Lecture 11: Swept Wings

Aerodynamics

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1 Subsonic/supersonic leading and trailing edges

This section is an expansion of the discussion on this topic appearing in Shapiro [1954] and Bertin and Cummings [2013]. Here, we simply explicate further on figure 1. Consider a wing of arbitrary planform in supersonic flow (i.e., $M_\infty > 1$). The portion of the leading edge (LE) where the Mach number normal to the LE is supersonic is called "supersonic LE." Conversely, if the flow normal to a leading edge is subsonic, then it is called a "subsonic LE." Similar terminology also applies to the trailing edge.

We make the construction as shown in figure 1. That is, we draw tangents to the LE at the Mach angle μ_∞ corresponding to the freestream. Let these touch the leading edge at points B and D. Then, by construction, the local sweep angle of the leading edge is equal to $\Lambda_\infty := 90° - \mu_\infty$ at these two points. The Mach number of the flow normal to the leading edge at these two points is $M_\infty \cos \Lambda_\infty = M_\infty \sin \mu_\infty = 1$. At all points between B and D, the local sweep angle is less than Λ_∞, so that the cosine of that is greater than $\cos \Lambda_\infty$, in turn implying that the LE-normal flow is supersonic. Hence, the portion BCD constitutes a supersonic LE. Similar arguments lead to the other labelling choices shown.

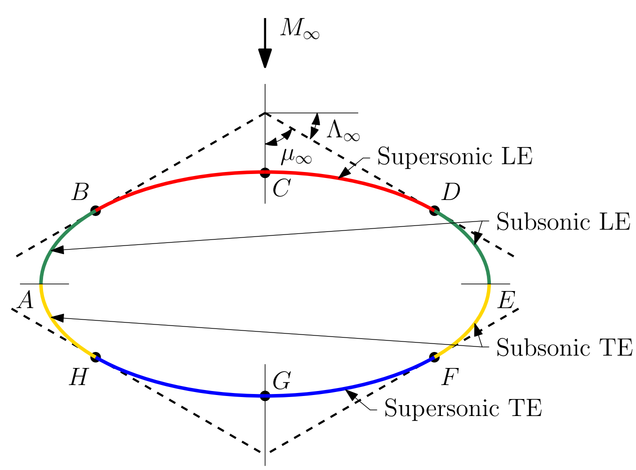

Figure 1: Schematic for determining supersonic vs. subsonic leading and trailing edges.

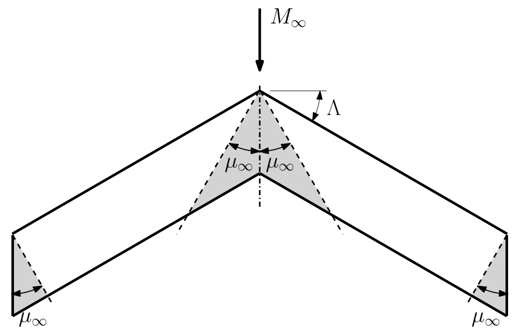

Figure 2: Shaded regions have 'conical' flow and unshaded regions have two-dimensional flow for a sweptback wing (reproduced from Bertin and Cummings [2013]).

2 Sweptback wings with 2D flow

We are interested in sweptback wings in supersonic flow, as in figure 2. If the sum of the Mach angle corresponding to the freestream flow (i.e., μ_∞) and the sweep angle Λ of the wing is less than 90°, then there is a region of the wing that has two-dimensional flow (as in a 2D airfoil). Essentially, the flow being supersonic in this region, information about its termination at the root and tip does not penetrate into the region itself. We will analyze the flow over these portions of the wing by idealizing them to be of infinite aspect ratio, as shown in figure 3. Further, we will assume the flow to be inviscid, and use shock-expansion theory. Our development essentially follows Ivey and Bowen [1947], with figures adapted from Shapiro [1954] and Bertin and Cummings [2013]. There is an error in the formulation of the original work, which was corrected in the two latter publications.

The geometry of the flow is explained in figure 3. Let the angle of attack be α and the sweep angle of the wing be Λ. The wing is not tapered. The freestream velocity vector is naturally resolved into two components – (i) a component in the plane of the wing (i.e., $M_\infty \cos\alpha$), and (ii) the residual component that is perpendicular to the plane of the wing (i.e., $M_\infty \sin\alpha$). In this problem, it is advantageous to further resolve the former into two components – (a) one perpendicular to the leading edge (i.e., $M_\infty \cos\alpha\cos\Lambda$), and (b) the remainder that is parallel to the leading edge (i.e., $M_\infty \cos\alpha\sin\Lambda$). If we neglect viscous effects, then this last component of the flow is unaffected by the presence of the wing. Thus, with reference to figure 3, it is advantageous to perform the initial analysis in section B-B, and then revert to section A-A for the final calculations. Parameters of the flow in section B-B are denoted by the subscript e; this signifies that these are 'effective' for the two-dimensional flow.

2.1 Expressions for the 'effective' flow parameters

The effective Mach number can be determined from the velocity vector diagram w.r.t. section B-B as

$$M_e = \sqrt{M_\infty^2 \sin^2\alpha + M_\infty^2 \cos^2\Lambda\cos^2\alpha} = M_\infty\sqrt{1-\sin^2\Lambda\cos^2\alpha}. \quad (1)$$

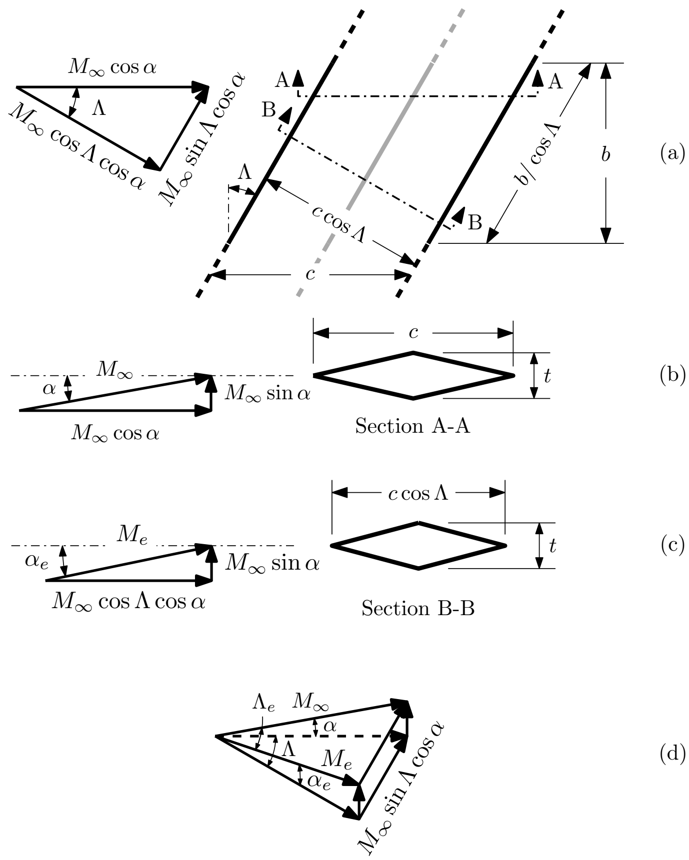

Figure 3: Nomenclature for flow around a sweptback wing of infinite aspect ratio showing (a) view in plane of wing, (b) view in plane parallel to direction of flight, (c) view in plane normal to the leading edge, and (d) complete velocity vector diagram (adapted from Shapiro [1954] and Bertin and Cummings [2013]).

The effective angle of attack can be determined likewise:

$$\tan\alpha_e = \frac{M_\infty \sin\alpha}{M_\infty \cos\Lambda\cos\alpha} = \frac{\tan\alpha}{\cos\Lambda} \qquad \Longrightarrow \alpha_e = \tan^{-1}\left(\frac{\tan\alpha}{\cos\Lambda}\right). \tag{2}$$

The effective chord, span and thickness in section B-B are

$$c_e = c\cos\Lambda, \qquad b_e = \frac{b}{\cos\Lambda}, \qquad t_e = t.$$

Thus, the effective thickness ratio is

$$\tau_e = \frac{t}{c\cos\Lambda} = \frac{\tau}{\cos\Lambda}, \tag{3}$$

where τ is the thickness ratio of the airfoil in the plane parallel to the direction of flight.

2.2 Analysis in 'effective' frame

Suppose that the section of the wing consists of a simple geometry made of planar facets – like the symmetric double-wedge airfoil shown in figure 3. Then the shock-expansion theory can be used to calculate the aerodynamic flow in this section, provided that the relevant assumptions of the theory hold – i.e.,

(a) all shocks are attached to corners of facets,

(b) flow remains attached to the surfaces, and

(c) preceding shocks do not reflect succeeding ones back on to the airfoil body.

We apply this theory in the plane normal to the leading edge of the wing. In this plane, the fractional thickness is τ_e, the Mach number is M_e and the angle of attack is α_e. The shock-expansion theory yields the in-plane lift and wave drag coefficients, denoted $c_{l,e}$ and $c_{d,w,e}$, respectively.

For later use, let us calculate the effective lift force experienced by the span b of the wing (with the effective dynamic head being $q_e = 0.5\gamma p_\infty M_e^2$):

$$L_e|_b = c_{l,e} q_e c_e b_e = c_{l,e} q_e \left(c \cos\Lambda\right) \left(\frac{b}{\cos\Lambda}\right) = c_{l,e} q_e c b.$$

This force is of course normal to the effective Mach number vector in section B-B. Similarly, the effective wave drag force experienced by the same span of the wing is

$$D_{w,e}|_b = c_{d,w,e} q_e c_e b_e = c_{d,w,e} q_e c b.$$

This force is parallel to the effective Mach number vector.

2.3 Analysis in wind frame

There being no other source of lift, the effective lift force on the span b of the wing found above is also the actual lift force experienced by this span, $L|_b$ (normal to the actual Mach number vector). But, by the definition of the sectional lift coefficient of the wing c_l, we have

$$L|_b = c_l q_\infty c b,$$

where $q_\infty = 0.5\gamma p_\infty M_\infty^2$ is the dynamic head of the freestream. Thus, we can find the sectional lift coefficient of the swept wing in terms of the previously-determined effective lift coefficient as

$$c_l = \frac{L|_b}{q_\infty c b} = \frac{q_e}{q_\infty}\frac{L_e|_b}{q_e c b} = c_{l,e}\left(\frac{M_e}{M_\infty}\right)^2. \tag{4}$$

The effective wave drag force on the span b of the wing found above is aligned with the effective Mach number vector (see figure 3(d)). It can be resolved into a component aligned with the freestream Mach number vector, and another component that is normal to it (like a side-slip force). In the assumed absence of viscous effects, the latter component of the effective drag force does not affect the wing.

Now, it will be noted from figure 3(d) that the angle between the effective and freestream Mach number vectors is Λ_e, which is different from the sweep angle of the wing. The sweep angle is of course defined in the plan of the wing, whereas Λ_e is defined in the plane of the wind vector. We can find Λ_e by invoking the cosine rule for the triangle consisting of arms M_∞, M_e and $M_\infty \sin\Lambda\cos\alpha$:

$$\cos\Lambda_e = \frac{M_\infty^2 + M_e^2 - (M_\infty \sin\Lambda\cos\alpha)^2}{2M_\infty M_e} = \frac{1 + (1-\sin^2\Lambda\cos^2\alpha) - (\sin^2\Lambda\cos^2\alpha)}{2\sqrt{1-\sin^2\Lambda\cos^2\alpha}}$$
$$\implies \Lambda_e = \cos^{-1}\sqrt{1-\sin^2\Lambda\cos^2\alpha} = \sin^{-1}(\sin\Lambda\cos\alpha).$$

Evidently, $\Lambda_e \leq \Lambda$, with the difference increasing with angle of attack.

Thus, the wave drag force on the span b of the wing is

$$D_w\big|_b = D_{w,e}\big|_b \cos\Lambda_e = c_{d,w,e} q_e c b \cos\Lambda_e.$$

Now, by the definition, the sectional wave drag coefficient of the wing $c_{d,w}$, we have

$$D_w\big|_b = c_{d,w} q_\infty c b.$$

Bringing everything together,

$$c_{d,w} = c_{d,w,e}\cos\Lambda_e \left(\frac{M_e}{M_\infty}\right)^2. \tag{5}$$

This final result is slightly different from Bertin and Cummings [2013], who took the angle between the effective drag and the actual drag as the sweep angle Λ itself. Shapiro [1954], on the other hand, did not explicitly state any relation between $c_{d,w}$ and $c_{d,w,e}$. The original work of Ivey and Bowen [1947] had errors in both the lift and wave drag expressions, as discussed in Appendix A.

The wave drag is of course just one component of drag. In two-dimensional flow, the other main component is skin-friction drag. It is assumed here that the skin-friction drag coefficient is independent of angle of attack, freestream Mach number and sweep angle. Thus, we can directly add it to the wave drag coefficient found above. We do not consider form drag in this analysis.

2.4 Results

In figure 4, we recreate the results presented originally by Ivey and Bowen [1947], and subsequently by Bertin and Cummings [2013]. These consider sweptback wings of the type under consideration, i.e., those with supersonic leading edge. The section is a symmetric double-wedge airfoil, where the effective thickness ratio τ_e is maintained constant with changes in sweep angle (reflecting typical structural design prerogative). These results demonstrate the substantial benefits accrued from sweeping the wing. The aerodynamic efficiency is increased, especially at higher Mach numbers. The lift slope is also increased with sweep, although this benefit is greater at lower Mach numbers (and at high sweep angles).

Comparison with the results presented in Bertin and Cummings [2013] reveals two main differences. First, the aerodynamic efficiency curves are slightly lower. This may be due to the increase in wave drag coefficient due to the correction of the effective sweep angle. Second, the curves do not continue for as long. For example, at $M_\infty = 2$, the $\Lambda = 45°$ curve

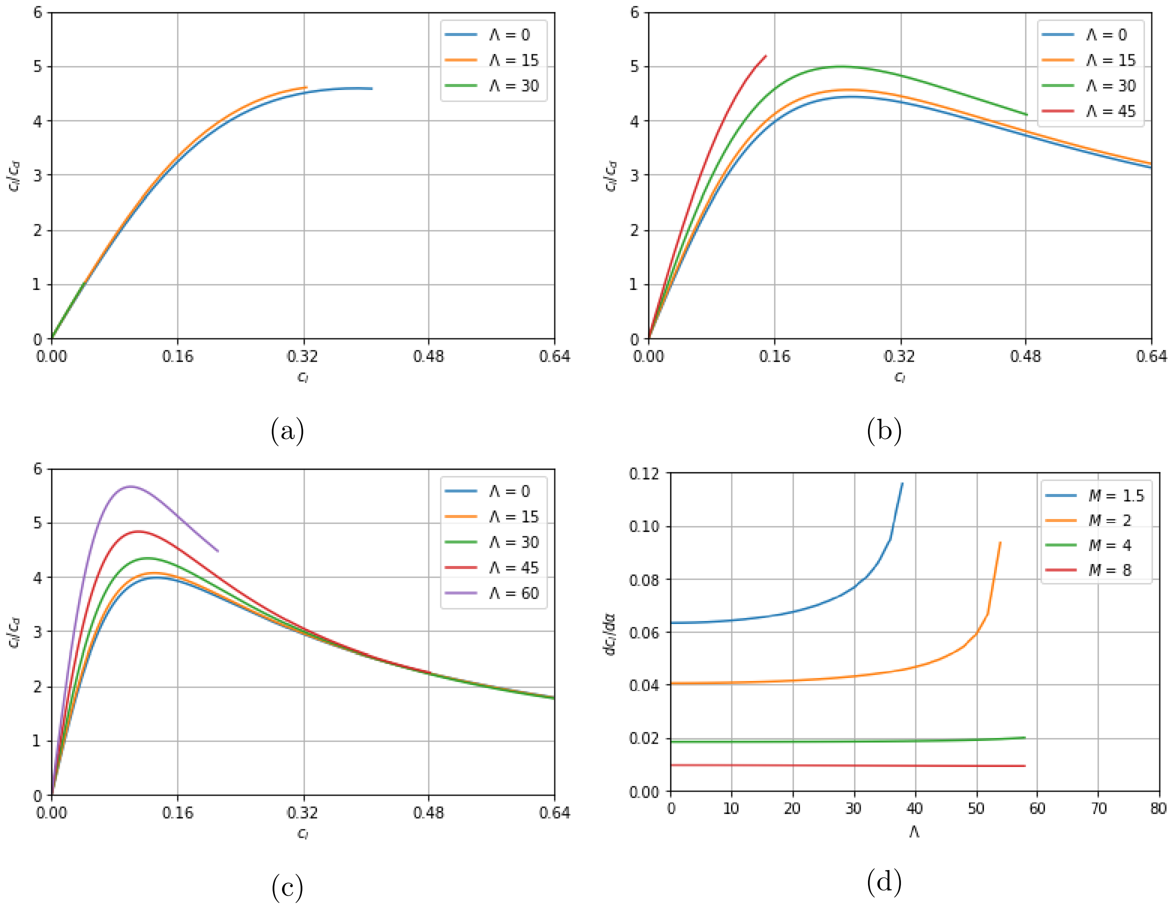

Figure 4: Theoretical effect of sweepback for symmetric double-wedge section airfoils of effective thickness ratio $\tau_e = 0.1$ with supersonic leading edges: lift-to-drag ratio for (a) $M_\infty = 1.5$, (b) $M_\infty = 2$, and (c) $M_\infty = 4$. (d) Lift slope for a $\tau_e = 0.05$ airfoil at $\alpha = 0°$. In all cases, a skin friction drag coefficient of 0.006 is assumed.

stops before $c_l = 0.16$, whereas it continues till past $c_l = 0.2$ in the reference. The curves are of course terminated when the solution is not possible with shock-expansion theory. This happens either when the shock is no longer attached to the leading edge, or when the flow is subsonic after the leading-edge oblique shock so that an expansion fan is not possible at the mid-chord corner. The reason for this discrepancy could not be found.

Appendix

A Error in Ivey and Bowen [1947]

There appears to be a mistake in the original work of Ivey and Bowen [1947], although the consequent discrepancy in the result is very slight. Those authors first found the pressure on the four facets of the double-wedge airfoil in the effective frame (section B-B), as we have done here too. However, instead of finding the effective lift and wave drag coefficients in the effective frame, they considered these pressures to act on the corresponding facets in the actual wind frame (section A-A) too, and directly found the lift and wave drag coefficient in this frame. The error is that the pressure forces on the facets are NOT in the wind frame (section A-A).

References

J. J. Bertin and R. M. Cummings. *Aerodynamics for engineers.* Pearson Education International, sixth edition, 2013.

H. R. Ivey and E. H. Bowen. Theoretical supersonic lift and drag characteristics of symmetrical wedge-shape-airfoil sections as affected by sweepback outside the Mach cone. Technical Report NACA Tech. Note 1226, 1947.

A. H. Shapiro. *The dynamics and thermodynamics of compressible fluid flow, Vol. 2.* Ronald Press, New York, 1954.