$\boxed{1}$

Passo 0: scelta delle variabili

x_i = numero di addetti assegnati al casello autostradale della città ~~dove~~ dalla i-esima con $i = A, B, C, T, M$ ~~corrispondente~~

y_i = variabile decisionale $i = A, B, T$ con valore $\in [0,1]$

Passo 1: vincoli

$x_T \geq (50 x_A + 75 x_B + 75 x_g) \cdot 0{,}01\%$

$x_M \geq (35 x_A + 70 x_B + 32{,}5 x_C) \cdot 0{,}01\%$

$x_A \geq \left((35 x_T + 28 x_M)/2 + (65 x_T + 42 x_M)/2\right) \cdot 0{,}01\%$

$x_B \geq \left((50 x_T + 60 x_M)/2 + (100 x_T + 80 x_M)/2\right) \cdot 0{,}01\%$

$x_C \geq \left((60 x_T + 20 x_M)/2 + (90 x_T + 45 x_M)/2\right)$

$x_C \geq (75 x_T + 32{,}5 x_M) \cdot 0{,}01\%$

$x_M + x_T \leq 35$

$x_B \geq 16$

$x_M + x_T \geq (x_A + x_B + x_C) \cdot \frac{1}{2}$

$x_M + x_T \leq 2 (x_B + x_C)$

$x_T \leq 19 y_T$

$x_B \leq 18 y_B$ $\quad y_i \in \{0,1\}$

$x_A \leq 9 y_A$

deve essere vero almeno 1

$y_T + y_B + y_A \geq 1$

Passo 2:

min $3{,}5 x_T + 3{,}6 x_M + 2{,}8 x_A + 2{,}9 x_B + 2{,}8 x_C$

$1 + \frac{1}{2}$

$\boxed{2}$

$\frac{1}{2}\cdot -2$ $\quad \frac{3}{2}\cdot -2 = -3 \quad -\frac{1}{4}\cdot -x \quad \leq \frac{1}{2}$ $\quad \frac{4}{\cancel{2}}\cdot 2$

$\min \; -x_1 + x_2 + x_3$

$-x_1 + 2x_2 - x_4 = -2$

$-\frac{1}{2}x_1 - x_2 + 2x_3 - 3x_5 \geq -4 \;\longrightarrow\; \frac{1}{2}x_1 + x_2 - 2x_3 + 3x_5 + x_6 = 4$

$x \geq 0$

$\frac{1}{2}x_1 + x_2 - 2x_3 + 3x_5 \leq 4$

F. standard

$x_1 - 2x_2 \quad + x_4 = 2$

$\frac{1}{2}x_1 + x_2 - 2x_3 + 3x_5 + x_6 = 4$

$x \geq 0$

il problema è in forma canonica; le variabili in base sono x_4 e x_6

$$
\begin{array}{cccccc|c}
1 & -2 & 0 & 1 & 0 & 0 & 2 \\
1/2 & 1 & -2 & 0 & 3 & 1 & 4 \\
\hline
-1 & 1 & 1 & 0 & 0 & 0 & 0
\end{array}
$$

~~iterazione 0:~~

$$
\begin{array}{cccccc|c}
1 & -2 & 0 & 1 & 0 & 0 & 2 \\
1/2 & 1 & -2 & 0 & 3 & 1 & 4 \\
\hline
1/2 & 0 & -1 & 1 & 3 & 1 &
\end{array}
$$

$x_B = \begin{pmatrix} x_4 \\ x_6 \end{pmatrix}$ $\quad x_N = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_5 \end{pmatrix}$

$c_B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad c_N = \begin{pmatrix} 1/2 \\ 0 \\ -1 \\ 3 \end{pmatrix}$

Iterazione 0

$r^0 = c_N - (B^{-1}\cdot N)^T \cdot c_B =$

$= \begin{pmatrix} 1/2 \\ 0 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 & 1/2 \\ -2 & 1 \\ 0 & -2 \\ 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 0 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 3/2 \\ -1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

$-5 + \frac{3}{2} = -\frac{7}{2} \qquad 2 + \frac{3}{2}$

$2 + \frac{3}{2} = \frac{7}{2}$

$= \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

ottimalità:

$r^0 \geq 0$ no

illimitatezza $\pi_1 \leq 0$ NO

~~Pivot~~

V. entrante: $h = 1 \rightarrow x_1$

V. uscente $\min \left\{\frac{2}{1}, 4\cdot 2\right\} = 2 \quad k = 1 \rightarrow x_4$

Pivot

$$
\begin{array}{cccccc|c}
1 & -2 & 0 & 1 & 0 & 0 & 2 \\
0 & 2 & -2 & -1/2 & 3 & 1 & 3 \\
\hline
0 & -1 & 1 & 1 & 0 & 0 & 2
\end{array}
$$

$x_B = \begin{pmatrix} x_1 \\ x_6 \end{pmatrix}$ $\quad x_N = \begin{pmatrix} x_4 \\ x_2 \\ x_3 \\ x_5 \end{pmatrix}$

Iterazione 1

$$y^1 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

ottimalità: $y^0 \geq 0$ NO

illimitatezza $\pi_2 \leq 0$ NO

v. entrante $\Rightarrow h = 2 \to x_2$

v. uscente $\Rightarrow \min \left\{ \frac{3}{2} \right\}$ $k = 2 \to x_6$

Pivot

$$\begin{array}{cccccc|c} 1 & 0 & -2 & 1/2 & 3 & 1 & 5 \\ 0 & 1 & -1 & -1/4 & 3/2 & 1/2 & 3/2 \\ \hline 0 & 0 & 0 & 3/4 & 3/2 & 1/2 & 7/2 \end{array}$$

$$x_B = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad x_N = \begin{pmatrix} x_4 \\ x_6 \\ x_3 \\ x_5 \end{pmatrix}$$

Iterazione 2

$y^2 \geq 0$ ottimo

$$\tilde{x} = \begin{pmatrix} 5 \\ 3/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\frac{5!}{3! \cdot 2!} = \frac{5 \cdot \cancel{4}^{2}}{\cancel{2} \cdot \cancel{2}} = 10$ $\frac{4!}{3! \cdot \cancel{1}!} = 4$

1-2-3
1-2-4
1-3-4
2-3-4

$y_2 = 5 - \frac{7}{3} = \frac{15-7}{3} = \frac{8}{3}$

$\frac{15}{3} = 5$ $\cancel{3} \cdot \frac{7}{\cancel{3}}$

$\cancel{y_2}$ $\frac{8}{3} - \cancel{2}$

$5 - \frac{7}{3} =$

$-y_3 = 2 - y_2$

$y_3 = y_2 - 2$

$y_3 = \frac{8}{3} - 2 = \frac{2}{3}$

$\frac{7}{3} + \frac{8}{3} = \frac{15}{3} = 5$

$3 \cdot \frac{7}{3} = 7$ $\frac{8}{3} - \frac{2}{3} = \frac{6}{3} = 2$

$2 + \frac{7}{3} = \frac{13}{3}$

$\begin{vmatrix} 0 & 0 & 1 \\ 3 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -3 \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} = 3 \neq 0$

$P_0 = \{(s,5),(5,t)\}$

$\delta^+ = \min\{c_{uv} - x_{uv}\} = \min\{(4-0),(8-5)\} = 3$

$\delta^- =$ non c'è

$\delta = \delta^+ = 3$

$\vartheta^0 = \bar{\vartheta} + \delta = 9+3 = 12$

$P_1 = \{(s,2),(2,3),(3,4),(4,t)\}$

$\cancel{P_0 = \{(s,5),(5,4),(4,t)\}}$

$\cancel{\delta^+ = \{(4-0),(2-0),(5-0)\} = 2}$

$\delta^+ = \min\{4,9,2,5\} = 2$

$\delta^- =$ non c'è

$\delta = \delta^+ = 2$

$\vartheta^1 = \vartheta^0 + \delta = 12+2 = 14$

$$\sum_{\substack{s \in W \\ u = \bar{W}}} c_{su} = \bar{\vartheta}$$

$P_2 = \{(s,1),(1,4),(4,t)\}$

$\delta^+ = \{2,3,3\} = 2$

$\delta = \delta^+ = 2$

$\vartheta^2 = \vartheta^1 + 2 = 14+2 = 16$

$P_3 = \{(s,2),(1,2),(1,t)\}$

$\delta^+ = \{2,4\} = 2$

$\delta^- = \{4\} = 4$

$\delta = \min\{\delta^+, \delta^-\} = 2$

$\vartheta^3 = \vartheta^2 + 2 = 16+2 = 18$

$P_4 = \{(s,5),(5,4),(4,t)\}$

$\delta^+ = \{1,2,1\} = 1$

$\delta = 1$

$\vartheta^4 = \vartheta^2 + \delta = 19$

$P_5 = \{$

f. convesse

$f[\alpha x+(1-\alpha)y] \leq \alpha f(x)+(1-\alpha)f(y)$

min

$f(x^*) \leq f(x) \quad \forall x \in$

$\leq f(y)$

$f(x)=$

$f(x^*)= f[\alpha x^* +(1-\alpha)y] \leq \alpha f(x^*)+(1-\alpha)f(y)$

$f(x^*)(1-\alpha) \leq (1-\alpha) f y$

$\gamma \geq 0$ per min $\qquad \gamma = C_N - (B^{-1}N)^T C_B$

$\gamma \leq 0$ per max

$-\gamma > 0$

$C_N-(B^{-1}N)^T C_B \leq 0$

max $C_B x_B + C_N x_N$

$B x_B + N x_N = b$

$x_B \geq 0 \quad x_N \geq 0$