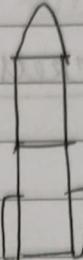
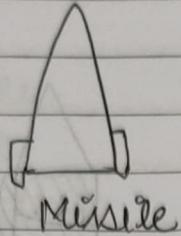
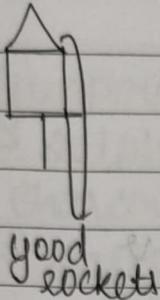


LECTURE-1

Launch vehicle

- 1) History classification → Ch 1
- 2) External Ballistics → Launch vehicles → Missiles → Chap 2, 3, 4
- 3) Rocket performance — Thermodynamics & comp flows
- 4) Heat transfer → conduction → convection → Chapter 6 new
- 5) Solid Rocket Motor → Ch 7
- 6) Liquid rocket engines → Ch 8 & 9

\* Textbook → Rocket propulsion  
Heister, Anderson  
Poupart & Causday

3 Quizzes →  $3 \times 20 = 60$

Endsem → 40

\* Grading statistics :-  
Total → 70

AA → 11

AB → 26

BB → 16

BC → 9

CC → 5

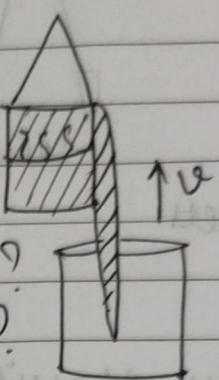
CD → 1

DD → 2

Lecture - 2★ Evolution of Rockets

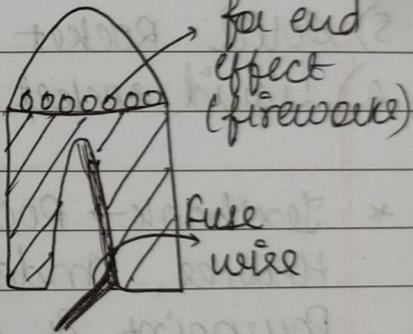
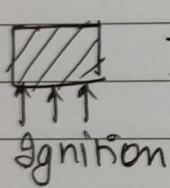
## 1) Diwali Rocket

for pressure  
 accumulation → why bottle?  
 (To gain velocity) why stick?  
 ↗ direction  
 ↗ Produces enough drag to  
 keep it straight  
 ↗ Just like



what is inside? gun powder

gun powder block :-



How far does it go? ~100 m

What is the thrust?

- Mass of components

Rocket → 41 gm

Propellant → 108 gm

Payload → 9 gm

Burntime → 5.5 s

Thrust → 1470 gm

Propellant is a combination of fuel & oxidiser.  
Fuel is a sube that can burn in presence of oxygen.

- Gunpowder
  - Sulphur } fuel
  - charcoal dust }
  - Saltpeter } oxidiser
    - ↪ Potassium nitrate ( $\text{KNO}_3$ )

- Early Rockets ~900 CE (common era)

- Diwali Rocket ~1272 CE

- Roger Bacon (Better gunpowder) (13<sup>th</sup> cent)
- Foyart (14<sup>th</sup> cent)
  - ↪ Fire from inside a tube  
better accuracy.

- Idea of cannon

D/D

- Galileo & Newton

- ↪ Laws of motion
- ↪ Trajectory

Brønsted  
Reptile

$$\frac{gMm}{r^2}$$

- Feymann's (last lecture)

- Tipu Sultan used Mysorean rockets (cast iron casing)

- ↪ go far but go low → chamber pressure

Range ~ 2 km

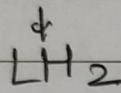
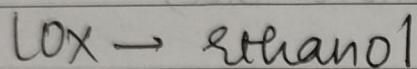
- Congreve (England)

Used them in American wars → used them on

- Hale (use spin to stabilise)
  - (no stick)

Mexico.

- Tsiolkovskii  
Rocket equation
- Goddard → American
- Oberth → orbital mechanics
- von Braun → present rocket FINALLY!!  
V-2 Rocket → LOX + Ethanol  
Liquid oxygen



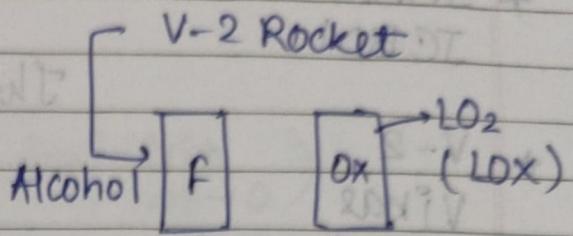
MOVIE → Apollo 13Lecture - 3

> WW-2  
Missiles

Range ~ 7000km by 1945

Missile  
Rocket

Launch vehicle,  
Spacecrafts



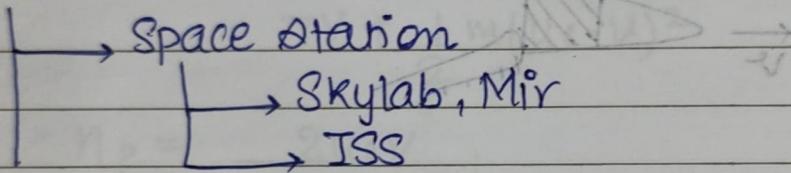
Range

~ 200 miles

liquid  
propellant

Rocket  
Engine

Space



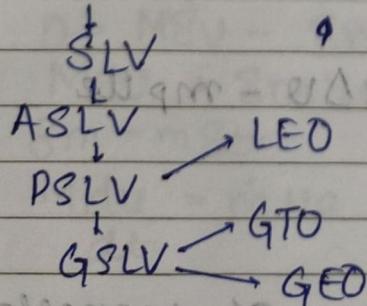
1957 → Sputnik

1958 → Explorer

Communication  
satellite

Dog  
Humans

Indian Space Program:- Sarabhai



IGMDP

Prithvi → 150-300 km  
digni → 2000-5000 km

sun = vΔm

unpublished, go Nayanwara to start = 18

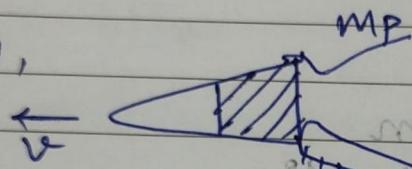
\* IRBM (Inter Range Ballistic missile)  
ICBM

	Thrust (kN)
V-2	265
Vikas	3735
(PSLV) PS-1	4500
(Sanjan-V) F-1	6860

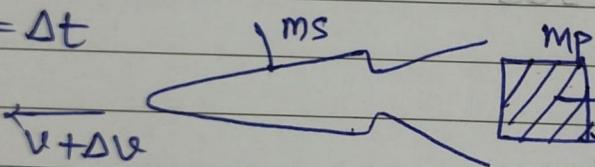
\* Thrust Equation

Heuristic (or N-S)

$$\text{At } t=0,$$



$$\text{At } t=\Delta t$$



Mom<sup>n</sup> conservation

$$(m_s + m_p)v = m_s(v + \Delta v) - m_p(u_e - \Delta v)$$

$u_e$  - speed at which propellant is ejected from nozzle.

$$(m_s + m_p)v = (m_s + m_p)v + m_s \Delta v - m_p u_e$$

$$m_s \Delta v = m_p u_e$$

$$m_p = m \Delta t$$

$m$  - rate of consumption of propellane

$$m_s \frac{\Delta u}{\Delta t} = \dot{m} u_e$$

$$T = m_s \frac{dv}{dt} = \dot{m} u_e \quad (\text{for } \lim_{\Delta t \rightarrow 0})$$

for long duration, Total impulse  $\rightarrow m_p u_e$

### • Propulsive efficiency

$$\eta_p = \frac{\text{Vehicle power}}{(\text{Vehicle power}) + (\text{Residual jet K.E. power})}$$

$$= \frac{TV}{TV + \frac{1}{2} m (u_e - v)^2}$$

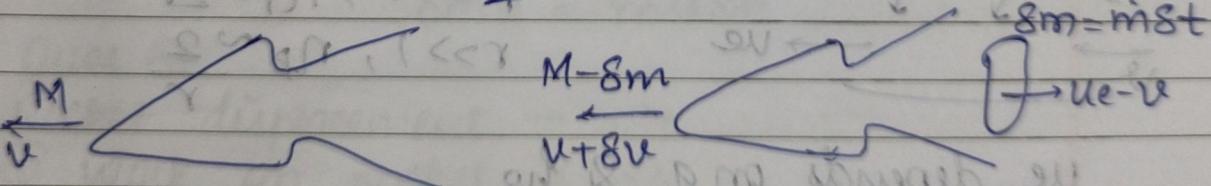
$$\eta_p = \frac{2 u_e v}{u_e^2 + v^2}$$

Work done on vehicle = Force  $\times$  disp

$$T \times v \Delta t$$

Jet ejected propellant

$$\text{K.E.} = \frac{1}{2} m_p (u_e - v)^2, m_p = \dot{m} \Delta t$$



$$Mv = (M-8m)(v+8v) + 8m(v-u_e)$$

$$0 = M8v - 8mv + 8m(v-u_e)$$

$$M8v = 8m u_e$$

$$8m = \dot{m} 8t$$

$$\frac{Mdv}{dt} = \dot{m} u_e$$

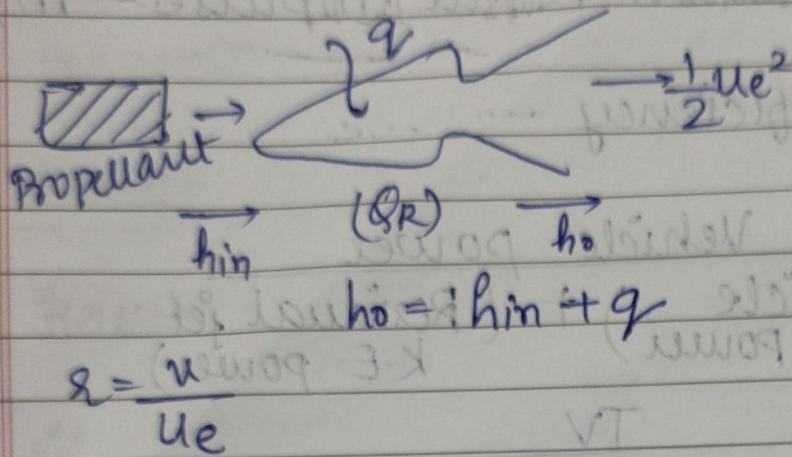
$$\lim_{8t \rightarrow 0} \frac{Mdv}{dt} = \dot{m} u_e$$

Smallness parameter

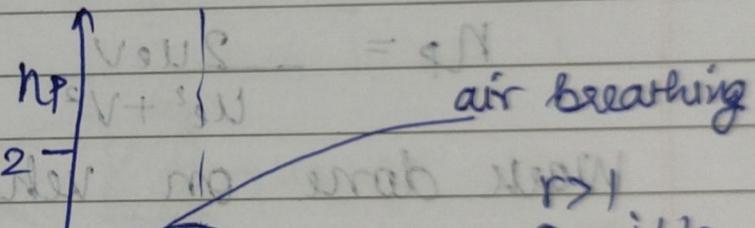
$$8m \ll M$$

$$(M - 8m) \ll M$$

$$8msv \ll Msv$$



$$n_p = \frac{2\lambda}{1 + \lambda^2} \frac{s(U - gW) + VT}{s}$$



$$T = m(U_e - V)$$

$$\lambda \ll 1, n_p \sim 2r$$

$$r \gg 1, n_p \sim \frac{2}{r}$$

$U_e$  depends on  $q$  &  $h_0$

$$(u - v)mB + (vB + v)(mB - M) = 2vM$$

$$(u - v)mB + vM - vB = 0$$

$$uM - vB = vB$$

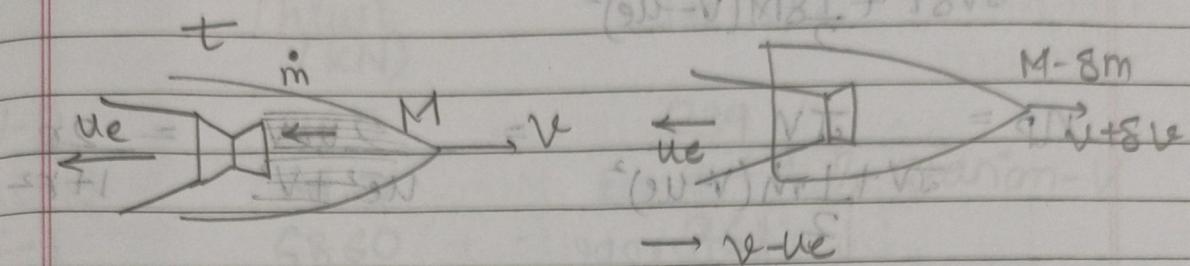
$$vB = mB$$

$$\frac{uM}{tB} = \frac{vB}{tB}$$

$$\frac{uM}{tB} = \frac{vB}{tB}$$

## LECTURE-4

### Thrust Equation



$$Sm = mst \quad \text{or} \quad Mv = (M-m)t(v-ue) + mst(v-ue)$$

No external force

$$MV = (M-m)(v+su) + (mst)(v-ue)$$

$$MSV = mst ue \xrightarrow{st \rightarrow 0} \frac{MdV}{dt} = mue$$

$$M8V = mue$$

$$MV + M8V - \cancel{8mv} - \cancel{sm8v} + \cancel{mst} - \cancel{mst}$$

Small  $st$

$$8mv = mst$$

Over  $st$

Rocket displacement  $\rightarrow vst$

Work on rocket  $JVst$

$$\text{Exhaust gases} \rightarrow \frac{1}{2} 8M(v-ue)^2 \leftarrow \textcircled{b}$$

(in absolute frame)

Propulsive efficiency  $\rightarrow \eta_p = \frac{①}{① + ②}$

$$= \frac{JVst}{JVst + \frac{1}{2} \rho M (V - U_e)^2}$$

$$\eta_p = \frac{JV}{JV + \frac{1}{2} \rho M (V - U_e)^2} = \frac{2 U_e V}{U_e^2 + V^2} = \frac{2r}{1+r^2}$$

$$J = m_e U_e + (p_e - p_a) A_e = m_e U_{eff} + 3 \dot{m} = m^2$$

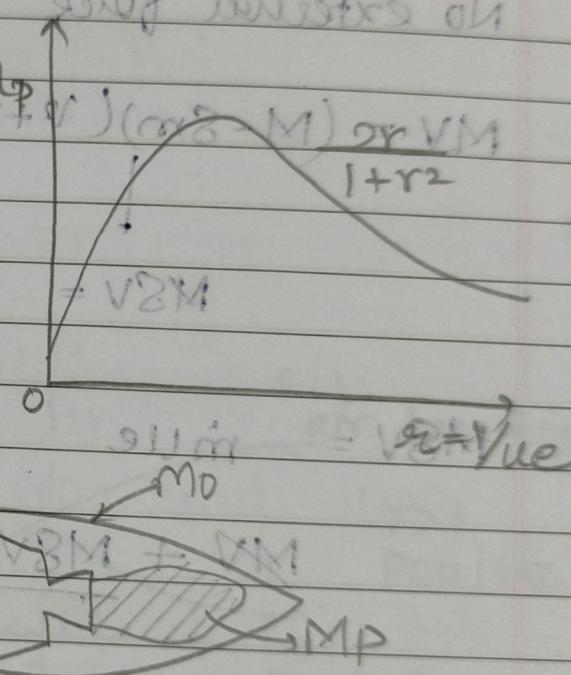
$$(g_0 - v)(t_0 \dot{m}) + (v_0^2 + v^2)(m_0 - M) \text{ or } VM$$

Total propellant mass is  $M_p$ .

Rocket mass at start =  $M_0$

After all propellant is consumed, Rocket mass =  $m_e$   
(empty mass)

$$M_0 \neq M_e \quad M_0 = M_p + M_e$$



Total impulse that the propellant can provide

$$\int J dt = M_p U_e$$

$$\int m_e U_e dt = U_e \int m dt$$

Specific Impulse

$$I_{sp} = \frac{\text{Total Impulse}}{\text{Total propellant weight at sea level}}$$

(in kg sec / N)

$$J_{sp} = \frac{U_e}{g_0}$$

Thrust  
(kN)

$J_{sp}(S)$

V-2	265	239
RL-10	66.7	44.47
F-1	6860	3610
VIKAS	735	295
PS-1	4500	250

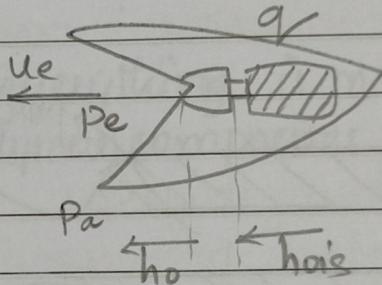
$g_0 \rightarrow$  acc. due to gravity at sea level

$$m h_{o1} = m h_{oin} + \dot{q}$$

$$\dot{q} = m Q_R$$

$$h_o = h_{oin} + Q_R$$

negligible



Temp of the gas that is coming in is negligible -  
 $h_{oin} \approx Q_R$

$h_o \rightarrow$  Total enthalpy

$h_{oin} \rightarrow$  before combustion

$$Q_R \rightarrow \frac{1}{2} U_e^2 \quad h_o, T_o \rightarrow \text{after combustion}$$

$$h_o \sim Q_R$$

$$U_e \approx \sqrt{2 C_p T_o}$$

$$C_p T_o \sim \frac{1}{2} U_e^2$$

$$U_e \sim \sqrt{\frac{2(\gamma)}{(\gamma+1)} M_w T_o}$$

highest for low gamma

## LECTURE-5

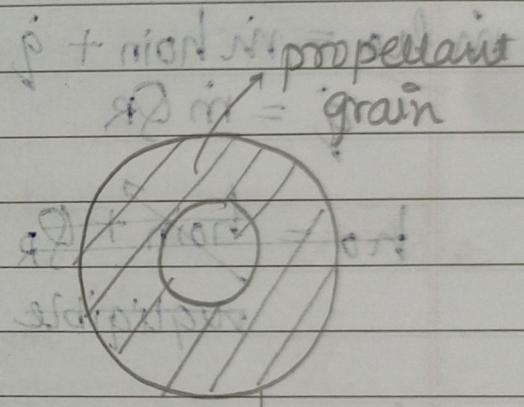
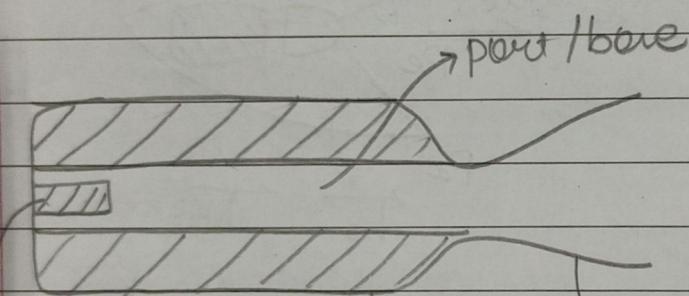
### SCHEMATICS

chemical (Energy)

Rockets

- solid (propellant)
- solid Rocket Motor (SRM)
- liquid (propellant)
- liquid rocket engines (LRE)

level SRM principle of sub prop



Propellant is fuel + oxidiser  
Can carry its own weight

PSLV Stage 1

PS-1 - S139 Motor

ht - 20m | dia - 2.8m |

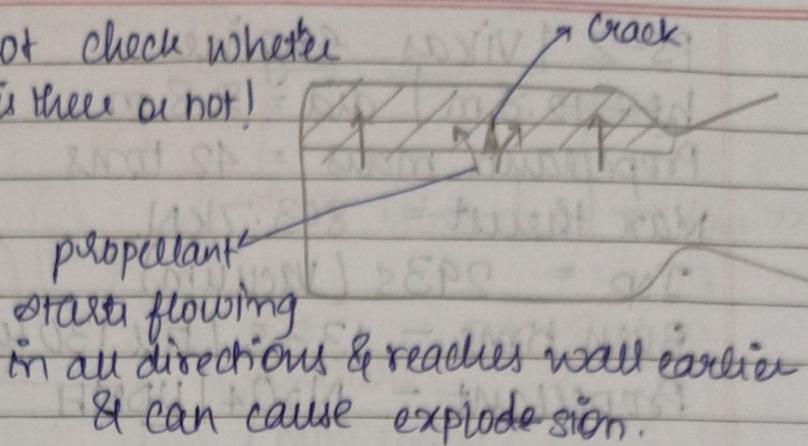
Propellant mass - 138.2 tons

Max. thrust - 4847 KN

Isp ~ 237 - 269s | Burn time = 110s till all ~ 55km

Propellant - HTPB

Cannot check whether crack is there or not!

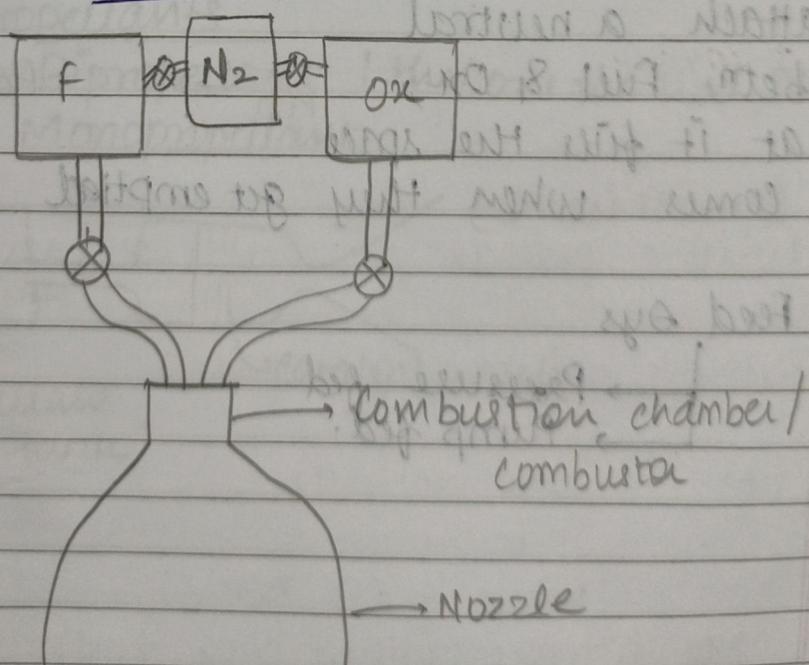


Once agreed DRDO gives its samples to private companies to manufacture further.

If there is a crack, In beam vibration, the frequency changes & we can detect it → NDT

The core & casing & everything is made of (multiple polymers) composites.

### LRE (Fuel & oxidiser are separate)



## PS-2 | Vikas engine

ht = 12.8 m | dia = 2.8 m

Propellant mass = 42 tons

Max thrust = 803.7 kN

Isp = 293 s (vacuum)

Burn time - 133 s (till 130 km)

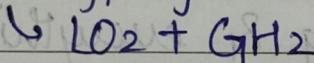
Propellant - N<sub>2</sub>O<sub>4</sub> | UDMH

Again all its parts are made of composite.

① Ignition → N<sub>2</sub>O<sub>4</sub> & UDMH

→ hypergolic (ignite as soon as fuel & ox are in contact)

→ Non-hypergolic (need an igniter)



② Feed system

Chamber pressure → Tank pr

$$\frac{\Delta p}{L} \propto \text{in}$$

We attach a neutral

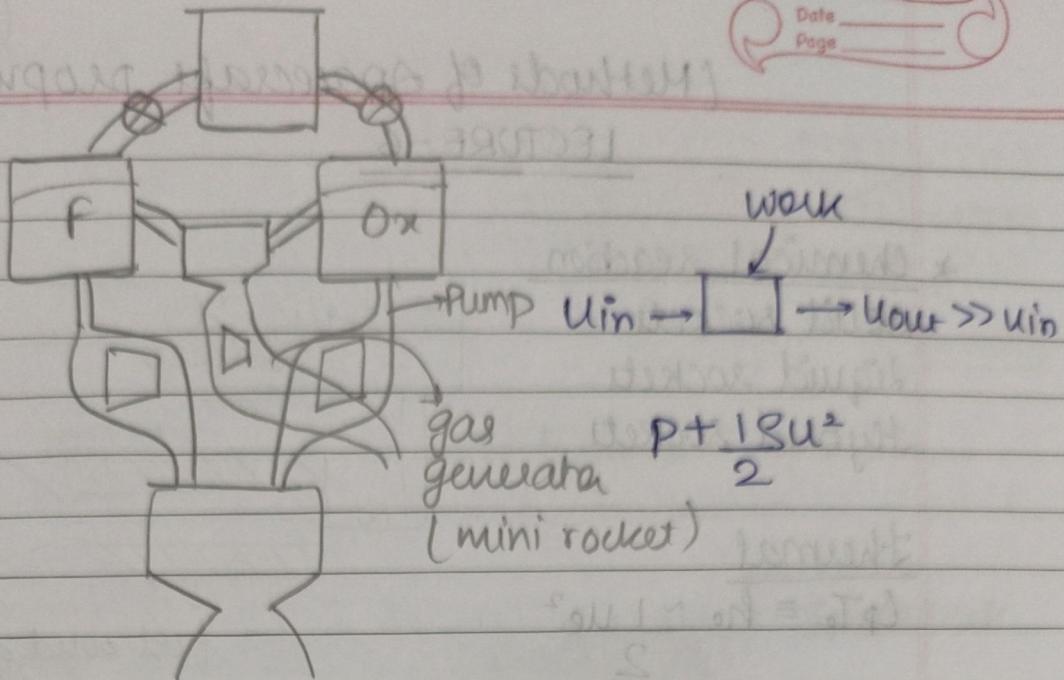
gas betw fuel & ox,  $P_{in} \rightarrow P_{out}$

so that it fills the space  
which comes when they get emptied

Feed sys

→ Pressure fed  
→ Pump fed.

(Minimum energy to start)



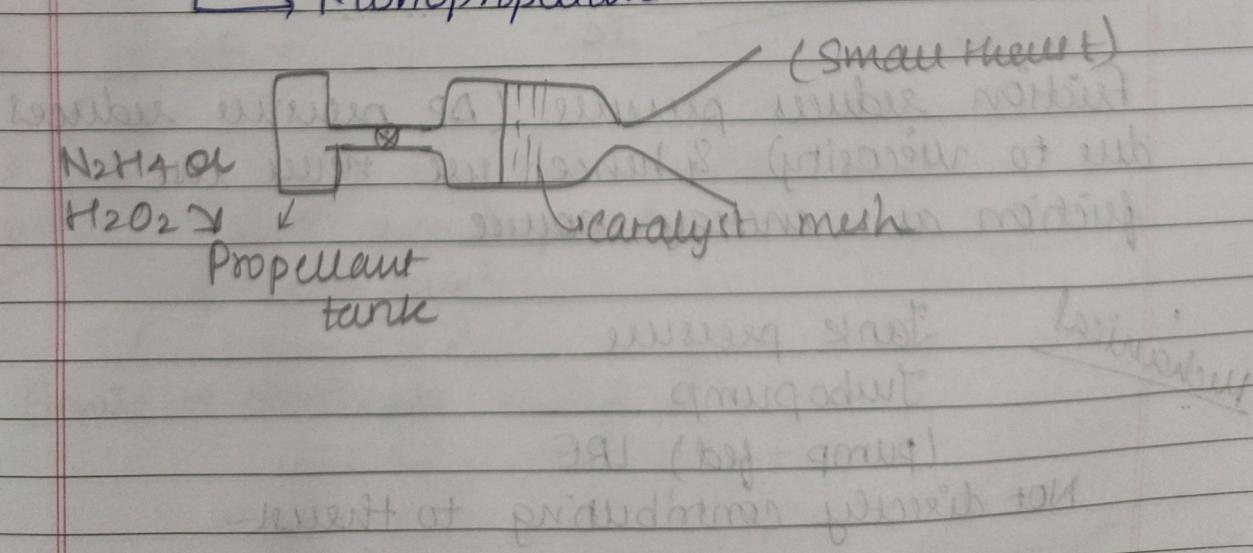
Pump gas generator gives high speed flow that helps in running turbine of pump

also to maintain the press. of three chambers above condenser press  $\rightarrow$  gas generator

Host of hydrodynamic instabilities  
If some problem in suppose On, fuel chamber  
It is transferred to combustion chamber.

### ③ No. of propellants

- Bipropellant (the one we had seen)
- Monopropellant



# (Methods of Spacecraft propulsion)

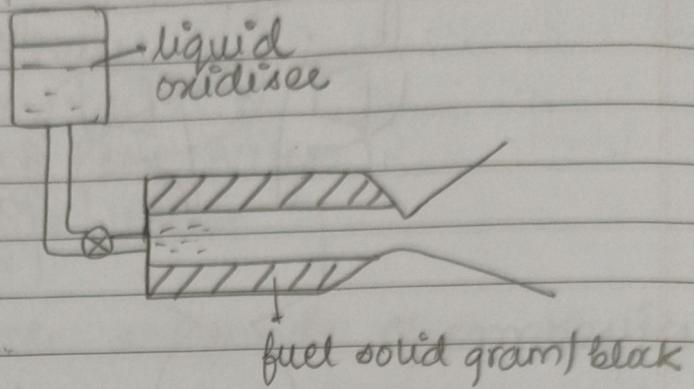
## LECTURE - 6

### \* Chemical reaction

Solid rockets

Liquid rockets

Hybrid rockets



### Thermal

$$C_p T_0 \equiv h_0 \approx \frac{1}{2} Ue^2$$

fuel solid grain block

→ Nuclear

→ Electric

→ Friction

→ Mechanical

$$\text{Friction } C_p T_0 \xrightarrow{\frac{1}{2} Ue^2} \text{Kinetic energy}$$

$$C_p T_0 \xrightarrow{\text{CPTe}} \text{Static energy}$$

$$P_{oe} < P_o \rightarrow \text{friction}$$

$$\frac{P_o}{T_0} \xrightarrow{\text{Thermal energy}} \frac{1}{2} Ue^2$$

Friction reduces pressure ( $\Delta P$  pressure reduces due to viscosity) & hence we never use friction as energy source.

### Mechanical

Tank pressure

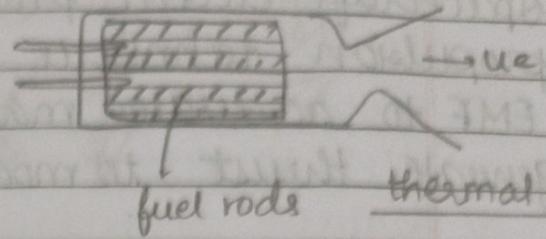
Turbopump

(pump-fed) IRE

Not directly contributing to thrust

Nuclear

Using nuclear reaction instead of chemical reaction to burn fuel & oxidiser to generate thrust.

NERVA

WDM - 93881 = 1100 H

TANDEM - 1001

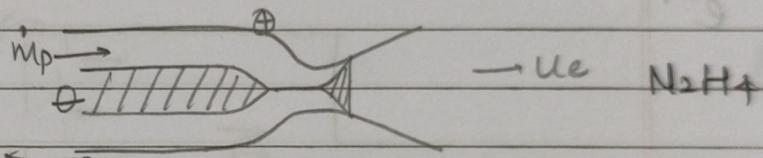
To &lt; 4000K

Resistojet

electrical arcjet → electric discharge → mp  
is created in flow of propellant. I → VMD  
additional energy to propellant.



Resistojet →  
Provides heating by sending electricity through arcjet resistor to a non reactive fluid.



	Isp (s)	Power (W)	Thrust (mN)
--	---------	-----------	-------------

Rev.	300	340	500
------	-----	-----	-----

arc	865	365	22500
-----	-----	-----	-------

electrostatic Flow does not change M.F.  
 $\vec{E}$  &  $\vec{B}$  are imposed  
 $q \sim C_p T_0 \sim \frac{V e^2}{2}$

Ionise → accelerate using  $\vec{E}$

200.1 - 8.970 10

Pass = 910 m/s

Vp0 = 1000

M

V = 9000 km/s

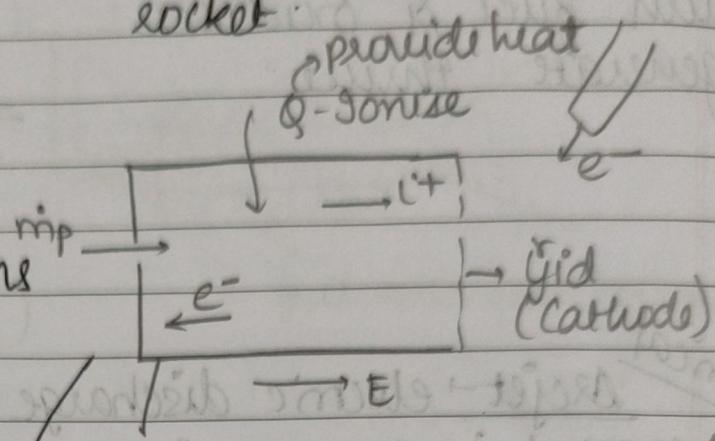
Vip = 1000

Spacecraft electric propulsion is a type of spacecraft propulsion technique that uses electrostatic or EMF to accelerate mass to high speed & thus generate thrust to modify velocity of spacecraft in rocket.

$$H_{max} = 1836 e^- \text{ mass}$$

ions  $\rightarrow$  thrust

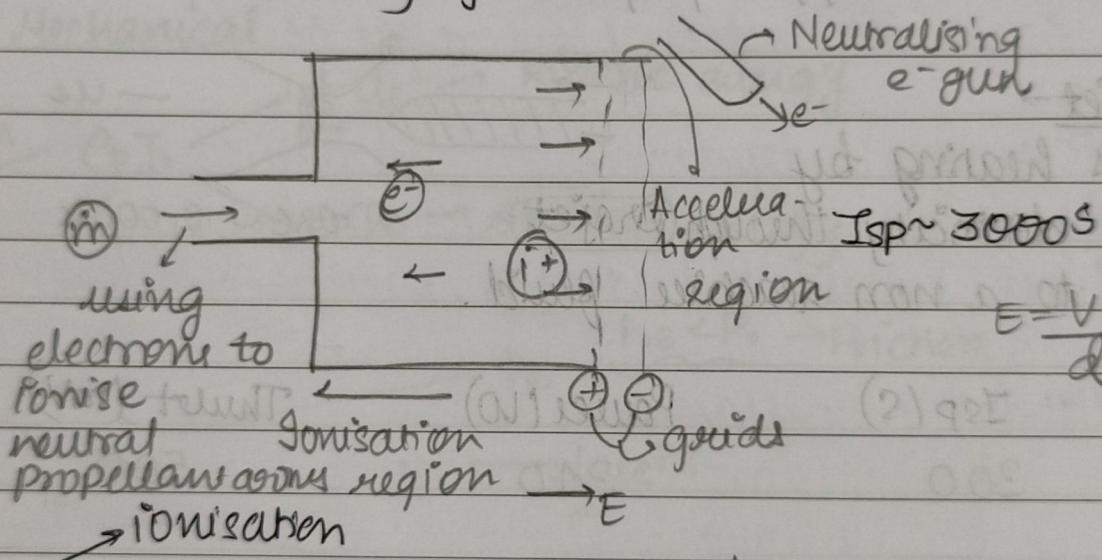
electrons  $\rightarrow$  neutralise ions



Neutral propellant

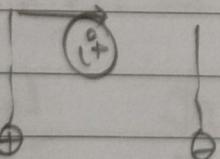
atoms are ionised using gridded ion thruster

$e^-$



Power  $\rightarrow$  force  
 $\rightarrow$  thrust

$$I = I_i + I_e \\ = q_i n_s u_i A \\ + q_e n_e A$$



$$T = \frac{\pi}{m} A U_i u_i$$

Potential drop  $\rightarrow V$

$$\frac{1}{2} M U_i^2 = q_i V$$

at STP,  $g \sim 1.225$

1 mole = 22.4

$$U_i = \sqrt{\frac{2qV}{M}}$$

Son beam current,  $I_b = q_i n_i A u^i = \frac{q_i m}{M}$

$$T = I_b \frac{M}{q} \sqrt{\frac{2qV}{M}} = \sqrt{\frac{2M}{q}} \frac{T}{I_b} \sqrt{V}$$

$$I_{sp} = \frac{U_i}{g_0} = \frac{1}{g_0} \sqrt{\frac{2qV}{M}}$$

$M$  → Mass of ions

$$T = \sqrt{\frac{2M}{q}} I_b \sqrt{V}$$

$$P = V_d I_d = \frac{V_b I_d}{\eta_d}$$

↳ Discharge voltage

$\eta_d$

↳ Discharge efficiency

$$T = \sqrt{\frac{2M}{q}} \frac{\eta_d}{\sqrt{V}}$$

Power

For a given power, high  $M$  is better

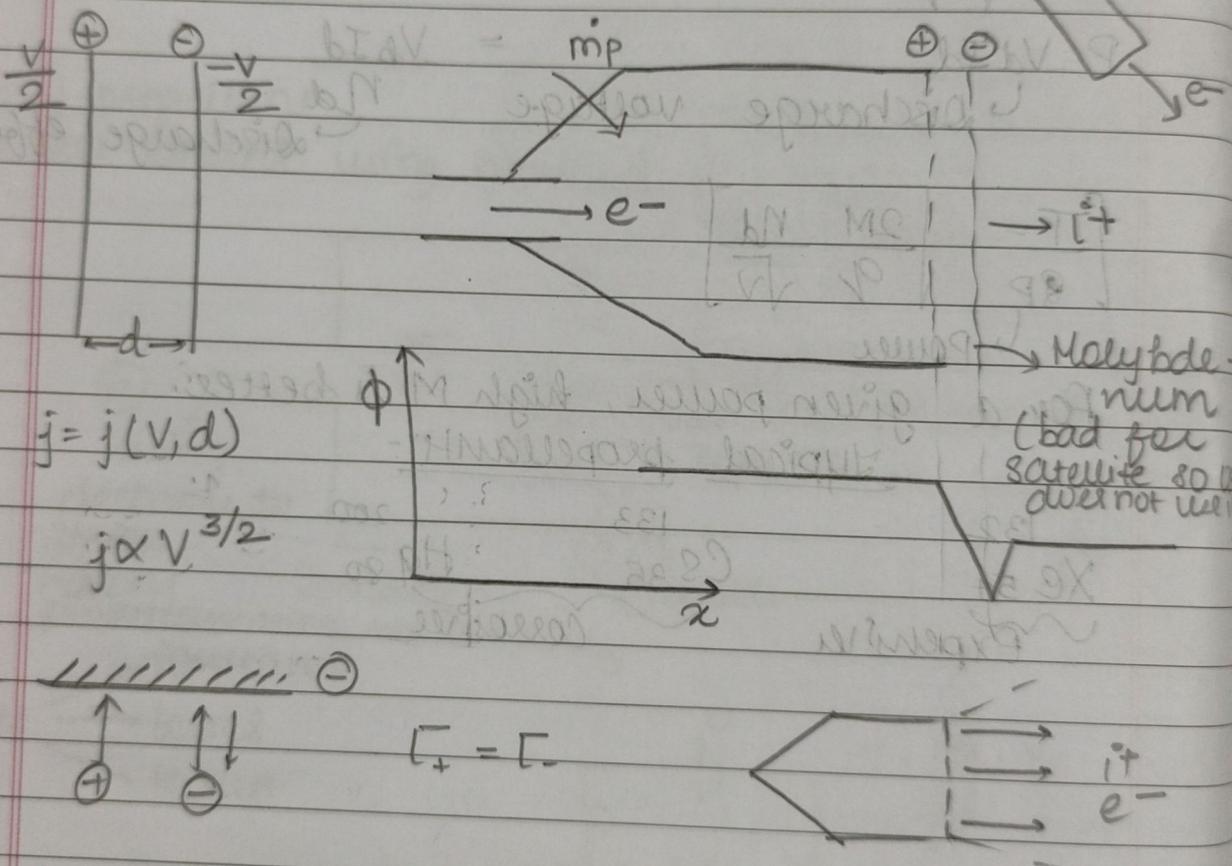
Typical propellants:-

Xe 54	132	133	200
Expensive	Cs 35	Hg 80	

Corrosive

lecture-7

- 6) Raptor Semi cryo engine (SpaceX)  $\rightarrow \text{LOx} + \text{CH}_4$   
 1) V-2 Rocket  $\rightarrow \text{LOx} + \text{Ethanol}$   
 2) Hybrid propellant rocket engine  $\rightarrow$  Hybrid prop  
 3) Arcjet thruster  $\rightarrow$  Electric propsys  
 4) Electrothermal thruster  
 5) Gridded Ion thruster  $\rightarrow$  Electrostatic



How to replace molybdenum grid?

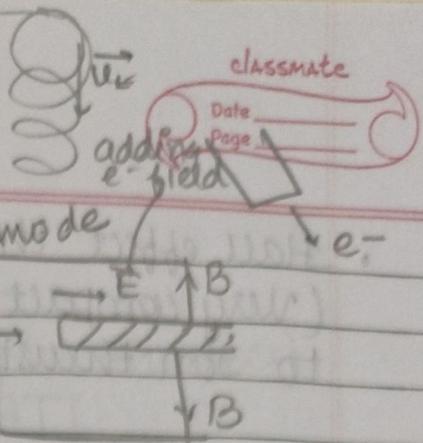
That sustains a large pot drop.

HALL THRUSTERS (To inc the area force)

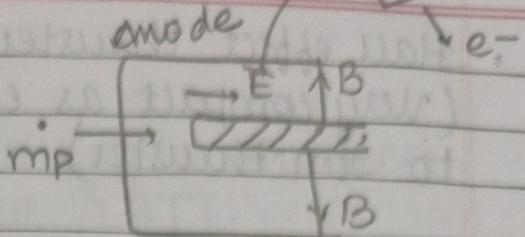
If the idea is the reduction of  $e^-$  flow inwards we can achieve this using a magnetic field. Only electrons are 'magnetised'.

(NASA's evolutionary XENON thruster (NEXT))

What are  $\vec{V}_c$  &  $\vec{V}_E$ ? components  
of vel of e- along  $\vec{E}$  &  $\vec{B}$



If we add E, F there are  
Lorentz forces.



Ions & should obey E.F,  
e- should obey M.F.

$$\vec{V} = \vec{V}_c + \vec{V}_E$$

$$\frac{m d\vec{V}}{dt} \neq q\vec{V}_c \times \vec{B}$$

Cyclotron

$$\frac{m d(\vec{V}_c + \vec{V}_E)}{dt} = q\vec{E} + q\vec{V}_c \times \vec{B} + q\vec{V}_E \times \vec{B}$$

$$\text{Define } \vec{V}_c := \frac{m d\vec{V}_c}{dt} = q\vec{V}_c \times \vec{B}$$

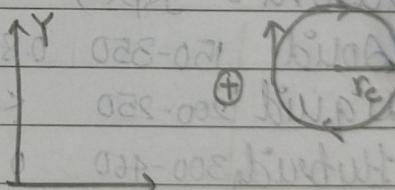
$$\frac{m d\vec{V}_E}{dt} = q\vec{E} + q\vec{V}_E \times \vec{B} = 0$$

$$\vec{V}_E = \vec{E} \times \vec{B}$$

$$\vec{E} = E \hat{e}_z$$

$$\vec{B} = B \hat{e}_r$$

$$\vec{V}_E = -\frac{E}{B} \hat{e}_\theta$$



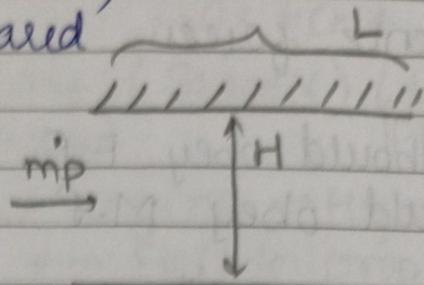
$$\frac{m dV_x}{dt} = qV_y B \rightarrow \frac{dV_x}{dt} = \frac{qB}{m} V_y$$

$$\frac{m dV_y}{dt} = -qV_x B$$

$$\frac{m V^2}{r_c} = qVB$$

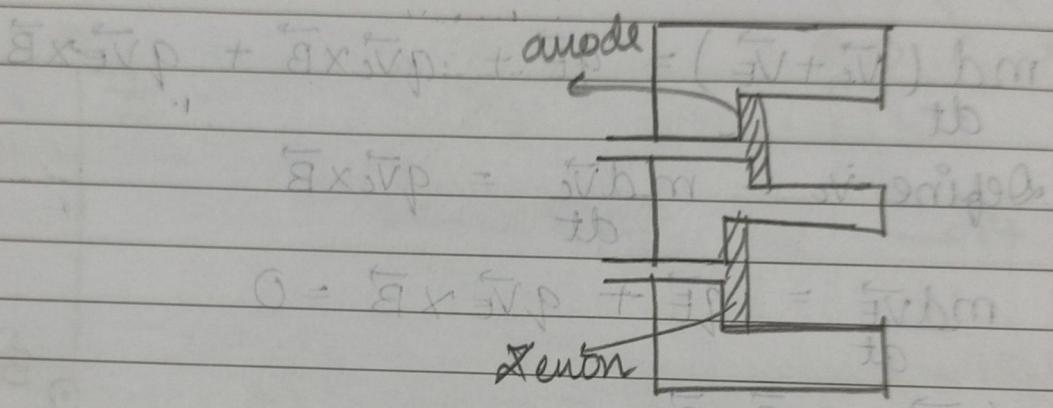
$$r_c = \frac{V}{\omega_c} = \frac{mv}{qB}, r_c \propto m$$

- Hall effect thruster (HET)  
(Very compact as compared to Ion thruster)



Propellants  $\rightarrow$  Xenon, Krypton

Uses an electrostatic potential  $V_f$  if  $r_{ce} < L, H < r_{ci}$   
to accelerate ions upto high speeds.



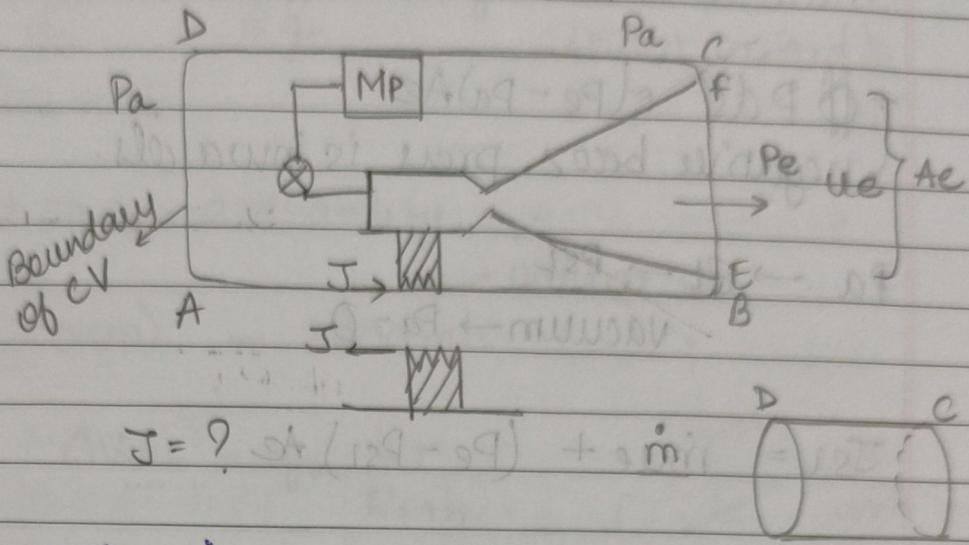
\* Comparison :-  $I_{sp}(s)$

	1	$mp \rightarrow mi$ Inert (Tank, pump, pipes etc)
Solid	150-350	0.8-0.95
mono prop	200-250	<0.5
lb prop	Hybrid	300-460
	Acet.	0.7-0.9
	Ion	300-350
		0.85-0.9
	Hall Thruster	460-1500
		<0.5
		1500-10000
		<0.5
		$\lambda = \frac{mp}{mp+mi}$ (Propellant mass fraction)

Product gases  $\rightarrow$   $CO_2, NOx, H_2O, SOx, Al_2O_3$ .

LECTURE-8

\* Thrust equation:- (of jet engine thrust)  
Using Newton's 2<sup>nd</sup> law & RTT.



Force  $\leftrightarrow$  acceleration

$$\frac{d}{dt} \left( \oint_S \vec{q} \vec{v} d\vec{s} \right) + \oint_S \vec{q} \vec{v} \cdot \vec{v} d\vec{s} = \text{Force} = - \oint_S P d\vec{s} + J_{ex}$$

Time interval \$\Delta t\$

Mom<sup>n</sup> of inflow  $\rightarrow 0$

Mom<sup>n</sup> of outflow  $\rightarrow m \Delta t U_e$

Body  
face  
exerted  
on it

$$AD \rightarrow Ae$$

Pressure force :-

(AD) @ inflow  $\rightarrow P_a A_0$

(BC) @ outflow  $\rightarrow P_e A_e + P_a (A_0 - A_e)$

$$J = (P_e - P_a) A_e \cdot \text{Total force}$$

$$\text{Force} \leftrightarrow \text{acceleration} \quad \left[ \frac{m \Delta t U_e - 0}{\Delta t} \right]$$

$$J = m \Delta t U_e + [P_e - P_a] A_e$$

$$dm = \rho v_n ds$$

For here we assume steady state

$$\oint \rho \vec{v} \cdot \vec{v} \cdot d\vec{s} = \dot{m}_{ue}$$

$$\oint P d\vec{s} \sim (P_e - P_a) A_e$$

negative becoz pres is invards.

$$P_a \rightarrow S_L \rightarrow P_{SL}$$

$$\text{vacuum} \rightarrow P_a = 0$$

$$J_{SL} = \dot{m}_{ue} + (P_e - P_{SL}) A_e$$

$$J_{vac} = \dot{m}_{ue} + P_e A_e$$

$$J_{vac} = J_{SL} + P_{SL} A_e$$

$$J = \dot{m}_{ue} + (P_e - P_a) A_e \equiv \dot{m} u_{eq}$$

$$u_{eq} = u_e + \frac{(P_e - P_a) A_e}{m}$$

$$I_{sp} = \frac{u_{eq}}{g_0} = \frac{u_e}{g_0} + \frac{(P_e - P_a) A_e}{m g_0}$$

$$J = \dot{m} u_e + (P_e - P_a) A_e \equiv \dot{m} u_{eq}$$

$$\dot{m} u_e \gg (P_e - P_a) A_e \rightarrow \text{Rocket}$$

$$\dot{m} u_e \ll (P_e - P_a) A_e$$

ROCKET :-  $I_{sp}, I$

~~mass~~

$$\frac{u_{eq}}{g_0} \text{ or } \frac{J}{m g}$$

$$\lambda = \frac{M_p}{M_p + M_i}$$

↑ Sweet mass like tanks

$$\frac{U_{eq}}{g_0} \propto \frac{1}{m g} \quad (\text{highest in vacuum} \\ \text{lowest on grd}).$$

Mission  $\rightarrow \Delta V$

→ ease of use availability

$$\text{LEO (300km)} \rightarrow 8000 \text{ m/s}$$

→ orbital vel

$$\Delta V_{orb} = g_0 \frac{R_o^2}{R_o + h}$$

$$\Delta V_{orb} = \Delta V_{alt} = \sqrt{2gh} = \sqrt{G} \text{ km/s.}$$

$$\Delta V = \Delta V_{alt} + \Delta V_{orb} \sim 10 \text{ km/s.}$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

Vikas engine

$$\frac{1}{2}mv^2 \approx mg_0h$$

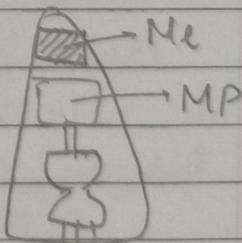
$J_{sp} \sim 300 \text{ s}$

Payload  $\sim 500 \text{ kg}$

$$V = \sqrt{2g_0h}$$

$$\lambda = \frac{M_p}{M_i + M_p}$$

$$\text{Sweet gas } M_i = \frac{1 - \lambda}{\lambda} M_p$$



For the rocket,

Force = (accel)  $\times$  mass

$$J = \dot{m}_p U_{eq} = \dot{M} \frac{dv}{dt}$$

$$m_p = -\frac{dM}{dt} *$$

assume  
near-unif

$$\frac{MdV}{dt} = -\frac{dM}{dt} = -\frac{dM}{dt} u_{eq} \quad \text{and} \quad dX = \frac{1}{M} u_{eq} dt$$

$$dV = -u_{eq} \frac{dM}{M}$$

assume  $u_{eq} = \text{const}$

For the rocket (in absence of gravity)

$$\Delta V = \int_{t=0}^{t=t_f} dV = u_{eq} \ln \left[ \frac{M_0}{M_f} \right] \quad (\text{Ansatz 03})$$

$$M_0 = M_p + M_i + M_e + S$$

$$M_f = M_i \rightarrow M_f = M_i + M_e$$

$$dM/dt = N \beta c L = N \rho V A = d\rho V A$$

$$dM/dt \rightarrow d\rho V A + dV A = VA$$

approx. work

$$dM/dt = -VM$$

$$dM/dt \sim q \rho L$$

$$d\rho M \sim \rho V m L$$

$$\frac{dM}{dM + iM} = A$$

$$\sqrt{\rho \rho L} = V$$

$$dM/A = iM \quad \text{and} \quad dM/A = V$$

$$dM/A = iM \quad \text{and} \quad dM/A = V$$

## LECTURE-9

$$\frac{Md\mathbf{v}}{dt} = \dot{m}_{eq}\mathbf{\dot{v}} - \textcircled{1} \quad [J = \dot{m} u_e + (P_e - P_a)A_e \\ \equiv \dot{m} u_{eq}]$$

$$\frac{dM}{dt} = \dot{m} \equiv \dot{m}_p$$

↓ Total change in rocket mass

$$\int_{t=t_0}^{t=t_f} \textcircled{1} dt \rightarrow \Delta V = u_{eq} \ln \left[ \frac{M_0}{M_p} \right]$$

$t_f = t_b \rightarrow$  final time or burn out time

Initial mass	$M_{ch}$	+ $M_p$	+ $M_i$	
	Payload	Propellant	Instrument / Structural mass	Chamber, nozzle, tank, guidance negligible

$$M_f = M_i + M_i$$

Depends on mission,  $\Delta V$  ↗ LEO (10 km/s)

Total thrust over one year that is provided to counter drag,

$$\Delta V = \int J dt$$

$$\Delta V = u_{eq} \ln [MR]$$

$$MR = \frac{M_0}{M_f} = \frac{M_p + M_i + M_e}{M_i + M_e}$$

For selected engine  $\rightarrow u_{eq}$  is given.

&  $\lambda = \frac{M_p}{M_i + M_p}$  is used to find  $M_i$  -

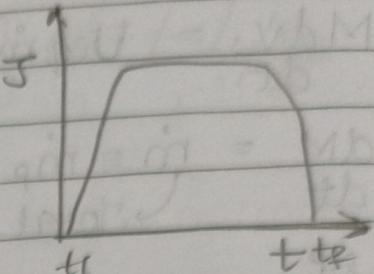
\* How to find  $u_{eq}$  for any engine? =

$$\text{Measure } \dot{m} = M_p$$

$$t_f - t_i$$

$$MR = \frac{M_p + (\lambda)M_e}{(1-\lambda)M_p + \lambda M_e}$$

Using this find  $M_p$ .



Measure thrust & mass flow rate, the ratio is  $u_{eq}$

\* Vikas engine:

$$u_{eq} = 3000 \text{ m/s}$$

$$\Delta V = 10 \text{ km/s}$$

$$\lambda = 0.9$$

$$M_e = 1 \text{ kg}$$

Find  $M_p$

Two cases  $\rightarrow \Delta V = 3 \text{ km/s}$

$$\Delta V = 10 \text{ km/s}$$

$$\Delta V = u_{eq} \ln [MR]$$

$$\frac{10 \times 1000}{3000} = \ln [MR] = \frac{100}{3}$$

$$MR = e^{100/3}$$

$$MR = \frac{x + 0.9}{0.1x + 0.9} = 1.001$$

$$0.1x + 0.9 = 1.001(0.1x + 0.9) = 0.1001x + 0.9009$$

$$0.0009 = (1 - 1.001)x$$

$$0.0009 = 0.8999 x$$

$$x = 1.0001 \text{ kg} \rightarrow M_p$$

# Difference b/w $U_{exp}$ & $U_{eq}$

classmate

Date \_\_\_\_\_  
Page \_\_\_\_\_

$$M_p + \lambda M_e = MR \{ (1-\lambda) M_p + \lambda M_e \}$$

$$M_p [1 - (1-\lambda) MR] = \lambda (MR-1) M_e$$

$$M_p = M_e \left[ \frac{(MR-1) \lambda}{1 - (1-\lambda) MR} \right]$$

$$1 - (1-\lambda) MR > 0$$

$$MR < \frac{1}{1-\lambda}$$

$$\Delta V < U_{exp} \ln \left( \frac{1}{1-\lambda} \right)$$

} limiting case.

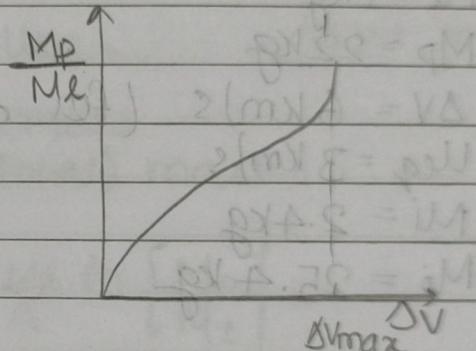
For first case,  $\Delta V = 3000 \text{ m/s}$ ,  $U_{exp} = 3000 \text{ m/s}$

$$M_p \approx 2.1 \text{ kg.}$$

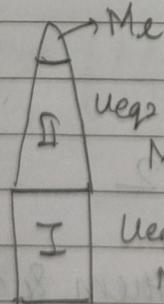
$$\Delta V = 2 U_{eq}$$

$$\lambda = 0.9$$

$$\frac{M_p}{M_e} = 22, \frac{M_i}{M_e} = 2.4$$



- Falcon 9 → Two stages & same engine for both engines.



$U_{eq2}, M_p, M_2, \text{ Tank, chamber, nozzle}$

$U_{eq1}, M_p, M_1$

\* Stage I (M<sub>0</sub> - Total initial mass)

$$M_{01} = M_{p1} + M_{i1} + M_e$$

$$M_{e1} = M_0$$

$$\Delta V_1 = U_{eq1} \ln [MR_1]$$

\* Stage II

$$M_{i2} = M_e$$

$$M_{02} = M_{p2} + M_{i2} + M_e$$

$$\Delta V_2 = U_{eq2} \ln [MR_2]$$

$$\Delta V = \Delta V_1 + \Delta V_2$$

\* Continuing with previous ex:-

$$M_e = 1 \text{ kg}$$

$$M_p = 22 \text{ kg}$$

$$\Delta V = 6 \text{ km/s} \text{ (for single stage)}$$

$$U_{eq} = 3 \text{ km/s}$$

$$M_i = 2.4 \text{ kg}$$

$$M_0 = 25.4 \text{ kg}$$

Split into 2, half goes into Stage 1 & other to Stage II.

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$M_{p1} \text{ & } M_{p2} = M_p/2 = 11$$

$$M_{i1} \text{ & } M_{i2} = M_i/2 = 1.2$$

Find  $\Delta V_1$  &  $\Delta V_2$ , add them & compare with ~~6 km/s~~ & check whether it increased or not

$$1.7 \text{ kg} \quad 5.6 \text{ kg}$$

$$\text{Total } 7 \text{ kg.}$$

LECTURE-10

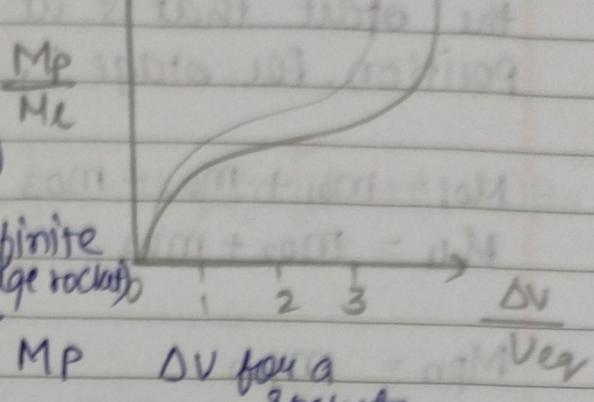
Discard unnecessary  
inert mass

→ discard once (2 stage  
rocket)

→ discard continuously (infinite  
stage rocket)

$$\lambda = \frac{M_p}{M_p + M_i}, M_i = \left[ \frac{1-\lambda}{\lambda} \right] M_p$$

$\Delta V$  for a rocket  
 $\lambda=0.9, \lambda=0.95$



$$m_t = m_p + m_i = \frac{m_p}{\lambda}$$

$m_i \rightarrow$  less in  
inert mass

$$\frac{MdV}{dt} = \dot{m} u_{eq} = \frac{m_p}{\lambda} u_{eq}$$

↓ less of total mass

→ rocket component (total) mass

$$\Delta V = \frac{u_{eq} \ln \left[ \frac{M_0}{M_e} \right]}{\lambda} = \frac{u_{eq} \ln \left[ \frac{M_0}{M_e} \right]}{\lambda}$$

$$\frac{M_0}{M_e} = \exp \left[ \frac{\lambda \Delta V}{u_{eq}} \right], \frac{M_1}{M_0} = \exp \left[ -\frac{\lambda \Delta V}{u_{eq}} \right]$$

$\infty$ -stage rocket

$$M_p = \lambda \left[ \exp \left( \frac{\lambda \Delta V}{u_{eq}} \right) - 1 \right] M_e$$

$$t \rightarrow t + \delta t$$

$$8m = m_p \delta t$$

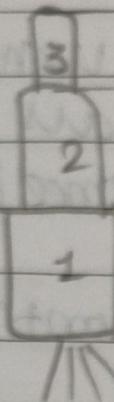
$$8m = 8m_p + 8m_i$$

} infinite stage rocket

Multi stage rockets (N)  
as 1<sup>st</sup> part is removed in  
the start that is at first  
position. For stage 1,

$$M_{01} = m_{01} + m_{02} + m_{03}$$

$$M_{10} = m_{02} + m_{03}$$



$$M_{pn} = m_{pn}$$

$$M_{fn} = m_{fn}$$

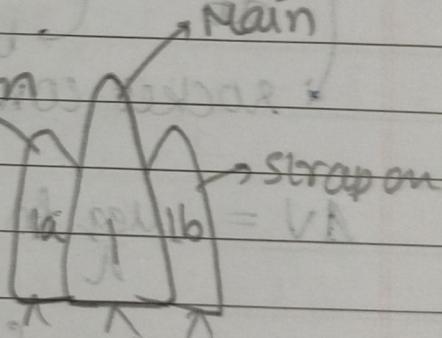
$$M_{0n} = \sum_{j=1}^N m_{0j} \quad \text{&} \quad M_{1n} = \sum_{j=n+1}^N m_{0j}$$

1<sup>st</sup> Stage → booster stage

$$J = J_1 + J_{10} + J_{1b} + J_{1n} \rightarrow \begin{array}{c} \text{Stage 1} \\ \text{Stage 10} \\ \text{Stage 1b} \\ \text{Stage 1n} \end{array}$$

$= m_{1,0eg_1} + m_{1,0eg_2} + \dots + m_{1,0eg_n}$

$\therefore m_{1,0eg_1} + m_{1,0eg_2} + \dots + m_{1,0eg_n}$

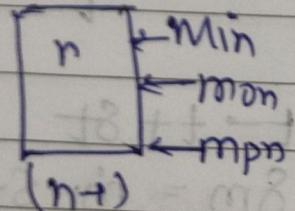


$$M_{eg} = m_{1,0eg_1} + \dots + m_{1,0eg_n}$$

$m_1 + m_2 + \dots + m_{1n}$

stage structural factor

$$\epsilon_n = \frac{m_{fn}}{m_{0n}} = \frac{m_{fn}}{m_{0n}}$$



$m_{fn}$  → final mass of stage  
 $n$  after stage  $n$  is completed.

## Payload factor

$$\beta_n = \frac{M_{fn}}{M_{on}} = \frac{M_{on+1}}{M_{on}}$$

$$\Delta V_n = \mu_{eqn} \ln \left( \frac{M_{on}}{M_{fn}} \right) = -\mu_{eq} \ln [e_n + (1-e_n)\beta_n]$$

$$M_e = 1 \text{ ton}$$

$$M_0 = 25.5 \text{ ton}$$

$$M_p = 22 - 1 -$$

$$M_i = 2.4 \text{ ton}$$

$$m_{02} = 12.2 \text{ ton}$$

$$m_{p2} = 11 \text{ ton}$$

$$m_{i2} = 1.2 \text{ ton}$$

$$m_{01} = 12.2 \text{ ton}$$

$$m_{p1} = 11 \text{ ton}$$

$$m_{i1} = 1.2 \text{ ton}$$

$$\beta_1 = \frac{M_{02}}{M_0}, \quad \beta_2 = \frac{M_e}{M_{02}}$$

$$\beta_1 = \frac{12.2}{25.4} = \frac{1}{2}, \quad \beta_2 = \frac{1}{13.2}, \quad e_1 = e_2 = \frac{1.2}{12.2}$$

$$\Delta V = \sum \Delta V_n$$

$$\Delta V = - \sum \mu_{eqn} \ln [e_n + (1-e_n)\beta_n]$$

Given  $\frac{M_e}{M_0}$  or  $\Delta V_{max}$

$$\frac{\partial \Delta V}{\partial \beta_n}, \quad \beta = \frac{M_e}{M_0} \quad (\text{Payload}) \quad (\text{in our control})$$

$$e_n = \frac{\min}{m_{on}}$$

$$e_n = \min_{mon}$$

$$\Delta V = - \sum_1^N \mu_{eq} \ln [e + (1-e)\beta_n]$$

Same rocket engine for  
all stages  $\Rightarrow u_{eqn} = u_{eq}$   
 $c_n = c$

$$\frac{M_e}{M_0} = \frac{N_{02}}{M_0} \cdot \frac{N_{03}}{M_{02}} \cdots \frac{M_p}{M_{0N-1}}$$

$$= \prod_{i=1}^N B_N$$

$$\frac{M_{02}M}{M_0M} = \frac{B_2M}{M_0M} = B_2$$

$$u_{eqn} = u_{eq}$$

$$c_n = c$$

$$\textcircled{1} \quad \Delta V = - \sum_{n=1}^N u_{eq} \ln [e + (1-e)B_n]$$

$$\frac{M_e}{M_0} = \prod_{n=1}^N B_N$$

$$\textcircled{2} \quad M \frac{M_e}{M_0} = \sum_{n=1}^N$$

$$= [\log(3 \cdot 1) + 3] \text{ m/s} = V_A$$

$$[\log(3 \cdot 1) + 3] \text{ m/s} = V_A$$

$$(\text{m/s}) \times (\text{m/s}) = \text{m/s}^2$$

$$m/M = n$$

$$[\log(3 \cdot 1) + 3] \text{ m/s} = V_A$$

LECTURE-11

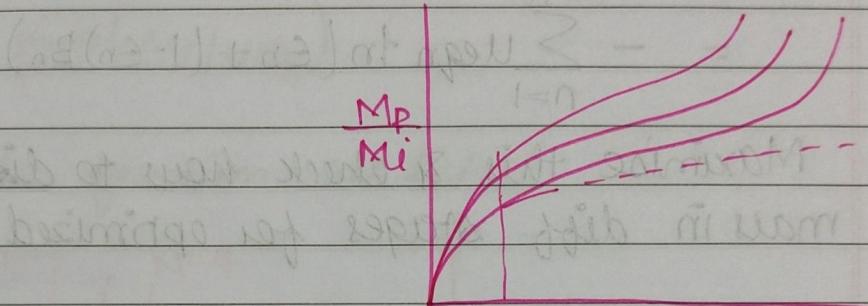
\* Single stage rocket

$$\Delta V = U_{eq} \ln \left( \frac{M_0}{M_f} \right) \equiv U_{eq} \ln (MR) \text{ or } MR = \exp \left( \frac{\Delta V}{U_{eq}} \right)$$

$$\Delta V > U_{eq}/2$$

$$\frac{MdV}{dt} = m_p U_{eq} = \dot{m} U_{eq}$$

$$\lambda = \frac{m_p}{M_p + M_i} \Rightarrow \dot{m} = \dot{m}_p + \dot{m}_i = \frac{\dot{m}_p}{\lambda}$$



\* Multistage rocket

For  $n^{th}$  stage:-

$$M_{on} = \sum_{j=n}^N m_{oj}$$

$$B_1 = \frac{M_{o2}}{M_{o1}}$$

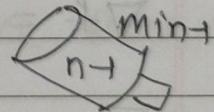
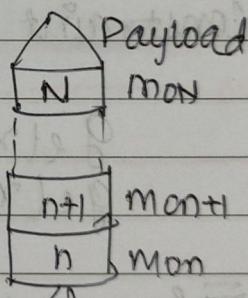
$$M_{el} = M_{o2}$$

$$B_2 = \frac{M_{o3}}{M_{o2}}$$

$$B_n = \frac{M_{on}}{M_{on}} = \frac{M_{on+1}}{M_{on}}$$

$$M_{on} = \sum_{j=n}^N m_{oj}, \quad M_{on} = \sum_{j=n+1}^N m_{onj}$$

$$\Delta V_n = U_{eqn} \ln \left[ \frac{M_{on}}{M_{on}} \right]$$



Stage mass :-  $m_{on} = m_{pn} + m_{in}$

$$e_n = \frac{m_{in}}{m_{on}}, m_{in} = e_n m_{on}$$

$$M_{fn} = m_{in} + M_{en} \rightarrow \text{final mass}$$

$$m_{in} = e_n [M_{on} - M_{en}]$$
$$= M_{on} [e_n + (1-e_n) \beta_n]$$

$$\Delta V = \sum_{n=1}^N \Delta V_n$$

$$= - \sum_{n=1}^N \mu_{eqn} \ln (e_n + (1-e_n) \beta_n)$$

Maximise this & check how to distribute mass in diff stages for optimized problem.

\* Maxima / Minima along a curve  
constraint functions or constraints

$$g_1(x, y) = x + y - 1$$

$$g_2(x, y) = x^2 + y^2 - 4 \quad g_1(x, y) = 0, g_2(x, y) = 0$$

$$\bar{\nabla} f = \frac{\partial f}{\partial n} \hat{e}_n + \frac{\partial f}{\partial j} \vec{e}_j^0 \text{ at extrema}$$

$$\bar{\nabla} g = \frac{\partial g}{\partial n} \hat{e}_n + \frac{\partial g}{\partial j} \vec{e}_j^0 \text{ everywhere}$$

At extrema

$$\bar{\nabla} f = \frac{\partial f}{\partial n} \times \bar{\nabla} g = \alpha \bar{\nabla} g$$
$$\frac{\partial g}{\partial n}$$

For  $\frac{\partial g}{\partial n} \neq 0$ ,

$$\nabla(f - \alpha g) = 0$$

we minimise  $f - \alpha g$  instead of  $f(x,y)$   
 $\alpha$  → Lagrange multiplier.

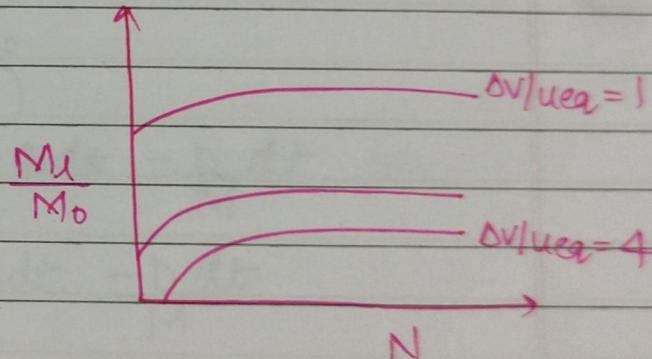
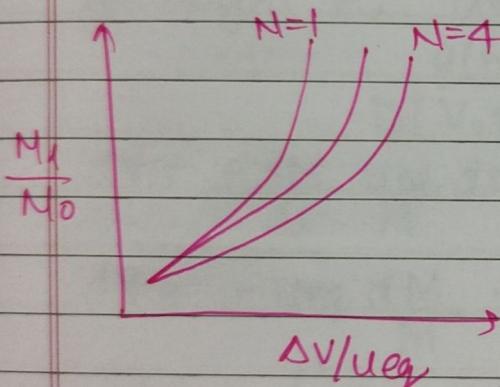
Optimum mass distribution:-

$$\frac{M_e}{M_0} = \prod_{n=1}^N \beta_n = \prod_{n=1}^N \frac{-\alpha}{(\alpha + u_{eqn})} \frac{\epsilon_n}{(1-\epsilon_n)}$$

Payload fraction:-  $\frac{M_e}{M_0} = \frac{(w^{-N} - \epsilon)}{(1-\epsilon)}^N$

$$w = \exp(\Delta V / u_{eq})$$

Payload fraction vs  $\Delta V$ .



Countries capable of launching Space Rockets.

\* PSLV (C53)

Four stages

First stage → PS1

$$\beta_1 = \beta_2 + \frac{M_{02}}{M_{01}} = \frac{M_e}{M_{01}} \rightarrow M_{02} = \sqrt{M_1 M_0}$$

\* GSLV (MK-II) R1

GSLV MK-III M1

\* Launch facilities

Sites → around the world.

SpaceX - Falcon 9

Payload (tons), LEO launch cost, height,  
diameter, lift-off mass.

lecture :- 12

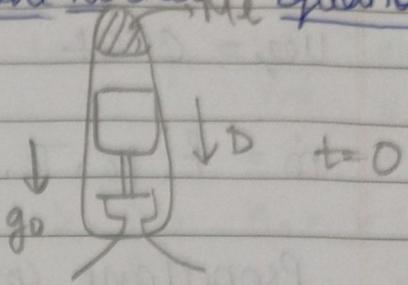
Vertical Trajectories - The Rocket Equation

$$M_0 = M_p + M_i + M_e$$

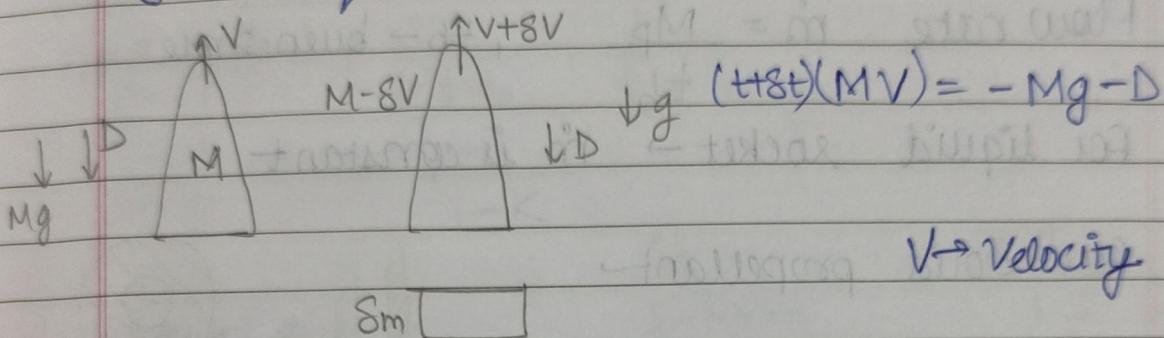
$t = t_b$  - burn-out

$$M_p = M_i + M_e$$

$$J - Mg - D = \frac{MdV}{dt}$$



$$J = \dot{m} u_{eq} = \dot{m} u_e + (p_e - p_a) A_e$$



at height  $h$ ,  $u_{eq}(h)$

$$g(h) = g_0$$

$$D(V, p, M)$$

$$dV = -\frac{dM}{dt} \frac{u_{eq}}{M} dt - g_0 dt - \frac{D}{M} dt$$

$$dV = -\frac{u_{eq} dM}{M} - g_0 dt - \frac{D}{M} dt$$

$$\Delta V = \int_0^h dV$$

$$= -\underbrace{\int u_{eq} \frac{dM}{M}}_{\Delta V_{ideal}} - \underbrace{g_0 t}_{\Delta V_g} - \underbrace{\int \frac{D}{M} dt}_{\Delta V_D}$$

(g + less)

$$\Delta V = \Delta V_{ideal} - \Delta V_g - \Delta V_D$$

$$J = \dot{m} u_{eq} = \dot{m} u_e + (p_e - p_a) A_e$$

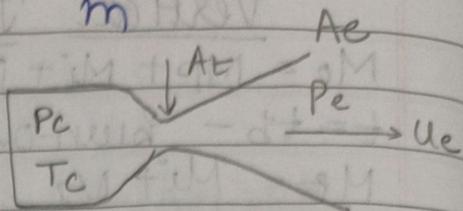
$$J = \dot{m} c^* C_f$$

$c^*$  → characteristic velocity  
 $C_f$  → thrust coefficient

$$\dot{m}c^* = P_c A_t \quad \text{or} \quad \dot{m} c^* = \frac{P_c A_t}{\dot{m}}$$

$$U_{eq} = C^* C_f$$

$$\dot{m} = f(P_c, T_c, y, A_t)$$



Propellant Comb  $\rightarrow T_c, y$ .

$$\text{Flow rate } \dot{m} = \frac{M_p}{t_b}, \quad t_b \rightarrow \text{burn time}$$

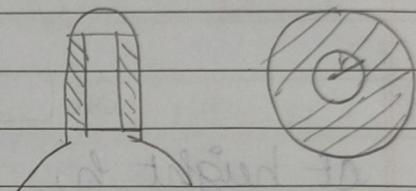
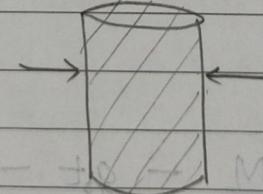
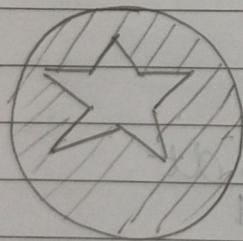
For liquid rocket  $\rightarrow \dot{m}$  is constant

for solid propellant -

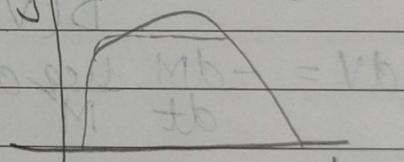
$$\frac{MdV}{dt} = J$$

$$a = \frac{dv}{dt} = \frac{J}{M}$$

$$n = \frac{g}{g} = \frac{J}{Mg}$$

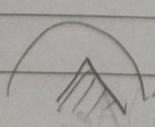


$2\pi r l$



$$A = 2\pi(r_i + r_o)L$$

area increases



area decreases

$$n = \frac{J \cdot V_A}{Mg}$$

$$(n = \frac{a}{g})$$

$$\pi (r_0^2 - r^2) L - 2\pi r L \dot{r}$$

$$n = \frac{J}{Mg} = \frac{M_p u_{eg}}{M g_0 t_b}$$

$$\text{At } t=0 \quad n_0 = \frac{M_p}{M_0} \frac{I_{sp}}{t_b}$$

$$k = \frac{M_p}{M_0}, \quad n_0 = \frac{k I_{sp}}{t_b}$$

$$\text{At } t=t_b \\ M_f = M_0 - M_p = M_0(1-k)$$

$$n_f \equiv n_{max} = \frac{J}{M_f g_0} = \frac{M_0}{M_f n_0}$$

$$n_{max} = \frac{n_0}{1-k}$$

$$\frac{t_b}{2-8 \text{ min}}$$

$$\frac{n_{max}}{1-2-6}$$

\* Large SLV  
(~~space~~ launch vehicle)  
satellite

Strap-on Booster

0.5-2 min

1.2-3

S/C orbit  
maint/maneuvers

<10 min

0.1-6

SAM (anti a/c missile)

2-75 s

5-20  
(<100)

Air-launched guided  
missile

2-5 s (boosted)  
+ 10-30 s (sustained)

<25

Rocket assisted proj  
(gun launched)

~few seconds

up to  
20000

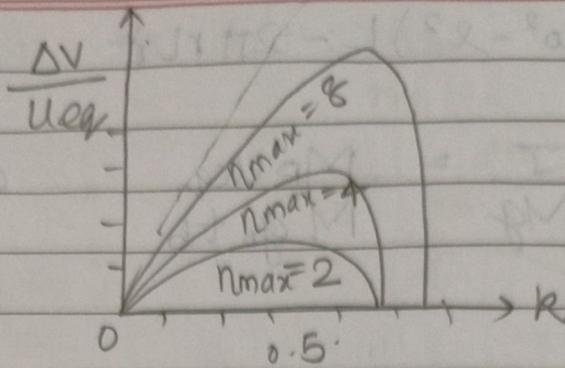
$$t_b = \frac{k}{1-k} \frac{I_{sp}}{n_{max}}$$

$$\dot{m} = \frac{M_p}{t_b}$$

$$\Delta V = u_{eg} \ln \left( \frac{M_0}{M - M_p} \right) - g_0 t_b = u_{eg} \ln \left( \frac{k}{1-k} \right) - \frac{u_{eg}}{n_{max}(1-k)} k$$

$f(s-N)$

$n_{\max} \approx 8$



$$\frac{d(\Delta V)}{dk} = 0$$

### Tsiolkovsky equation

$$V_b \equiv \Delta V = U_{\text{esc}} \log \left( \frac{M_0}{M_f} \right) - g_0 t_b$$

Lecture - 13Drag

$$\eta_0 = \frac{F}{m_0 g_e}$$

Force req  
for lift off

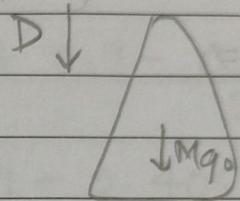
$$K_{opt} = 1 - \frac{1}{\eta_{max}}$$

$$\Delta V_{opt} = u_{eq} \left[ \ln(\eta_{max}) + \frac{1}{\eta_{max}} - 1 \right]$$

$$t_{bopt} = \frac{K I_{sp}}{\eta_0} = \frac{K_{opt}}{1 - K_{opt}} \frac{I_{sp}}{\eta_{max}} = K_{opt} I_{sp}$$

$$\eta_{0opt} = 1$$

$$\frac{MdV}{dt} = J - D - Mg$$



Neglect drag

$$\frac{MdV}{dt} = m u_{eq} - Mg_0$$

$$dV = -\frac{dM}{M} u_{eq} dt$$

Beginning, at  $t=0, V=0, h=0$ 

$$V(t) = \int_0^t dV = u_{eq} \ln \left[ \frac{M_0}{M(t)} \right] - g_0 t$$

$$M(t) = M_0 - mt = M_0 - \frac{Kt}{t_b}$$

$$M_p = m t_b \quad | t_b \rightarrow \text{total burn time}$$

$$V(t) = -u_{eq} \ln \left[ 1 - \frac{Kt}{t_b} \right] - g_0 t$$

$$h(t) = \int_0^t V(t) dt = -u_{eq} \int_0^t \ln \left[ 1 - \frac{Kt}{t_b} \right] dt$$

$$-\frac{1}{2} g_0 t^2$$

$$h(t) = \int_0^t v(t) dt = -u_{eq} \int_0^t \ln \left[ \frac{1-kt}{t_b} \right] dt - \frac{1}{2} g_0 t^2$$

$$= u_{eq} \left\{ t + \left[ \frac{1}{k} \ln \left( \frac{1-kt}{t_b} \right) \right] \right\} - \frac{1}{2} g_0 t^2$$

We can use  $v(t) = -g_e - u_{eq} \ln \left[ \frac{m(t)}{m_0} \right]$  - get

$$h_{bo} = u_{eq} t_b \left( \frac{m_f}{m_0 - m_f} \ln \left( \frac{m_f}{m_0} \right) + 1 \right) - \frac{g_0 t_b^2}{2}$$

$$h_{bo} = u_{eq} t_b \left( \frac{1-k \ln(1-k)}{k} + 1 \right) - \frac{g_0 t_b^2}{2}$$

$$\frac{dh_{bo}}{dk} =$$

$$t_b = \frac{M_p}{m} = \frac{M_p u_{eq}}{h_{max} [M_0 - M_p] g_0}$$

$$h_{bo} = \frac{g_e I_{sp}^2}{g_0} \left[ (1-k) \ln(1-k) + k - \frac{k^2}{2} \right] - \frac{k}{1-k} \frac{I_{sp}}{h_{max}}$$

$$k = k(t_b)$$

$$\frac{dh_b}{dk} = \frac{\partial h_b}{\partial k} + \frac{\partial h_b}{\partial t_b} \cdot \frac{\partial t_b}{\partial k} = 0$$

$$N_0 = 1, V_{bo} = u_{eq} \left[ \ln \left( \frac{1}{1-k} \right) - k \right]$$

$$h_{bo} = \frac{u_{eq}^2}{g_0} \left[ (1-k) \ln(1-k) + k - \frac{k^2}{2} \right]$$

$$m_0 h_{max} = m_0 h_b + \frac{1}{2} m V_{bo}^2$$

$$h_{max} = h_b + \frac{V_{bo}^2}{2g}$$

$$\text{Ex:- } U_{eq} = 3000 \text{ m/s}$$

$$K = 0.9$$

$$V_b = 4000 \text{ m/s}$$

$$h_{b0} = 240 \text{ km}$$

$$h_c = 900 \text{ km}$$

$$h_{max} = 1140 \text{ km}$$

$$V_{b0} = 3000 [f_n(10) - 0.9]$$

$$= 3000 [0.303 - 0.9]$$

$$= 3000 \times 1.403$$

$$= 1403 \times 3 = 4209 \text{ m/s}$$

$$h_{b0} = \frac{g \times 10^6}{10} \left[ 0.1 \times f_n(0.1) + 0.9 - \frac{0.81}{2} \right]$$

$$= g \times 10^5 \left[ -0.1 \times 0.303 + 0.9 - 0.405 \right]$$

### \* Sounding Rockets

Launch  $\angle \rightarrow 85^\circ$

(~20 min above  
100 km)

Can reach an apogee upto 1600 km

for  $M_e \sim 500 \text{ kg}$ ,  $h_{max} \leq 600 \text{ km}$

(< 10 min above 100 km)

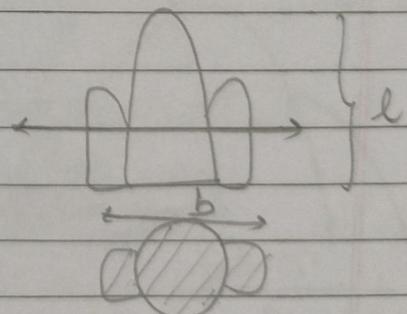
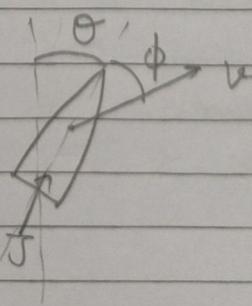
$$D = \frac{1}{2} \rho V^2 A C_D$$

Drag  $D \sim L^2$

$$\text{Diameter } d = \sqrt{\frac{4A}{\pi}}$$

$$M \sim \nabla P \sim L^3$$

$$\frac{D}{M} \sim \frac{1}{L}$$



$$dV = \frac{-dM}{u_{\text{ex}} g} - g \cos \theta dt - \frac{D}{M} dt$$

Stability:  $L = \rho A C_D$

Movable fins

Gimbled Thrust

Venier rockets

Thrust vane.

Which chapter from book?  
No chap from book.

classmate

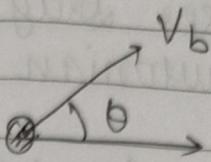
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## Projectile Basics

$$\frac{dV_x}{dt} = 0, \quad \frac{dx}{dt} = V_x$$



$$\frac{dV_y}{dt} = -g_0, \quad \frac{dy}{dt} = V_y$$

$$V_x(t) = V_b \cos \theta$$

$$V_y(t) = V_b \sin \theta - g_0 t$$

$$x(t) = V_b \cos \theta t$$

$$h(t) \equiv y(t) = V_b \sin \theta t - \frac{1}{2} g_0 t^2$$

$$h_{\max} = \frac{V_b^2 \sin^2 \theta}{2g_0} = R \tan \theta$$

$$R = \frac{V_b^2 \sin 2\theta}{g_0}$$

$$\text{Peak} \rightarrow V_y = 0 \rightarrow t_p = \frac{V_b \sin \theta}{g_0}$$

$$\text{Flight time} \rightarrow t_f = 2t_p$$

\* Trajectory without drag ( $h_b = 0$ )

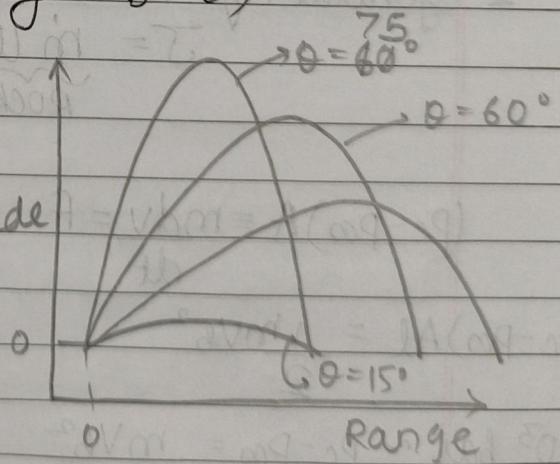
$$\theta_1 < \frac{\pi}{4} \rightarrow \tan \theta_1 < 1$$

$$h_{\max 1} = \tan^2 \theta_1$$

$$h_{\max 2}$$

2 angles for max ht

For same range 'R',  $\theta_1 \approx \frac{\pi}{2} - \theta$



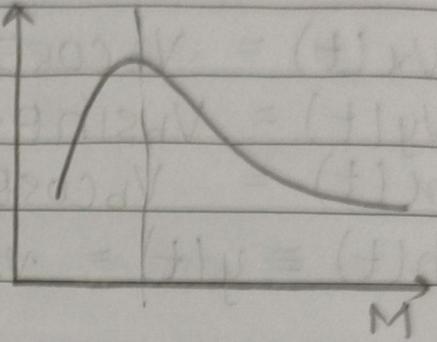
If we consider drag  
 Stokes Drag  $\propto vV$   
 Newtonian drag  $\propto \alpha V^2$

$$D = A C_D \frac{1}{2} \rho V^2 = A C_D \frac{\rho M^2}{2}$$

$$M > 2, D \propto \rho M$$

$$\frac{d\vec{v}}{dt} = -g_0 \hat{e}_y - D \hat{e}_x$$

$\alpha V^2$  or  $\tau V$  (Stokes)  
 (Newton)



STOKES drag:- It decreases & Range increases.

The peak  $\angle$  that gives max altitude has reduced.

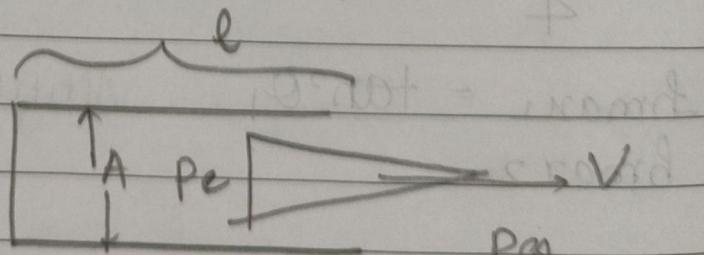
Generating  $V_b$

$$J = \underbrace{m u_e}_{\text{Rocket}} + \underbrace{(p_e - p_a) A_e}_{\text{Projectile}}$$

$$(p_e - p_\infty) A = m \frac{dv}{dt} = F$$

$$AL = \frac{1}{2} m V_b^2$$

$$\sim p_e - p_\infty = \frac{m V_b^2}{2 A L}$$



## \* Ballistic Missiles

Without drag,  $R' = \frac{V_b^2 \sin 2\theta}{g_0}$

$$V_b = \sqrt{\frac{R g_0}{\sin 2\theta}} \quad \text{for } \theta = 45^\circ$$

Intercontinental  
Ballistic missile

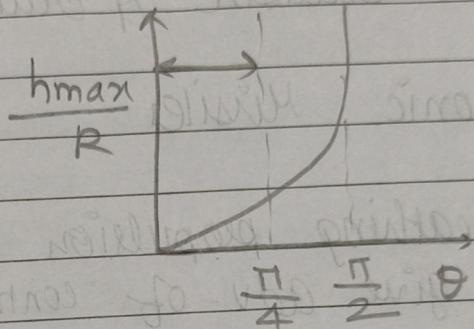
SRBM, MRBM, IRBM, ICBM

$< 100 \text{ km}$        $1 \text{ k} - 3 \text{ k km}$        $3 - 8 \text{ k km}$        $> 5,500 \text{ km}$

$3.13 \text{ km/s}$

• Avoid Drag

$$h_{\max} = \frac{R \tan \theta}{4}$$

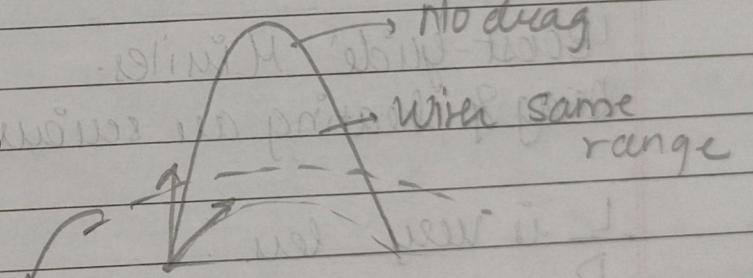


$R = 8000 \text{ km}$

$\theta = \pi/4$

$h_{\max} = 2000 \text{ km}$

$$h_k = y(t) = V_0 \sin \theta_k \frac{t^2}{2} - \frac{1}{2} g t_k^2$$



ht of Karman line

Karman line

$t_p \rightarrow$  time to reach peak altitude

$$\frac{t_{h_k}}{2h_{\max}} \sim \frac{2t_k}{2t_{ps}} < 1$$

\* ICBM Trajectories (Depressed angle, lofted)

atmospheric Re entry  
 empirical correlations for stagnation region heat flux.

$$q^n = C \frac{g_0}{R_c} \frac{V_{\infty}^3}{h_0} (1 - h_w)$$

Blunt bodies have lower heat flux.

Re entry thermal protection sys (TPS)  
 stiff materials are used.

Indian Ballistic & cruise missiles

Hypersonic Missiles

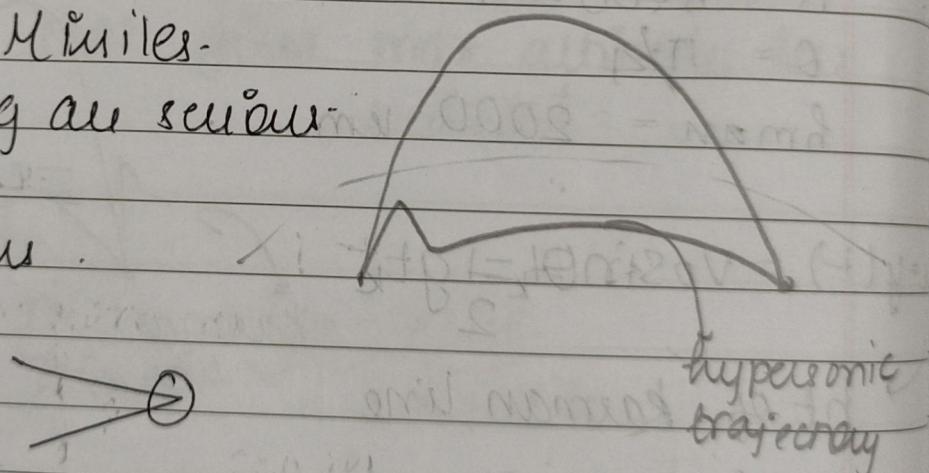
Air-breathing propulsion at similar speeds ( $M \times$ ) would give adv. of control.

'Boost-glide' Missiles.

Drag & heating are serious.

$\frac{L}{D}$  is very less.

Sharp edge



BRAHMOS, at  $M = 2$

$$T_0 = T \left[ 1 + \frac{\gamma - 1}{2} M^2 \right]$$

$T_0 = 8T \sim 1600K$ .  
 Ramjet is used.

# PROJECTILE BASICS FROM SLIDES

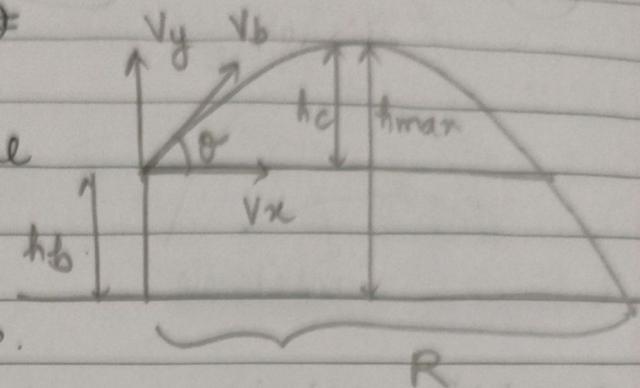
$h_b \rightarrow$  burnout ht  
when drag = 0,  $h_b = 0$

→  $h_b$  can be 0

→ Range of  $0^\circ$  &  $75^\circ$  are same.

If  $h_b > 0$ ,

Range of  $0^\circ >$  of  $75^\circ$ .

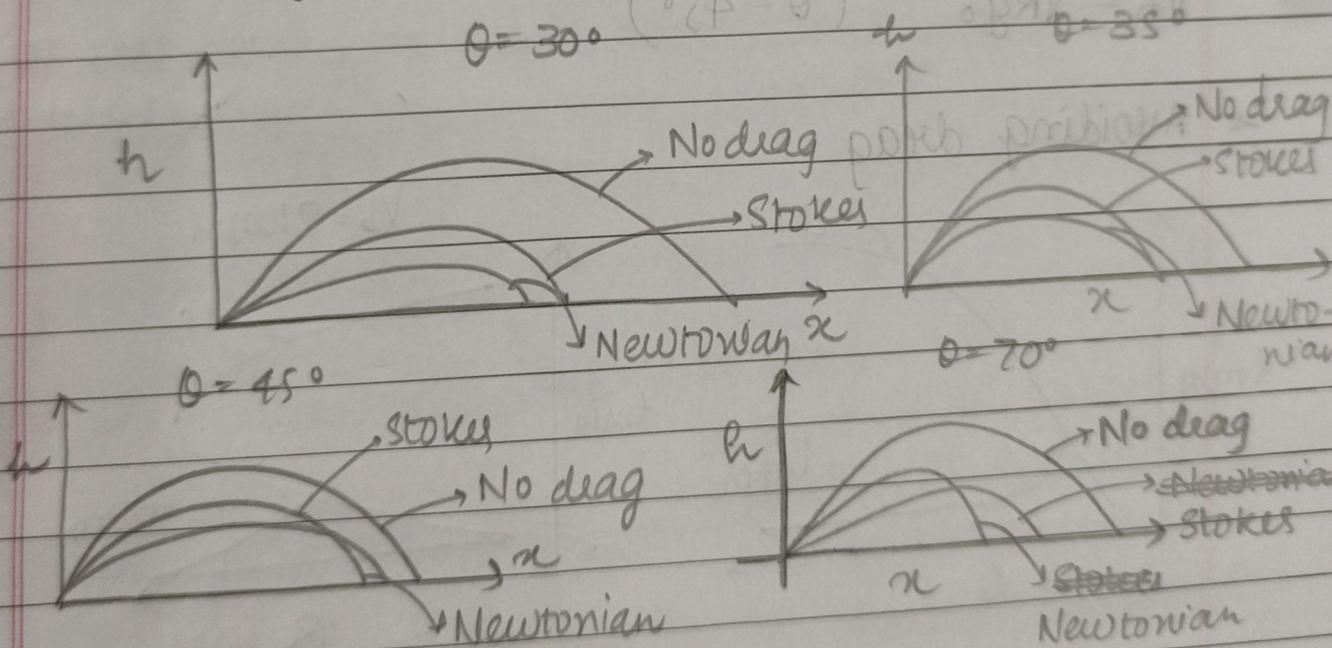


With Drag →  $\rightarrow$  coeff of viscosity

1) Stokes (viscous)  $\propto VV$

2) Newtonian  $\propto V^2$

Both Range & ht reduce (Stokes,  $h_b = 0$ )



\* Generating  $V_b$

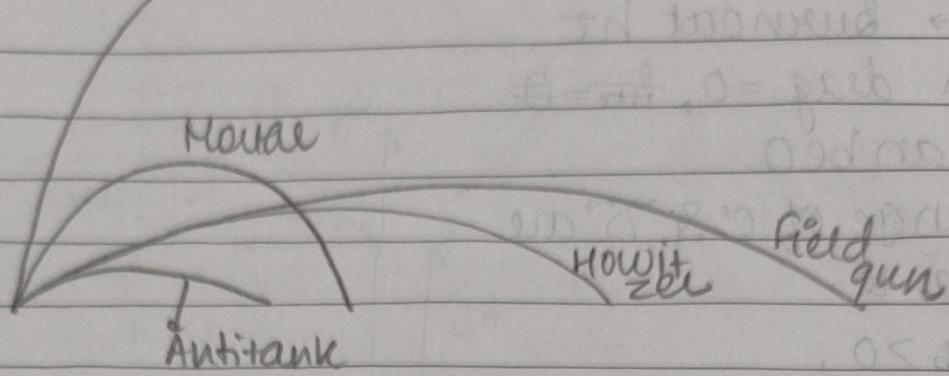
$$J = m u_e + (\rho_e - \rho_a) A e$$

rocket projectile

Two types → gun sys with recoil +  
Recoilless gun sys  
(gases ~~expelled~~ reward.)

## \* Classification of guns

Anti Aircraft

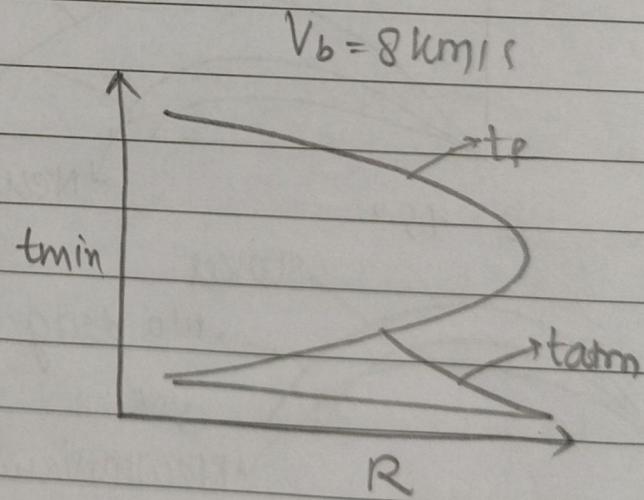
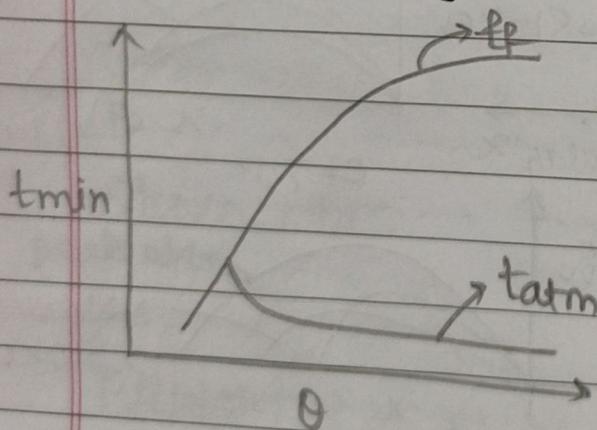


## \* Ballistic Missiles:- $\underline{V_b}$

- 1) SRBM  $3.13 \text{ km/s}$
- 2) MRBM  $3.13 - 5.4$
- 3) IRBM  $5.4 - 7.3$
- 4) ICBM (Intercontinental Ballistic missiles)  $> 7.3$

$$V_b = \sqrt{R g_0} \quad (\theta = 45^\circ)$$

## \* Avoiding drag



# BOOK NOTES

CLASSMATE

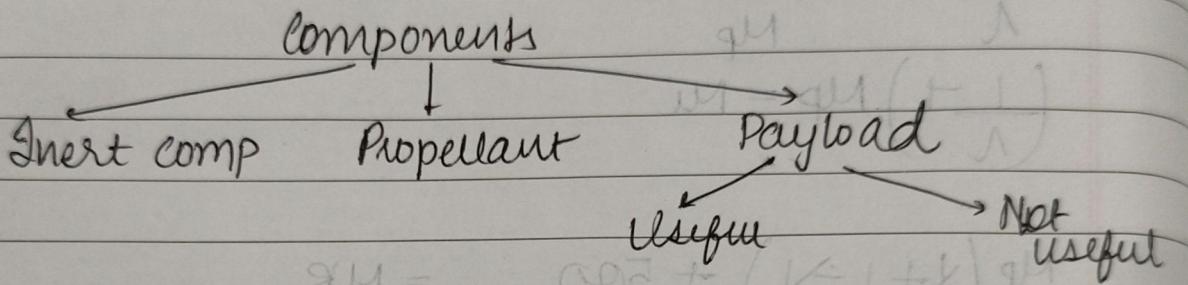
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## CHAPTER-2

### Mission analysis fundamentals

\* Rocket-propellant types:

- 1) Launch vehicles
- 2) Upper stage or orbital transfer vehicles
- 3) Interceptors
- 4) Ballistic missiles



• Propellant mass fraction →

$$\lambda = \frac{m_p}{m_p + m_i}, \text{ as size inc, thick dec to } \lambda \rightarrow 1.$$

Process:-

- 1) Select a propulsion sys
- 2) Define mission eq.
- 3) Prelim design & trade studies
- 4) Detailed design
- 5) Demonstration & Qualification testing

## CHAPTER - 3

$$\Delta V = V_c - V_{surf}$$

$$F_g = mge \left(\frac{r_e}{r}\right)^2 \rightarrow \text{at dist } r \text{ from origin}$$

$$P.E. = - \int \frac{mge r_e^2 dz}{(r_e + z)^2} = \frac{mge r_e z}{(r_e + z)}$$

$$P.E. \text{ on surface} = 0$$

$$K.E. = \frac{mv_z^2}{2}$$

$$v_z = \sqrt{\frac{2g_e r_e z}{(r_e + z)}}$$

$v_z$  → initial vel of mass at surface that will attain altitude  $z$ .

$$\text{If } z \rightarrow \infty, v_\infty = \sqrt{2g_e r_e} = 11.2 \text{ km/s}$$

\* Mission requirements for upper stage or OTV.

Vacuum of space

- 1) Length & size (Package ability)
- 2) Propulsion sys weight.
- 3) Guidance considerations. (Three axis control sys)

Two stage → Interstage in berm

Extendible exit cone (EEC) → skirt to inc size  
(Inertial Upper Stage) (IUS)

Liquid sys → Centaur upper stage (RL10 engine)

- Placing satellite into GEO.

If  $\tau = 1$ ,  $r = 35800 \text{ km}$ .

Launch vehicle transfers spacecrafts.

Initial GTO (Apogee at GEO & perigee at LEO)

Liquid Apogee engines (LAES) → provide final apogee impulse.

- First stage → creates an elliptic orbit with  $A \rightarrow \text{rad}$  of final orbit.

Perigee kick manoeuvre (PKM), apogee kick  $\mu$ .

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V_1 = V_{C1} \left[ \sqrt{\frac{2r_1}{r_1+r_2}} - 1 \right], \Delta V_2 = V_{C1} \left[ \sqrt{\frac{r_1}{r_2}} - \sqrt{\frac{2r_1}{r_2(r_1+r_2)}} \right]$$

$V_{C1}$  → vel in initial circular orbit

If solar resources,  $t_b \gg T$  (NO hohmann)

$$\Delta V = \sqrt{V_{C1}^2 - 2V_{C1}V_{C2}\cos(\pi\alpha) + V_{C2}^2}$$

(No plane change)

\* with Plane change (NOT hohmann)

$$\Delta V = \sqrt{V_{C1}^2 - 2V_{C1}V_{C2}\cos\left(\frac{\pi\alpha}{2}\right) + V_{C2}^2}$$

for Hohmann,  $\Delta V = 2V_C \sin(\alpha/2)$

Better to change plane at higher altitude.

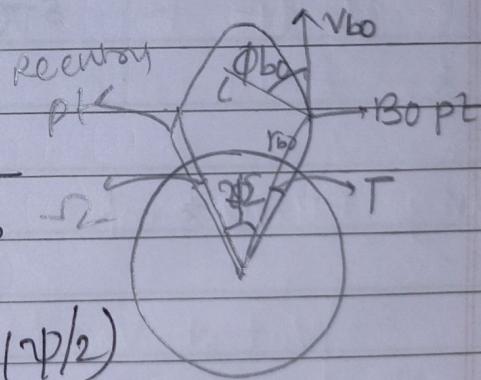
mission requirements for Ballistic missiles.  
 Deliver a nuclear warhead to fixed location.  
 Long range - 3 propulsive stages + post boost vehicle  
 Parameters:-  
 Burning time of each stage  
 Short range (i.e. why deploy SRMs)  
 Inert weights  
 Guidance sys  
 Reentry heating (composite materials)

Zero-lift trajectory  $\rightarrow 0 \angle$  of attack.  
 Elliptical orbit with focal pt at the center.  
 Free flight range  $\angle \psi$ .  
 $r_{\text{fp}} \rightarrow \text{Range}$ .  
 Typical burnout altitudes  $\sim 250-500$  miles  
 Uses gravity to steer vehicle onto desired trajectory. Adv  $\rightarrow$  uses vehicle's own thrust  
 Initial ascent phase  $\rightarrow$  even zero  $\angle$  of attack.  
 Minimizes aerodynamic drag.

$$Q_{bo} = \frac{V_{bo}^2 r_{bo}}{\mu} = \left( \frac{V_{bo}}{V_c} \right)^2$$

$$\cos\left(\frac{\psi}{2}\right) = \frac{1 - Q_{bo} \cos^2 \phi_0}{\sqrt{1 + Q_{bo}(Q_{bo}-2)\cos^2 \phi_{bo}}}$$

$$\sin(2\phi_{bo} + \psi/2) = \frac{2 - Q_{bo} \sin(\psi/2)}{Q_{bo}}$$



$$Q_{bo} = \frac{2 \sin(\psi/2)}{1 + \sin(\psi/2)} \rightarrow \text{Max range}$$

high trajectory &  
 low trajectory  
 solution

$$R_{\text{max}} = \frac{s_{bo}}{2 - Q_{bo}} [1 + \sqrt{1 + Q_{bo}(Q_{bo}-2)\cos^2 \phi_{bo}}] - r_c$$

## \* Interception

defend

Unique rocket designed to close in on a moving target. (kinetic kill vehicle → KKV)

Boosted by two axial propulsion SRMs.

Two pulses, two coast phases.

1-3 stage

Solid or liquid with storable propellants

Liquid → gel propellants

~~considerations~~ →

- 1) Efficient packing (carried on the aircraft)  
↳ low size.
- 2) Target maneuverability

## CHAPTER 3 → Trajectory Analysis & Rocket Design

- \* Chapter 2 →  $\Delta v$  → mission requirements
- Chapter 4 → Isp
- Chap 3 → size & Mass

The Rocket eqn → The Tsiolkovsky eqn.

### Trajectory optimization

- 1) Vertical Trajectory
- 2)  $t_b$  & acc levels
- 3) why staging
- 4) general form of trajectory eqn's

\* Vertical Trajectory  $T = \dot{m} g I_{sp}$

$$-mg - D + \dot{m} g I_{sp} = \frac{mdv}{dt}$$

If the rocket is expelling only propellant gas

$$dv = -g I_{sp} \frac{dm}{m} - \frac{D}{m} dt - g dt$$

$$\Delta v = g e I_{sp} \ln \left( \frac{m_0}{m_f} \right) - \underbrace{\int_0^{t_b} g dt}_{\text{grav}} - \underbrace{\int_0^{t_b} \left( \frac{D}{m} \right) dt}_{\text{drag loss}} \quad (\text{if } I_{sp} \text{ is const})$$

$$\Delta v_{ideal} = g e I_{sp} \ln \left( \frac{m_0}{m_f} \right)$$

$$D = -C_D A \frac{\rho V^2}{2}$$

$A_f$  → frontal area

$$q = \frac{\rho V^2}{2} \rightarrow \text{dynamic press}$$

$$\frac{D \propto L}{m}$$

$$\Delta V_D = \int_{t_0}^{t_B} \left( \frac{D}{m} \right) dt$$

$$= \frac{(D)}{m} \cdot t_B - \frac{(D)}{m} \cdot t_0$$

$$= \frac{(D)(t_B - t_0)}{m}$$

$$= \frac{(D)(\Delta t)}{m}$$

- Steering losses - When thrust is not aligned with vehicle.

$$\Delta V_D = \frac{1}{2} g m^2 \Delta t$$

$$N_{\infty} = \frac{V}{\sqrt{\frac{Y_P(h)}{S(h)}}}$$

$$\Delta V_{\text{ideal}} = \Delta V + \Delta V_g + \Delta V_D + \Delta V_s$$

actual vel gain to be imparted to payload

- Rocket comprises of payload, inert, propellant mass

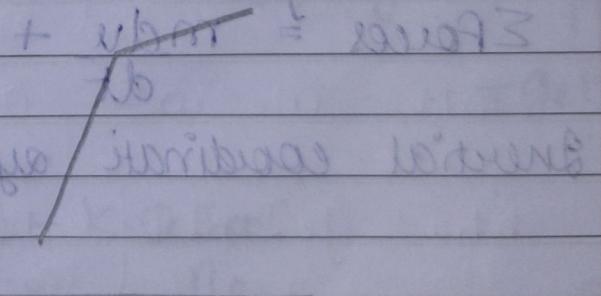
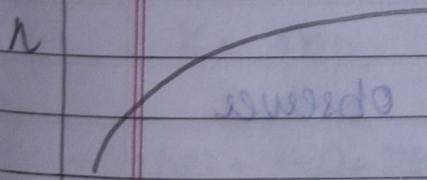
Structural efficiency is determined

$$\text{by } \Lambda = \frac{m_p}{m_p + m_i}$$

as sys gets bigger,  $\Lambda$  increases  
bcoz of cubed-squared law.

We can't make it as thin as it needs to be to retain design loads.

$$(c/v) m_b + v m_t = 20000^3$$



Small motors

- \* Storable propellants  $\rightarrow$  higher  $\lambda$  than cryogenic
- Solid stages  $\rightarrow$  higher  $\lambda$  than liq.

$$MR = \frac{m_0}{m_f} = \frac{m_{pe} + m_p + m_i}{m_{pe} + m_i} = \frac{\Delta m_{pe} + m_p}{\Delta m_{pe} + m_p(1-\lambda)}$$

$$m_p = m_{pe} \frac{MR - 1}{MR - (MR - 1)/\lambda}$$

$$MR = e^{(\Delta v / g_e I_{sp})}$$

avg mass flow rate  $\rightarrow m_p$

$$A_t = \frac{m_p c^*}{\rho_e A_t}, c^* \rightarrow \text{characteristic vel}$$

- Small change in performance i.e.  $\Delta v$  can cause big change in payload.

Any growth in structural weight can lead to huge losses in payload. So multiple stages

- Considering Newton's 2nd Law

$$\sum \text{Forces} = \frac{d}{dt} m \frac{du}{dt} + \frac{dm}{dt} (u)$$

Inertial coordinate sys w.r.t. observer

$I_{sp} = 300$ Example

$m_p = 100 \text{ lb}$

$\Delta V_{id} = 3000 \text{ ft/s}, n = 0.8$

$C^* = 4900 \text{ ft/s}, \gamma = 1.2$

$t_b = 3s, P_c = 1000 \text{ psi}$

$\frac{P_e}{P_c} = 0.0147, P_e = P_a, P_a = 14.7 \text{ psi}$

$\epsilon = g C_f = C_{f, opt} = 1.6$

$I_{sp} = \frac{C_{f, opt} C^*}{g} = 243 \text{ s}$

$\Delta V_{id} = g I_{sp} \ln(MR)$

$\min A_t = m_p \left( \frac{1}{n} - 1 \right), g = 32.$

$\min = m_p \left( \frac{1}{n} - 1 \right), \dot{m} = \frac{m_p}{t_b}, A_e = \epsilon A_t$

\* Burning time & Acc effects :-

$n_o = \frac{F}{m o g_e} > 1$

$t_b = \frac{m_p}{\dot{m}} = \frac{m_p I_{sp}}{F}$

$\text{Trajectory near earth: } \Delta V = g_e I_{sp} \ln \left( \frac{m_0}{m_f} \right)$

$K = \frac{m_p}{m_0}$

$\Delta V = -g_e I_{sp} \ln \left( \frac{1-K}{K} \right) + K \cdot g_e I_{sp} \frac{n_o}{n_0} = \text{odd}$

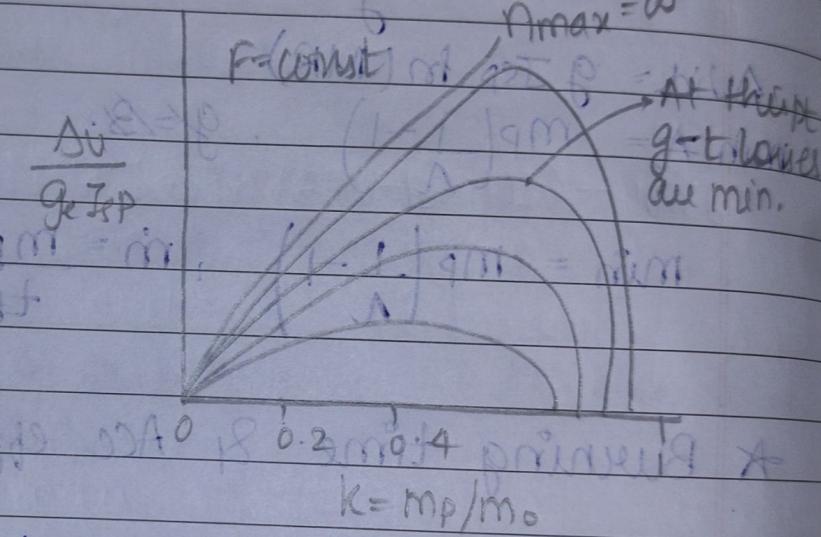
Peak acc occurs  
at burnout

$$m_f \eta_{\max} g_e = F$$

$$\begin{aligned} m_f &= m_0 - m_p \\ &= m_0(1-k) \end{aligned}$$

$$\eta_{\max} = \frac{\eta_0}{1-k}$$

$$\Delta V = -g_e I_s p \ln(1-k) - \frac{k g_e I_s p}{(1-k) \eta_{\max}}$$



### \* Vertical ht calculations

Launch from earth's surface,  $v_0 = h_0 = 0$

$$h(t) = -g_e I_s p \ln \left( \frac{m(t)}{m_0} \right) - \frac{g_e I_s p t^2}{2}$$

$$m(t) = m_0 - m \quad 0 < t < t_b$$

$$h_{bo} = g_e I_s p t_b \left[ \frac{m_f}{m_0 - m_f} \ln \left( \frac{m_f}{m_0} \right) + 1 \right] - \frac{g_e I_s p t_b^2}{2}$$

$$\text{for } h_{\text{bom}} = \frac{1}{2} g e t_b^2 \text{ (max)}$$

consider fixed payload & inert mass

$$K(t_b) = \frac{m_p}{m_p + m_i + m_{pl}} = \frac{m_{tb}}{m_{tb} + m_i + m_{pl}}$$

For taking  $K$  as varying,

$$h_{\text{bom}} = g e I_s p^2 \left[ (1-K) \ln(1-K) + K - \frac{K^2}{2} \right]$$

$$\text{Burnout vel} = V_{bo} = g e I_s p \left[ \ln \left( \frac{1}{1-K} \right) - \frac{1}{K} \right]$$

Additional ht during coining

K.E at burnout = P.E. at apogee.

$$h_c = \frac{1}{2} g e I_s p^2 \left[ \ln(1-K) + \frac{K-1}{K} \right]^2$$

\* Multistage systems (N stages rocket),  $n \leq N$

- $m_{fn}$  = burnout mass of stage  $n$
- $m_{on}$  = initial mass of stage  $n$
- $\epsilon_n$  = stage structural factor  $m_{fn}/m_{on}$
- $\beta_n$  = payload factor =  $\frac{\sum_{i=1}^n m_{on}}{\sum_{i=1}^n m_{on}}$

$$\epsilon_N = \frac{m_{RN} - m_{pl}}{m_{on} - m_{pl}}, \quad \beta_N = \frac{m_{pl}}{m_{on}}$$

Small  $\epsilon$  values.

$$V_{boN} = \sum_{j=1}^N \Delta v_j$$

$$\Delta V_j = g_e I_{spj} \ln(MR_j) - \bar{g}_j t_b$$

$$\bar{g}_j = \int_0^{t_b} g dt / t_b, MR_j \rightarrow \text{stage mass ratio}$$

$$MR_j = \frac{\sum_{i=j}^N moi}{\sum_{i=j+1}^N moi + m_f}$$

$$MR_j = [\epsilon_j(1-\beta_j) + \beta_j]^{-1}$$

$$\Delta V_j = -g_e I_{spj} \ln(\epsilon_j(1-\beta_j) + \beta_j) - \bar{g}_j t_b$$

Special case:-

$$V_{b0N} = -N g_e I_{sp} \ln(\epsilon(1-\beta) + \beta) - g_e \sum_i$$

$$T_b = N t_b \rightarrow \text{Total power on time}$$

$$\Delta V_j = \frac{V_{b0N}}{N}$$

$$G = \frac{\text{lift off mass}}{\text{pay load mass}} = \frac{\sum_{i=1}^N moi}{m_p}$$

$$S = \frac{V_{b0N} + g_e \sum_i}{g_e I_{sp}} = -N \ln(\epsilon(1-\beta) + \beta)$$

$$G = \prod_{j=1}^N \frac{1}{B_j} = \beta^{-N}$$

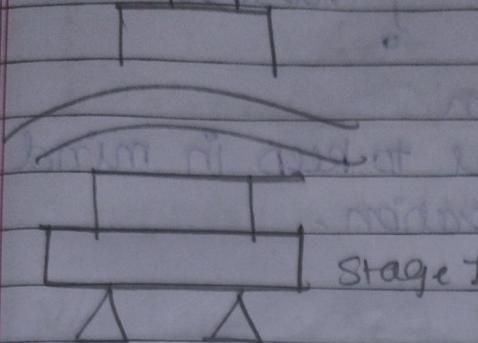
$$G = [(1-\epsilon) / (e^{-S/N} - \epsilon)]^N$$

$$\lim_{N \rightarrow \infty} \ln G = e^{S(1-\epsilon)}$$

$$\sum_{i=1}^N = N \ln G$$

$m_{PL} = m_{ON} B_N$

Stage N



then  $N=1$

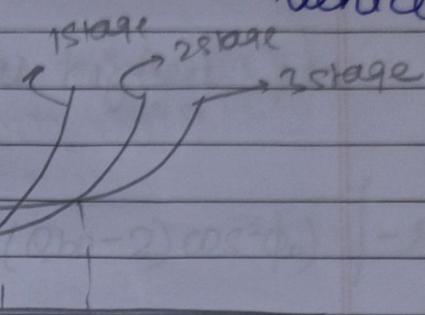
$N=2 \quad \epsilon=0.1$

$J_{sp} = 300$

$V_{bon}$

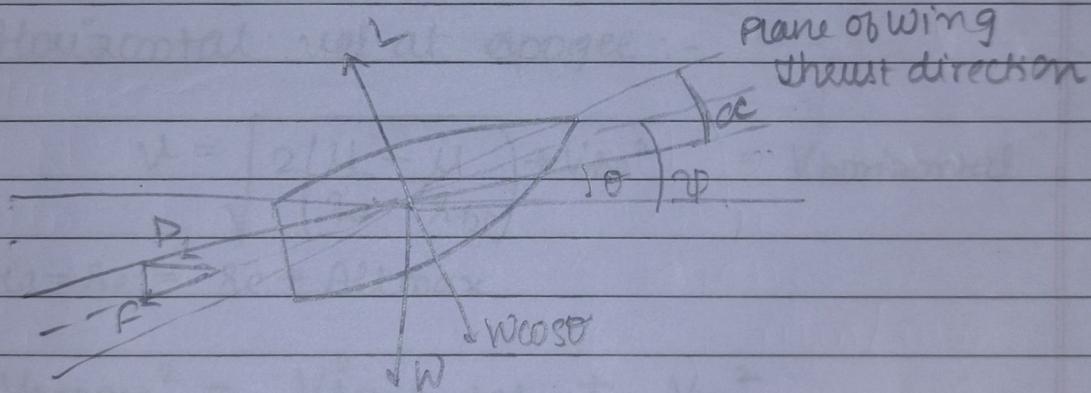
law of diminishing results for multistage vehicles.

GLOW



$\Delta V$

## \* Generalized Trajectories



$$\frac{mdv}{dt} = F \cos(\psi - \theta) - C_D \frac{\rho v^2 A}{2} - mg \sin \theta \quad (\text{horizontal})$$

$$\frac{v^2}{R} = \frac{vd\theta}{dt}$$

$$\frac{mv d\theta}{dt} = F \sin(\psi - \theta) + C_L \frac{\rho v^2 A}{2} - mg \cos \theta \quad (\text{vertical})$$

Re not included in  $C_L$  &  $C_D$  as turbulent range

SOME FORMULAE WHILE SOLVING CHAP 2 QUES

$$1) \int \frac{1}{ax^2 + bx + c} dx = \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \left( \frac{2ax + b}{\sqrt{4ac - b^2}} \right)$$

2)  $V_{surf} = r_e \times w$  (Vel at which missile is launched from surface)

$$V_c = \sqrt{\frac{\mu_c}{r_c}}, r_c \rightarrow \text{The final dist from center.}$$

$$3) Q_{bo} = \left( \frac{V_{bo}}{V_c} \right)^2 \quad (V_c \rightarrow \text{Vel at highest pt})$$

$$4) Altmax = \frac{r_{bo}}{2 - Q_{bo}} \left[ 1 + \sqrt{1 + Q_{bo} (Q_{bo} - 2) \cos^2 \phi_0} \right] - r_e$$

$r_{bo} \rightarrow$  burnout altitude

5) Horizontal vel at apogee :-

$$v = \sqrt{\left( \frac{\mu}{r_a} - \frac{\mu}{r_{bo}} \right) + V_{bo}^2} = V_{horizontal}$$

$$r_a = r_c = r_e + Altmax$$

$$6) V_{apogee}^2 = V_{horizontal}^2 + V_{bo}^2$$

7) When boost phase ends, it has burnout velocity

$$8) \frac{dV_r}{dt} = -\frac{1}{2m_e} C_d A_s \rho_g V_r^2$$

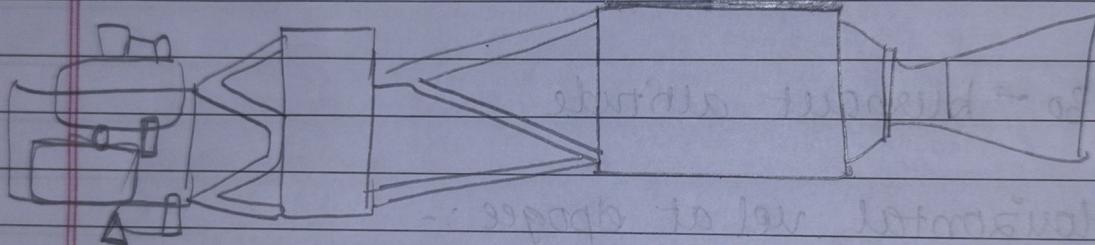
g) Range =  $r_0 \sin \phi$

$$\cos(\phi/2) = \frac{1 - Q_{bo} \cos^2 \phi_{bo}}{\sqrt{1 + Q_{bo} (\phi_{bo} - 2) \cos^2 \phi_{bo}}}$$

$$Q_{bo} = \frac{V_{bo}^2 r_{bo}}{\mu}$$

10) For max range,  $Q_{bo} = \frac{2 \sin(\phi/2)}{1 + \sin(\phi/2)}$

11)  $\left( \frac{\text{Dist from center}}{\text{Time period}} \right)^2 = \frac{\mu}{4\pi^2}$



$$mmf_{airgap} = \frac{BoeI_a}{(R + R_s)}$$

$$mmf_{airgap} + mmf_{core} = BoeI_a$$

$$mmf_{airgap} + mmf_{core} = BoeI_a$$

$$mmf_{airgap} = BoeI_a$$

Confusion about  $c^*$  &  $C_f$

$$\Delta V = \int_0^t dv$$

$$\Delta V = - \int \frac{m_{eq} dM}{M} - g_0 t - \int \frac{D dt}{m}$$

$$\Delta V = \Delta V_{id} - \Delta V_{g} - \Delta V_D$$

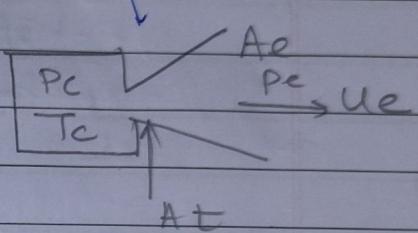
$$J = \dot{m} u_{eq} = \dot{m} u_e + (P_e - P_a) A_e$$

$$J = \dot{m} c^* C_f -$$

$$\dot{m} c^* = \underset{\sim}{P_e A_t} \quad \text{or} \quad c^* = \underset{m}{\cancel{P_e A_t}}$$

$$u_{eq} = c^* C_f$$

$$\dot{m} = f(P_c, T_c, \lambda, A_t)$$



$$\frac{dV}{dt} = \dot{m} u_{eq}$$

$$\text{const } \dot{m} = \frac{M_p}{t_b}, \quad t_b \rightarrow \text{burn time}$$

$$c^* = \frac{P_e A_t}{\dot{m}}$$

## CHAPTER-1

### Classification of Rocket propulsion systems & historical perspectives.

- \* Fathers of Modern Rocketry:-
  - 1) Konstantin Tsiolkovsky
  - 2) Herman Oberth
  - 3) Dr Robert Goddard

- \* V-2 Rocket (Germans used)
  - ↳ LOX & alcohol as propellants
  - Dr von Braun (Apollo)

- \* Intercontinental ballistic missiles (ICBMs) (Russia & America in cold war).

Atlas / Titan / Thor or Delta → US unmanned launch vehicle

Jet assisted take off (JATO)

October 4, 1957 (SS-6 ICBM launched (Sputnik) into orbit) → Russia

NASA in 1958 (Saturn V → Neil Armstrong on Moon on 20 July, 1969)

- \* Ratio of payload mass to GLOW - less than 2% very large devices to place smaller devices in orbit.

$$\text{Ratio of payload mass to GLOW} = \frac{\text{mass of payload}}{\text{mass of rocket + payload}}$$

\* Classification of Rocket Prop sys.

Specific Impulse Isp.  
 $Isp = \frac{F}{m}$  (Propellant efficiency)

Proper measure should be N-s/kg but it's

only s.

$$\text{Total Impulse} = \int_0^{t_b} F dt = F t_b .$$

$$I = Isp \int_0^{t_b} m dt = Isp m_p$$

→ propellant mass fraction

$$\Lambda = \frac{m_p}{m_0} = \frac{\text{Initial Imp}}{\text{Initial Imp} + \text{mass of empty rocket}}$$

$m_0 \rightarrow$  overall mass of loaded prop sys.

mass of prop mass + rocket body mass

high Isp  $\Rightarrow$  high flame temp, high comb pres.

$\rightarrow$  thicker chamber  $\rightarrow$  lower mass fraction.

### ★ Solid Rocket Motor (SRM)

Ignition

→ burning of binder + solid fuel

→ sust

→ generation

→ main propellant

→ propellant

High mass fraction of SRM

$\rightarrow$  attractive for first stage thrust.

stage thrust. Inexpensive propellant nozzle quenching

is required for low Jc

burnout time

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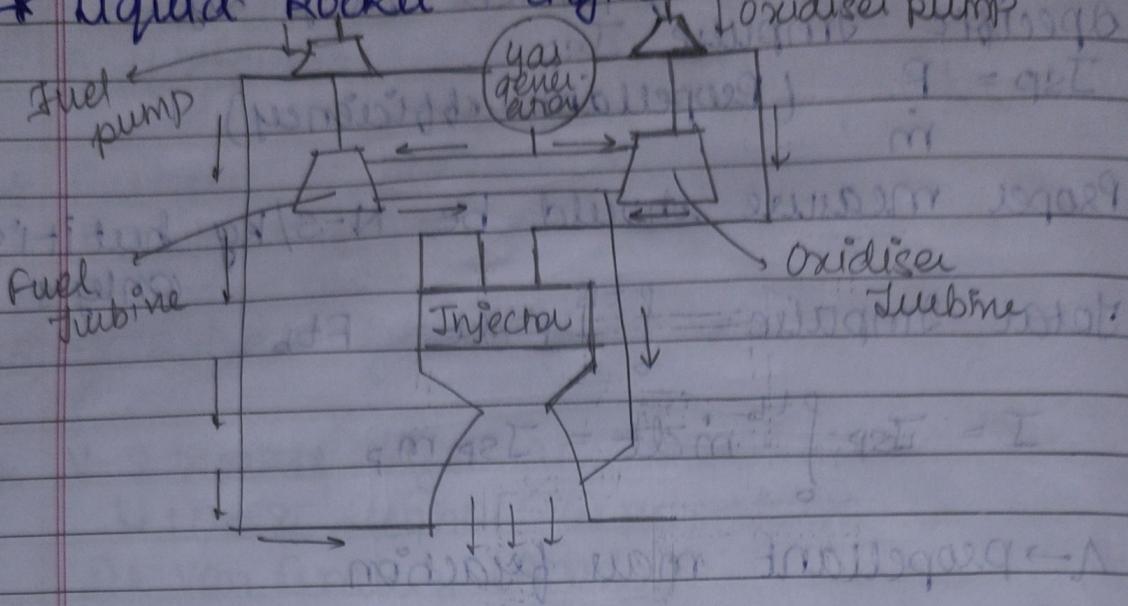
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 very large devices to place smaller devices  
 in orbit.

$$\text{thrust} = \rho A v_e (F_d - D) = \rho A v_e (F_d - D - m)$$

## \* Liquid Rocket engine



engine power cycle → method used to obtain energy to drive turbine

Pressure fed → contains inert gas to force propellants into chamber

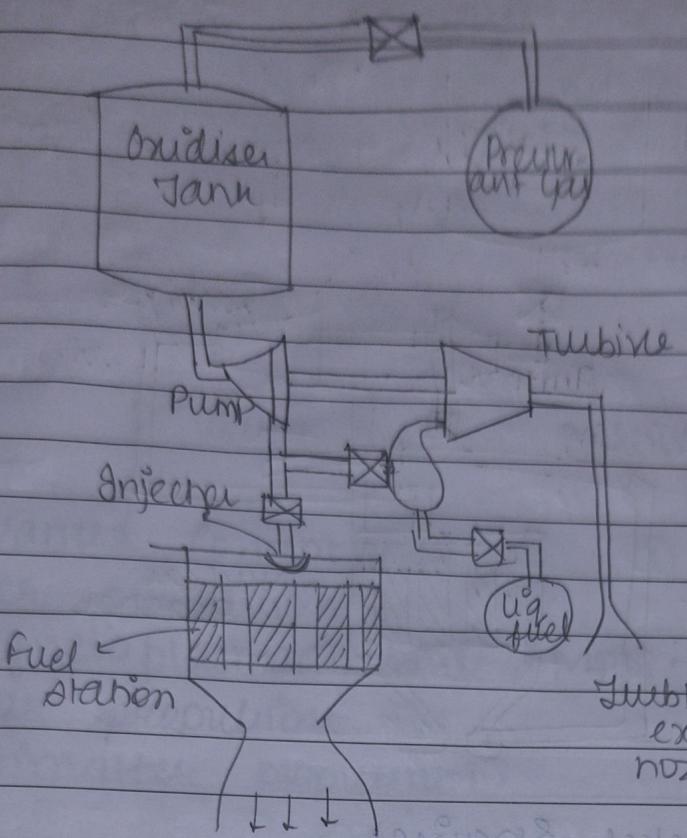
1 is lower.

## \* Hybrid Rocket Engines.

Oxidiser & fuel stored separately → simpler than bipropellant liq sys.

Both performance & propellant mass fraction lie b/w SRB & LRE

Relative amounts of fuel & oxidiser are not controlled.



### \* Nuclear rocket engines

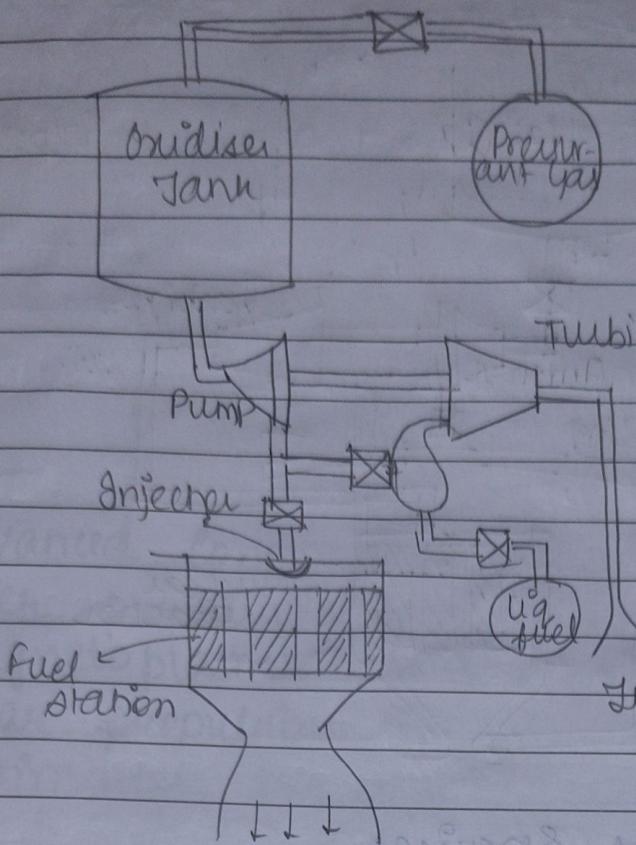
- Nuclear thermal rocket  
Energy from nuclear process is used to heat a working fluid to high temp & hot gases then expanded through a de Laval Nozzle.

### \* Nuclear electric rocket

Utilises electricity to generate thrust

→ high Isp (Interplanetary missions) (low n)

NERVA (Nuclear engine for Rocket vehicle app)



## \* Nuclear rocket engines

- Nuclear thermal rocket

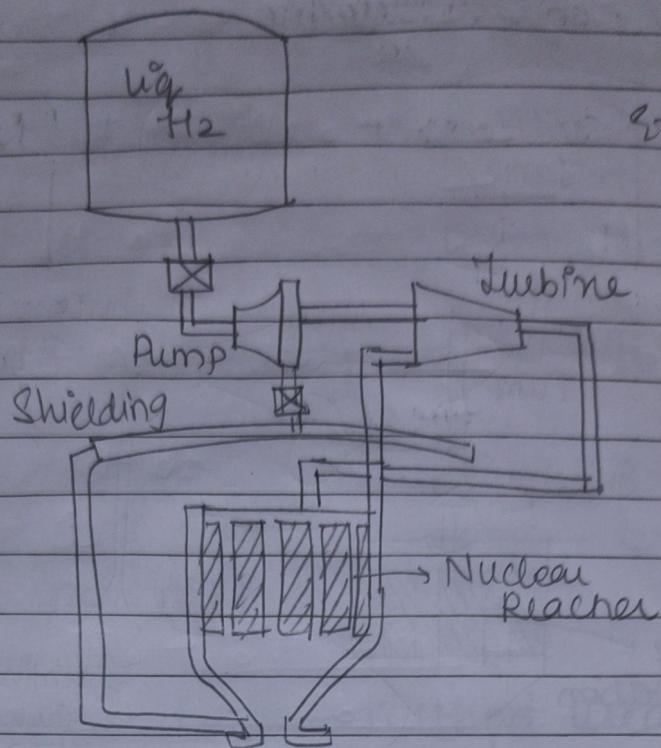
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Utilises electricity to generate thrust.

high Isp (Interplanetary missions) (low n)

NERVA (Nuclear engine for Rocket vehicle app)



## \* Electric Rocket Engines

Arcjet / plasma thermal thruster

Electric pot is used b/w electrodes

Isp ~ 400 - 1500 s.

Fluid passing through arc is superheated

$H_2, NH_3, He \rightarrow$  used successfully

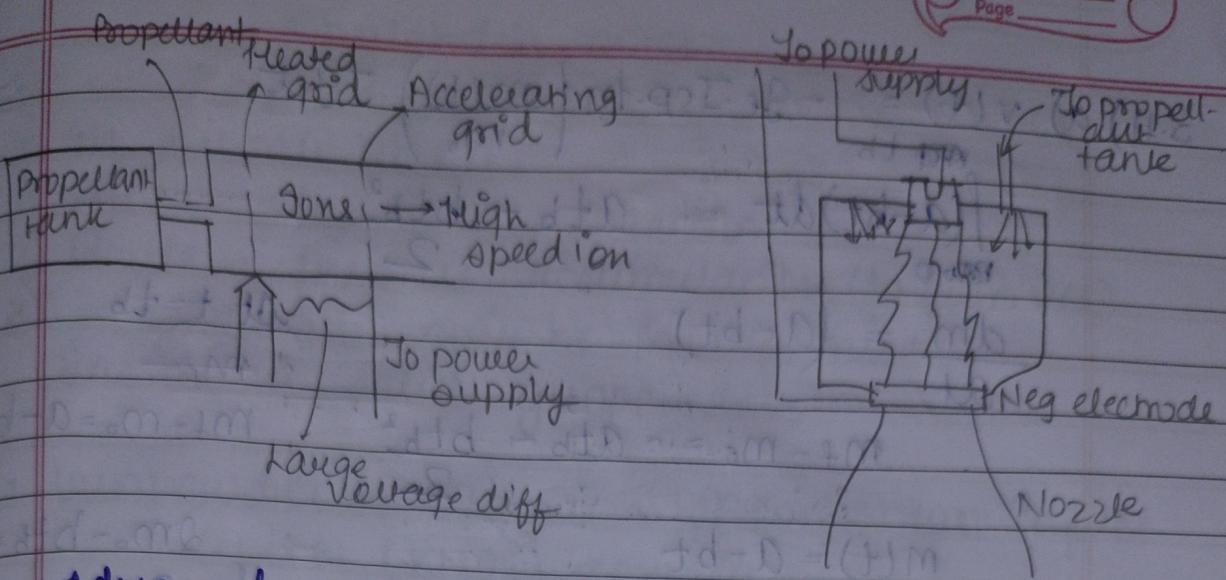
## Electrostatic / Ion thrusters :-

Heavy atoms ( $Xe$ ) are ionised via electric heating coils, +vely charged ions accelerated using electric pot. to high veloc.

Isp ~ 10000 s

Large amt of electric power

Satellite maneuvering



## \* Advanced concepts

- 1) Laser propulsion
- 2) Magneto plasmadynamics (MPD) (fleutere)
- 3) Solar propulsion sys.
- 4) Antimatter propulsion

$$N = \frac{f(4m - m_e)}{dt \cdot m}$$

$$\frac{4m - m_e}{dt \cdot m} = \frac{ab}{dt \cdot m}$$

$$ab \cdot dt \cdot m = ab \quad ab$$

$$4m - m_e$$

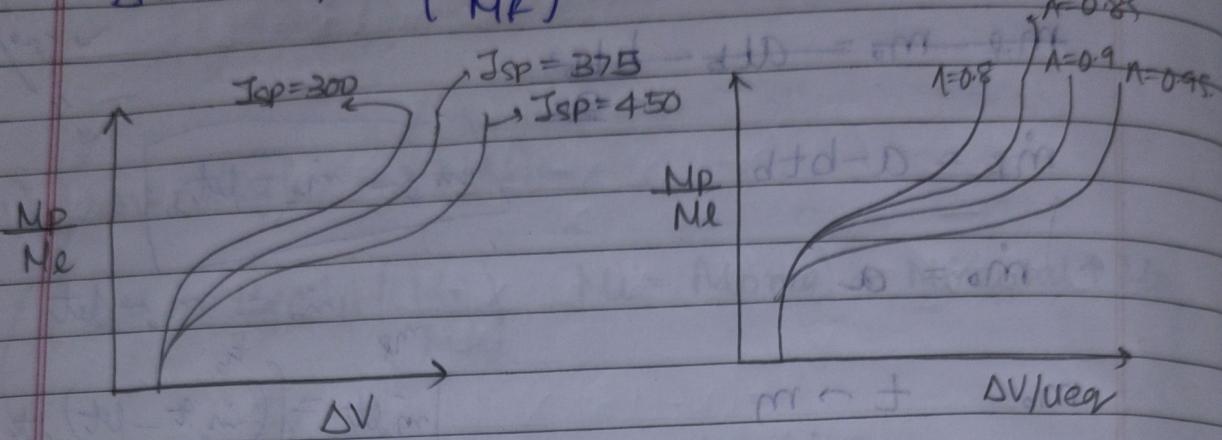
$$) + x - x \cancel{x} = x \cancel{b} \cancel{x} )$$

$$) + x - x \cancel{x}$$

## Multi stage Rocket notes of sir

(10/11.8)

- Rocket eqn for single stage
 
$$\Delta V = \lambda u_{eq} \ln \left( \frac{M_0}{M_f} \right) = u_{eq} \ln (MR)$$



- continuous ejection

$$\dot{m} = \dot{m}_p + \dot{m}_i = \frac{\dot{m}_p}{\lambda}$$

$$\frac{dV}{dt} = \dot{m}_p u_{eq} = (\dot{m}_p + \dot{m}_i) \lambda u_{eq} = - \frac{dM}{dt} \lambda u_{eq}$$

$$\Delta V_\infty = \lambda u_{eq} \ln \left( \frac{M_0}{M_e} \right)$$

$$(\text{Mass Ratio})_\infty \equiv \frac{M_0}{M_e} = \exp \left( \frac{\Delta V}{\lambda u_{eq}} \right)$$

$$1 + \frac{M_p + M_i}{M_e} \equiv 1 + \frac{M_p}{\lambda M_e} = MR_\infty$$

$$\frac{M_p}{M_e} = \lambda \left( \exp \left( \frac{\Delta V}{\lambda u_{eq}} \right) - 1 \right), \frac{M_i}{M_e} = (1 - \lambda) \left( \exp \left( \frac{\Delta V}{\lambda u_{eq}} \right) - 1 \right)$$

### \* Multi-stage Rocket

$$\text{Stage mass } m_{on} = m_{pn} + m_{in}$$

$$\text{Structural factor : } \epsilon_n = \frac{m_{fn}}{m_{on}} = \frac{m_{in}}{m_{in} + m_{pn}}$$

$$\text{Propellant fraction } \lambda_n = \frac{m_{pn}}{m_{in} + m_{pn}} = 1 - \epsilon_n$$

$$\cdot M_{\text{on}} = \sum_{j=n}^N M_{\text{obj}}$$

$$B_n = \frac{M_{\text{on}}}{M_{\text{on}}}, M_{\text{on}} = m_{\text{un}} + M_{\text{in}}$$

$$M_{\text{in}} = \sum_{n+1}^N M_{\text{obj}}$$

$$\Delta V_n = \text{Ueqn ln} \left( \frac{M_{\text{on}}}{M_{\text{fn}}} \right)$$

For a given  $\frac{M_e}{M_0}$ , when is  $\Delta V$  max?

\* Staging -  $(1 + (x-1) \ln \frac{x}{N})^{\frac{1}{N}}$

Obj function

$$f(U_{\text{eqn}}, \epsilon_n, B_n) = \Delta V = \sum_{n=1}^N \Delta V_n$$

$$= - \sum_{n=1}^N \text{Ueqn ln} (\epsilon_n + (1-B_n) B_n)$$

constraint func

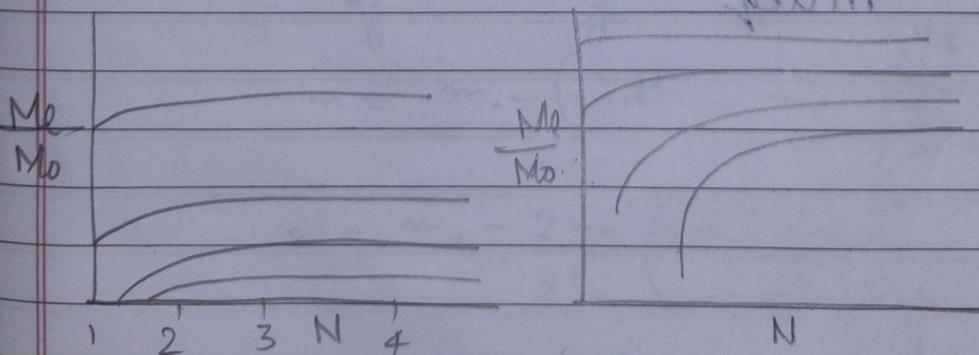
$$\frac{M_e}{M_0} = \frac{M_{e1}}{M_0} \cdot \frac{M_{e2}}{M_0} \cdot \frac{M_{e3}}{M_0} \cdots \frac{M_{eN}}{M_0} = \prod_{n=1}^N B_n$$

$$g(B_n) = \ln \left( \frac{M_e}{M_0} \right) = \sum_{n=1}^N \ln B_n$$

One constraint

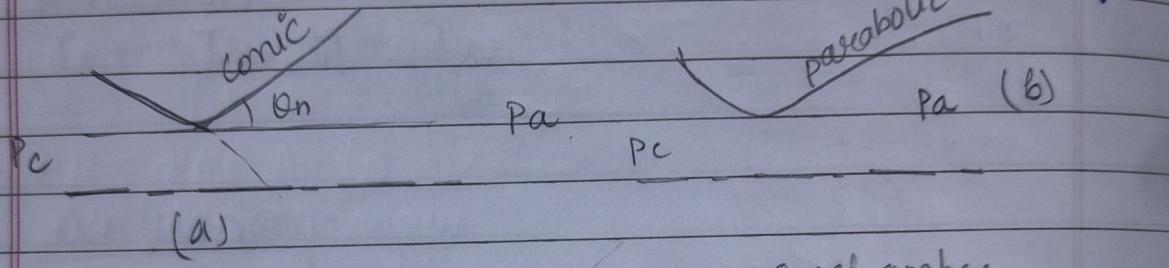
$$\frac{M_e}{M_0} = \prod_{n=1}^N B_n = B^N \Rightarrow B = \left( \frac{M_e}{M_0} \right)^{1/N}$$

$$\Delta V_{\text{opt}} = - N \text{Ueqn ln} \left( \epsilon + (1-\epsilon) \left( \frac{M_e}{M_0} \right)^{1/N} \right)$$

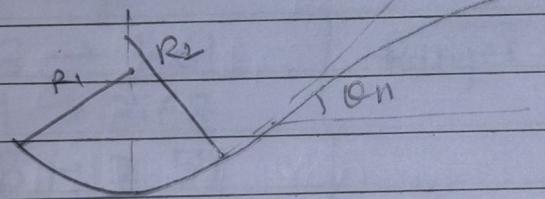


## Rocket Nozzle performance - Class 2

- \* Multiphase flow
  - ↳ particles / drops need not follow 'streamlines'.
  - ↳ a 'minor effect' for flow modeling
  - ↳ 'quasi 1-D' flow  $A = A(x)$
- \* Validity of 'quasi 1-D'
  - ↳ no 2D effects
  - ↳ oblique shock
  - ↳ expansion fan
  - ↳ parabolic

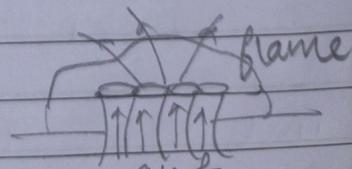
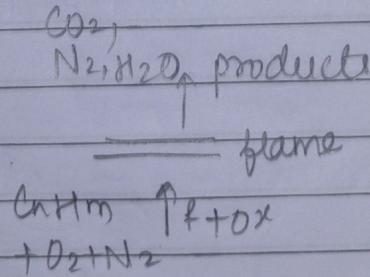


c → chamber  
a → ambient



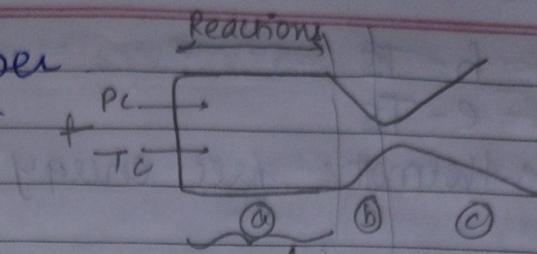
characteristics  
waves  
direction of info propagation

- \* Constant properties (thermodynamic & transport)



(only fast reactions)

- (a) Combustion chamber  
 (b) Chamber - throat  
 (c) Throat - exit



fast-slow (relative)

$$t_c = L/V$$

convective (or flow) time scale

'residence time'

$$t_{ch} \equiv t_r = \frac{L}{W_f} \rightarrow \text{fuel cone}$$

per length ('each')

per sec per unit vol ("")

Damköhler No.

$$Da = \frac{t_c}{t_r}$$

equilibrium flow  
 (chemical, composition, shifting)

Infinitely fast reaction,  $t_r \rightarrow 0$ ,  $Da \rightarrow \infty$

Infinitely slow reaction,  $t_r \rightarrow \infty$ ,  $Da \rightarrow 0$   
 ↳ chemically frozen flow

frozen composition flow / frozen flow.

$I_{sp}$  ↳ equilibrium  
 ↳ frozen

Finite Da, Finite rate chemistry

$I_{sp}$  depends on Da

$$h_t = \sum_{j=1}^N \left( h_f + \int_{T_{ref}}^T \dot{C}_p j dT \right)$$

combinations

↳ STANJAN, CEA, Chemkin, Cantera

↳ finite rate chemistry

# Helmholtz free energy

Nozzle theory assumptions

- Steady, quasi 1D
- Homogeneous perfect gas
- Isoentropic - no reaction, no heat transfer, no friction

Perfect gas

$$P = \rho R T$$

$$C_P = \frac{\gamma}{\gamma-1} R T, C_V = \frac{1}{\gamma-1} R T$$

$$R = \frac{R_u}{M_w}$$

Dalton partial pressures,  $P = \sum p_j$

$$p_j = P \rho_j R_j T$$

$$P = \sum \rho_j R_j T$$

Thermal equilibrium

$$T = T_t = T_r = T_v$$

$$(T_{v,0} + \rho d) \leq \rho d$$

# Rocket Nozzles → class 3

$u \rightarrow$  speed

$v \rightarrow$  specific volume

$$v = 1/\rho$$

$$A_c = \pi r_c^2, A_t = \pi r_t^2$$

$$A_e = \pi r_e^2$$

ASSUMPTIONS:

- 1) Quasi - 1D, steady
- 2) Homogeneous perfect gas
- 3) Nozzle flow is isentropic (no friction, heat transfer)
- 4) No chemical reactions in nozzle → frozen flow

$P_c, T_c \rightarrow$  stagnation properties for nozzle flow

total properties

$$\dot{m} = \rho u A_e + (P_e - P_a) A_e \quad (\text{express in terms of stagnation properties})$$

$$\dot{m} = \rho A u = \frac{\rho}{RT} A M \sqrt{YRT}$$

$$= \frac{P_c}{RT_c} \left[ 1 + \frac{Y-1}{2} M^2 \right]^{\frac{Y}{Y-1} + 1 - \frac{1}{2}} A M \sqrt{YRT}$$

any area of CS in nozzle

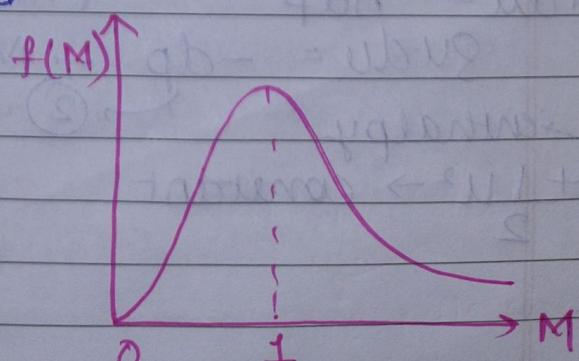
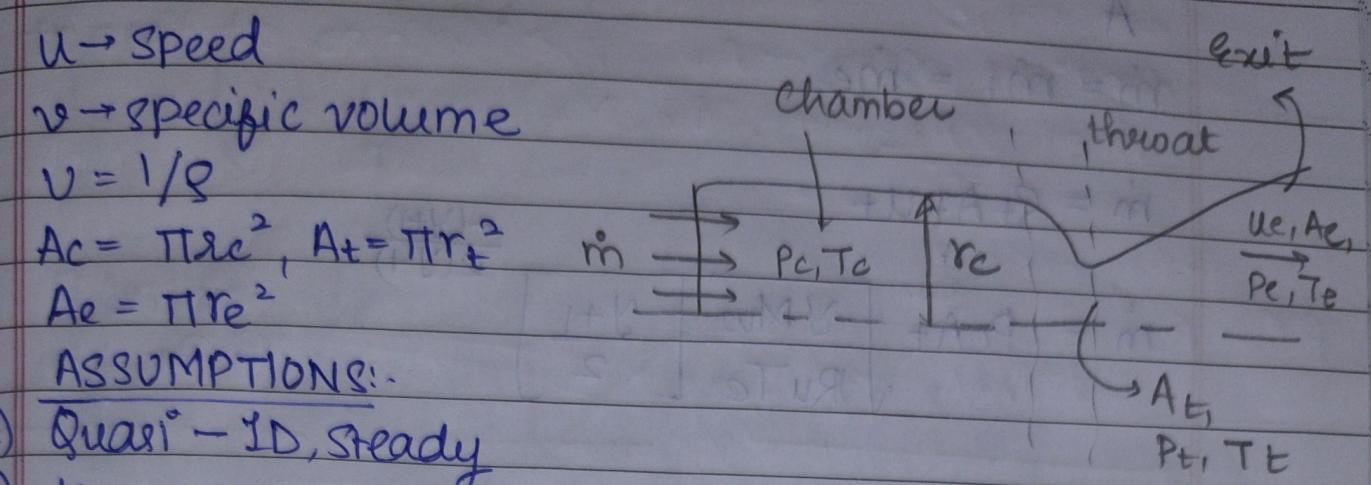
$$\dot{m} = \frac{P_c}{\sqrt{T_c}} A_e f(Y, M_w, M)$$

$$f(Y, M_w, M) = \sqrt{\frac{Y M_w}{R_u}} \left[ 1 + \frac{Y-1}{2} M^2 \right]^{\frac{1}{2(Y-1)}}$$

$M_w \rightarrow$  molecular weight

$$\frac{\dot{m}}{A} = \rho u \Rightarrow \text{Max for } M = 1$$

$$\dot{m} = \rho A_t u_t$$



$$\frac{m}{A} = \rho u \rightarrow \text{Max for } M=1$$

$$\dot{m} = \dot{m}_t = \dot{m}_e$$

$$\dot{m} = \dot{S} + A_t u_t$$

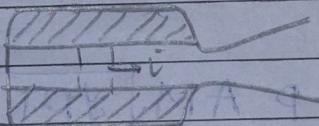
$$\dot{m} = \rho_c A_t \sqrt{\frac{\gamma M_w}{R_u T_c}} \left[ \frac{\gamma+1}{2} \right]^{-\frac{1}{\gamma-1}}$$

\* Why  $M=1$  at exit?

$$\dot{m} = \rho A u = (p + dp)(A + dA)(u + du)$$

$$0 = \rho A du + \rho u dA + A du p + \rho c u dA + u dp dA + A dp du + dp dA du$$

$$\frac{dp}{P}, \frac{dA}{A}, \frac{du}{u}, \frac{dp}{P} \ll 1$$



$$\frac{dp}{P} + \frac{du}{u} + \frac{dA}{A} = 0 \quad \text{--- (1)}$$

$$\dot{m}(u + du) = \dot{m}u + \rho A - (p + dp)(A + dA) + \left( \frac{p + dp}{2} \right) dA$$

$$\dot{m} du = \frac{dp}{2} dA - dp(A + dA)$$

$$\dot{m} du = -Adp \quad (\dot{m} = \rho A u)$$

$$\rho u du = -dp \rightarrow \text{Momentum conservation}$$

$$\underbrace{h + \frac{1}{2} u^2}_{\text{enthalpy}} \rightarrow \text{constant}$$

$$C_p dT = dh = -udu$$

$$C_p dT = -udu \quad \text{--- (3)}$$

$$P = \rho R T$$

$$\frac{dp}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

$$S C_p dT = dp$$

$$\frac{dA}{A} = - \left[ \frac{dp}{S} + \frac{du}{u} \right] = - \left[ \frac{dp}{P} - \frac{dT}{T} + \frac{du}{u} \right]$$

$$= - \left[ - \frac{\gamma u^2}{P} \frac{du}{u} + \frac{u^2}{C_p T} \frac{du}{u} + \frac{du}{u} \right]$$

$$\frac{dA}{A} = [M^2 - 1] \frac{du}{u} = \left[ \frac{M^2 - 1}{1 + \frac{\gamma - 1}{2} M^2} \right] \frac{dM}{M}$$

$$- \left[ \frac{\gamma u^2}{P} + \frac{u^2}{C_p T} + 1 \right] \frac{du}{u}$$

$$\frac{\gamma u^2}{\gamma P} = M^2 \quad \hookrightarrow (\gamma - 1) M^2$$

$$- \left[ -y M^2 + (y-1) M^2 + 1 \right] = - [1 - M^2]$$

$$M < 1 \rightarrow dA > 0 \leftrightarrow dM < 0$$

$$dA < 0 \leftrightarrow dM > 0$$

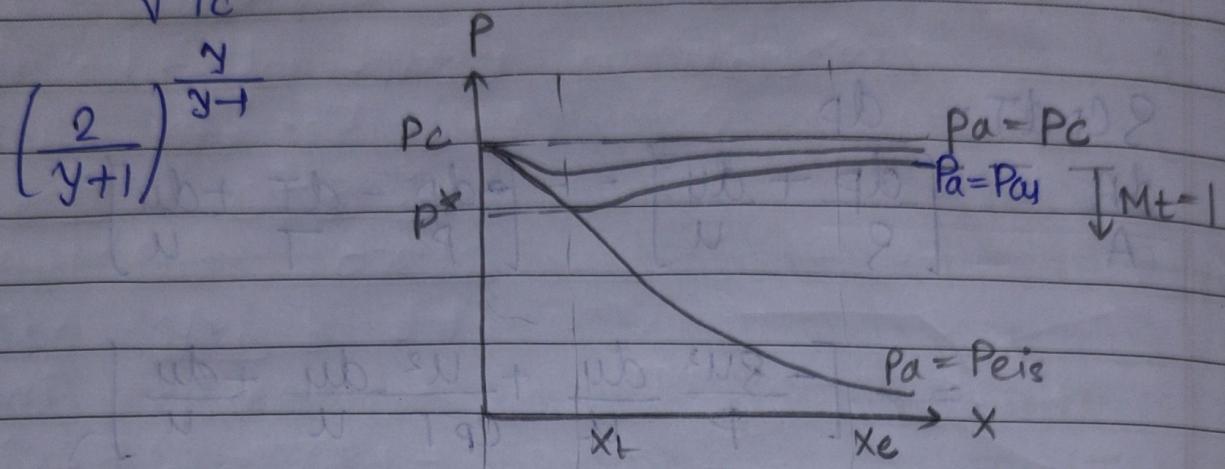
$$M > 1 \rightarrow dA > 0 \leftrightarrow dM > 0$$

$$dA < 0 \leftrightarrow dM < 0$$

These expressions give the reason of why Mach number cannot be 1 at the exit.

mallent Area  $\rightarrow A_t$  Mach no. at throat

$$m = \frac{S_c A_t}{\sqrt{T_c}} f(y, M_w, M_t)$$



$$\frac{M_b}{M} \left[ \frac{1-M}{1+M} \right] = \frac{u_b}{u} \left[ \frac{1-M}{1+M} \right] = \frac{u_b}{u} = \frac{A_b}{A}$$

$$P_a \quad P_c$$

$$\left[ \frac{u_b}{u} \left[ 1 + s_u \right] + s_u \right] -$$

$$0 > M_b \Leftrightarrow 0 < A_b \Leftrightarrow 1 > M$$

$$0 < M_b \Leftrightarrow 0 > A_b$$

$$0 < M_b \Leftrightarrow 0 < A_b \Leftrightarrow 1 < M$$

$$0 > M_b \Leftrightarrow 0 > A_b$$

for go nozzle wif minimum area  
fixe wif w + pe form w minumum area

$$\star P = P_a + \tilde{P} \quad (\text{Perturbation})$$

$$T = T_a + \tilde{T}$$

$$S = S_a + \tilde{S}$$

$$U \equiv \tilde{U}$$

$$\frac{\tilde{S}, \tilde{T}, \tilde{P}}{S_a T_a P_a} \ll 1$$

\* Conservation eqns:-

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (S U) = 0$$

$$\frac{\partial (S U)}{\partial t} + \frac{\partial}{\partial x} (P U^2 + P) = 0$$

$$\frac{\partial (S E)}{\partial t} + \frac{\partial}{\partial x} (S H + U) = 0$$

$$S \rho S_a U + \tilde{\rho} U = S_a U \left( 1 + \frac{\tilde{S}}{S_a} \right)$$

$$\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial}{\partial x} (S_a U) = 0$$

$$\frac{\partial (S_a U)}{\partial t} + \frac{\partial}{\partial x} (\tilde{P}) = 0$$

$$\frac{\partial}{\partial t} (S_a \tilde{E}) + \frac{\partial}{\partial x} (P_a h a u) = 0$$

$$S_a U^2 + P_a + \tilde{P} \neq 0$$

2nd Order 0' order 1st Order

$$e + \frac{U^2}{2} = e_a + \tilde{e} + \frac{U^2}{2}$$

$$h + \frac{U^2}{2} = h_a + \tilde{h} + \frac{U^2}{2}$$

$$S \left( h + \frac{U^2}{2} \right) U \equiv S_a h a u$$

$$\frac{\partial}{\partial t} \left( \frac{\partial \tilde{g}}{\partial t} \right) = - \frac{\partial}{\partial x} \left( \frac{\partial (P_a \tilde{u})}{\partial t} \right)$$

$$\frac{\partial^2 \tilde{g}}{\partial t^2} = \frac{\partial^2 \tilde{g}}{\partial x^2} = \left( \frac{\partial \tilde{p}}{\partial \tilde{g}} \right) \frac{\partial^2 \tilde{g}}{\partial x^2}$$

constant.

$\tilde{g} = \text{const}$  for small perturbations

$$\tilde{p} = \tilde{g}^*$$

$$\frac{\partial^2 \tilde{p}}{\partial t^2} = C_s^2 \frac{\partial^2 \tilde{g}}{\partial x^2}$$

$$C_s^2 = \frac{Y_p a}{S_a}$$

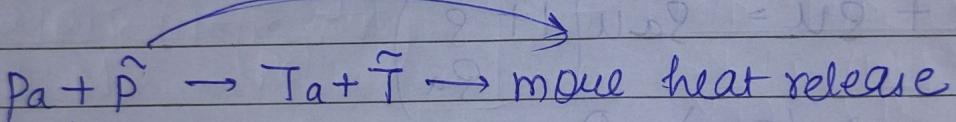
reaction  $\leftrightarrow$  acoustic  
(flow)

$$\tilde{p} \leftrightarrow \tilde{g}, \quad \tau_a = \frac{L}{c_a}$$

$$T \gg \tau_a$$

$\Delta p$  causes  $\Delta u$

$\Delta T$  causes  $\Delta g$ .



$\curvearrowleft$  reaction rate

★  $J = m u_e + (P_e - P_a) A_e$

$m \Rightarrow \text{Max for } M=1$

A

↳ can happen at min area

location

$$\dot{m} = P_e A_e + \sqrt{\frac{Y_p a}{R_u T_c} \left( \frac{Y_f}{2} \right)^{\frac{N_f}{2(N_f)}}}$$

$$\sqrt{\frac{Y}{R_u T_c}} \sim \frac{Y}{C_s}$$

$$C^* = \frac{P_c A_t}{m} = \sqrt{\frac{R u T_e}{\gamma M_a}} = \left( \frac{2}{\gamma + 1} \right)^{2(\gamma+1)}$$

$$J = - \frac{P_c A_t}{C^*} U_e + (P_e - P_a) A_e$$

$$J = - \frac{P_c A_t}{C^*} \left\{ 2 C_p T_c \left( 1 - \left( \frac{P_c}{P_e} \right)^{\frac{1}{\gamma-1}} \right)^{\frac{1}{\gamma-1}} \right\}^{1/2} + (P_e - P_a) A_e$$

$$\frac{J}{P_c A_t} = \frac{1}{C^*} \left[ 2 C_p T_c \left( 1 - \left( \frac{P_e}{P_c} \right)^{\frac{1}{\gamma-1}} \right)^{\frac{1}{\gamma-1}} \right]^{1/2} + \left( \frac{P_e - P_a}{P_c} \right) \frac{A_e}{A_t} = C_J$$

$C_J \rightarrow$  Throat coeff

$$J = P_c A_t C_J \equiv m C^* C_J \xrightarrow{\text{Nozzle pert.}}$$

Comb chamber  
perturbation

$$U_{eq} = C^* G$$

Characteristic vel / speed ( $C^*$ )  $\rightarrow$  func of propellant combination

$$h_c + \frac{U_c^2}{2} = h_T \rightarrow U_e = \sqrt{2 h_T \left( 1 - \frac{T_e}{T_T} \right)}$$

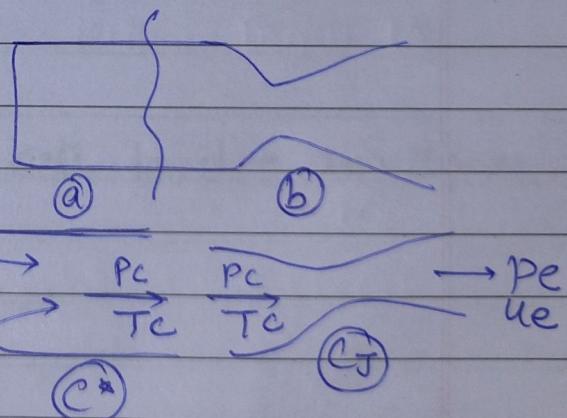
$$J = [P_e(1 + \gamma M_e^2) - P_a] A_e$$

$$P_a = 0, J = J_{vac}$$

Max thrust

for given  $A_e/A_t$

For a given  $(\frac{A_e}{A_t})$



Peak  $C_J$  = Optimum expansion ( $P_e = P_a$ )

Book Chapter 4 continued

\* Thrust coefficient:-

$$C_f = \frac{F}{P_c A_t} = \left[ \frac{2\gamma^2}{\gamma-1} \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \left( 1 - \left( \frac{P_e}{P_c} \right)^{\frac{\gamma-1}{\gamma+1}} \right) \right]^{\frac{1}{2}} + \left( \frac{P_e}{P_c} \right) \epsilon$$

$\epsilon = A_e / A_t \rightarrow$  nozzle expansion ratio

$$\text{If } P_a = 0, C_{fV} = \left[ \frac{2\gamma^2}{\gamma-1} \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \left( 1 - \left( \frac{P_e}{P_c} \right)^{\frac{\gamma-1}{\gamma+1}} \right) \right]^{\frac{1}{2}} + \left( \frac{P_e}{P_c} \right) \epsilon$$

Optimal expansion  $\rightarrow P_e = P_a$

$$C_f = C_{fV} - \left( \frac{P_a}{P_c} \right) \epsilon$$

$$F = C_f P_c A_t : \dot{m} = \frac{P_c A_t}{C^*} = \frac{g P_c A_t}{C^*}$$

$$I_{sp} = \frac{F}{\dot{m}g} = C_f C^*$$

$$I_{sp} = \sqrt{\frac{2\gamma R T_c}{(\gamma+1)M} \left( 1 - \left( \frac{P_e}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right)} \left[ 1 + \frac{\left( \frac{P_e}{P_c} - \frac{P_a}{P_c} \right) \left( \frac{P_c}{P_e} \right)^{\frac{1}{\gamma}}}{\frac{2\gamma}{\gamma-1} \left( 1 - \left( \frac{P_e}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right)} \right]$$

$$I_{sp} \propto \sqrt{\frac{T_c}{M}}$$

CLASS NOTESRocket propulsion nozzles

\*  $\epsilon = \frac{A_e}{A_i}$  given gives  $C_J \text{vac}$  ( $P_a = 0$ )

$$C_J = C_J \text{vac} - \left( \frac{P_a}{P_c} \right) \epsilon$$

$P_e < P_a < P_c$   $\rightarrow$  Normal shock at exit

Nozzles  $\rightarrow$  Over expanded  $\rightarrow$  Vulcain

Perfectly expanded  $\rightarrow$  RL-10

Under expanded  $\rightarrow$  Saturn rocket

Optimum expansion  $\rightarrow P_e = P_a$

\* Flow separation  $\rightarrow$  Correlation.

$$\frac{P_{sep}}{P_a} = (1.88 M_{sep}^{-1})^{-0.64}$$

If  $P_e < 0.4 P_a$ ,

separation occurs.

$$P_a = P_{sep} \exp\left(\frac{-h_1}{h_2}\right)$$

$$h_2 = 701.04 \text{ m}$$

$$\frac{P_{sep}}{P_a} = 0.3 - 0.4 \quad \text{at } \Delta \text{ level} \rightarrow P_a = 1 \text{ bar}$$

$$C^* = 1850 \text{ m/s}$$

$$P_c = 48.3 \text{ bar}$$

$$\epsilon = 20, \gamma = 1.2$$

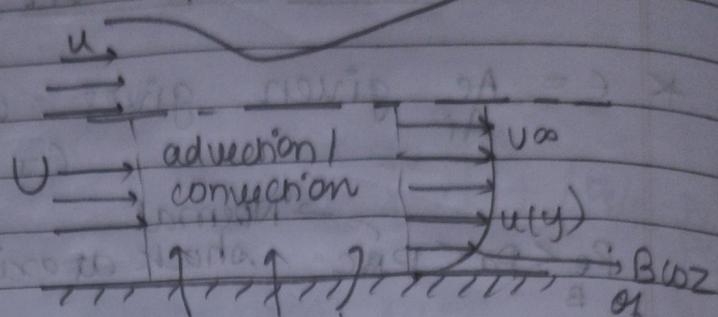
$$\frac{P_e}{P_a} = 0.005$$

$$\frac{d}{dt} \left[ \int_{\text{vessel}} \vec{u} \cdot d\vec{v} \right] = - \oint_{\text{vessel}} \vec{u} \cdot d\vec{s}$$

$$\frac{d\vec{u}}{dt} + g\vec{u} \cdot \vec{u}$$

$$= -\nabla p + \mu \nabla^2 \vec{u}$$

Why Laplacian?



Pressure → Using the force that particles exert on a wall that is kept in their way.

Momentum flux of random motion

$$u, T, \gamma \rightarrow \text{meanings similar}$$

Collisions →  $\mu, k, D$

$$\Gamma_x \propto -\nabla x$$

$$(1 - \phi^2 M^{88.1}) = \frac{\partial \phi}{\partial x}$$

$$\vec{\Gamma}_n = -D_n \vec{\nabla} x$$

$$\nabla T_{xy} = -\mu \left( \frac{\partial y}{\partial x} + \frac{\partial x}{\partial y} \right), \quad T_{xx} = -\mu \frac{\partial y}{\partial x}$$

At what dist particle sees the effect of wall?

$$\frac{du}{dt} + g\vec{u} \cdot \vec{\nabla} \vec{u} + \frac{1}{\rho} \nabla P = \mu \nabla^2 \vec{u}$$

$$T_{\text{conv}} = l/\mu$$

$$T_{\text{visc}} = b/c = \lambda y^2/\mu$$

$$\frac{\partial n}{\partial t} = -\vec{\nabla} \cdot \vec{\Gamma}_x \\ \approx D_x \nabla^2 x$$

$$\frac{\partial n}{\partial T_{\text{diff}}} = \frac{D_x \frac{\partial x}{\partial y}}{l y^2}$$

$$T_{\text{diff}} = \frac{l y^2}{D_x}$$

$$ly = \frac{Re}{10}$$

$$Re = \frac{ul}{\eta}$$

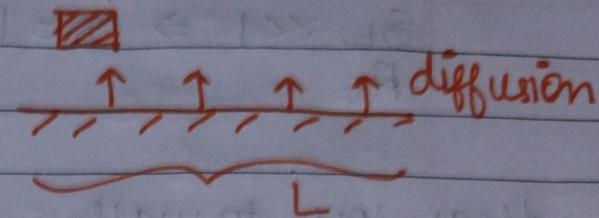
## Lecture

Boundary layer - separation

Time scales →

$$\tau_{\text{conv}} \sim \tau_{\text{diff}}$$

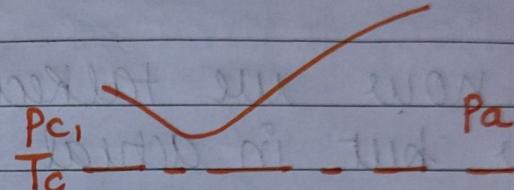
$$\frac{\delta}{L} = \frac{5}{\sqrt{Re_L}}$$



how long it takes to travel  $\frac{\delta}{L}$  in horizontal direction  
 (convection)  $Re = \frac{UL}{v}$

$$y = 8 \rightarrow u(y) = 0.99U$$

horizontal vel



v → vertical velocity

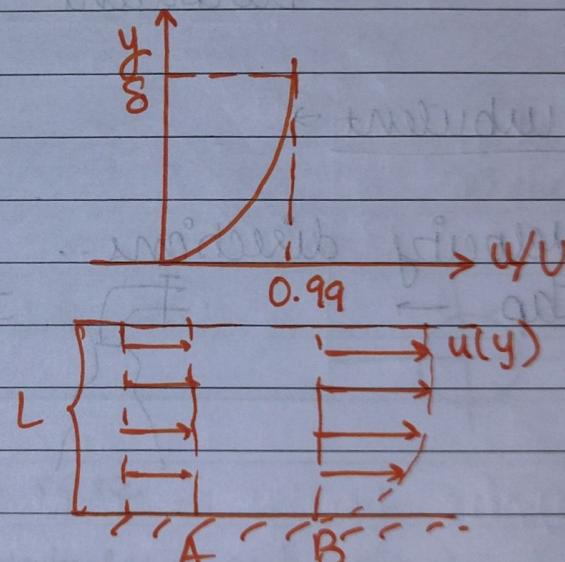
↪ it is zero outside

$$m = \rho w L$$

Area

$$m = \int_0^L \rho w u dy$$

$$= \rho w L u$$



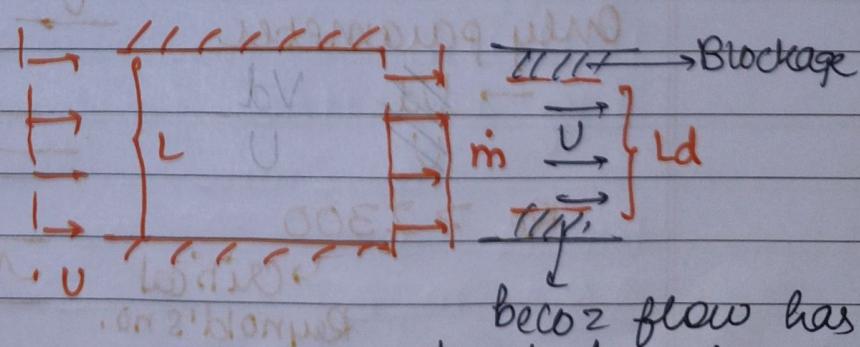
For channel flow →

$$S^* = L - l_d$$

displacement thickness

$$\frac{s^*}{l} = \frac{1.72}{\sqrt{Re_L}}$$

(Blasius Boundary Layer)



becoz flow has slowed down here & mass can't be accomodated.

$$m = \rho \pi R^2 U$$

$$C_D = \frac{\text{actual}}{\text{ideal}} = \left( \frac{R_t - S_t}{R_t} \right)^2$$

Discharge coeff.

$$S_t \ll 1 \Rightarrow C_D \approx 1 - \frac{2S_t}{R_t}$$

$R_t \uparrow \downarrow$

$$M=1$$

$$\Delta E$$

At throat

$$R_d = R_t - S_t$$

heat loss to wall

↳ low T  $\Rightarrow$  high g

↳ higher m

$$0.93 < C_D < 1.15$$

$$M \xrightarrow[T]{P} \text{compressible}$$

|||||| zw

$$V_{EB,0} = (U)U - \beta = 0$$

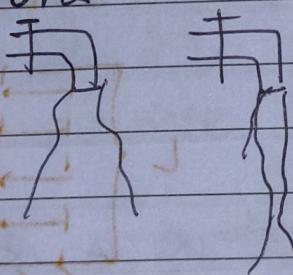
[Till now we talked only about laminar flow but in actual rocket nozzles, there is turbulent flow!]

Turbulent  $\Rightarrow$

Velocity directions..

Tap  $\rightarrow$

PP.O



Only parameter

$$\frac{V_d}{U}$$

$$\frac{V_d}{U}$$

$$\frac{U}{U} - \frac{U}{U} - \frac{U}{U} \quad f_d$$

transit

$$Re > 2300$$

↳ critical  
Reynold's no.

Turbulent

(w.r.t some restoring force)

"Too much energy" in turbulent flow  
 Unstable (destabilizes)      ↗ entropy  
 ↗ "free energy"  
 ↗ h-Ts

Perturbations  
 ↓  
 ↗ changes (unstable)  
 stays same ⇒ stable

Turbulence → fluctuating fields

Not a Gaussian → statistics are 'steady'

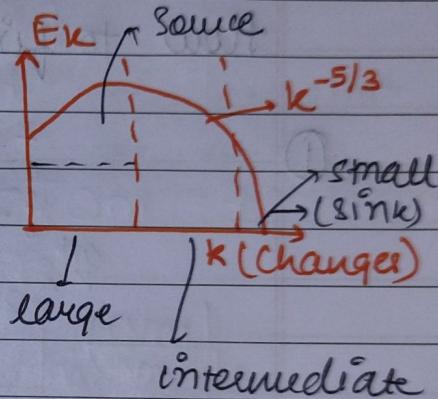
$E(k) \rightarrow F(E)$  ↗ Fourier transform

$$E = k \cdot E = \frac{1}{2} U^2 + \frac{1}{2} V^2 + \frac{1}{2} W^2$$

$$E = \int E_k e^{ikx}$$

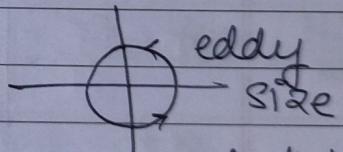
l → length scale

k → wave number.



U, V & θ

$$U = \cos(\omega t), \quad V = \sin(\omega t)$$



given k, E

↳ dissipation rate

→  $\tau_{\text{eff}} t$  (Turbulent viscosity)

$$\mu \nabla^2 U \rightarrow \mu k^2 U t$$

$$l \sim \left(\frac{1}{k}\right)$$

$$T = 8 \times 10^2 \text{ s} \quad \Rightarrow \quad \tau_{\text{eff}} = \frac{T}{U} = \frac{8 \times 10^2}{20} = 40 \text{ s}$$

# LECTURE

## HEAT TRANSFER

Combustion chamber heat + Nozzle  $\rightarrow q'' \rightarrow T_h \rightarrow T_c$  cold

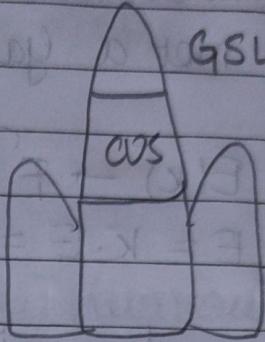
→ Internal

Re-entry → External

Ballistic

Manned missions

GSLV Mk-III



How to find  $q'' = ?$

$T \approx 2500\text{K}$

$f$   
 $f$   
 $f$

$T \approx 3000\text{K}$

$q''$   
(MW)  
 $m^2$

10

20

exp

xt

Units for  $q'' \rightarrow \frac{10\text{MW}}{m^2} = 1\text{kw/cm}^2$

$10\text{MW/m}^2$

1 kg of water for from  $30^\circ\text{C}$  to  $100^\circ\text{C}$  in 2 min

Burner area  $\sim 300\text{cm}^2$

$$\Delta Q = m c \Delta T = 3 \times 10^5 \text{ J}$$

$1 \downarrow \quad \downarrow 70 \quad \downarrow$   
4186

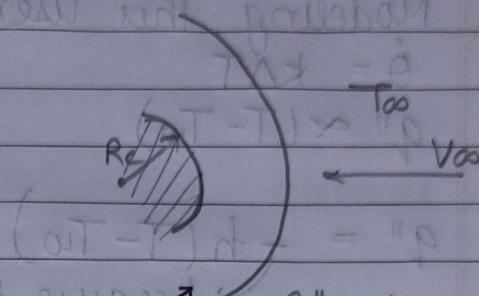
$$q'' = \frac{\Delta Q}{At} = \frac{3 \times 10^5}{3 \cdot 10^{-2} \cdot 120} = 0.08 \text{ MW/m}^2$$

Density  $\rightarrow 10^3 \frac{\text{kg}}{\text{m}^3} \sim 1 \frac{\text{kg}}{\text{m}^3}$

This was about engine combustion

### \* Re-entry

$$q'' = C \sqrt{\frac{P_\infty}{R}} V_\infty^3 \left[ 1 - \frac{h_w}{h_0} \right]$$



High  $R_c \rightarrow$  low  $q'' \rightarrow$  blunt leading edge.  $q'' = 10 \text{ MW/m}^2$  for manned

$h_w = C_p T_w \rightarrow$  wall temperature

$$h_0 = h_\infty + \frac{1}{2} V_\infty^2$$

$R_c \rightarrow$  Radius of curvature of leading edge.

### \* Entry

$$V_\infty \left( \frac{\text{km}}{\text{s}} \right) \quad \frac{V_\infty^2}{2 \left( \frac{\text{MJ}}{\text{kg}} \right)} = h_0 \left( \frac{\text{MJ}}{\text{kg}} \right)$$

Apollo

$$11.4 \quad (wT - T) \quad 66 \quad = 0.2$$

Man Return

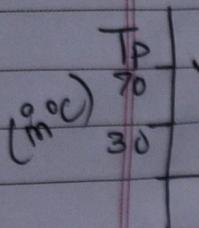
$$14 \quad (wT - T) \quad 98 \quad = 0.2$$

Water  $\rightarrow$  23 MJ/kg to boil

Carbon  $\rightarrow$  60 MJ/kg to evaporate.

$$\begin{aligned} \text{sm} &= \omega T + q_{\text{f}}, & \left[ + \text{Ad} - \right] \text{q}_{\text{rx}} &= (+10) \\ \text{AN} & & \text{sm} & \\ & & \left( \frac{+}{-} \right) \text{q}_{\text{rx}} & \end{aligned}$$

\* Example of Potatoes in water  
 $\downarrow$   
 $\Delta t = 0, 30^\circ\text{C}$   
 $\Delta t = 0, 70^\circ\text{C}$



Modeling this using Newton's equations

$$\dot{\theta} = k \Delta T$$

$$q'' \propto (T - T_w)$$

$$q'' = -h(T - T_w)$$

because the difference is decreasing

$$E = mcT$$

specific heat

$$\frac{dE}{dt} = \dot{q} = q'' A$$

$$\frac{mcdT}{dt} = q'' A = -hA(T - T_w)$$

$$\frac{dT}{dt} = -\frac{hA}{mc}(T - T_w)$$

$$T - T_w = 8(T_0 - T_w) \exp\left[\frac{-hA}{mc}t\right]$$

$T \rightarrow$  Potato temp

$$\Theta = \frac{T - T_w}{T_0 - T_w} \rightarrow \text{Normalised}$$

$$\Theta(t) = \exp\left[\frac{-hA}{mc}t\right], \text{ let } \Sigma_c = \frac{mc}{hA}$$

$\downarrow \exp\left(\frac{-t}{\Sigma_c}\right)$

$$\frac{dE_w}{dt} = -hA(T_w - T)$$

What is  $h$ ?

$h$

$A$

$T_w$

$$\text{Newton's Law} \rightarrow q'' \propto (T - T_w)$$

How is the heat getting transferred?

↳ Convection. (Is it?)

No, conduction just around the surface

When we heat water

$$\Delta P = \rho gh$$

Buoyancy → natural convection currents

Fouier's law -  $\vec{q}'' \propto \nabla T$

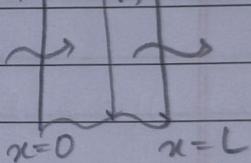
$$\frac{\partial T}{\partial x}$$

$$\vec{q}'' = -k \nabla T$$

fluid

$T_n$

$T_c$



$$E = mcT$$

$$m = \rho(L)A$$

$$\frac{dE}{dt} = AK \frac{dT}{dx} \Big|_{x=0} - AK \frac{dT}{dx} \Big|_{x=L}$$

$$SCL \frac{dT}{dt} = K \left( \frac{dT}{dx} \right)_{x=0} - \left( \frac{dT}{dx} \right)_{x=L}$$