American Computer Science League

2021 Finals ● Solutions to Short Problems ● Intermediate/Classroom Divisions

1. Boolean Algebra

$\overline{\overline{A} + B} \oplus \overline{A + \overline{B}} = \overline{\overline{A}}\,\overline{B} \oplus \overline{A}\,\overline{\overline{B}}$

$= A\,\overline{B} \oplus \overline{A}\,B$

$= (A\,\overline{B})(\overline{\overline{A}\,B}) + (\overline{A\,\overline{B}})(\overline{A}\,B)$

$= (A\,\overline{B}\,(A + \overline{B})) + ((\overline{A} + B)\,\overline{A}\,B)$

$= (A\,\overline{B}\,(A + \overline{B})) + ((\overline{A} + B)\,\overline{A}\,B)$

$= A\,\overline{B} + \overline{A}\,B$ Choice d

$= A \oplus B$ Choice b

Choice a simplifies to the Boolean expression B.

$A \oplus B$ cannot be equal to its negation so choice c is invalid.

1. b, d (D)

2. Boolean Algebra

$(\overline{AB + C})(\overline{A}\,\overline{B} + \overline{C}) = (\overline{A\,B}\,\overline{C})(\overline{A}\,\overline{B} + \overline{C})$

$= (\overline{A} + \overline{B})\,\overline{C}\,(\overline{A}\,\overline{B} + \overline{C})$

To be TRUE $\overline{C} = 1$, *so* $C = 0$.

The expression simplifies to: $(\overline{A} + \overline{B})\,1\,(\overline{A}\,\overline{B} + 1)$

$= \overline{A} + \overline{B}$

This is FALSE only when $A = B = 1$.

Therefore 3 ordered triples make it TRUE:

(0, 0, 0), (0, 1, 0), and (1, 0, 0)

2. 3 (B)

3. Bit-String Flicking

(LSHIFT-3 (RCIRC-2 (NOT (LCIRC-10 011010))))
= (LSHIFT-3 (RCIRC-2 (NOT 100110)))
= (LSHIFT-3 (RCIRC-2 011001))
= (LSHIFT-3 010110)
= 110000

3. 110000 (C)

4. Bit-String Flicking

Let X = abcde

(X AND (LSHIFT-2 01011)) = (RCIRC-3 00011)

(abcde AND (LSHIFT-2 01011)) = (RCIRC-3 00011)

LHS = (abcde AND (LSHIFT-2 01011))
= (abcde AND 01100)
= 0bc00

RHS = (RCIRC-3 00011)
= 01100

LHS = RHS ⇒ 0bc00 = 01100
⇒ a = *, b = 1, c = 1, d = *, e = *
⇒ *11** (8 answers)

4. 8 (C)

5. Recursive Functions

Work backwards from the answer choices to see which one evaluates $f(3,-2) = 1/9$.

$b/f(b,e-1)$: $f(3,-2) = 3/f(3,-3) = 3/(3/f(3,-4)\ldots$
This does not work since the exponent decreases each time, causing an infinite loop.

$f(b,e+1)/b$: $f(3,-2) = f(3,-1)/3 = f(3,0)/3/3 = 1/9$
This one calculates the correct answer.

$f(b,e-1)/b$: $f(3,-2) = f(3,-3)/3 = f(3,-4)/3/3\ldots$
Numerator never stops.

$b/f(b,e+1)$: $f(3,-2) = 3/f(3-1) = 3/3/f(3,0) = 1$

5. $f(b,e+1)/b$ (B)

6. Recursive Functions

$f(1,20) = f(1+3, 20-2) - (1-20) = f(4,18) + 19 = 52 + 19 = 71$
$f(4,18) = f(4+3, 18-2) - (4-18) = f(7,16) + 14 = 38 + 14 = 52$
$f(7,16) = f(7+3, 16-2) - (7-16) = f(10,14) + 9 = 29 + 9 = 38$
$f(10,14) = f(10+3, 14-2) - (10-14) = f(13,12) + 4 = 25 + 4 = 29$
$f(13,12) = 13 + 12 = 25$

6. 71 (D)

7. Digital Electronics

The circuit translates to: $\overline{(A + \overline{B\,C})(\overline{B\,C} + C)}$

$= \overline{(A + \overline{B} + \overline{C})(\overline{B} + \overline{C} + C)}$
$= \overline{A + \overline{B} + \overline{C}}$
$= \overline{A}\,B\,C$
$\Rightarrow A = 0, B = 1, C = 1 \quad (0, 1, 1)$

7. 1 (A)

8. Digital Electronics

A	B	C	$A\,B$	$B + C$	$\overline{B + C}$	AND	OR	NAND	NOR
0	0	0	0	0	1	0	1	1	0
0	0	1	0	1	0	0	0	1	1
0	1	0	0	1	0	0	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	0	0	0	1	0	1	1	0
1	0	1	0	1	0	0	0	1	1
1	1	0	0	1	0	0	0	1	1
1	1	1	1	1	0	0	1	1	0

8. NAND (C)

9. Prefix-Infix-Postfix

/ - + / # 2 5 3 1 # 4 1 + # 1 3 1 = / - + / (# 2 5) 3 1 (# 4 1) + (# 1 3) 1

9. 3 (B)

= / - + (/ 57 3) 1 5 (+ 4 1)
= / - (+ 19 1) 5 5
= / (- 20 5) 5
= / 15 5
= 3

10. Prefix-Infix-Postfix

$\frac{A+\sqrt{B^2-C}}{3\,D}$ = $(A\ +\ (B\uparrow 2\ -\ C)\uparrow(1/2))\ /\ (3\ *\ D)$ in Infix notation

= $(A\ +\ (B\ 2\uparrow C\ -)\uparrow(1\ 2\ /))\ /\ (3\ D\ *)$
= $(A\ +\ (B\ 2\uparrow C\ -\ 1\ 2\ /\uparrow))\ /\ (3\ D\ *)$
= $(A\ B\ 2\uparrow C\ -\ 1\ 2\ /\uparrow\ +)\ /\ (3\ D\ *)$
= $A\ B\ 2\uparrow C\ -\ 1\ 2\ /\uparrow\ +\ 3\ D\ *\ /$

10. $AB2\uparrow C\ -\ 12/\uparrow\ +\ 3D\ *\ /$ (D)

11. Computer Number Systems

$3B4A_{16}$ OR $9D2E_{16}$ = $0011\ 1011\ 0100\ 1010_2$
OR $\underline{1001\ 1101\ 0010\ 1110_2}$
$1011\ 1111\ 0110\ 1110_2$
$1\ 011\ 111\ 101\ 101\ 110_2$
$1\ \ 3\ \ 7\ \ \ 5\ \ 5\ \ 6_8$

11. 137556 (B)

12. Computer Number Systems

2021 = 11111100101_2 2 consecutive 0s
2020 = 11111100100_2 2 consecutive 0s
2019 = 11111100011_2 3 consecutive 0s
2018 = 11111100010_2 3 consecutive 0s
2017 = 11111100001_2 4 consecutive 0s

12. 2017 (C)

13. Data Structures

ACSL*FINALS**COMPETITION***THIS*YEAR*

The queue (FIFO) is built as follows:

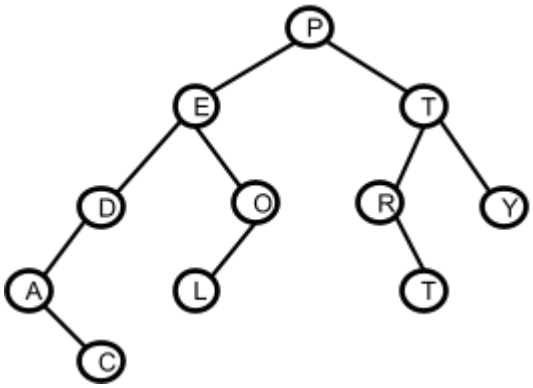
C, CS, CSL, L, LF, LFN, LFNL, LFNLS, NLS, S, SC, SCM, SCMP, SCMPT, SCMPTT, SCMPTTN, MPTTN, TTN, N, NT, NTH, NTHS, HS, HSY, HSYR, YR

The longest length of the queue is 7: SCMPTTN

13. 7 (C)

14. Data Structures

The binary search tree for **PTERODACTYL is:**



The preorder transversal (root, left, right) is: PEDACOLTRTY

14. PEDACOLTRTY (B)

15. Graph Theory

Squaring the adjacency matrix gives the number of paths of length 2.

$$\begin{vmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{vmatrix}^2 = \begin{vmatrix} 1 & 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{vmatrix}$$

There are 12 paths of length 2 that do not exist: AB, AE, BA, BB, BC, BE, CB, CC, CD, CE, DA and DD. However if only pairs are counted (order does not count), there are 5: AB/BA, BB, BC/CB, CC, DD

The others are eliminated because there is a path in the other order-

15. 5 (C)

AE has none but EA has 1.	
16. Graph Theory There are 5 cycles from A: ABCDA, ACDA, AEA, AECDA, AEDA	16. 5 (C)
17. What Does This Program Do? There are 6 possible original arrangements of 3 numbers. Choices (A), (B) and (C) work for some of the 6 arrangements but only (A) works on all 6.	17. a > b (A)
18. LISP (CDDADDR '(P (R O) (G (R A) (M M) I) (N G))) = (CDDADR '((R O) (G (R A) (M M) I) (N G))) = (CDDAR '((G (R A) (M M) I) (N G))) = (CDDR '(G (R A) (M M) I)) = (CDR '((R A) (M M) I)) = ((M M) I)	18. ((M M) I) (B)
19. FSAs and Regular Expressions **[a-l]*[e-u]*(cs|y)** The expressions that are not accepted are: c. physics - fails at the "y" d. botany - fails at the "a" g. mathematics - fails at the first "a"	19. c, d, g (B)
20. Assembly Language This program sums the proper factors of a number and outputs that sum. For 6: 1 + 2 + 3 = 6 which makes 6 a perfect number.	20. 6 (C)
