

American Computer Science League

2020-2021 • Contest 4: Solutions • Intermediate Division

1. Graph Theory

Cubing the adjacency matrix will give the number of paths of length 3 in the directed graph.

$$\begin{array}{c}
 A \quad B \quad C \quad D \\
 \begin{array}{c}
 A \\
 B \\
 C \\
 D
 \end{array}
 \begin{bmatrix}
 1 & 0 & 0 & 1 \\
 0 & 1 & 1 & 0 \\
 1 & 0 & 1 & 0 \\
 0 & 0 & 1 & 1
 \end{bmatrix}^3 = \begin{bmatrix}
 3 & 0 & 3 & 3 \\
 3 & 1 & 3 & 1 \\
 3 & 0 & 2 & 3 \\
 3 & 0 & 3 & 2
 \end{bmatrix}
 \end{array}$$

A, C and D do not have paths of length 3 to vertex B.

1. A, C, D (A)

2. Graph Theory

There are 7 cycles from B in the directed graph: BAB, BCAB, BDB, BADB, BAEDB, BCADB, and BCAEDB.

2. 7 (C)

3. Digital Electronics

The Boolean expression represented by the digital circuit is:

$$\begin{aligned}
 & \overline{(A + (\overline{AB})(B + C))C} \\
 &= \overline{\overline{A}((\overline{AB})(B + C))C} \\
 &= \overline{A}C((\overline{AB}) + (\overline{B + C})) \\
 &= \overline{A}C(AB + \overline{B}\overline{C}) \\
 &= A\overline{A}BC + \overline{A}\overline{B}C\overline{C} \\
 &= 0 + 0 \\
 &= 0
 \end{aligned}$$

Since the expression is always FALSE, none, or 0, make it TRUE.

3. 0 (A)

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INTERMEDIATE DIVISION SOLUTIONS

4. Digital Electronics

The Boolean expression represented by the digital circuit is:

$$\begin{aligned}(A \oplus B)(B \oplus C) &= (\overline{A}B + A\overline{B})(\overline{B}C + B\overline{C}) \\ &= \overline{A}B\overline{B}C + \overline{A}BB\overline{C} + A\overline{B}\overline{B}C + A\overline{B}B\overline{C} \\ &= 0 + \overline{A}B\overline{C} + A\overline{B}C + 0 \\ &= \overline{A}B\overline{C} + A\overline{B}C\end{aligned}$$

To be TRUE one of the terms must be TRUE.

$$\overline{A}B\overline{C} = 1 \Rightarrow A = 0, B = 1, C = 0 \Rightarrow (0, 1, 0)$$

$$A\overline{B}C = 1 \Rightarrow A = 1, B = 0, C = 1 \Rightarrow (1, 0, 1)$$

4. (0, 1, 0) (1, 0, 1) (E)

5. Assembly Language

This is a translation of the assembly language program:

```
x = 21
y = 49
while y != 0
    r = int(x / y) * y
    r = x - r
    x = y
    y = r
end while
output x
```

This is the Euclidean Algorithm to find the greatest common divisor of two numbers. The GCF of 21 and 49 is 7.

5. 7 (B)