Intro to Algorithms - Homework 1

Q1.

(a): $f(n)$ and $g(n)$ are $O(n)$, therefore $f = \Theta(g)$

(b): $\frac{1}{2} < \frac{2}{3}$, therefore $f = O(g)$

(c): $f(n)$ and $g(n)$ are both $O(n)$, therefore $f = \Theta(g)$

(d): $f(n)$ and $g(n)$ are both $O(n \log n)$, therefore $f = \Theta(g)$

(e): $f(n)$ and $g(n)$ are both $O(\log n)$, therefore $f = \Theta(g)$

(f): $f(n)$ and $g(n)$ are both $O(\log n)$, therefore $f = \Theta(g)$

(g): $n^{0.01} > \log^2 n$, therefore $f = \Omega(g)$

(h): $\frac{n^2}{\log n} \cdot \frac{\log n}{n} = n$, $n(\log n)^2 \cdot \frac{\log n}{n} = (\log n)^3$
therefore $f = \Omega(g)$

(i) $n^{0.1} > (\log n)^{10}$; therefore $f = \Omega(g)$

(j) $(\log n)^{\log n} = n^{\log \log n}$; therefore $f = \Omega(g)$

Q2. $T(n)$ is 2^n.

Base case: $T(0) = 2^0 = 1$; when n is 0, the function will print 1 *
which is true.

Induction step: show $T(n) \to T(n+1)$ for all $n \geq 0$.

Assume $T(n)$ is true, show $T(n+1) = 2^{n+1}$ is True.

for $n = 0$, it print 1 * $\quad 2^0 = 1$

$T(2) = 4 \quad n = 1$, it print 2 * $\quad 2^1 = 2$

$T(3) = 8 \quad n = 2$, it print 4 * $\quad 2^2 = 4$

$n = 3$, it print 8 * $\quad 2^3 = 8$

$T(n+1) = 2^n \cdot 2 = 2^{n+1}$

which is exactly what was to be shown.

So, $T(n+1)$ is True.

By induction, $T(n)$ is 2^n for all $n \geq 0$.

Q3. $\max(f(n), g(n)) = \Theta(f(n) + g(n))$

The basic definition of Θ-notation is $c_1 g(n) \leq f(n) \leq c_2 g(n)$

Since $f(n)$ and $g(n)$ are asymptotically nonnegative functions and the definition of Θ-notation, we can see that $f(n) \leq c_2 g(n)$ and $f(n) \geq c_1 g(n)$ if there exist constant c_1, c_2.

Hence, by combining this, we can get

$$\max(f(n), g(n)) \leq f(n) + g(n) \leq 2\max(f(n), g(n))$$

therefore, $\max(f(n), g(n)) = \Theta(f(n) + g(n))$ proved.

Q4

(a) $2^{2n} != O(2^n)$; which is mean that 2^{2n} is not $O(2^n)$

(b) Because $O(2^n)$ will be in form of $2^{n+1} \longrightarrow (2^n)2 \implies$ This will result in $O(2^n)$

However, $2^{2n} \rightarrow (2^n)^2$, which is $(2^n)^2 > (2^n)2$.

Hence, $2^{2n} != O(2^n)$.