Multiple linear regression

Grading the professor

Many college courses conclude by giving students the opportunity to evaluate the course and the instructor anonymously. However, the use of these student evaluations as an indicator of course quality and teaching effectiveness is often criticized because these measures may reflect the influence of non-teaching related characteristics, such as the physical appearance of the instructor. The article titled, "Beauty in the classroom: instructors' pulchritude and putative pedagogical productivity" (Hamermesh and Parker, 2005) found that instructors who are viewed to be better looking receive higher instructional ratings. (Daniel S. Hamermesh, Amy Parker, Beauty in the classroom: instructors pulchritude and putative pedagogical productivity, *Economics of Education Review*, Volume 24, Issue 4, August 2005, Pages 369-376, ISSN 0272-7757, 10.1016/j.econedurev.2004.07.013. http://www.sciencedirect.com/science/article/pii/S0272775704001165.)

In this lab we will analyze the data from this study in order to learn what goes into a positive professor evaluation.

The data

The data were gathered from end of semester student evaluations for a large sample of professors from the University of Texas at Austin. In addition, six students rated the professors' physical appearance. (This is aslightly modified version of the original data set that was released as part of the replication data for *Data Analysis Using Regression and Multilevel/Hierarchical Models* (Gelman and Hill, 2007).) The result is a data frame where each row contains a different course and columns represent variables about the courses and professors.

```r
load("more/evals.RData")
```

variable	description
`score`	average professor evaluation score: (1) very unsatisfactory - (5) excellent.
`rank`	rank of professor: teaching, tenure track, tenured.
`ethnicity`	ethnicity of professor: not minority, minority.
`gender`	gender of professor: female, male.
`language`	language of school where professor received education: english or non-english.
`age`	age of professor.

variable	description
`cls_perc_eval`	percent of students in class who completed evaluation.
`cls_did_eval`	number of students in class who completed evaluation.
`cls_students`	total number of students in class.
`cls_level`	class level: lower, upper.
`cls_profs`	number of professors teaching sections in course in sample: single, multiple.
`cls_credits`	number of credits of class: one credit (lab, PE, etc.), multi credit.
`bty_f1lower`	beauty rating of professor from lower level female: (1) lowest - (10) highest.
`bty_f1upper`	beauty rating of professor from upper level female: (1) lowest - (10) highest.
`bty_f2upper`	beauty rating of professor from second upper level female: (1) lowest - (10) highest.
`bty_m1lower`	beauty rating of professor from lower level male: (1) lowest - (10) highest.
`bty_m1upper`	beauty rating of professor from upper level male: (1) lowest - (10) highest.

variable	description
`bty_m2upper`	beauty rating of professor from second upper level male: (1) lowest - (10) highest.
`bty_avg`	average beauty rating of professor.
`pic_outfit`	outfit of professor in picture: not formal, formal.
`pic_color`	color of professor's picture: color, black & white.

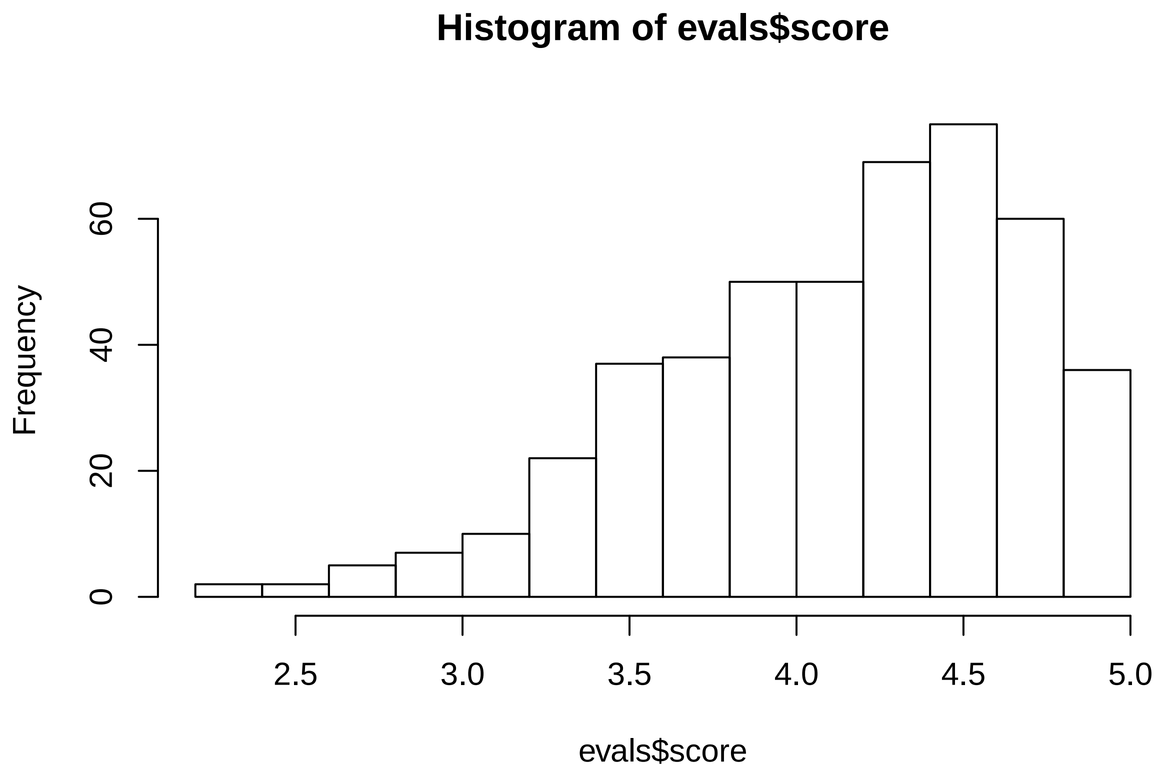
Exploring the data

1. Is this an observational study or an experiment? The original research question posed in the paper is whether beauty leads directly to the differences in course evaluations. Given the study design, is it possible to answer this question as it is phrased? If not, rephrase the question.

This is an experiment. It sounds like the study design could be used to determine if a causal relationship exists, however, without knowing more about the study design we cannot be sure.

2. Describe the distribution of `score`. Is the distribution skewed? What does that tell you about how students rate courses? Is this what you expected to see? Why, or why not?

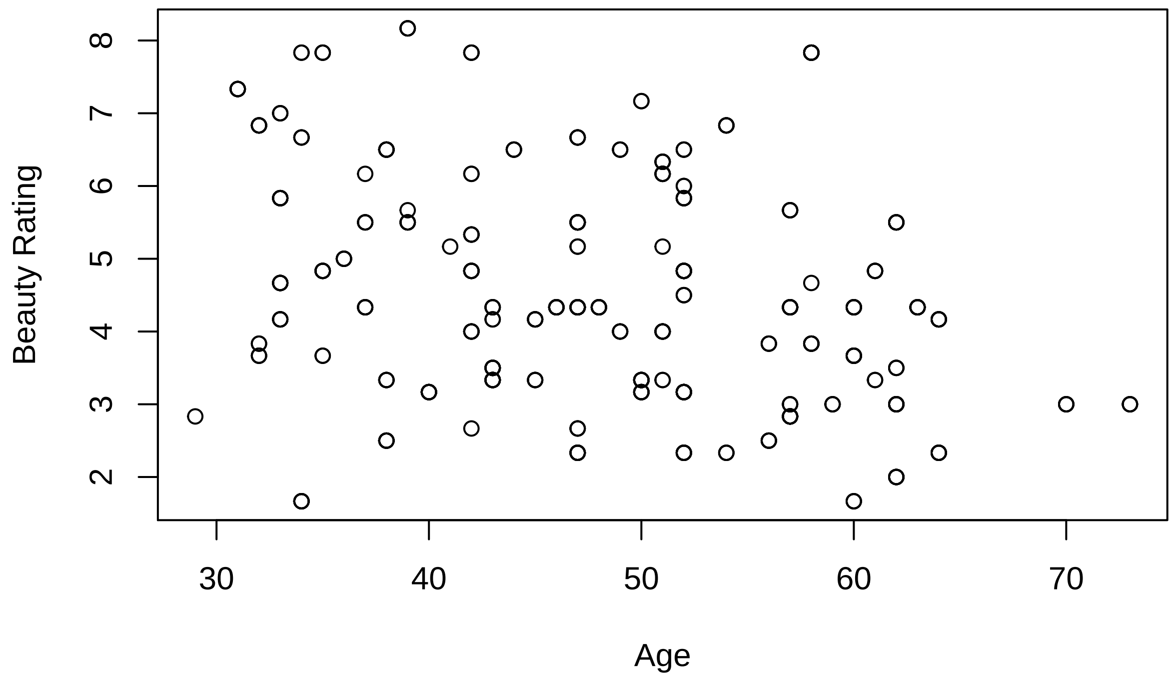
```
hist(evals$score)
```



The `score` variable is left-skewed, and tells me that most students are decently happy with their courses. I think it is more or less what I expected to see.

3. Excluding `score`, select two other variables and describe their relationship using an appropriate visualization (scatterplot, side-by-side boxplots, or mosaic plot).

```
plot(x=evals$age, y=evals$bty_avg, ylab = "Beauty Rating", xlab = "Age")
```



The `age` and `bty_avg` variables do not seem to have any discernable relationship.

Simple linear regression

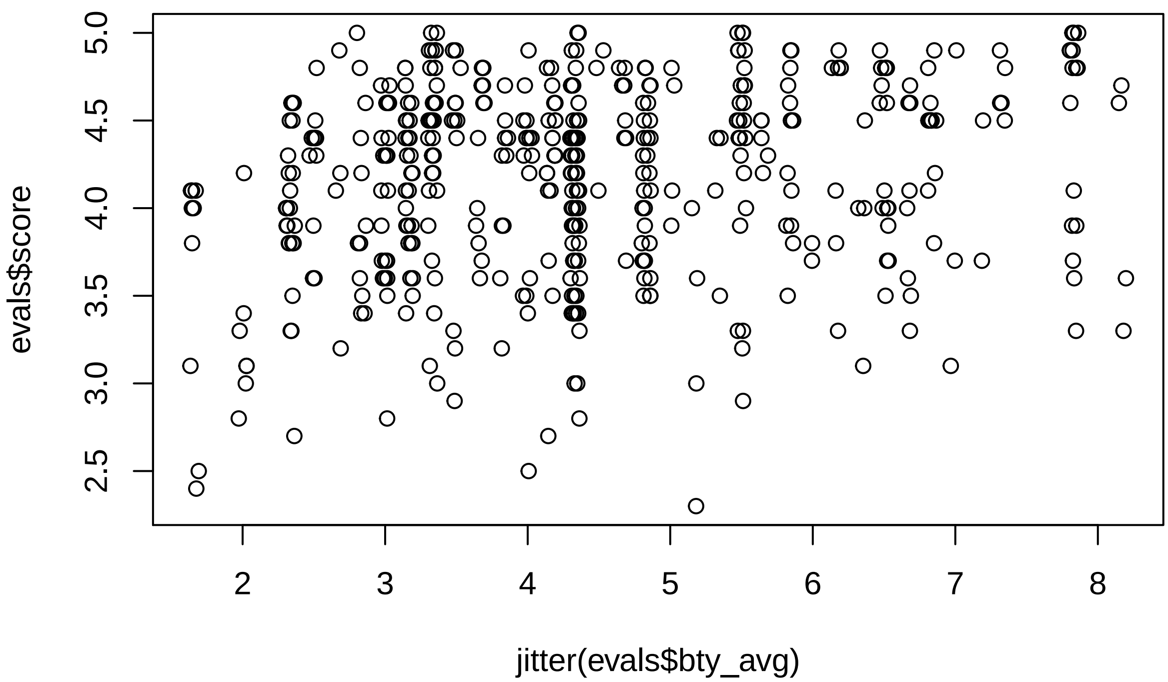
The fundamental phenomenon suggested by the study is that better looking teachers are evaluated more favorably. Let's create a scatterplot to see if this appears to be the case:

```
plot(evals$score ~ evals$bty_avg)
```

Before we draw conclusions about the trend, compare the number of observations in the data frame with the approximate number of points on the scatterplot. Is anything awry?

4. Replot the scatterplot, but this time use the function `jitter()` on the y- or the x-coordinate. (Use `?jitter` to learn more.) What was misleading about the initial scatterplot?

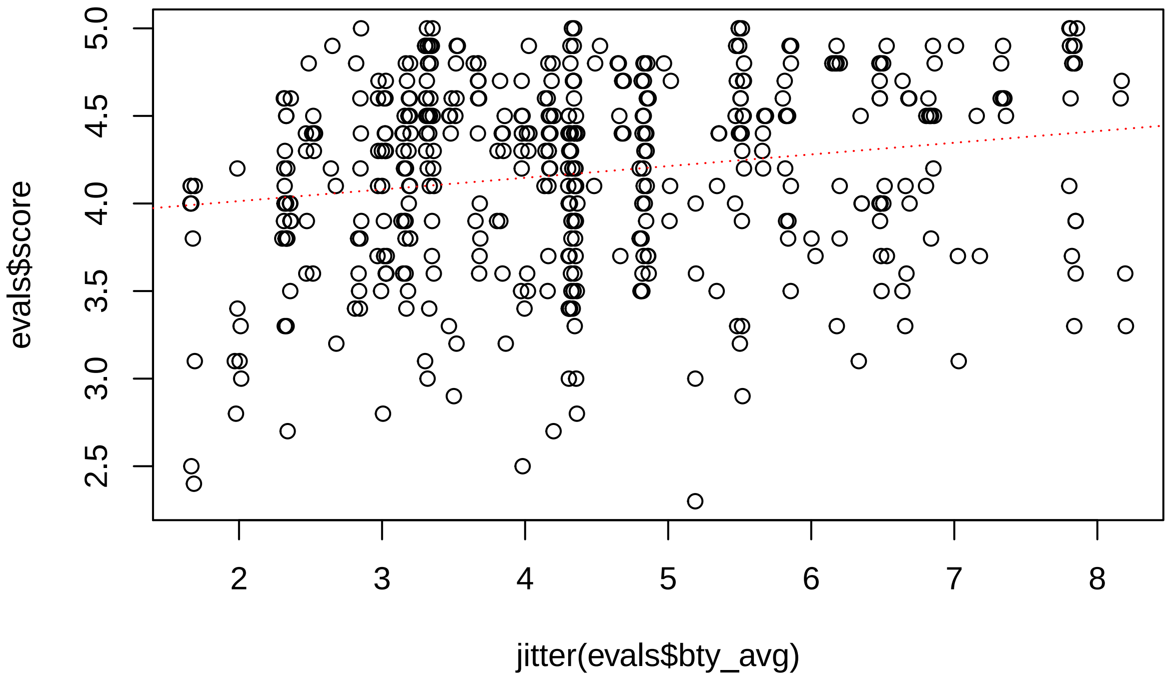
```
plot(evals$score ~ jitter(evals$bty_avg))
```



In the initial scatterplot, many points overlapped, which presented a misleading visual representation.

5. Let's see if the apparent trend in the plot is something more than natural variation. Fit a linear model called `m_bty` to predict average professor score by average beauty rating and add the line to your plot using `abline(m_bty)`. Write out the equation for the linear model and interpret the slope. Is average beauty score a statistically significant predictor? Does it appear to be a practically significant predictor?

```
m_bty <- lm(evals$score ~ evals$bty_avg)
plot(evals$score ~ jitter(evals$bty_avg))
abline(m_bty, lty = 3, col = "red")
```




```r
summary(m_bty)
```

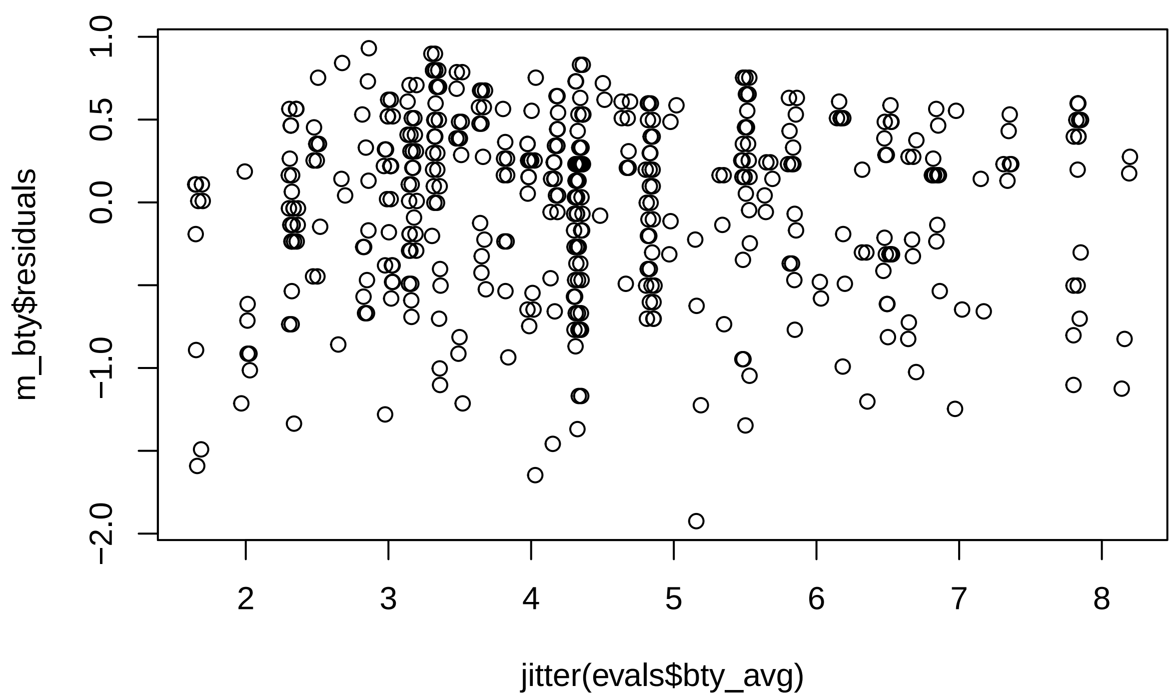
```
##
## Call:
## lm(formula = evals$score ~ evals$bty_avg)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9246 -0.3690  0.1420  0.3977  0.9309
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3.88034    0.07614   50.96  < 2e-16 ***
## evals$bty_avg  0.06664    0.01629    4.09 5.08e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5348 on 461 degrees of freedom
## Multiple R-squared:  0.03502,    Adjusted R-squared:  0.03293
## F-statistic: 16.73 on 1 and 461 DF,  p-value: 5.083e-05
```

The linear model equation is: $\hat{y} = 0.06664x + 3.88034$. The slope value indicates that each point increase on the rating of beauty, the overall score is estimated to increase by 0.06664. Although this predictor shows as statistically significant, it is certainly not practically significant, as the amount of rating it explains is very, very small.

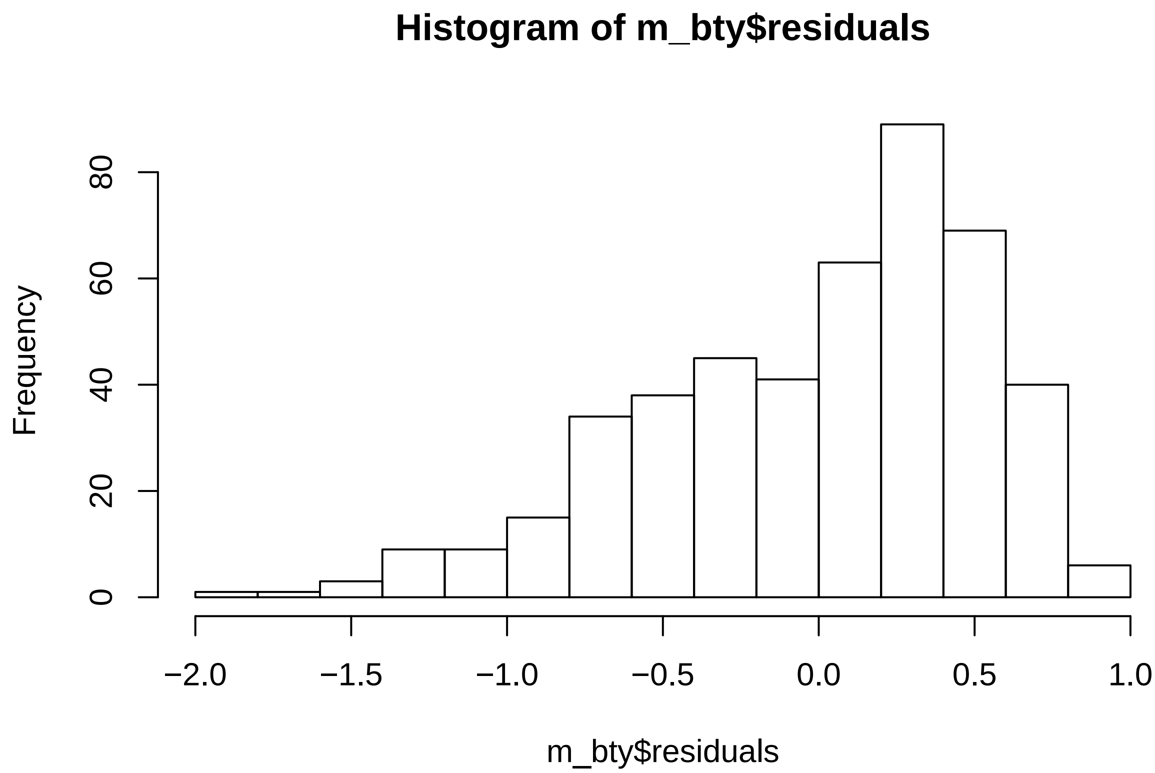
6. Use residual plots to evaluate whether the conditions of least squares regression are reasonable. Provide

plots and comments for each one (see the Simple Regression Lab for a reminder of how to make these).

```
plot(jitter(evals$bty_avg), m_bty$residuals)
```




```
hist(m_bty$residuals, breaks=20)
```



The residuals do not display any particular pattern. Looking at the distribution of the residuals on the histogram, however, they are decently far from normal.

Multiple linear regression

The data set contains several variables on the beauty score of the professor: individual ratings from each of the six students who were asked to score the physical appearance of the professors and the average of these six scores. Let's take a look at the relationship between one of these scores and the average beauty score.

```
plot(evals$bty_avg ~ evals$bty_f1lower)
cor(evals$bty_avg, evals$bty_f1lower)
```

As expected the relationship is quite strong - after all, the average score is calculated using the individual scores. We can actually take a look at the relationships between all beauty variables (columns 13 through 19) using the following command:

```
plot(evals[,13:19])
```

These variables are collinear (correlated), and adding more than one of these variables to the model would not add much value to the model. In this application and with these highly-correlated predictors, it is reasonable to use the average beauty score as the single representative of these variables.

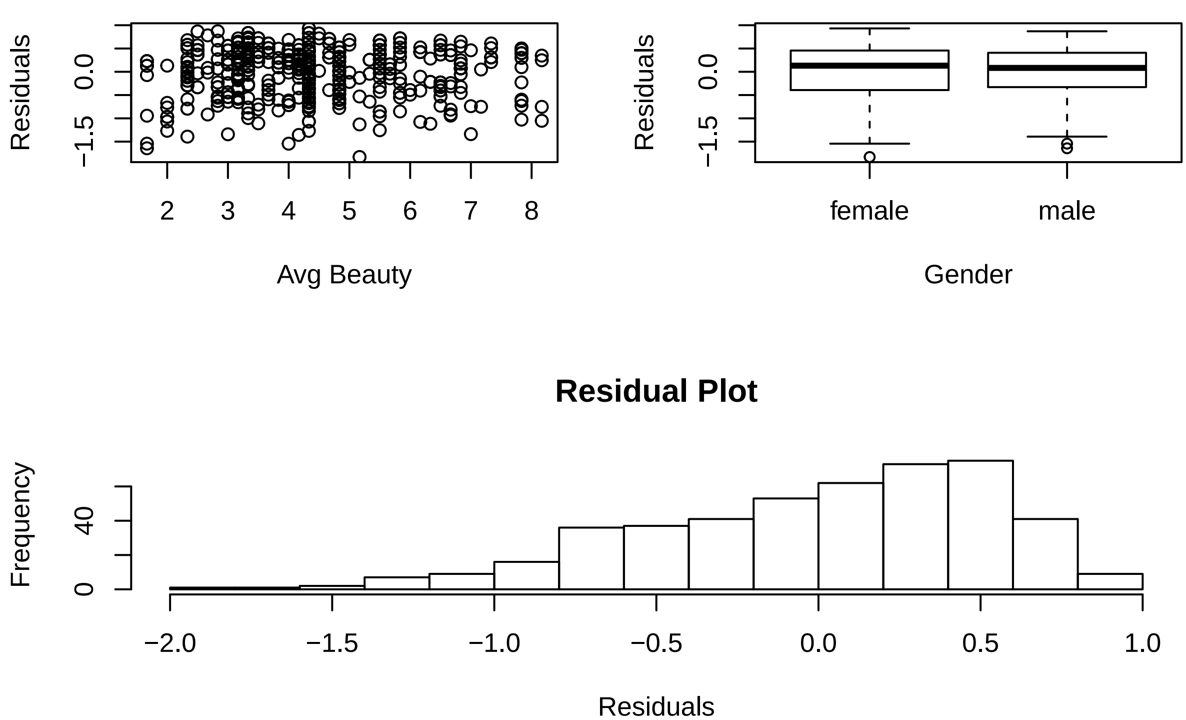
In order to see if beauty is still a significant predictor of professor score after we've accounted for the gender of the professor, we can add the gender term into the model.

```
m_bty_gen <- lm(score ~ bty_avg + gender, data = evals)
summary(m_bty_gen)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + gender, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8305 -0.3625  0.1055  0.4213  0.9314
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.74734    0.08466  44.266  < 2e-16 ***
## bty_avg      0.07416    0.01625   4.563 6.48e-06 ***
## gendermale   0.17239    0.05022   3.433 0.000652 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5287 on 460 degrees of freedom
## Multiple R-squared:  0.05912,    Adjusted R-squared:  0.05503
## F-statistic: 14.45 on 2 and 460 DF,  p-value: 8.177e-07
```

7. P-values and parameter estimates should only be trusted if the conditions for the regression are reasonable. Verify that the conditions for this model are reasonable using diagnostic plots.

```r
layout(matrix(c(1,2,3,3), 2, 2, byrow = TRUE))
plot(evals$bty_avg, m_bty_gen$residuals, ylab="Residuals",xlab="Avg Beauty")
plot(evals$gender, m_bty_gen$residuals, xlab="Gender",ylab="Residuals")
hist(m_bty_gen$residuals, xlab="Residuals", main="Residual Plot")
```



The residuals are not really normal and there is some left skew. As far as variability, the residuals compared to each predictor seem to have relatively constant variance. I think the conditions are mostly met.

8. Is `bty_avg` still a significant predictor of `score`? Has the addition of `gender` to the model changed the parameter estimate for `bty_avg`?

Yes `bty_avg` is a statistically significant predictor of `score`. With the addition of `gender` the parameter estimate for `bty_avg` has increased. Practically-speaking, `bty_avg` doesn't seem to be all that useful as a predictor.

Note that the estimate for `gender` is now called `gendermale`. You'll see this name change whenever you introduce a categorical variable. The reason is that R recodes `gender` from having the values of `female` and `male` to being an indicator variable called `gendermale` that takes a value of 0 for females and a value of 1 for males. (Such variables are often referred to as "dummy" variables.)

As a result, for females, the parameter estimate is multiplied by zero, leaving the intercept and slope form familiar from simple regression.

$$\begin{aligned}\widehat{score} &= \hat{\beta}_0 + \hat{\beta}_1 \times bty\_avg + \hat{\beta}_2 \times (0) \\ &= \hat{\beta}_0 + \hat{\beta}_1 \times bty\_avg\end{aligned}$$

We can plot this line and the line corresponding to males with the following custom function.

```r
multiLines(m_bty_gen)
```

9. What is the equation of the line corresponding to males? (*Hint:* For males, the parameter estimate is multiplied by 1.) For two professors who received the same beauty rating, which gender tends to have the higher course evaluation score?

The equation of the line corresponding to males is $\widehat{score} = (0.07416 \times$ beauty$) + 3.91973$. So, for the same professor, males would tend to rate higher than females according to this model.

The decision to call the indicator variable `gendermale` instead of`genderfemale` has no deeper meaning. R simply codes the category that comes first alphabetically as a 0. (You can change the reference level of a categorical variable, which is the level that is coded as a 0, using the`relevel` function. Use `?relevel` to learn more.)

10. Create a new model called `m_bty_rank` with `gender` removed and `rank` added in. How does R appear to handle categorical variables that have more than two levels? Note that the rank variable has three levels: `teaching`, `tenure track`, `tenured`.

```r
m_bty_rank <- lm(score ~ bty_avg + rank, data = evals)
summary(m_bty_rank)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + rank, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8713 -0.3642  0.1489  0.4103  0.9525
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)       3.98155    0.09078  43.860  < 2e-16 ***
## bty_avg           0.06783    0.01655   4.098 4.92e-05 ***
## ranktenure track -0.16070    0.07395  -2.173   0.0303 *
```

```
## ranktenured      -0.12623    0.06266  -2.014   0.0445 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5328 on 459 degrees of freedom
## Multiple R-squared:  0.04652,    Adjusted R-squared:  0.04029
## F-statistic: 7.465 on 3 and 459 DF,  p-value: 6.88e-05
```

With variables that have more than 2 levels, it creates $n - 1$ dummy variables. So with 3 levels, it makes 2 variables.

The interpretation of the coefficients in multiple regression is slightly different from that of simple regression. The estimate for `bty_avg` reflects how much higher a group of professors is expected to score if they have a beauty rating that is one point higher *while holding all other variables constant*. In this case, that translates into considering only professors of the same rank with `bty_avg` scores that are one point apart.

The search for the best model

We will start with a full model that predicts professor score based on rank, ethnicity, gender, language of the university where they got their degree, age, proportion of students that filled out evaluations, class size, course level, number of professors, number of credits, average beauty rating, outfit, and picture color.

11. Which variable would you expect to have the highest p-value in this model? Why? *Hint:* Think about which variable would you expect to not have any association with the professor score.

I expect that `pic_color` would have the highest p-value and not have any association to professor score.

Let's run the model…

```r
m_full <- lm(score ~ rank + ethnicity + gender + language + age + cls_perc_eval
             + cls_students + cls_level + cls_profs + cls_credits + bty_avg
             + pic_outfit + pic_color, data = evals)
summary(m_full)
```

12. Check your suspicions from the previous exercise. Include the model output in your response.

My guess was wrong. The p-value for picture color was actually significant in the new model. The least significant was whether there was a single or multiple professors.

13. Interpret the coefficient associated with the ethnicity variable. **The coefficient predicts higher scores for non-minority professors. It predicts an increase in score of approximately 0.123 for non-minority professors versus minority professors, assuming all other variables are equal.**

14. Drop the variable with the highest p-value and re-fit the model. Did the coefficients and significance of the other explanatory variables change? (One of the things that makes multiple regression interesting is that coefficient estimates depend on the other variables that are included in the model.) If not, what does this say about whether or not the dropped variable was collinear with the other explanatory variables?

```r
m_minus_one <- lm(score ~ rank + ethnicity + gender + language + age + cls_perc_eval
             + cls_students + cls_level + cls_credits + bty_avg + pic_outfit
             + pic_color, data = evals)
summary(m_minus_one)
```

```
##
## Call:
## lm(formula = score ~ rank + ethnicity + gender + language + age +
```

```
##     cls_perc_eval + cls_students + cls_level + cls_credits +
##     bty_avg + pic_outfit + pic_color, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.7836 -0.3257  0.0859  0.3513  0.9551
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)           4.0872523  0.2888562  14.150  < 2e-16 ***
## ranktenure track     -0.1476746  0.0819824  -1.801 0.072327 .
## ranktenured          -0.0973829  0.0662614  -1.470 0.142349
## ethnicitynot minority  0.1274458  0.0772887   1.649 0.099856 .
## gendermale            0.2101231  0.0516873   4.065 5.66e-05 ***
## languagenon-english  -0.2282894  0.1111305  -2.054 0.040530 *
## age                  -0.0089992  0.0031326  -2.873 0.004262 **
## cls_perc_eval         0.0052888  0.0015317   3.453 0.000607 ***
## cls_students          0.0004687  0.0003737   1.254 0.210384
## cls_levelupper        0.0606374  0.0575010   1.055 0.292200
## cls_creditsone credit  0.5061196  0.1149163   4.404 1.33e-05 ***
## bty_avg               0.0398629  0.0174780   2.281 0.023032 *
## pic_outfitnot formal  -0.1083227  0.0721711  -1.501 0.134080
## pic_colorcolor       -0.2190527  0.0711469  -3.079 0.002205 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4974 on 449 degrees of freedom
## Multiple R-squared:  0.187,  Adjusted R-squared:  0.1634
## F-statistic: 7.943 on 13 and 449 DF,  p-value: 2.336e-14
```

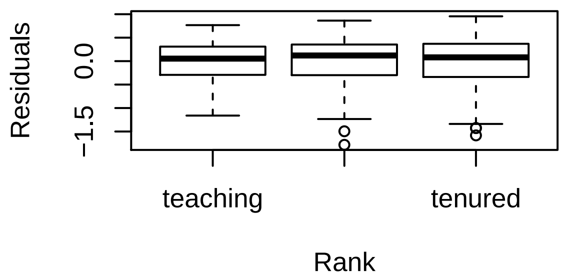
The removal of `cls_profs` did not seem to significantly impact the model. The parameter estimates only changed by a very small amount, and the adjusted R-squared only went up very slightly. I suspect that the variable was very collinear with another variable.

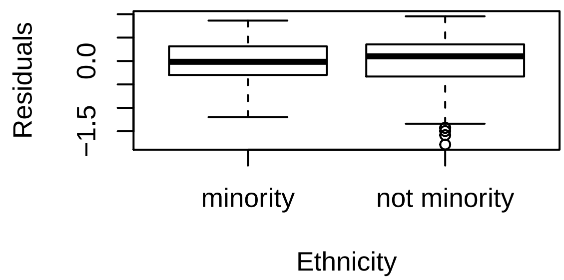
15. Using backward-selection and p-value as the selection criterion, determine the best model. You do not need to show all steps in your answer, just the output for the final model. Also, write out the linear model for predicting score based on the final model you settle on.

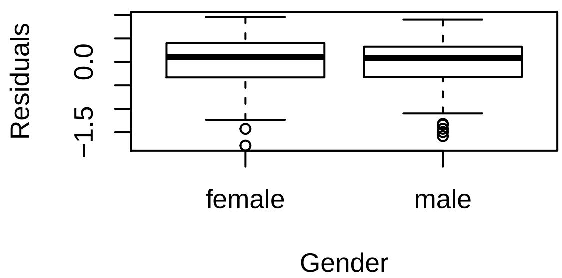
The model above `m_minus_one` was the best model. Removing another variable reduced the adjusted r-squared.

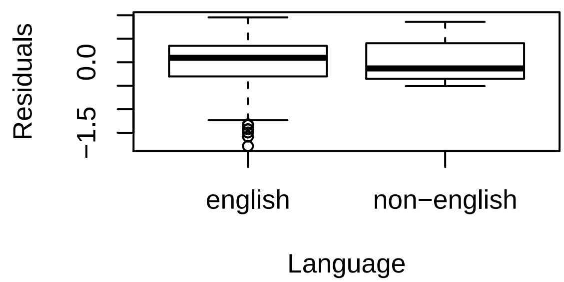
16. Verify that the conditions for this model are reasonable using diagnostic plots.

```r
layout(matrix(c(1,2,3,4), 2, 2, byrow = TRUE))
plot(evals$rank, m_minus_one$residuals, ylab="Residuals",xlab="Rank")
plot(evals$ethnicity, m_minus_one$residuals, ylab="Residuals",xlab="Ethnicity")
plot(evals$gender, m_minus_one$residuals, ylab="Residuals",xlab="Gender")
plot(evals$language, m_minus_one$residuals, ylab="Residuals",xlab="Language")
```

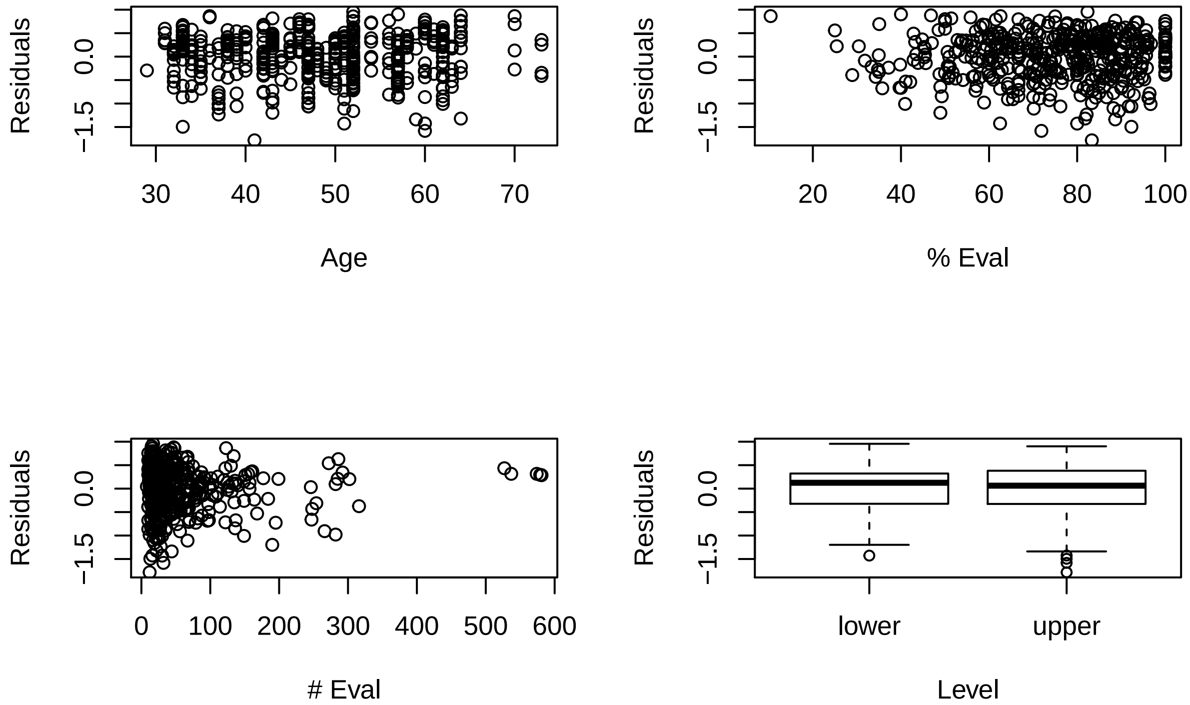




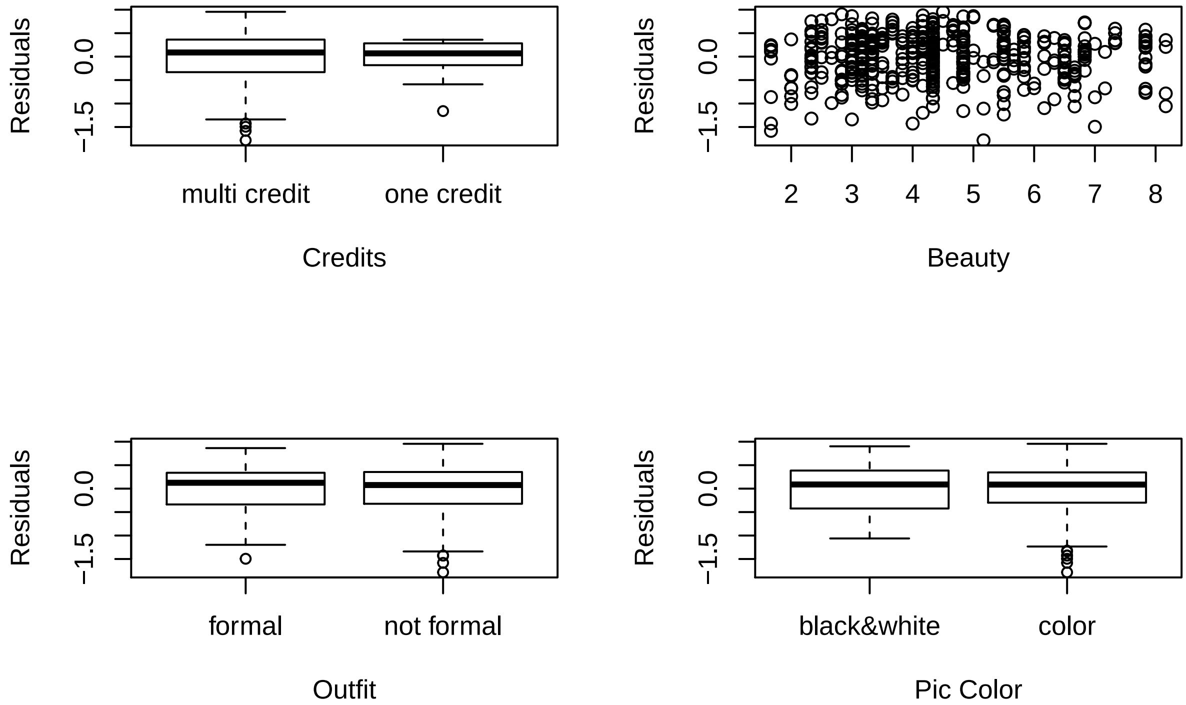




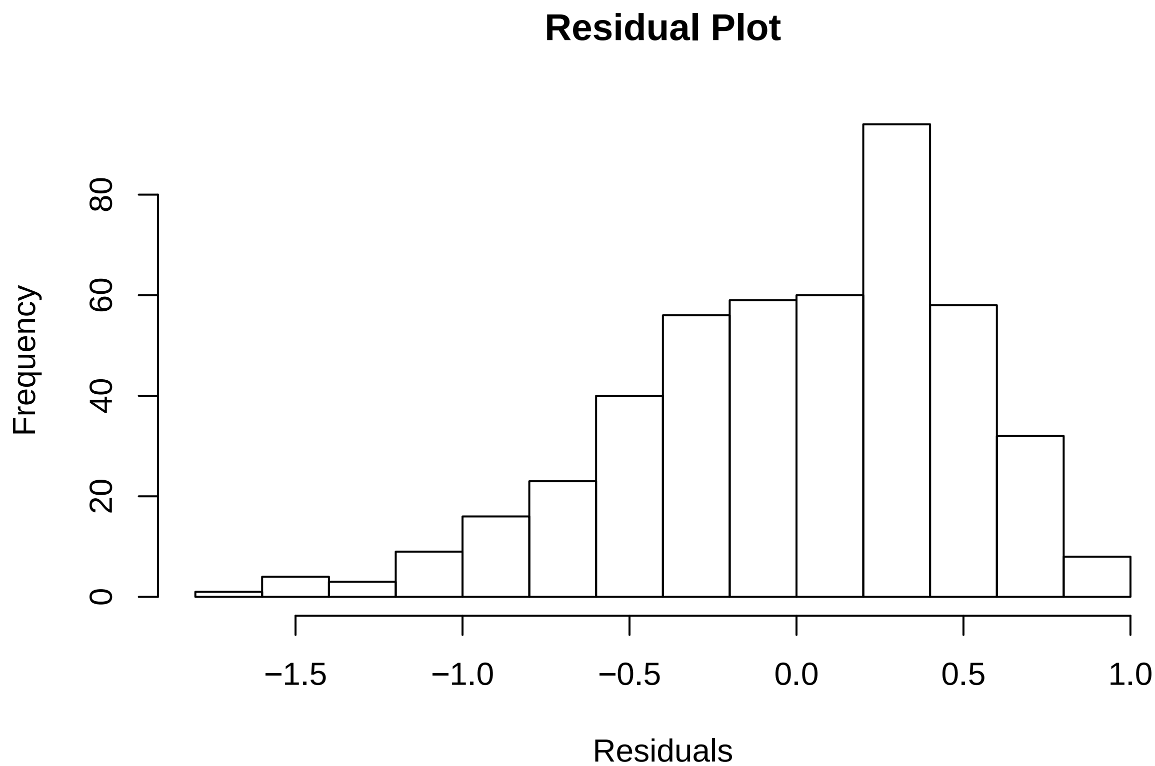

```
plot(evals$age, m_minus_one$residuals, ylab="Residuals",xlab="Age")
plot(evals$cls_perc_eval, m_minus_one$residuals, ylab="Residuals",xlab="% Eval")
plot(evals$cls_students, m_minus_one$residuals, ylab="Residuals",xlab="# Eval")
plot(evals$cls_level, m_minus_one$residuals, ylab="Residuals",xlab="Level")
```




```
plot(evals$cls_credits, m_minus_one$residuals, ylab="Residuals",xlab="Credits")
plot(evals$bty_avg, m_minus_one$residuals, ylab="Residuals",xlab="Beauty")
plot(evals$pic_outfit, m_minus_one$residuals, ylab="Residuals",xlab="Outfit")
plot(evals$pic_color, m_minus_one$residuals, ylab="Residuals",xlab="Pic Color")
```




```
par(mfrow=c(1,1))
hist(m_minus_one$residuals, xlab="Residuals", main="Residual Plot")
```



The conditions seem to be met as residuals are somewhat close to normally-distributed, and exhibit somewhat constant variability (except for some interesting patterns in `cls_students` and differing variability in `cls_credits`).

17. The original paper describes how these data were gathered by taking a sample of professors from the University of Texas at Austin and including all courses that they have taught. Considering that each row represents a course, could this new information have an impact on any of the conditions of linear regression?

Yes. Rather than each row be a rating, we're summarizing across each course and we'd lose individual-level data such as gender of the rater.

18. Based on your final model, describe the characteristics of a professor and course at University of Texas at Austin that would be associated with a high evaluation score.

A highly-rated professor would be a non-minority non-tenured young professor teaching a single-credit course, rated by a male student.

19. Would you be comfortable generalizing your conclusions to apply to professors generally (at any university)? Why or why not?

No. Students differ vastly at different institutions and so do their professors. Also, the types of courses differ.