

AMERICAN COMPUTER SCIENCE LEAGUE

2019-2020

Contest #3

Junior Division - Veitch

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PROBLEM: The Veitch Diagram is a method to represent a Boolean expression. The method places an X in the cell that describes each term and groups adjacent cells. For this program, the X's are already placed. Each diagram will represent an expression of at most 3 variables. Variables are eliminated from a term's representation if the variable and its negation are included. Terms are always joined by the OR symbol (+). AND within terms will always be implied. Each X can be used just once in the forming of groups. The ~ symbol will be used to indicate NOT.

X's are grouped according to the following priorities:

1. Group 4 adjacent X's. This is shown in Diagrams 1 and 2. When 4 adjacent X's are grouped, the representation is one term. The representation of Diagram 1 is $\neg A$. Note B and $\neg B$ and then C and $\neg C$ are included in the grouped X's and are eliminated. The priority for grouping 4 adjacent X's is top row, then bottom row, and finally columns from left to right. The representation of Diagram 2 is $\neg B$.
2. Group 4 end column adjacent X's. That is X's fill the first and last column. There is only one way that this can be done which yields $\neg C$.
3. Group 2 adjacent X's. When 2 adjacent X's are grouped, the result is a term of 2 variables. In Diagram 3 there are 2 groups of 2 adjacent X's. The priority for grouping 2 adjacent X's is top row from left to right, bottom row from left to right, and then columns from left to right. The top row representation is $\neg AB$. The bottom row representation is $A\neg B$. The expression for Diagram 3 is $\neg AB + A\neg B$.
4. Group 2 end column adjacent X's. The priority is top row adjacent X's first. This is shown in Diagram 4. The top row in the diagram has end column adjacent X's. That part of the representation is $B\neg C$.
5. Represent single X's in the priority top row, bottom row, and then left to right. Diagram 4 has a single X that is not already used by the above rules. Single X's always translate to a term of 3 variables. It translates to $A\neg BC$. The expression for Diagram 4 is $B\neg C + A\neg BC$.

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INPUT: There will be 5 lines of input. Each line will contain a 2-character string. The 2 characters will each represent a hexadecimal digit. When each is converted to a 4-digit binary number with leading zeros and placed into the diagram top row then bottom row, the 1's will represent the placement of the X's in the Veitch Diagram. The input for Diagram 4 is 94 which represents 1001 and 0100.

OUTPUT: For each line of input, print the expression using the rules and priorities above. Since the listed priorities apply, the terms must be listed in the order specified. Also, the factors must always be in ABC order. Spacing between terms and within terms will not affect the answer.

SAMPLE INPUT:

33
3C
94
77
95

SAMPLE OUTPUT:

1. $\sim A$
2. $\sim AB + A\sim B$
3. $B\sim C + A\sim BC$
4. $C + \sim A\sim C$
5. $\sim A\sim C + AB\sim C + A\sim BC$

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TEST DATA

TEST INPUT:

F0

1D

9D

E9

E7

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Figure 1 illustrates the step-by-step construction of a 2D array for the 4-queens problem. The grids show the state of the array after placing queens in columns A, A, -A, and -A. The rows are labeled B, -B, and -C. The columns are labeled A, A, -A, and -A. The cells are colored yellow to indicate the positions of the queens and blue to indicate the positions of the queens that are not valid due to conflicts.

	A	A	-A	-A
B			X	X
-B			X	X
	-C	C	C	-C

	A	A	-A	-A
B				
-B	X	X	X	X
	-C	C	C	-C

	A	A	-A	-A
B			X	X
-B	X	X		
	-C	C	C	-C

	A	A	-A	-A
B	X			X
-B		X		
	-C	C	C	-C

PROBLEM (问题) : 维奇图 (Veitch Diagram) 是布尔表达式的一种表示方法。通过把X填入方格内, 并与相邻的已填入X的方格组成一组来描述。上面这些维奇图都已经填入X。每个图都表示一个布尔表达式, 且最多有3个变量。如果变量及其否定都出现在一项中, 则将该变量从项的表示形式中删除。每一项始终以OR符号 (+) 连接, 且其中每一项中的变量都是用AND连接, 但AND总是隐含的。每个X在形成组时只能使用一次。~符号表示NOT。

X按以下优先级进行分组:

1. 相邻的4个X组成一组。如图1和图2所示，当4个相邻的X组成一组时，表示为一项。图1表示的是 $\sim A$ 。注意如果B和 $\sim B$ ，C和 $\sim C$ 都出现在有X的组内，应删除。4个相邻的X组成一组的优先顺序为，先顶行，然后是底行，最后是从左列到右列。图2表示的是 $\sim B$ 。
2. 4个X在分别位于两端列组成一组，也就是第一列和最后一列都被X填满。只有一种方法可以做到这一点，即产生 $\sim C$ 。
3. 2个相邻的X组成一组。当2个相邻的X组成一组时，就会形成一个由2个变量组成的项。在图3中，有2组2个相邻的X。2个相邻的X组成一组的优先顺序为，从左到右的顶行，从左到右的底行，然后从左列到右列。顶行那组表示的是 $\sim AB$ ，底行那组表示的是 $A \sim B$ ，因此图3表示的是 $\sim AB + A \sim B$ 。
4. 位于同行的最左列和最右列的2个X组成一组。优先顺序为，与X相邻的顶行，如图4所示。该图的顶行在最左列和最右列与X相邻，因此那个部分表示的是 $B \sim C$ 。
5. 单个X的优先顺序为，先顶行，后底行，最后从左列至右列。例如图4中存在一个单个X无法使用上述规则。单个X始终表示一个有3个变量的项，按照字母表的顺序，转换后为 $A \sim BC$ 。因此图4的表达式为 $B \sim C + A \sim BC$ 。

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INPUT (输入): 将会有5行输入，每一行输入都会包含一个2个字符的字符串。这两个字符各自代表一个十六进制数值。每个字符都需要被转换成四位二进制数，若位数不够，前面补0，并分别放在图中的第一行和最后一行，1表示维奇图中X的位置。如图4的输入值是94，它表示1001和0100。

OUTPUT (输出): 对于每一行输入，使用上面的规则和优先顺序打印输出表达式。由于上述列出的优先级顺序，每一项必须按照指定的顺序列出。此外，必须始终按ABC顺序排列。项与项之间和项以内的空格不会影响答案。

SAMPLE INPUT:

33
3C
94
77
95

SAMPLE OUTPUT:

1. $\sim A$
2. $\sim AB + A\sim B$
3. $B\sim C + A\sim BC$
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