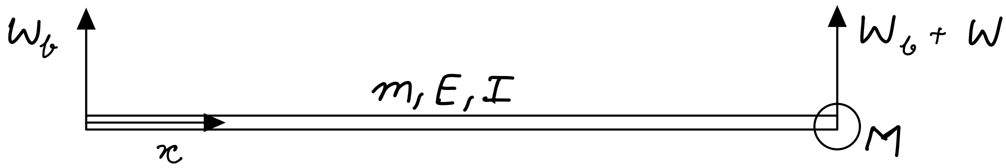
Solution 1 :-



Kinetic energy $= \frac{1}{2} M \left(\dot{W}_b + \dot{W}(L)\right)^2 + \frac{1}{2}\int_0^L m\left(\dot{W} + \dot{W}_b\right)^2 dx$

Potential energy $= \frac{1}{2}\int_0^L EI \frac{\partial^2 W}{\partial x^2} dx$

Essential Boundary condition

$$W_b(t) + W(0,t) = 0$$

$$\frac{\partial W(0,t)}{\partial x} = 0$$

Solution 2

$m = 2000$ kg
$y_0 = 0.2$ m
$\lambda = 2.5$ m
$v = 62.5$ m/s
$k = 5 \times 10^6$ N/s

$x_0 = 0.1$ m

(a)

$$y = y_0 \cos \frac{2\pi z}{\lambda} \qquad z = vt$$

$$= 0.2 \cos \frac{2\pi}{2.5} 62.5\, t$$

$$= 0.2 \cos \omega t$$

$$\omega = \frac{2\pi\, 62.5}{2.5} = 157.07 \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{50 \times 10^6}{2000}} = 50 \text{ rad/s}$$

$$r = \frac{\omega}{\omega_n} = 3.14$$

Steady state amplitude

$$x_0 = y_0 \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}$$

$$\frac{0.1}{0.2} = \sqrt{\frac{1 + 39.43\zeta^2}{78.29 + 39.43\zeta^2}}$$

$$\zeta = 0.7935$$

$$c = 2m\zeta\omega_n = 158700 \text{ Ns/m}$$

(b) Acceleration amplitude

$$A = \omega^2 x_0$$

3) Given:-

$m = 10^5 \text{ Kg}$, $t_0 = 4 \text{ sec}$

$$F(t) = F_0 \sin\left(\frac{\pi t}{t_0}\right) \quad 0 < t \le t_0$$

0 else

$$x(t) = \frac{F_0}{K}\left(\frac{1}{1-r^2}\sin\left(\frac{\pi t}{t_0}\right) - \frac{r}{1-r^2}\sin \omega_n t\right)$$

where $r = \frac{\pi}{t_0 \omega_n}$

Given: $x_{max} = 0.5 \text{m}$, $K = ?$

x_{max} occurs when $\frac{dx}{dt} = 0$ ($\because$ Similar to Calculus)

f_{max} occurs when $\frac{df}{dx} = 0$, $\frac{d^2 f}{dx^2} < 0$

$$\frac{dx}{dt} = \frac{F_0}{K}\left(\frac{1}{1-r^2}\frac{\pi}{t_0}\cos\left(\frac{\pi t}{t_0}\right) - \frac{r\,\omega_n}{1-r^2}\cos \omega_n t\right) = 0$$

$$\Rightarrow \frac{\pi}{t_0}\cos\left(\frac{\pi t}{t_0}\right) = r\,\omega_n \cos \omega_n t \qquad \left(r\,\omega_n = \frac{\pi}{t_0 \omega_n}\omega_n = \frac{\pi}{t_0}\right)$$

$$\Rightarrow \cos\left(\frac{\pi t}{t_0}\right) = \cos \omega_n t$$

$\cos A = \cos B$

Sol^n is $A = B + 2n\pi$

$n = 0, 1, 2, 3, \ldots$

$$\therefore \frac{\pi t}{t_0} = \omega_n t + 2n\pi$$

$$\Rightarrow t = \frac{2n\pi}{\frac{\pi}{t_0} - \omega_n}$$

$$\therefore t = \frac{2\pi n}{\frac{\pi}{4} - \omega_n}$$

$$x_{max} = \frac{F_0}{K}\left[\left(\frac{1}{1-r^2}\sin\left(\omega_n t + 2n\pi\right) - \frac{r}{1-r^2}\sin\omega_n t\right)\right]$$

$$\sin(2n\pi + \theta) = \sin\theta$$

$$\therefore x_{max} = \frac{F_0}{K}\left(\frac{1-r}{1-r^2}\sin\omega_n t\right) = \frac{F_0}{K(1+r)}\sin\omega_n t.$$

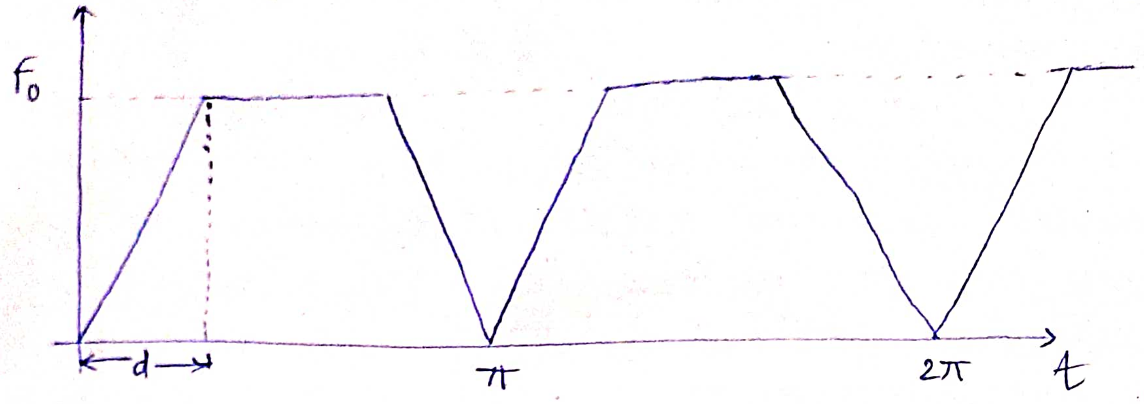
$$\Rightarrow 0.5 = \frac{F_0}{K\left(1 + \frac{\pi}{t_0\omega_n}\right)}\sin\omega_n\left(\frac{2\pi n}{\frac{\pi}{4} - \omega_n}\right)$$

$$\boxed{0.5 = \frac{F_0}{K\left(1 + \frac{\pi}{t_0\sqrt{\frac{K}{m}}}\right)}\sin\frac{2\pi n\sqrt{\frac{K}{m}}}{\frac{\pi}{4} - \sqrt{\frac{K}{m}}}}$$

If we solve for K in this eq^n using

non-linear root finding techniques such as Newton Raphson method, we get the value of K. Writing this eq^n will ensure you maximum marks.

AE - 326, QUIZ - 2 : Q4 SOLUTION



$$f(t) = \begin{cases} \frac{f_0}{d} t , & n\pi \leq t \leq n\pi + d \\ f_0 , & (n\pi + d) \leq t \leq (n+1)\pi - d \\ \frac{f_0}{d}(\pi - t) , & (n+1)\pi - d \leq t \leq (n+1)\pi \end{cases}$$

[3M]

$n \in [0, 1, 2, \ldots]$

fourier series :

$$f(t) = \frac{a_0}{2} + \sum_{j=1}^{\infty} (a_j \cos j\omega t + b_j \sin j\omega t)$$

where, $a_j = \frac{2}{T} \int_0^T F(t) \cos(j\omega t) dt$, $j = 0, 1, 2, \ldots$

$b_j = \frac{2}{T} \int_0^T F(t) \sin(j\omega t) dt$, $j = 1, 2, 3, \ldots$

Here, $\boxed{T = \pi \quad \omega = \frac{2\pi}{T} = 2}$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} F(t)\, dt$$

$$= \frac{2}{\pi}\left[\int_0^{d}\left(\frac{F_0}{d}\right) t\, dt + \int_d^{\pi-d} F_0\, dt + \int_{\pi-d}^{\pi} \frac{F_0}{d}(\pi - t)\, dt\right]$$

$$= \frac{2}{\pi}\left[\frac{F_0 d}{2} + F_0[\pi - 2d] + \left[\frac{F_0 \pi}{d}(\pi - (\pi - d)) - \left[\frac{F_0}{d}\left(\frac{t^2}{2}\right)\right]_{\pi-d}^{\pi}\right]\right]$$

$$= \frac{2}{\pi}\left[\frac{F_0 d}{2} + F_0\pi - 2F_0 d + F_0\pi - \left(\frac{F_0\cancel{\pi^2}}{2d} - \frac{F_0}{2d}(\cancel{\pi^2} - 2\pi d + d^2)\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{F_0 d}{2} - 2F_0 d + 2F_0\pi - \left(\frac{2\pi F_0}{2} - \frac{F_0 d}{2}\right)\right]$$

$$\boxed{a_0 = \frac{2}{\pi}\left[F_0\pi - F_0 d\right] = \frac{2F_0}{\pi}(\pi - d)}$$ (1M)

$$a_j = \frac{2}{\pi}\int_0^{\pi} F(t)\cos(j2t)\, dt$$

$\{j = 1, 2, 3, \ldots\}$

$$= \frac{2}{\pi}\left[\overbrace{\int_0^{d}\frac{F_0 t}{d}\cos(2jt)\, dt}^{A} + \overbrace{\int_d^{\pi-d} F_0\cos(2jt)\, dt}^{B} + \overbrace{\int_{\pi-d}^{\pi}\frac{F_0}{d}(\pi - t)\cos(2jt)\, dt}^{C}\right]$$

$$= \frac{2}{\pi}\left[\frac{F_0}{d}\left[\frac{+t\sin(2jt)}{2j} + \frac{\cos(2jt)}{4j^2}\right]_0^{d} + \left(\frac{F_0\sin(2jt)}{2j}\right)_d^{\pi-d} + \int_{\pi-d}^{\pi}\frac{F_0}{d}(\pi - t)\cos(2jt)\, dt\right]$$

$$A = \frac{2}{\pi}\left[\frac{F_0}{d}\left(\frac{d\sin(2jd)}{2j} + \frac{\cos(2jd)}{4j^2} - \left(0 + \frac{1}{4j^2}\right)\right)\right]$$

$$\boxed{\therefore A = \frac{2}{\pi}\left[\frac{F_0}{d}\left(\frac{d\sin(2jd)}{2j} + \frac{\cos(2jd)}{4j^2} - \frac{1}{4j^2}\right)\right]}$$

$$B = \frac{2}{\pi}\int_{d}^{\pi-d} F_0 \cos(2jt)\,dt = \frac{2}{\pi}\left(\frac{F_0 \sin(2jt)}{2j}\right)_{d}^{\pi-d}$$

$$B = \frac{2}{\pi}\left[\frac{F_0}{2j}\left(\sin(2j(\pi-d)) - \sin 2jd\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{F_0}{2j}\left(\cancel{\sin 2j\pi \cos 2jd} - \cos 2j\pi \sin 2jd - \sin 2jd\right)\right]$$

$$\boxed{B = \frac{2}{\pi}\left[\frac{F_0}{2j}(-2\sin 2jd)\right] = \frac{2}{\pi}\left[-\frac{F_0 \sin 2jd}{j}\right]}$$

$$C = \frac{2}{\pi}\left[\int_{\pi-d}^{\pi} \frac{F_0}{d}(\pi-t)\cos(2jt)\,dt\right]$$

$$= \frac{2}{\pi}\frac{F_0}{d}\left[\frac{\pi \sin 2jt}{2j} - \left(\frac{t \sin 2jt}{2j} + \frac{\cos 2jt}{4j^2}\right)\right]_{\pi-d}^{\pi}$$

$$= \frac{2}{\pi}\frac{F_0}{d}\left[\frac{\pi \sin 2j\pi}{2j} - \left(\frac{\pi \sin 2j\pi}{2j} + \frac{\cos 2j\pi}{4j^2}\right) - \frac{\pi \sin 2j(\pi-d)}{2j} + \left(\frac{(\pi-d)\sin 2j(\pi-d)}{2j} + \frac{\cos 2j(\pi-d)}{4j^2}\right)\right]$$

$$= \frac{2F_0}{\pi d}\left[\frac{-1}{4j^2} - \frac{d \sin 2j(\pi-d)}{2j} + \frac{\cos 2j(\pi-d)}{4j^2}\right]$$

$$= \frac{2F_0}{\pi d}\left[\frac{-1}{4j^2} - \frac{d}{2j}\left(\sin 2j\pi \cos 2jd - \cos 2j\pi \sin 2jd\right) + \frac{\cos 2j\pi \cos 2jd + 0}{4j^2}\right]$$

$$\boxed{C = \frac{2F_0}{\pi d}\left[-\frac{1}{4j^2} + \frac{d}{2j}\sin 2jd + \frac{\cos 2jd}{4j^2}\right]}$$

$$\therefore a_j = A + B + C$$

$$= \frac{2}{\pi}\left[\frac{F_0}{d}\left(\cancel{\frac{d\sin(2jd)}{2j}} + \frac{\cos(2jd)}{4j^2} - \frac{1}{4j^2}\right) - \cancel{\frac{F_0 \sin 2jd}{j}} + \frac{F_0}{d}\left(-\frac{1}{4j^2} + \cancel{\frac{d\sin 2jd}{2j}} + \frac{\cos 2jd}{4j^2}\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{2F_0}{d}\left(\frac{\cos 2jd}{4j^2} - \frac{1}{4j^2}\right)\right]$$

$$\boxed{\therefore a_j = \frac{F_0}{\pi j^2 d}\left(\cos 2jd - 1\right)}$$

$$b_j = \frac{2}{\pi}\int_0^{\pi} f(t)\sin 2jt\, dt$$

Now, the given forcing function is odd, hence, sinusoidal integration will yield all $\boxed{b_j = 0}$

(OR)

$$b_j = \frac{2}{\pi}\Bigg[\underbrace{\int_0^d \frac{F_0}{d} t\sin 2jt\, dt}_{A} + \underbrace{\int_d^{\pi-d} F_0 \sin(2jt)\, dt}_{B} + \underbrace{\int_{\pi-d}^{\pi} \frac{F_0}{d}(\pi-t)\sin 2jt\, dt}_{C}\Bigg]$$

$$A = \frac{2}{\pi}\left[\int_0^d \frac{F_0}{d} t\sin 2jt\, dt\right] = \frac{2}{\pi}\left[\frac{F_0}{d}\left(\frac{-t\cos 2jt}{2j} + \frac{\sin 2jt}{4j^2}\right)\right]_0^d$$

$$\boxed{\therefore A = \frac{2}{\pi}\left[\frac{F_0}{d}\left(\frac{-d\cos 2jd}{2j} + \frac{\sin 2jd}{4j^2}\right)\right]}$$

$$B = \frac{2}{\pi}\left[\int_{d}^{\pi-d} F_0 \sin(2jt)\,dt\right]$$

$$= \frac{2}{\pi}\left[F_0\left(\frac{-\cos 2jt}{2j}\right)_{d}^{\pi-d}\right] = \frac{2F_0}{\pi}\left[\frac{-\cos 2j(\pi-d) + \cos 2jd}{2j}\right]$$

$$= \frac{2F_0}{\pi}\left[\frac{-\cos 2j\pi \cos 2jd - \sin 2j\pi \sin 2jd + \cos 2jd}{2j}\right]$$

$$\therefore \boxed{B = \frac{2F_0}{\pi}\left[\frac{-\cos 2jd + \cos 2jd}{2j}\right] = 0}$$

$$C = \frac{2}{\pi}\int_{\pi-d}^{\pi} \frac{F_0(\pi-t)}{d} \sin 2jt\,dt$$

$$= \frac{2F_0}{\pi d}\left[\frac{-\pi\cos 2jt}{2j} - \left(\frac{t(-\cos 2jt)}{2j} + \frac{\sin 2jt}{4j^2}\right)\right]_{\pi-d}^{\pi}$$

$$C = \frac{2F_o}{\pi d}\left[\left(\frac{-\pi\cos 2j\pi}{2j} + \frac{\pi\cos 2j(\pi-d)}{2j}\right) - \left(\frac{-\pi}{2j} + 0 + \frac{(\pi-d)\cos 2j(\pi-d)}{2j} - \frac{\sin 2j(\pi-d)}{4j^2}\right)\right]$$

$$= \frac{2F_o}{\pi d}\left[\cancel{\frac{-\pi}{2j}} + \cancel{\frac{\pi}{2j}\cos 2j(\pi-d)} + \cancel{\frac{\pi}{2j}} - \cancel{\frac{\pi}{2j}\cos 2j(\pi-d)} + \frac{d\cos 2j(\pi-d)}{2j} + \frac{\sin 2j(\pi-d)}{4j^2}\right]$$

$$= \frac{2F_o}{\pi d}\left[\frac{d\cos 2jd}{2j} + \frac{(-\sin 2jd)}{4j^2}\right]$$

$$\therefore \boxed{C = \frac{2}{\pi}\left[\frac{F_o}{d}\left(\frac{d\cos 2jd}{2j} - \frac{\sin 2jd}{4j^2}\right)\right] = -A}$$

$\because A = -C$ & $B = 0 \Rightarrow \boxed{b_j = 0}$

→ SOLUTION/RESPONSE OF SYSTEM:

$$m\ddot{x} + c\dot{x} + Kx = \frac{a_0}{2}$$

$$m\ddot{x} + c\dot{x} + Kx = a_j \cos j\omega t$$

$$m\ddot{x} + c\dot{x} + Kx = b_j \sin j\omega t$$

$$x_p(t) = \left\{\frac{a_0}{2K} + \sum_{j=1}^{\infty} \frac{a_j/K}{\sqrt{(1-j^2r^2)^2 + (2\xi jr)^2}}\cos(j\omega t - \phi_j) + \cancel{\frac{b_j/K}{\sqrt{(1-j^2r^2)^2 + (2\xi jr)^2}}}\sin(j\omega t - \phi_j)\right\}$$

$$\therefore x_p(t) = \left\{\frac{a_0}{2K} + \sum_{j=1}^{\infty} \frac{a_j/K}{\sqrt{(1-j^2r^2)^2 + (2\xi jr)^2}}\cos(j\omega t - \phi_j)\right\}$$

$$\phi_j = \tan^{-1}\left\{\frac{2\xi jr}{1-j^2r^2}\right\}, \quad r = \frac{\omega}{\omega_n}$$

$$\therefore\ x_p(t) = \left\{ \frac{F_0}{\pi k}(\pi - d) + \sum_{j=1}^{\infty} \left[\frac{\frac{F_0}{\pi j^2 d k}(\cos 2jd - 1)\cos(2jt - \phi_j)}{\sqrt{(1-j^2r^2)^2 + (2\xi j r)^2}} \right] \right\}$$