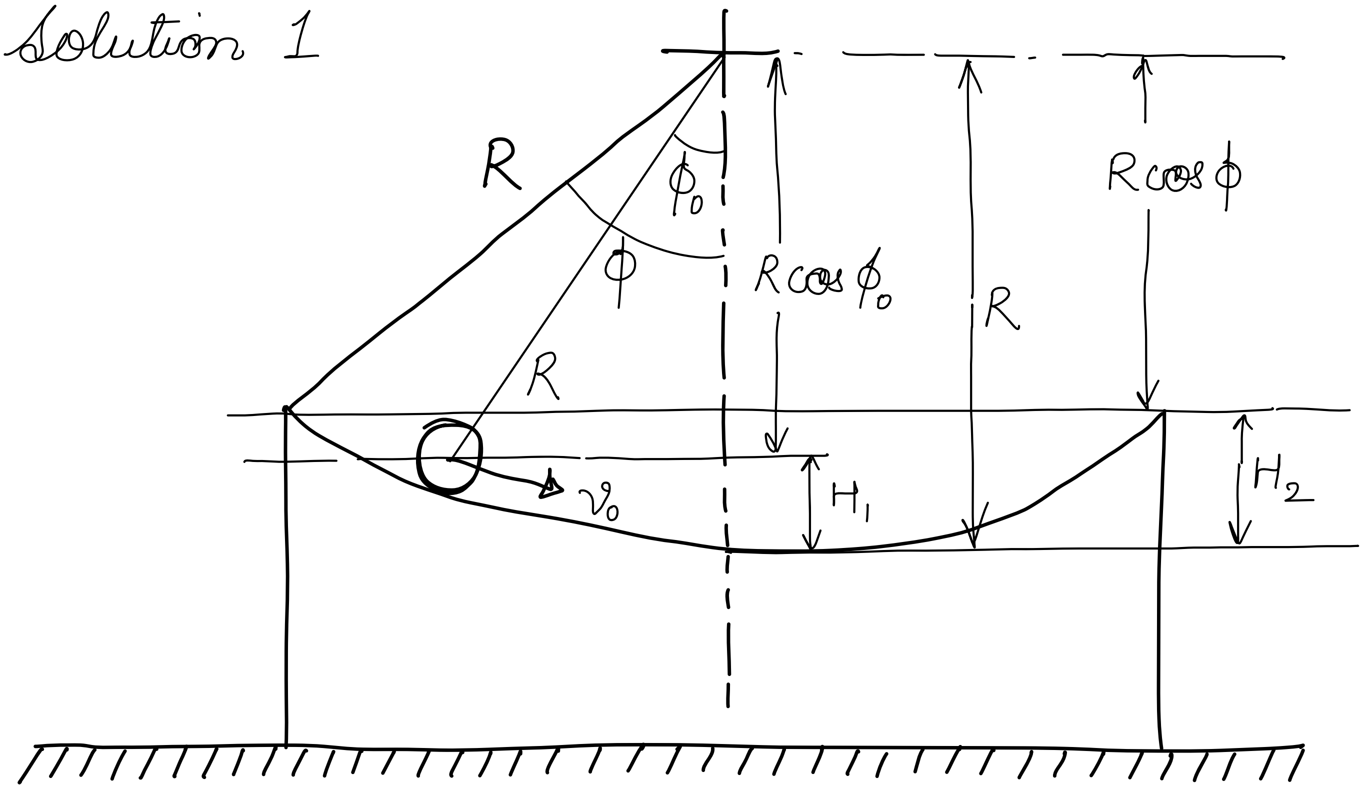
Solution 1



$$H_1 = R(1-\cos\phi_0)$$
$$H_2 = R(1-\cos\phi)$$

Case 1 :- If $v_0 > \sqrt{2gH_2}$,

the ball will leave the surface of bowl and projectile there after.

So only ϕ as DoF is not sufficient

Case 2 : If $v_0 \leq \sqrt{2gH_2}$

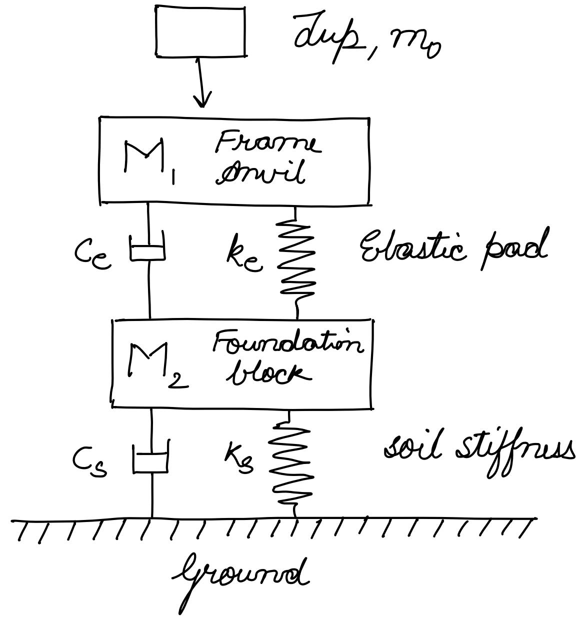
the ball will be in contact with bowl and its single degree of freedom.

We can take ϕ as DOF.

This is one solution.
There can be many.

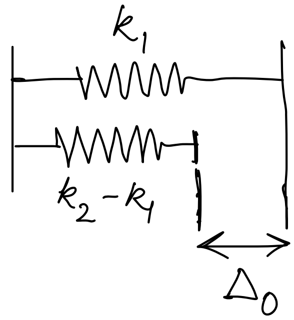
Solution 2 :- Part (a)

As the anvil and foundational block are quite stiff as compared to the elastic pad and soil

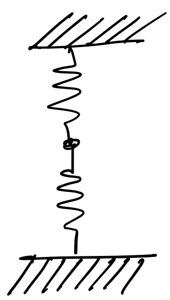


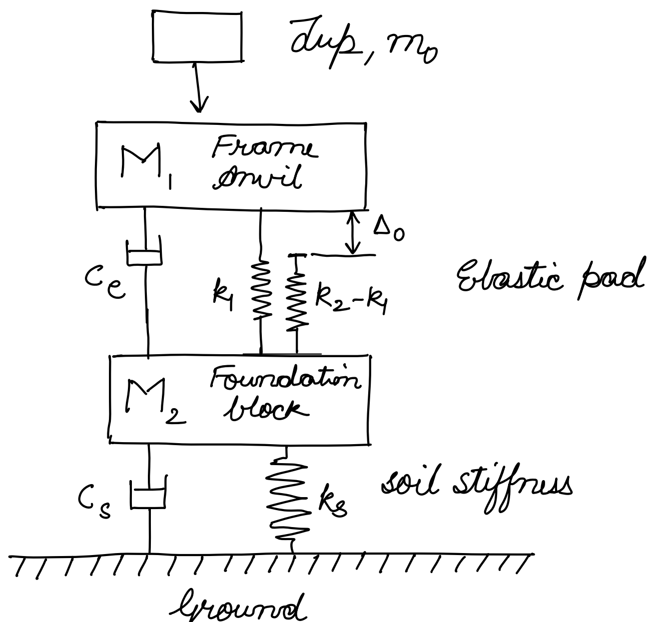
Part (b)

Bilinear stiffness



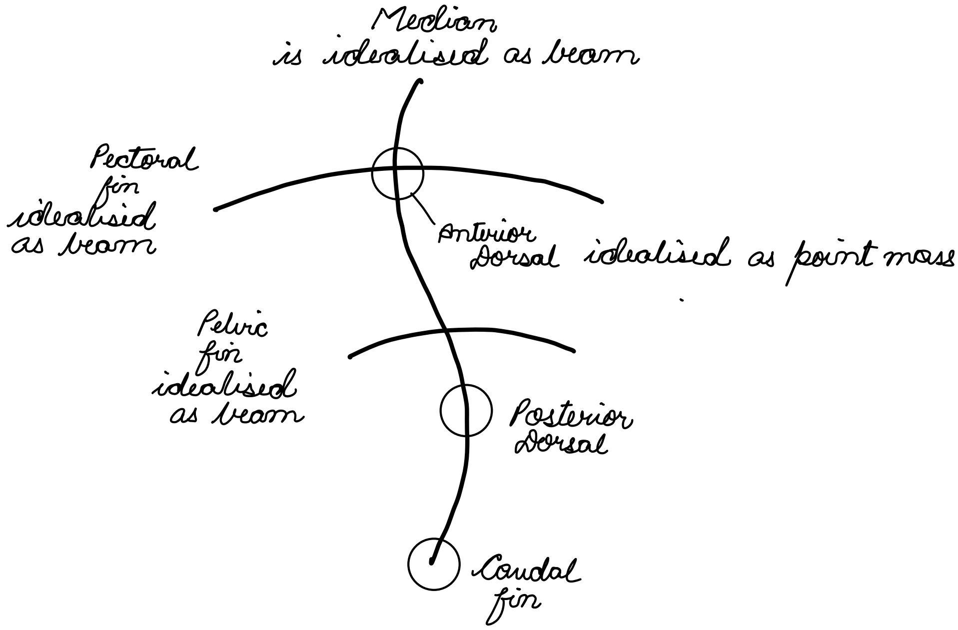
Modified Results



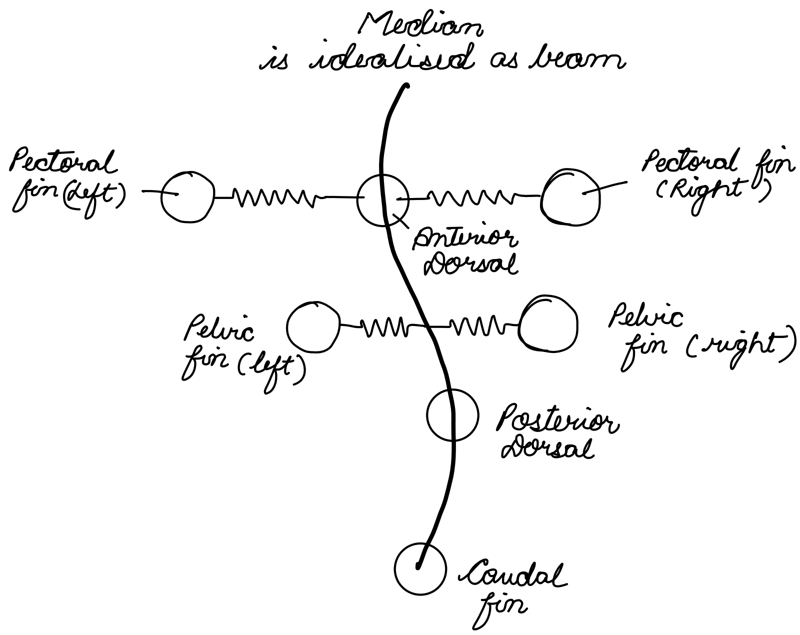


Solution 3

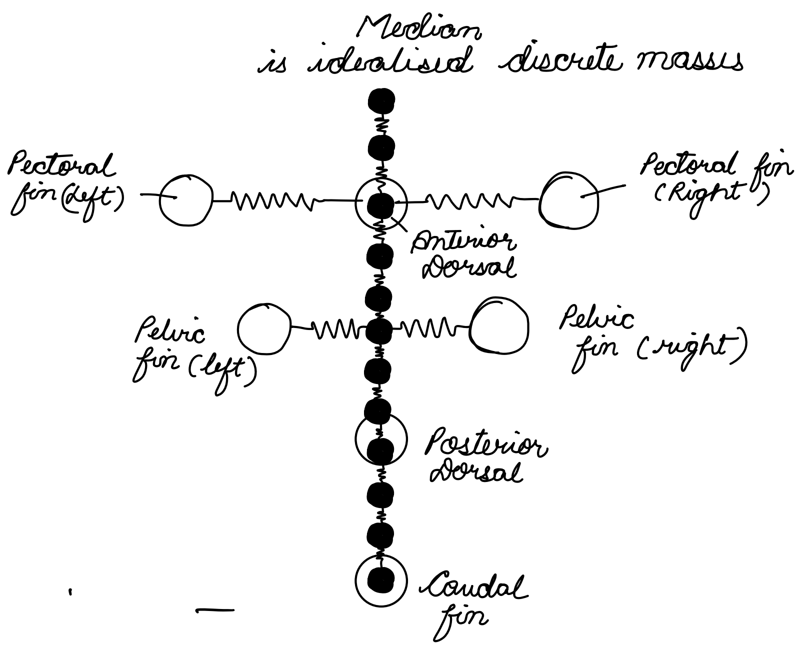
Abstraction 1



Abstraction 2



Abstraction 3



Abstraction 4

Solution 4 :

Hamilton's principle

$$\int_{t_1}^{t_2} (\delta T - \delta V + \delta W)\, dt = 0$$

Here $\delta W = 0$

$$\int_{t_1}^{t_2} \delta T\, dt = \int_{t_1}^{t_2} \delta \left[\frac{1}{2} \int_0^L \rho A \left(\frac{\partial u}{\partial t}\right)^2 dx + \frac{1}{2} M \left[\frac{\partial u(L,t)}{\partial t}\right]^2 \right] dt$$

$$= \int_{t_1}^{t_2} \left(\int_0^L \rho A \frac{2}{2} \frac{\partial u}{\partial t} \delta\left(\frac{\partial u}{\partial t}\right) dx + \frac{2}{2} M \frac{\partial u(L,t)}{\partial t} \delta\left(\frac{\partial u(L,t)}{\partial t}\right) \right) dt$$

$$= \int_{t_1}^{t_2} \left(\int_0^L \rho A \frac{\partial u}{\partial t} \left(\frac{\partial \delta u}{\partial t}\right) dx + M \frac{\partial u(L,t)}{\partial t} \left(\frac{\partial \delta u(L,t)}{\partial t}\right) \right) dt$$

$$= \underbrace{\boxed{\int_0^L \int_{t_1}^{t_2} \rho A \frac{\partial u}{\partial t} \left(\frac{\partial \delta u}{\partial t}\right) dt\, dx}}_{\text{Expression 1}} + \underbrace{\boxed{\int_{t_1}^{t_2} M \frac{\partial u(L,t)}{\partial t} \left(\frac{\partial \delta u(L,t)}{\partial t}\right) dt}}_{\text{Expression 2}}$$

Using integration by parts to expression 1

$$\int_0^L \int_{t_1}^{t_2} \rho A \frac{\partial u}{\partial t}\left(\frac{\partial \delta u}{\partial t}\right) dt\, dx = -\int_0^L \int_{t_1}^{t_2} \frac{\partial}{\partial t}\left(\rho A \frac{\partial u}{\partial t}\right) \delta u \, dt\, dx$$

$$+ \int_0^L \cancelto{0}{\left[\rho A \frac{\partial u}{\partial t}\, \delta u\right]_{t_1}^{t_2}} \quad \left(\begin{array}{c} u \text{ at} \\ \text{endpoints} \\ (t_1, t_2) \\ \text{are known} \end{array}\right)$$

Using integration by parts to expression 2

$$\int_{t_1}^{t_2} M \frac{\partial u(L,t)}{\partial t}\left(\frac{\partial \delta u(L,t)}{\partial t}\right) dt = -\int_{t_1}^{t_2} M \frac{\partial^2 u(L,t)}{\partial t^2}\, \delta u(L,t)\, dt$$

$$+ \cancel{\left[M \frac{\partial u(L,t)}{\partial t}\, \delta u(L,t)\right]_{t_1}^{t_2}} \quad \left(\begin{array}{c} u \text{ at} \\ \text{endpoints} \\ (t_1, t_2) \\ \text{are known} \end{array}\right)$$

Hence

$$\int_{t_1}^{t_2} \delta T\, dt = -\int_0^L \int_{t_1}^{t_2} \frac{\partial}{\partial t}\left(\rho A \frac{\partial u}{\partial t}\right) \delta u\, dt\, dx$$

$$- \int_{t_1}^{t_2} M \frac{\partial^2 u(L,t)}{\partial t^2}\, \delta u(L,t)\, dt$$

$$= -\int_{t_1}^{t_2}\left(\int_0^L \frac{\partial}{\partial t}\left(\rho A \frac{\partial u}{\partial t}\right) dx\, \delta u + M \frac{\partial^2 u(L,t)}{\partial t^2}\, \delta u(L,t)\right) dt$$

Now

$$\int_{t_1}^{t_2} \delta V\, dt = \int_{t_1}^{t_2} \delta\left(\int_0^L \frac{EA}{2}\left(\frac{\partial u}{\partial x}\right)^2 dx + \frac{k}{2}\left(u(L,t)\right)^2\right) dt$$

$$= \int_{t_1}^{t_2}\int_0^L \left(EA \frac{\partial u}{\partial x}\, \frac{\partial \delta u}{\partial x}\, dx + k\, u(L,t)\, \delta\left(u(L,t)\right)\right) dt$$

$$= \int_{t_1}^{t_2}\left(-\int_0^L \frac{\partial}{\partial x}\left(EA \frac{\partial u}{\partial x}\right) dx\, \delta u + \left[EA \frac{\partial u}{\partial x}\, \delta u\right]_0^L + k\, u(L,t)\, \delta u(L,t)\right) dt$$

$$= \int_{t_1}^{t_2}\left[-\int_0^L \frac{\partial}{\partial x}\left(EA \frac{\partial u}{\partial x}\right) dx\, \delta u + \left[EA \frac{\partial u}{\partial x} + k u\right]_{x=L} \delta u(L,t) - \cancel{\delta u(0,t)} \left[EA \frac{\partial u}{\partial x}\right]_{x=0}\right] dt$$

fixed at $x = 0$

$$\int_{t_1}^{t_2} \delta V\, dt = \int_{t_1}^{t_2}\left[-\int_0^L \frac{\partial}{\partial x}\left(EA \frac{\partial u}{\partial x}\right) dx\, \delta u + \left[EA \frac{\partial u}{\partial x} + k u\right]_{x=L} \delta u(L,t)\right] dt$$

Hamilton Principle

$$\int_{t_1}^{t_2} (\delta T - \delta V)\, dt = 0$$

$$\Rightarrow -\int_{t_1}^{t_2}\left(\int_0^L \frac{\partial}{\partial t}\left(\rho A \frac{\partial u}{\partial t}\right) dx\, \delta u + M \frac{\partial^2 u(L,t)}{\partial t^2}\, \delta u(L,t)\right) dt$$

$$+ \int_{t_1}^{t_2}\left[\int_0^L \frac{\partial}{\partial x}\left(EA\frac{\partial u}{\partial x}\right) dx\, \delta u - \left[EA\frac{\partial u}{\partial x} + k u\right]_{x=L} \delta u(L,t)\right] dt$$

$$\Rightarrow \int_{t_1}^{t_2}\left(\int_0^L \left(-\frac{\partial}{\partial t}\left(\rho A \frac{\partial u}{\partial t}\right) + \frac{\partial}{\partial x}\left(EA\frac{\partial u}{\partial x}\right)\right) dx\, \delta u\right) dt$$

$$+ \int_{t_1}^{t_2}\left(-M\frac{\partial^2 u(L,t)}{\partial t^2} - \left[EA\frac{\partial u}{\partial x} + k u\right]_{x=L}\right) \delta u(L,t)\, dt = 0$$

Equation of motion $\quad \frac{\partial}{\partial t}\left(\rho A \frac{\partial u}{\partial t}\right) = \frac{\partial}{\partial x}\left(EA\frac{\partial u}{\partial x}\right)$

Boundary condition at $x = L$

$$M\frac{\partial^2 u(L,t)}{\partial t^2} + EA\frac{\partial u(L,t)}{\partial t} + k u(L,t) = 0$$

Solution 5 :-

$$L = T - V$$

Euler Lagrange Equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}_i}\right) - \frac{\partial L}{\partial \phi_i} = 0 \qquad i = 1, 2$$

$$L = \frac{1}{6} m l^2 \left(\dot{\phi}_2^2 + 4\dot{\phi}_1^2 + 3\dot{\phi}_1 \dot{\phi}_2 \cos(\phi_1 - \phi_2)\right) - \frac{mgl}{2}\left(3\cos\phi_1 + \cos\phi_2\right)$$

Taking $i = 1$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}_1}\right) - \frac{\partial L}{\partial \phi_1} = 0$$

$$ml^2 \frac{d}{dt}\left(\frac{4\times \cancel{2}}{\cancel{6}3} \dot{\phi}_1 + \frac{\cancel{3}\dot{\phi}_2}{\cancel{6}2} \cos(\phi_1 - \phi_2)\right) - \frac{3mgl}{2} \sin\phi_1 = 0$$
$$+ \frac{1}{6} ml^2 \left(3\dot{\phi}_1 \dot{\phi}_2 \sin(\phi_1 - \phi_2)\right)$$

$$\frac{4ml^{2}}{3}\ddot{\phi}_1 + \frac{ml^2}{2}\left(\ddot{\phi}_2 \cos(\phi_1 - \phi_2) - \dot{\phi}_2 \sin(\phi_1 - \phi_2)(\dot{\phi}_1 - \dot{\phi}_2)\right)$$
$$+ \frac{1}{2} ml^2 \left(\dot{\phi}_1 \dot{\phi}_2 \sin(\phi_1 - \phi_2)\right) - \frac{3mgl}{2} \sin\phi_1 = 0$$

$$\boxed{\frac{4}{3} l \ddot{\theta}_1 + \frac{l}{2}\left(\ddot{\theta}_2 \cos(\theta_1-\theta_2) - \dot{\theta}_2 \sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2) + \dot{\theta}_1\dot{\theta}_2 \sin(\theta_1-\theta_2)\right) - \frac{3g}{2}\sin\theta_1 = 0}$$

Taking $i = 2$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0$$

$$\cancel{m}l^{\cancel{2}} \frac{d}{dt}\left(\frac{2}{6}\dot{\theta}_2 + \frac{3\dot{\theta}_1}{6}\cos(\theta_1-\theta_2)\right) - \frac{\cancel{mgl}}{2}\sin\theta_2 - \frac{1}{\cancel{6}\,2}\cancel{m}l^{\cancel{2}}\left(\cancel{3}\dot{\theta}_1\dot{\theta}_2 \sin(\theta_1-\theta_2)\right) = 0$$

$$\Rightarrow \frac{l}{3}\ddot{\theta}_2 + \frac{l}{2}\left(\ddot{\theta}_1 \cos(\theta_1-\theta_2) + \dot{\theta}_1 \sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2) - \dot{\theta}_1\dot{\theta}_2 \sin(\theta_1-\theta_2)\right) - \frac{g}{2}\sin\theta_2 = 0$$

$$\boxed{\frac{l}{3}\ddot{\theta}_2 + \frac{l}{2}\left(\ddot{\theta}_1 \cos(\theta_1-\theta_2) + \dot{\theta}_1 \sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2) - \dot{\theta}_1\dot{\theta}_2 \sin(\theta_1-\theta_2)\right) - \frac{g}{2}\sin\theta_2 = 0}$$