American Computer Science League

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1. Graph Theory

The directed graph is as follows:
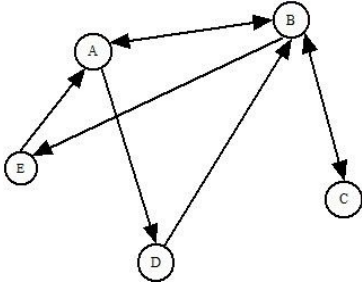
The paths of length 2 can be found by inspection: BAB, BCB, BAD, BEA.
The number of paths can also be found by squaring the adjacency matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Adding the numbers in the second row gives 4 paths from B of length 2.

1. 4 (D)

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2. Graph Theory

The cycles from A are: ABA, ACA, ABCA, ABDA, ABDCA.

2. 5 (C)

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3. Digital Electronics

The digital circuit can be represented by the Boolean expression:

$A + (AB + B)$
$= A + B(A + 1)$
$= A + B$

3. $A + B$ (B)

4. Digital Electronics

The digital circuit can be represented by the Boolean expression:

$$\overline{((A + B)\,(BC))}\,\overline{C} \;=\; \overline{(A + B + \overline{BC})}\,\overline{C}$$
$$=\; (\overline{A}\,\overline{B} + \overline{B} + \overline{C})\,\overline{C}$$
$$=\; \overline{A}\,\overline{B}\,\overline{C} + \overline{B}\,\overline{C} + \overline{C}\,\overline{C}$$
$$=\; \overline{C}\,(\overline{A}\,\overline{B} + \overline{B} + 1)$$
$$=\; \overline{C}$$

To be TRUE, $\overline{C} = 1$, so $C = 0$, $A = *$, $B = *$.

There are 4 ordered pairs: (1, 1, 0), (1, 0, 0), (0, 1, 0), (0, 0, 0).

5. What Does This Program Do? (Strings)

The program creates a new string, b, by comparing each character of string a to the letter "O" starting at the second character and ending at the next to last character. If it is alphabetically before "O", then the previous character of a is added to the beginning of b. If it is alphabetically after "O", then the next character of a is added to the end of b. If the character is "O", then it is skipped. The final result for b = "TOLSAHTRTHLET". The longest substring in ascending alphabetical order is "AHT" of length 3.