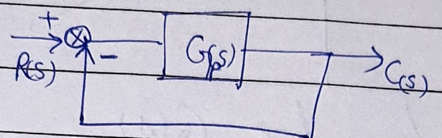
AE308
Project Report

① Introduction -

Given :- Openloop transfer function = $G_p(s) = \dfrac{1}{s^2(1+0.1s)}$

assuming unity feedback



Closed loop transfer function $= \dfrac{C(s)}{R(s)} = T(s) = \dfrac{G_p(s)}{1+G_p(s)} = \dfrac{1}{0.1s^3+s^2+1}$

So, poles/~~roots~~ of T(s) are given by,

the roots of $0.1s^3 + s^2 + 1 = 0$

$$\Rightarrow \quad s_1 = -10.09807$$

$$s_2 = 0.04903 + 0.99392j$$

$$s_3 = 0.04903 \cancel{+} - 0.99392j$$

Hence T(s) can be re-written as -

$$T(s) = \frac{10}{(s-s_1)(s-s_2)(s-s_3)}$$

Since s_2 and s_3 lie in right half side of the s plane so the given system is unstable.

Also s_2 and s_3 are dominant poles of the system

For $G(s)$ we can also plot the root locus to analyze it in a better way -

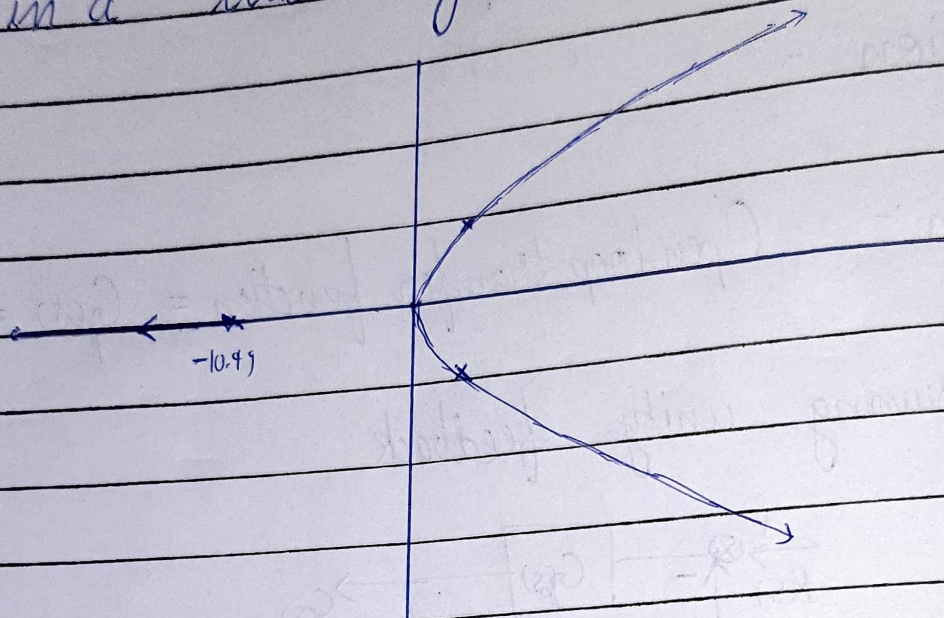


fig root locus for $G_p(s)$

Same ~~A similar~~ plot has been plotted in Matlab for easier calculations

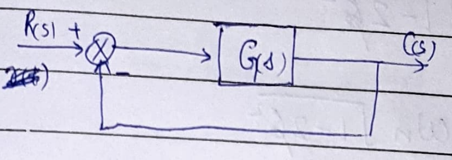
(2) Control objectives -

Requirements stated in the question.

Resonant amplitude = 1.5

Resonant frequency = 1.4 rad/s

Firstly we need to understand what these terms signify and then we will calculate the values for a general second order system (amplitude = E)



if $r(t) = A\sin(\omega_0 t)$
then for the resonance with a 2nd order sys.

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$c(t)$ is given by $A|G(j\omega_0)| \sin(\omega_0 t + \angle G(j\omega_0))$

For the resonance we need to maximise $|T(j\omega)|$

$$T(j\omega) = \frac{\omega_n^2}{-\omega^2 + 2j\zeta\omega_n + \omega_n^2}$$

$$\Rightarrow T(j\omega) = \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + j\left(\frac{2\zeta\omega}{\omega_n}\right)} = \frac{1}{(1-u^2) + j\,2\zeta u}$$

So magnitude,

$$\Rightarrow M = |T(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2 + (2\zeta u)^2}}$$

phase,

$$\Rightarrow \angle T(j\omega) = -\tan^{-1}\left(\frac{2\xi u}{1-u^2}\right)$$

Now we need to calculate / find relation b/w u & ξ at M_{max}

$$\left.\frac{dM}{du}\right|_{u_r} = -\frac{1}{2}\left[(1-u_r^2)^2+(2\xi u_r)^2\right]^{-3/2}\left[2(1-u_r^2)(-2u_r)+2(2\xi u_r)2\xi\right]$$

$$\Rightarrow 0 = ($$

$$\Rightarrow \quad u_r = \sqrt{1-2\xi^2}$$

$$\Rightarrow \quad \omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$\& \quad \left. M\right|_{\omega_r} = M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

so, $$\boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} \quad \& \quad \omega_r = \omega_n\sqrt{1-2\xi^2}}$$

our requirements

$$1.5 = \frac{1}{2\xi\sqrt{1-\xi^2}} \quad \& \quad 1.4 = \omega_n\sqrt{1-2\xi^2}$$

$\Rightarrow \quad \xi = 0.357,\ 0.934$

but at 0.934 ω_n becomes a complex no.

so

$\xi = 0.357$

$\omega_n = 1.621885$

so $T(s) = \dfrac{\omega_n^2}{\omega^2 + 2\zeta\omega\omega_n + \omega_n^2} = \dfrac{2.63}{\omega^2 + 1.158\omega + 2.63}$

poles of this ~~eqn~~ transfer function are

$s_{1D} = -0.579 + 1.51485j$

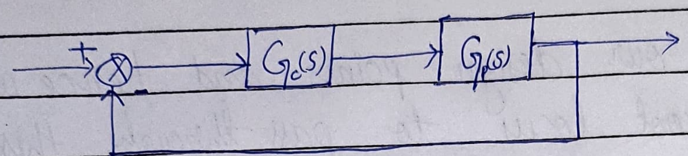
$s_{2D} = -0.579 - 1.51485j$

These are our design points and hence we want our root locus to pass through this point. Also we want to stablize our system.

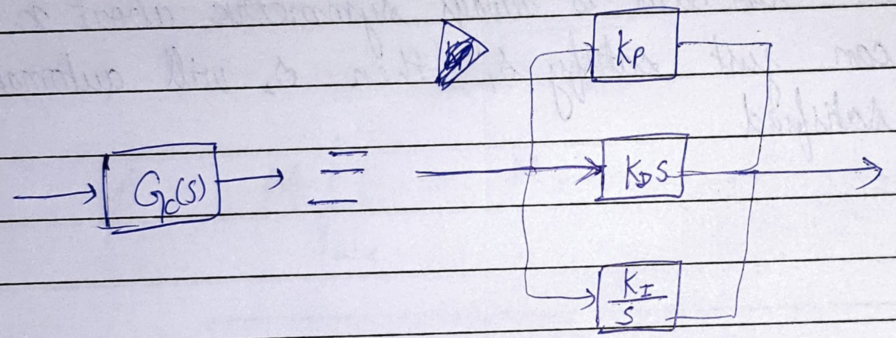
Since root locus is always symmetric about x axis, we can just satisfy s_1, then s_2 will automatically be satisfied.

③ Controller Design

Since we want to stablize our system as well as shift our root locus towards left so, we have decided to use a PID (Proportional derivative Integral) controller. Only PD controller also works because & if we ignore unstability then system has 0 steady state error, ie



where $G_c(s)$ is a parallel combination of



so, $$G_c(s) = K_P + K_D s + \frac{K_I}{s}$$

$$= \frac{K_D\left(s^2 + \frac{K_P}{K_D}s + K_I\right)}{s}$$

so, $G_c(s)$ has two zeroes & one pole at 0 it can be rewritten as

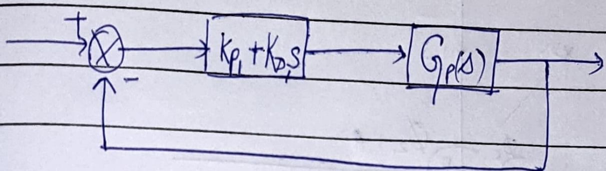
$$= \frac{K\,(s - s_{1c})\,(s - s_{2c})}{s}$$

$$= \underbrace{K\,(s - s_{1c})}_{\text{PD controller}}\ \underbrace{\left(\frac{s - s_{2c}}{s}\right)}_{\text{PI controller}}$$

* So PID controller is basically a combination of PI & PD controllers in series

We can use the above statement for our benefit.

Assume that we will use only PD controller



$$T(s) = \frac{(K_{P_1} + K_{D_1}s)\, G_P(s)}{1 + (K_{P_1} + K_{D_1}s)\, G_P(s)}$$

to find poles / root locus

$$1 + (K_{P_1} + K_{D_1}s)\, G_P(s) = 0$$

$$F(s) = \frac{(K_{P_1} + K_{D_1}s)}{s^2\left(1 + \frac{s}{10}\right)} = -1$$

so mag $= 1$

phase $= 180^\circ$

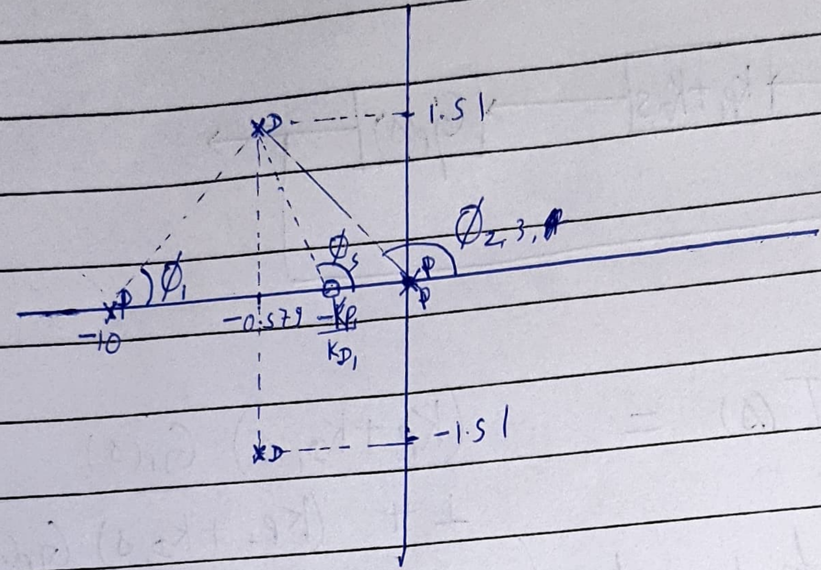
i.e. if we want rootlocus to pass through s_1

then

we have two conditions

$$|F(s_D)| = 1$$

$$\angle F(s_D) = 180^\circ$$



$$\phi_5 - \phi_1 - \phi_2 - \phi_3 - \cancel{\phi_4} = 180^\circ$$

$$\Rightarrow \cancel{180}$$

$$\Rightarrow \phi_5 = 180^\circ + \sum_{1}^{4} \phi_i$$

$$\Rightarrow \phi_5 = 180^\circ + 9.13^\circ + \cancel{2} \times 110.97$$

$$\Rightarrow \phi_5 = \cancel{182}\ 50.97^\circ$$

$$\Rightarrow \phi_5 = \cancel{17.}\ \cancel{180 - 17.9329}$$

$$\Rightarrow -\frac{K_P}{K_D} - (-(0.579)) = -\cancel{4.68}\ \frac{-1.51}{\tan 50.97}$$

$$\Rightarrow \cancel{\frac{K_{P_1}}{K_{D_1}} = 5.26}$$

$$-\frac{K_{P_1}}{K_{D_1}} = -1.803 \Rightarrow \boxed{\frac{K_{P_1}}{K_{D_1}} = 1.803}$$

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Now using magnitude criderio

$$\frac{K_{D_1}\left(s+\frac{K_{P_1}}{K_{D_1}}\right)}{s^2\left(1+s/10\right)}$$

$s \to -0.579 + 1.51485j$

$$\frac{K_{D_1}\sqrt{(1.22)^2+(1.51)^2}}{\sqrt{0.579^2+1.51^2}\ \sqrt{(1-0.0579)^2+(0.151)^2}} = 1$$

$$\boxed{\begin{aligned} K_{D_1} &= 0.79 \\ K_{P_1} &= 0.96\ \cancel{1.65} \end{aligned}}$$

$\cancel{G_{PDC} \Rightarrow 1.65 + 0.313 s\ 0.96\ 0.86 + 0.79 s}$

$G_{PDC} = 0.96 + 0.79 s$

Now for PI controller we can again perform same analysis but

$$G_{PI} = \left(\frac{s + s_{PI}}{s}\right)$$

so it will add one pole at 0 and one zero at $-s_{PI}$ if we try to compensate the effect by placing s_{PI} near to 0 ~~general~~

$-s_{PI}$ = general rule of thumb says it should be $\frac{Re(s_D)}{20} = 0.025$

So, final controller

$$G_C = (0.96 + 0.79 s)\left(\frac{s + 0.025}{s}\right)$$

$$= 0.024(1 + 0.82 s)\left(\frac{1 + 40 s}{s}\right)$$

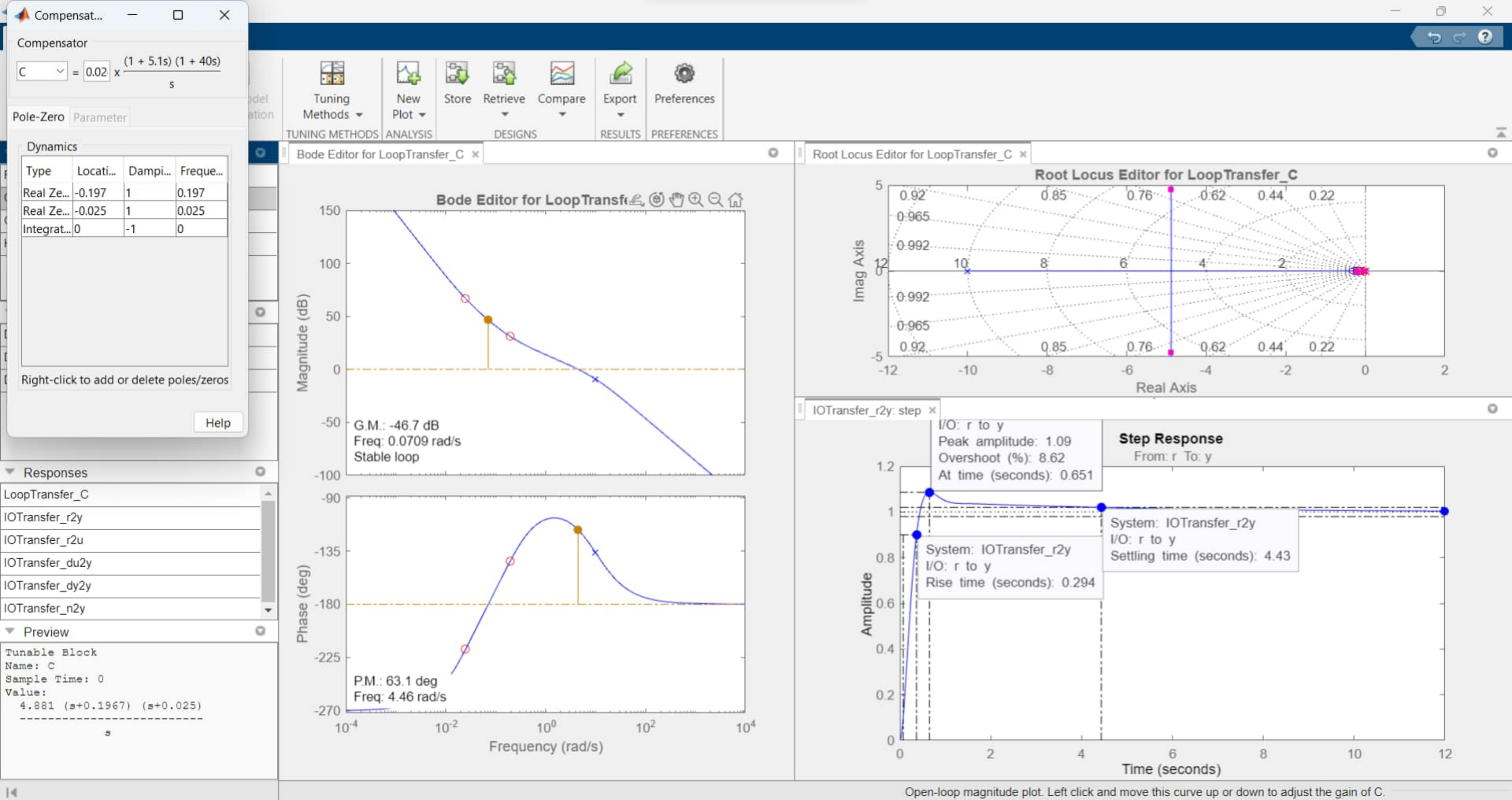
(5) Conclusion

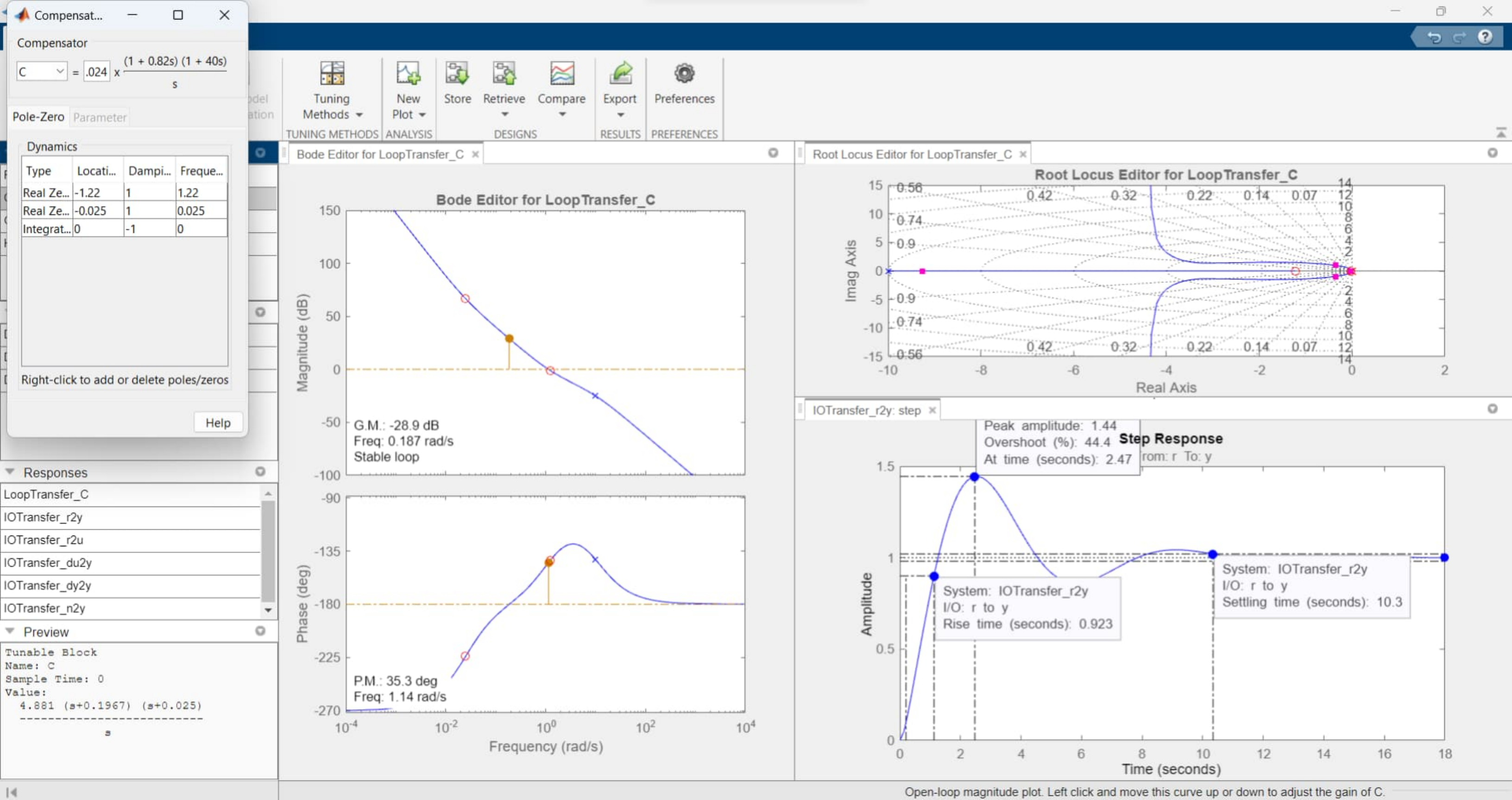
at the above expression of $G_c(s)$

the % overshoot was 44%
2% settling time was 10.3s
bandwidth was $\frac{1}{0.923}\ s^{-1}$

but when we tuned and shifted first PD ~~zer~~ zero
from −1.22 to −0.66
we obtained
% overshoot 8.62 %
2 % settling time 4.43s
bandwidth is $\frac{1}{0.294}\ s^{-1}$

Hence the system became more sharp and stable


Compensat...
Compensator
C = 0.02 x (1 + 5.1s) (1 + 40s) / s
Pole-Zero
Parameter
Dynamics
Type | Locati... | Dampi... | Freque...
Real Ze... | -0.197 | 1 | 0.197
Real Ze... | -0.025 | 1 | 0.025
Integrat... | 0 | -1 | 0
Right-click to add or delete poles/zeros
Help
Tuning Methods
New Plot
Store
Retrieve
Compare
Export
Preferences
TUNING METHODS
ANALYSIS
DESIGNS
RESULTS
PREFERENCES
Bode Editor for LoopTransfer_C
Root Locus Editor for LoopTransfer_C
Magnitude (dB)
G.M.: -46.7 dB
Freq: 0.0709 rad/s
Stable loop
Phase (deg)
P.M.: 63.1 deg
Freq: 4.46 rad/s
Frequency (rad/s)
Imag Axis
Real Axis
IOTransfer_r2y: step
Step Response
From: r To: y
I/O: r to y
Peak amplitude: 1.09
Overshoot (%): 8.62
At time (seconds): 0.651
System: IOTransfer_r2y
I/O: r to y
Rise time (seconds): 0.294
System: IOTransfer_r2y
I/O: r to y
Settling time (seconds): 4.43
Amplitude
Time (seconds)
Responses
LoopTransfer_C
IOTransfer_r2y
IOTransfer_r2u
IOTransfer_du2y
IOTransfer_dy2y
IOTransfer_n2y
Preview
Tunable Block
Name: C
Sample Time: 0
Value:
4.881 (s+0.1967) (s+0.025)
s
Open-loop magnitude plot. Left click and move this curve up or down to adjust the gain of C.


Compensat...
Compensator
C = .024 x (1 + 0.82s) (1 + 40s) / s
Pole-Zero
Parameter
Dynamics
Type | Locati... | Dampi... | Freque...
Real Ze... | -1.22 | 1 | 1.22
Real Ze... | -0.025 | 1 | 0.025
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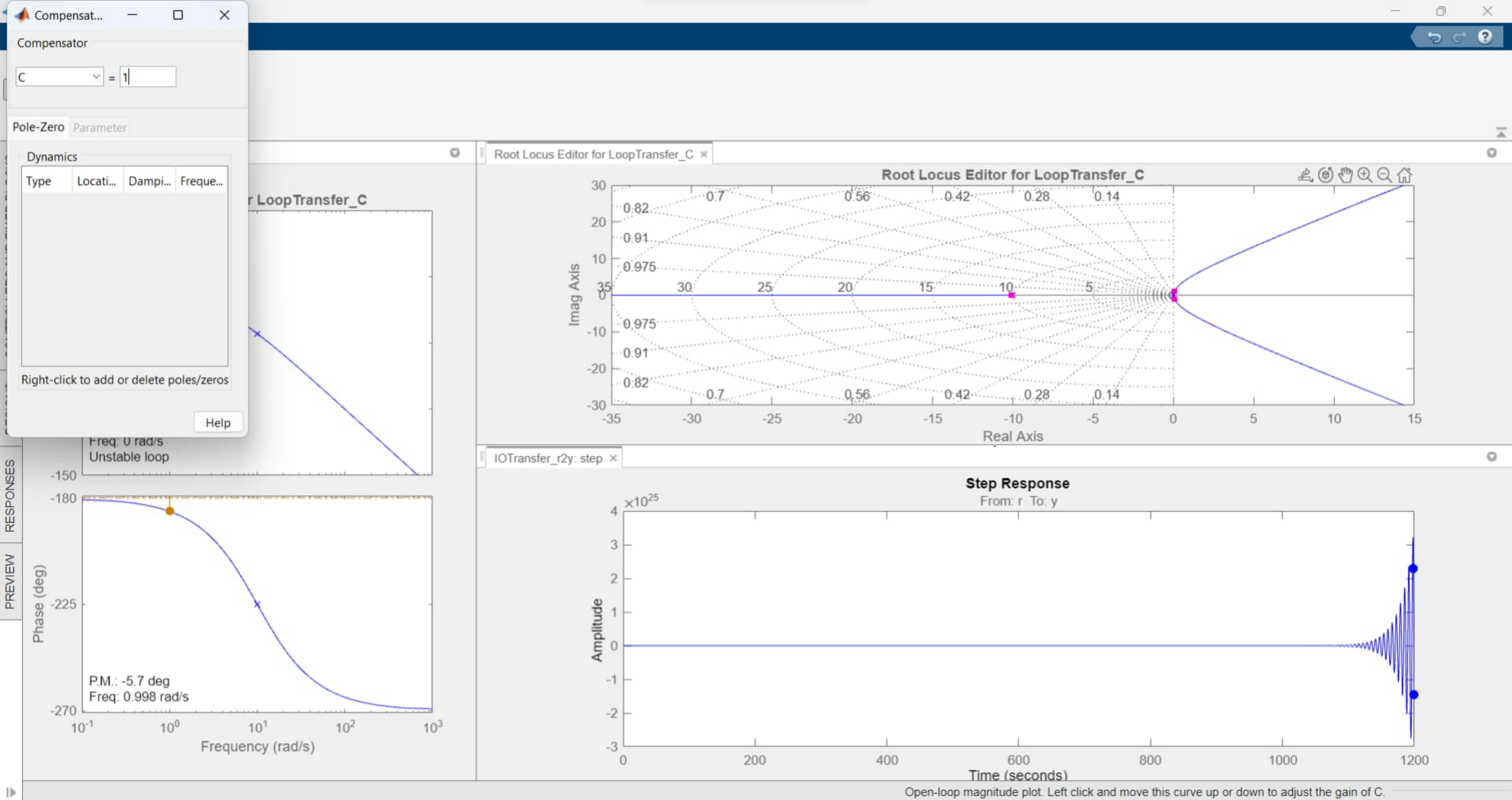
Right-click to add or delete poles/zeros
Help
Tuning Methods
New Plot
Store
Retrieve
Compare
Export
Preferences
TUNING METHODS
ANALYSIS
DESIGNS
RESULTS
PREFERENCES
Bode Editor for LoopTransfer_C
Magnitude (dB)
G.M.: -28.9 dB
Freq: 0.187 rad/s
Stable loop
Phase (deg)
P.M.: 35.3 deg
Freq: 1.14 rad/s
Frequency (rad/s)
Root Locus Editor for LoopTransfer_C
Imag Axis
Real Axis
IOTransfer_r2y: step
Step Response
From: r To: y
Peak amplitude: 1.44
Overshoot (%): 44.4
At time (seconds): 2.47
System: IOTransfer_r2y
I/O: r to y
Rise time (seconds): 0.923
System: IOTransfer_r2y
I/O: r to y
Settling time (seconds): 10.3
Amplitude
Time (seconds)
Responses
LoopTransfer_C
IOTransfer_r2y
IOTransfer_r2u
IOTransfer_du2y
IOTransfer_dy2y
IOTransfer_n2y
Preview
Tunable Block
Name: C
Sample Time: 0
Value:
4.881 (s+0.1967) (s+0.025)
s
Open-loop magnitude plot. Left click and move this curve up or down to adjust the gain of C.


Compensat...
Compensator
C
= 1
Pole-Zero
Parameter
Dynamics
Type
Locati...
Dampi...
Freque...
Right-click to add or delete poles/zeros
Help
r LoopTransfer_C
Freq: 0 rad/s
Unstable loop
P.M.: -5.7 deg
Freq: 0.998 rad/s
Phase (deg)
Frequency (rad/s)
RESPONSES
PREVIEW
Root Locus Editor for LoopTransfer_C
Imag Axis
Real Axis
IOTransfer_r2y: step
Step Response
From: r To: y
Amplitude
Time (seconds)
Open-loop magnitude plot. Left click and move this curve up or down to adjust the gain of C.