AE 308: Control Theory

AE 775: System Modelling, Dynamics & Control

Lecture 10: Tracking and Disturbance Rejection

Dr. Arnab Maity
Department of Aerospace Engineering
Indian Institute of Technology Bombay
Powai, Mumbai 400076, India

Introduction

Performance Expectations

- Our system must give desired performance about operating point.
- In addition to that our system must withstand disturbances of reasonable size, without significant changes to their performance.
- **Example**:

I want to ride at constant speed but potholes are acting as **disturbances**.

Introduction

Performance Attributes

- Expectations are commonly stated in terms of
  - Stability
  - Reference tracking
  - Disturbance rejection

Stability

- Ability to return to its equilibrium, if disturbed

Disturbance rejection

- Ability to minimize the impact of a disturbance

Tracking

- Ability to follow a desired trajectory with minimum or no errors

Concept of Tracking

Concept of tracking

Figure: Autonomous car trying to track the path generated by trajectory planner

Concept of Tracking

Can you think any other example?

Tracking - Examples

1^{st} order system

- Consider three plants, subjected to the unit step input

$$G_1(s) = \frac{1}{s+5} \qquad G_2(s) = \frac{4}{s+5} \qquad G_3(s) = \frac{5}{s+5}$$

- The corresponding step response are as follows

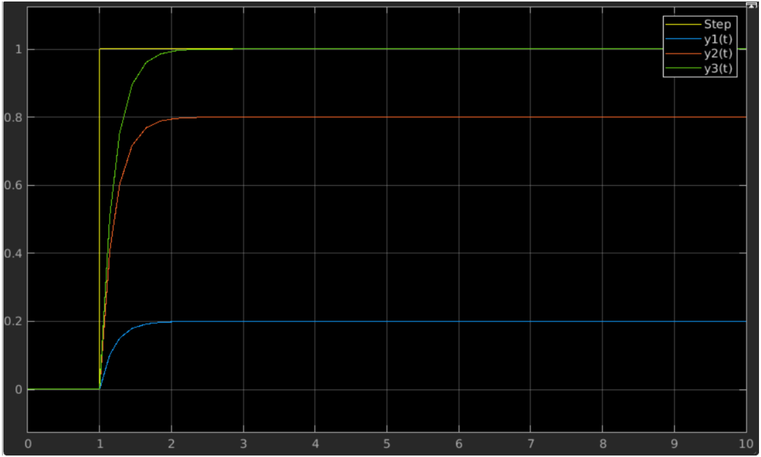
$$y_1(t) = \frac{1}{5}(1 - e^{-5t})$$

$$y_2(t) = \frac{4}{5}(1 - e^{-5t})$$

$$y_3(t) = (1 - e^{-5t})$$

Tracking - Examples

1^{st} order system (cont...)



Tracking - Examples

2^{nd} **order system**

- Consider two plants, subjected to the unit step input

$$G_1(s) = \frac{1}{s^2 + 2s + 5} \qquad G_2(s) = \frac{4}{s^2 + 2s + 5}$$

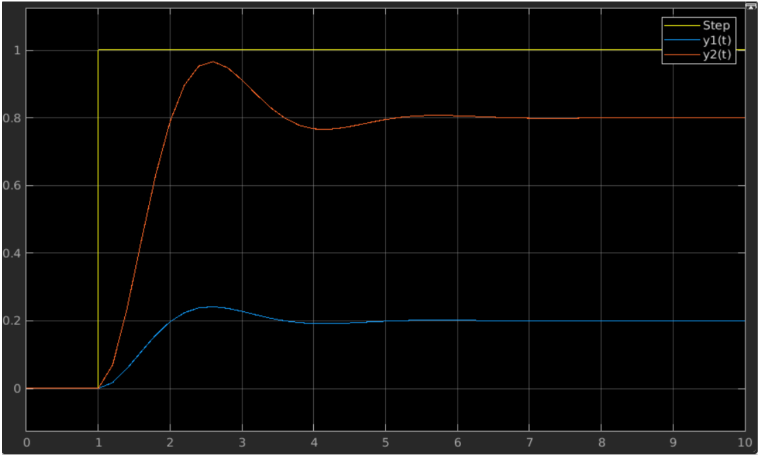
- The corresponding step response are as follows

$$y_1(t) = \frac{1}{5}\left[1 - \frac{2}{\sqrt{5}}e^{-t}\sin(2t + 1.11)\right]$$

$$y_2(t) = \frac{4}{5}\left[1 - \frac{2}{\sqrt{5}}e^{-t}\sin(2t + 1.11)\right]$$

Tracking - Examples

2^{nd} order system (cont...)



Tracking Observations

What do you observe?

Tracking - Analysis and Summary

Analysis

- Modified system shows better tracking performance.
- Simple multiplication with a constant is able to reduce the steady state error.

Summary

- Tracking task involves following a desired input.
- Its quality is rated based on the error with respect to the desired reference input.

Concept of Disturbance Rejection

Concept of disturbance rejection

- Disturbance rejection relates to the ability to nullify the impact of a disturbance in shortest possible time and with least possible departure from equilibrium.
- Aircraft during cruise may experience a gust, but it should not generate significant departure from its speed and altitude.

Disturbance Rejection - Examples

1^{st} **order system**

- Consider two plants, subjected to unit impulse (as disturbance)

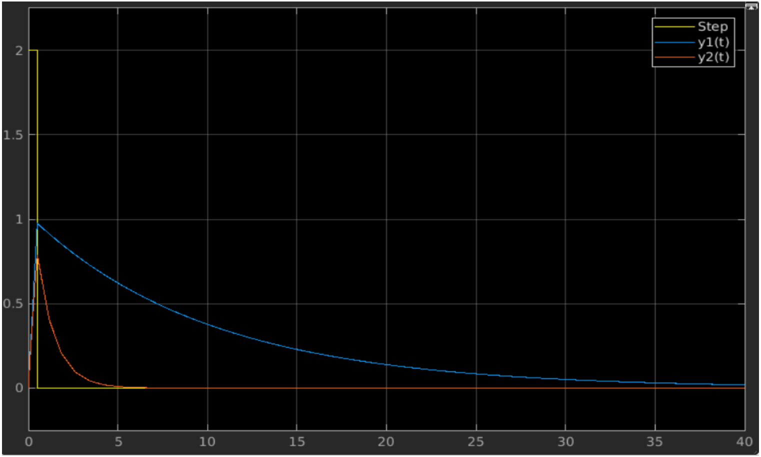
$$G_1(s) = \frac{1}{s+0.1} \qquad G_2(s) = \frac{1}{s+1}$$

- Their corresponding responses are

$$y_1(t) = e^{-0.1t}$$
$$y_2(t) = e^{-t}$$

Disturbance Rejection - Examples

1^{st} order system (cont...)



Disturbance Rejection - Examples

2^{nd} **order system (cont...)**

- Consider two plants, subjected to unit impulse (as disturbance)

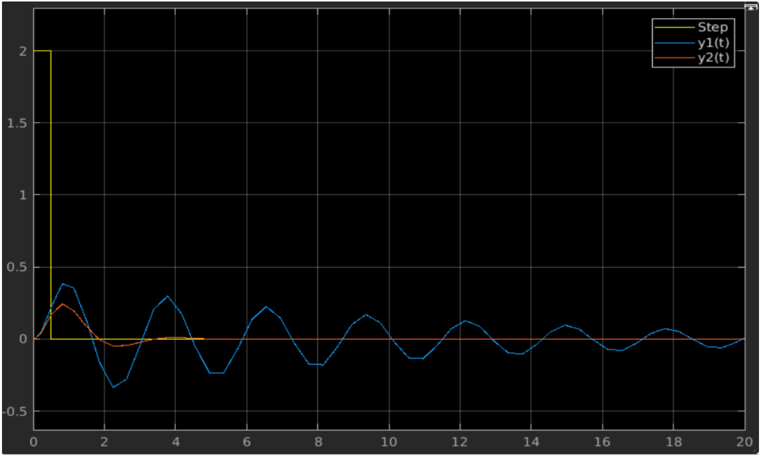
$$G_1(s) = \frac{1}{s^2 + 0.2s + 5} \qquad G_2(s) = \frac{1}{s^2 + 2s + 5}$$

- Their corresponding responses are

$$y_1(t) = \frac{1}{2.23} e^{-0.1t} \sin(2.23t + 1.53)$$
$$y_2(t) = \frac{\sqrt{5}}{2} e^{-t} \sin(2t + 1.11)$$

Disturbance Rejection - Examples

2^{nd} order system (cont...)



Disturbance Rejection Observations

Can you comment on the observations?

Disturbance Rejection - Analysis and Summary

Analysis

- Modified system shows better rejection performance
- In these cases we see that a more difficult analysis is needed to reduce the departures as well as settling time.

Summary

- Rejection task aims to reduce departures with minimal time to achieve the steady-state and is essentially a transient response attribute.

Question???

Do you think a system with good tracking capability will also have good disturbance rejection capabilities?

Examples - Tracking vs Disturbance Rejection

1^{st} **order system**

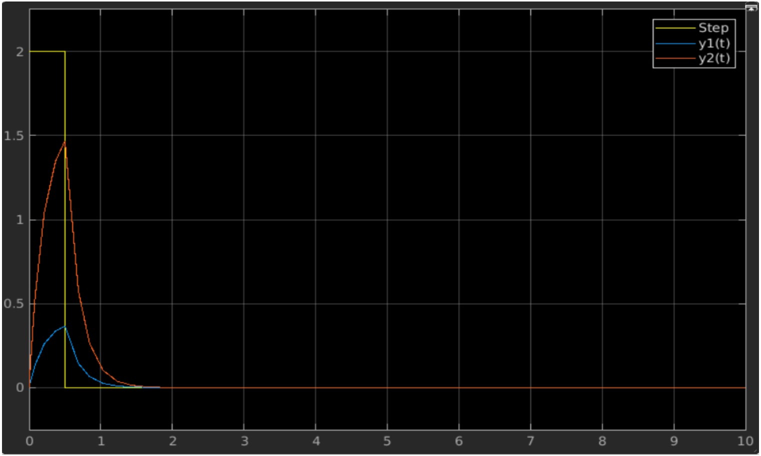
- Consider two plants,

$$G_1(s) = \frac{1}{s+5} \qquad G_2(s) = \frac{4}{s+5}$$

- We know that $G_2(s)$ has better tracking capability.
- Lets do their impulse response analysis.

Examples - Tracking vs Disturbance Rejection

1^{st} order system (cont...)



Conclusion

1^{st} order system (cont...)

- Simple multiplication by a constant may not work in all situations.
- We need to come up with a different strategy, such as a controller design.

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