AE 330/708

AEROSPACE PROPULSION

Instructor

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Review of thermodynamic relations

Perfect gas law

$$p_x V_x = R T_x$$

Ratio of specific heats – exponent in isentropic process

$$k = c_p / c_v$$

Specific heat at constant pressure > Specific heat at constant volume

$$c_p - c_v = R/J$$

Specific gas constant, R – ratio of the universal gas constant to the molecular weight of the gas

$$c_p = kR/(k-1)J$$

Universal gas constant = 8.3145 J/mol.K

$$T_x/T_y = (p_x/p_y)^{(k-1)/k} = (V_y/V_x)^{k-1}$$

Isentropic process relations:

pV^k = constant

$$a = \sqrt{kRT}$$

a = speed of sound

$$M = v/a = v/\sqrt{kRT}$$

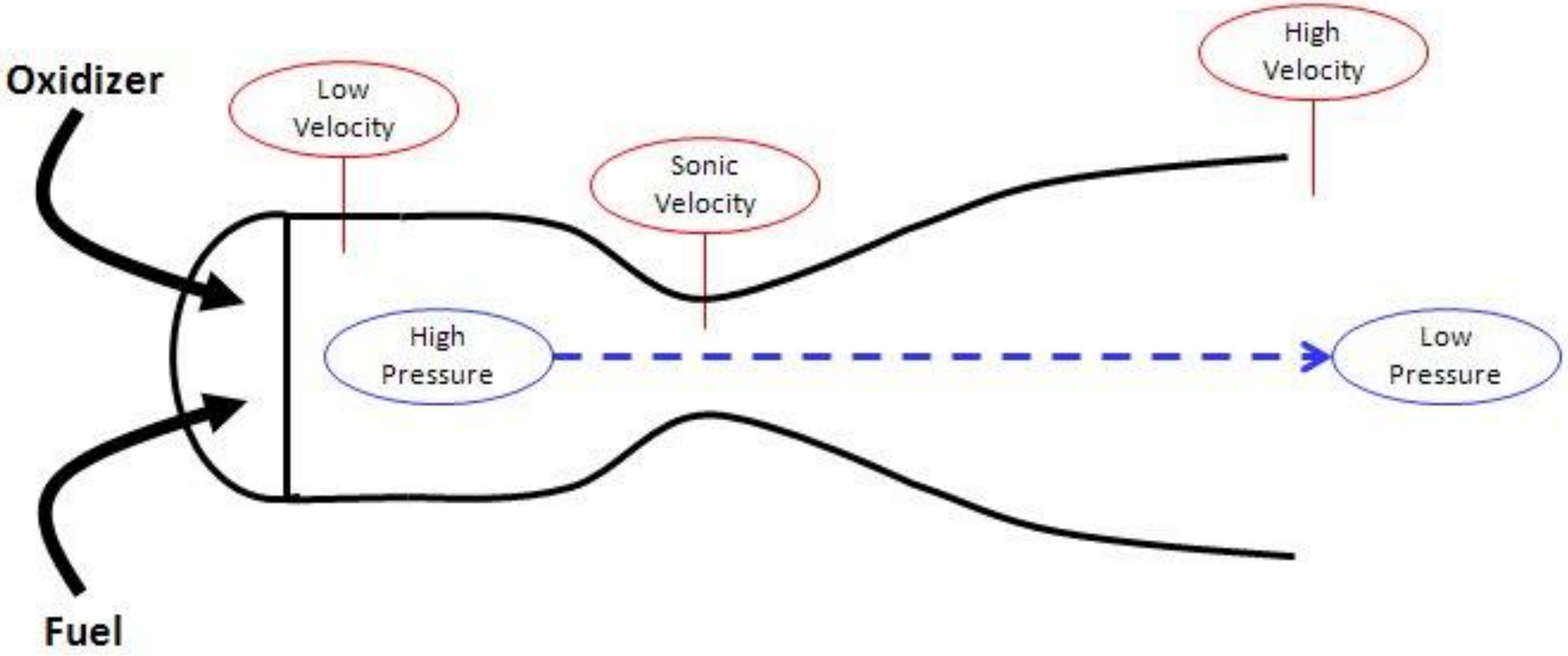
M = Mach number

$$T_0 = T\left[1 + \frac{1}{2}(k-1)M^2\right]$$

Stagnation temperature and Stagnation pressure

$$p_0 = p\left[1 + \frac{1}{2}(k-1)M^2\right]^{k/(k-1)}$$

Stagnation and static properties are related through Mach number and k



A device that converts the thermal energy released by the combustion of propellants into the kinetic energy through the process of expansion

$$F = \dot{m}v_2 + (P_2 - P_3)A_2$$

Ideal Rocket Analysis – Assumptions

1. The working substance (combustion gases) is homogeneous
2. All the species of the working fluid are gaseous. Any condensed phase adds a negligible amount to the total mass
3. The working substances follow the perfect gas law
4. No heat transfer at the rocket walls; hence, the flow is adiabatic
5. There is no appreciable friction and the boundary layer effects are small
6. There are no shock waves or discontinuities in the nozzle flow
7. The flow is steady, constant and uniform
8. All exhaust gases leaving the nozzle have an axially directed velocity
9. The gas velocity, temperature, density and pressure are uniform across any section normal to the axis
10. Chemical equilibrium has reached and the gas composition does not change in the nozzle (frozen flow)
11. Stored propellants are at room temperature and cryogenic propellants are at their boiling points

For steady, one-dimensional flow,

Euler equation : $dp = -\rho v dv$

Continuity equation: $\rho v A = \text{constant}$

Differentiating continuity equation & dividing by $\rho v A$,

$$\frac{d\rho}{\rho} + \frac{dv}{v} + \frac{dA}{A} = 0$$

$$\frac{dp}{\rho} = \frac{dp}{d\rho} \cdot \frac{d\rho}{\rho} \Rightarrow -v dv = \frac{dp}{d\rho} \cdot \frac{d\rho}{\rho}$$

Further, $\frac{dp}{d\rho} = a^2$ (speed of sound, a)

$$\therefore \quad a^2 \frac{d\rho}{\rho} = -v dv \Rightarrow \frac{d\rho}{\rho} = -\frac{v}{a^2} dv = -\frac{v^2}{a^2} \frac{dv}{v}$$

$$\therefore \quad \frac{d\rho}{\rho} = -M^2 \frac{dv}{v}$$

$$\frac{dA}{A} + (1 - M^2) \frac{dv}{v} = 0$$

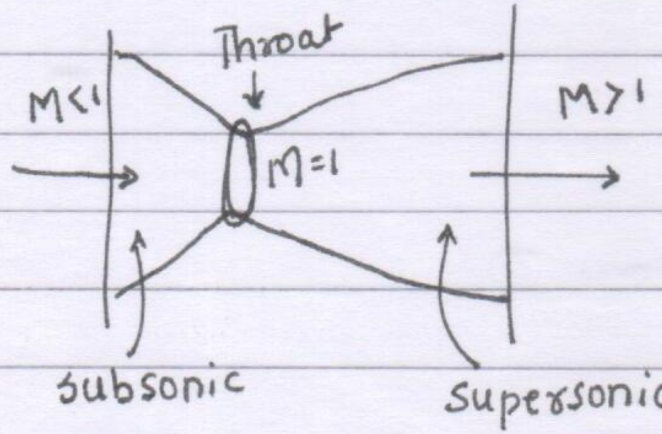
or $$\boxed{\frac{dA}{A} = (M^2 - 1) \frac{dv}{v}}$$

For $M < 1$ (subsonic flow)

flow accelerates (V - increases) if 'A' decreases (convergent)

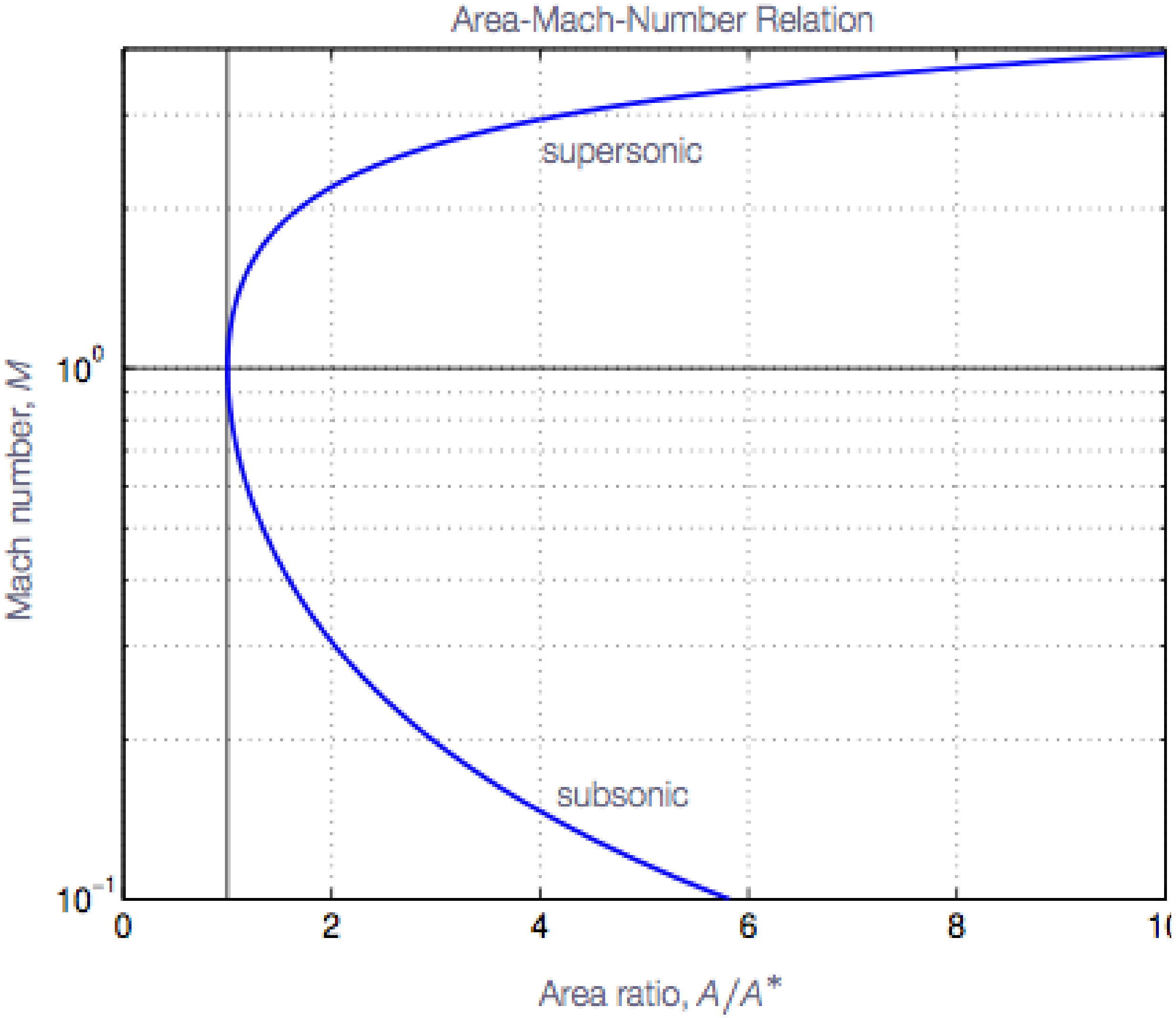
For $M > 1$ (supersonic flow)

Flow accelerates if 'A' increases (divergent)



$$\left(\frac{A}{A_t}\right)^2 = \frac{1}{M^2}\left[\frac{2}{k+1}\left(1 + \frac{k-1}{2}M^2\right)\right]^{\frac{k+1}{k-1}}$$

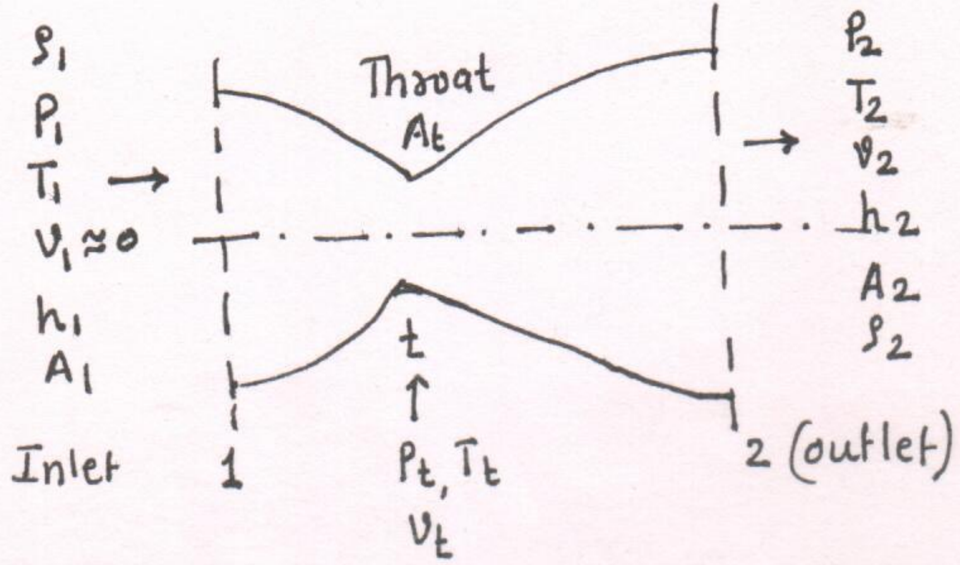
Area-Mach number relation: $M = f(A/At)$



There are two values of M that correspond to a given A/At (>1)

$$\frac{A_y}{A_x} = \frac{M_x}{M_y}\sqrt{\left\{\frac{1+[(k-1)/2]M_y^2}{1+[(k-1)/2]M_x^2}\right\}^{(k+1)/(k-1)}}$$

Isentropic flow through nozzle



For isentropic flow in nozzle, total enthalpy $\left(h + \frac{v^2}{2}\right)$ must be conserved

$$h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2}$$

$\Rightarrow$ Velocity at the nozzle exit, $v_2 = \sqrt{2(h_1 - h_2) + v_1^2}$

We use, $h = C_p \cdot T$, $\quad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$ and $C_p = \frac{kR}{k-1}$

$$v_2 = \sqrt{\frac{2k}{k-1} R \cdot T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right] + v_1^2}$$

Isentropic flow through nozzle

In most cases, inlet area >> Throat area &

hence, the inlet gas velocity can be neglected. ($V_1 \approx 0$)

Approximation of inlet conditions same as stagnation conditions.

$$\therefore \boxed{T_1 \approx T_0}$$

$$V_2 = \sqrt{\frac{2k}{k-1} R \cdot T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} = \sqrt{\frac{2k}{k-1} \frac{R_u}{MW} T_1 \left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right)}$$

Isentropic flow through nozzle

R_u = Universal gas constant. $= R \cdot MW$

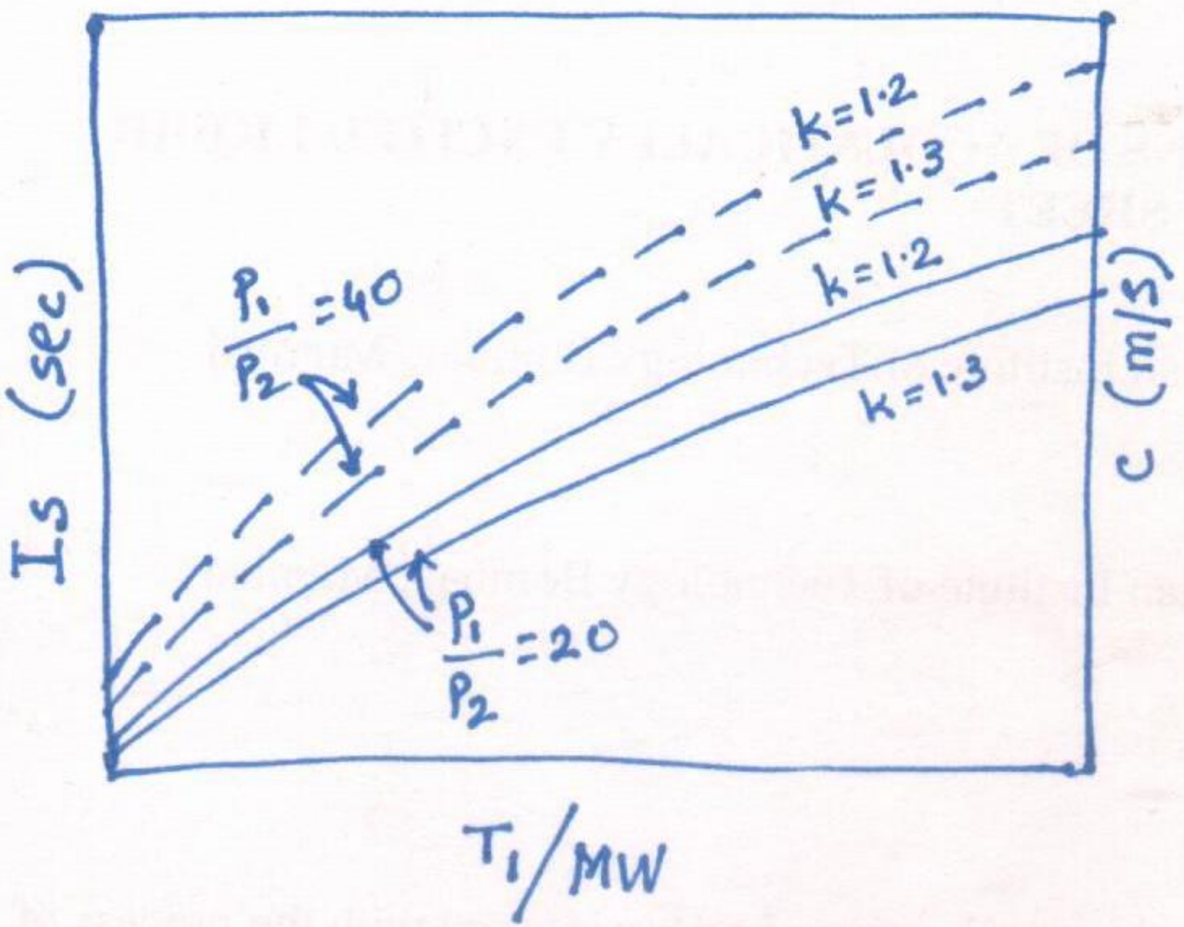
MW = Molecular weight of gases.

$$v_2 = f\left(\frac{P_2}{P_1}, k, T_1, R\right) \quad \text{or} \quad v_2 \;\&\; I_s \sim \sqrt{\frac{T_1}{MW}}$$

Max. exit velocity, $(v_2)_{max} = \sqrt{\frac{2k}{k-1} R \cdot T_1}$ } Exit velocity is always finite.

(When $P_2 = 0$, i.e. expansion in vacuum)

Isentropic flow through nozzle

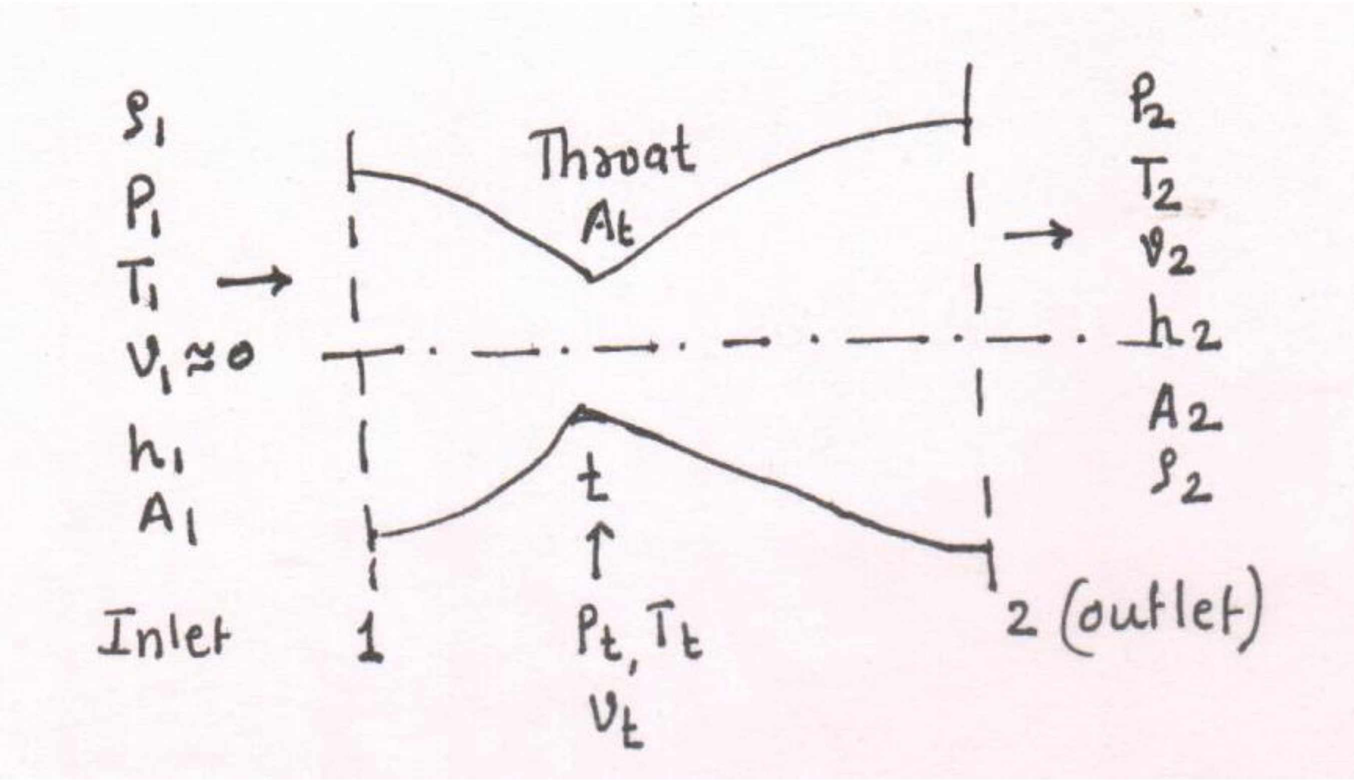


Nozzle performance (in terms of c or Is) improves with

1. increase in nozzle pressure ratio
2. decrease in the value of k
3. Increase in the value of T1/MW

The combination of the propellants should be such that they should give high flame temperature and low molecular weight

Choked nozzle and nozzle area expansion ratio



Important geometric parameter in the design of the nozzle

Ratio of the exit area to the throat area of the nozzle

Important consequences in case of the choked nozzle

Nozzle area expansion ratio, $\varepsilon = \dfrac{A_2}{A_t}$

Choked nozzle and nozzle area expansion ratio

For supersonic expansion in convergent-divergent nozzle, the Mach number at the throat is 1.

This corresponds to maximum possible mass flow rate through nozzle.

Throat pressure (P_t) for which the mass flow rate is maximum is called as critical pressure

Using $P_0 = P\left[1 + \frac{1}{2}(k-1)M^2\right]^{\frac{k}{k-1}}$ & Setting $M=1$ at throat,

$$\frac{P_t}{P_1} = \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}} \quad \text{and then} \quad \frac{T_t}{T_1} = \frac{2}{k+1} \; ; \; \frac{V_t}{V_1} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}$$

where, V = Specific volume = $\frac{1}{\rho}$

For $k = 1.2 \Rightarrow \frac{P_t}{P_1} \approx 0.56$; $\frac{T_t}{T_1} \approx 0.91$; $\frac{V_t}{V_1} \approx 1.61$

Choked nozzle and nozzle area expansion ratio

Critical throat velocity

Throat velocity, $v_t = a_t = \sqrt{kRT_t} = \sqrt{\frac{2k}{k+1} R \cdot T_1}$

Choked throat → A unique condition → Mass flow rate can't be further increased by lowering the downstream pressure

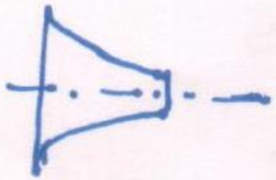
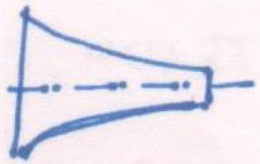
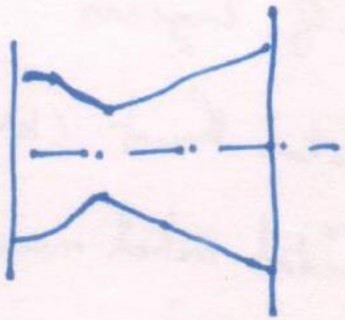
In rocket nozzles, the throat is always choked and this has important consequences on the performance

The properties at the choked throat are frozen and depend only on the upstream conditions

Hence, the performance of nozzle in terms of mass flowrate is unchanged when the rocket goes through various backpressure conditions

Choked nozzle and nozzle area expansion ratio

The sonic and supersonic flow condition is attained only if the critical pressure ratio exists at the throat or P2/P1 is less than or equal to Pt/P1

	Subsonic	Sonic	Supersonic
Throat velocity	$V_t < a_t$	$V_t = a_t$	$V_t = a_t$
Exit velocity	$V_2 < a_2$ $M_2 < 1$	$V_2 = V_t$ $M_2 = M_t = 1$	$V_2 > a_2$ $M_2 > 1$
Pressure ratio	$\frac{P_1}{P_2} < \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}$	$\frac{P_1}{P_2} = \frac{P_1}{P_t} = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}$	$\frac{P_1}{P_2} > \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}$
Nozzle Shape			

Choked nozzle and nozzle area expansion ratio

Impossible to increase the throat velocity of mass flow rate by lowering the exit pressure (even evacuating the nozzle)

Mass flow rate for choked throat $\Rightarrow$

$$\dot{m} = \frac{A_t \cdot v_t}{V_t} = \frac{A_t \cdot P_1 \cdot k}{\sqrt{kRT_1}} \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

$$\dot{m} \propto P_1 \ \& \ \frac{1}{\sqrt{T_1/MW}}$$

Choked nozzle and nozzle area expansion ratio

For any downstream location x,

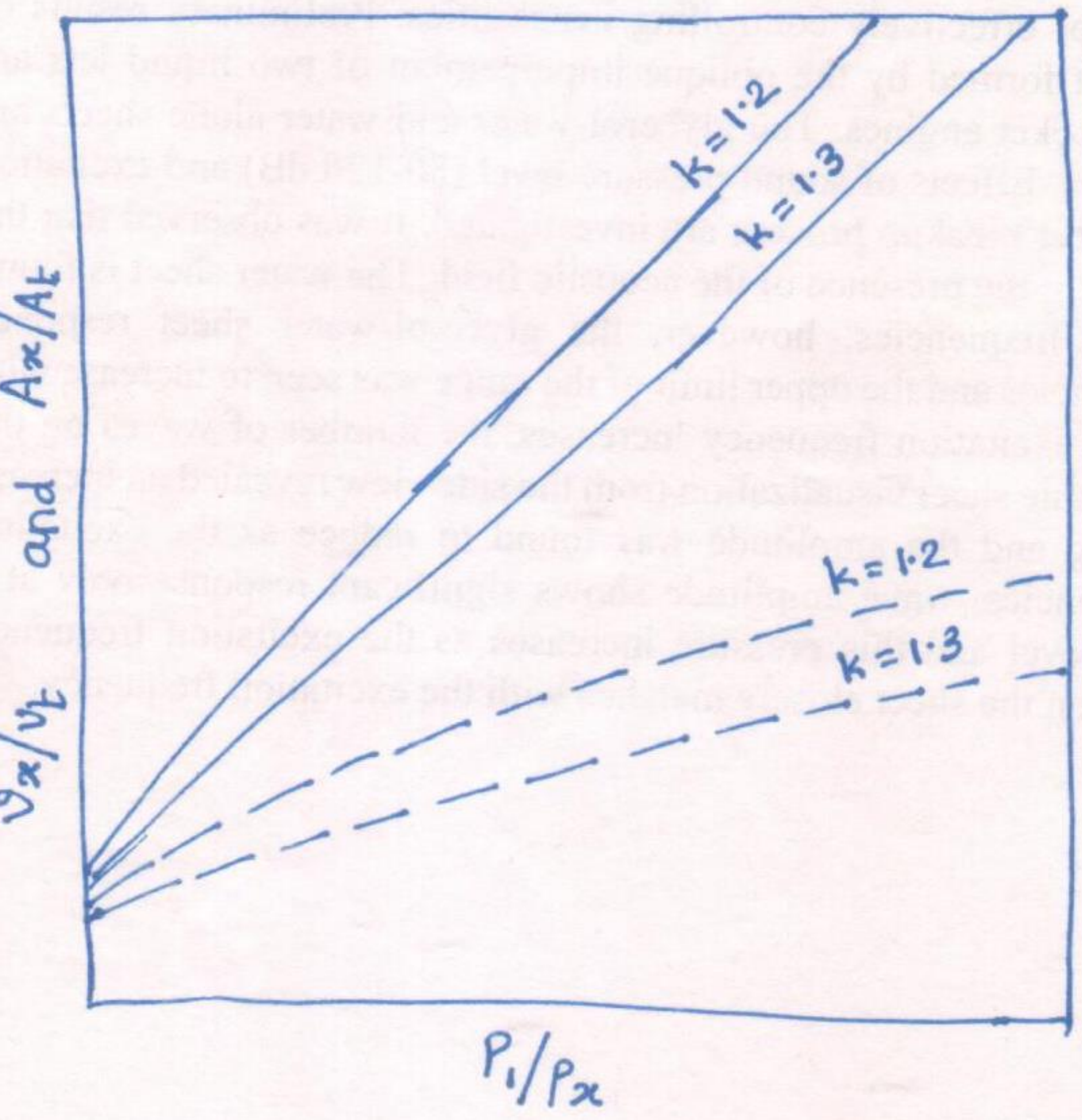
$$\frac{A_t}{A_x} = \frac{V_t \cdot \vartheta_x}{V_x \cdot \vartheta_t} = \left(\frac{k+1}{2}\right)^{1/k-1} \cdot \left(\frac{P_x}{P_1}\right)^{1/k} \sqrt{\frac{k+1}{k-1}\left[1 - \left(\frac{P_x}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

When $P_x = P_2 \Rightarrow \dfrac{A_x}{A_t} = \dfrac{A_2}{A_t} = \varepsilon$

Similarly, $\dfrac{\vartheta_x}{\vartheta_t} = \sqrt{\dfrac{k+1}{k-1}\left[1 - \left(\dfrac{P_x}{P_1}\right)^{\frac{k-1}{k}}\right]}$ .... velocity ratio

For low altitude operation (h < 10 km), ε ~ 3 – 25

For high altitude (h > 100 km), ε ~ 40 – 200 (sometimes as high as 400)



$k = 1{\cdot}2$
$k = 1{\cdot}3$
$k = 1{\cdot}2$
$k = 1{\cdot}3$
v_x/v_t and A_x/A_t
P_1/P_x

Characteristic velocity

$$c^* = \frac{P_1 \cdot A_t}{\dot{m}} = \frac{\sqrt{k R T_1}}{k \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}}$$

Characteristic velocity → function of only combustion chamber

- temperature (T1)
- Molecular weight of the gases (MW)
- Ratio of specific heat (k)

c* → thus independent of nozzle geometry, pressure ratio

Indicative of combustion performance of the propellant combination

Used extensively for comparison of propellant combinations

$c^* \sim (T1/MW)^{0.5}$ ➔ High flame temperature and low molecular weight increase characteristic velocity