

American Computer Science League

2020-2021 • Contest 1: Solutions • Intermediate Division

1. Computer Number Systems

$$\begin{aligned}2021_{10} &= 1024 + 512 + 256 + 128 + 64 + 32 + 4 + 1 \\&= 1 * 2^{10} + 1 * 2^9 + 1 * 2^8 + 1 * 2^7 + 1 * 2^6 + 1 * 2^5 \\&\quad + 0 * 2^4 + 0 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0 \\&= 11111100101_2 \\&\text{There are 8 1's.}\end{aligned}$$

1. 8 (C)

2. Computer Number Systems

Method 1: Subtract in octal and convert the answer to hex. $72126_8 - 54127_8$
 $= 15777_8 = 110111111111_2 = 1\text{BFF}_{16}$

Method 2: Convert each number to hex and then subtract in hex.
 $72126_8 - 54127_8 = 111010001010110_2 - 101100001010111_2$
 $= 7456_{16} - 5857_{16} = 1\text{BFF}_{16}$

2. 1BFF_{16} (A)

3. Recursive Functions

The function evaluates as follows:

$$\begin{aligned}f(1) &= 2 \cdot 1 + 2 = 4 & f(2) &= 2 \cdot 2 + 2 = 6 \\f(3) &= f(2) - f(1) = 2 \\f(4) &= f(3) - f(2) = 2 - 6 = -4 \\f(5) &= f(4) - f(3) = -4 - 2 = -6 \\f(6) &= f(5) - f(4) = -6 - (-4) = -2 \\f(7) &= f(6) - f(5) = -2 - (-6) = 4 \\f(8) &= f(7) - f(6) = 4 - (-2) = 6 \\f(9) &= f(8) - f(7) = 6 - 4 = 2 \\f(10) &= f(9) - f(8) = 2 - 6 = -4\end{aligned}$$

The sequence of numbers repeats: 4, 6, 2, -4, -6, -2, ...

3. -4 (B)

4. Recursive Functions**4. 9 (A)**

The function evaluates as follows:

$$f(17) = 2 + f(17 - 2) = 2 + f(15) = 2 + 11 = 13$$

$$f(15) = 2 + f(15 - 2) = 2 + f(13) = 2 + 9 = 11$$

$$f(13) = 2 + f(13 - 2) = 2 + f(11) = 2 + 7 = 9$$

$$f(11) = 2 + f(11 - 2) = 2 + f(9) = 2 + 5 = 7$$

$$f(9) = 2 + f(9 - 2) = 2 + f(7) = 2 + 3 = 5$$

$$f(7) = 7 - 4 = 3$$

Therefore, $f(f(17)) = f(13) = 9$

5. What Does This Program Do?**5. 7 (D)**

The program can be traced using the following table:

a	b	c	d
5	2	25	4
5	6	25	4
5	6	10	4
5	6	10	1
5	6	10	1
5	6	10	5

There are 7 unique values of the variables: 1, 2, 4, 5, 6, 10, 25.
