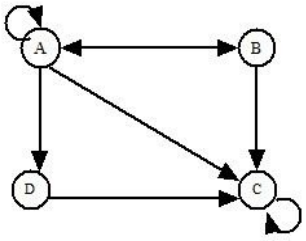
American Computer Science League

2020-2021 ● Contest 4: Solutions ● Senior Division

1. Graph Theory

The directed graph is as follows:



The number of paths can also be found by cubing the adjacency matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}^3 = \begin{bmatrix} 3 & 2 & 8 & 2 \\ 2 & 1 & 5 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Adding the entries gives the numbers of paths of length 3 which is 26.

1. 26 (D)

2. Graph Theory

There are 17 cycles in the directed graph: ABDA, ABDCA, ABEA, ABECA, ABECDA, ABEDA, ABEDCA, AEA, AECA, AEDA, AEDCA, AECDA, AECBDA, BDCB, BECB, BEDCB, CDC

2. 17 (C)

3. Digital Electronics

The Boolean expression represented by the circuit is:

$$\overline{\overline{A} + ((A(\overline{B + C})) \oplus \overline{C})} = \overline{\overline{A} + (A\,\overline{B}\,\overline{C} \oplus \overline{C})}$$

$$= \overline{\overline{A} + (A\,\overline{B}\,\overline{C}\,\overline{C} + A\,\overline{B}\,\overline{C}\,\overline{\overline{C}})}$$

$$= \overline{\overline{A} + ((\overline{A} + B + C)\,\overline{C} + 0)}$$

$$= \overline{\overline{A} + (\overline{A}\,\overline{C} + B\,\overline{C} + C\,\overline{C})}$$

$$= \overline{\overline{A} + \overline{A}\,\overline{C} + B\,\overline{C}} = \overline{\overline{A}\,(1 + \overline{C}) + B\,\overline{C}}$$

$$= \overline{\overline{A} + B\,\overline{C}} = \overline{\overline{A}}\,\overline{B\,\overline{C}} = A\,(\overline{B} + C)$$

This is TRUE for (1, 1, 1), (1, 0, 0), (1, 0, 1).

3. (1, *, 1), (1, 0, 0) (A)

4. Digital Electronics

The Boolean expression for the circuit is:

$(A + B)(\overline{B + C}) + (\overline{B + C})(C + D)$

$= (\overline{B + C})(A + B + C + D)$

$= \overline{B}\,\overline{C}\,(A + B + C + D)$

$= A\,\overline{B}\,\overline{C} + B\,\overline{B}\,\overline{C} + \overline{B}\,C\,\overline{C} + \overline{B}\,\overline{C}\,D$

$= A\,\overline{B}\,\overline{C} + 0 + 0 + \overline{B}\,\overline{C}\,D$

$= A\,\overline{B}\,\overline{C} + \overline{B}\,\overline{C}\,D$

$= \overline{B}\,\overline{C}\,(A + D)$

This expression uses 2 NOT gates, 1 OR gate, and 2 AND gates. Using the Distributive Property adds 2 more AND gates.

4. $\overline{B}\,\overline{C}\,(A + D)$ (B)

5. Assembly Language

This program adds the factors in the prime factorization of 60.

60 = 2 * 2 * 3 * 5 and 2 + 2 + 3 + 5 = 12

A translation to our ACSL language yields the following:

```
n = 60
while n >= 2
        f = 2
        p = int(n / f)
        while n - p * f != 0
                f = f + 1
                p = int(n / f)
        end while
        s = s + f
        n = p
end while
output s
```

5. 12 (D)