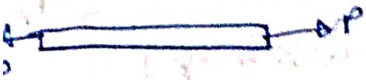

$U = \frac{1}{2}\int_0^l EA\left(\frac{\partial u}{\partial x}\right)^2 dx \qquad \frac{\partial}{\partial x}\left[EA\frac{\partial u}{\partial x}\right] + P_x(x,t) = m\frac{\partial^2 u}{\partial t^2}$

$U = \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \qquad \frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 w}{\partial x^2}\right] + m\frac{\partial^2 w}{\partial t^2} = P$

$U = \frac{1}{2}\int_0^l GJ\left(\frac{\partial \phi}{\partial x}\right)^2 dx \qquad \frac{\partial}{\partial x}\left[GJ\frac{\partial \phi}{\partial x}\right] + t_0 = \rho I_p\frac{\partial^2 \theta}{\partial t^2}$

$T = \frac{1}{2}\int_0^l I_p \dot{\phi}^2 dx$

$U = \frac{1}{2}\int_0^l T\left(\frac{\partial w}{\partial x}\right)^2 dx \qquad \frac{\partial}{\partial x}\left[T\frac{\partial w}{\partial x}\right] + 6 = m\ddot{w}$

For uniform string $w(x,t) = W(x)T(t)$: $\frac{c^2}{W}\frac{d^2W}{dx^2} = \frac{1}{T}\frac{d^2T}{dt^2} = -\omega^2$

$$\omega^2 = \frac{\int_0^l EI\left(\frac{d^2 W(x)}{dx^2}\right)^2 dx}{\int_0^l \rho A (W(x))^2 dx}$$

$T = \int_0^l \frac{1}{2} m\left(\frac{\partial w}{\partial t}\right)^2 dx$

$= \frac{1}{2}\dot{g}(t)^2 \underbrace{\int_0^l m\,6(x)^2 dx}_{\tilde{m}}$

$V = \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx$

$\frac{1}{2}g(t)^2 \underbrace{\int_0^l EI\,6''(x)^2 dx}_{k}$

1 dof damp No load under damp

$x(t) = Xe^{-\xi\omega_n t}\cos(\omega_d t - \phi)$

$X = \frac{\sqrt{x_0^2\omega_n^2 + \dot{x}_0^2 + 2x_0\dot{x}_0\xi\omega_n}}{\omega_d} \qquad \omega_n = \sqrt{\frac{k}{m}}$

$\phi = \tan^{-1}\left[\frac{\dot{x}_0 + \xi\omega_n x_0}{x_0\omega_n\sqrt{1-\xi^2}}\right] \qquad \xi = \frac{c}{2\sqrt{mk}}$

Critically damped

$x(t) = (C_1 + C_2 t)e^{-\omega_n t}$

$C_1 = x_0, \quad C_2 = \dot{x}_0 + \omega_n x_0$

Over damped

$C_1 e^{(-\xi + \sqrt{\xi^2-1})\omega_n t} + C_2 e^{(-\xi - \sqrt{\xi^2-1})\omega_n t}$

$C_1 = \frac{x_0\omega_n(\xi + \sqrt{\xi^2-1}) + \dot{x}_0}{2\omega_n\sqrt{\xi^2-1}}$

$C_2 = \frac{-x_0\omega_n(\xi - \sqrt{\xi^2-1}) - \dot{x}_0}{2\omega_n\sqrt{\xi^2-1}}$

$f_D = \frac{F_0}{k}$ Amp $\to x_0, x_0 - 2f_D$

$x_0 - 4f_D \cdots$

$\dot{x} < 0$

$x(t) \Rightarrow (x_0 - f_D)\cos\omega_n t + f_D$

$x(t) = (x_0 - 3f_D)\cos\omega_n t - f_D$

$\dot{x} > 0$

For force $\Rightarrow F_0\cos\omega t$

$\left(x_0 - \frac{F_0}{k - m\omega^2}\right)$

$\phi = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right)$

For force $= F_0\cos\omega t$

$x_p(t) = X\cos(\omega t - \phi) \qquad x_h = X_0 e^{-\xi\omega_n t}\cos(\omega_d t - \phi_0)$

$X = \frac{F_0}{[(k - m\omega^2)^2 + c^2\omega^2]^{1/2}}$

$\frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \qquad \phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$

$\frac{F_0}{k}$

$\frac{\omega}{\omega_n}$

$$X_0 = \left[(x_0 - X\cos\phi)^2 + \frac{1}{\omega_d^2}(\xi\omega_n x_0 + \dot{x}_0 - \xi\omega_n X\cos\phi - \omega X\sin\phi)^2\right]^{1/2}$$

$$\tan\phi_0 = \frac{\xi\omega_n x_0 + \dot{x}_0 - \xi\omega_n X\cos\phi - \omega X\sin\phi}{\omega_d(x_0 - X\cos\phi)}$$

$$Q_{trac} = \frac{1}{2\xi}$$

For $F(t) = F_0 e^{i\omega t}$

$$x_p = X e^{i\omega t}$$

$$X = \frac{F_0}{[(k - m\omega^2)^2 + c^2\omega^2]^{1/2}} e^{-i\phi}$$

$$Q_{trac} = \frac{\omega_n}{\omega_2 - \omega_1}$$

$$\phi = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right)$$

Force on base (Steady state case)

$$f_T(t) = F_T\cos(\omega t - \phi_T)$$

$$F_T = \frac{F_0\sqrt{1 + (2\xi r)^2}}{\sqrt{(1 - r^2)^2 + (2\xi r)^2}} = kx + c\dot{x}$$

$$\tan\phi_T = \frac{2\xi r^3}{1 - r^2 + (2\xi r)^2}$$

$$F(t) = \frac{a_0}{2} + \sum_{j=1}^{\infty} a_j\cos j\omega t + \sum_{j=1}^{\infty} b_j\sin j\omega t$$

$$a_j = \frac{2}{\tau}\int_0^{\tau} F(t)\cos j\omega t\,dt \qquad b_j = \frac{2}{\tau}\int_0^{\tau} F(t)\sin j\omega t \qquad \tau = \frac{2\pi}{\omega}$$

$$g(t) = \frac{e^{-\xi\omega_n t}}{m\omega_d}\sin\omega_d t$$

$$x(t) = \int_0^{t} g(t - \tau)F(\tau)\,d\tau$$

Base excitation problem

Relative $m\ddot{z} + c\dot{z} + kz = -m\ddot{y}$

$z = x - y$

absolute $m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$

For moving base

force on base

$$\frac{F_T}{kY} = r^2\left[\frac{1 + (2\xi r)^2}{(1 - r^2)^2 + (2\xi r)^2}\right]^{1/2}$$

phase is same as that of $x(t)$

base moving with $Y\sin\omega t$

$$\frac{X}{Y} = \left[\frac{1 + (2\xi r)^2}{(1 - r^2)^2 + (2\xi r)^2}\right]^{1/2}$$

$$\phi = \tan^{-1}\left[\frac{2\xi r^3}{1 + (4\xi^2 - 1)r^2}\right]$$

For relative case $Z = \dfrac{Y r^2}{\sqrt{(1 - r^2)^2 + (2\xi r)^2}}$

$$\phi = \tan^{-1}\left(\frac{2\xi r}{1 - r^2}\right)$$

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$$a_0 = \frac{2}{N}\sum_{i=1}^{N} F_i$$

$$a_j = \frac{2}{N}\sum_{i=1}^{N} F_i\cos\frac{2j\pi t_i}{\tau}$$

$$b_j = \frac{2}{N}\sum_{i=1}^{N} F_i\sin\frac{2j\pi t_i}{\tau}$$

Travelling wave

soln. $y = G(x - ct) + g(x + ct)$

$$G(x) = \frac{1}{2}\left[w_0(x) - \frac{1}{c}\int_{x_0}^{x}\dot{w}_0(x')\,dx'\right]$$

$$g(x) = \frac{1}{2}\left[w_0(x) + \frac{1}{c}\int_{x_0}^{x}\dot{w}_0(x')\,dx'\right]$$

$$w(x) = \sin\frac{n\pi x}{l}\left[C_n\cos\frac{nc\pi t}{l} + D_n\sin\frac{nc\pi t}{l}\right]$$

$$u(x,t) = \left(A\cos\frac{\omega x}{c} + B\sin\frac{\omega x}{c}\right)(C\cos\omega t + D\sin\omega t)$$

$$\int_{t_1}^{t_2}(\delta T - \delta V + \delta W)\,dt = 0$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$$

$$\frac{y_j}{y_{j+n}} = \frac{e^{-\xi\omega_n T}}{e^{-\xi\omega_n n T}} \qquad T = \frac{2\pi}{\omega_d}$$