GEOG 4/5/7 9073: Environmental Analysis in R

Week 9.01: Localized spatial analysis

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Today's schedule

- Open discussion
- Autocorrelation and its metrics (lecture)

Anything to discuss? Questions?

Remaining topics

- Week 10: Localized Spatial Analysis (Intro lab 3), update presentations
- Week 11: AAG
- Week 12: Rasters
- Week 13: Making maps (Intro lab 4)
- Week 14: Interactive mapping (Intro lab 5)
- Week 15: Applications
- Week 16: Project presentations

This week's plan

- Today: lecture on formalizing space (it may look familiar to some of you)
- Thursday:
  - update presentations
  - lab that builds on lecture and demonstrates how to calculate spatial autocorrelation in R

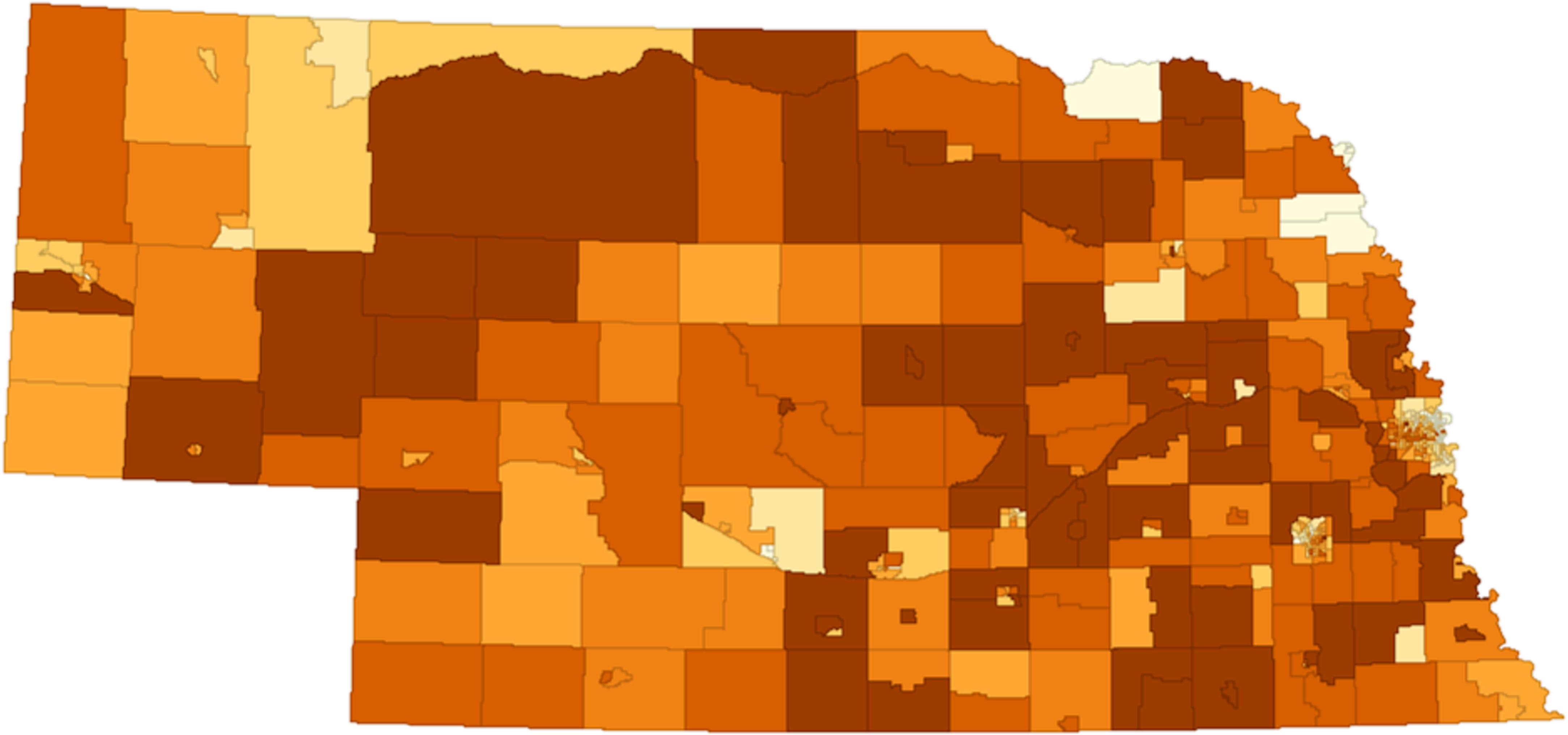
Today is about formalization

- Why?
- But also, WHY???
- Formalizing spatial relationships is foundational
    - "Global" spatial autocorrelation
    - Local spatial autocorrelation metrics
    - Spatial clustering
    - Hot spots/cold spots

Tobler's first law of Geography

- Everything is related to everything else, but nearer things are more related than farther (or something like that)
- Formally, *spatial autocorrelation*

Spatial autocorrelation of areal units (% white, 2010 Census)



Relevant questions

- Which areas are important?
- Which areas are unusual?
- Are there “hotspots” of some phenomena?
- How much influence do neighbors have?
- How should we measure/ conceptualize “neighbors”?
- Implications of our choices?

Spatial autocorrelation

the degree to which a variable is correlated with itself across space. It indicates whether similar values cluster together or are dispersed in a given spatial distribution

Types of Spatial Autocorrelation:

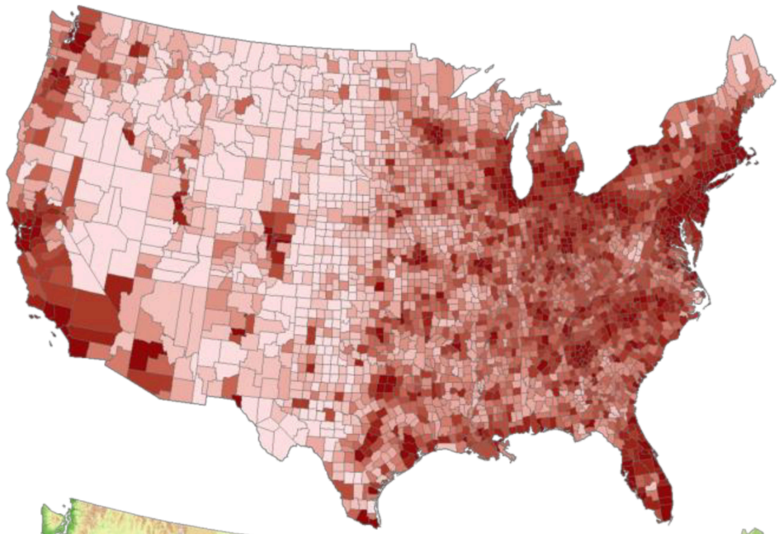
- Positive Spatial Autocorrelation: Nearby locations have similar values (e.g., high-income neighborhoods clustering together)
- Negative Spatial Autocorrelation: Nearby locations have dissimilar values (e.g., urban-rural income disparities)
- Zero Spatial Autocorrelation: No spatial pattern, values are randomly distributed

Spatial autocorrelation foundations

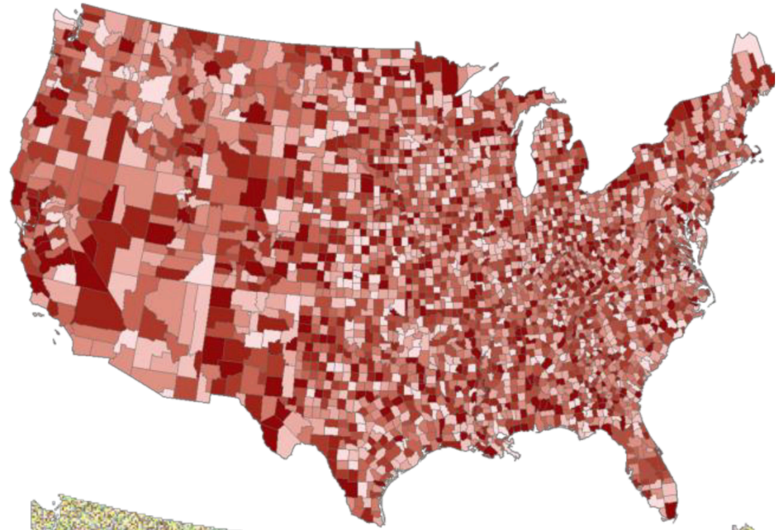
Tobler's First Law of Geography: "Everything is related to everything else, but near things are more related than distant things."

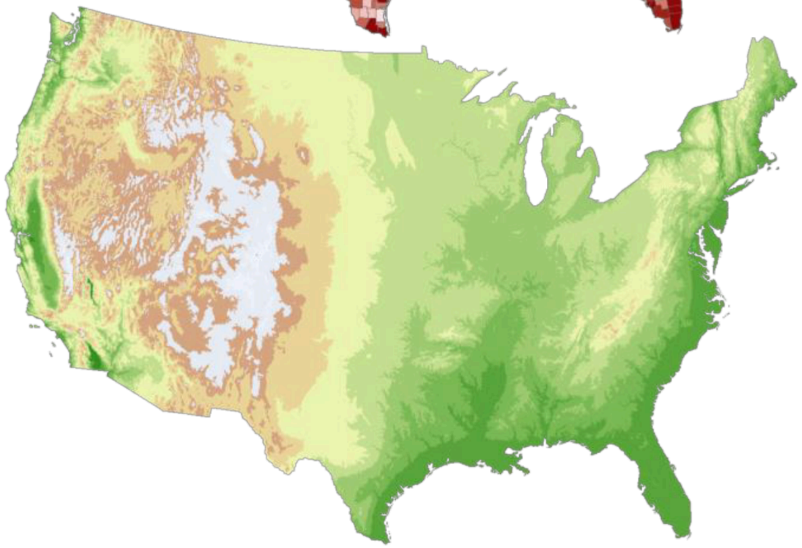
- Positive Spatial Autocorrelation: Nearby locations have similar values
  - Example: Cyanobacteria blooms tend to cluster in specific parts of Lake Erie
- Negative Spatial Autocorrelation: Nearby locations have opposite values
  - Example: Temperature inversion effects—low temperatures in valleys, high temperatures on ridges

If features were randomly distributed ...

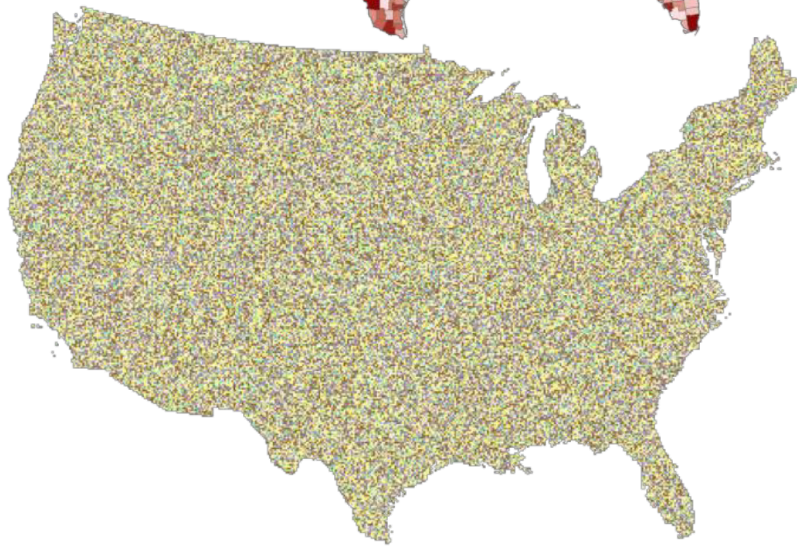


... population density map of the US would look like this





... elevation map of the US would look like this



Why does SA matter?

1. Helps identify spatial patterns (e.g., clustering of environmental pollutants)
2. Impacts statistical modeling (e.g., regression models that assume independence may be invalid)
3. Affects interpolation methods used for predicting missing data.

A digression into formalization of neighborhoods

Spatial context matters

- For a statistical method to be explicitly spatial, it needs to contain some representation of the geography, or spatial context
- One of the most common ways is through *spatial weights matrices*

Formalizing processes

- **(Geo)Visualization**: translating numbers into a (visual) language that the human brain “speaks better”
- **Spatial Weights Matrices**: translating geography into a (numerical) language that a computer “speaks better”

Spatial weights matrices

Core element in several spatial analysis techniques:

- Spatial autocorrelation
- Spatial clustering / geodemographics
- Spatial regression

Formalization

W as a formal representation of space

W (the spatial weights matrix)

- *N x N positive matrix* that contains **spatial relations** between all the observations in the sample
- FORMALLY, w_{ij}… the weight from zone *i* to zone *j*
- Core concept in statistical analysis of areal data
- Two steps involved:
    - define which relationships between observations are to be given a nonzero weight, i.e., define spatial neighbors
    - assign weights to the neighbors

w_{ii} = 0 (by convention)

…what is a neighbor?

How would you define a "neighbor"?

- Making the neighbors and weights is not easy as it seems to be
- Which states are near Ohio?


Washington
Seattle
Spokane
Portland
Missoula
Montana
Oregon
Idaho
Boise
Idaho Falls
Billings
Minot
North Dakota
Bismarck
Minnesota
Fargo
Duluth
Lake Superior
Eureka
Wyoming
Casper
South Dakota
Rapid City
Minneapolis
St. Paul
Wisconsin
Lake Michigan
Lake Huron
Michigan
Toronto
Lake Ontario
New York
Burlington
Concord
Maine
Bangor
Portland
Boston
Massachusetts
Hartford
Rhode Island
Connecticut
Sioux Falls
Madison
Milwaukee
Detroit
Lake Erie
Buffalo
Reno
Nevada
Salt Lake City
Utah
Cheyenne
Nebraska
Omaha
Lincoln
Iowa
Cedar Rapids
Chicago
Ft Wayne
Cleveland
Pennsylvania
Pittsburgh
New York
New Jersey
Philadelphia
San Francisco
San Jose
Denver
Colorado
Pueblo
U. S. A.
Kansas
Topeka
Kansas City
Peoria
Illinois
Indianapolis
Indiana
Columbus
Ohio
Cincinnati
Baltimore
Delaware
Maryland
Washington D. C.
West Virginia
Virginia
Richmond
Norfolk
St. Louis
Missouri
Louisville
Frankfort
Lexington
Kentucky
California
Las Vegas
Wichita
Springfield
Raleigh
North Carolina
Charlotte
Los Angeles
Flagstaff
Arizona
Santa Fe
Albuquerque
New Mexico
Amarillo
Oklahoma
Oklahoma City
Tulsa
Nashville
Tennessee
San Diego
Phoenix
Tucson
Lubbock
Little Rock
Arkansas
Memphis
Birmingham
Atlanta
South Carolina
Charleston
Georgia
Mississippi
Alabama
Savannah
Dallas
El Paso
Jackson
Louisiana
Texas
Mobile
Tallahassee
Jacksonville
Baton Rouge
New Orleans
Austin
San Antonio
Houston
Orlando
Tampa
Florida
Ft Myers
Miami
Key West
Beaufort Sea
Alaska
Anchorage
Juneau
Berring Sea
Gulf of Alaska
0 500 Miles
0 500 Km
Pacific Ocean
Honolulu
Hawaii
0 100 Miles
0 100 Km

What IS a neighbor?

A neighbor is “somebody” who is:

- Next door → **Contiguity**-based Ws
- Close → **Distance**-based Ws

Spatial neighbors

Contiguity-based neighbors

- Zone i and j are neighbors if zone i is contiguous or adjacent to zone j
- But what constitutes contiguity?

Distance-based neighbors

- Zone i and j are neighbors if the distance between them are less than the threshold distance
- But what distance do we use?

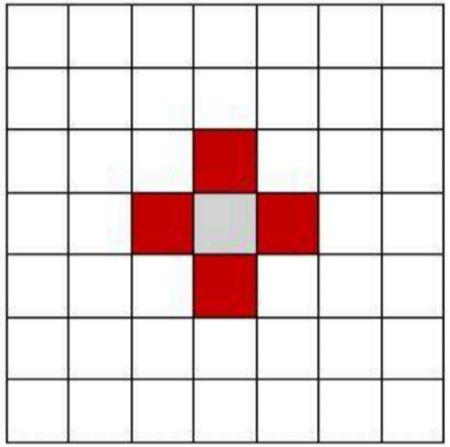
Choice of W

- Should be based on and reflect the underlying channels of interaction for the question at hand. Examples:
  - Processes propagated by immediate contact (e.g. disease contagion) → Contiguity weights
  - Accessibility → Distance weights
  - Effects of county differences in laws → Block weights

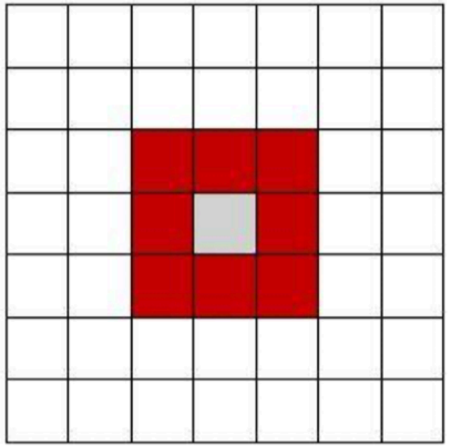
Contiguity

- Common boundaries: if two polygons share boundaries to some degree, they will be labeled as neighbors under these kinds of weights
    - **Queen**: only need to share a vertex (a common POINT)
    - **Rook**: share a vertex AND a line segment
- Depending on the level of irregularity, queen and rooks contiguity may be *very* similar (if not identical)

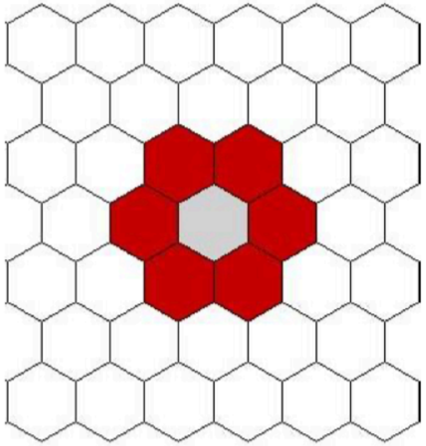
Contiguity-based Spatial Neighbors



Rook



Queen



Hexagons

Irregular

Example

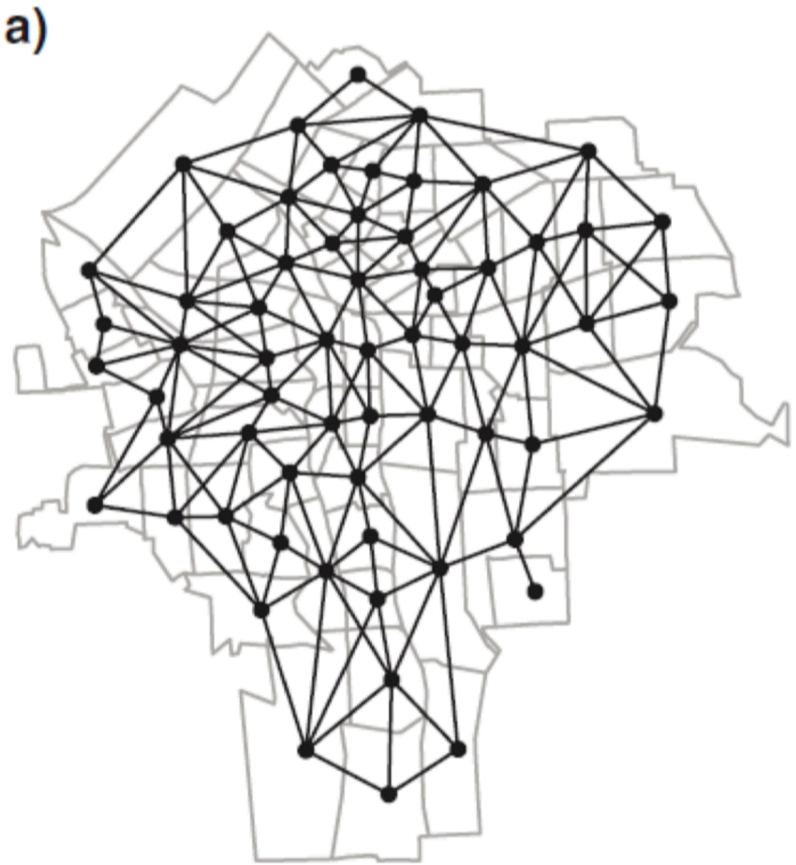
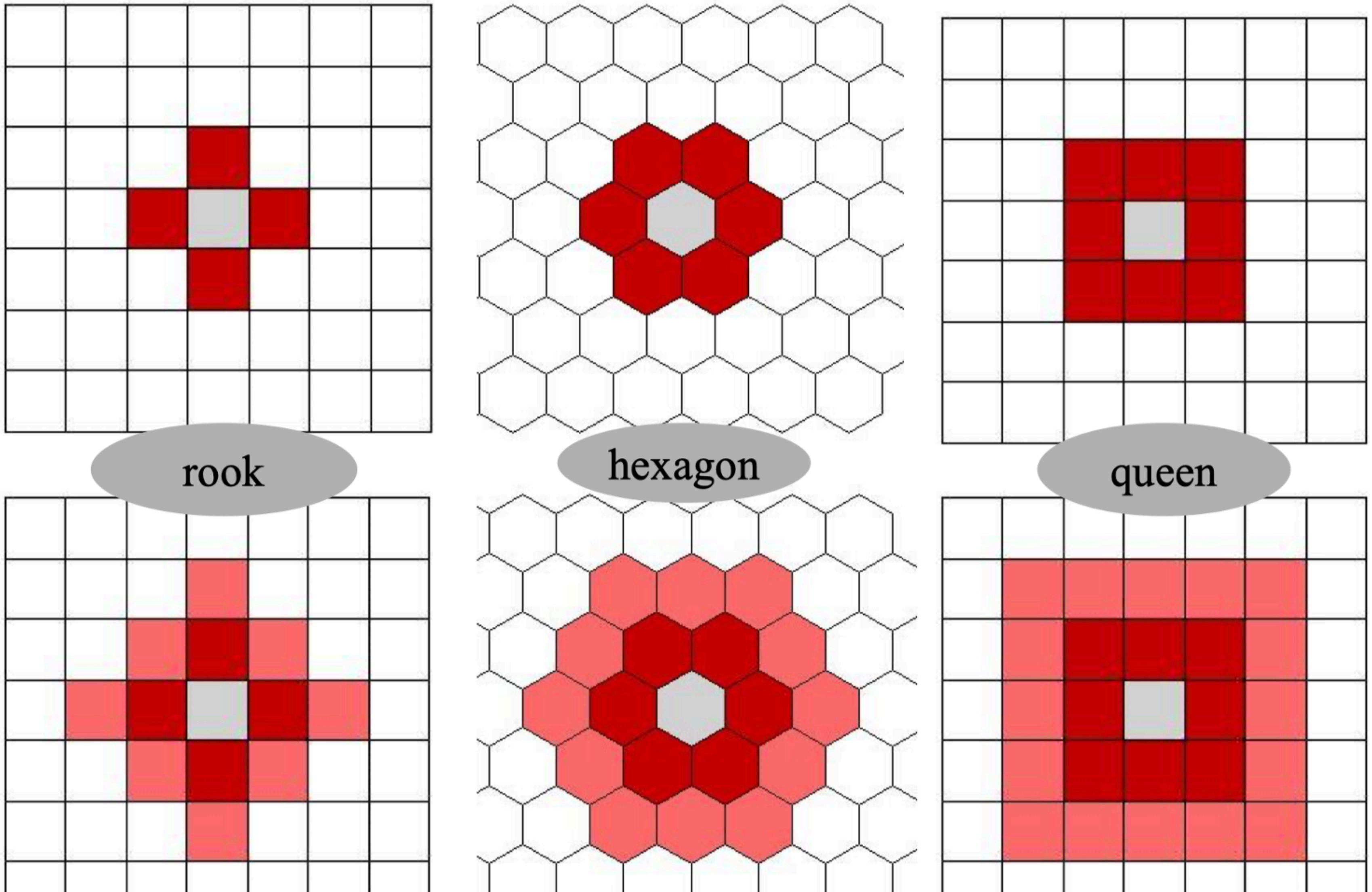


Fig. 9.3. (**a**) Queen-style census tract contiguities, Syracuse; (**b**) Rook-style contiguity differences shown as thicker lines

Higher-order contiguity

1st order
Nearest neighbor

2nd order
Nearest neighbor



Distance-based neighbors

- How do we measure distance between polygons?
- Distance metrics
  - 2D Cartesian distance (projected data)
  - 3D spherical distance/great-circle distance (lat/long data)
- *But where do we measure from?*
- *Any implications of our choices?*

Distance-based neighbors (k-nearest)

Fig. 9.5. (**a**) $k = 1$ neighbours; (**b**) $k = 2$ neighbours; (**c**) $k = 4$ neighbours

Distance-based neighbors (threshold distance)

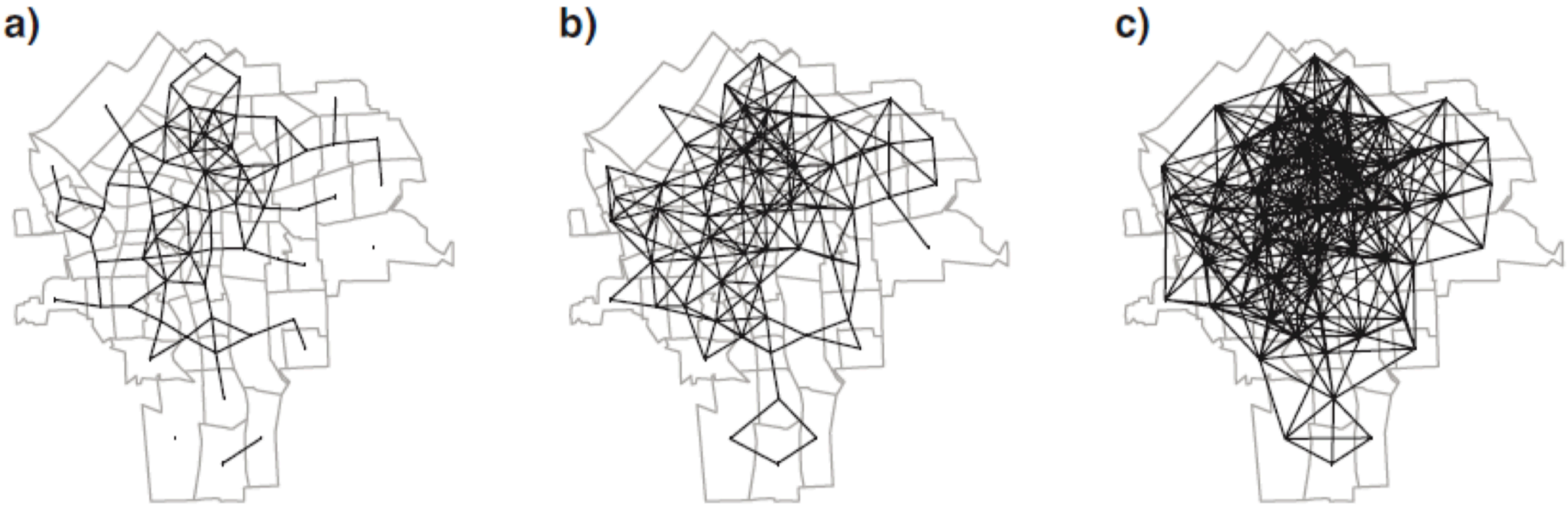
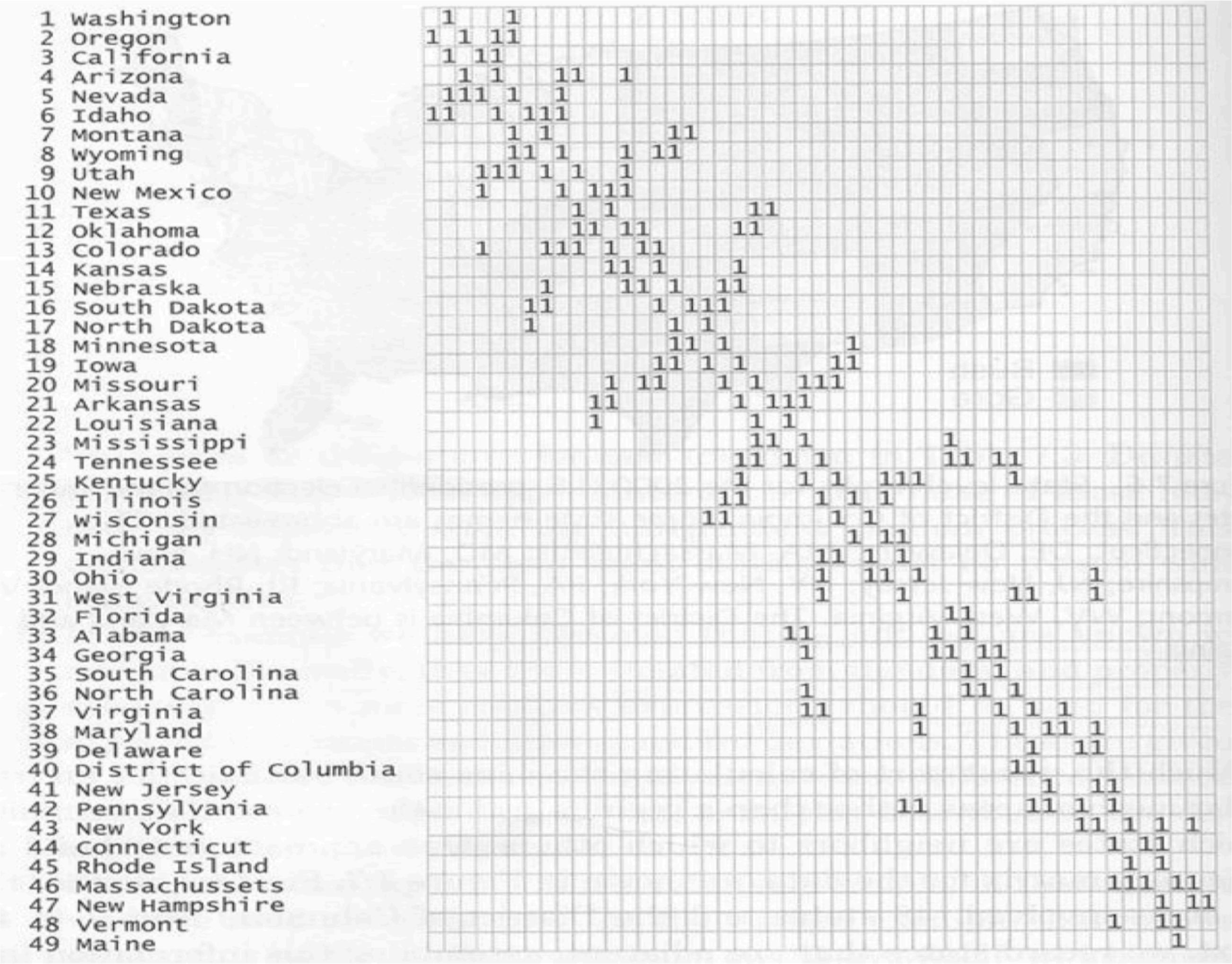


Fig. 9.6. (**a**) Neighbours within 1,158 m; (**b**) neighbours within 1,545 m; (**c**) neigl
bours within 2,317 m

A simple spatial weights matrix

1 if adjacent,
2 if not



Decay functions of distance

- Most common choice is the multiplicative inverse (reciprocal) of the distance between locations *i* and *j*
- Other functions also used
  - inverse of squared distance
  - Or negative exponential

Standardization

- In some applications (e.g. spatial autocorrelation) it is common to standardize W
- The most widely used standardization is row-based: divide every element by the sum of the row

Back to spatial autocorrelation...

Measuring SA

Moran's I – A ***global*** measure indicating overall spatial autocorrelation in a dataset. Values range from -1 (negative) to +1 (positive), with 0 indicating no spatial pattern

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \times \frac{\sum_i \sum_j w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\sum_i (x_i - \bar{x})^2}$$

What Each Part Means

- N = Total number of locations (e.g., cities, counties, pixels)
- x_i = Value of the variable at location i (e.g., population density, crime rate)
- $\bar{x}$ = Mean (average) of the variable across all locations
- w_{ij} = Spatial weight between location i and location j (defines how "close" or "connected" two locations are)

Breaking It Into Two Main Parts

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \times \frac{\sum_i \sum_j w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\sum_i (x_i - \bar{x})^2}$$

- The numerator
  - Measures whether locations with similar values are near each other
  - If nearby locations have similar values, the sum will be high → positive spatial autocorrelation
  - If nearby locations have very different values, the sum will be negative → negative spatial autocorrelation
- The denominator
  - This is just a standard variance measure, which helps us scale the statistic so it is comparable across datasets

Testing for spatial autocorrelation in R isn't too hard

```
moran.test(sf_object$variable, weights_file)
```

In practice (see lab 3)

```
moran.test(ohio.projected$B01001e47, lw)
```

Transitioning to Local Measures of Spatial Autocorrelation

We’ve explored Moran’s I as a global measure of spatial autocorrelation…

- It tells us whether spatial autocorrelation exists across the entire study area…
- but what if patterns vary within the study region?

A single global statistic may miss localized clusters and variations

To better understand spatial heterogeneity, we need local measures

Local Indicators of Spatial Association (LISAs)

LISAs help answer key questions:

1. Where are significant clusters of high or low values?
2. Are there spatial outliers—areas with values different from their neighbors?
3. How does spatial autocorrelation vary across a region?

LISAs allow us to detect and map local patterns that global measures might overlook

Spatial lag (can be confusing)

Formally...

The product of a spatial weights matrix *W* and a given variable *y*

$Wy_i = \sum w_{ij}y_{ij}$

more generally:

- Measure that captures the behavior of a variable in the neighborhood of a given observation *i*
- If *W* is standardized, the spatial lag is the average value of the variable in the neighborhood of *i*
- Common notation: the spatial lag of *y* is expressed as W_y
- With a neighbor structure defined by the non-zero elements of the spatial weights matrix W, a spatially lagged variable is a weighted sum or a weighted average of the neighboring values for that variable

Back to the formalization

The product of a spatial weights matrix *W* and a given variable *y*

$Wy_i = \sum w_{ij}y_{ij}$

This allows us to compare the value of y to W_y

or in other words, the value of the feature to the weighted average of the feature's neighbors

Still a bit confused? Think of it this way:

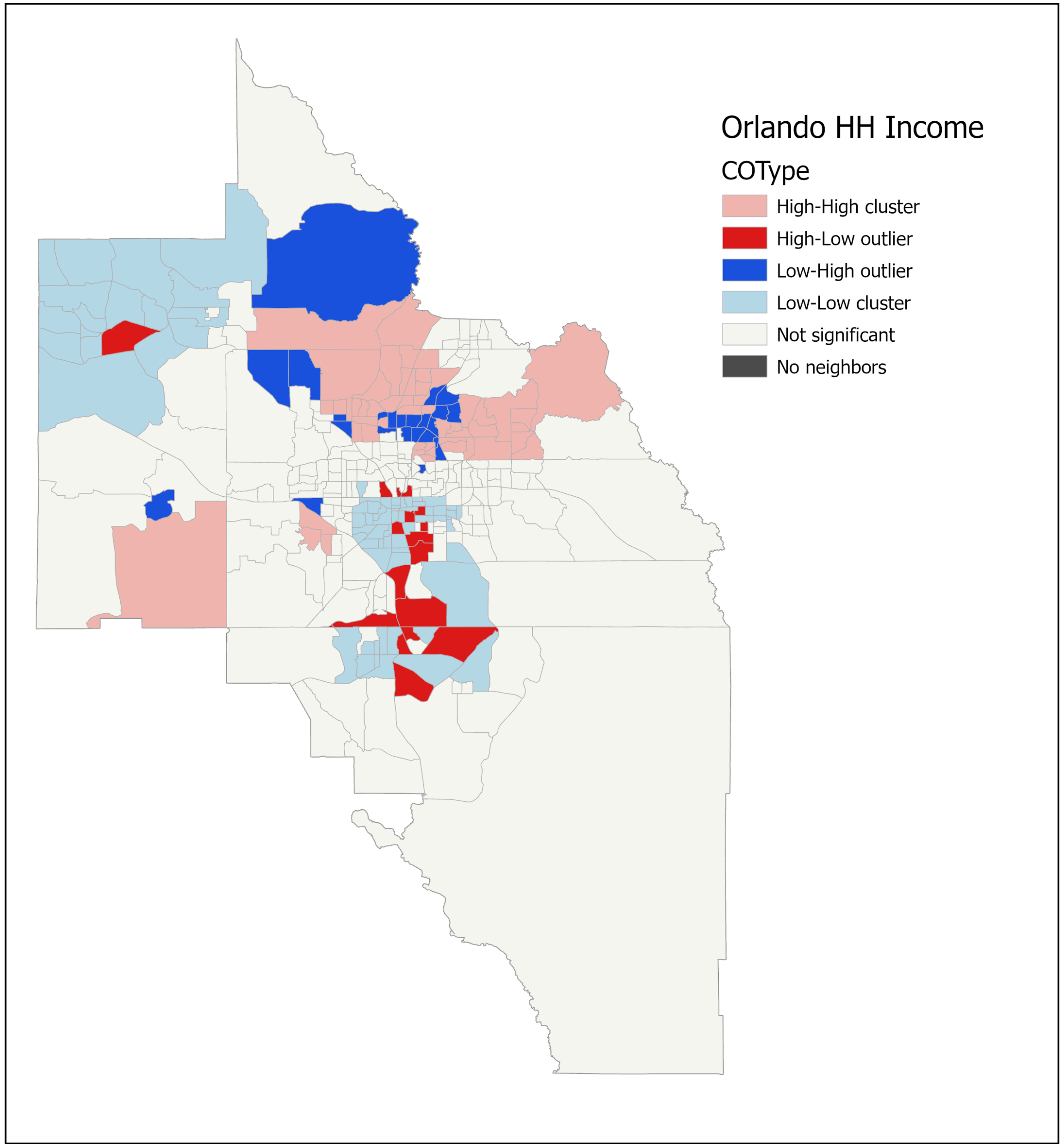
How different is the per capita income of Portage County to its neighbors?

Assumes we have formalized a neighborhood (W) and weighted those neighborhood relationships

Moran plot

A standardized Moran Plot also partitions the space into four quadrants that represent different situations:

1. High-High (HH): high values above average surrounded by values above average
2. Low-Low (LL): low values below average surrounded by values below average
3. High-Low (HL): high values above average surrounded by values below average
4. Low-High (LH): low values below average surrounded by values above average



Clusters and outliers

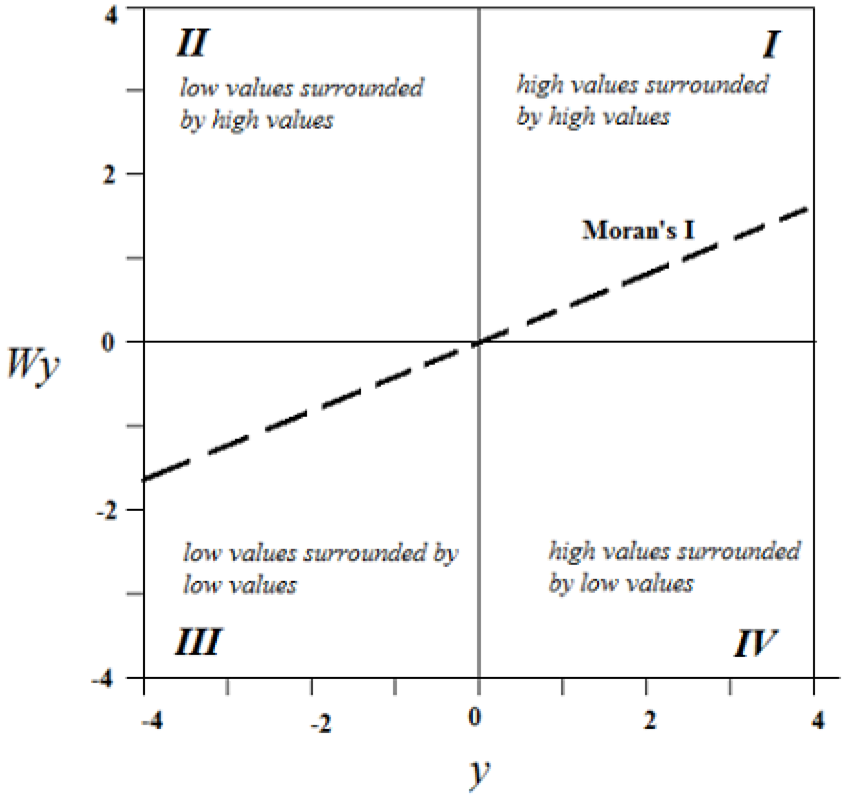
- **"Clusters":** high values surrounded by high (H-H) OR low surrounded by low (L-L)
- **"Outliers":** low surrounded by high (L-H) OR high surrounded by low (H-L)

Requires Monte Carlo test of statistical significance

More about Moran statistics

- A standardized Moran Plot implies that average values are centered in the plot (as they are zero when standardized) and dispersion is expressed in standard deviations
- General rule: values greater or smaller than two standard deviations are considered *outliers*

Moran plot



II
low values surrounded by high values
I
high values surrounded by high values
Moran's I
low values surrounded by low values
III
high values surrounded by low values
IV
Wy
y
4
2
0
-2
-4

For this week

- Thursday:
  - Lab 3
  - In-class presentations
- Chapter 8 from your textbook
- Practice, practice, practice