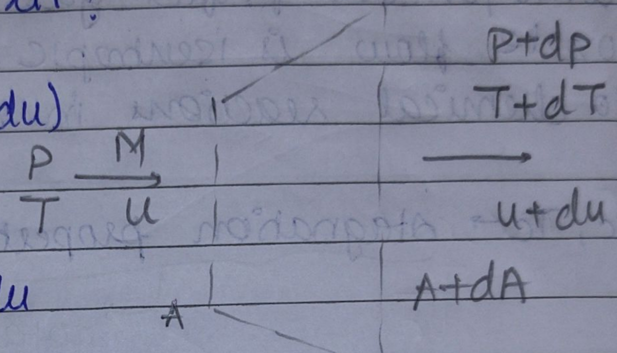
$$\dot{m} = \rho_t A_t u_t$$

$$\dot{m} = p_c A_t \sqrt{\frac{\gamma M_w}{R_u T_c}\left[\frac{\gamma+1}{2}\right]^{\frac{-(\gamma+1)}{2(\gamma-1)}}}$$

---

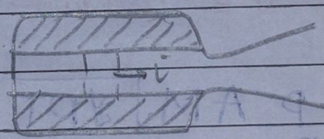
* Why $M \neq 1$ at exit?

$$\dot{m} = \rho A u = (\rho+d\rho)(A+dA)(u+du)$$



$$0 = \rho A du + \rho u dA + A u d\rho + \rho du dA + u d\rho dA + A d\rho du + d\rho dA du$$

$$\frac{dp}{p}, \frac{dA}{A}, \frac{du}{u}, \frac{d\rho}{\rho} \ll 1$$



$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0 \quad -(1)$$

$$\dot{m}(u+du) = \dot{m}u + pA - (p+dp)(A+dA) + \left(p+\frac{dp}{2}\right)dA$$

$$\dot{m}du = \frac{dp}{2}dA - dp(A+dA)$$

$$\dot{m}du = -Adp \qquad (\dot{m} = \rho A u)$$

$$\rho u du = -dp \rightarrow \text{Momentum conservation} \rightarrow (2)$$

enthalpy

$$h + \frac{1}{2}u^2 \rightarrow \text{constant}$$

$$C_p dT = dh = -u\,du$$

$$C_p dT = -u\,du \quad — ③$$

$$p = \rho RT$$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

$$\rho C_p dT = dp$$

$$\frac{dA}{A} = -\left[\frac{d\rho}{\rho} + \frac{du}{u}\right] = -\left[\frac{dp}{p} - \frac{dT}{T} + \frac{du}{u}\right]$$

$$= -\left[-\frac{\rho u^2}{p}\frac{du}{u} + \frac{u^2}{C_pT}\frac{du}{u} + \frac{du}{u}\right]$$

$$\frac{dA}{A} = [M^2-1]\frac{du}{u} = \left[\frac{M^2-1}{1+\frac{\gamma-1}{2}M^2}\right]\frac{dM}{M}$$

$$-\left[-\frac{\rho u^2}{p} + \frac{u^2}{C_pT} + 1\right]\frac{du}{u}$$

$$\frac{\rho u^2}{\gamma p} = M^2 \qquad \hookrightarrow (\gamma-1)M^2$$

$$-\left[-\gamma M^2 + (\gamma-1)M^2 + 1\right] = -\left[1-M^2\right]$$

$M<1 \rightarrow \quad dA>0 \leftrightarrow dM<0$

$\qquad\qquad dA<0 \leftrightarrow dM>0$

$M>1 \rightarrow \quad dA>0 \leftrightarrow dM>0$

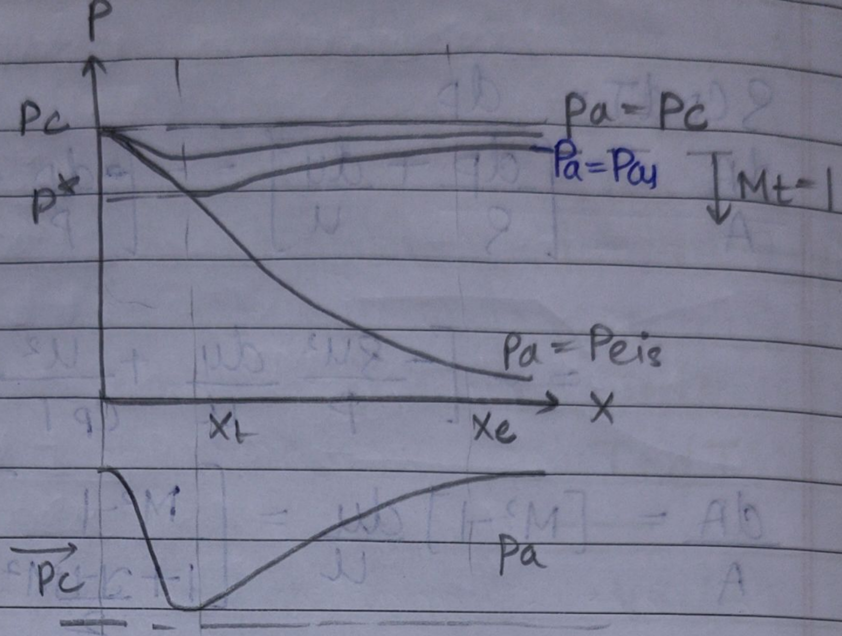
$\qquad\qquad dA<0 \leftrightarrow dM<0$

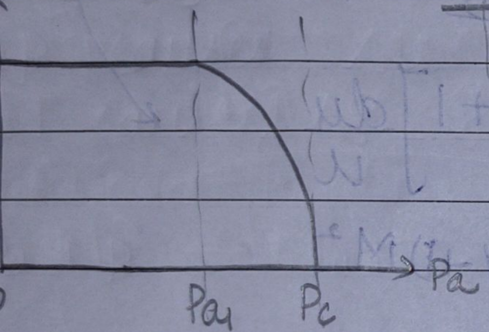
These expressions give the reason of why Mach number cannot be 1 at the exit.

mallest Area → A_t → Mach no. at throat

$$\dot{m} = \frac{\rho_c A_t}{\sqrt{T_c}} f(\gamma, M_w, M_t)$$

$$\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$





$$p = p_a + \tilde{p} \qquad \text{(Perturbation)}$$
$$T = T_a + \tilde{T}$$
$$\rho = \rho_a + \tilde{\rho} \qquad \frac{\tilde{\rho}}{\rho_a}, \frac{\tilde{T}}{T_a}, \frac{\tilde{p}}{p_a} << 1$$
$$u \equiv \tilde{u}$$

* Conservation eqⁿs:-

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0$$

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial}{\partial x}(\rho u^2 + p) = 0$$

$$\frac{\partial}{\partial t}(\rho e_t) + \frac{\partial}{\partial x}(\rho h_t u) = 0$$

$$\rho_a u + \tilde{\rho} u = \rho_a u\left(1 + \frac{\tilde{\rho}}{\rho_a}\right)$$

$$\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial}{\partial x}(\rho_a u) = 0$$

$$\frac{\partial}{\partial t}(\rho_a u) + \frac{\partial}{\partial x}(\tilde{p}) = 0$$

$$\frac{\partial}{\partial t}(\rho_a \tilde{e}) + \frac{\partial}{\partial x}(\rho_a h_a u) = 0$$

$$\underset{\text{2nd order}}{\rho_a u^2} + \underset{\text{0 order}}{p_a} + \underset{\text{1st order}}{\tilde{p}}$$

$$e + \frac{u^2}{2} = e_a + \tilde{e} + \frac{u^2}{2}$$

$$h + \frac{u^2}{2} = h_a + \tilde{h} + \frac{u^2}{2}$$

$$\rho\left(h + \frac{u^2}{2}\right)u \equiv \rho_a h_a u$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \tilde{\rho}}{\partial t}\right) = -\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t}(\rho_a \bar{u})\right)$$

$$\frac{\partial^2 \tilde{\rho}}{\partial t^2} = \frac{\partial^2 \tilde{\rho}}{\partial x^2} \equiv \underbrace{\left(\frac{\partial \tilde{p}}{\partial \tilde{\rho}}\right)}_{\text{constant}} \frac{\partial^2 \tilde{\rho}}{\partial x^2}$$

$\tilde{\rho}$ = const for small perturbations

$$\tilde{p} = \tilde{\rho}^{\gamma}$$

$$\frac{\partial^2 \tilde{p}}{\partial t^2} = C_s^2 \frac{\partial^2 \tilde{\rho}}{\partial x^2}$$

$$C_s^2 = \frac{\gamma P_a}{\rho_a}$$

reaction ↔ acoustic
(flow)

$\tilde{p} \leftrightarrow \tilde{\rho}$, $\tau_a = \frac{L}{c_a}$

$T \gg \tau_a$

Δp causes Δu

ΔT causes $\Delta \rho$.

$P_a + \tilde{p} \rightarrow T_a + \tilde{T} \rightarrow$ more heat release

Reaction rate

★ $J = \dot{m} u_e + (P_e - P_a) A_e$

$\frac{\dot{m}}{A} \Rightarrow$ Max for $M=1$

↳ can happen at min area location

$$\dot{m} = P_c A_t \sqrt{\frac{\gamma M_a}{R_u T_c}} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\sqrt{\frac{\gamma}{R_u T_c}} \sim \frac{\gamma}{C_s}$$

$$C^* = \frac{p_t A_t}{\dot{m}} = \sqrt{\frac{R_u T_c}{\gamma M_a}} = \left(\frac{2}{\gamma+1}\right)^{2(\gamma-1)}$$

$$J = \frac{P_c A_t}{C^*} u_e + (p_e - p_a) A_e$$

$$J = \frac{P_c A_t}{C^*} \left\{ 2 C_p T_c \left(1 - \left(\frac{P_t}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \right) \right\}^{1/2} + (p_e - p_a) A_e$$

$$\frac{J}{P_c A_t} = \frac{1}{C^*} \left\{ 2 C_p T_c \left(1 - \left(\frac{p_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \right) \right\}^{1/2} + \left(\frac{p_e}{P_c} - \frac{p_a}{P_c} \right) \frac{A_e}{A_t} = C_J$$

$C_J \rightarrow$ Thrust coeff

$J = P_c A_t C_J \equiv \dot{m}\, C^*\, C_J$ → Nozzle perf.

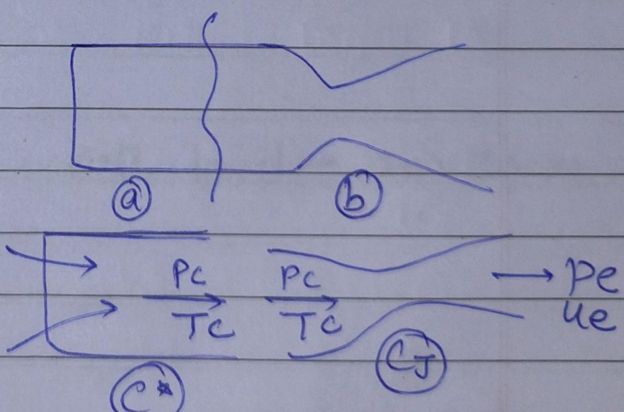
C^* → Comb chamber perturbation

$u_{eq} = C^* C_J$

Characteristic vel / speed (C^*) → func of propellant combination

$h_c + \frac{u_c^2}{2} = h_t \rightarrow u_e = \sqrt{2h_t\left(1 - \frac{T_e}{T_t}\right)}$

$J = [\, p_e(1 + \gamma M_e^2) - p_a \,] A_e$



$p_a = 0$, $J = J_{vac}$
↳ Max thrust for given A_e/A_t

For a given $\left(\frac{A_e}{A_t}\right)$

Peak C_J = Optimum expansion ($p_e = p_a$)

Book Chapter 4 continued..

* Thrust coefficient:-

$$C_f = \frac{F}{P_c A_t} = \left[\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(1-(P_e/P_c)^{(\gamma-1)/\gamma}\right)\right]^{1/2} + \left(P_e/P_c - \frac{P_a}{P_c}\right)\epsilon$$

$\epsilon = A_e/A_t$ ~~at~~ → nozzle expansion ratio

If $P_a = 0$, $$C_{fv} = \left[\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right)\right]^{1/2} + \left(\frac{P_e}{P_c}\right)\epsilon$$

Optimal expansion → $P_e = P_a$

$$C_f = C_{fv} - (P_a/P_c)\epsilon$$

$$F = C_f P_c A_t \quad ; \quad \dot{m} = \frac{P_c A_t}{C^*} = \frac{g P_c A_t}{C^*}$$

$$I_{sp} = \frac{F}{\dot{m} g} = \frac{C_f C^*}{g}$$

$$I_{sp} g = \sqrt{\frac{2\gamma R_u T_c}{(\gamma-1)M}\left(1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right)}\left[1 + \frac{(P_e/P_c - P_a/P_c)(P_c/P_e)^{1/\gamma}}{\frac{2\gamma}{\gamma-1}\left(1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right)}\right]$$

$$I_{sp} \propto \sqrt{\frac{T_c}{M}}$$

classmate
Date
Page 10/10/22

CLASS NOTES
Rocket propulsion nozzles

* $\epsilon = \frac{A_e}{A_i}$ given gives $C_{J vac}$ $(P_a = 0)$
$$C_J = C_{J vac} - \left(\frac{P_a}{P_c}\right)\epsilon$$

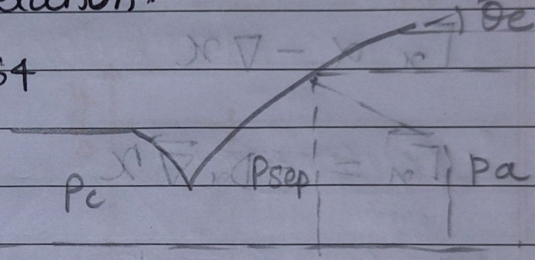
$P_{eB} < P_a < P_{ec}$ → Normal shock at exit

Nozzles → Over expanded → Vulcain
Perfectly expanded → RL-10
Under expanded → Saturn rocket

Optimum expansion → $p_e = p_a$

* Flow separation → Correlation.

$$\frac{P_{sep}}{P_a} = (1.88 M_{sep} - 1)^{-0.64}$$



If $p_e < 0.4 p_a$,
separation occurs.

$$p_a = p_{sl} \exp\left(\frac{-h}{h_2}\right)$$

$h_2 = 701.04 m$

$\frac{P_{sep}}{P_a} = 0.3 - 0.4$ sea level → $p_a = 1$ bar

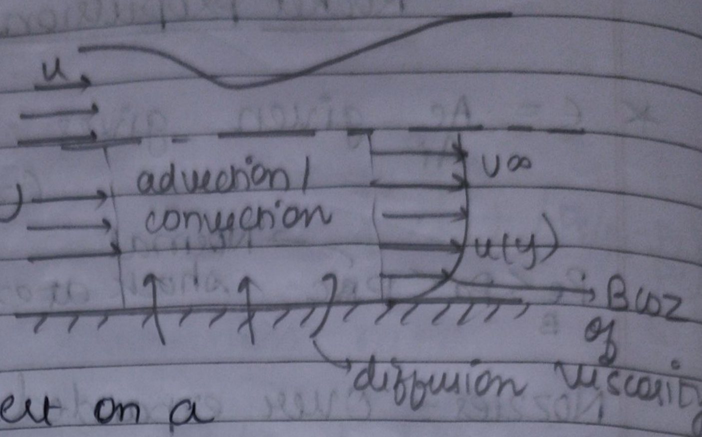
$C^* = 1850$ m/s
$p_c = 48.3$ bar
$\epsilon = 20$, $\gamma = 1.2$
$\frac{p_e}{p_a} = 0.005$

$$\frac{d}{dt}\left[\iiint_V \rho \vec{v}\, d\forall\right] = -\oiint_S \vec{v}\rho(\vec{v}\cdot d\vec{s})$$

$$\rho\frac{d\vec{u}}{dt} + \rho\vec{u}\,\vec{\nabla}\vec{u}$$

$$= -\nabla p + \mu\nabla^2 u$$

Why laplacian?



Pressure → Using the force that particles exert on a wall that is kept in their way.

Momentum flux of random motion

μ, T, γ

Collisons → μ, K, D

$$\Gamma_x \propto -\nabla x$$

$$\vec{\Gamma}_x = -D_x\vec{\nabla} x$$

$$\nabla \quad \tau_{xy} = -\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \quad \tau_{xx} = -2\mu\frac{\partial u}{\partial x}$$

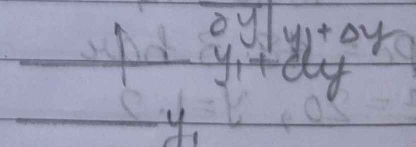
At what dist particle sees the effect of wall?

$$\rho\frac{\partial u}{\partial t} + \rho\vec{u}\cdot\vec{\nabla}\vec{u} + \cancel{\rho\,\vec{u}\,\vec{\nabla}} \quad \nabla P = \mu\nabla^2 u$$

$\tau_{conv} = l/u$

$\tau_{visc} = b/c = l_y^2/\mu$

$D = \frac{\partial x}{\partial y}\Big|_{y_1+\Delta y}$



$$\frac{\partial x}{\partial t} = -\vec{\nabla}\cdot\Gamma_x$$

$$= D_x\nabla^2 x$$

$$\frac{\partial x}{\partial \tau_{diff}} = D_x\frac{\delta x}{l_y^2}$$

$$\tau_{diff} = \frac{l_y^2}{D_x}$$

$$l_y \sim \sqrt{\frac{\nu l}{u}} \qquad Re_l = \frac{ul}{\nu}$$

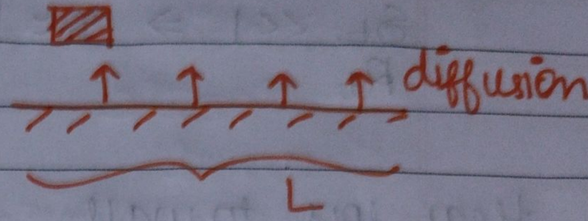
Lecture

Boundary Layer – separation

Time scales →

$\tau_{conv} \sim \tau_{diff}$

$$\frac{\delta}{L} = \frac{5}{\sqrt{Re_L}}$$

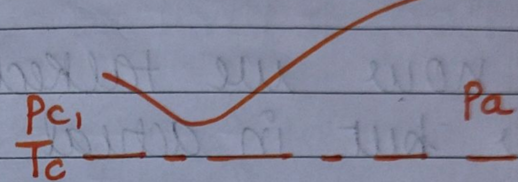


how long it takes to travel in horizontal direction (convection)

$Re = \frac{UL}{\nu}$

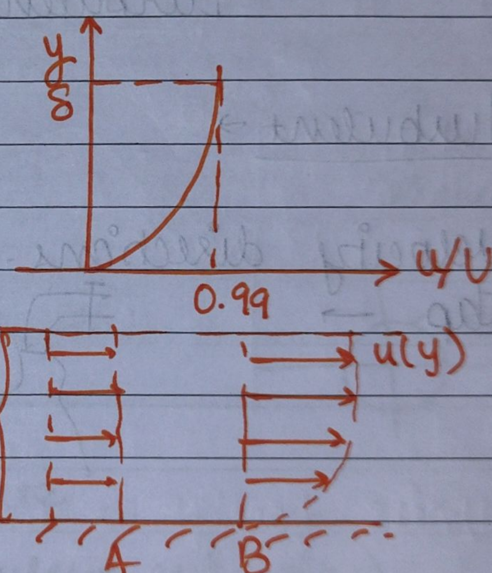
$y = \delta \rightarrow u(y) = \cancel{U}\, 0.99U$

horizontal vel



$v \rightarrow$ vertical velocity

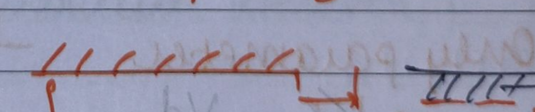
↳ It is zero outside



$\dot{m} = \rho U \underbrace{WL}_{Area}$

$$m = \int_0^L \rho W u\, dy$$

$$= \rho W L_d v$$



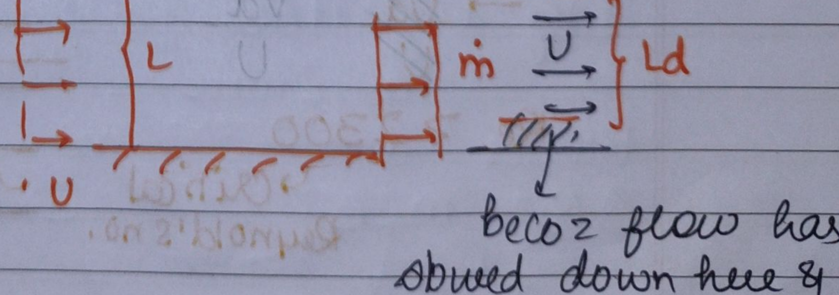
For channel flow →

$$\delta^* = \frac{L - L_d}{L}$$

displacement thickness

$$\frac{\delta^*}{L} = \frac{1.72}{\sqrt{Re_L}}$$

(Blasius Boundary Layer)



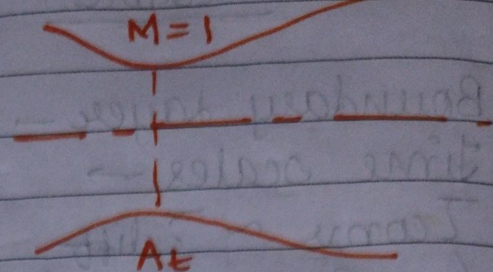
becoz flow has slowed down here & mass can't be accomodated.

$m = \rho \pi r^2 U$

$C_D = \dfrac{\dot{m}_{actual}}{\dot{m}_{ideal}} = \left(\dfrac{R_t - \delta_t}{R_t}\right)^2$

↳ Discharge coeff.

$\dfrac{\delta_t}{R_t} << 1 \Rightarrow C_D \approx 1 - \dfrac{2\delta_t}{R_t}$



~~At throat~~

$R_d = R_t - \delta_t$

Heat loss to wall

↳ Low T ⇒ high ρ

↳ higher $\dot{m}$

$M \xrightarrow[P]{T}$ compressible

τ_w

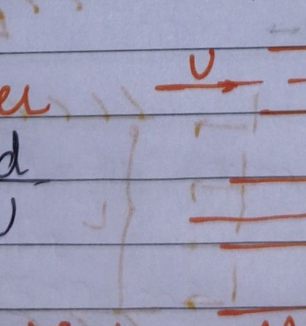
$0.93 < C_D < 1.15$

[Till now we talked only about Laminar flow but in actual rocket nozzles, there is turbulent flow!]

Turbulent →

Velocity directions..

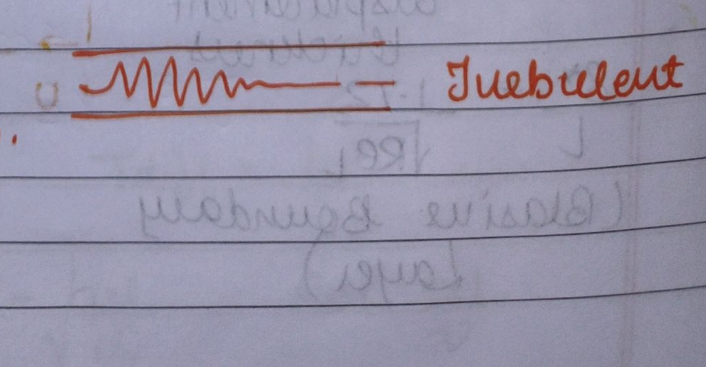
Tap →



Only parameter → $\cancel{\frac{Vd}{\nu}}$ $\dfrac{Vd}{\nu}$

Re > 2300

↳ Critical Reynold's no.



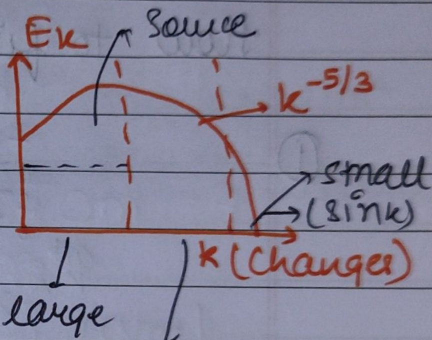
(w.r.t some restoring force)
"Too much energy" in turbulent flow
Unstable
(destabilizes)
↳ entropy
"Free energy"
$h - TS$

Perturbations ↳ changes (unstable)
↓
stays same ⇒ stable

Turbulence → fluctuating fields
Not a Gaussian → statistics are 'steady'

→ Fourier transform
$E(k) \rightarrow F(E)$

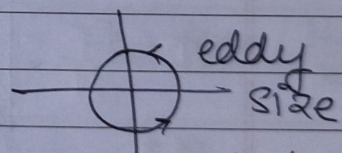
$E = K.E = \frac{1}{2}u^2 + \frac{1}{2}v^2 + \frac{1}{2}w^2$



$E = \int E_k e^{ikx}$

l → length scale
k → wave number.

u, v & θ
$u = \cos(\omega t)$, $v = \sin(\omega t)$



$l \sim \left(\frac{1}{k}\right)$

given k, ϵ
↳ dissipation rate
→ ν_t (Turbulent viscosity)

$\mu \nabla^2 u \rightarrow \mu k^2 u_t$

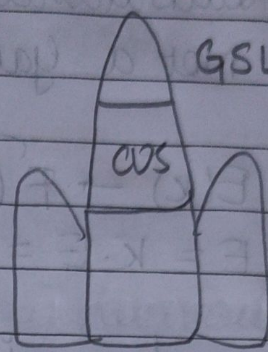
LECTURE

HEAT TRANSFER

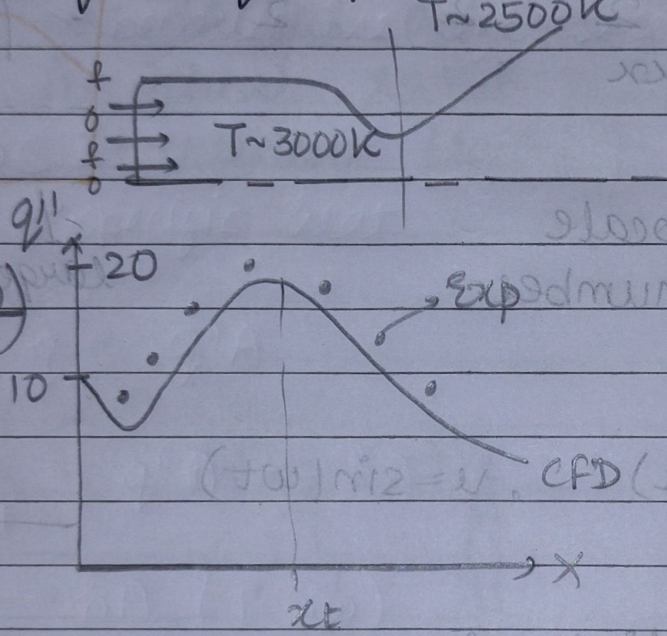
	T_h	K $\xrightarrow{\;q''\;}$	T_c
Combustion chamber + Nozzle	Heat		cold

→ Internal

Re-entry → External
- → Ballistic
- → Manned missions



How to find $q'' = ?$



Units for $q'' \rightarrow \frac{10\,MW}{m^2} \equiv \frac{1\,kW}{cm^2}$

$10\,MW/m^2$

1 kg of water ~~fr~~ from 30°C to 100°C in 2 min

Burner area ~ 300 cm^2

$\Delta Q = mC\Delta T = 3\times10^5\,J$

$m \rightarrow 1$, $C \rightarrow 4186$, $\Delta T \rightarrow 70$

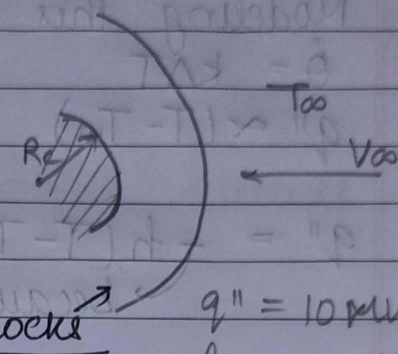
$$q'' = \frac{\Delta Q}{At} = \frac{3\times10^5}{3.10^{-2}.120} = 0.08\ MW/m^2$$

Density $\rightarrow 10^3 \frac{kg}{m^3} \sim 1 \frac{kg}{m^3}$

This was about engine combustion

* Re-entry

$$q'' = C\sqrt{\frac{\rho_\infty}{R}}\, V_\infty^3 \left[1 - \frac{h_w}{h_o}\right]$$



High $R_c \rightarrow$ low $q'' \rightarrow$ blunt leading edge

$q'' = 10\ MW/m^2$ for manned missions

$h_w = C_p T_w \rightarrow$ wall temperature

$h_o = h_\infty + \frac{1}{2} V_\infty^2$

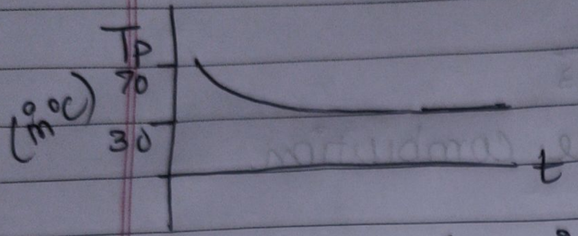
$R_c \rightarrow$ Radius of curvature of leading edge.

* Entry	$V_\infty \left(\frac{km}{s}\right)$	$\frac{V_\infty^2}{2} \left(\frac{MJ}{kg}\right)$	$h_\infty \left(\frac{MJ}{kg}\right)$
Apollo	11.4	66	0.2
Mars Return	14	98	0.2

Water $\rightarrow$ 23 MJ/kg to boil
Carbon $\rightarrow$ 60 MJ/kg to evaporate.

* Example of Potatoes in water

$\swarrow$ At $t=0, 70°C$ ↳ At $t=0, 30°C$



Modeling this using Newton's equations

$\dot{\theta} = k\Delta T$

$q'' \propto (T - T_w)$

$q'' = -h(T - T_w)$

↳ because the difference is decreasing

$E = mcT$

↳ specific heat

$\frac{dE}{dt} = \dot{q} = q''A$

$mc\frac{dT}{dt} = q''A = -hA(T - T_w)$

$\frac{dT}{dt} = -\frac{hA}{mc}(T - T_w)$

$T - T_w = (T_0 - T_w)\exp\left[\frac{-hA}{mc}t\right]$

T → Potato temp

$\theta = \frac{T - T_w}{T_0 - T_w}$ → Normalized

$\theta(t) = \exp\left[\frac{-hA}{mc}t\right]$, let $\tau_c = \frac{mc}{hA}$

↳ $\exp\left(\frac{-t}{\tau_c}\right)$

$$\frac{dE_w}{dt} = -hA(T_w - T)$$

What is h?

h

A

T_w

Newton's Law → $q'' \propto (T - T_w)$

How is the heat getting transferred?

↳ Convection. (Is it?)

No, conduction just around the surface

When we heat water

~~$\Delta\rho$~~ $\Delta P = \rho g h$

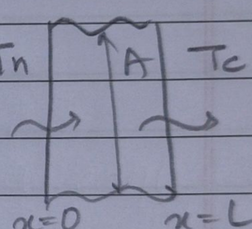
Buoyancy → natural convection currents

Fourier's law → $\vec{q}'' \propto \overline{\nabla} T$

$\frac{\Delta T}{\Delta x}$

$\vec{q}'' = -k\overline{\nabla} T$

↳ fluid



$E = mcT$

$m = \rho(AL)$

$$\frac{dE}{dt} = AK\frac{dT}{dx}\bigg|_{x=0} - AK\frac{dT}{dx}\bigg|_{x=L}$$

$$\rho c L\frac{dT}{dt} = K\left\{\left(\frac{dT}{dx}\right)_{x=0} - \left(\frac{dT}{dx}\right)_{x=L}\right.$$