Assignment - 5 (AE-330)

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Ans 1) Given, $F_{sea} = 8896.4\,N$, $P_1 = 6.89\,MPa$, $t_b = 10\,sec$, $T_{amb} = 294\,K$

$T_1 = 1755.3\,K$, $r = 2.54\times10^{-3}\,m/s$, $k = 1.22$, $MW = 22$, $\rho_P = 1550\,kg/m^3$

$C_F = 1.57$, $\varepsilon = 7.8$,

We know that $\boxed{C_F = \dfrac{F}{P_1 A_t}}$

$$A_t = \frac{F}{C_F P_1} = \frac{8896.4}{6.89\times10^6 \times 1.57}$$

$$\boxed{A_t = 8.224\times10^{-4}\,m^2}$$

Now, $\dot{m} = \dfrac{A_t P_1 k}{\sqrt{kRT_1}}\sqrt{\left(\dfrac{2}{k+1}\right)^{\frac{k+1}{k-1}}} = \rho_b A_b r$

$$\dot{m} = \frac{8.224\times10^{-4}\times 6.89\times10^6 \times 1.22}{\sqrt{1.22\times\frac{8314}{22}\times 1755.3}} \times \sqrt{\left(\frac{2}{2.22}\right)^{\frac{2.22}{0.22}}} = 1550 \times A_b\, 2.54\times10^{-3}$$

$$\dot{m} = \frac{6.913\times10^3}{8.996\times10^2}\times(0.59) = 3.937\times A_b$$

$$\dot{m} = 4.534\,kg/sec$$

$$\Rightarrow \boxed{\begin{array}{c} \dot{m} = 4.534\,kg/sec \\ \hline A_b = 1.1516\,m^2 \end{array}}$$

Now, Specific Impulse $\Rightarrow \boxed{I_{sp} = \dfrac{F}{\dot{m}g} = 200\,sec}$

$m_{propellant} = \dot{m} \times t_b$

$m_p = 4.534 \times 10$

$\boxed{m_p = 45.34 \text{ kg}}$

Total Impulse $(I_t) = I_{sp} \times m_p \times g$

$\boxed{I_t = 88957.08 \text{ kg-m/s}}$

Hence the answers are,

$$\boxed{\begin{aligned} I_{sp} &= 200 \text{ sec} \\ A_t &= 8.224 \times 10^{-4} \text{ m}^2 \\ \dot{m} &= 4.534 \text{ kg/s} \\ m_p &= 45.34 \text{ kg} \\ I_t &= 88957.08 \\ A_b &= 1.1516 \text{ m}^2 \end{aligned}}$$

$\Leftarrow$ Answers

For A_{exit} (A_2), we use,

$A_2 = \varepsilon A_t$

$\boxed{A_2 = 6.414 \times 10^{-3} \text{ m}^2}$

Ans 2) Given, $\rho_b = 1710 \text{ kg/m}^3$, $P_1 = 14 \text{ MPa}$, $a = 17.21 \times 10^{-3}$, $k = 1.27$,

$T_1 = 2220 \text{ K}$, $MW = 23 \Rightarrow R = \frac{8314}{23} \Rightarrow R = 361.48$, $n = 0.3$

$F_{req} = 5000 \text{ N}$, $\Rightarrow$ Optimum expansion condition at 0.1 MPa, $t_b = 15$ sec

Firstly, we evaluate A_t,

$$F = P_1 A_t \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} + 0$$

$5000 = 14 \times 10^6 \times A_t \times 1.6328$

$$\boxed{A_t = 2.187 \times 10^{-4} \, m^2}$$

Now $\dot{m} = \frac{A_t P_1 k}{\sqrt{kRT_1}} \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$

$$\dot{m} = \frac{2.187 \times 10^{-4} \times 14 \times 10^{6}}{1009.53} \times 0.587$$

$$\boxed{\dot{m} = 1.78 \, kg/s}$$

Also $\dot{m} = \rho_b A_b (a P_1^n) \times 10^{-3}$

$$1.78 = 1710 \times A_b \left(17.21 \times (14)^{0.3}\right) \times 10^{-3}$$

$$\frac{1.78}{1.71} = A_b \times 37.986$$

$$\boxed{A_b = 2.74 \times 10^{-2} \, m^2}$$

Now, $\frac{A_t}{A_2} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}} \sqrt{\frac{k+1}{k-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$

$\frac{1}{\varepsilon} = 0.0762 \Rightarrow \boxed{\varepsilon = 13.123} \Rightarrow \boxed{A_2 = 2.87 \times 10^{-3} \, m^2}$

Also, $m_{propellant} = \dot{m} t_b$

$\Rightarrow \boxed{m_p = 26.7 \, kg}$

Hence the designed SRM has the following characteristics

mass of the propellant stored	$m_p = 26.7$ kg
Area of burning propellant	$A_b = 2.74 \times 10^{-2} \, m^2$
Area of Throat section	$A_t = 2.187 \times 10^{-4} \, m^2$
Area of Exit Section	$A_2 = 2.87 \times 10^{-3} \, m^2$

And $\boxed{\varepsilon = 13.123}$

Ans 3) The reaction of Hydrazine is,

$$N_2H_4 \rightarrow \alpha NH_3 + \left(1-\frac{\alpha}{2}\right)N_2 + \left(2-\frac{3\alpha}{2}\right)H_2 + \text{heat}$$

As it is assumed that no NH_3 decomposes,

$$2 - \frac{3\alpha}{2} = 0 \Rightarrow \boxed{\alpha = \frac{4}{3}}$$

$$\Rightarrow \quad N_2H_4 \longrightarrow \frac{4}{3}NH_3 + \frac{1}{3}N_2 \Leftarrow$$

Now, $\Delta H_{reactants} = \Delta H_{products}$

$$1 \times 50.3 \times 10^3 = \frac{4}{3}(-45.9) \times 10^3 + \frac{4}{3}(45.06)(T_{ad} - 298) + \frac{1}{3}(0) + \frac{1}{3}(38)(T_{ad} - 298)$$

$$\Rightarrow \quad 111.5 \times 10^3 = 72.747 \times (T_{ad} - 298)$$

$$\Rightarrow \boxed{T_{ad} = 1830.71\ K}$$

Now, given, $T_{vacuum} = 1000\ N$, $P_1 = 1\ MPa$, $P_2 = 2\ kPa$,

$k = 1.25$.

$$MW_{products} = \frac{\frac{4}{3}(17) + \frac{1}{3}(28)}{\frac{5}{3}} = \underline{\underline{19.2}}$$

$$\Rightarrow R = \frac{8314}{19.2} \Rightarrow \boxed{R = 433\ J/kg\text{-}mol}$$

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Hence, $c^* = \dfrac{\sqrt{kRT_1}}{k\sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}}$

$$c^* = \frac{995.42}{1.25 \times 0.5886}$$

$$\boxed{c^* = 1352.93 \ m/s}$$

Nozzle expansion ratio can be found as,

$$\frac{A_t}{A_2} = \frac{1}{\varepsilon} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}} \sqrt{\frac{k+1}{k-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$\frac{1}{\varepsilon} = 1.6018 \times 6.931 \times 10^{-3} \times 2.53$$

$$\frac{1}{\varepsilon} = 2.809 \times 10^{-2} \Rightarrow \boxed{\varepsilon = 35.596}$$

$$V_2 = \sqrt{\frac{2kRT_1}{k-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$\boxed{V_2 = 2374.8 \ m/sec}$$

First we will find A_t & A_2.

(Vacuum $P_3 = 0$)

$$F = P_1 A_t \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} + (P_2 - 0)\varepsilon A_t$$

$$1000 = A_t\left[1.7553 \times 10^6 + 2 \times 10^3 \times 35.596\right]$$

$$A_t = \frac{1000}{1.8265 \times 10^6} \Rightarrow \boxed{A_t = 5.475 \times 10^{-4} \ m^2}$$

$$A_2 = \varepsilon A_t$$

$$= 35.596 \times 5.475 \times 10^{-4}$$

$$\boxed{A_2 = 1.9379 \times 10^{-2}\ m}$$

Now, as $\frac{\pi d^2}{4} = A$, $d_t = \sqrt{\frac{4A_t}{\pi}}$ & $d_2 = \sqrt{\frac{4A_2}{\pi}}$

$$\boxed{d_t = 2.64\ cm} \quad \boxed{d_2 = 15.7\ cm}$$

Now lets calculate $\dot{m}$ as,

$$\dot{m} = \frac{A_t P_1 k}{\sqrt{kRT_1}} \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

$$\dot{m} = \frac{5.475 \times 1 \times (0.5886) \times 10^{-4} \times 10^{6}}{995.425}$$

$$\boxed{\dot{m} = 0.319\ kg/s}$$

$$I_{sp,vaccum} = \frac{F_{vaccum}}{\dot{m} g} = \frac{1000}{0.319 \times 9.81} \Rightarrow \boxed{I_{sp,v} = 319.55\ s}$$

Hence, the answers are,

$\boxed{MW = 19.2g}$, $\boxed{c^* = 1352.9\ m/s}$, $\boxed{\varepsilon = 35.596}$,

$\boxed{V_2 = 2374.8\ m/s}$, $\boxed{\dot{m} = 0.319\ kg/s}$, $\boxed{d_t = 2.64 cm}$ $\boxed{d_2 = 15.7\ cm}$

$$\boxed{I_{sp,\ vaccum} = 319.55\ s}$$

Ans 4) Reaction $\Rightarrow$ Between gaseous kerosene (C_8H_{18}) and oxygen at $\underline{\phi = 1}$ (complete combustion)

$$\Rightarrow \quad C_8H_{18} + \frac{25}{2}O_2 \longrightarrow 8CO_2 + 9H_2O$$

$$MR = \frac{\frac{25}{2} \times 32}{8 \times 12 + 18 \times 1} = \frac{400}{114} = \frac{200}{57}$$

Hence, $\frac{\dot{m}_{ox}}{\dot{m}_{fuel}} = \frac{200}{57}$ $\Rightarrow$ We will use this later.

for the combustion, $\Delta H_{reactants} = \Delta H_{product}$, Hence

$$1 \times (-249.95) = 8(-393.978) + 8(T_{ad} - 298)\, 60.43 \times 10^{-3}$$
$$+ 9(-241.997) + 9(T_{ad} - 298)\, 53.9 \times 10^{-3}$$

$$5079.874 \times 10^{3} = 968.54\,(T_{ad} - 298)$$

$$T_{ad} - 298 = 5244.88$$

$$\boxed{T_{ad} = 5542.88 \text{ K}}$$

Due to cooling processes, T_{flame} drops to 80% of T_{ad}

$$\Rightarrow \quad T_{flame} = 0.8\,T_{ad}$$

$$\boxed{T_{flame} = 4434.3 \text{ K}}$$

Now we will use this value as T_1 for further calculations

Given, $F_{vacuum} = 100 \times 10^3 N$, $P_1 = 6 MPa$, $P_2 = 30 KPa$

and $k = 1.22$

Now before all these, let's calculate MW

$$MW = \frac{8(44) + 9(18)}{17}$$

$$MW = 30.235 \, g \Rightarrow R = \frac{8314}{30.235} =$$

$$\Rightarrow \boxed{R = 274.977 \, J/kg\text{-}K}$$

$$\boxed{R \approx 275 \, J/kg\text{-}K}$$

$$C^* = \frac{\sqrt{kRT_1}}{k\sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} = \frac{1219.71}{1.22 \times 0.5906}$$

$$\boxed{C^* = 1692.8 \, m/s}$$

ε can be found as,

$$\frac{A_t}{A_2} = \frac{1}{\varepsilon} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}} \sqrt{\frac{k+1}{k-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$\frac{1}{\varepsilon} = (1.607)(0.013)(2.492)$$

$$\frac{1}{\varepsilon} = 0.052 \Rightarrow \boxed{\varepsilon = 19.21}$$

Now considering Thrust (vaccum),

$$F = P_1 A_t \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} + P_2 \varepsilon A_t$$

$$10^5 = A_t\left[6\times10^6(1.704) + 30\times10^3(19.21)\right]$$

$$10^5 = A_t * 10.8\times10^6$$

$$\boxed{A_t = 9.259\times10^{-3}\ m^2}$$

$$\dot{m} = \frac{P_1 A_t k}{\sqrt{kRT_1}}\sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} = \frac{P_1 A_t}{c^*}$$

$$\dot{m} = \frac{6\times10^6 \times 9.259\times10^{-3}}{1692.8}$$

$$\boxed{\dot{m} = 32.818\ kg/s}$$

Now, $\dot{m}_{ox} + \dot{m}_{fuel} = \dot{m}$

& $\frac{\dot{m}_{ox}}{\dot{m}_{fuel}} = \frac{200}{57}$

$$\Rightarrow \quad \dot{m}_{ox} = \frac{200}{257}\dot{m} \quad ; \quad \dot{m}_{fuel} = \frac{57}{257}\dot{m}$$

$$\boxed{\dot{m}_{ox} = 25.539\ kg/s} \quad ; \quad \boxed{\dot{m}_{fuel} = 7.2787\ kg/s}$$

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$$I_{sp,\,vacuum} = \frac{F_{,\,vacuum}}{\dot{m} g}$$

$$= \frac{100 \times 10^3}{32.818 \times 9.81}$$

$$\boxed{I_{sp} = 310.612 \text{ sec}}$$

<u>Answers</u> :-

$$C^* = 1692.8 \text{ m/s}$$

$$\varepsilon = 19.12$$

$$\dot{m}_{ox} = 25.539 \text{ kg/s} \quad ; \quad \dot{m}_{fuel} = 7.2787$$

$$I_{sp} = 310.612 \text{ sec}$$

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