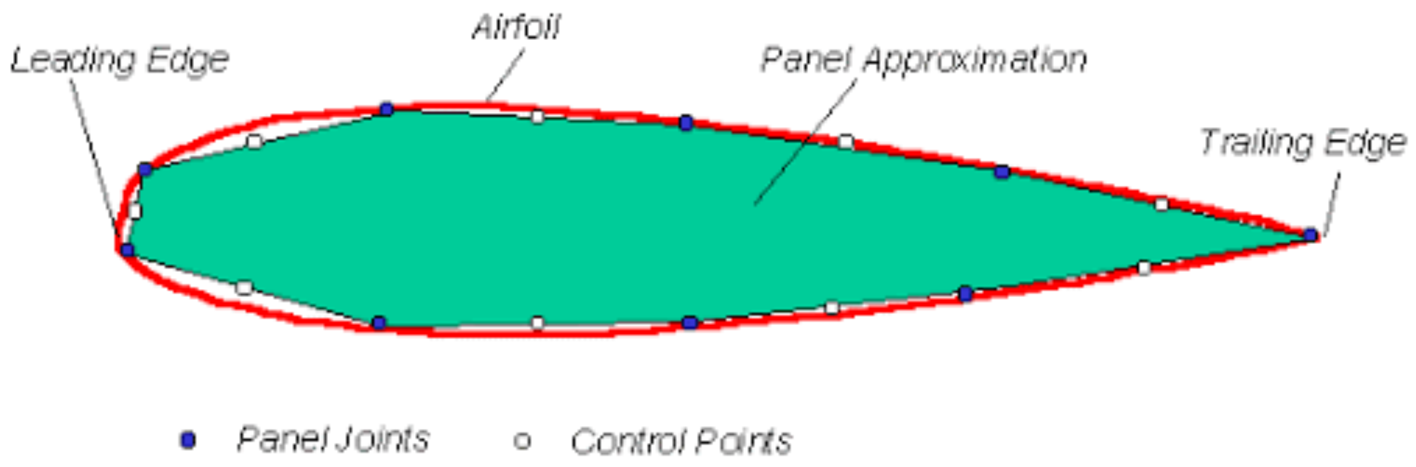
Panel Methods in Aerodynamics

Aniruddha Sinha



Progressing from Thin Airfoil Theory

We wish to solve for aerodynamics of immersed bodies w/ attached flow in the setting of

- Steady, incompressible, 2-D & inviscid (irrotational, potential) flow

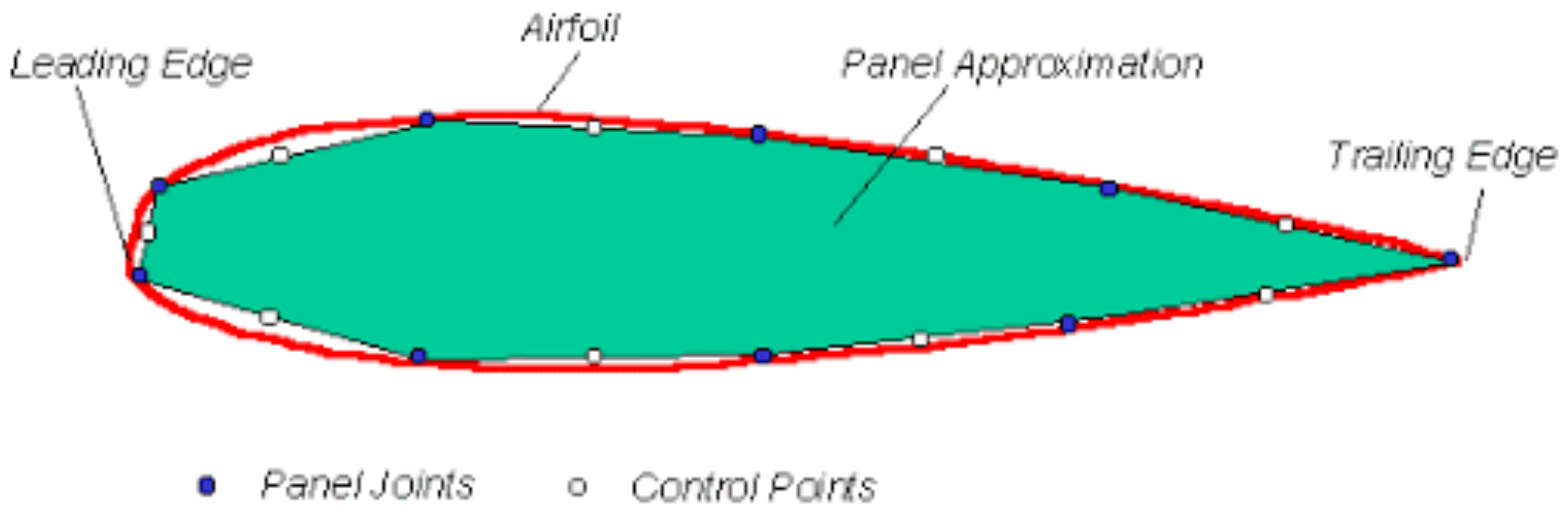
But, we wish to relax additional assumptions of thin airfoil theory:

- Thin airfoil with small camber
- Operating at small angle of attack

We cannot expect analytical solution anymore

- Instead, we look for numerical solution
- We typically have to solve ~100s of coupled linear eqns. (easy!)

Discretizing airfoil into panels (infinite slats in 2D)



http://www.engapplets.vt.edu/fluids/vpm/vpminfo.html

Sheet singularities on panels

Options for placing vortex/source sheet singularities on each panel:

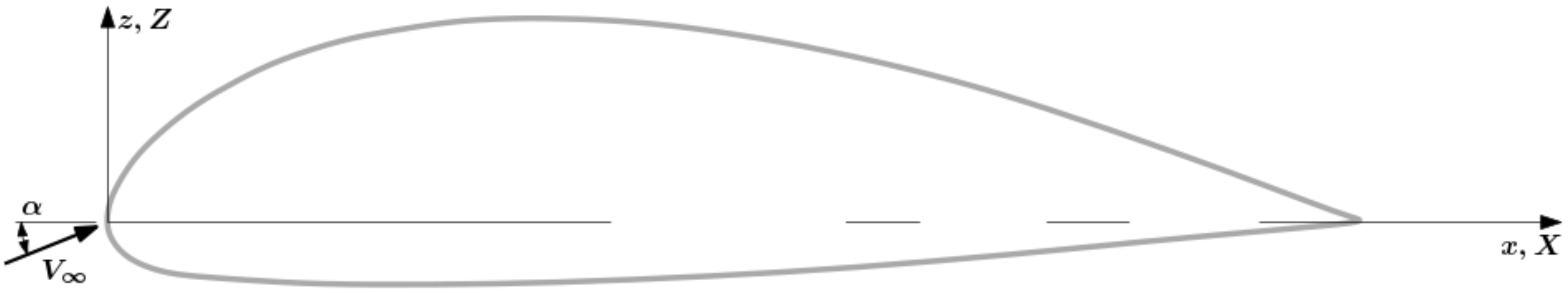
1. Original (Hess-Smith) approach:
   - Uniform-density source sheets on each panel, different on each
   - Collocated uniform-density vortex sheets on each panel, all with same density
2. Linear vortex panels:
   - Vortex sheets with linearly-varying circulation density from one end of panel to other; different distributions for different panels
   - No source sheets

Numerical stability and accuracy of method depends on choice

Formulation of Linear Vortex Panel Method

Details in Lecture Notes and Kuethe & Chow Textbook

Linear vortex panel method (LVPM)* – Setup



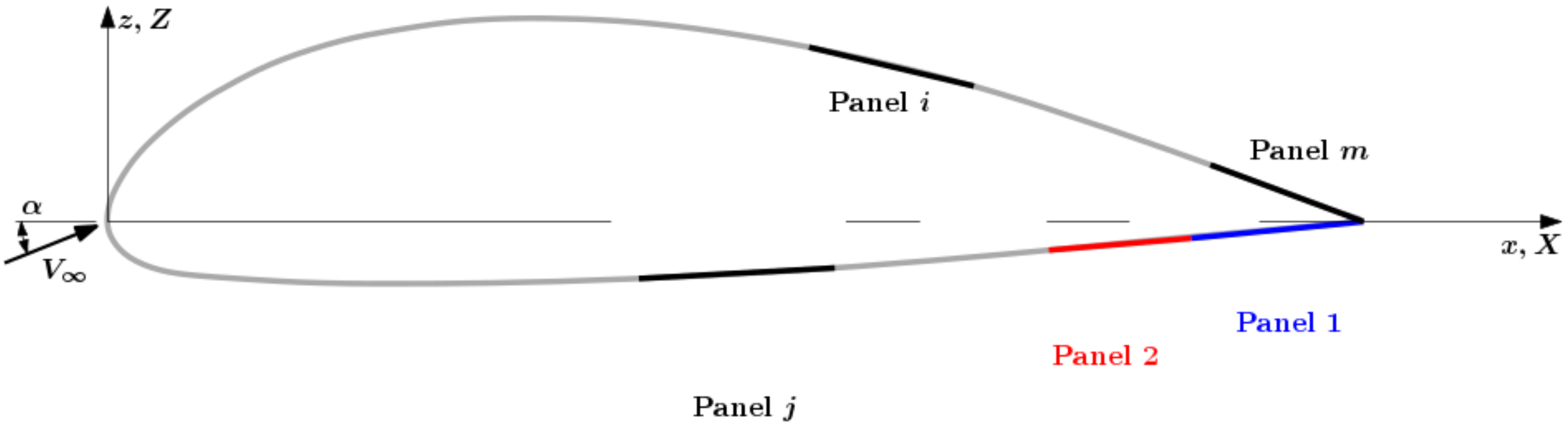
Airfoil (or any immersed body) placed in freestream at angle of attack

- Coordinate frame fixed to airfoil
- Direction of freestream can vary as desired

X, *Z*: coordinates on airfoil; *x*, *z*: coordinates of arbitrary point in flow

**Kuethe & Chow, Foundations of Aerodynamics, John Wiley & Sons, 1998*

LVPM – Panel order

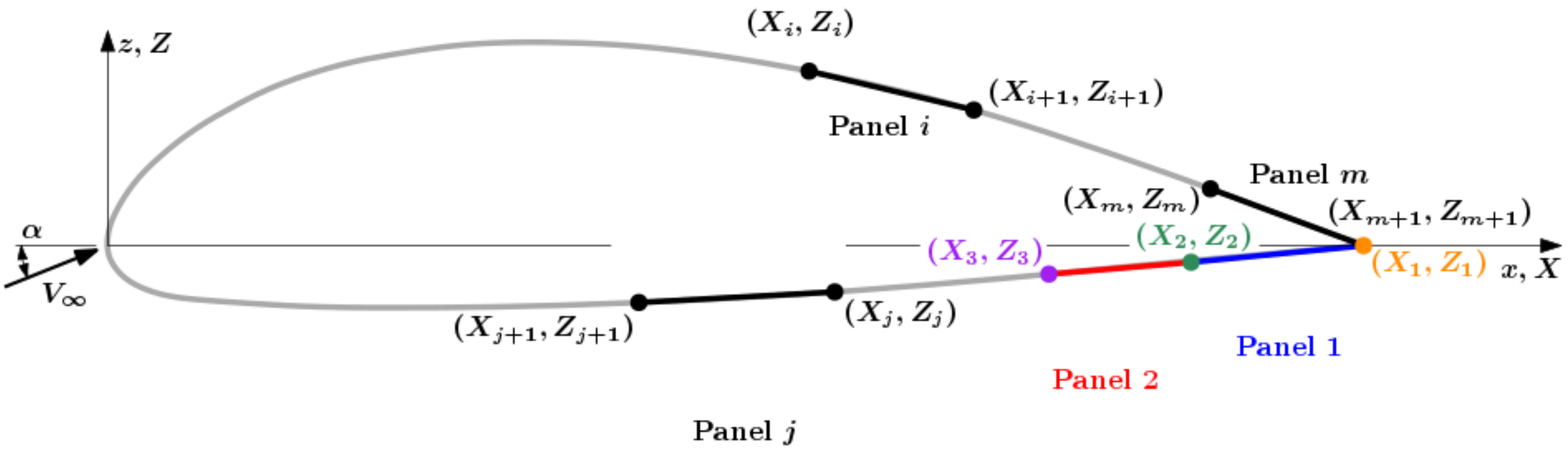


Panels ordered from 1 to *m*, starting and ending at TE, forming polygon

- Going around airfoil, first along lower surface, and then along upper

Indices of panels (and associated quantities) are in superscripts

LVPM – Nodes

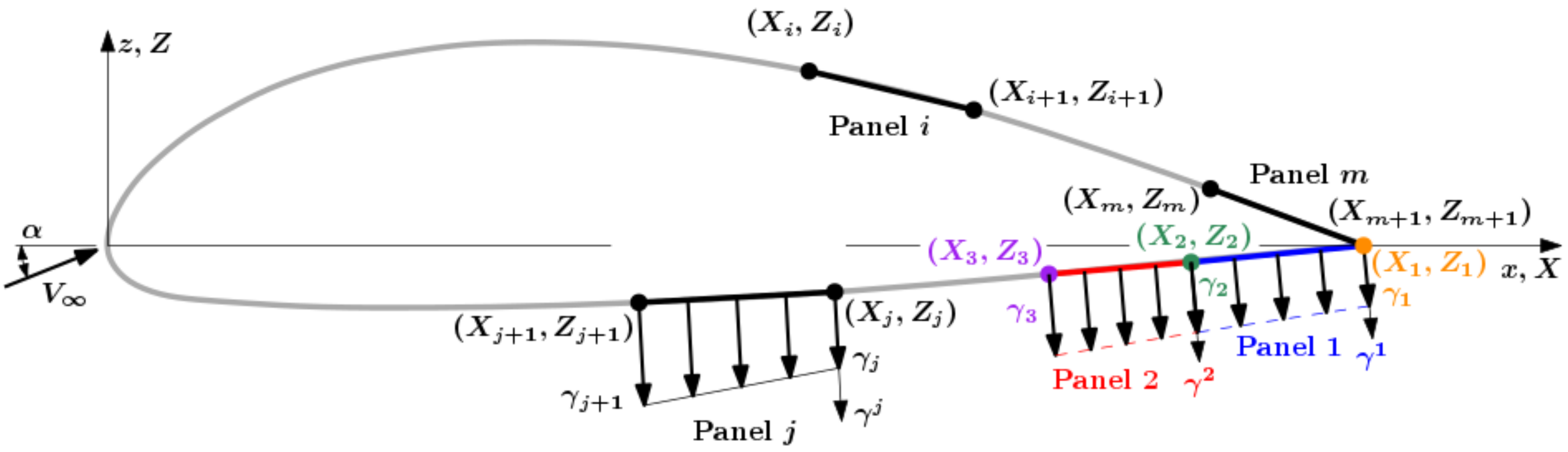


Nodes are numbered from 1 to (m+1)

- First and last nodes are coincident

Indices of nodes (and associated quantities) are in subscripts

LVPM – Parameterizing panels' circulation density

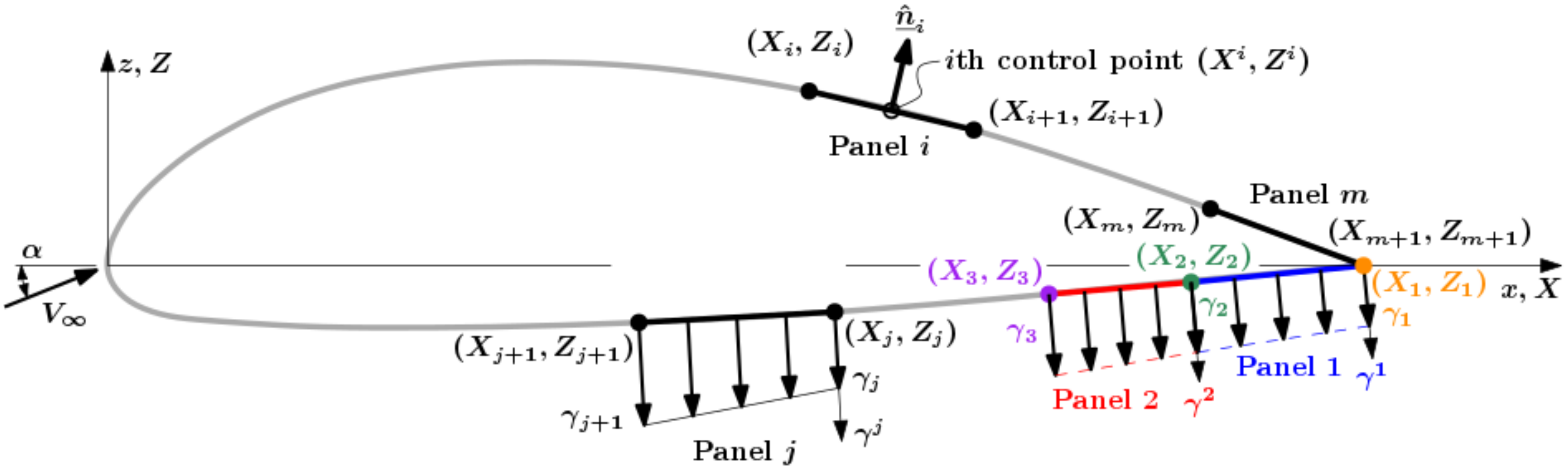


(Unknown) circulation densities specified at nodes: $\gamma_1, \gamma_2, \cdots, \gamma_j, \cdots, \gamma_{m+1}$

- Although nodes 1 and (m+1) coincide, $\gamma_1 \neq \gamma_{m+1}$!

Circulation density on panel j (i.e., γ^j) varies linearly from value specified at first node (γ_j) to the value specified at the second one (γ_{j+1})

LVPM – Control points to apply flow tangency



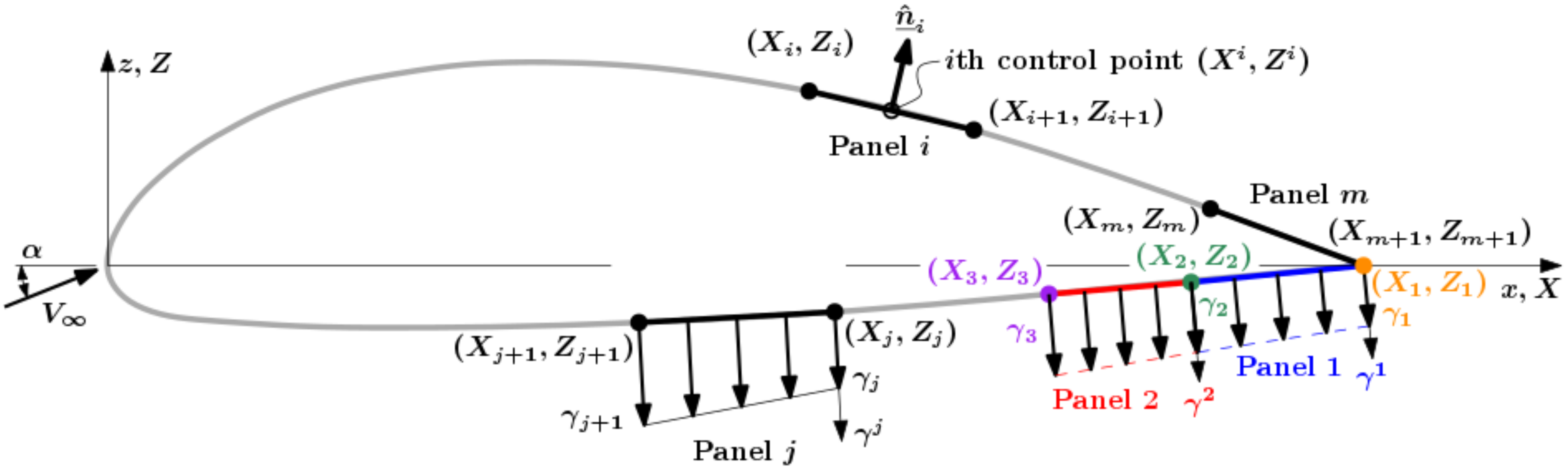
There are finite no. of unknowns (m+1 unknown γ's at as many nodes)

- So, we cannot satisfy flow tangency at all points of airfoil surface

Satisfy at 'control points' – mid-points of the m panels (m equations)

- Control point on ith panel has coordinates $\left(X^i, Z^i\right)$

LVPM – Kutta condition



As in all potential theories of airfoil aerodynamics, we need additional Kutta condition to complete problem formulation

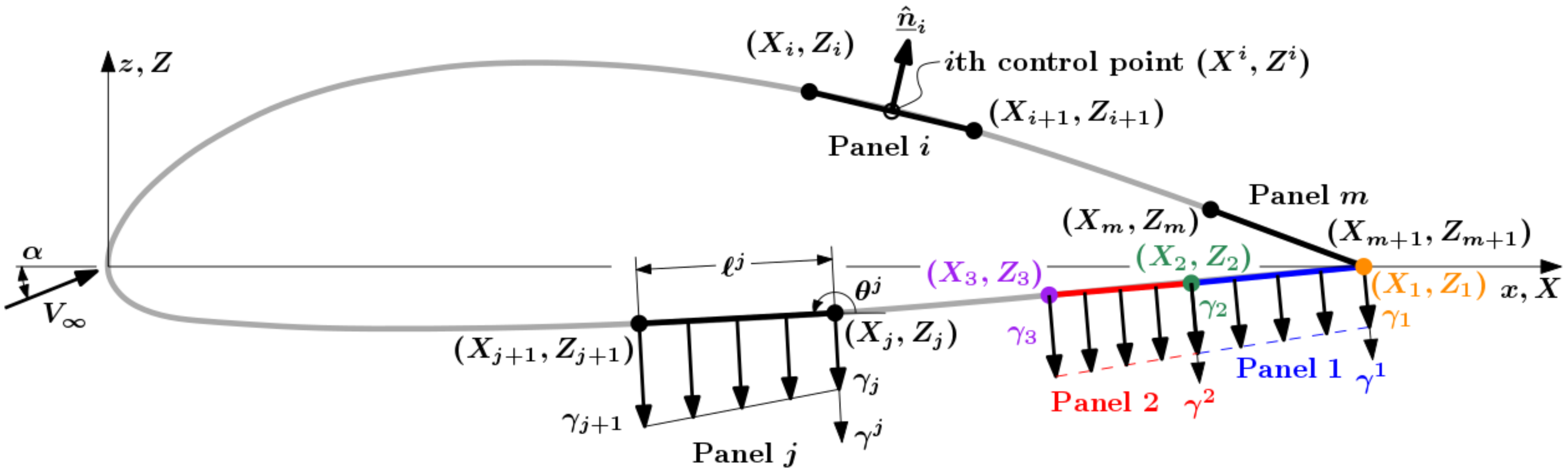
- Here, we set overall γ to 0 at TE; i.e., $\gamma_1 + \gamma_{m+1} = 0$

This completes the set of *m*+1 (linear) equations in as many unknowns

Mathematical Formulation of Linear Vortex Panel Method

Details in Lecture Notes and Kuethe & Chow Textbook

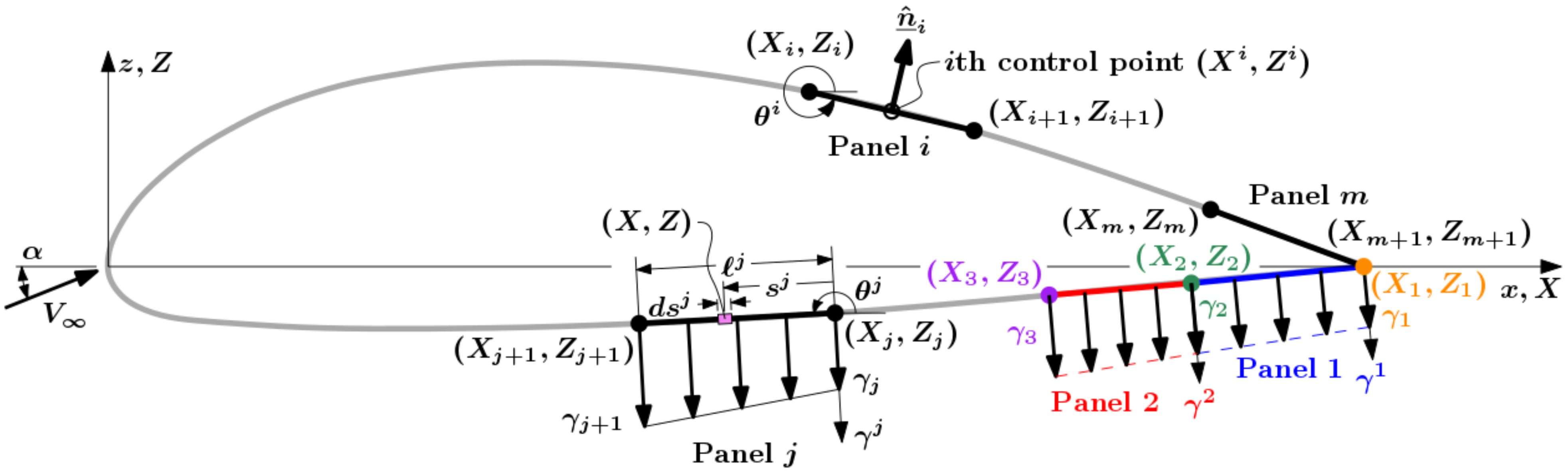
LVPM – Panels’ geometry specifications



Panels’ lengths and orientation (w.r.t. x-axis) can be calculated once and for all from geometry

Angle of attack (w.r.t. x-axis) can vary

LVPM – Parameterizing arbitrary point on panel



Let $s^j \in [0,1]$ parameterize the position of an arbitrary point on panel j

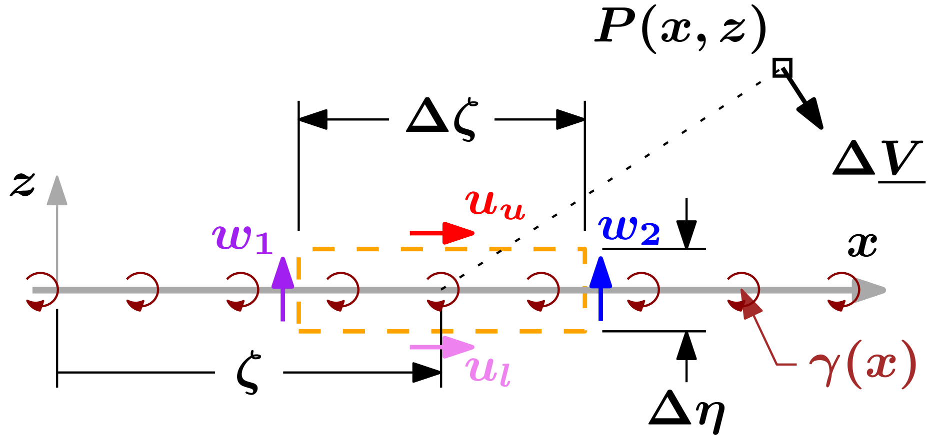
$$\Longrightarrow X(s^j) = X_j + s^j l^j \cos\theta^j \,; \quad Z(s^j) = Z_j + s^j l^j \sin\theta^j$$

Circulation density on panel j: $\gamma^j(s^j) = (1 - s^j)\gamma_j + s^j \gamma_{j+1}$

LVPM – Velocity at a point due to vortex panel *j*

Found this in context of thin airfoil theory)

- Now, only difference is that (planar) sheet is not along *x*-axis



Velocity at (x, z) due to vortex panel *j*:

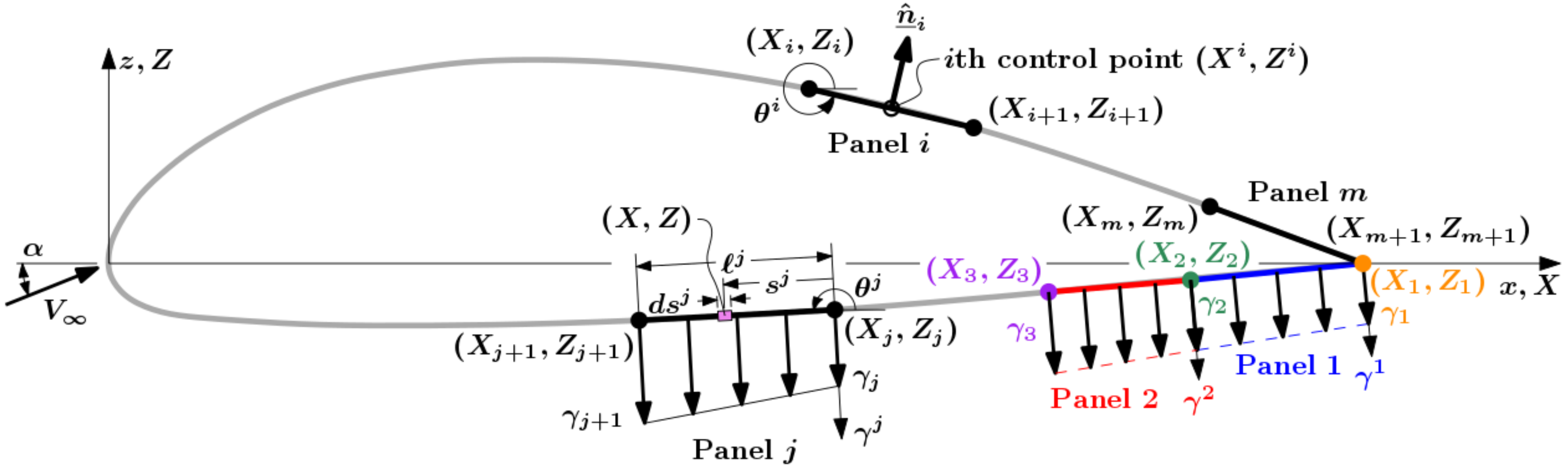
$$\underline{V}^j(x,z) = \int_0^1 \frac{\gamma^j(s^j)}{2\pi} \frac{\left(z - Z(s^j)\right)\hat{\underline{\imath}} - \left(x - X(s^j)\right)\hat{\underline{k}}}{\left(x - X(s^j)\right)^2 + \left(z - Z(s^j)\right)^2} l^j ds^j$$

$$= \underline{A}_j(x,z)\gamma_j + \underline{B}_j(x,z)\gamma_{j+1}$$

Velocity at control point of panel *i* due to vortex panel *j*:

$$\underline{V}^j(X^i,Z^i) = \underline{A}_j(X^i,Z^i)\gamma_j + \underline{B}_j(X^i,Z^i)\gamma_{j+1} =: \underline{A}_{i,j}\gamma_j + \underline{B}_{i,j}\gamma_{j+1}$$

LVPM – Flow tangency b.c. implementation



Flow must be tangential to each panel at its respective control point

$$\underline{V}(X^i, Z^i) \cdot \underline{\hat{n}}_i = 0 \quad \forall i \in [1, m]$$

$$\Longrightarrow \left[V_\infty \left(\cos\alpha\, \underline{\hat{\imath}} + \sin\alpha\, \underline{\hat{k}} \right) + \sum_{j=1}^{m} \left(\underline{A}_{i,j}\gamma_j + \underline{B}_{i,j}\gamma_{j+1} \right) \right] \cdot \underline{\hat{n}}_i = 0 \quad \forall i \in [1, m]$$

LVPM – Pressure coefficient on panels

Bernoulli's eqn. gives pressure coefficient at control point of panel *i*:

$$p^i + \frac{1}{2}\rho_\infty \left(V^i\right)^2 = p_\infty + \frac{1}{2}\rho_\infty V_\infty^2 \qquad \Longrightarrow C_p^i = 1 - \left(\frac{V^i}{V_\infty}\right)^2$$

As flow tangency is enforced at control point, $V^i = V_t^i = \underline{V}\left(X^i, Z^i\right) \cdot \underline{\hat{t}}^i$

$$V_t^i = \left[V_\infty\left(\cos\alpha\, \underline{\hat{\imath}} + \sin\alpha\, \underline{\hat{k}}\right) + \sum_{j=1}^{m}\left(\underline{A}_{i,j}\gamma_j + \underline{B}_{i,j}\gamma_{j+1}\right)\right] \cdot \underline{\hat{t}}^i$$

To obtain c_l, we integrate C_p^i

- Alternatively we integrate γ^j's, and apply Kutta-Joukowski theorem

Similarly, c_m can be found by integrating C_p

End of Topic

Panel Methods in Aerodynamics