

American Computer Science League

2021 Finals • Short Problems • Senior Division

1. Boolean Algebra

Simplify the following expression:

$$\overline{ABC(A + B)(\overline{B} + \overline{C})}$$

- A. \overline{ABC}
- B. $A + \overline{B} + C$
- C. \overline{ABC}
- D. $\overline{A} + B + \overline{C}$
- E. None of the above

2. Boolean Algebra

How many ordered triples make the following expression TRUE?

$$(\overline{AB} + \overline{C}) + (\overline{A} * \overline{B} + \overline{C})$$

- A. 1
- B. 3
- C. 5
- D. 7
- E. None of the above

3. Bit-String Flicking

Evaluate the following expression:

$$((\text{LCIRC-25 } 011001) \text{ OR } (\text{RCIRC-16 } 101101) \text{ AND } (\text{NOT } 011100))$$

- A. 110010
- B. 100010
- C. 100011
- D. 110110
- E. None of the above

4. Bit-String Flicking

Which bit string values of X (a bitstring of 5 bits) make the following equation TRUE?

$$((\text{LSHIFT-1 } X) \text{ OR } (\text{NOT } (\text{RSHIFT-1 } 10111))) = X$$

- A. 11111
- B. 11100
- C. 1110*
- D. 111*0
- E. None of the above

5. Recursive Functions

Find $f(20)$ given:

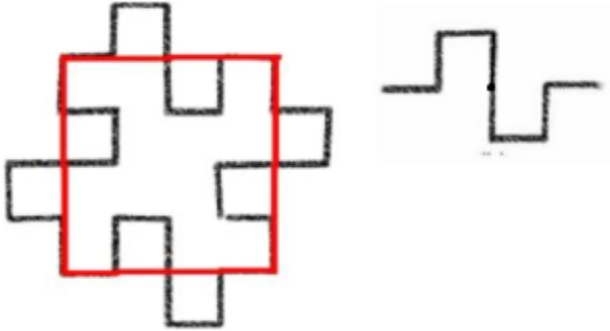
$$f(x) = \begin{cases} f(f(x-3)) + 3 & \text{if } x \geq 10 \\ f(x-2) - 2 & \text{if } 4 < x < 10 \\ x - 1 & \text{if } x \leq 4 \end{cases}$$

- A. 1
- B. 3
- C. 5
- D. 7
- E. None of the above

6. Recursive Functions

The squareflake curve is generated by beginning with a square (Stage 1), then replacing each side of the square with the figure on the right using 8 smaller segments of the same length to form a new curve (Stage 2).

Continue the process on each segment of the new curve that is created.



The rule for generating the squareflake fractal curve is to replace each line segment with the zig-zag curve consisting of 8 segments shown above on the right.

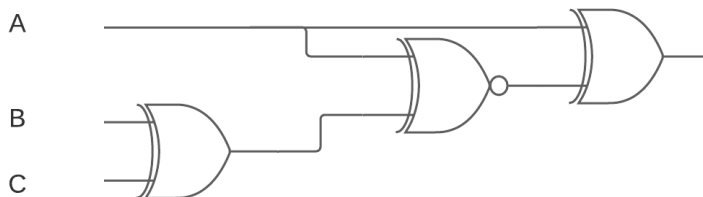
The diagram is taken from <https://nrich.maths.org/>

If the original red square has a perimeter of length 64, during the construction of this fractal, at which stage will the length of the entire perimeter of the new curve exceed 4000?

- A. 5
- B. 6
- C. 7
- D. 8
- E. None of the above

7. Digital Electronics

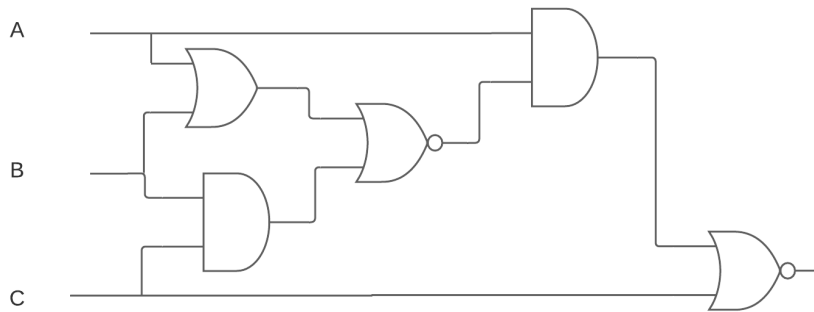
How many ordered triples make the following circuit TRUE?



- A. 0
- B. 2
- C. 4
- D. 8
- E. None of the above

8. Digital Electronics

Simplify the Boolean expression represented by the digital circuit below:



- A. $\overline{A} C$
- B. \overline{C}
- C. $B C$
- D. C
- E. None of the above

9. Prefix-Infix-Postfix

Convert the following postfix expression to prefix if & is a trinary operator and ! is a unary operator (numbers are single digits):

3 4 5 + 7 2 1 & 6 ! * * 7 8 - 4 3 + 2 & 3 2 ^ ! / &

- A. $\&3^{**}+45\&721!6/\&-78+432!^{^32}$
- B. $\&3^{*}+45^{*}\&721!6/\&-78+432!^{^32}$
- C. $\&3^{*}+45\&^{*}721!6/\&-78+432!^{^32}$
- D. $\&3^{*}+45^{*}\&721!6/\&-78+432!^{^32}$
- E. None of the above

10. Prefix-Infix-Postfix

Evaluate this prefix expression if all numbers are single digits:

* - + 4 7 - 1 8 / + ^ 5 2 * 5 - 6 2 ^ 3 2

- A. -90
- B. -20
- C. 90
- D. 120
- E. None of the above

11. Computer Number Systems

Evaluate the following expression in hexadecimal:

$A74F2C_{16} / 4_{16}$

- A. $A74F2B_{16}$
- B. $29D3CB_{16}$
- C. 12351714_{16}
- D. $57A8E2_{16}$
- E. None of the above

12. Computer Number Systems

In the octal representation for every year from 2001 to 2021 inclusive, how many total digits are greater than 4?

- A. 25
- B. 26
- C. 27
- D. 28
- E. None of the above

13. Data Structures

What nodes (left to right) are at a depth of 2 in the min-heap created for the string:

TRICERATOPS

- A. CERA
- B. OERA
- C. OIRR
- D. TIRR
- E. None of the above

14. Data Structures

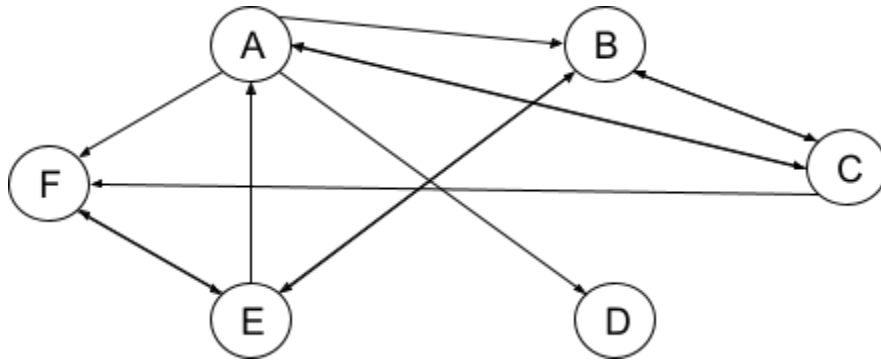
Accountants use two ways of accounting for calculating taxes on stock distributions: LIFO and FIFO. Suppose you bought 100 shares of a stock in each of the months March, April, and May for \$10 per share, \$15, and \$5 respectively. If you sold 100 shares in July when the price was \$20 per share, that means you now have 200 shares left. With FIFO accounting, you would calculate the profit based on the 100 shares you bought in March. With LIFO accounting, you would calculate the profit based on the 100 shares you bought in May.

Calculate the total profit each way and determine which method would be to your advantage as the taxpayer (i.e. less profit to pay taxes on) and how much would you save if the rate is 20% on all profits?

- A. FIFO, saving \$100
 - B. LIFO, saving \$100
 - C. FIFO, saving \$500
 - D. LIFO, saving \$500
 - E. No difference
-

15. Graph Theory

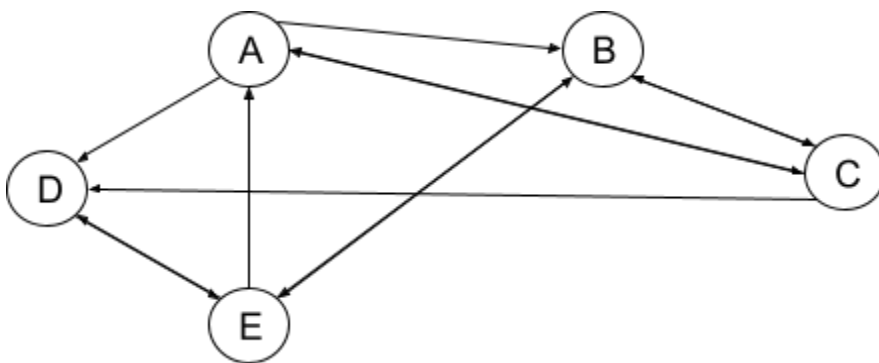
Given the directed graph below, how many unique pairs of vertices do not have a path of length 2 in either direction? (X,X) could be included.



- A. 3
- B. 4
- C. 5
- D. 6
- E. None of the above

16. Graph Theory

How many cycles are there in the directed graph below?



- A. 8
- B. 9
- C. 10
- D. 11
- E. None of the above

17. What Does This Program Do?

Pascal's Triangle is created as follows:

```
      1
     1 1
    1 2 1
   1 3 3 1
  1 4 6 4 1
```

Given that row X (starting at row 0) is stored in a one-dimensional array named **pt** of $X+1$ items, and the first (location 0) and last (location X) items are always assigned the value 1, which of the following loops will create the other values in row $X+1$ successfully?

- A. for $i = 1$ to X
 $pt(i) = pt(i) + pt(i-1)$
 next i
- B. for $i = X$ to 1 step -1
 $pt(i) = pt(i) + pt(i-1)$
 next i
- C. for $i = X$ to 1 step -1
 $pt(i+1) = pt(i) + pt(i-1)$
 next i
- D. for $i = 1$ to X
 $pt(i-1) = pt(i-1) + pt(i)$
 next i
- E. None of the above

18. LISP

After the following LISP program is run, what is the value of the CONS expression?

```
(SETQ S '(H ((A C) K) (E (R R (A))) (N (K))))
(SETQ G (CADDADADDR S))
(SETQ Q '((P L) (A (T (F O) R)) M))
(SETQ H (CADADADR Q))
(CONS G H)
```

- A. ((A) F O)
- B. (A F O)
- C. ((A) (F O))
- D. (A (F O))
- E. None of the above

19. FSA and Regular Expressions

A new school supply store just opened and is offering a 50% discount on any item that matches the following regular expression:

$[^aeiou][aeiou][^a-j][p-t]?.(er|s)^*$

How many different items will be discounted from the following items?

crayons	markers	rulers	erasers
staples	staplers	notebooks	desks
computers	tablets	boards	compass

- A. 7
- B. 6
- C. 5
- D. 4
- E. None of the above

20. Assembly Language

What is printed when the following program is run if the data is 8 and 3?

```

      READ      X
      READ      Y
      LOAD      X
      SUB       Y
      STORE     Z
      LOAD      =1
      STORE     C
      STORE     B
      STORE     A
T1    LOAD      C
      MULT      Z
      STORE     C
      LOAD      Z
      SUB       =1
      STORE     Z
      BG        T1
T2    LOAD      B
      MULT      Y
      STORE     B
      LOAD      Y
      SUB       =1
      STORE     Y
      BG        T2
T3    LOAD      A
      MULT      X
      STORE     A
      LOAD      X
      SUB       =1
      STORE     X
      BG        T3
      LOAD      A
      DIV       B
      DIV       C
      STORE     D
      PRINT     D
      END
```

- A. 6,720
- B. 336
- C. 56
- D. 24
- E. None of the above

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1. 布尔代数

化简以下表达式：

$$\overline{ABC(A + B)(\overline{B} + \overline{C})}$$

- A. \overline{ABC}
- B. $A + \overline{B} + C$
- C. \overline{ABC}
- D. $\overline{A} + B + \overline{C}$
- E. 以上都不正确

2. 布尔代数

有多少个有序三元组可使以下表达式成立？

$$(\overline{AB + C}) + (\overline{A * B + C})$$

- A. 1
- B. 3
- C. 5
- D. 7
- E. 以上都不正确

3. 比特位操作

计算下列表达式：

$$((\text{LCIRC-25 } 011001) \text{ OR } (\text{RCIRC-16 } 101101) \text{ AND } (\text{NOT } 011100))$$

- A. 110010
- B. 100010
- C. 100011
- D. 110110
- E. 以上都不正确

4. 比特位操作

X (一个长度为 5 比特的位串) 取哪个值可使下面的等式成立？

$$((\text{LSHIFT-1 } X) \text{ OR } (\text{NOT } (\text{RSHIFT-1 } 10111))) = X$$

- A. 11111
- B. 11100
- C. 1110*
- D. 111*0
- E. 以上都不正确

5. 递归函数

已知下列函数，求 $f(20)$ ：

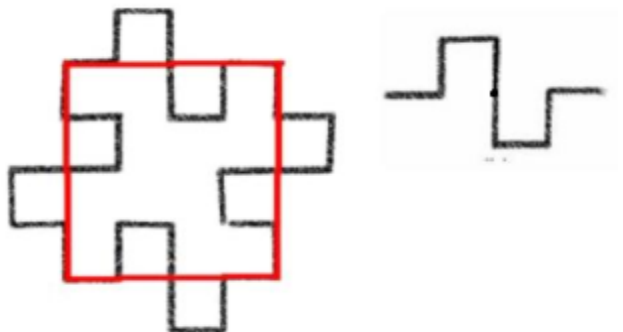
$$f(x) = \begin{cases} f(f(x-3)) + 3 & \text{设 } x \geq 10 \\ f(x-2) - 2 & \text{设 } 4 < x < 10 \\ x - 1 & \text{设 } x \leq 4 \end{cases}$$

- A. 1
- B. 3
- C. 5
- D. 7
- E. 以上都不正确

6. 递归函数

构建方形曲线的步骤为：从一个正方形开始（步骤 1），用右侧的图形来替换正方形的每边（步骤 2），右侧图形由 8 条长度相同的线段构成，从而形成新曲线。

在构建的新曲线的每条线段上继续这一过程。



方形曲线的构建方式是将每条线段替换为右上方所示的由 8 条线段组成的 Z 字形曲线。

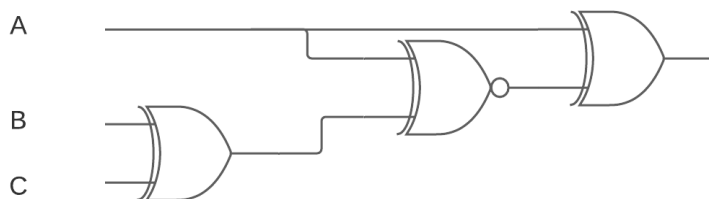
图片来自<https://nrich.maths.org/>

如果原来红色正方形的周长为 64，那么在构建方形曲线的过程中，进行到哪一步骤时周长会超过 4000？

- A. 5
- B. 6
- C. 7
- D. 8
- E. 以上都不正确

7. 数字电路

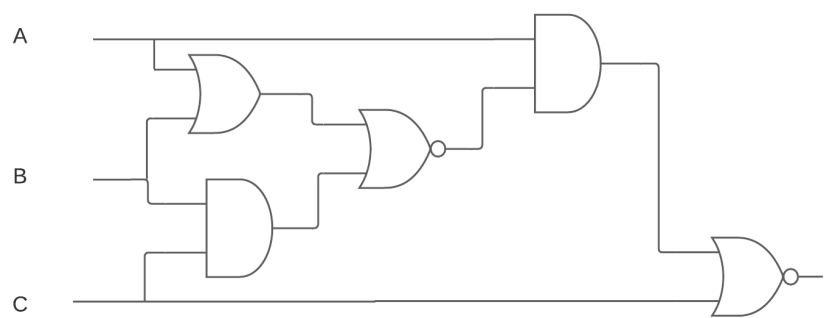
有多少个有序三元组可使下列电路成立？



- A. 0
- B. 2
- C. 4
- D. 8
- E. 以上都不正确

8. 数字电路

简化以下数字电路表示的布尔表达式：



- A. $\overline{A} C$
- B. \overline{C}
- C. $B C$
- D. C
- E. 以上都不正确

9. 前缀-中缀-后缀

设 & 为一个三元运算符，! 为一个一元运算符 (所有数字都是个位数)，将下列后缀表达式转化为前缀表达式：

$3\ 4\ 5 + 7\ 2\ 1\ \&\ 6!\ *\ 7\ 8 - 4\ 3 + 2\ \&\ 3\ 2\ ^!\ /\ \&$

- A. $\&3^{**}+45\&721!6/\&-78+4\ 32!^{\wedge}32$
- B. $\&3^{*}+4\ 5*\&721!6/\&-78+432!^{\wedge}32$
- C. $\&3^{*}+45\&*721!6/\&-78+432!^{\wedge}32$
- D. $\&3^{*}+45*\&721!6/\&-78+432!^{\wedge}32$
- E. 以上都不正确

10. 前缀-中缀-后缀

设所有的数字都是个位数，计算以下前缀表达式：

$*-+4\ 7-1\ 8/\wedge 5\ 2*5-6\ 2^{\wedge}3\ 2$

- A. -90
- B. -20
- C. 90
- D. 120
- E. 以上都不正确

11. 计算机计数系统

计算下列表达式，并将结果表示为十六进制：

$A74F2C_{16} / 4_{16}$

- A. $A74F2B_{16}$
- B. $29D3CB_{16}$
- C. 12351714_{16}
- D. $57A8E2_{16}$
- E. 以上都不正确

12. 计算机计数系统

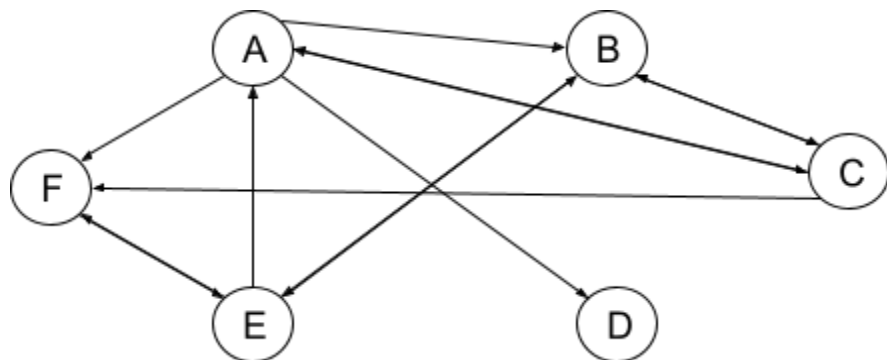
从 2001 到 2021（包括 2001 和 2021），将年份用八进制表示后，共有多少个数位大于 4？

- A. 25
- B. 26
- C. 27
- D. 28
- E. 以上都不正确

<p>13. 数据结构</p> <p>在下列字符串创建的最小堆中，哪些节点（从左到右）的深度为 2：</p> <p style="text-align: center;">TRICERATOPS</p>	<p>A. CERA B. OERA C. OIRR D. TIRR E. 以上都不正确</p>
<p>14. 数据结构</p> <p>会计师用两种方法来计算股权分配税，分别是：LIFO 和 FIFO。假设你在 3 月、4 月和 5 月分别以每股 10 美元、15 美元和 5 美元的价格购买了 100 股股票。如果你在 7 月份以每股 20 美元的价格卖了 100 股，这意味着你现在还剩下 200 股。如果用 FIFO，你可以根据在 3 月份购买的 100 股股票计算利润。如果用 LIFO，你可以根据在 5 月份购买的 100 股股票计算利润。</p> <p>计算每种方法的总利润，判断哪种方法对纳税人有利 (即需要交税的利润变少)。如果税率是 20%，纳税人会节省多少钱？</p>	<p>A. FIFO, 节省 \$100 B. LIFO, 节省 \$100 C. FIFO, 节省 \$500 D. LIFO, 节省 \$500 E. 没有差别</p>

15. 图论

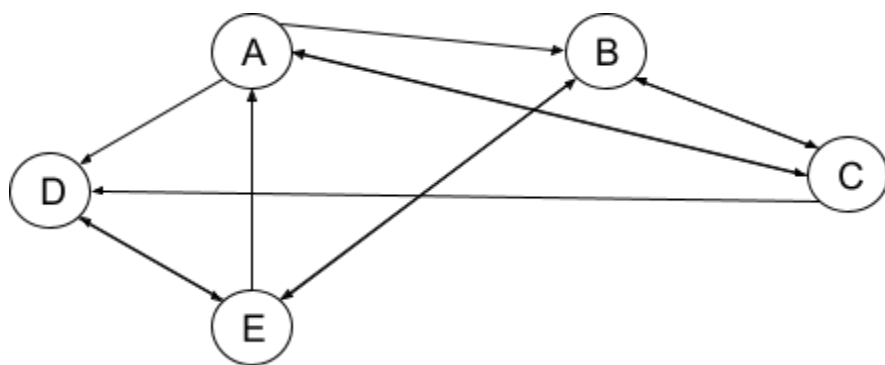
下面的有向图中，有多少对顶点在任意方向都不存在长度为 2 的路径？包括 (X,X) 。



- A. 3
- B. 4
- C. 5
- D. 6
- E. 以上都不正确

16. 图论

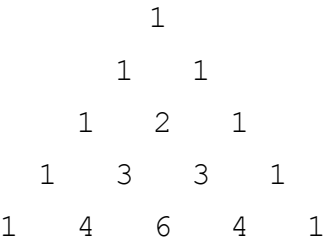
下面的有向图中有多少个环？



- A. 8
- B. 9
- C. 10
- D. 11
- E. 以上都不正确

17. 程序填空

杨辉三角形的结构如下：



假设一个命名为 **pt** 的一维阵列有 $X+1$ 项，阵列中储存有 X 行 (从第 0 行开始)，第一 (储存单元 0) 和 最后 (储存单元 X) 一项总被赋值为 1，以下哪个循环可以在第 $X+1$ 行成功创建其他值？

- A. for i = 1 to X
pt(i) = pt(i) + pt(i-1)
next i
- B. for i = X to 1 step -1
pt(i) = pt(i) + pt(i-1)
next i
- C. for i = X to 1 step -1
pt(i+1) = pt(i) + pt(i-1)
next i
- D. for i = 1 to X
pt(i-1) = pt(i-1) + pt(i)
next i
- E. 以上都不正确

18. LISP 语言

下面的 LISP 程序运行后，CONS 表达式的值是多少？

```
(SETQ S `(H ((A C) K) (E (R R (A))) (N (K))))  
(SETQ G (CADDADADDR S))  
(SETQ Q `((P L) (A (T (F O) R)) M))  
(SETQ H (CADADADR Q))  
(CONS G H)
```

- A. ((A) F O)
- B. (A F O)
- C. ((A) (F O))
- D. (A (F O))
- E. 以上都不正确

19. FSA 与正则表达式

一家新的文具店刚刚开业，并且为可以匹配以下正则表达式的所有商品提供五折优惠：

[^aeiou][aeiou][^a-j][p-t]?.(er|s)*

下面所有商品中，有多少种可以打折？

crayons	markers	rulers	erasers
staples	staplers	notebooks	desks
computers	tablets	boards	compass

- A. 7
- B. 6
- C. 5
- D. 4
- E. 以上都不正确

20. 汇编语言

设数据为 8 和 3，以下程序运行后会输出什么？

```

      READ      X
      READ      Y
      LOAD      X
      SUB       Y
      STORE     Z
      LOAD      =1
      STORE     C
      STORE     B
      STORE     A
T1    LOAD      C
      MULT      Z
      STORE     C
      LOAD      Z
      SUB       =1
      STORE     Z
      BG        T1
T2    LOAD      B
      MULT      Y
      STORE     B
      LOAD      Y
      SUB       =1
      STORE     Y
      BG        T2
T3    LOAD      A
      MULT      X
      STORE     A
      LOAD      X
      SUB       =1
      STORE     X
      BG        T3
      LOAD      A
      DIV       B
      DIV       C
      STORE     D
      PRINT     D
      END
```

- A. 6,720
- B. 336
- C. 56
- D. 24
- E. 以上都不正确