Thrust from a rocket engine

* Thrust & thrust coefficient :- $F = \dot{m} v_2 + (P_2 - P_3) A_2$

$$F = F_{opt} + P_1 A_t \left(\frac{P_2}{P_1} - \frac{P_3}{P_1} \right) \frac{A_2}{A_t}$$

$\left\{ \because F_{opt} \text{ when } P_2 = P_3 \right\}$

$$I_s = I_{s_{opt}} + \frac{c^* \varepsilon}{g_0} \left(\frac{P_2}{P_1} - \frac{P_3}{P_1} \right)$$

$\left\{ \begin{array}{l} I_s \text{ for new } P_2 \text{ for new} \\ \varepsilon \text{ can be calculated.} \end{array} \right.$

Thrust from a rocket engine

$$F = \frac{A_t v_t}{V_t} v_2 + (P_2 - P_3) A_2 \quad \ldots \text{(General for all rockets)}$$

$$\therefore \; F = A_t P_1 \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} + (P_2 - P_3) A_2$$

$\ldots$ (Ideal rocket
k- constant)

$\therefore \; F \propto A_t,\; P_1 \;\&\; f(P_1/P_2) \;\&\; f(k) \Rightarrow$ Ideal Thrust Equation.

Thrust from a rocket engine

The thrust coefficient, $\boxed{C_F = \frac{F}{P_1 \cdot A_t}}$

* Represents amplification factor w.r.t. an imaginary rocket with pressure P_1 acting on area A_t.

$$\therefore \quad C_F = \frac{v_2^2 A_2}{P_1 A_t V_2} + \frac{P_2}{P_1} \cdot \frac{A_2}{A_t} - \frac{P_3}{P_1} \frac{A_2}{A_t}$$

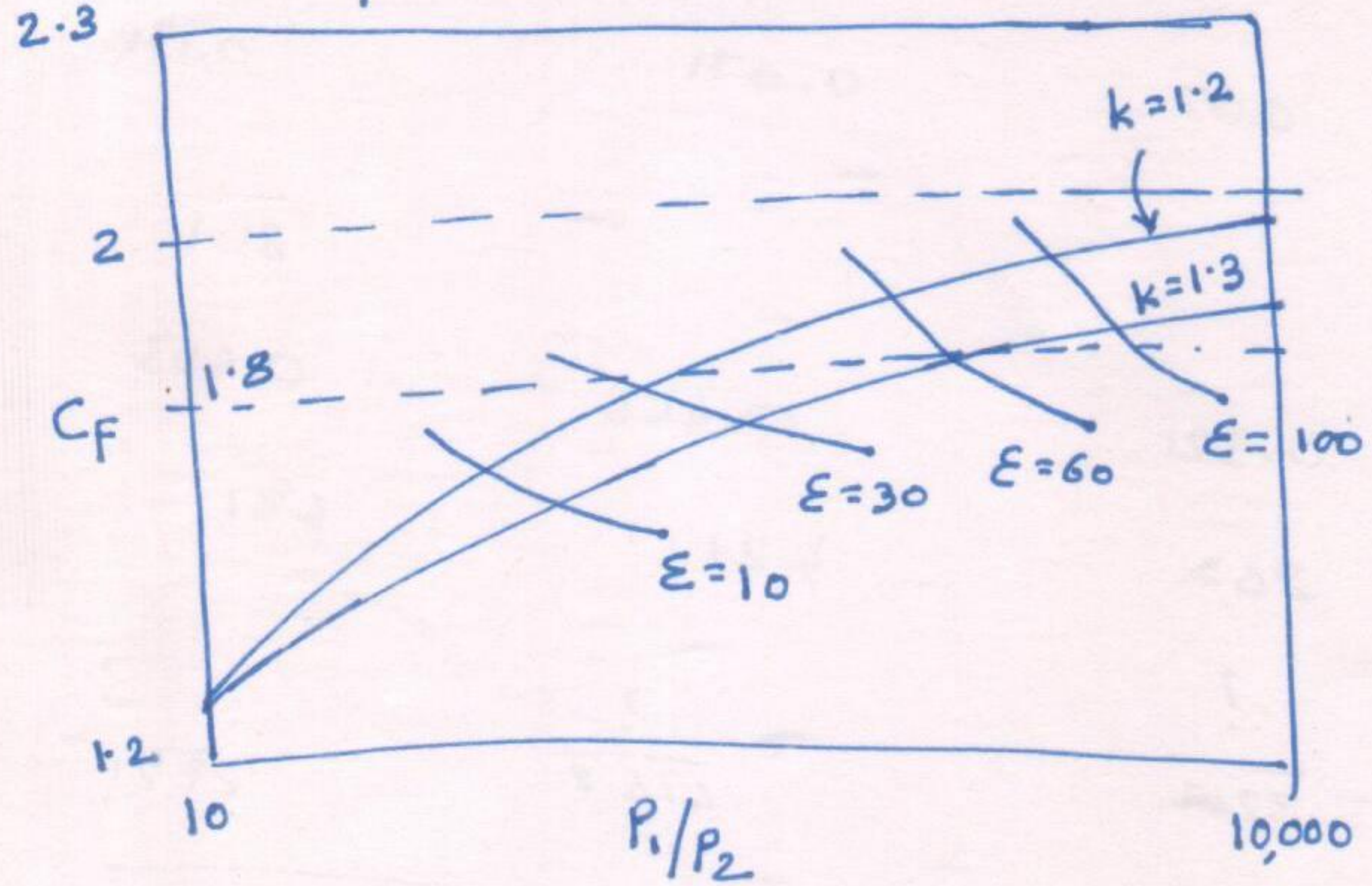
$$C_F = \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} + \frac{P_2 - P_3}{P_1} \cdot \frac{A_2}{A_t}$$

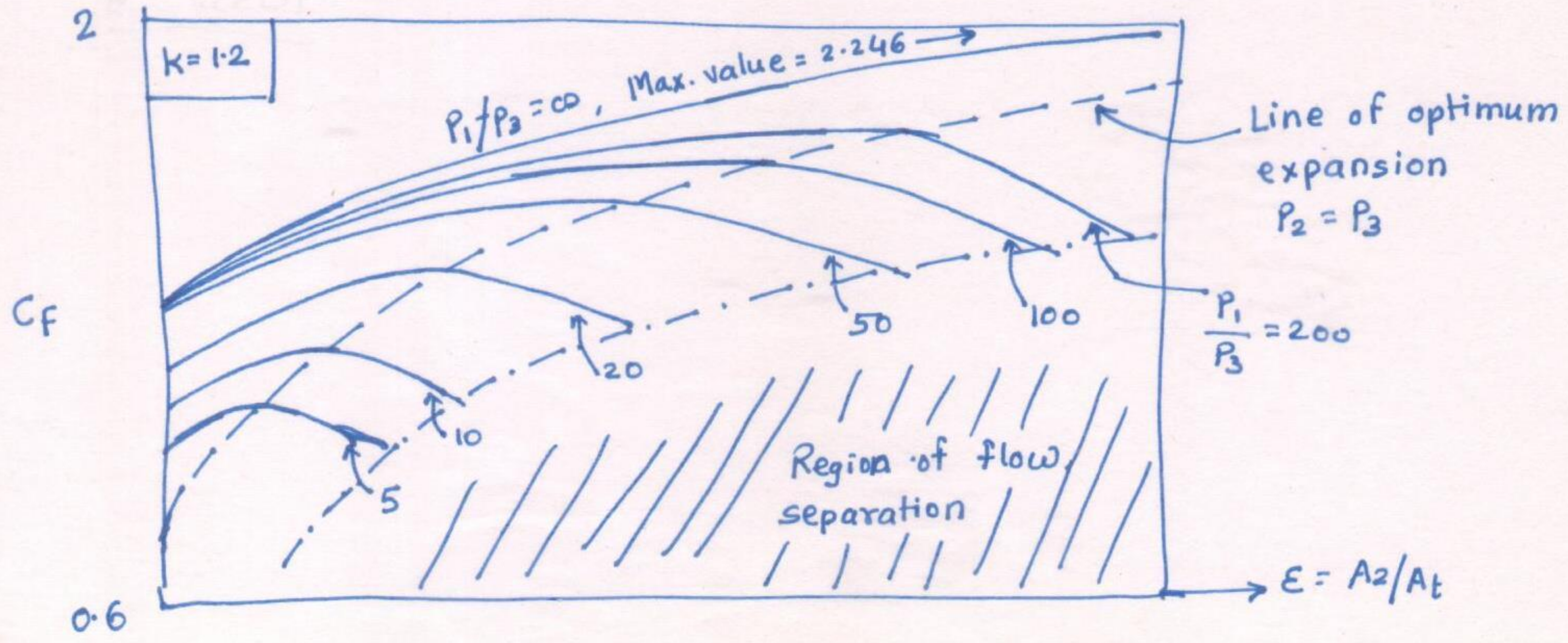
Experimental determination of C_F

$$C_F = f(k, \varepsilon, P_1/P_2) \quad \& \quad \boxed{F = C_F P_1 A_t}$$

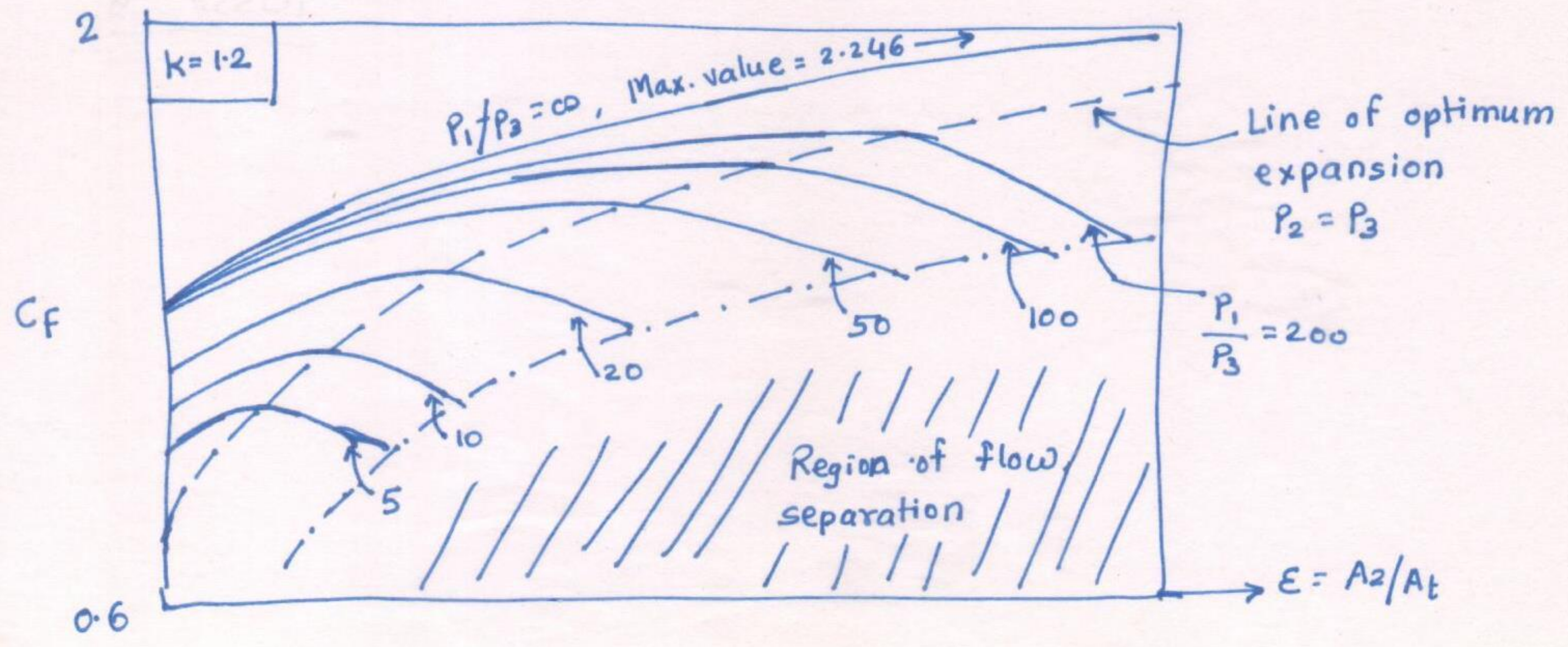
$$C_F \sim 0.8 - 1.9$$

Optimum expansion condition



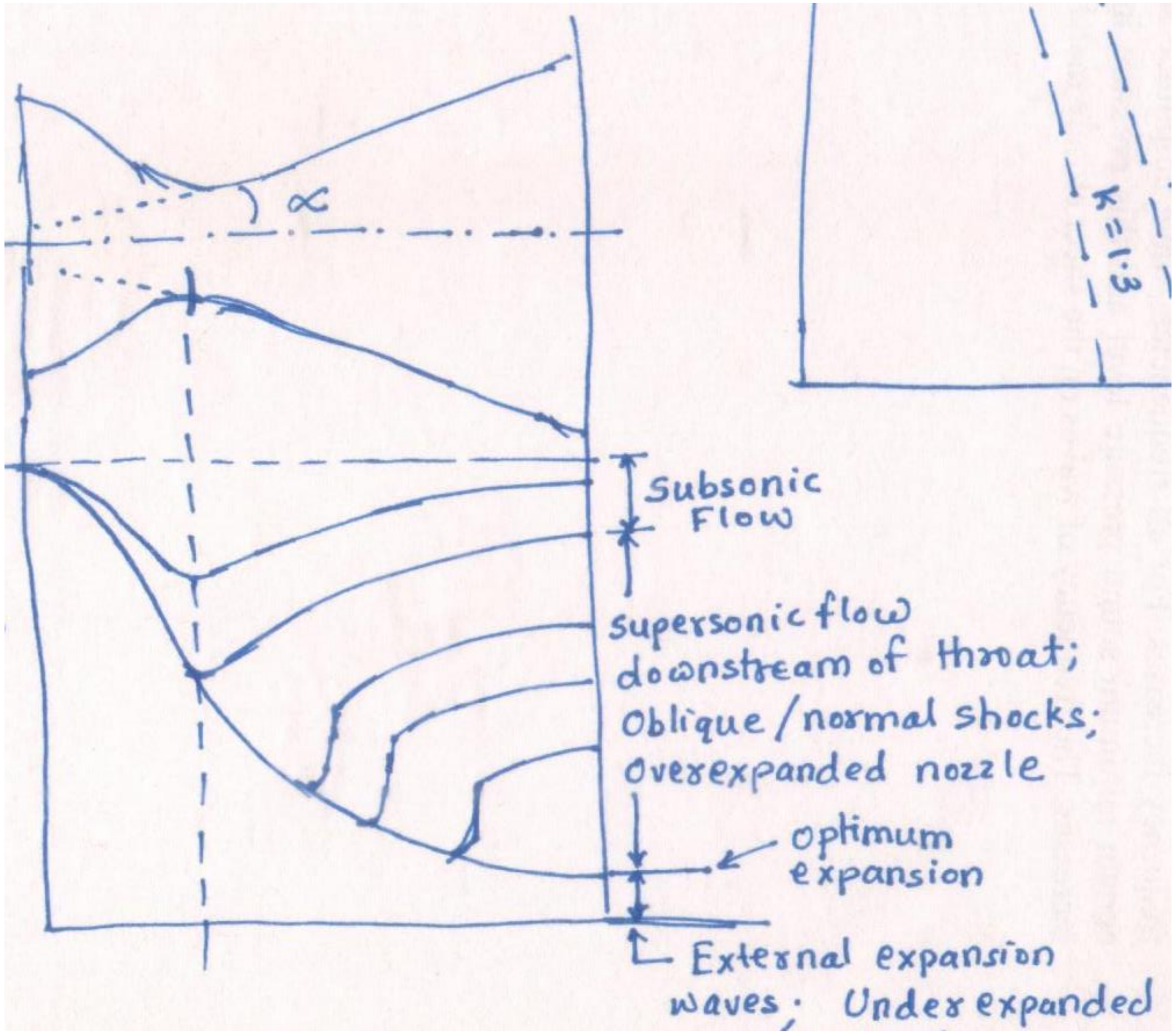


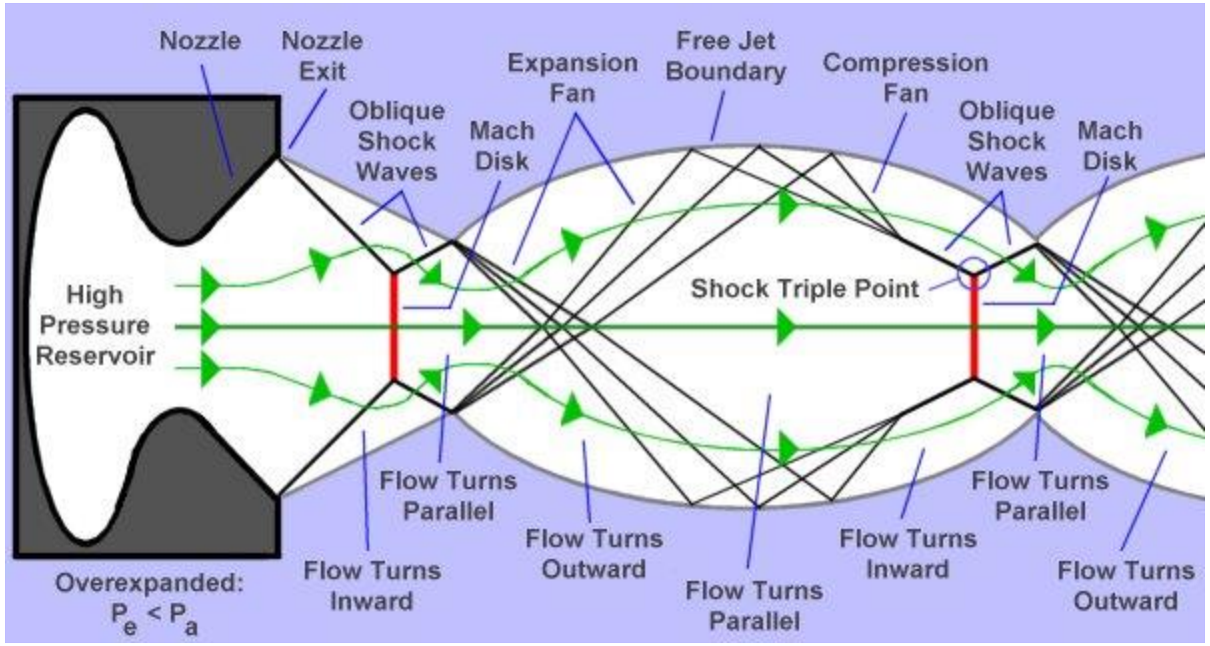
2
0·6
k = 1·2
$P_1/P_3 = \infty$, Max. value = 2·246
Line of optimum expansion $P_2 = P_3$
C_F
5
10
20
50
100
$\frac{P_1}{P_3} = 200$
Region of flow separation
$\varepsilon = A_2/A_t$

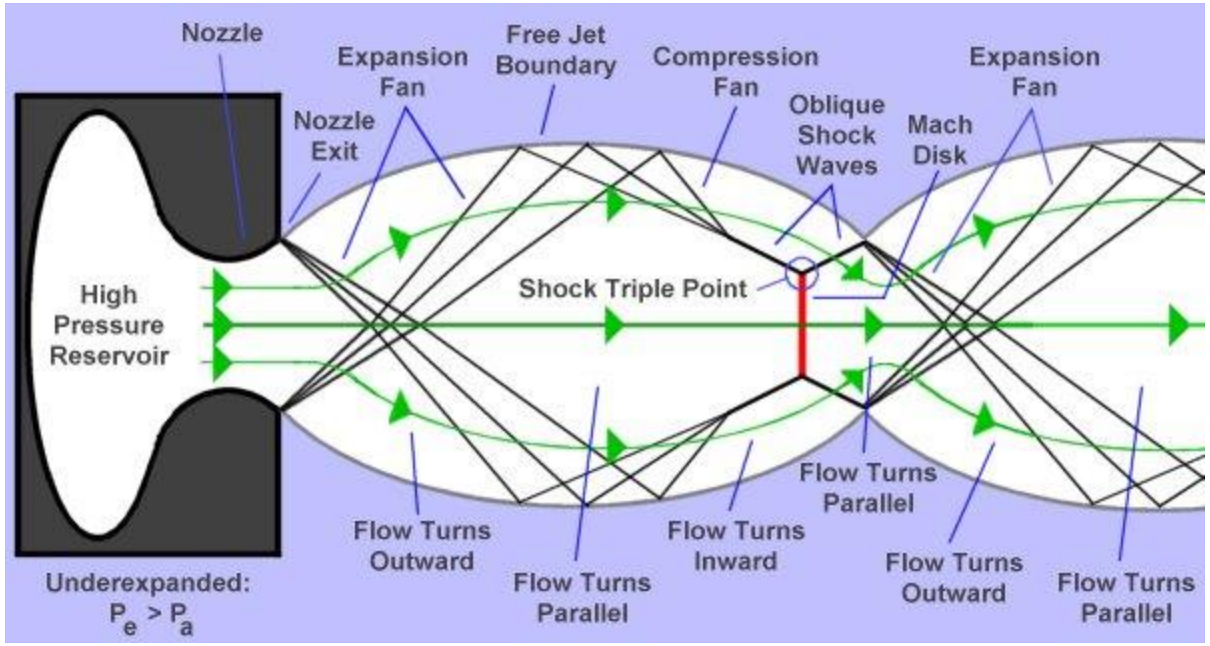


$$F = \dot{m}v_2 + (P_2 - P_3)A_2$$

Over-expanded and Under-expanded Nozzles




Nozzle
Nozzle Exit
Oblique Shock Waves
Mach Disk
Expansion Fan
Free Jet Boundary
Compression Fan
Oblique Shock Waves
Mach Disk
High Pressure Reservoir
Shock Triple Point
Flow Turns Parallel
Flow Turns Inward
Flow Turns Outward
Flow Turns Parallel
Flow Turns Inward
Flow Turns Parallel
Flow Turns Outward
Overexpanded:
$P_e < P_a$

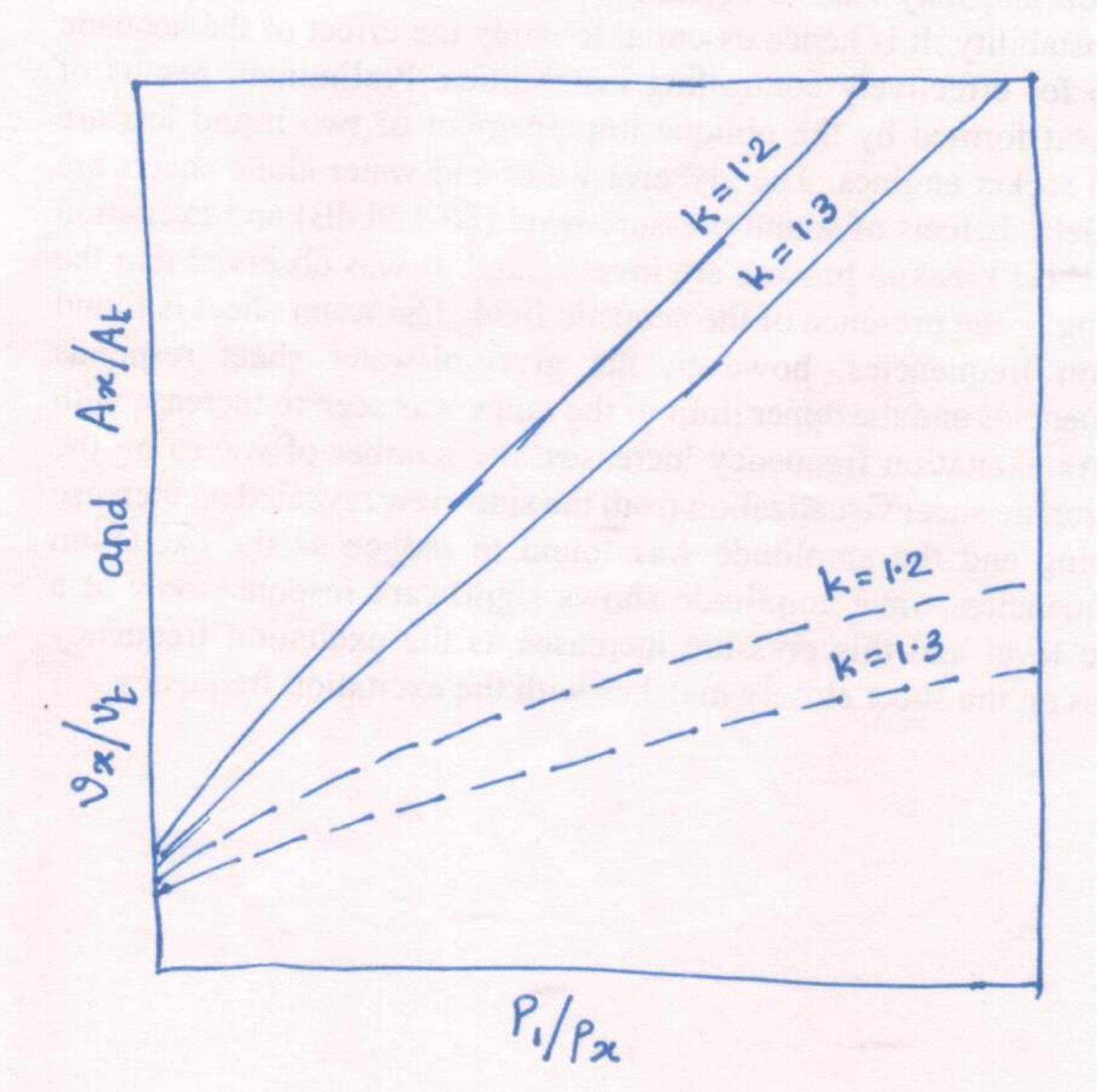

Nozzle
Expansion Fan
Free Jet Boundary
Compression Fan
Expansion Fan
Nozzle Exit
Oblique Shock Waves
Mach Disk
High Pressure Reservoir
Shock Triple Point
Flow Turns Parallel
Flow Turns Outward
Flow Turns Inward
Flow Turns Parallel
Flow Turns Outward
Flow Turns Parallel
Underexpanded:
$P_e > P_a$

CASE STUDY: Thrust variation in underexpanded, optimally expanded and overexpanded nozzle in a given ambient condition – case study

k = 1.2, MW = 24, T_1 = 3600 K, P_1 = 5 MPa, P_3 = 0.05 Mpa, mass flow rate = 100 kg/s

	P_2 (MPa)	v_2 (m/s)	F_M (N)	A_2 (m^2)	F_P (N)	F (N)
Underexpanded nozzle (P2>P3)	0.1	2677	267700	0.243	12150	279850
Optimum expansion (P2=P3)	0.05	2832	283200	0.409	0	283200
Overexpanded nozzle (P2<P3)	0.025	2963	296300	0.696	-17400	278900

Over-expanded and Under-expanded Nozzles



$F = m.v_2 + (P_2 - P_3).A_2$

Under-expansion to optimum expansion

$$v_2 = \sqrt{\frac{2k}{k-1} R \cdot T_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right]}$$

Optimum to over-expansion

Area increases at much faster rate than the velocity of the jet

Real Nozzles

Flow separation

Observed in case of over-expanded nozzles

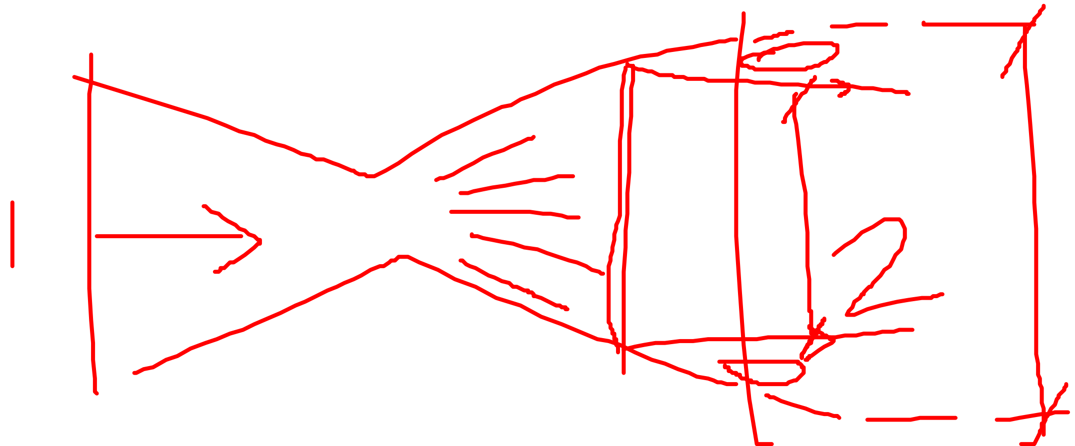
Separation occurs when the flow passes through the normal shock wave in divergent section

The jet leaves as reduced diameter from the point of separation

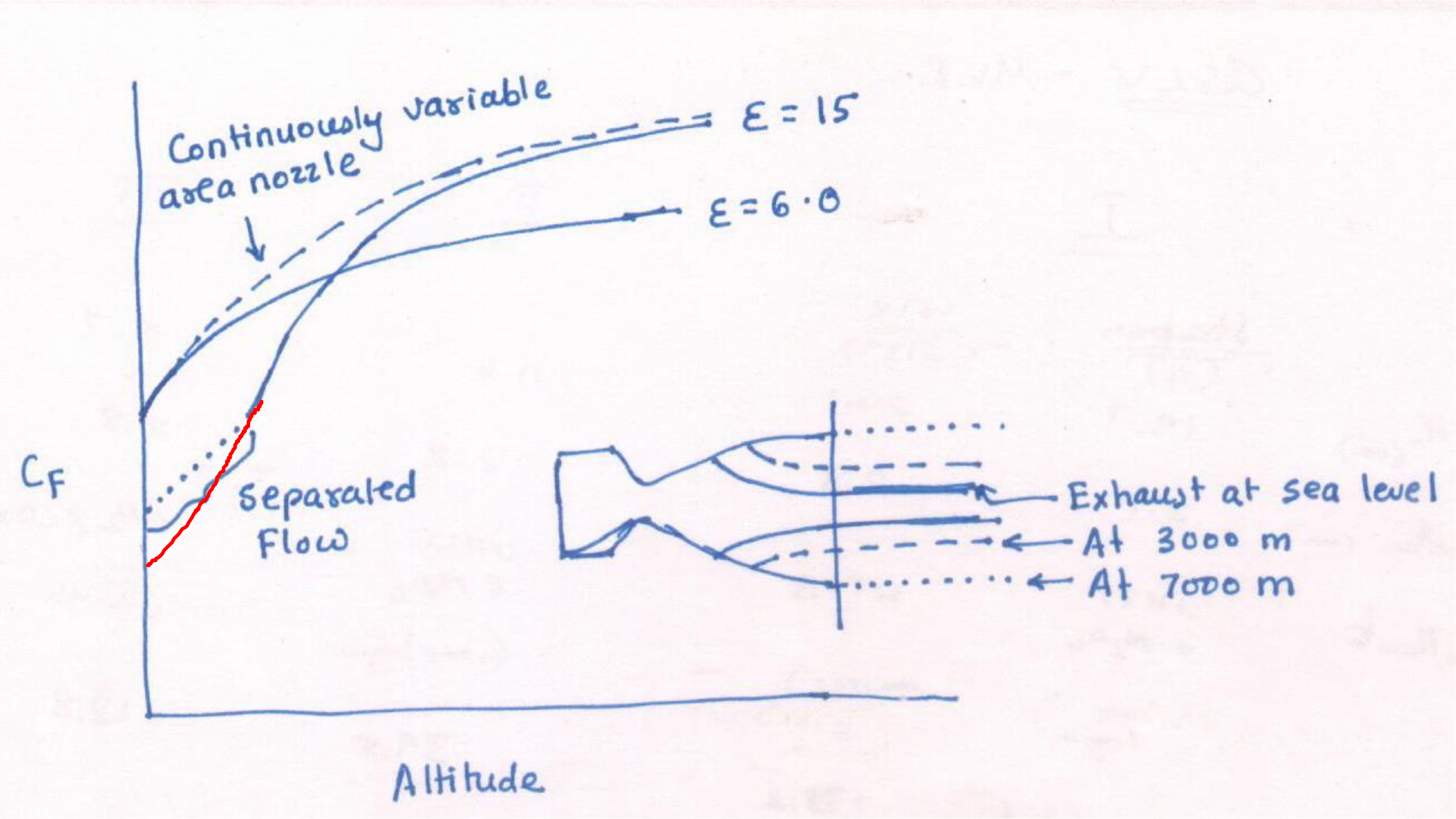
Separation point depends on the exact conditions at the exit plane

Summerfield criterion : Flow separation generally occurs when $P2 <= 0.4\ P3$

It also depends on the local Mach number; but the formulations are mostly empirical



Flow separation

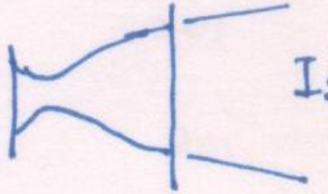
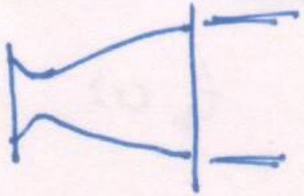
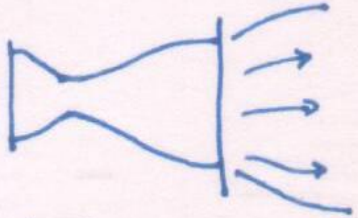
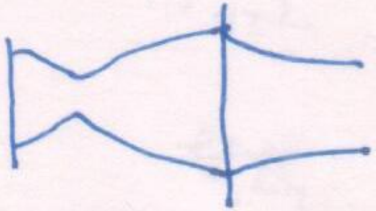
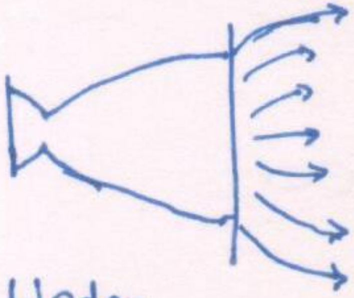
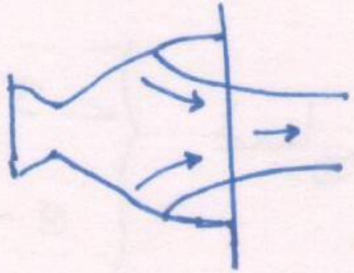


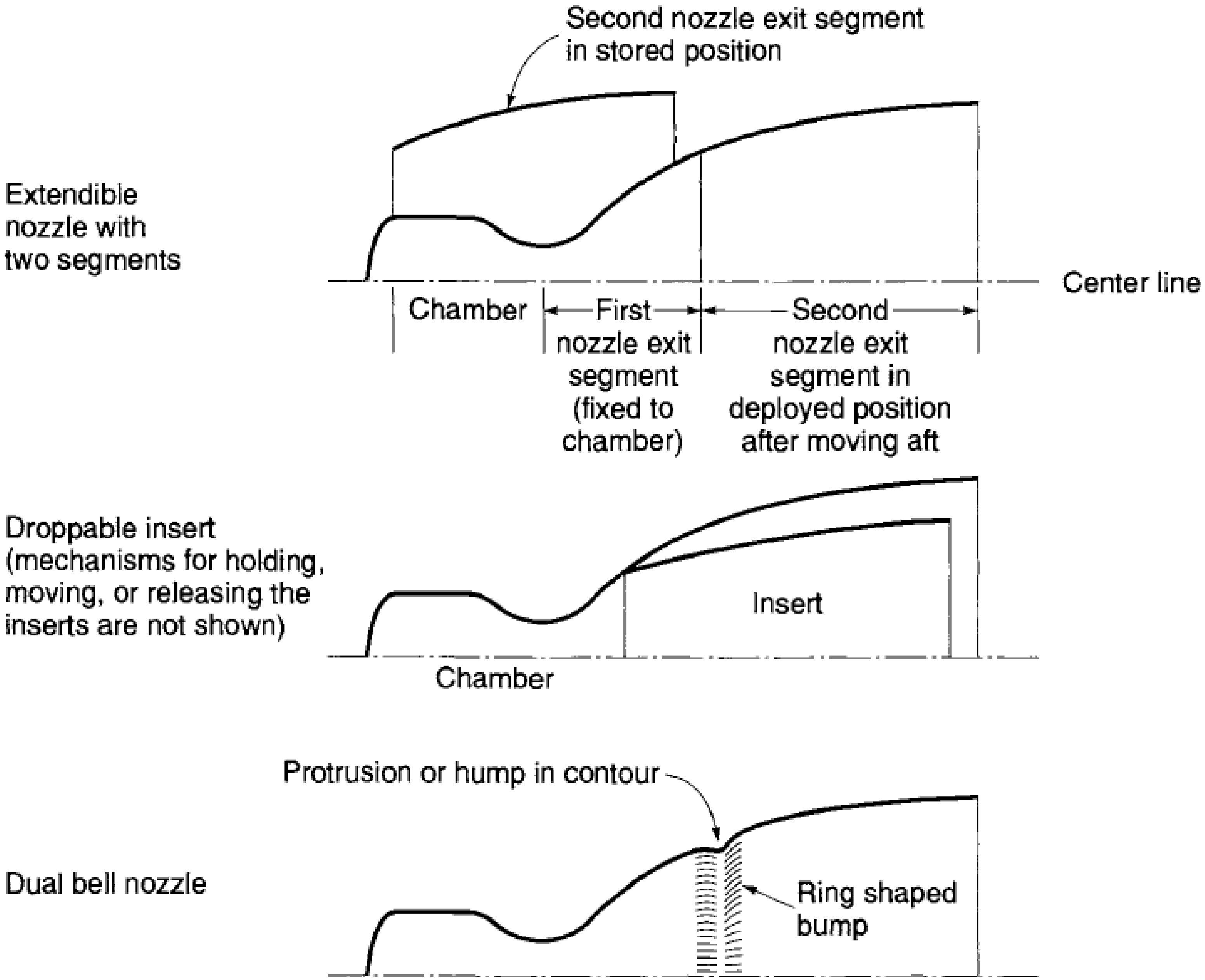
CF for separated flow is larger than the corresponding attached flow; but smaller than the optimum thrust

Flow separation is not desirable for following reasons:

Unnecessary long nozzle (large weight)

Uneven flow separation is possible which may lead to side forces

Stage	A_2/A_t	During flight	Sea level static test
Booster/ First stage	6	 $I_s = 267$	 $I_s = 267$ sec.
Second stage	10	 312 sec Under expansion	 $I_s = 254$ sec. Over expansion
Third stage	40	 334 sec Under expansion	 245 sec Flow separation caused by over-expansion.


Second nozzle exit segment in stored position
Extendible nozzle with two segments
Center line
Chamber
First nozzle exit segment (fixed to chamber)
Second nozzle exit segment in deployed position after moving aft
Droppable insert (mechanisms for holding, moving, or releasing the inserts are not shown)
Insert
Chamber
Protrusion or hump in contour
Dual bell nozzle
Ring shaped bump

Inlet gas velocity

When $A_1/A_t > 4$, the inlet velocity of the gases may be neglected ($v_1 \sim 0$)

However, space and weight constraints sometimes require smaller combustion chambers

The inlet gas velocity cannot be neglected in such cases

The gases in the combustion chamber expand as heat is added → energy necessary to accelerate expanding gases within the combustion chamber causes a pressure drop → reduction in thrust

Throatless rocket motor: Energy loss is maximum when throat diameter and chamber diameter are equal

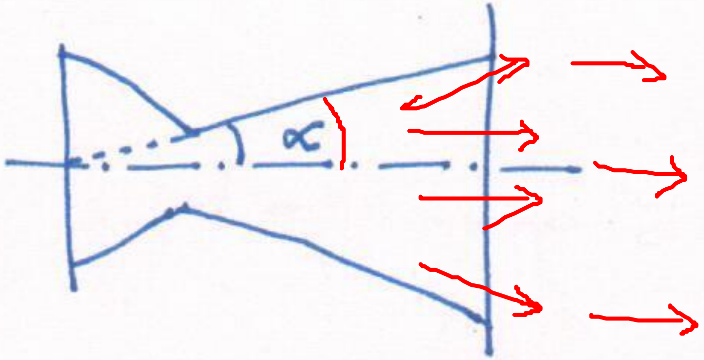
Chamber pressure at the nozzle inlet is smaller for smaller values of A1/At

Inlet gas velocity

Chamber to throat area ratio	Throat Pressure (%)	Thrust reduction (%)	Specific Impulse reduction (%)
∞	100	0	0
3.5	~ 99	1.5	0.31
2.0	~ 96	5.0	0.55
1.0	~ 81	19.5	1.34

For $k = 1.2$ and $p_1/p_2 = 1000$

Divergence Loss



$$\lambda = \frac{\text{Exhaust gas momentum with angle } \alpha}{\text{Ideal gas momentum (axial)}}$$

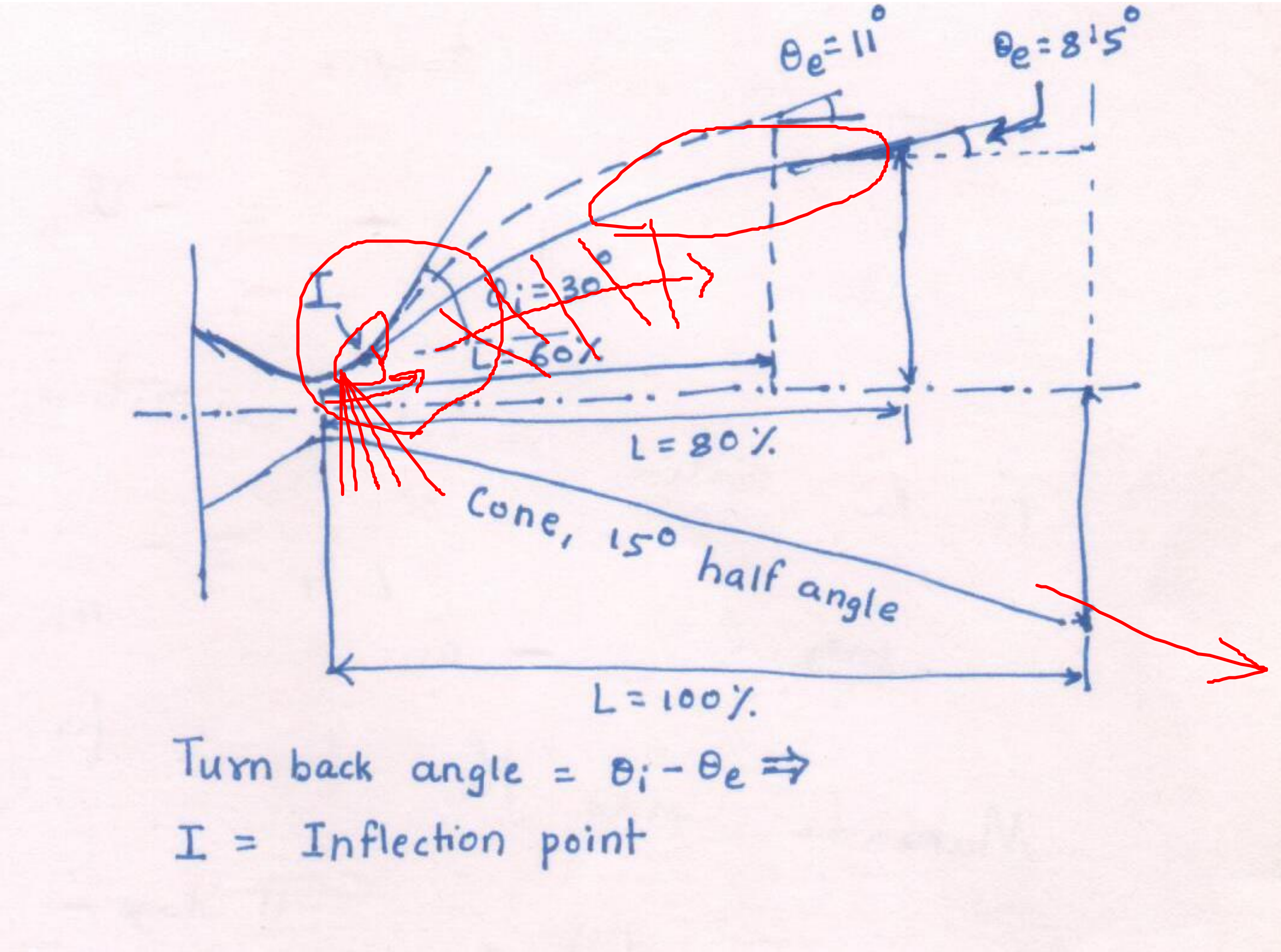
$$\boxed{\lambda = \frac{1}{2}(1+\cos\alpha)}$$

Half angle (α)	λ
0	1
2	0.9997
6	0.9972
10	0.9924
15	0.983
20	0.9698

High value of half divergence angle to attain required value of area expansion ratio in shorter length → more divergence loss

Typical values of half divergence angle for conical nozzles – 12-18 degrees

Bell shaped nozzle



80% bell nozzle: Length of the nozzle is 80% of the conical nozzle for similar area expansion ratio

The rate of increase in the area decreases along the length of the nozzle

Large expansion just after the throat through the expansion waves (strong favorable pressure gradient)

After inflection point, weak compression waves form when the flow is turned along the axial direction

Hence, the design of the contour should be such that the weak compression waves should not merge to form a strong shock

Bell shaped nozzle

– Cone (15°) $\varepsilon = 50$

Length 100% $22.66\, r_t$

λ 0.9829

– 80% bell contour

Length (80%) $18.12\, r_t$

λ 0.988

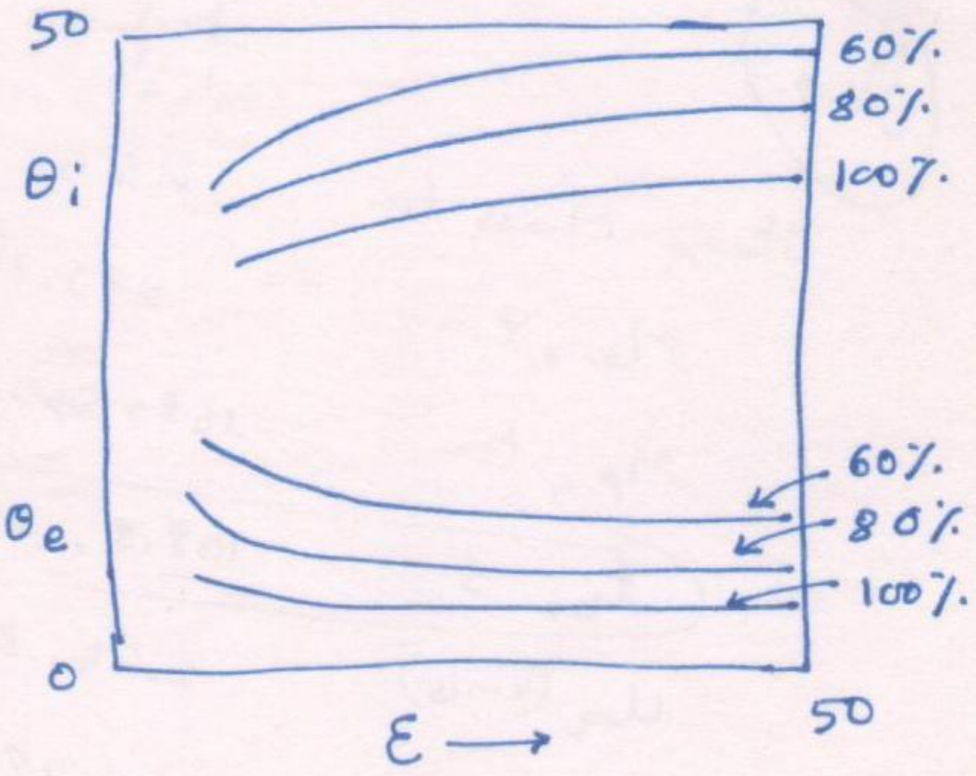
Angles $32^\circ / 7.5^\circ$

– 60% bell

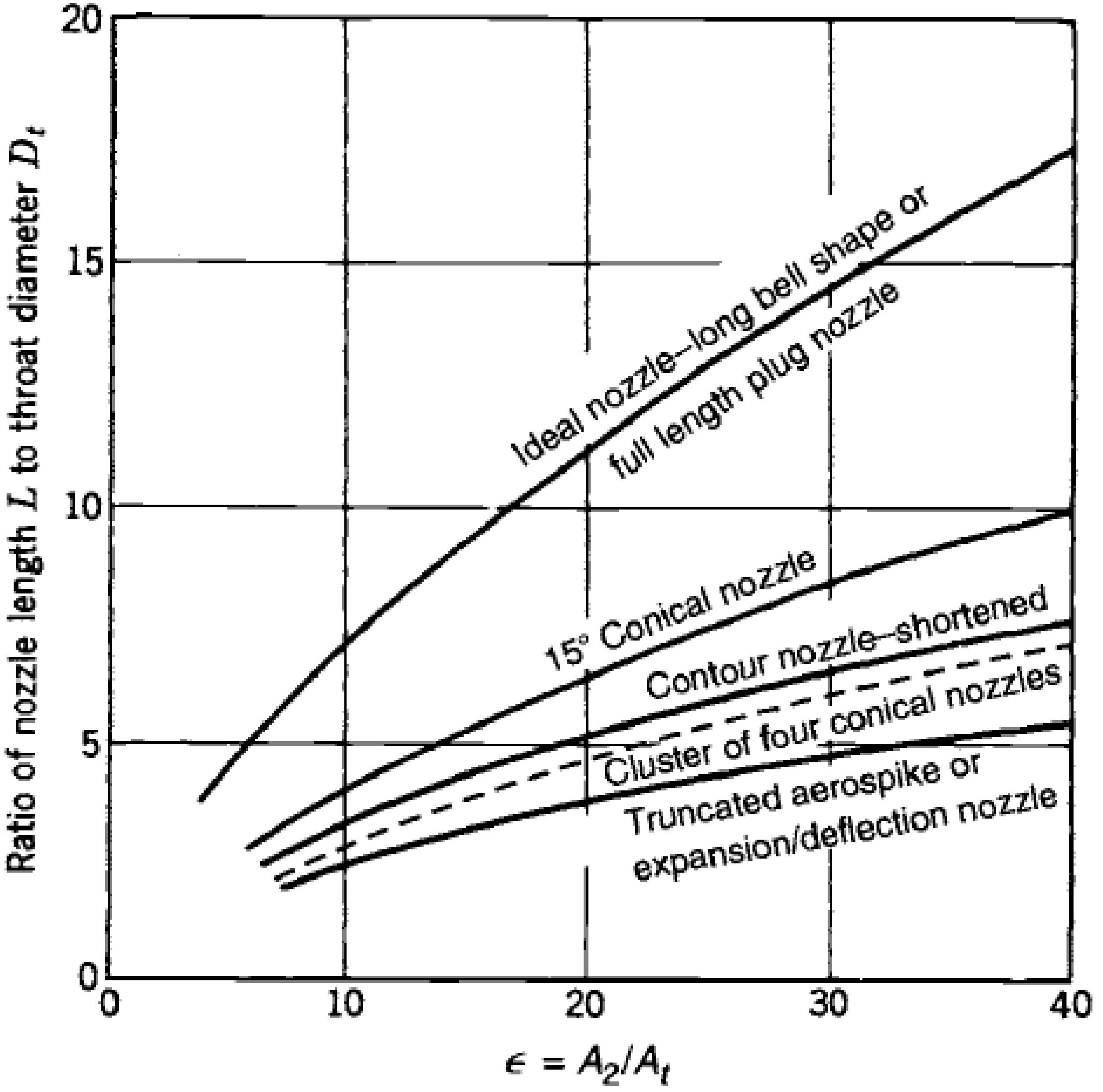
Length (60%) $13.59\, r_t$

λ 0.974

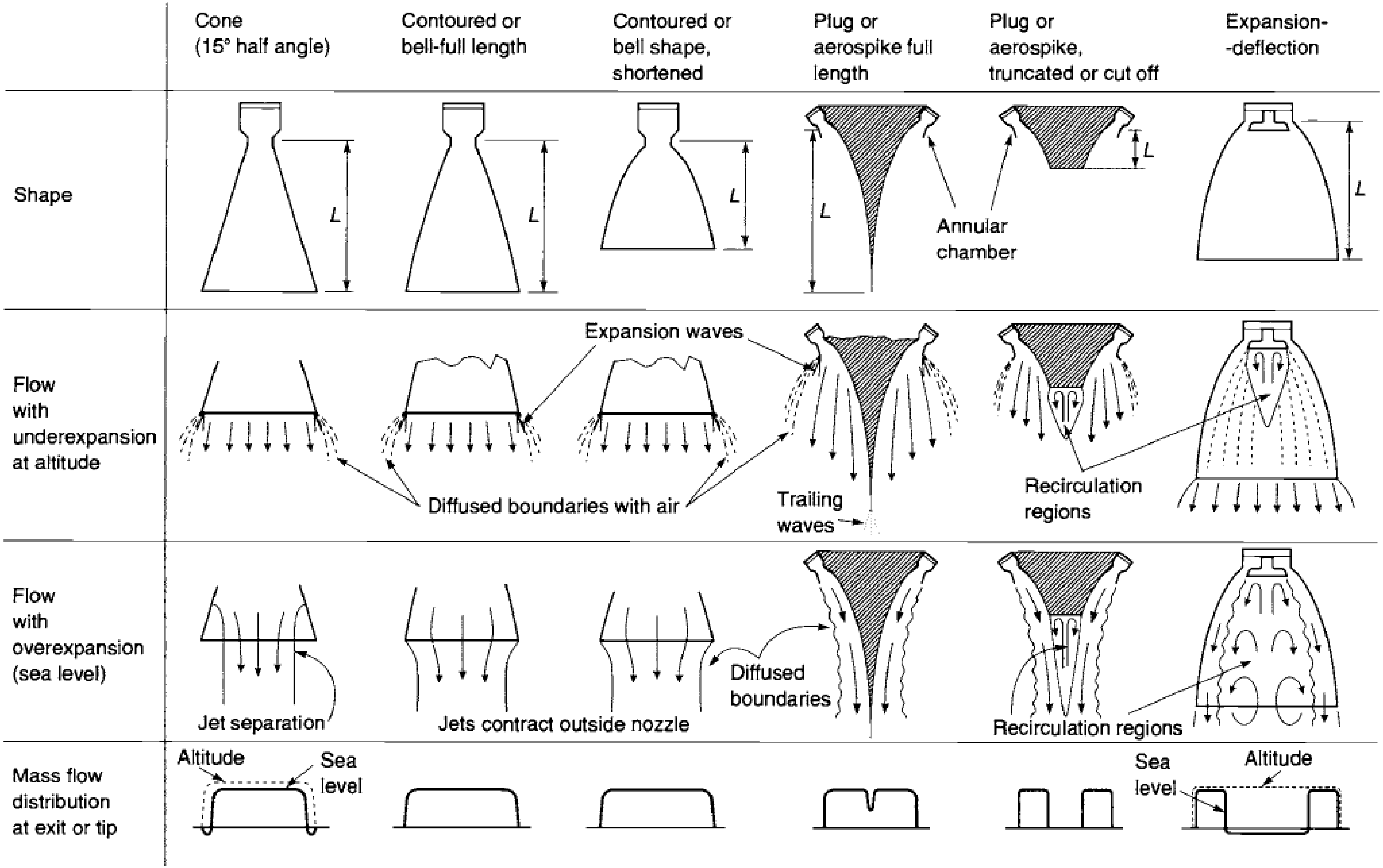
Angles $39^\circ / 18^\circ$



Minimize the divergence loss; Reduce the length of the nozzle and hence the dead weight of the hardware



Ratio of nozzle length L to throat diameter D_t
$\epsilon = A_2/A_t$
0
5
10
15
20
0
10
20
30
40
Ideal nozzle–long bell shape or
full length plug nozzle
15° Conical nozzle
Contour nozzle–shortened
Cluster of four conical nozzles
Truncated aerospike or
expansion/deflection nozzle


Cone
(15° half angle)
Contoured or
bell-full length
Contoured or
bell shape,
shortened
Plug or
aerospike full
length
Plug or
aerospike,
truncated or cut off
Expansion-
-deflection
Shape
L
Annular
chamber
Flow
with
underexpansion
at altitude
Expansion waves
Diffused boundaries with air
Trailing
waves
Recirculation
regions
Flow
with
overexpansion
(sea level)
Jet separation
Jets contract outside nozzle
Diffused
boundaries
Recirculation regions
Mass flow
distribution
at exit or tip
Altitude
Sea
level
Sea
level
Altitude

Other minor losses in the nozzle

Lower flow velocity in boundary layer: Wall friction dissipates some energy. Typical reduction in c ~ 0.5 – 1.5%

Multiphase flow: solid particles and liquid droplets ~ up to 5% loss

Gradual erosion of throat region increases the throat diameter by ~ 1-6% → reduction in chamber pressure and thrust by ~ 1-6 %

Real gas behaviour: Changes in k, Cp, MW due to non-equilibrium flow → typical loss in thrust by 0.2 – 0.7%