American Computer Science League

SENIOR DIVISION SOLUTIONS

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1. Boolean Algebra

$$A\,\overline{B} + C\left(\overline{A} + B\right) + \overline{A}\,(B + \overline{C})$$

$$= A\,\overline{B} + \overline{A}\,C + B\,C + \overline{A}\,B + \overline{A}\,\overline{C}$$

$$= \overline{A}\,(B + C + \overline{C}) + A\,\overline{B} + B\,C$$

$$= \overline{A} + A\,\overline{B} + B\,C$$

$$\overline{A} + A\,\overline{B} + B\,C = 0 \rightarrow \overline{A} = 0 \rightarrow A = 1$$

$$0 + 1\,\overline{B} + B\,C = 0 \rightarrow \overline{B} = 0 \rightarrow B = 1 \wedge C = 0 \Rightarrow (1,\,1,\,0)$$

1. 1

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2. Boolean Algebra

$$C\,(A \oplus \overline{B}) + B\,(A \oplus C) + \overline{A}\,(B \oplus \overline{C})$$

$$= C(A\,B + \overline{A}\,\overline{B}) + B(A\,\overline{C} + \overline{A}\,C) + \overline{A}(B\,C + \overline{B}\,\overline{C})$$

$$= A\,B\,C + \overline{A}\,\overline{B}\,C + A\,B\,\overline{C} + \overline{A}\,B\,C + \overline{A}\,B\,C + \overline{A}\,\overline{B}\,\overline{C}$$

$$= B\,C\,(A + \overline{A}) + \overline{A}\,C\,(B + \overline{B}) + \overline{A}\,\overline{B}\,\overline{C}$$

$$= B\,C + \overline{A}\,C + \overline{A}\,\overline{B}\,\overline{C}$$

$$B\,C + \overline{A}\,C + \overline{A}\,\overline{B}\,\overline{C} = 1$$

$$C = 0 \rightarrow \overline{A}\,\overline{B} = 1 \rightarrow A = 0 \wedge B = 0 \Rightarrow (0,\,0,\,0)$$

$$C = 1 \rightarrow B + \overline{A} = 1 \Rightarrow (1,\,1,\,1),(0,\,0,\,1),(0,\,1,\,1)$$
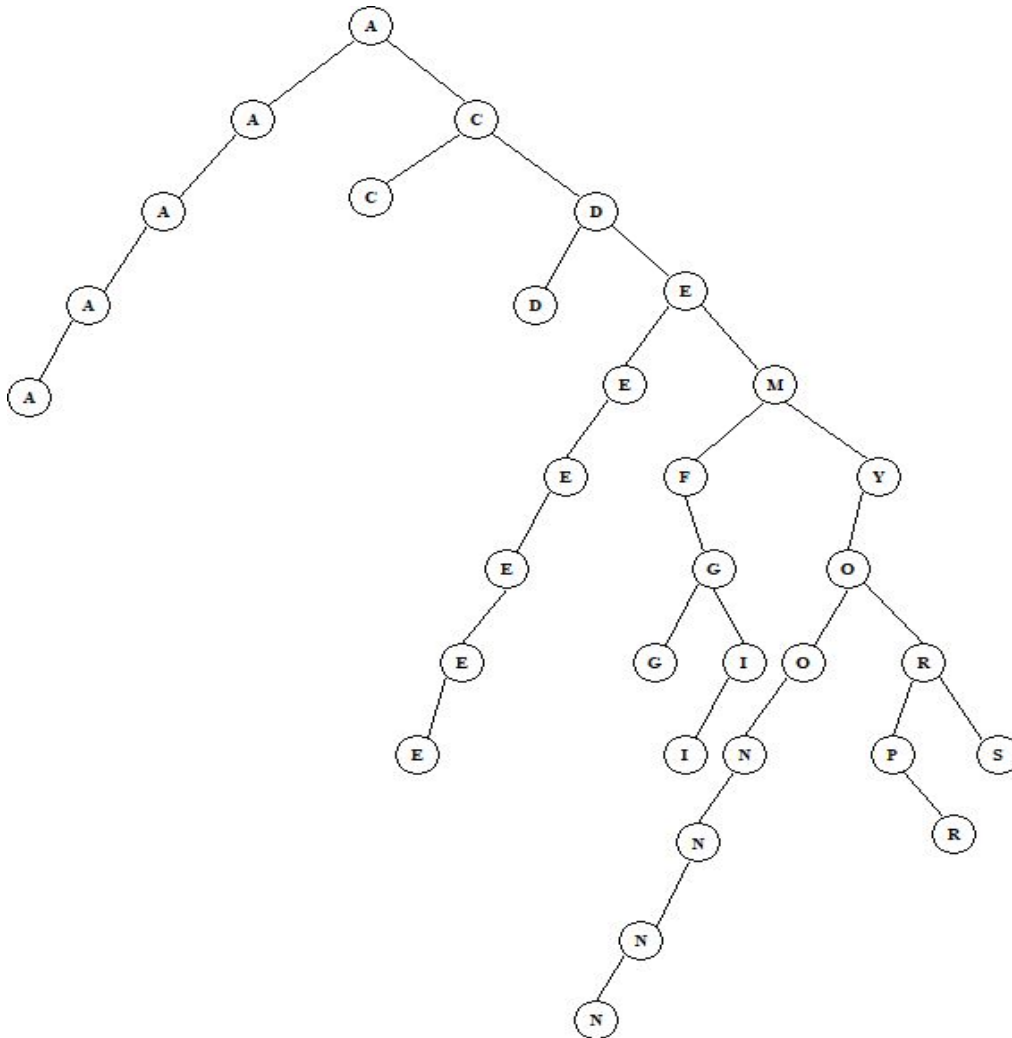
2. (0, 0, 0)
(0, 0, 1)
(0, 1, 1)
(1, 1, 1)
(1, 1, 0)

SENIOR DIVISION SOLUTIONS

3. Data Structures

3.   176

The binary search tree is:

The internal path length is 2 * 1 + 3 * 2 + 3 * 3 + 3 * 4 + 3 * 5 + 3 * 6 +
5 * 7 + 5 * 8 + 2 * 9 + 1 * 10 + 1 * 11 = 2 + 6 + 9 + 12 + 15 + 18 + 35 +
40 + 18 + 10 + 11 = 176.

4. Data Structures The stack is constructed using LIFO as follows: T, TH, THE, TH, THP, THPH, THP, THPA, THPAN, THPANT, THPAN, THPA, THP, THPO, THPOM, THPO, THP, THPO, THPOF, THPOFT, THPOFTH, THPOFT, THPOF, THPOFE, THPOFEO, THPOFE, THPOFEP, THPOFEPE, THPOFEPER, THPOFEPE, THPOFEP, THPOFE, THPOFEA, THPOFE, THPOF. The next item popped would be F.	**4.** F
5. FSA's and Regular Expressions The FSA translates to: $$01*1(11*00*0 \cup 00*11*01)1*1$$	**5.** As shown