American Computer Science League

2021 Finals ● Solutions to Short Problems ● Elementary Division

1. Boolean Algebra

The only option that includes only these 6 numbers is if a > 3 and a < 7 which includes 4, 5, and 6 OR a > 15 and a < 19 which includes 16, 17, and 18 only. D does not include the boundary conditions and switching the AND with the OR yields all possible numbers in A and C.

1. (a>3 AND a<7) OR (a>15 AND a<19) (B)

2. Boolean Algebra

To simplify ~AB + ~A + B * ~(A + ~B)
= ~A(B + 1) + B(~AB)
= ~A + ~AB
= ~A

2. ~A (D)

3. Boolean Algebra

The truth table used to evaluate A * B % A + ~A % B is:

A	B	~A	B % A	A * B%A	~A % B	A*B%A + ~A%B
0	0	1	1	0	1	1
0	1	1	1	0	0	0
1	0	0	1	1	1	1
1	1	0	0	0	1	1

Therefore, there are 3 ordered pairs that make the expression TRUE.

3. 3 (C)

4. Prefix-Infix-Postfix

The postfix expression 4 3 * 6 2 / - 2 ^ can be evaluated as:
(4 * 3) (6 / 2) - 2 ^
(12 - 3) 2 ^
9 ^ 2 = 81

4. 81 (A)

5. Prefix-Infix-Postfix

The prefix expression - ^ @ 1 9 2 @ * 3 7 / 9 3 can be evaluated as:
- ^ (@ 1 9) 2 (@ (3 * 7) (9 / 3))
- ^ ((1 + 9) / 2) 2 ((21 + 3) / 2)
- (5 ^ 2) 12 = 25 - 12 = 13

5. 13 (B)

6. Prefix-Infix-Postfix

The postfix expression 4 5 * 8 2 / + 7 * 4 2 ^ - can be translated to prefix by moving only the operators as follows:

(((4 5 *) (8 2 /) +) 7 *) (4 2 ^) -
(- (* (+ (* 4 5) (/ 8 2)) 7) (^ 4 2))
- * + * 4 5 / 8 2 7 ^ 4 2

6. - * + * 4 5 / 8 2 7 ^ 4 2 (A)

7. Computer Number Systems

$DEAD_{16}$ D - E = F ⇒ (13 + 16) - 14 = 29 - 14 = 15 and A becomes 9.
- ABE_{16} 9 - B = E ⇒ (9 + 16) - 11 = 25 - 11 = 14 and E becomes D.
---------- D - A = 3 ⇒ 13 - 10 = 3 so the D stays the same.
$D3EF_{16}$ When subtracting in base 16, you must borrow 16 if needed.

7. $D3EF_{16}$ (D)

8. Computer Number Systems

To evaluate $20_{16} + 21_8$ you must convert each to base 10 and add them.
20 = 2 * 16 + 0 and 21 = 2 * 8 + 1 so 32 + 17 = 49

8. 49 (C)

9. Computer Number Systems

Each of these numbers can be converted to any common base by grouping 3 binary digits for base 8 and 4 bits for base 16.

$10110111_2 = 267_8 = B7_{16}$	$11001010_2 = 312_8 = CA_{16}$
$BE_{16} = 10111110_2 = 276_8$	$267_8 = 10110111_2 = B7_{16}$
$356_8 = 11101110_2 = EE_{16}$	$CA_{16} = 11001010_2 = 312$
$11101111_2 = 357_8 = EF_{16}$	$11011000_2 = 330_8 = D8_{16}$
$B7_{16} = 10110111_2 = 267_8$	$357_8 = 11101111_2 = EF_{16}$

The one that occurs most often occurs 3 times: $10110111_2 = 267_8 = B7_{16}$

9. 3 (B)

10. Graph Theory Traversals must start and end with the 2 vertices that have odd degree which are A and E. They must use each edge once and only once. Therefore, the only possible answer given these choices is ACBADCE.	10. ACBADCE (D)
11. Graph Theory If you draw the graph, the edges are BC, DS, CA, AP, DP, PS, CD, and PB so Pittsburgh is the one-stop layover from Buffalo to Seattle.	11. Pittsburgh (C)
12. Graph Theory The 6 unique cycles ABDCA, ABDEA, ABDCFA, ACDEA, ACFA, AEDCFA, and, but they are counted both ways so there are 12 total.	12. 12 (D)