American Computer Science League

2021 Finals ● Short Problems ● Elementary Division

1. Boolean Algebra

Given that the variable **a** can only be a whole number, which Boolean expression is TRUE for all of the numbers in the set {4, 5, 6, 16, 17, 18} and only these 6 numbers?

A. (a > 4 OR a < 6) AND (a > 16 OR a < 18)
B. (a > 3 AND a < 7) OR (a > 15 AND a < 19)
C. (a > 3 OR a < 7) AND (a > 15 OR a < 19)
D. (a > 4 AND a < 6) OR (a > 16 AND a < 18)

A. (a>4 OR a<6) AND (a>16 OR a<18)
B. (a>3 AND a<7) OR (a>15 AND a<19)
C. (a>3 OR a<7) AND (a>15 OR a<19)
D. (a>4 AND a<6) OR (a>16 AND a<18)
E. None of the above

2. Boolean Algebra

Simplify the following Boolean expression:

~AB + ~A + B * ~(A + ~B)

A. B
B. ~A+B
C. ~AB
D. ~A
E. None of the above

3. Boolean Algebra

Define a new binary operator % by the following truth table:

A	B	A%B
0	0	1
0	1	1
1	0	1
1	1	0

The % has a higher priority than both * and +, but a lower priority than ~. How many ordered pairs make the following TRUE?

A * B % A + ~A % B

A. 1
B. 2
C. 3
D. 4
E. None of the above

4. Prefix-Infix-Postfix

Evaluate the following postfix expression (assume all single digits):

4 3 * 6 2 / - 2 ^

A. 81
B. 30
C. 18
D. 1
E. None of the above

5. Prefix-Infix-Postfix

Evaluate the following prefix expression if all numbers are single digits and the @ operator is defined as the average of a and b (i.e. a @ b = (a + b) / 2):

- ^ @ 1 9 2 @ * 3 7 / 9 3

A. 12
B. 13
C. 14
D. 15
E. None of the above

6. Prefix-Infix-Postfix

Translate the following postfix expression to prefix (assume all single digits):

4 5 * 8 2 / + 7 * 4 2 ^ -

A. -*+*45/827^42
B. *+*45/827-^42
C. -+*45/82*7^42
D. *45/82-7+*^42
E. None of the above

7. Computer Number Systems

Evaluate and express the result in hexadecimal:

$\mathbf{DEAD_{16} - ABE_{16}}$

A. $D3DF_{16}$
B. $C3EF_{16}$
C. $D4EF_{16}$
D. $D3EF_{16}$
E. None of the above

8. Computer Number Systems

What is the value of the following expression in base 10?

$\mathbf{20_{16} + 21_{8}}$

A. 41
B. 48
C. 49
D. 641
E. None of the above

9. Computer Number Systems

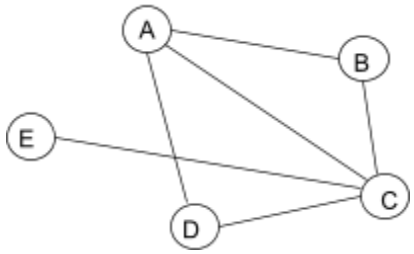
Given all of the following base 2, base 8, or base 16 numbers, find which value occurs most often. How many numbers below represent that value?

10110111_2 BE_{16} 356_8 11101111_2 $B7_{16}$
11001010_2 267_8 CA_{16} 11011000_2 357_8

A. 4
B. 3
C. 2
D. 1
E. None of the above

10. Graph Theory

Which of the following is a valid traversal of the following graph?



A. ACABCDACE
B. EDCABCA
C. ECBACDE
D. ACBADCE
E. None of the above

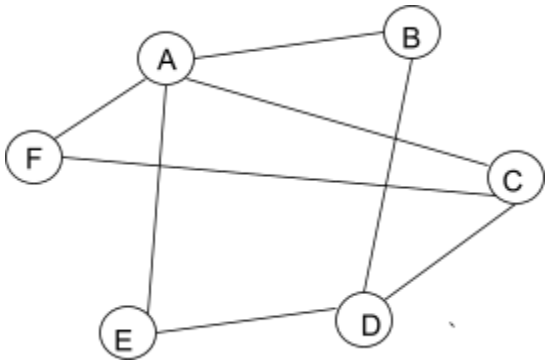
11. Graph Theory

Given that there are airplane routes between Buffalo and Cleveland, Denver and Seattle, Cleveland and Atlanta, Atlanta and Pittsburgh, Denver and Pittsburgh, Pittsburgh and Seattle, Cleveland and Denver, and Pittsburgh and Buffalo, which of the following cities must be a single layover on a 1-stop flight from Buffalo to Seattle? In other words, find the paths of length 2 and identify the city that is in the middle.

A. Cleveland
B. Denver
C. Pittsburgh
D. Atlanta
E. None of the above

12. Graph Theory

Determine the number of cycles that exist in the following graph.



A. 5
B. 8
C. 10
D. 12
E. None of the above

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1. 布尔代数

假设变量 **a 只能是一个整数**，以下哪个布尔表达式对于集合 {4, 5, 6, 16, 17, 18}中的数字均成立，且仅对于这 6个数字成立？

A. (a > 4 OR a < 6) AND (a > 16 OR a < 18)
B. (a > 3 AND a < 7) OR (a > 15 AND a < 19)
C. (a > 3 OR a < 7) AND (a > 15 OR a < 19)
D. (a > 4 AND a < 6) OR (a > 16 AND a < 18)

A. (a>4 OR a<6) AND (a> 16 OR a<18)
B. (a>3 AND a<7) OR (a> 15AND a<19)
C. (a>3 OR a<7) AND (a> 15 OR a<19)
D. (a>4 AND a<6) OR (a> 16 AND a<18)
E. 以上都不正确

2. 布尔代数

简化以下布尔表达式：

~AB + ~A + B * ~(A + ~B)

A. B
B. ~A+B
C. ~AB
D. ~A
E. 以上都不正确

3. 布尔代数

根据以下真值表来定义一个新的二元运算符 %：

A	B	A%B
0	0	1
0	1	1
1	0	1
1	1	0

% 的优先级高于 * 和 +，但低于 ~。有多少个有序对可使以下表达式成立？

A * B % A + ~A % B

A. 1
B. 2
C. 3
D. 4
E. 以上都不正确

4. 前缀-中缀-后缀

计算下列后缀表达式 (假设所有数字都是个位数) :

4 3 * 6 2 / - 2 ^

A. 81
B. 30
C. 18
D. 1
E. 以上都不正确

5. 前缀-中缀-后缀

假设所有的数字都是个位数，且运算符 @ 定义为 a 和 b 的平均值 (例如：a @ b = (a + b) / 2)，计算下列前缀表达式 ：

- ^ @ 1 9 2 @ * 3 7 / 9 3

A. 12
B. 13
C. 14
D. 15
E. 以上都不正确

6. 前缀-中缀-后缀

将下列后缀表达式转换为前缀表达式 (假设所有数字都是个位数)：

4 5 * 8 2 / + 7 * 4 2 ^ -

A. -*+*45/827^42
B. *+*45/827-^42
C. -+*45/82*7^42
D. *45/82-7+*^42
E. 以上都不正确

7. 计算机计数系统

计算下列表达式，并将结果表示为十六进制。

$DEAD_{16}$ - ABE_{16}

A. $D3DF_{16}$
B. $C3EF_{16}$
C. $D4EF_{16}$
D. $D3EF_{16}$
E. 以上都不正确

8. 计算机计数系统

以下表达式用 10 进制表示的值是多少？

20_{16} + 21_{8}

A. 41
B. 48
C. 49
D. 641
E. 以上都不正确

9. 计算机计数系统

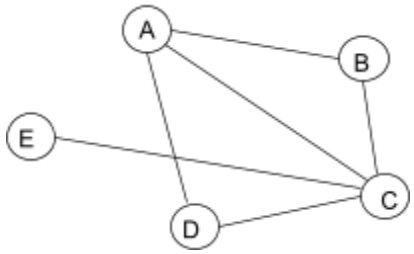
以下均为 2 进制，8 进制或16 进制的数，找出频率出现最高的值。下列数字中有多少个可以表示该值？

10110111_2 BE_{16} 356_8 11101111_2 $B7_{16}$
11001010_2 267_8 CA_{16} 11011000_2 357_8

A. 4
B. 3
C. 2
D. 1
E. 以上都不正确

10. 图论

下列哪个选项是以下图的有效遍历过程？



A. ACABCDACE
B. EDCABCA
C. ECBACDE
D. ACBADCE
E. 以上都不正确

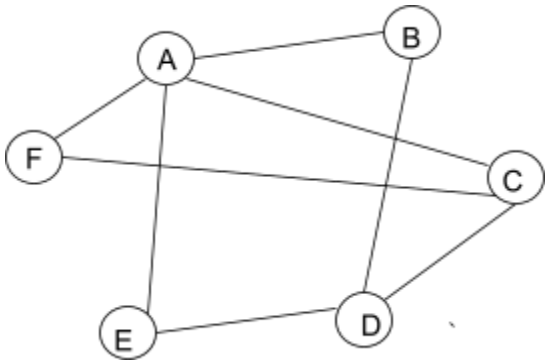
11. 图论

假设有航班往返于布法罗和克利夫兰，丹佛和西雅图，克利夫兰和亚特兰大， 亚特兰大和匹兹堡，丹佛和匹兹堡，匹兹堡和西雅图，克利夫兰和丹佛，匹兹堡和布法罗。从布法罗飞往西雅图的单程航班必须在下列哪个城市中转? 即找出路径长度为 2 ，且位于中间的城市。

A. 克利夫兰
B. 丹佛
C. 匹兹堡
D. 亚特兰大
E. 以上都不正确

12. 图论

求下列图中环的个数。



A. 5
B. 8
C. 10
D. 12
E. 以上都不正确