

American Computer Science League

2021 Finals • Short Problems • Junior Division

1. Boolean Algebra How many ordered triples make the following expression TRUE? $\overline{A \overline{AB} + B\overline{C}}$	A. 0 B. 3 C. 4 D. 6 E. None of the above
2. Boolean Algebra Simplify the following Boolean expression: $\overline{\overline{A}(A + \overline{B}) + B(\overline{A} + B)}$	A. $\overline{A} + B$ B. $\overline{A} B$ C. $A \overline{B}$ D. $A + \overline{B}$ E. None of the above
3. Bit-String Flicking Evaluate the following expression: $((\text{LSHIFT-1}(\text{NOT}(\text{RCIRC-2 } 01101)) \text{ AND } (\text{LCIRC-2 } 01101))) \\ \text{OR } (\text{RSHIFT-1 } 01101))$	A. 10101 B. 10100 C. 01011 D. 01110 E. None of the above
4. Bit-String Flicking Evaluate the following expression: $(\text{NOT} (01101 \text{ OR } 01010) \text{ AND } (01100 \text{ OR } 10110))$	A. 01010 B. 01111 C. 11110 D. 10000 E. None of the above
5. Recursive Functions Find $f(25)$ given: $f(x) = \begin{cases} x + f(x - 3) & \text{if } x \geq 15 \\ 2 \cdot f(x + 2) & \text{if } 12 < x < 15 \\ x + 2 & \text{if } x \leq 12 \end{cases}$	A. 140 B. 93 C. 58 D. 14 E. None of the above

6. Recursive Functions

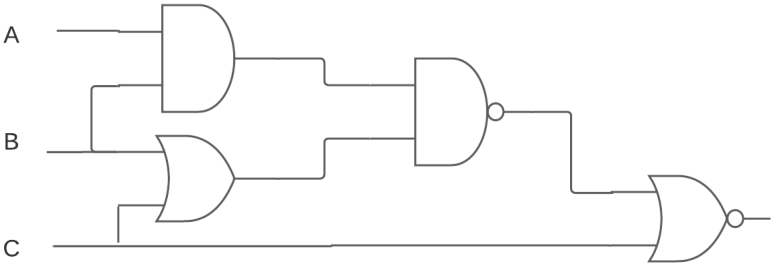
Given the following recursive function, what is the smallest value of y where $f(5, y) > 1000$?

$$f(x, y) = \begin{cases} 1 & \text{if } y = 0 \\ x \cdot f(x, y - 1) & \text{if } y > 0 \end{cases}$$

- A. 4
- B. 5
- C. 10
- D. 15
- E. None of the above

7. Digital Electronics

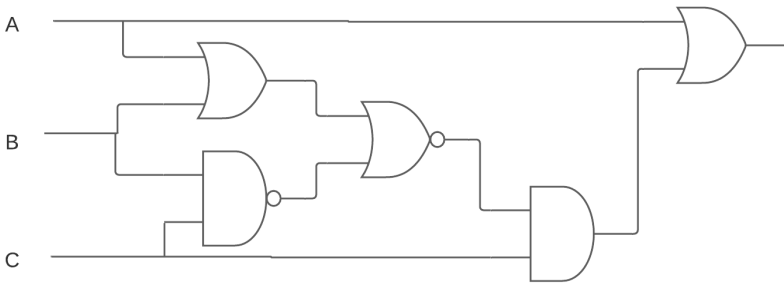
Simplify the Boolean expression represented by the circuit below:



- A. $A B C$
- B. $\overline{A} B C$
- C. $A \overline{B} C$
- D. $A B \overline{C}$
- E. None of the above

8. Digital Electronics

How many ordered triples make the following circuit TRUE?



- A. 0
- B. 2
- C. 4
- D. 8
- E. None of the above

9. Prefix-Infix-Postfix

Evaluate this prefix expression. (Note: all numbers are single digits.)

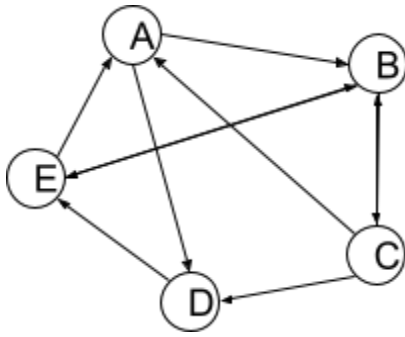
$- + / + 1\ 8\ 3 * 4\ 3 \uparrow 2\ 3$

- A. 6
- B. 7
- C. 8
- D. 9
- E. None of the above

<p>10. Prefix-Infix-Postfix</p> <p>Given the equation written in infix, convert it to postfix.</p> $(x - h)^2 + (y - k)^2 = r^2$	<p>A. $xh - 2 \uparrow yk - 2 \uparrow + r 2 \uparrow =$ B. $xh - 2 \uparrow yk - 2 \uparrow r 2 \uparrow + =$ C. $xh 2 \uparrow - yk 2 \uparrow - + r 2 \uparrow =$ D. $xh - 2 \uparrow + yk - 2 \uparrow r 2 \uparrow =$ E. None of the above</p>
<p>11. Computer Number Systems</p> <p>Given the year 2021, what is the next year that has 2 more 1s in its binary representation than 2021 has in its binary representation?</p>	<p>A. 2031 B. 2039 C. 2022 D. 2029 E. None of the above</p>
<p>12. Computer Number Systems</p> <p>Evaluate the following and express the result in hexadecimal.</p> $AB_{16} + 74_8 - 1101_2$	<p>A. EA_{16} B. DA_{16} C. $B3_{16}$ D. FB_{16} E. None of the above</p>
<p>13. Data Structures</p> <p>Build a binary search tree for:</p> <p style="text-align: center;">NODINOSAURS</p> <p>What is the depth of the tree?</p>	<p>A. 2 B. 3 C. 4 D. 5 E. None of the above</p>
<p>14. Data Structures</p> <p>Given an initially empty stack and the following commands on the stack, what will the value of Z be?</p> <p>PUSH(3), PUSH(7), PUSH(2), PUSH(4), X = POP(), Y = POP(), PUSH(X+Y), X = POP(), Y = POP(), PUSH(X-Y), X = POP(), Y = POP(), PUSH(X*Y), PUSH(9), PUSH(3), X = POP(), Y = POP(), PUSH(Y/X), X = POP(), Y = POP(), PUSH(Y^X), Z = POP()</p>	<p>A. -9 B. 3 C. -27 D. 1 E. None of the above</p>

15. Graph Theory

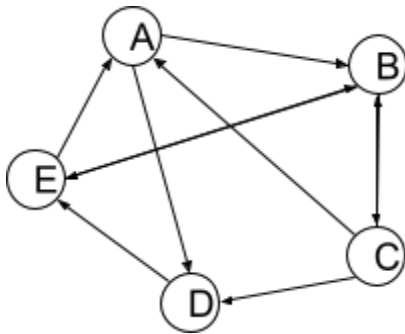
How many paths of length 2 are in the directed graph below?



- A. 10
- B. 15
- C. 19
- D. 21
- E. None of the above

16. Graph Theory

How many cycles are there in the directed graph (same as #15)?



- A. 7
- B. 8
- C. 9
- D. 10
- E. None of the above

17. What Does This Program Do?

Given a 6 x 8 two-dimensional array, a, that is populated by the numbers from 1 to 48 in row-major order, which of the following statement(s) inside the nested loops will print only those that are divisible by 3 or divisible by 5 exactly once?

```
for x = 0 to 5
  for y = 0 to 7
    .....
  next y
next x
```

- A. if $a(x,y) \% 3 == 0$ then
output(a(x,y))
if $a(x,y) \% 5 == 0$ then
output(a(x,y))
- B. if $a(x,y) \% 15 == 0$ then
output(a(x,y))
- C. if $a(x,y) \% 3 == 0$ or
 $a(x,y) \% 5 == 0$ then
output(a(x,y))
- D. if $a(x,y) \% 3 == 0$ and
 $a(x,y) \% 5 == 0$ then
output(a(x,y))
- E. None of the above

18. What Does This Program Do?

What is the output for the following program?

```
r = 1
p = 1
t = 1
for x = 1 to 8
    r = r * x
next x
for x = 1 to 3
    p = p * x
next x
for x = 1 to 5
    t = t * x
next x
output r / (p * t)
```

- A. 1
- B. 42
- C. 48
- D. 56
- E. None of the above

19. What Does This Program Do?

Given that array **arr** contains the values below, how many values will be output by the following program?

11	16	24	39	43	56	69	81	97
----	----	----	----	----	----	----	----	----

```
for x = 0 to 8
    check = 1
    for y = 3 to int(arr(x)/2) step 2
        if arr(x) % y == 0 then
            check = 0
        end if
    next y
    if check == 1 then
        output(arr(x))
    end if
next x
```

- A. 2
- B. 3
- C. 4
- D. 5
- E. None of the above

20. What Does This Program Do?

Using ACSL code, which of the following statements can be used where the blank line is to produce the following output in the given program?

Output:

```
F
_FI
_L_FIN
_SL_FINAL
CSL_FINAL
ACSL_FINALS
```

```
s = "ACSL_FINALS"
for x = (len(s)-1)/2 to 0 step -1
    output(_____)
next x
```

- A. `s[int(len(s) / 2) - x : x]`
- B. `s[x+1 : len(s) - x]`
- C. `s[x : len(s) - x + 1]`
- D. `s[x : len(s) - x - 1]`
- E. None of the above

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<p>1. 布尔代数</p> <p>有多少个有序三元组可使以下表达式成立？</p> $\overline{A \overline{AB} + B\overline{C}}$	<p>A. 0 B. 3 C. 4 D. 6 E. 以上都不正确</p>
<p>2. 布尔代数</p> <p>化简下列布尔表达式：</p> $\overline{\overline{A}(A + \overline{B}) + B(\overline{A} + B)}$	<p>A. $\overline{A} + B$ B. $\overline{A} B$ C. $A \overline{B}$ D. $A + \overline{B}$ E. 以上都不正确</p>
<p>3. 比特位操作</p> <p>计算下列表达式：</p> <p>((LSHIFT-1 (NOT (RCIRC-2 01101) AND (LCIRC-2 01101))) OR (RSHIFT-1 01101))</p>	<p>A. 10101 B. 10100 C. 01011 D. 01110 E. 以上都不正确</p>
<p>4. 比特位操作</p> <p>计算下列表达式：</p> <p>(NOT (01101 OR 01010) AND (01100 OR 10110))</p>	<p>A. 01010 B. 01111 C. 11110 D. 10000 E. 以上都不正确</p>
<p>5. 递归函数</p> <p>已知下列函数，求 $f(25)$：</p> $f(x) = \begin{cases} x + f(x-3) & \text{设 } x \geq 15 \\ 2 \cdot f(x+2) & \text{设 } 12 < x < 15 \\ x + 2 & \text{设 } x \leq 12 \end{cases}$	<p>A. 140 B. 93 C. 58 D. 14 E. 以上都不正确</p>

6. 递归函数

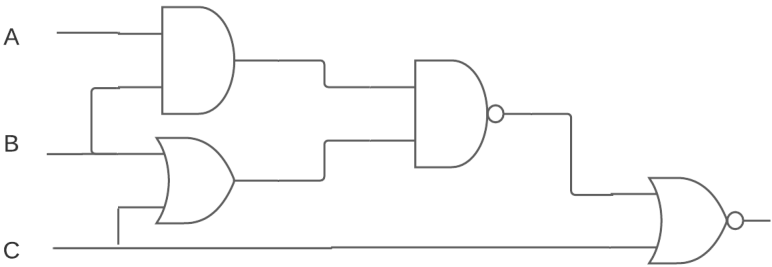
对于下列递归函数，当 $f(5,y)>1000$ 时， y 的最小值是多少？

$$f(x,y)=\begin{cases}1 & \text{设 } y=0 \\ x\cdot f(x,y-1) & \text{设 } y>0\end{cases}$$

- A. 4
- B. 5
- C. 10
- D. 15
- E. 以上都不正确

7. 数字电路

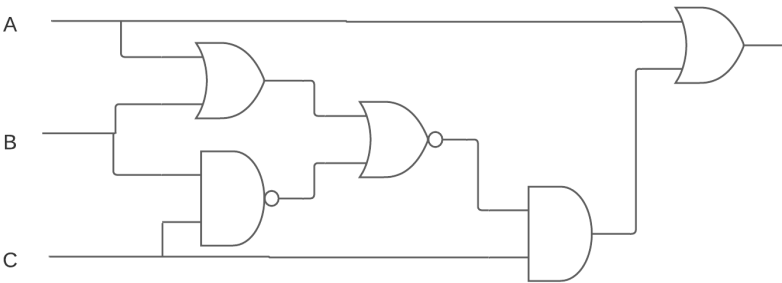
化简下列电路表示的布尔表达式：



- A. $A B C$
- B. $\overline{A} B C$
- C. $A \overline{B} C$
- D. $A B \overline{C}$
- E. 以上都不正确

8. 数字电路

有多少个有序三元组可使下列电路成立？



- A. 0
- B. 2
- C. 4
- D. 8
- E. 以上都不正确

9. 前缀-中缀-后缀

计算下列前缀表达式。(注：所有数字都是个位数。)

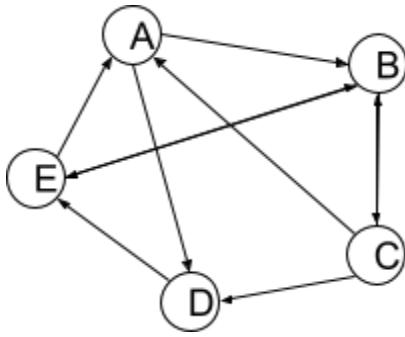
$- + / + 183 * 43 \uparrow 23$

- A. 6
- B. 7
- C. 8
- D. 9
- E. 以上都不正确

<p>10. 前缀-中缀-后缀</p> <p>假设以下方程用中缀形式写出，将其转换为后缀形式。</p> $(x - h)^2 + (y - k)^2 = r^2$	<p>A. $xh - 2 \uparrow yk - 2 \uparrow + r 2 \uparrow =$ B. $xh - 2 \uparrow yk - 2 \uparrow r 2 \uparrow + =$ C. $xh 2 \uparrow - yk 2 \uparrow - + r 2 \uparrow =$ D. $xh - 2 \uparrow + yk - 2 \uparrow r 2 \uparrow =$ E. 以上都不正确</p>
<p>11. 计算机计数系统</p> <p>从 2021 年开始，找出转换为二进制后比 2021 转换成的二进制数多 2 个 1 的下一个年份？</p>	<p>A. 2031 B. 2039 C. 2022 D. 2029 E. 以上都不正确</p>
<p>12. 计算机计数系统</p> <p>计算下列表达式，并将结果表示为十六进制：</p> $AB_{16} + 74_8 - 1101_2$	<p>A. EA_{16} B. DA_{16} C. $B3_{16}$ D. FB_{16} E. 以上都不正确</p>
<p>13. 数据结构</p> <p>构建一个二叉搜索树：</p> <p style="text-align: center;">NODINOSAURS</p> <p>该二叉树的深度是多少？</p>	<p>A. 2 B. 3 C. 4 D. 5 E. 以上都不正确</p>
<p>14. 数据结构</p> <p>给定一个初始空栈并对其执行以下命令，那么 Z 的值是多少？</p> <p>PUSH(3), PUSH(7), PUSH(2), PUSH(4), X = POP(), Y = POP(), PUSH(X+Y), X = POP(), Y = POP(), PUSH(X-Y), X = POP(), Y = POP(), PUSH(X*Y), PUSH(9), PUSH(3), X = POP(), Y = POP(), PUSH(Y/X), X = POP(), Y = POP(), PUSH(Y^X), Z = POP()</p>	<p>A. -9 B. 3 C. -27 D. 1 E. 以上都不正确</p>

15. 图论

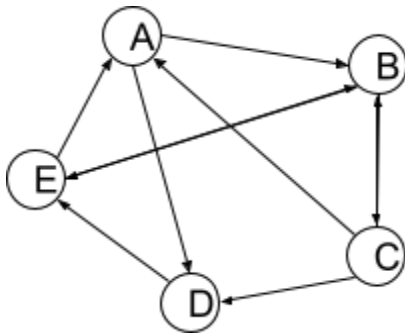
下方有向图中有多少条长度为 2 的路径？



- A. 10
- B. 15
- C. 19
- D. 21
- E. 以上都不正确

16. 图论

下方有向图中有多少个环？(图同#15)?



- A. 7
- B. 8
- C. 9
- D. 10
- E. 以上都不正确

17. 程序填空

设一个 6×8 的二维数组 a 以行主序方式排列着 1 到 48，
以下嵌套循环中的哪个/哪些语句可以输出只能被 3 或 5 整除一次的数字？

```

for x = 0 to 5
  for y = 0 to 7
    .....
  next y
next x
  
```

- A. 设 $a(x,y) \% 3 == 0$ 则
输出(a(x,y))
设 $a(x,y) \% 5 == 0$ 则
输出(a(x,y))
- B. 设 $a(x,y) \% 15 == 0$ 则
输出(a(x,y))
- C. 设 $a(x,y) \% 3 == 0$ 或
 $a(x,y) \% 5 == 0$ 则
输出(a(x,y))
- D. 设 $a(x,y) \% 3 == 0$ 和
 $a(x,y) \% 5 == 0$ 则
输出(a(x,y))
- E. 以上都不正确

18. 程序填空

运行以下程序会输出什么？

```
r = 1
p = 1
t = 1
for x = 1 to 8
    r = r * x
next x
for x = 1 to 3
    p = p * x
next x
for x = 1 to 5
    t = t * x
next x
output r / (p * t)
```

- A. 1
- B. 42
- C. 48
- D. 56
- E. 以上都不正确

19. 程序填空

设数组 **arr** 包含以下值，下面的程序会输出多少个值？

11	16	24	39	43	56	69	81	97
----	----	----	----	----	----	----	----	----

```
for x = 0 to 8
    check = 1
    for y = 3 to int(arr(x)/2) step 2
        if arr(x) % y == 0 then
            check = 0
        end if
    next y
    if check == 1 then
        output(arr(x))
    end if
next x
```

- A. 2
- B. 3
- C. 4
- D. 5
- E. 以上都不正确

20. 程序填空

使用ACSL编码，哪个语句填入空行后，可以在下面的程序中输出以下结果？

输出：

```
F
_FI
_L_FIN
_SL_FINAL
_CSL_FINAL
_ACSL_FINALS
```

```
s = "ACSL_FINALS"
for x = (len(s)-1)/2 to 0 step -1
    output(_____)
next x
```

- A. `s[int(len(s) / 2) - x : x]`
- B. `s[x+1 : len(s) - x]`
- C. `s[x : len(s) - x + 1]`
- D. `s[x : len(s) - x - 1]`
- E. 以上都不正确