AE 707: Tutorial on Camber Problem. Total = 0.

This is for your practice only; it will NOT be graded.

1. Consider a thin airfoil with a simple parabolic-arc camber line $y_c(x) = 4\varepsilon x(1 - x/c)$, having maximum camber height εc. Show that

$$c_l = 2\pi(\alpha + 2\varepsilon), \qquad c_{m,c/4} = -\pi\varepsilon, \qquad \alpha_{L=0} = -2\varepsilon.$$

2. Consider the camber function for the NACA 4-digit airfoil series:

$$\frac{z_c(x)}{c} = m \begin{cases} \frac{1}{p^2}\left(2p\frac{x}{c} - \frac{x^2}{c^2}\right), & 0 \le x/c \le p, \\ \frac{1}{(1-p)^2}\left((1-2p) + 2p\frac{x}{c} - \frac{x^2}{c^2}\right), & p \le x/c \le 1. \end{cases}$$

Here p is the fraction of chord where the maximum camber occurs, and the value of z_c/c at this point is m.

(a) Plot this function and its derivative in your favourite plotting software, to get a feel for it.

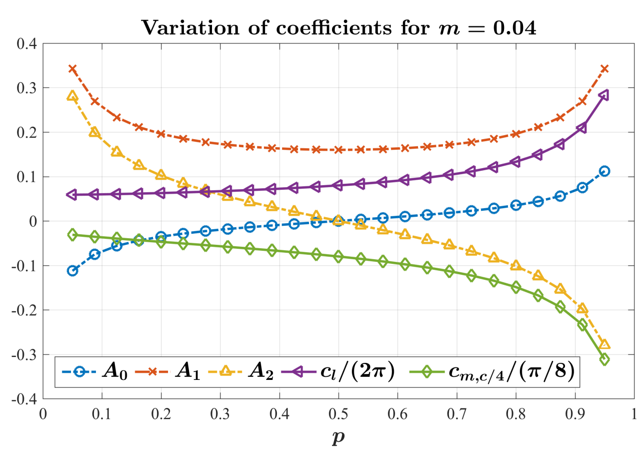
(b) Let the aerodynamic coordinate (θ) corresponding to the point of maximum camber be θ_p. Then, show that the first few coefficients of the circulation density function (per the lecture notes) are given as

$$A_0 = -\frac{1}{\pi}\int_0^\pi \frac{dz_c}{dx}(\theta)d\theta = \frac{m\cos\theta_p}{(1-p)^2}\left[1 - \frac{\sin\theta_p - \theta_p\cos\theta_p}{\pi p^2}\right],$$

$$A_1 = \frac{2}{\pi}\int_0^\pi \frac{dz_c}{dx}(\theta)\cos\theta d\theta = \frac{m}{(1-p)^2}\left[1 + \frac{\cos\theta_p}{\pi p^2}\left(\theta_p - \frac{\sin 2\theta_p}{2}\right)\right],$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz_c}{dx}(\theta)\cos n\theta d\theta = \frac{m\cos\theta_p}{\pi p^2(1-p)^2}\left[\frac{\sin(n-1)\theta_p}{n(n-1)} - \frac{\sin(n+1)\theta_p}{n(n+1)}\right], \quad n > 1.$$

(c) Plot the variation of A_0, A_1, A_2, c_l and $c_{m,c/4}$ vs. p to get a feel for them (see below).



(d) Plot the circulation density distribution (using a few terms in the above series) for a few values of p to try to understand the behaviour of c_l and $c_{m,c/4}$ above.

(e) It will appear that c_l increases monotonically with p. Then why do you think NACA 4-digit airfoils typically have p limited to within the fore portion of the airfoil? Where have you seen this principle being used that the camber is most effective towards the trailing edge?

3. A thin airfoil theory model of a single-element airfoil has the circulation density of the vortex sheet as

$$\gamma(\theta) = 2V_\infty \left((\alpha - B_0)\frac{1 + \cos\theta}{\sin\theta} + B_1 \sin\theta + B_2 \sin 2\theta \right),$$

where B_0, B_1, B_2 are the first three Fourier cosine coefficients of the camber derivative function dz_c/dx in the aerodynamic trigonometric coordinate $\theta := \cos^{-1}(1 - 2x/c)$:

$$\frac{dz_c}{dx}(\theta) = B_0 + B_1 \cos\theta + B_2 \cos 2\theta.$$

It is given that the airfoil has $c_{l,i} = 1/2$ and $c_{m,c/4} = 5/32$. Here, $c_{l,i}$ is the value of c_l achieved at the *ideal* angle of attack α_i corresponding to zero strength of the vortex sheet at the leading edge (such that the flow comes onto the airfoil smoothly thereat). Determine the shape of the camber line from this information. Also find α_i.