End-Sem Solution

PAGE No.

DATE / /

Q.1 $F = 6\,kN$ Max. acceleration, $a_{max} = 10 \cdot g$

Empty mass of vehicle = 40 kg

Let M_b = Burn-out mass.

Max. acceleration ⇒ when the rocket reaches burn-out stage.

$$\frac{F - M_b \cdot g}{M_b} \leq a_{max}$$

$$\frac{F - M_b \cdot g}{M_b} \leq 10 \cdot g \Rightarrow F \leq 11 \cdot M_b \cdot g$$

$$\therefore M_b \geq 55.6\ kg$$

M_b = Mass of payload (M_L) + Empty vehicle mass

⇒

$\cancel{M_L = 55.6 - 40}$ $\boxed{M_L \geq 15.6\ kg}$

Payload must be more than 15.6 kg to restrict the acceleration to less than $10 \cdot g$.

Q2

Payload ~~of~~ satellite mass = 700 kg

stage	M_s	M_p	M_L	M_o	R
Fourth	2200	1200	700	4100	1.41
Third	1400	7000	4100	12500	2.27
Second	8200	40000	12500	60700	2.93
First	25000	140000	60700	225700	2.63

	R_i	I_{s_i}	$u_i = g \cdot I_{s_i} \ln(R_i)$
Fourth	1.41	304	1024.67
Third	2.27	293	2356.31
Second	2.93	292	3079.36
First	2.63	264	2504.33

m/s

Total Δu by the vehicle = $\boxed{8964.67}$ m/s

Thrust of first stage, $F = 4800$ kN

Total mass of vehicle $M_o = 225700$ kg

Initial acceleration, $a = \dfrac{F - M_o \cdot g}{M_o}$

$$\boxed{a = 11.45 \text{ m/s}^2}$$

PAGE No.
DATE / /

Q.3 $P_1 = 6\ MPa = 60\ bar$; SG of propellant $= 1.75 \Rightarrow$

$\rho_p = 1750\ kg/m^3$

Nozzle optimum expansion at 20 km altitude.

$$P_2\ (\text{at } 20km) = 101325 \cdot \exp\left(\frac{-h}{8435}\right) \qquad \{h \text{ in meter}\}$$

$$P_2 = 9461.78\ Pa$$

Burn rate, $r = 4.5 \times P_1^{0.3} = 15.37\ mm/s = 15.37 \times 10^{-3}\ m/s$

Burning surface area, $A_b = 1\ m^2$

Mass flow rate, $\dot{m} = \rho_p \cdot A_b \cdot r = \boxed{26.89\ kg/s}$

Throat area, $A_t = 0.009\ m^2$

Characteristic velocity, $c^* = \frac{P_1 \cdot A_t}{\dot{m}} = \boxed{2008.2\ m/s}$

Nozzle area expansion ratio, (for $k = 1.2$)

$$\frac{A_t}{A_2} = \left(\frac{k+1}{2}\right)^{1/k-1} \left(\frac{P_2}{P_1}\right)^{1/k} \sqrt{\frac{k+1}{k-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}$$

$$\frac{A_t}{A_2} = 1.61 \times 4.62 \times 10^{-3} \times 2.69 = 20 \times 10^{-3} = \frac{1}{\varepsilon}$$

$$\therefore \boxed{\varepsilon = 50}$$

$R = \frac{R_u}{MW}$

$MW = 19$

$$c^* = \frac{\sqrt{kRT_1}}{k\sqrt{\left(\frac{2}{k+1}\right)^{k+1/k-1}}} \Rightarrow \boxed{T_1 = 3876.31\ K}$$

Gas jet velocity, $v_2 = \sqrt{\frac{2k}{k-1} \frac{R_u}{MW} T_1 \left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right)}$

$$\boxed{v_2 = 3661.97\ m/s}$$

Thrust, $F = \dot{m}\, v_2$ (optimum expansion at cruise altitude)

$\therefore$ $\boxed{F = 98.47 \text{ kN}}$

Specific impulse $= \frac{F}{\dot{m} g} = \boxed{373.9 \text{ sec}}$

Drawback of end-burning configuration: –

There is a major shift in the CG (Center of gravity) of the vehicle as the propellant gets consumed.

Alternative grain geometry $\Rightarrow$ star-burning grain.

Suitable thrust vectoring technique $\Rightarrow$ Jet-vane type

Q4 $C_{12}H_{24} + n \cdot O_2 \longrightarrow x\, H_2O + y\, CO_2 + z\, CO$

$$MR = 2.5 = \frac{m_{ox}}{m_F} = \frac{n \times 32}{(144+24)} \Rightarrow n = \boxed{13.125}$$

$$\boxed{C_{12}H_{24} + 13.125\, O_2 \longrightarrow 12\, H_2O + 2.25\, CO_2 + 9.75\, CO}$$

Flame Temperature $\Rightarrow$ $[H]_R = [H]_P$

Reactants $\Rightarrow H = -159$ kJ/mol

Products $\Rightarrow$

Species	n_i	Δh_{f_i}	$C_{p_i}(T_{ad} - 298)$
H_2O	12	-241	$0.058\,(T_{ad} - 298)$
CO_2	2.25	-390	$0.063\,(T_{ad} - 298)$
CO	9.75	-112	$0.037\,(T_{ad} - 298)$

$$-159 = -4861.5 + 1.1985\,(T_{ad} - 298)$$

$$\boxed{T_{ad} = 4221.65 \text{ K} = \boxed{T_1}}$$

Molecular weight, $MW = \frac{12}{24} \times 18 + \frac{2.25}{24} \times 44 + \frac{9.75}{24} \times 28$

$$\boxed{MW = 24.5 \text{ kg/kmol}}$$

Average $C_p = 49.93$ J/mol-K

$C_v = C_p - R_u = 41.62$ J/mol-K

$$\boxed{k = \frac{C_p}{C_v} = 1.2}$$

PAGE No. DATE / /

$T_1 = 4221.65$ K , $MW = 24.5$, $k = 1.2$

$P_1 = 6$ MPa , $\dot{m} = 200$ kg/s

$[I_s]_{SL} = 270$ sec

$[I_s]_{vac} = 300$ sec

$$c^* = \frac{\sqrt{kRT_1}}{k\sqrt{\left(\frac{2}{k+1}\right)^{k+1/k-1}}} = \frac{1311.15}{0.71} = \boxed{1846.7 \text{ m/s}}$$

$$A_t = \frac{\dot{m}c^*}{P_1} \Rightarrow \boxed{A_t = 0.0615 \text{ m}^2}$$

$(F)_{SL} = \dot{m}v_2 + (P_2 - P_{SL})A_2$ $(F)_{vac} = \dot{m}v_2 + P_2 \cdot A_2$

① $(I_s)_{SL} = \frac{v_2}{g} + \frac{(P_2 - P_{SL})A_2}{\dot{m}\cdot g}$ $(I_s)_{vac} = \frac{v_2}{g} + \frac{P_2 A_2}{\dot{m}g}$ ②

Substracting ① from ②

$$(I_s)_{vac} - (I_s)_{SL} = \frac{P_{SL} \times A_2}{\dot{m}\cdot g}$$

$$\therefore \boxed{A_2 = 0.581 \text{ m}^2}$$

Nozzle expansion ratio, $\varepsilon = \frac{A_2}{A_t} = \boxed{9.44}$

From nozzle map, For $\varepsilon = 9.44 \Rightarrow \boxed{M_2 \approx 3.5}$

For $M_2 = 3.5$, ~~Exit pressure~~,

$$P_1 = P_2\left[1 + \frac{1}{2}(k-1)M_2^2\right]^{k/k-1}$$

$$\Rightarrow \boxed{P_2 = 49450.5 \text{ Pa}}$$

$$P = 101325 \exp\left(\frac{-h}{8435}\right)$$

$$\Rightarrow h \text{ (for optimum expansion)} = \boxed{6050.94 \text{ m}}$$

Best strategy for engine cooling ⇒ Film cooling + Regenerative cooling

Preferred atomization technique ⇒ Shear co-axial atomizer or Swirl- coaxial atomizer