Intro to Algorithms - Homework 2

Q1.

Assume $n = 3$; and $2^{2^3} = 2^8 = 256$

when $n = 0$, $= 2$

$= 1$, $= 4$

$= 2$, $= 16$

$= 3$, $= 256$

In this case, we can see a pattern that the program only run n time to get the answer of 2^{2^n}.

code :

```
result = 2
for i in range(0, n):
    result *= result
print ( result)
```

At first, we set the variable to 2, then we run n-time to get 2^{2^n} answer.

Q2

(a) Base on the factorial rule, the running time is $O(n)$, Because suppose we need to get $4!$, it is equal to $4! = \underbrace{4 \times 3 \times 2 \times 1}_{4\text{-times}}$. Heree, It is $O(n)$ time.

Then, if N is a n-bit number, we can do it by giving example base on the log rule. Suppose we have N is 7. it is $111_2 = 7_{10}$, it is 3-bit number, so the n is 3.

Then, we try it by using log rule, $\log_2 7 \approx 2.8 \approx 3$. Therefore, the running time in this will be $O(\log N)$

By combining $N!$ and n-bit together, we can get

$O(N \log N)$ as our answer.

(b)

```
result = 1
for i in range(1, n+1):
    result = result * i
```

The running time is $O(n)$ Because we all the way multiply from 1 to the number n.

$$7! = \underbrace{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7}_{7\text{-times}}$$

Which is $O(n)$ running time.

Q3.

(a)

Prime factorization:

1492: $1492 \div 2 = 746$

$746 \div 2 = 373$

$1492 = 2 \times 2 \times 373 = 2^2 \times 373$

1776: $1776 \div 2 = 888$

$888 \div 2 = 444$

$444 \div 2 = 222$

$222 \div 2 = 111$

$111 \div 3 = 37$

$1776 = 2 \times 2 \times 2 \times 2 \times 3 \times 37 = 2^4 \times 3 \times 37$

$GCD(1492, 1776) = 2^2 = 4$

Euclid's method:

$1776 = 1492 \times 1 + 284$

$1492 = 284 \times 5 + 72$

$284 = 72 \times 3 + 68$

$72 = 68 \times 1 + 4$

$68 = 4 \times 17$

$GCD(1492, 1776) = 4$

(b) From what we just wrote from part (a),

In linear combination of the two inputs, we can get it from the bottom to top.

$$
\begin{aligned}
4 &= 72 - (68 \times 1) \\
&= 72 - (284 - 72 \times 3) \\
&= 72 - 284 + 72 \times 3 \\
&= 72 \times 4 - 284 \\
&= (1492 - 284 \times 5) \times 4 - 284 \\
&= 1492 \times 4 - 284 \times 20 - 284 \\
&= 1492 \times 4 - 284 \times 21 \\
&= 1492 \times 4 - (1776 - 1492) \times 21 \\
&= 1492 \times 4 - 1776 \times 21 + 1492 \times 21 \\
&= 1776 \times -21 + 1492 \times 25
\end{aligned}
$$