Aerospace Engineering Department, IIT Bombay
AE 308 & AE 775 - Control Theory
Tutorial 4 Solution

Q1

For a unity feedback system with the forward transfer function

$$G(s) = \frac{K(s+20)}{s(s+2)(s+3)},$$

find the range of K to make the system stable.

Solution:

$$0 < K < 2.$$

Q2

Use the Routh-Hurwitz criterion to find how many poles of the following closed-loop system, $T(s)$, are in the rhp, in the lhp, and on the jw-axis:

$$T(s) = \frac{s^3 + 7s^2 - 21s + 10}{s^6 + s^5 - 6s^4 - s^2 - s + 6}.$$

Solution:

Two poles in the right-half plane, two in the left-half plane, and two on the jw-axis.

Q3

A unity feedback control system has the open-loop transfer function

$$G(s) = \frac{A}{s(s+a)}.$$

Compute the sensitivity of the closed-loop transfer function to changes in the parameters A and a.

Solution:

Closed-loop transfer function:

$$T(s) = \frac{A}{s^2 + as + A}.$$

Sensitivity of $T(s)$ w.r.t A is denoted by $\mathcal{S}_A^T$, and defined as:

$$\mathcal{S}_A^T := \frac{\frac{\partial T}{T}}{\frac{\partial A}{A}}, \qquad \mathcal{S}_A^T = \frac{s^2 + as}{s^2 + as + A}.$$

Similarly,

$$\mathcal{S}_a^T := \frac{\frac{\partial T}{T}}{\frac{\partial a}{a}}, \qquad \mathcal{S}_a^T = \frac{-as}{s^2 + as + A}.$$

Q4

Consider the second-order plant with the transfer function

$$G(s) = \frac{1}{(s+1)(5s+1)},$$

in a unity feedback structure. Determine the system type and error constant with respect to tracking polynomial reference inputs of the system for PID $[D_c = 19 + \frac{0.5}{s} + \frac{4}{19}s]$.

Solution:

System type is 1. Error constant $K_v = 0.5$.

Q5

Let the transfer function of the plant is $G(s) = \frac{K_0}{4s+1}$. The system is in a unity feedback structure. Compute the steady-state error of the closed-loop plant when reference is a step input. Design a controller to make the steady-state error zero.

Solution:

Steady-State error: $\frac{1}{1+K_0}$.

An integral controller is used to make the steady-state error zero.

Controller: $D_c = \frac{K_I}{s}$, where K_I is any non-zero real value.

with this controller steady-state error is 0.

Q6

Consider a plant with nominal model given by $G(s) = \frac{1}{s+2}$. Compute the parameters of a PI controller so that the natural modes of the closed loop response decay at least as fast as e^{-5t}.

Solution:

A PI controller has transfer function given by: C(s) = $\frac{as+b}{s}$, Where a = K_p, b = $\frac{K_p}{T_r}$,

The closed loop characteristic polynomial, $A_c(s)$ = numerator of 1+$G_o(s)$C(s) = s^2 + (2+a)s + b

We choose the controller to obtain a pair of complex conjugate poles. To achieve a closed loop transient as fast as $e^{-}5t$, those poles must have real parts equal to -5. This requires a = -8 and take b = 49. Hence

$C(s) = \frac{(8s+49)}{s}$

Q7

Consider the system shown in Figure 1. This is a PID control of a second-order plant $G(s)$. Assume that disturbances $D(s)$ enter the system as shown in the diagram. It is assumed that the reference input $R(s)$ is normally held constant, and the response characteristics to disturbances are a very important consideration in this system. Design a control system such that the response to any step disturbance will be damped out quickly (in 2 to 3 sec in terms of the 2% settling time). Choose the configuration of the closed-loop poles such that there is a pair of dominant closed-loop poles.

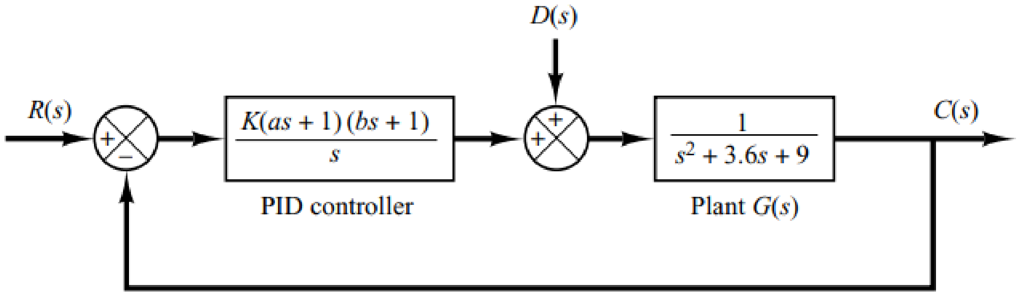

Figure 1:

Solution:

The PID controller has the transfer function $G_c(s)$: $\dfrac{k(as+1)(bs+1)}{s}$,

For the disturbance input in the absence of the reference input, the closed-loop transfer function becomes:

$$\frac{C_d(s)}{D(s)} = \frac{s}{s(s^2+3.6s+9)+k(as+1)(bs+1)} = \frac{s}{s^3+(3.6+kab)s^2+(9+ka+kb)s+k}$$

The specification requires that the response to the unit-step disturbance be such that the settling time be 2 to 3 sec and the system have a reasonable damping. We may interpret the specification as $\zeta = 0.5$ and $\omega_n = 4$ rad/s for the dominant closed-loop poles. We may choose the third pole at s = -10 so that the effect of the real pole on the response is small. Then the desired characteristic equation can be written as

$$(s+10)(s^2+2\times 0.5\times 4s+4^2) = (s+10)(s^2+4s+16) = s^3+14s^2+56s+160$$

The characteristic equation of the system is

$s^3 + (3.6 + Kab)s^3 + (9 + Ka + Kb)s + K$

Hence, we require

3.6 + Kab = 14

9 + Ka + Kb = 56

K = 160

Which yields ab = 0.065, a + b = 0.29375

The PID controller now becomes

$G_c(s) = \frac{K(abs^2)+(a+b)s+1}{s} = \frac{160(0.065s^2+0.29375s+1)}{s} = \frac{10.4(s^2+4.5192s+15.385)}{s}$

Q8

The block diagram of a control system with a series controller is shown in Figure 2. Find the transfer function of the controller $G_c(s)$ so that the following specifications are satisfied:

1. The ramp error constant K_v is 5.
2. The closed loop transfer function is of the form $M(s) = \dfrac{Y(s)}{R(s)} = \dfrac{K}{(s^2 + 20s + 200)(s + a)}$ where K and a are real constants. Find the values of K and a.

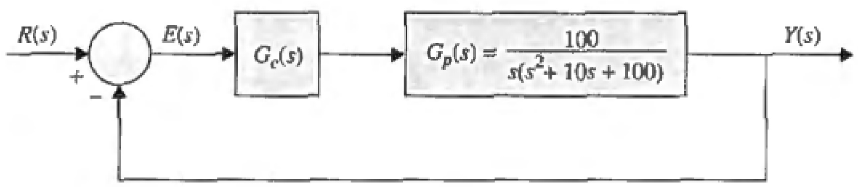

Figure 2:

Solution:

The forward-path transfer function

$G(s) = \frac{M(s)}{1-M(s)} = \frac{K}{s^3+(20+a)s^2+(200+20a)s+200a-K}$

For type 1 system, 200a - K = 0

Thus K = 200a

Ramp-error constant:

$K_v = \lim_{s\to 0} sG(s) = \frac{K}{200+20a} = \frac{200a}{200+20a} = 5$

Thus a = 10, K = 2000

The forward-path transfer function is

$G(s) = \frac{2000}{s(s^2+30s+400)}$

The controller transfer function is

$G_c(s) = \frac{G(s)}{G_p(s)} = \frac{20(s^2+10s+100)}{s^2+30s+400}$