STATISTICA ESERCIZIARIO FOGLIO 1.

1) | 2N 1R | | 1B 2R |

a) $\Omega = \{(x\,y),\ x \in \{2N, 1R\}\ y \in \{1B, 2R\}\}$

b) SONO TUTTI I POSSIBILI $E = P(X \subseteq \Omega)$ DOVE

$|\Omega| = 3 \cdot 3 = 9$

$|E| = 2^{|\Omega|} = 2^9 = 512$

c) $P(x = y) = \frac{2}{9}$ N.B SI ORDINE SI RIPETIZIONE.

d) $P(x \neq y) = 1 - \frac{2}{9} = \frac{7}{9}$

2) $|\text{MAZZO CARTE}| = 52$, $\Omega = \{x \mid x \in \text{MAZZO DI CARTE}\}$

P(QUALSIASI FIGURA) = P(ESTRARRE UNA FIGURA) + P(UNA CARTA A FIORI) − P(∩B) = 11/26
(A = ESTRARRE UNA FIGURA, B = UNA CARTA A FIORI)

$P(A) = 12/52$

$P(B) = 13/52$ N.B: SEGUENDO LOGICA

$P(B \cap A) = 3/52$

3) $|\Omega^*| = 40$ $\Omega = \{(x_1, \dots, x_{10}) \mid x_i \text{ con } i \in [1, 10]\ x_i \in \Omega^*\}$

$P(4\text{ ASSI}) = |4\text{ ASSI}| / |\Omega| = \binom{36}{6} / \binom{40}{10}$

$P(\text{ALMENO UN DANARO}) = 1 - |\text{NESSUN DANARO}| / |\Omega| = 1 - \binom{30}{10} / \binom{40}{10}$

$P(\text{AL PIU' DUE ASSI}) = 1 - [P(4\text{ ASSI}) + P(3\text{ ASSI})] = P(4\text{ ASSI}) + |3\text{ ASSI}| / |\Omega| = \binom{36}{6} / \binom{40}{10} + \binom{37}{7}\binom{4}{3} / \binom{40}{10}$

$|\Omega| = C_{40,10} = \binom{40}{10} = \frac{40!}{10!\,30!}$

$|4\text{ ASSI}| = C_{4,4} \cdot C_{36,6} = \cancel{\binom{4}{4}}\binom{36}{6} = \binom{36}{6}$ N.B. NO ORDINE NO RIPETIZIONE

$|\text{ALMENO UN DANARO}| \overset{c}{=} |\text{NESSUN DANARO}| = \binom{30}{10}$

$|\text{AL PIU' DUE ASSI}| \overset{c}{=} |\text{PIU' DI DUE ASSI}|$ = COMBINO P(A) CHE HO CALCOLATO E P(3 ASSI)

N.B EVITO DI DOVER CONTARE 3 COSI, IN SEGUITO COMPLEMENTERO IL RISULTATO CON L'EVENTO CERTO Ω TROVANDO LA SUA NEGAZIONE.

4) $\Omega = \{(x_1, x_2, \dots, x_{25}) \mid x \in [1, 365]\}$ $|\Omega| = D'_{365,25} = 365^{25}$

CALCOLO LA NEGAZIONE DI "ALMENO DUE CON DATA COINCIDENTE" (A)

$P(\bar{A}) = |\text{NESSUNA DATA COINCIDENTE}| / |\Omega| = D_{365,25} / |\Omega| = \dfrac{\frac{365!}{(365-25)!}}{365^{25}}$

N.B: SI ORDINE SI RIPETIZIONE, QUANDO VOGLIO INVECE NESSUNA DATA COINCIDENTE SI ORDINE NO RIPETIZIONE

5) $P(\text{TESTA}) = \frac{1}{2}$ $\quad \Omega = \{x \mid x \in \{\text{TESTA, CROCE}\}\}$ $\quad |\Omega| = D'_{m,k} = 2^5 = 32$

$P(\text{5 LANCI ALMENO UNO TESTA}) = 1 - P(\text{5 LANCI NESSUNO TESTA}) = 1 - |\text{NESSUNO TESTA}| / |\Omega|$
$\overset{!}{=} 1 - 1/32 = 31/32$

6) $P(K \leq m) = \underbrace{\prod_{i=1}^{K-1} \left(\frac{m-i-1}{m-i}\right)}_{\simeq 1} \cdot \frac{1}{m} = \frac{1}{m}$

7) $P(\text{UNO DIFETTOSO}) = \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \binom{3}{2} = \frac{3 \cdot 4}{27} = \frac{12}{27} = \frac{4}{9}$
$P(\underset{A}{\text{UNO ALMENO DIFETTOSO}}) = 1 - P(\bar{A}) = 1 - \frac{2}{3}\frac{2}{3}\frac{2}{3} = 1 - \frac{8}{27} = \frac{19}{27}$

8) $P(\text{ALMENO 3 LANCI}) = 1 - (P(\text{MENO DI 3 LANCI}) = 1 - \left(\frac{5}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}\right) =$
$= 0{,}579$

9) $P(\text{TUTTE ROTTE}) = \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} = \frac{24}{720} = \frac{1}{30}$

10) $P(\text{NON PARI}) = \text{CASI FAVOREVOLI} / \#\text{CASI TOTALI} =$ $\qquad$ N.B 2 DADI 6 FACCE
$\overset{!}{=} 2 \cdot 9/36 = 18/36 = 1/2$ $\qquad \#\text{CASI TOTALI} = 6^2 = 36$
N.B $(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)$ $\times 2$ IN QUANTO CONTA L'ORDINE DI USCITA DEL DADO
$P(\text{UGUALE A 5}) = \text{CASI FAVOREVOLI} / \#\text{CASI TOTALI} = 2 \cdot 2/36 = 1/9$
N.B $(1,4), (2,3)$ $\times 2$ IN QUANTO CONTA L'ORDINE DI USCITA
$P(|D_1 - D_2| = 3) = 2 \cdot 3/36 = 6/36 = 1/6$
N.B $(1,4), (2,5), (3,6)$ $\times 2$ IN QUANTO CONTA L'ORDINE

11)
1N	
2N	4R
3N	5R

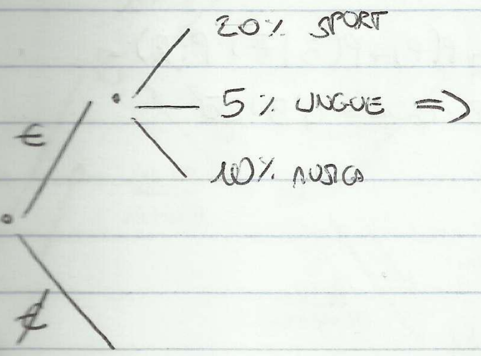
$\xrightarrow{CR}$ 2 PALLINE $\qquad \Omega = \{P \mid P \in \{1N, 2N, 3N, 4R, 5R\}\}$
$\left.\begin{array}{l}|B_1| = 3 \\ |B_2| = 3\end{array}\right\}$ VISTO IL REINSERIMENTO $\qquad |B_1 \cap B_2| = 3^2 = 9$

$\#\text{CASI TOTALI} = 5$ $\qquad \#\text{CASI TOTALI}(B_1 \cap B_2) = 5^2 = 25$
$P(B_1) = |B_1| / \#\text{CASI TOTALI} = 3/5$
$P(B_2) = |B_2| / \#\text{CASI TOTALI} = 3/5$
$P(B_1 \cap B_2) = |B_1 \cap B_2| = 9/25$

12) $\Omega = \{ (x_1, \dots, x_3) \mid x_i \text{ con } i \in [0,3] \in \{Y, N\} \}$

$|\text{ALMENO DUE DONNE USANDO IL PRODOTTO}| = \binom{3}{2} + \binom{3}{3} = 3$

13)



N.B
SPORT-MUSICA = 5%
MUSICA-LINGUA = 3%
SPORT-LINGUA = 2%
TUTTO = 1%

$P(\text{SOLO SPORT}) = P(S) - P(S \cap M) - P(S \cap L) + P(S \cap M \cap L) =$
$= 0{,}20 - 0{,}05 - 0{,}02 + 0{,}01 = 0{,}14$

$P(\text{MUSICA LINGUA NO SPORT}) = P(M \cap L) - P(M \cap L \cap S) = 0{,}03 - 0{,}01 = 0{,}02$

14)

1	4
2	5
3	6

2 PROVE C.R

$\Omega = \{(i, j) : i, j \in \{1..6\}\}^2$

N.B: $\#\text{CASI TOTALI} = 6^2 = 36$

$P(i+j=7 \,||\, i+j=8) = P(i+j=8) + P(i+j=7) = 6/36 + 5/36 = 11/36$

N.B 7: (1,6), (2,5), (3,4) ×2 CONTA ORDINE

8: (2,6), (3,5), (4,4) ×2 CONTA L'ORDINE MA LA COPPIA (4,4) NON VIENE CONSIDERATO DOPPIO

$P(7 \mid D_1 = 5, D_2 = 2) = 1/36$

$P(i+j=7 \,||\, i+j=11) = P(i+j=7) + P(i+j=11) = 6/36 + 2/36 = 8/36 = 2/9$

N.B 7: COME PRIMA

11: (5,6) ×2 CONTA L'ORDINE

$P(i+j>7) = P(8) + P(9) + P(10) + P(11) + P(12) = \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{15}{36} = \frac{5}{12}$

N.B 9: (3,6), (4,5) ×2 CONTA ORDINE

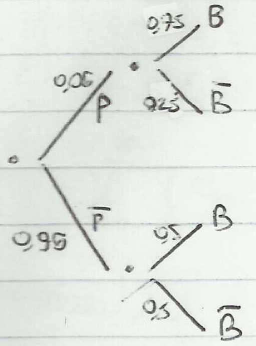
10: (4,6), (5,5) ×2 CONTA ORDINE (5,5) UNA SOLA VOLTA

11: (5,6) ×2 CONTA ORDINE

12: (6,6).

FOGLIO 2

1)



$$P(P|B) = \frac{P(B|P)\cdot P(P)}{P(B)} =$$

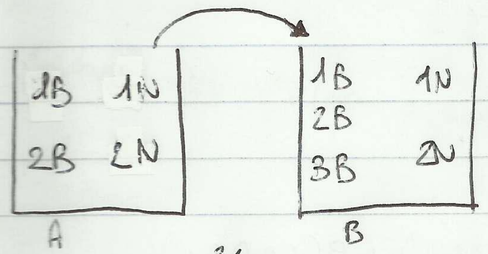
$$= \frac{0{,}75\cdot 0{,}05}{0{,}5152} = 0{,}0732$$

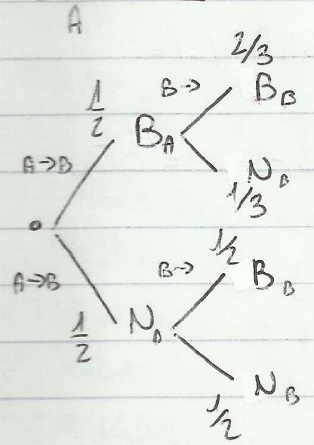
CALCOLO:

$$P(B) = P(B|P)\cdot P(P) + P(B|\bar{P})\cdot P(\bar{B}) =$$

$$= 0{,}75\cdot 0{,}05 + 0{,}5\cdot 0{,}95 = 0{,}5125$$

2)





$$P(B_A|B_B) = \frac{P(B_B|B_A)\cdot P(B_A)}{P(B_B)} =$$

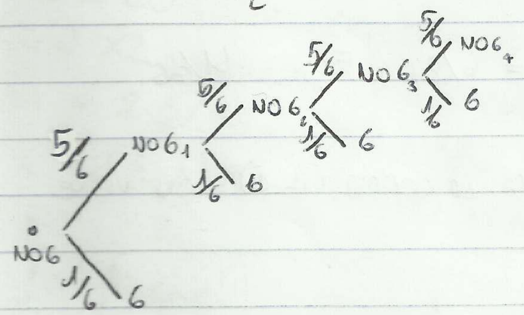
$$= \frac{\frac{1}{2}\cdot\frac{2}{3}}{7/12} = \frac{1}{\cancel{2}}\cdot\frac{\cancel{12}^{4}}{7} = \frac{4}{7}$$

CALCOLO:

$$P(B_B) = P(B_B|B_A)\, P(B_A) + P(B_B|B_B)\cdot P(B_B) =$$

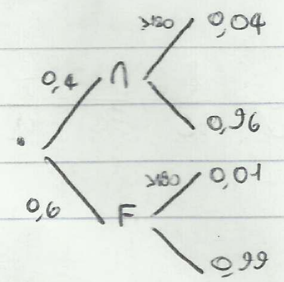
$$= \frac{2}{3}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

3)



$$P(6\ 4°\text{LANCIO}) = \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} = 0{,}579$$

4)



$$P(F|>180) = \frac{P(>180|F)\cdot P(F)}{P(>180)} =$$

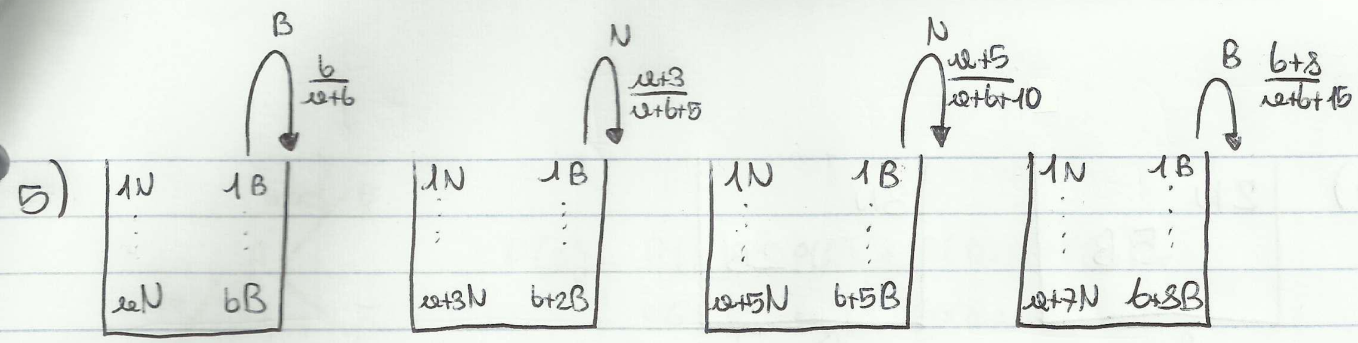
$$= \frac{0{,}01\cdot 0{,}6}{0{,}022} = 0{,}273$$

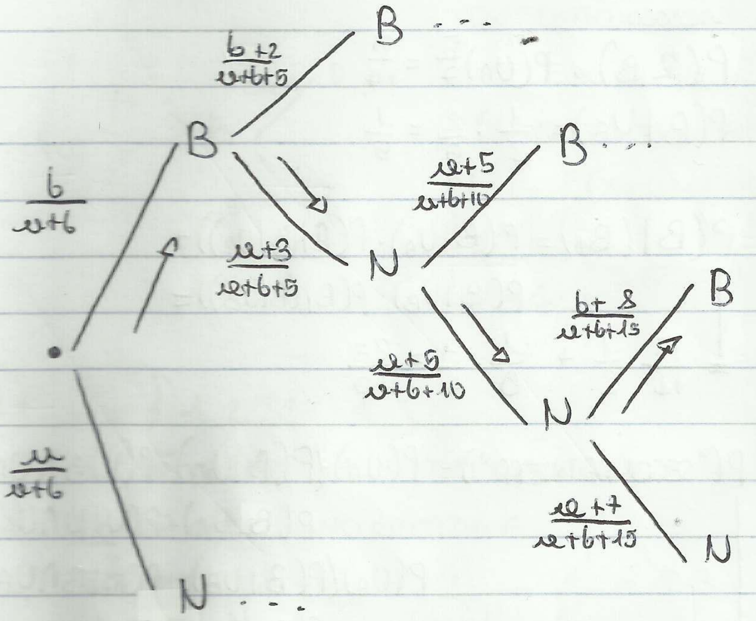
CALCOLO:

$$P(>180) = P(M)\cdot P(>180|M) + P(F)\cdot P(>180|F) =$$

$$= 0{,}4\cdot 0{,}04 + 0{,}6\cdot 0{,}01 = 0{,}016 + 0{,}006 = 0{,}022$$

5)

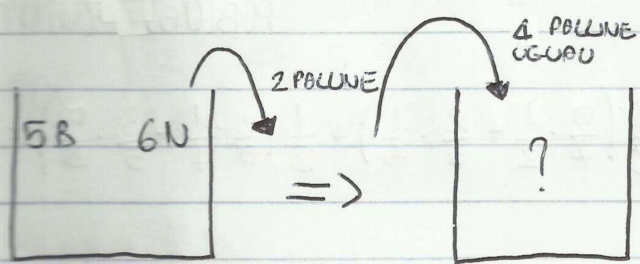




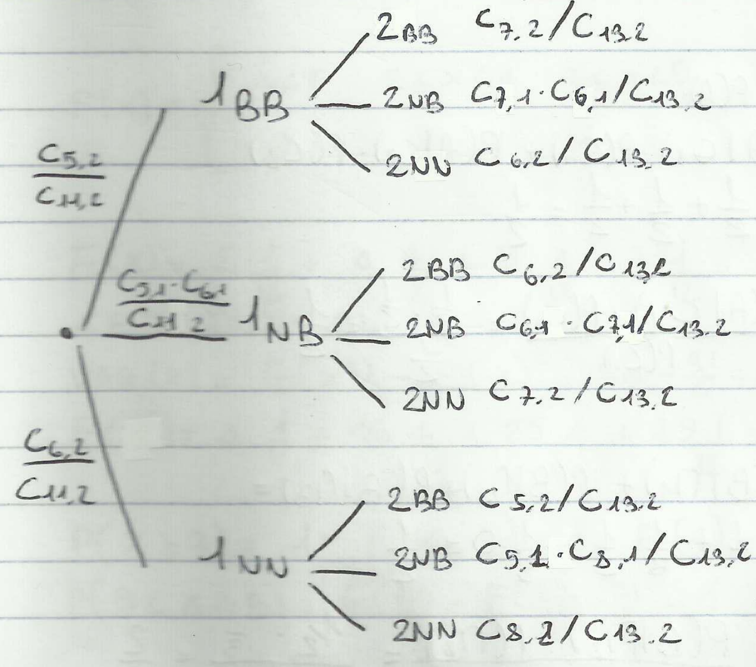
P("SEQ BNNB") =

$$= \frac{b}{n+b} \cdot \frac{n+3}{n+b+5} \cdot \frac{n+5}{n+b+10} \cdot \frac{b+8}{n+b+15}$$

6)



N.B $1_{BB}, 1_{BN}, 1_{NN}$ SONO LE POSSIBILI PRIME ESTRAZIONI. $2_{BB}, 2_{BN}, 2_{NN}$ SONO LE POSSIBILI SECONDE ESTRAZIONI



$$P(2BB) = P(2BB|1BB) \cdot P(1BB) + P(2BB|1NB) \cdot P(1BN) + P(2BB|1NN) \cdot P(1NN) =$$

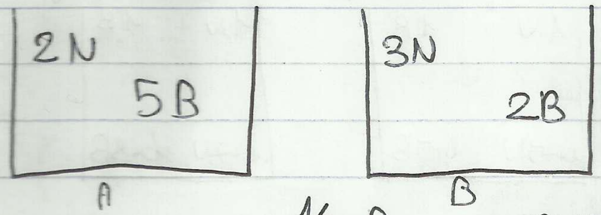
$$= \frac{C_{7,2}}{C_{13,2}} \cdot \frac{C_{5,2}}{C_{11,2}} + \frac{C_{6,2}}{C_{13,2}} \cdot \frac{C_{5,1} C_{6,1}}{C_{13,2}} + \frac{C_{5,2}}{C_{13,2}} \frac{C_{6,2}}{C_{11,2}}$$

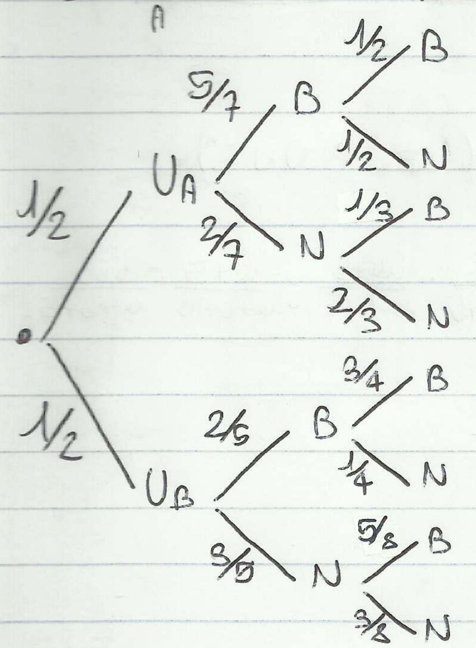
$$= \frac{27}{143}$$

$$P(1NN|2BB) = \frac{P(2BB|1NN) \cdot P(1NN)}{P(2BB)} =$$

$$= \frac{\left(\frac{C_{5,2}}{C_{13,2}} \cdot \frac{C_{6,2}}{C_{11,2}}\right) \cdot 143}{27} = \frac{5}{27}$$

7)





CALCOLO:

$P(B|U_A) = \frac{1}{2} \cdot \frac{5}{7} = \frac{5}{14}$

$P(B|U_B) = \frac{1}{2} \cdot \frac{2}{5} = \frac{1}{5}$

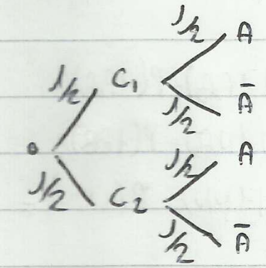
$$P(B_2, B_1) = P(B|U_A) \cdot P(B|(B|U_A)) + P(B|U_B) \cdot P(B|(B|U_B)) = \frac{5}{14} \cdot \frac{1}{2} + \frac{1}{5} \cdot \frac{3}{4} = \frac{23}{70}$$

$$P(\text{"SECONDO NERO"}) = P(U_A)(P(B|U_A) \cdot P(N|B \cap U_A) + P(N|U_A) \cdot P(N|N \cap U_A)) + P(U_B)(P(B|U_B) \cdot P(N|B \cap U_B) + P(N|U_B) \cdot P(N|N \cap U_B)) =$$

$$= \frac{1}{2}\left(\frac{5}{7} \cdot \frac{1}{2} + \frac{2}{7} \cdot \frac{1}{3}\right) + \frac{1}{2}\left(\frac{2}{5} \cdot \frac{1}{4} + \frac{3}{5} \cdot \frac{5}{8}\right) =$$

$= 0,436$

8) CREO L'ALBERO DI PROBABILITÀ

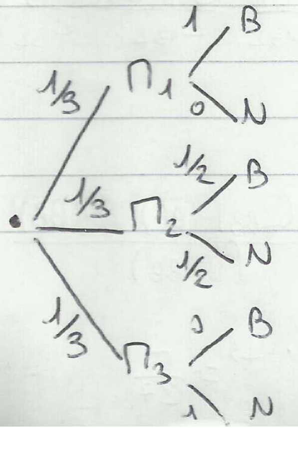


CALCOLO P(A):

$$P(A) = P(A|C_1) \cdot P(C_1) + P(A|C_2) \cdot P(C_2) = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

$$P(C_1|A) = \frac{P(A|C_1) \cdot P(C_1)}{P(C_1)} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{1}{2}$$

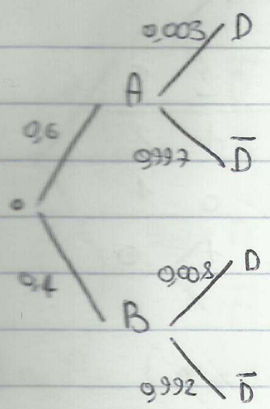
9)



$$P(B) = P(B|\Pi_1) + P(B|\Pi_2) + P(B|\Pi_3) = \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 0 = \frac{1}{2}$$

$$P(\Pi_2|B) = \frac{P(B|\Pi_1) \cdot P(\Pi_1)}{P(B)} = \frac{1 \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

10)



$P(D) = P(D|A)\cdot P(A) + P(D|B)\cdot P(B) =$

$\overset{!}{=} 0{,}6 \cdot 0{,}003 + 0{,}4 \cdot 0{,}008 = 0{,}005$

CONSIDERO INFINITI O QUASI I PEZZI DEL MAGAZZINO: E SI DISTRIBUISCE COME UNA BINOMIALE: $\sim Bi(20, 0{,}005)$

P(ALMENO 3 SU 20 DIFETTOSI) $= 1 - P(\bar{A}) = 1 - 1{,}34 \cdot 10^{-6}$

$P(\bar{A}) = P(X=0) + P(X=1) + P(X=2) =$

$\overset{!}{=} \binom{20}{0}(0{,}005)^0(0{,}995)^{20} + \binom{20}{1}(0{,}005)^1(0{,}995)^{19} + \binom{20}{2}(0{,}005)^2(0{,}995)^{18}$

$\overset{!}{=} 1{,}34 \cdot 10^{-6}$

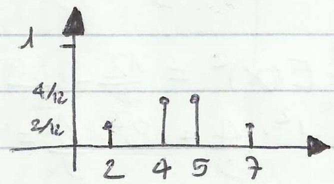
1) $U = \{1\ 2, 3, 4\}$ FOGLIO 3

X = "SOMMA DEI DUE NUMERI ESTRATTI"

NB SI ORDINE, NO RIPETIZIONE

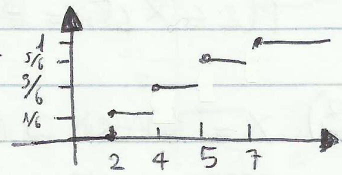
CASI TOTALI $= D_{4,2} = \frac{4!}{(4-2)!} = \frac{4!}{2!} = 4 \cdot 3 = 12$

$P(x) = \begin{cases} 2 & 4 & 5 & 7 \\ 2/12 & 4/12 & 4/12 & 2/12 \end{cases}$



CALCOLO F(x):

$F(x) = \begin{cases} x<2 & 2 \le x < 4 & 4 \le x < 5 & 5 \le x < 7 & x \ge 7 \\ 0 & 1/6 & 1/2 & 5/6 & 1 \end{cases}$



$E(x) = 2 \cdot \frac{1}{12} + 4 \cdot \frac{4}{12} + 5 \cdot \frac{4}{12} + 7 \cdot \frac{1}{12} = 4.5$

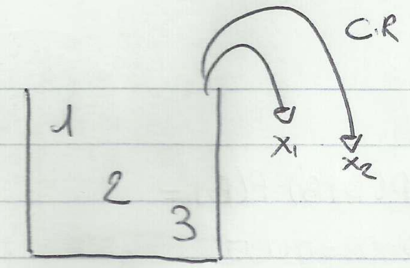
4 E 5 SONO MODA.

$VAR(x) = E(x^2) - E(x)^2 = \frac{45}{2} - \frac{81}{4} = 2{,}25$

$E(x^2) = 4 \cdot \frac{1}{12} + 16\,\frac{4}{12} + 25\,\frac{4}{12} + 49\,\frac{1}{12} = \frac{45}{2}$

$P(X \ge 7) = 1 - P(X \le 7) = 1 - F(7) = 1 - \frac{5}{6}$

$P(3 < X \le 5) = F(5) - F(3) = \frac{5}{6} - \frac{1}{6} = \frac{2}{3}$

2)  $\Rightarrow X = X_1 - X_2$

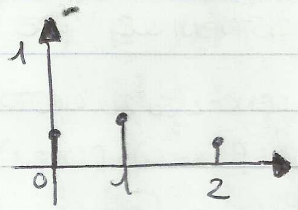
N.B

$\lvert X_1 - X_2 \rvert$	1	2	3
1	0	1	2
2	1	0	1
3	2	1	0

(colonne: X_1; righe: X_2)

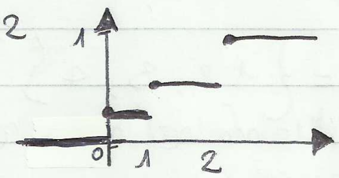
X = "DIFFERENZA NUMERI IN MODULO"

$$P(X) = \begin{cases} 0 & 1 & 2 \\ 3/9 & 4/9 & 2/9 \end{cases}$$



CALCOLO $F(X)$:

$$F(x) = \begin{cases} x<0 & 0 \le x < 1 & 1 \le x < 2 & x \ge 2 \\ 0 & 3/9 & 7/9 & 1 \end{cases}$$



$$E(X) = \frac{3}{9}\,0 + \frac{4}{9} \cdot 1 + 2 \cdot \frac{2}{9} = \frac{4}{9} + \frac{4}{9} = \frac{8}{9}$$

LA MODA È 1

$$VAR(X) = E(X^2) - E(X)^2 = \frac{12}{9} - \frac{64}{81} = \frac{108-64}{81} = \frac{44}{81} = 0{,}54$$

$$E(X^2) = 0^2 \cdot \frac{3}{9} + 1^2 \cdot \frac{4}{9} + 2^2 \cdot \frac{2}{9} = \frac{4}{9} + \frac{8}{9} = \frac{12}{9}$$

$$P(X \le 2) = F(2) = 1$$

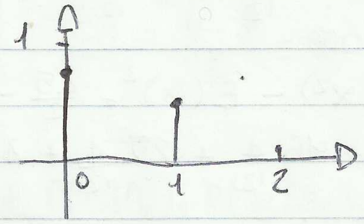
$$P(2 \le X < 5) = P(1 < X \le 4) = F(4) - F(1) = 1 - \frac{7}{9} = \frac{2}{9}$$

3) X = "NUMERO DI PALLINE NEL PRIMO CONTENITORE"

OSSERVO CHE:

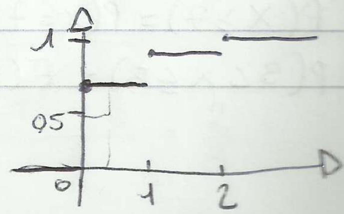
$X \sim Bi(2, 1/4) \Rightarrow$

X	P(X)
0	$P(X=0) = \binom{2}{0} \left(\frac{3}{4}\right)^2 = 9/16$
1	$P(X=1) = \binom{2}{1} \frac{1}{4} \cdot \frac{3}{4} = 6/16$
2	$P(X=2) = \binom{2}{2} \left(\frac{1}{4}\right)^2 = 1/16$



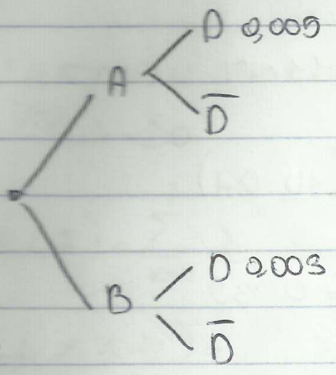
CALCOLO $F(X) =$

$$F(x) = \begin{cases} x<0 & 0 \le x < 1 & 1 \le x < 2 & x \ge 2 \\ 0 & 9/16 & 15/16 & 1 \end{cases}$$



MODA 0, MEDIANA 0

4)



$P(D) = P(D|A)\cdot P(A) + P(D|B)\cdot P(B)$

$= \frac{5}{12}\cdot 0{,}005 + \frac{7}{12}\cdot 0{,}003$

X = "SU 10 PEZZI CONTA PEZZI DIFETTOSI"

$X \sim Bi(n=10,\ p=0{,}00384)$

$P(X>1) = 1 - P(X\leq 1) = 1 - (P(X=0) + P(X=1)) =$

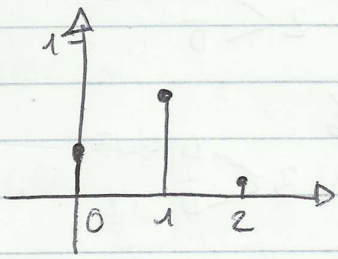
$= 1 - \left(\binom{10}{0} 0{,}99616^{10} + \binom{10}{1} 0{,}00384^{1}\cdot 0{,}99616^{9}\right) = 0{,}00056$

5) X = "NUMERO DI TESTE SEGUITE DA CROCE"   4 LANCI

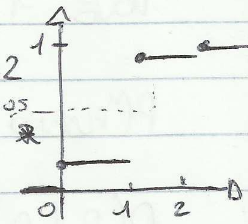
L_3	L_2	L_1	L_0	
C	C	C	C	1
C	C	C	T	1
C	C	T	C	-
C	C	T	T	1
C	T	C	C	-
C	T	C	T	-
C	T	T	C	-
C	T	T	T	1
T	C	C	C	-
T	C	C	T	-
T	C	T	C	X
T	C	T	T	-
T	T	C	C	-
T	T	C	T	-
T	T	T	C	-
T	T	T	T	1

$\Rightarrow$

	X	f(x)
1	0	5/16
-	1	10/16
X	2	1/16



$$\Rightarrow F(x) = \begin{cases} x<0 & 0\leq x<1 & 1\leq x<2 & x\geq 2 \\ 0 & 5/16 & 15/16 & 1 \end{cases}$$



$E(x) = 0\cdot\frac{5}{16} + 1\cdot\frac{10}{16} + 2\cdot\frac{1}{16} = \frac{12}{16} = \frac{3}{4}$

$E(x^2) = \frac{10}{16} + \frac{4}{16} = \frac{14}{16}$

$VAR(x) = E(x^2) - E(x)^2 = \frac{14}{16} - \frac{9}{16} = \frac{5}{16}$

MODA = $10/16 \Rightarrow 1$ MEDIANA 1 *

6) $A_1 = 20 \quad A_2 = 10$ SI RISERVA: 22 POSTI 11 POSTI

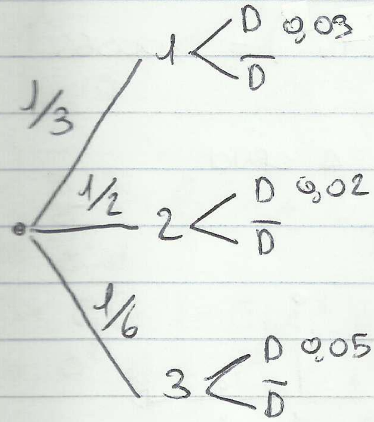
$X_{A_1} \sim Bi(22, 0{,}1) \qquad X_{A_2} \sim Bi(11, 0{,}1)$

$P(X_A \geq 21) = \binom{22}{21} \cdot 0{,}1^1 \cdot 0{,}9^{21} + \binom{22}{22} 0{,}9^{22} = 0{,}339$

$P(X_B = 11) = \binom{11}{11} 0{,}1^0 \, 0{,}9^{11} = 0{,}314$

NEL VOLO DA 20 POSTI.

7)



$P(D) = P(D|1) \cdot P(1) + P(D|2) \cdot P(2) + P(D|3) \cdot P(3) =$
$= \frac{1}{3} \cdot 0{,}03 + \frac{1}{2} \cdot 0{,}02 + \frac{1}{6} \cdot 0{,}05 = 0{,}0284$

$P(\bar{D}) = \frac{1}{3}\left(P_1(D)\right) + \frac{1}{2}\left(P_2(D)\right) + \frac{1}{6}\left(P_3(D)\right) =$
$= \frac{1}{3}\left(\binom{20}{0} 0{,}97^{20}\right) + \frac{1}{2}\left(\binom{20}{0} 0{,}98^{20}\right) + \frac{1}{6}\left(\binom{20}{0} 0{,}95^{20}\right) = 0{,}575$

DOVE 1 2 3 $\sim Bi\left(n = 20,\ p = \{0{,}03,\ 0{,}02,\ 0{,}05\}\right)$

P("ALMENO UN AUTORADIO ROTTO") $= 1 - P(\bar{D}) = 0{,}425$

$P(3|D) = \dfrac{P(D|3) \cdot P(3)}{P(D)} =$

Y = "AUTORADIO DIFETTOSE"
$Y \sim Bi(n = 15,\ p = 0{,}0284)$

$P(Y=0) = \binom{15}{0} \cdot 0{,}9716^{10} = 0{,}6496$
$P(Y=1 \mid Y \geq 1) =$

8) X = "AUTOMOBILI PER ORA"

$X \sim \mathcal{P}(\lambda = 20)$

$$P(X \leq 7) = \sum_{k=0}^{7} \frac{\lambda^k e^{-\lambda}}{k!} = e^{-20} \sum_{k=0}^{7} \frac{20^k}{k!} = 7,78 \cdot 10^{-4}$$

$$P(6 \leq X \leq 12) = \sum_{k=6}^{12} \frac{\lambda^k e^{-\lambda}}{k!} = e^{-20} \sum_{k=6}^{12} \frac{20^k}{k!} = 3,894 \cdot 10^{-2}$$

9) X = "PIOGGIA IN I-ESIMO GIORNO DI AGOSTO"

$X \sim Ge(0{,}05)$

$$P(X = 15) = (1 - 0{,}05)^{14} \cdot (0{,}05)^1 = 0{,}0244$$

$$P(X < 15) = 1 - P(X \geq 15) = 1 - (1 - 0{,}05)^{14} = 1 - 0{,}95^{14} = 0{,}5123$$

$$P(X < 25 \mid X \leq 10) = P(X \leq 15) = 1 - 0{,}95^{15} = ?$$

10) P(TESTA) = 0,6   P(CROCE) = 0,4

$\boxed{6B \quad 4N}$

SE:

- T: ESTRAGGO 4 PALLINE C.R
- C: ESTRAGGO 3 PALLINE S.R

$\mu = 12/5$ $\qquad\qquad$ $\mu = 9/5$

$T \sim Bi(4, 3/5)$ $\qquad$ $C \sim IG(10, 6, 3)$

$$P(T = k \mid T) = \binom{4}{k} \left(\frac{3}{5}\right)^k \cdot \left(\frac{2}{5}\right)^{4-k}$$

$$P(C = k \mid C) = \left(\frac{\binom{6}{k}\binom{10-6}{3-k}}{\binom{10}{3}} \right)$$

$$P(X = k) = P(T = k \mid T)\, P(T) + P(C = k \mid C) \cdot P(C).$$

$$E(X) = E(T \mid T)\, P(T) + E(C \mid C) \cdot P(C) =$$

14) $X =$ "# FIORITURE FRUTTETI"

$X \sim Bi(n = 200,\ p = 0{,}04)$

$P(X=7) = \binom{200}{7} 0{,}04^{7} \cdot 0{,}96^{200-7} = 0{,}14$

$P(\text{"MENO DI DUE"}) = P(X=0) + P(X=1) =$

$= \binom{200}{0} 0{,}04^{0} \cdot 0{,}96^{200} + \binom{200}{1} \cdot 0{,}04^{1} \cdot 0{,}96^{200-1} = 0{,}005 = 5\ 10^{-3}$

$X \sim P(\lambda = n \cdot p)$.

$\lambda = n \cdot p = 200 \cdot 0{,}04 = 8$

$P(\text{"MENO DI DUE"}) = P(X=0) + P(X=1) =$

$= e^{-\lambda} + \lambda e^{-\lambda} = e^{-8} + 8 e^{-8} = \frac{1}{e^{8}} + \frac{8}{e^{8}} = \frac{1}{e^{8}}(1+8) = 9 e^{-8} = 0{,}005$

1) $X \sim f(x) = K e^{-|x|} \quad \forall x \in \mathbb{R}$ FOGLIO 4.

OSSERVO CHE $|x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$

$\int_{\Omega} f(x)\, dx = 1 \Rightarrow$

$\int_{-\infty}^{+\infty} K e^{-|x|} dx = \left[\int_{-\infty}^{0} K e^{x} dx + \int_{0}^{+\infty} K e^{-x} dx\right] =$

$= K\left[\int_{-\infty}^{0} e^{x} dx + \int_{0}^{+\infty} e^{-x} dx\right] = K\left[e^{x}\Big|_{-\infty}^{0} + e^{-x}\Big|_{0}^{+\infty}\right] =$

$= 2K \Rightarrow 2K = 1 \Rightarrow K = \frac{1}{2}$

$f(x) = \frac{1}{2} e^{-|x|}$

$E(X) = \int_{-\infty}^{+\infty} f(x) \cdot x\, dx = \frac{1}{2} \int_{-\infty}^{+\infty} e^{-|x|} \cdot x\, dx =$

$= \frac{1}{2}\left[\int_{-\infty}^{0} e^{x} x\, dx + \int_{0}^{+\infty} e^{-x} x\, dx\right] = \frac{1}{2}\left[\underset{=0}{e^{x} x\Big|_{-\infty}^{0}} - \cancel{\int_{-\infty}^{0} e^{x} dx} + \underset{=0}{x e^{-x}\Big|_{0}^{+\infty}} + \cancel{\int_{0}^{+\infty} e^{-x}}\right]$

$= 0$

$$F(k) = \int_{-\infty}^{k} f(x)\,dx = \int_{-\infty}^{k} \frac{1}{2} e^{-|x|}\,dx =$$

$$= \int_{-\infty}^{0} \frac{1}{2} e^{x}\,dx + \int_{0}^{k} \frac{1}{2} e^{-x}\,dx =$$

$$= \frac{1}{2}\left[e^{x}\Big|_{-\infty}^{0} + \left(-e^{-x}\right)\Big|_{0}^{k} \right] = \frac{1}{2}\left[1 - \left(e^{-k} - 1\right)\right] =$$

$$= \frac{1}{2}\left(2 - e^{-k}\right) = 1 - \frac{e^{-k}}{2}$$

MEDIANO: $F(x) = \frac{1}{2} \Rightarrow 1 - \frac{e^{-x}}{2} = \frac{1}{2} \Rightarrow e^{-x} = 1 \Rightarrow x = 0$

$$E(X^2) = \int_{\mathbb{R}} x^2 f(x)\,dx = \int_{-\infty}^{+\infty} x^2 \frac{1}{2} e^{-|x|}\,dx =$$

$$= \frac{1}{2}\left[\int_{-\infty}^{0} x^2 e^{x}\,dx + \int_{0}^{+\infty} x^2 e^{-x}\,dx \right] =$$

$$= \frac{1}{2}(2+2) = 2$$

$VAR(X) = E(X^2) - E(X)^2 = 2 - 0 = 2$

3) $X \sim f(x) = \frac{c}{x^2}\,\mathbb{1}_{(100,+\infty)}(x)$

$$\int_{100}^{+\infty} \frac{c}{x^2}\,dx = 1 \Rightarrow c\int_{100}^{+\infty} \frac{1}{x^2}\,dx = c\int_{100}^{+\infty} x^{-2}\,dx = c\,\frac{x^{-1}}{-1}\Big|_{100} =$$

$$= -\frac{1}{x}\Big|_{100}^{+\infty} = c\frac{1}{100} \Rightarrow \frac{1}{100}c = 1 \Rightarrow c = 100$$

$$F(x) = \int_{100}^{k} \frac{100}{x^2}\,dx = -\frac{100}{x}\Big|_{100}^{k} = -\frac{100}{k} + 1$$

$E(X) = \int_{100}^{+\infty} \frac{100}{x^2}\cdot x\,dx = 100\log x\Big|_{100}^{+\infty} = +\infty$ NON ESISTE MEDIA NE VARIANZA

$F(x) = \frac{1}{4} \Rightarrow$ $400/3$

$F(x) = \frac{1}{2} \Rightarrow$ 200

$F(x) = \frac{3}{4} \Rightarrow$ 400

$P(x>500) = 1 - P(x \le 500) = 1 - \left(-\frac{100}{500} + 1\right) = \cancel{1} - \cancel{1} + \frac{1}{5} = \frac{1}{5}$

4) y = "SOGLIA FIUME"

$F(y) = 1 - \frac{1}{y^2} \mathbb{1}_{(1,+\infty)}(y)$

$F(+\infty) = 1$ ✓

$F'(y) = f(y) = \left(1 - \frac{1}{y^2}\right)' = \frac{2}{y^3} = 2y^{-3}$

$E(y) = \int_1^{+\infty} \frac{2}{y^{\cancel{3}2}} \cdot y \, dy = \int_1^{+\infty} 2y^{-2} dy = -2\frac{1}{y}\Big|_1^{+\infty} = 2$

$E(y^2) = \int \frac{2}{y^3} \cdot y^2 dy = \int_1^{+\infty} \frac{2}{y} dy = \log y\Big|_1^{+\infty}$ = NON ESISTE

VAR(y) = NON ESISTE

$f(y)' = -6y^{-2} \Rightarrow$ 1 MAX = MODA IN QUANTO $f(y) \searrow$

$F(y) = \frac{1}{2} \Rightarrow 1 - \frac{1}{y^2} = \frac{1}{2} \Rightarrow y = \sqrt{2}$

5) $Z \sim N(0,1)$ TROVARE:

- $P(Z \le a) = 0{,}9686 \overset{\text{SULLE TAVOLE}}{\Rightarrow} a = 1{,}86$

- $P(Z \ge b) = 0{,}1788 \Rightarrow 1 - P(Z < b) = 1 - 0{,}1788 = 0{,}8212 \Rightarrow$
$\Rightarrow b = 0{,}92$

6) $x \sim N(3, 5^2)$

- $P(4 \le x \le 6) = P(x \le 6) - P(x \le 4) = \phi\left(\frac{6-3}{5}\right) - \phi\left(\frac{4-3}{5}\right) = 0{,}146$
- $P(1 \le x \le 5) = P(X \le 5) - P(x \le 1) = \phi\left(\frac{5-3}{5}\right) - \phi\left(\frac{1-3}{5}\right) = \phi\left(\frac{2}{5}\right) - \left(1 - \phi\left(\frac{2}{5}\right)\right) =$
$= 2\phi\left(\frac{2}{5}\right) - 1 = 0{,}3108$

$P(-1 \leq x \leq 2) = P(x \leq 2) - P(x \leq -1) = \phi\left(\frac{2-3}{5}\right) - \phi\left(\frac{-1-3}{5}\right)$

$= \phi\left(-\frac{1}{5}\right) - \phi\left(-\frac{4}{5}\right) = 1 - \phi\left(\frac{1}{5}\right) - \left(1 - \phi\left(\frac{4}{5}\right)\right) = -\phi\left(\frac{1}{5}\right) + \phi\left(\frac{4}{5}\right) =$

$= \phi\left(\frac{4}{5}\right) - \phi\left(\frac{1}{5}\right) = 0,2088.$

7) $x \sim N(4, 4^2)$

$P(|x-4| \leq c) = 0,9505$

$P(-c \leq x-4 \leq c) = 0,9505$

$P\left(-\frac{c}{4} \leq \frac{x-4}{4} \leq \frac{c}{4}\right) = 0,9505$

$\phi\left(\frac{c}{4}\right) - \phi\left(-\frac{c}{4}\right) = \phi\left(\frac{c}{4}\right) - 1 + \phi\left(\frac{c}{4}\right) = 2\phi\left(\frac{c}{4}\right) - 1 = 0,9505$

$\Rightarrow 2\phi\left(\frac{c}{4}\right) = 1,9505 \Rightarrow \phi\left(\frac{c}{4}\right) = 1,9505/2 = 0,97525$

$\Rightarrow \frac{c}{4} = 1,96 = c = 1,96 \cdot 4 = 7,84$

8) $x \sim N(\mu, \sigma^2)$

- $P(x \leq 2,45) = 0,15$ $\quad 1 - P(x > 2,45) = 0,85 \quad 1 - \phi\left(\frac{2,45-\mu}{\sigma}\right) = 0,85$ $\quad 1,04$
- $P(x \geq 2,6) = 0,06$ $\quad 1 - P(x < 2,6) = 0,94 \quad 1 - \phi\left(\frac{2,6-\mu}{\sigma}\right) = 0,94$ $\quad 1,56$

$$\begin{cases} \frac{2,45-\mu}{\sigma} = 1,04 \Rightarrow \mu = 2,51 \\ \frac{2,6-\mu}{\sigma} = 1,56 \Rightarrow \sigma = 0,00336 \end{cases}$$

9) $x \sim U(a, b)$ $\quad E(x) = 6 \quad VAR(x) = 2$

$$\begin{cases} E(x) = \frac{a+b}{2} \\ VAR(x) = \frac{(b-a)^2}{12} \end{cases} \Rightarrow \begin{cases} 6 = \frac{a+b}{2} \\ 2 = \frac{(b-a)^2}{12} \end{cases} \Rightarrow \begin{cases} \end{cases}$$

10) X = "VOTO ESAME STATISTICA"
$X \sim N(\mu = 20, \sigma^2 = ?)$

• $P(X \geq 18) = 0,7 \Rightarrow 1 - P(X \leq 18) = 0,7 \Rightarrow 1 - \phi\left(\frac{18-20}{\sigma}\right) = \cancel{1} - \cancel{1} + \phi\left(\frac{2}{\sigma}\right)$

$\Rightarrow \phi\left(\frac{2}{\sigma}\right) = 0,7 \Rightarrow \frac{2}{\sigma} = 0,7 \Rightarrow \sigma = 3,846$

$X \sim N(\mu = 20, \sigma^2 = 3,846^2)$

• $P(X > x) = 0,1 \Rightarrow P(X < x) = 0,9 \Rightarrow \phi\left(\frac{x-20}{3,846}\right) = 0,9$

$\frac{x-20}{3,846} = 1,28 \Rightarrow x = 1,28 \cdot 3,846 + 20 = 24,923$

11) $X \sim U[2,6]$ $Y \sim \mathcal{P}(\lambda)$

$P(X \leq 4) = P(Y \leq 4) \Leftrightarrow$

$\Leftrightarrow \frac{4-2}{6-2} = \frac{1}{2} = \sum_{i=0}^{4} e^{-\lambda} \frac{\lambda^i}{i!} = e^{-\lambda} + e^{-\lambda}\lambda + \frac{e^{-\lambda}\lambda^2}{2} + \frac{e^{-\lambda}\lambda^3}{6} + \frac{e^{-\lambda}\lambda^4}{24} = e^{\lambda}$

$\Leftrightarrow \frac{1}{2} = e^{\lambda} \Rightarrow \lambda = \log \frac{1}{2}$

12) X = "DURATA IN MESI DI UNA BATTERIA"
$X \sim P(\lambda)$
$F(x) = 1 - e^{-7x}$ $x \geq 0$

$P(X > 3 | X > 1) = P(X > 2) = 1 - P(X \leq 2) = 1 - (1 - e^{-7 \cdot 2}) = e^{14}$

CALCOLO
$P(X > \frac{1}{3}) = P(X \leq \frac{1}{3}) = 1 - (1 - e^{-7 \cdot \frac{1}{3}}) = e^{-\frac{7}{3}}$

$Y \sim Bi(n = 30, p = e^{-7/3})$ Y = "BATTERIE CHE DURANO PIÙ DI 1/3 DEL MESE"

$P(Y < 5) = \sum_{i=0}^{4} \binom{30}{i} (e^{-7/3})^i (1 - e^{-7/3})^{30-i} = 0,8397$

13) X = "QUANTITÀ DI POMODORO NEI BARATTOLI"
$X \sim Bi(\mu = 300, \sigma^2 = ?)$

$P(X > 304) = 0{,}1 \Rightarrow P(X \leq 304) = 0{,}9$

$\frac{304-300}{\sigma} = 1{,}28 \Rightarrow \frac{4}{\sigma} = 1{,}28 \Rightarrow \sigma = \frac{4}{1{,}28} = 3{,}125$

$\sigma^2 = 9{,}76$

$P(X \geq 297) = \phi\left(\frac{297-300}{3{,}125}\right) = 1 - \phi(0{,}95) = 1 - 0{,}82894 = 0{,}17106$

y = "SU 25 BARATTOLI CONTO I NON CONFORMI"
$y \sim Bi(m = 25, p = 0{,}82894)$

$P(y > 2) = 1 - P(y \leq 2) = 1 - \left|\sum_{i=0}^{2} \binom{25}{i} 0{,}8294^{i} \cdot 0{,}17106^{25-i}\right| = 0{,}8165$

15) X = "TEMPO VITA LAMPADINE"
$X \sim Exp(\lambda = 0{,}05)$

$P(X \geq 300) = e^{-0{,}05 \cdot 300} = 0{,}2231$
$P(X > 270 \mid X > 150) = P(X > 120) = e^{-0{,}05 \cdot 120} = 0{,}5488$

y = "CONTO # LAMPADINE DURATE > 300"
$y \sim Bi(p = 0{,}223 \quad m = 50)$

$P(y = 3) = \binom{50}{3} 0{,}223^{3} \cdot 0{,}777^{47} = 0{,}00153$

16) X = "DIAMETRI RUOTE BICICLETTE"
$X \sim Bi(58{,}5, 0{,}9)$

• $P(58{,}1 < X < 58{,}8) = \phi\left(\frac{58{,}8-58{,}5}{\sqrt{0{,}9}}\right) - \phi\left(\frac{58{,}1-58{,}5}{\sqrt{0{,}9}}\right) = \phi\left(\frac{0{,}3}{\sqrt{0{,}9}}\right) - \phi\left(\frac{-0{,}4}{\sqrt{0{,}9}}\right)$
$= \phi(0{,}32) - \phi(-0{,}42) = \phi(0{,}32) + \phi(0{,}42) - 1 = 0{,}2874$

• $P(x > 58{,}7 \mid x > 58{,}5) = \frac{P(x \geq 58{,}7 \mid x > 58{,}5)\, P(x > 58{,}7)}{P(x \geq 58{,}7)} =$

$= \frac{P(x > 58{,}7)}{P(x > 58{,}5)} = \frac{1 - P(x < 58{,}7)}{1 - P(x < 58{,}5)} = \frac{1 - \Phi\left(\frac{0{,}3}{\sqrt{0{,}9}}\right)}{1 - \Phi(0)} =$

$= 0{,}8330$

• $P(x < 58{,}3) = \Phi\left(\frac{58{,}3 - 58{,}5}{\sqrt{0{,}9}}\right) = \Phi(-0{,}21082) = 1 - \Phi(0{,}21082) = 0{,}41651$

y = "CONTA # RUOTE DI DIAMETRO INFERIORE A 58,3"

$y \sim Bi(m = 20, p = 0{,}41651)$

$P(y = 1) = \binom{10}{1} 0{,}41651^1 \cdot (1 - 0{,}41651)^9 = 0{,}03265$

• $P(\text{2 DIFETTOSE PESCATE SU 20}) = \frac{1}{C_{20,2}} = \frac{1}{\binom{20}{2}} = 0{,}00526$

1) X = "APPOGGIO ALLA MOZIONE AUMENTO DELLE TASSE" FOGLIO 5

$X \sim Bi(p = 0{,}65,\ m = 100) \sim N(65,\ 22{,}75)$

$\mu = mp = 65 \quad \sigma^2 = mp \cdot (1 - p) = 65 \cdot 0{,}35 = 22{,}75$

• $P(x \geq 50) = \Phi\left(\frac{49{,}5 - 65}{\sqrt{22{,}75}}\right) = \Phi(-0{,}69) = 1 - \Phi(0{,}69) = 1 - 0{,}549$

$= 0{,}2551$

• $P(60 < x < 70) = P(x < 70) - P(x < 60) = \Phi\left(\frac{70 - 65}{\sqrt{22{,}75}}\right) - \Phi\left(\frac{60 - 65}{\sqrt{22{,}75}}\right) =$

$= \Phi(0{,}22) - \Phi(-0{,}22) = 2\Phi(0{,}22) - 1 = 2 \cdot 0{,}58706 - 1 = 0{,}17452$

• $P(X < 75) = \Phi\left(\frac{75 - 65}{\sqrt{22{,}5}}\right) = \Phi\left(\frac{10}{\sqrt{22{,}5}}\right) = \Phi(0{,}44) = 0{,}67003$

2) $\Pi_x(t) = e^{2t(1+t)}$

$\Pi'_x(t) = e^{2t^2 + 2t} \cdot (4t + 2) = 2$ $\quad E(X) = 2$

$\Pi''_x(t) = e^{2t^2 + 2t}(4t + 2)^2 + e^{2t^2 + 2t}\, 4 = 8$ $\quad \Rightarrow VAR(X) = 8 - 2^2 = 4$

$P(0 < x < 4)$ con $x \sim N(2, 4)$

$= P(x < 4) - P(x < 0) = \phi\left(\frac{4-2}{4}\right) - \phi\left(\frac{-2}{4}\right) = \phi\left(\frac{1}{2}\right) - \phi\left(-\frac{1}{2}\right) =$

$= 2\phi\left(\frac{1}{2}\right) - 1 = 2 \cdot 0{,}69146 - 1 = 0{,}38292$

3) $X \sim U[-1, 1]$ $\quad y = |x|$

• $P\left(|x| > \frac{1}{2}\right) = P\left(y > \frac{1}{2}\right) = 1 - P\left(y \leq \frac{1}{2}\right) = 1 - P\left(|x| \leq \frac{1}{2}\right) =$

$= 1 - P\left(\frac{1}{2} < x < \frac{1}{2}\right) = 1 - \left(F_x\left(\frac{1}{2}\right) - F_x\left(-\frac{1}{2}\right)\right) = 1 - \frac{3}{4} + \frac{1}{4} = \frac{1}{2}$

$f_y(y) = P(Y \leq y) = P(|x| \leq y) = P(-y \leq x \leq y) =$

$= F_x(y) - F_x(-y) = \frac{y+1}{2} - \left(\frac{-y+1}{2}\right) = y$

4) $x \sim Exp(\lambda)$

$Y = F_x(x) \Rightarrow Y = 1 - e^{-\lambda x}$

$$f_y(y) = \begin{cases} f_x[Y^{-1}(y)] \left| \frac{d(Y(y))}{dy} \right| \\ 0 \end{cases}$$

INVERTO $Y \Rightarrow x = \frac{\log(1-y)}{-\lambda} = Y^{-1}(y)$

$\left| \frac{dY^{-1}(y)}{dy} \right| = \frac{1}{(1-y)\lambda}$

$f_y(y) = \cancel{\lambda} e^{-\cancel{\lambda} \frac{\log(1-y)}{\cancel{-\lambda}}} \cdot \frac{1}{(1-y)\cancel{\lambda}} = \frac{1-y}{1-y} = 1$

5) $x \sim f(x) = \frac{x^2}{9}$ $\quad 0 \leq x \leq 3$

$\downarrow 3^3$

$y = x^3 \Rightarrow 0 \leq y \leq 27$

INVERTO $y \Rightarrow x = \sqrt[3]{y} = y^{-1}(y)$

$\left| \frac{dy^{-1}(y)}{dy} \right| = \frac{1}{3\sqrt[3]{y^2}} = \frac{1}{3} y^{-2/3}$

$\Rightarrow f_y(y) = \frac{\cancel{\sqrt[3]{y^2}}}{9} \cdot \frac{1}{3\cancel{\sqrt[3]{y^2}}} = \frac{1}{27}$

8) SUPPONGO I 200 PESATI DALLA POPOLAZIONE INFINITA O QUASI, POI SUPPONGO CHE X CONTA I MANDINI:

$$X \sim Bi(200, 0{,}12)$$

$$\mu = mp = 200 \cdot 0{,}12 = 24 \quad \sigma^2 = m \cdot p(1-p) = 21{,}12 \geq 10$$

$$X \sim N(24, \sqrt{21{,}12}^{\,2})$$

$$P(X \geq 20) = 1 - P(X < 19{,}5) = 1 - \phi\left(\frac{20-24}{\sqrt{21{,}12}}\right) = 1 - \phi\left(\frac{-4}{\sqrt{21{,}12}}\right) = 1 - \phi(-0{,}87) =$$

$$= \cancel{1} - \cancel{1} + \phi(0{,}87) = 0{,}80785$$

9) $$M_X(t) = \frac{9}{(3-t)^2}$$

$$M'_X(t) = \frac{18\cancel{(3-t)}}{(3-t)^{\cancel{4}}} = \frac{18}{(3-t)^3} = \frac{2}{3} = E(X)$$

$$M''_X(t) = \frac{54\cancel{(3-t)^2}}{(3-t)^{\cancel{6}}} = \frac{54}{(3-t)^4} = \frac{54}{81} = \frac{2}{3} = E(X^2)$$

$$VAR(X) = E(X^2) - E(X)^2 = \frac{2}{3} - \frac{4}{9} = \frac{6-4}{9} = \frac{2}{9}$$

1) ① ② ③

ESTRAGGO 2 PALLINE SENZA REINSERIMENTO. FOGLIO 6

X = "NUMERO PIÙ GRANDE" => $\{2, 3\}$

Y = "SOMMA NUMERI ESTRATTI" => $\{3, 4, 5\}$

CASI TOTALI = $|X| \cdot |Y| = 2 \cdot 3 = 6$

X \ Y	3	4	5	P_X (MARGINALE)
2	2/6	0	0	2/6
3	0	2/6	2/6	4/6
P_Y (MARGINALE)	2/6	2/6	2/6	1

(X, Y)	CASI FAVOREVOLI
(2, 3)	2 => {(1,2), (2,1)}
(2, 4)	0
(2, 5)	0
(3, 3)	0
(3, 4)	2 => {(3,1), (1,3)}
(3, 5)	2 => {(3,2), (2,3)}

CALCOLO:

$$\cdot P_{Y|X}(y \mid X=3) = \begin{cases} 0 & y=3 \\ 2/6 / 4/6 & y=4 \\ 2/6 / 4/6 & y=5 \end{cases} \qquad \text{N.B. } P_{Y|X}(y \mid X=x) = \frac{P(y \mid X=x)}{P_X(x)}$$

$$F_{Y|X}(y \mid X=3) = P(Y < y \mid X=3) = \qquad \text{N.B. } F_{Y|X}(y \mid X=x) = P(Y<y \mid X=x)$$

$$= \begin{cases} 0 & y \leq 3 \\ 1/2 & 3 < y \leq 4 \\ 1 & y \geq 5 \end{cases}$$

$$\text{COV}(X,Y) = E[XY] - E(X) \cdot E(Y) = 11 - \frac{32}{3} = \frac{1}{3}$$

$$E[X,Y] = \sum_x \sum_y x \cdot y \cdot P(x,y) = 6 \cdot \frac{1}{3} + 12 \cdot \frac{1}{3} + 15 \cdot \frac{1}{3} = 2+4+5 = 11$$

$$E[X] = \sum_x x \cdot P_X(x) = 2 \cdot \frac{1}{3} + 3 \cdot \frac{2}{3} = 8/3$$

$$E[Y] = \sum_y y \cdot P_Y(y) = 3 \cdot \frac{1}{3} + 4 \cdot \frac{1}{3} + 5 \cdot \frac{1}{3} = 4$$

$$\text{COR}(X,Y) = \frac{\text{COV}(X,Y)}{\sqrt{\text{VAR}(X) \cdot \text{VAR}(Y)}} = \frac{\frac{1}{3}}{\sqrt{22/3 \cdot 50/3}} = \frac{1}{3\sqrt{\frac{4}{27}}} = \frac{1}{2\sqrt{3}}$$

$$\text{VAR}(X) = E(X^2) - E(X)^2 = 22/3 - 64/9 = 2/9$$

$$\text{VAR}(Y) = E(Y^2) - E(Y)^2 = 50/3 - 16 = 2/3$$

$$E(X^2) = \sum_x x^2 P_X(x) = 2^2 \cdot \frac{1}{3} + 3^2 \frac{2}{3} = 22/3$$

$$E(Y^2) = \sum_y y^2 P_Y(y) = 3^2 \cdot \frac{1}{3} + 4^2 \frac{1}{3} + 5^2 \frac{1}{3} = 50/3$$

$$E[2X+3Y] = E[2X] + E[3Y] = 2E[X] + 3E[Y] = 2\frac{8}{3} + 3 \cdot 4 =$$

$$= \frac{16}{3} + 12 = \frac{42}{3} = 14$$

$$X \perp\!\!\!\perp Y \iff P_{XY}(x,y) = P_X(x) \cdot P_Y(y) \implies P_{XY}(3,3) = 0 \neq \frac{2}{6} \cdot \frac{4}{6} = \frac{8}{36}$$

2) $P_{X,Y}(x,y) = K(2y + x)$ con $x = 2,4$ e $y = 0,1,2$

$x \backslash y$	0	1	2	
2	2K	4K	6K	12K
4	4K	6K	8K	18K
	6K	10K	14K	30K

$\Rightarrow 30K = 1 \Rightarrow K = \frac{1}{30}$

$\Downarrow$

$x \backslash y$	0	1	2	
2	1/15	2/15	3/15	2/5
4	2/15	3/15	4/15	3/5
	1/5	1/3	7/15	1

$P(y \geq x) = P(2,2) = \frac{3}{15} = \frac{1}{5}$

$F_{x,y}(2,1) = P\{x \leq 2, y \leq 1\} = P(2,0) + P(2,1) = \frac{1}{15} + \frac{2}{15} = \frac{3}{15} = \frac{1}{5}$

$F_{x,y}(4,1) = P\{x \leq 4, y \leq 1\} = P(2,0) + P(2,1) + P(4,0) + P(4,1) =$
$= \frac{1}{5} + \frac{1}{3} = \frac{8}{15}$

$$P_{x|y}(x|y=1) = \frac{P_{x|y}(x=x, y=1)}{P_y(y)} = \begin{cases} 2/15 / 1/3 = \frac{2}{5} & x=2 \\ 3/15/1/3 = \frac{3}{5} & x=4 \end{cases}$$

$x \perp\!\!\!\perp y \Leftrightarrow P_{xy}(x,y) = P_x(x) \cdot P_y(y) \Rightarrow P_{xy}(2,0) = \frac{1}{15} \neq \frac{2}{5} \cdot \frac{1}{5} = \frac{2}{25}$

3) $f_{xy}(x,y) = 12xy(1-y) \qquad 0<x<1 \quad 0<y<1$

$f_x(x) = \int_0^1 f_{xy}(x,y)\,dy = \int_0^1 12xy(1-y)\,dy = 12x \int_0^1 y - y^2\,dy =$
$= 12x \left[y^2/2 - y^3/3 \right]_0^1 = 12x \left(\frac{1}{2} - \frac{1}{3} \right) = 12x \frac{1}{6} = 2x$

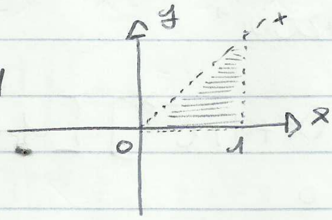
$f_y(y) = \int_0^1 f_{xy}(x,y)\,dx = \int_0^1 12xy(1-y)\,dx = 12y(1-y) \int_0^1 x\,dx =$
$= 12y(1-y) \cdot \left[x^2/2 \right]_0^1 = 12y(1-y) \cdot \frac{1}{2} = 6y(1-y)$

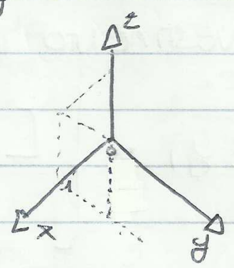
$f_{x,y}(x,y) = f_x(x) \cdot f_y(y) = 6y(1-y) \cdot 2x = 12xy(1-y)$ ✓

$x \perp\!\!\!\perp y$

4) $f_{x,y}(x,y) = K = 2 \qquad 0<x<1 \qquad 0<y<x$

$$\int_0^1 \left(\int_0^x f_{x,y}(x,y)\,dy\right)dx = 1$$





$$= \int_0^1 \left(\int_0^x K\,dy\right)dx =$$

$$= K\int_0^1 \left(\int_0^x 1\,dy\right)dx = K\int_0^1 [y]_0^x\,dx = K\int_0^1 x\,dx = K\left[x^2/2\right]_0^1 =$$

$$= K\frac{1}{2} \Rightarrow \frac{K}{2} = 1 \Rightarrow K = 2$$

$$f_x(x) = \int_0^x f_{xy}(x,y)\,dy = \int_0^x 2\,dy = 2[y]_0^x = 2x$$

$$f_y(y) = \int_0^1 f_{xy}(x,y)\,dx = \int_0^1 2\,dx = 2[x]_0^1 = 2$$

$$f_{xy}(x,y) = 2 \neq f_x(x)\cdot f_y(y) = 4x$$

5) $f_{xy}(x,y) = \frac{1}{8}(6-x-y)\,\mathbb{1}_{(0,2)}(x)\,\mathbb{1}_{(2,4)}(y)$

$$f_x(x) = \int_2^4 f_{xy}(x,y)\,dy = \int_2^4 \frac{1}{8}(6-x-y)\,dy = \frac{1}{8}\int_2^4 6 - x - y\,dy =$$

$$= \frac{1}{8}\left[6y - xy - y^2/2\right]_2^4 = (12-(4x-2x)-6)\frac{1}{8} = \frac{6}{8} - \frac{1}{4}x = \frac{3}{4} - \frac{1}{4}x$$

$$f_y(y) = \int_0^2 f_{xy}(x,y)\,dx = \int_0^2 \frac{1}{8}(6-x-y)\,dx = \frac{1}{8}\int_0^2 6-x-y\,dx =$$

$$= \frac{1}{8}\left[6x - \frac{x^2}{2} - yx\right]_0^2 = \frac{1}{8}(12-2-2y) = \frac{5}{4} - \frac{1}{4}y$$

$$f_{xy}(x,y) = \frac{f(x,y)}{f_y(y)} = \frac{\overset{1/2}{\cancel{\frac{1}{8}}}(6-x-y)}{\cancel{\frac{1}{4}}(5-y)} = \frac{1}{2}\frac{(6-x-y)}{(5-y)}$$

6) X = "INVESTIMENTO IN TITOLI DI STATO"
Y = "INVESTIMENTO IN AZIONI"

$$F_{(X,Y)}(x,y) = \begin{cases} \left[1-(x+1)e^{-x}\right]\left[1-(y+1)e^{-y}\right] & x \geq 0 \ y \geq 0 \\ 0 & \text{ALTRIMENTI} \end{cases}$$

PER VERIFICARE CHE LA DEFINIZIONE SIA BEN DATA CALCOLO I DUE SEGUENTI LIMITI:

- $\lim_{x\to -\infty} \lim_{y \to -\infty} F_{XY}(x,y) = 0$ ✓
- $\lim_{x\to +\infty} \lim_{y \to +\infty} F_{XY}(x,y) = 1$ ✓

LA DENSITÀ CONGIUNTA È

- $f_{XY}(x,y) = \frac{\partial^2}{\partial x \partial y} F_{XY}(x,y) = \frac{\partial}{\partial x}\left[1-(x+1)e^{-x}\right]\frac{\partial}{\partial y}\left[1-(y+1)e^{-y}\right] =$
$= \left(-e^{-x} + (x+1)e^{-x}\right)\left(e^{-y} + (y+1)e^{-y}\right) = x e^{-x} y e^{-y}$

$$f_X(x) = \int_0^{+\infty} f_{XY}(x,y)\,dy = \int_0^{+\infty} x e^{-x} y e^{-y}\,dy = x e^{-x}\int_0^{+\infty} y e^{-y}\,dy =$$
$$= x e^{-x}\left[-e^{-y}(1+y)\right]_0^{+\infty} = x e^{-x}$$

$$f_Y(y) = \int_0^{+\infty} f_{XY}(x,y)\,dx = \int_0^{+\infty} x e^{-x} y e^{-y}\,dx = y e^{-y}\int_0^{+\infty} x e^{-x}\,dx$$
$$= y e^{-y}\left[-e^{-x}(1+x)\right]_0^{+\infty} = y e^{-y}.$$

$$f_{XY}(x,y) = f_X(x)\cdot f_Y(y) = y e^{-y}\, x e^{-x} = x y e^{-y} e^{-x}. \checkmark$$

$$P(X \geq 2Y) = \int_0^{+\infty}\left(\int_{2y}^{+\infty} f_{XY}(x,y)\,dx\right)dy = \int_0^{+\infty}\left(\int_{2y}^{+\infty} x e^{-x} y e^{-y}\,dx\right)dy$$
$$= \int_0^{+\infty} y e^{-y}\left(\int_{2y}^{+\infty} x e^{-x}\,dx\right)dy = \int_0^{+\infty} y e^{-y}\left[-e^{-x}(1+x)\right]_{2y}^{+\infty} dy = \int_0^{+\infty} y e^{-y} e^{-2y}(1+2y)\,dy$$

$$= \int_0^{+\infty} y(1+2y) \cdot e^{-3y} dy = \left[y(1+2y) \frac{e^{-3y}}{-3} \right]_0^{+\infty} - \int_0^{+\infty} \frac{1+4y}{-3} e^{-3y} dy =$$

$$= \frac{1}{3} \int_0^{+\infty} (1+4y) e^{-3y} dy = \frac{1}{3} \left[\left[-\frac{e^{-3y}}{3}(1+4y) \right]_0^{+\infty} - 4 \int_0^{+\infty} \frac{e^{-3y}}{-3} dy \right] =$$

$$= \frac{1}{3} \left[\frac{1}{3} + \frac{4}{3} \cdot \left[\frac{e^{-3y}}{-3} \right]_0^{+\infty} \right] = \frac{1}{3} \left[\frac{1}{3} + \frac{4}{3} \frac{1}{3} \right] = \frac{1}{9} + \frac{4}{27} = \frac{7}{27}$$

7) $$f_{(x,y)}(x,y) = \begin{cases} 2e^{-(x+y)} & 0 \leq x \leq y \\ 0 & \text{ALTRIMENTI} \end{cases}$$

$$f_x(x) = \int_0^{+\infty} 2e^{-(x+y)} dy = 2\int_0^{+\infty} e^{-(x+y)} dy = 2 \int_0^{+\infty} e^{-x} \cdot e^{-y} dy =$$

$$= 2 \int_0^{+\infty} e^{-y} dy = 2e^{-x} \left[-e^{-y} \right]_0^{+\infty} = 2e^{-x}$$

$$f_y(y) = \int_0^y 2e^{-(x+y)} dx = 2 \int_0^y e^{-x} e^{-y} dx = 2e^{-y} \int_0^y e^{-x} dx =$$

$$= 2e^{-y} \cdot \left[-e^{-x} \right]_0^y = 2e^{-y}(1-e^{-y})$$

NON SONO INDIPENDENTI

$$E[X \cdot Y] = \int_0^{+\infty} \left(\int_0^y xy\, f_{xy}(x,y)\, dx \right) dy = \int_0^{+\infty} \left(y e^{-y} \int_0^y x e^{-x} dx \right) dy =$$

$$= \int_0^{+\infty} y e^{-y} \left[\left[-xe^{-x} - e^{-x} \right]_0^y \right] dx = \int_0^{+\infty} y e^{-y} (-e^{-y}(y+1) + 1) dy =$$

$$= -\int_0^{+\infty} y^2 e^{-2y} dy - \int_0^{+\infty} y e^{-2y} dy + \int_0^{+\infty} y e^{-y} dy =$$

$$= \left[\frac{1}{4} e^{-2y} (2y^2 + 2y + 1) \right]_0^{+\infty} - \left[e^{-2x} \left(-\frac{x}{2} - \frac{1}{4} \right) \right]_0^{+\infty} + \left[-e^{-x}(x+1) \right]_0^{+\infty}$$

$$= -\frac{1}{4} - \frac{1}{4} + 1 = \frac{1}{2}$$

8/9) $f_{xy}(x,y) = \begin{cases} k(x+y) & 0 \leq x \leq 2,\ 0 \leq y \leq 2 \\ 0 & \text{ALTRIMENTI} \end{cases}$

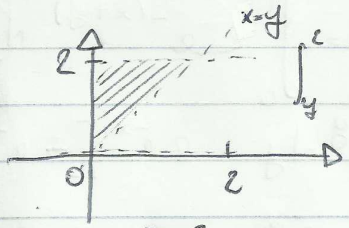
$$\int_0^2\left(\int_0^2 k(x+y)\,dx\right)dy = 1$$

$$= \int_0^2\left(\int_0^2 k(x+y)\,dx\right)dy = k\int_0^2\left(\int_0^2 x + y\,dx\right)dy =$$

$$= k\int_0^2\left[\frac{x^2}{2} + yx\right]_0^2 = k\int_0^2 2y+2\,dy = k\left[y^2+2y\right]_0^2 =$$

$$= 8k \Rightarrow 8k = 1 \Rightarrow k = \frac{1}{8}$$

$$P(X>Y) = \int_0^2\left(\int_y^2 \frac{1}{8}(x+y)\,dx\right)dy$$



$$= \frac{1}{8}\int_0^2\left(\int_y^2 (x+y)\,dx\right)dy = \frac{1}{8}\int_0^2\left[\frac{x^2}{2}+yx\right]_y^2 dy =$$

$$= \frac{1}{8}\int_0^2 2+2y-\frac{y^2}{2}-y^2\,dy = \frac{1}{8}\int_0^2 -\frac{3}{2}y^2+2y+2\,dy =$$

$$= \frac{1}{8}\left[-\frac{3}{2}\frac{y^3}{3}+y^2+2y\right]_0^2 = \frac{1}{8}\left(\cancel{-4}+\cancel{4}+4\right) = \frac{1}{2}$$

$$f_x(x) = \int_0^2 f_{xy}(x,y)\,dy = \int_0^2 \frac{1}{8}(x+y)\,dy = \frac{1}{8}\left[xy+\frac{y^2}{2}\right]_0^2 =$$

$$= \frac{1}{8}(2x+2) = \frac{1}{4}(x+1)$$

ANALOGAMENTE $f_y(y) = \frac{1}{4}(y+1)$

$f_{xy}(x,y) \neq f_x(x)\cdot f_y(y)$ NON SONO INDIPENDENTI

$$f_{x|y}(x|y) = \frac{f_{xy}(x,y)}{f_y(y)} = \frac{\frac{1}{8}(x+y)}{\frac{1}{4}(y+1)} = \frac{1}{2}\,\frac{x+y}{y+1}$$

$$F_{x|y}(x|y) = \int_0^x f_{x|y}(x|y)\,dx = \int_0^x \frac{1}{2}\,\frac{x+y}{y+1}\,dx =$$

$$= \frac{1}{2(y+1)} \int_0^x x+y\,dx = \frac{1}{2(y+1)} \left[x^2/2 + xy \right]_0^x = \frac{x^2/2 + xy}{2(y+1)} =$$

$$= \frac{1/2(x^2 + 2xy)}{2(y+1)} = \frac{(x^2 + 2xy)}{4(y+1)}$$

1) X = "DURATA TELEFONATE URBANE" FOGLIO 7

$X \sim N(10, 3^2)$

$X_1, \dots, X_{20}$ CAMPIONE CASUALE 20 TELEFONATE

$\bar{X}_{20} \sim N(\mu, \sigma) \Rightarrow \mu = 10 \quad \sigma^2 = \frac{3^2}{20}$

$$E[\bar{X}_{20}] = E\left[\sum_{i=0}^{20} \frac{x_i}{20}\right] = \sum_{i=1}^{20} \frac{1}{20} E[x_i] = \sum_{i=1}^{20} \frac{1}{20}\mu = \frac{\cancel{20}\mu}{\cancel{20}} = \mu$$

$$VAR[\bar{X}_{20}] = VAR\left[\sum_{i=1}^{20} \frac{x_i}{20}\right] = \sum_{i=1}^{20} \frac{1}{400} VAR[x_i] = \frac{1}{400}\sum_{i=1}^{20} \sigma^2 = \frac{20\sigma^2}{400} = \frac{\sigma^2}{20}$$

$$P(9{,}5 \le \bar{X}_{20} \le 13{,}3) = \phi\left(\underbrace{\frac{13{,}3-10}{\frac{3}{\sqrt{20}}}}_{z_1}\right) - \phi\left(\underbrace{\frac{9{,}5-13{,}3}{\frac{3}{\sqrt{20}}}}_{z_2}\right) =$$

$z_2 < 0$

$$= \phi(z_1) - \phi(z_2) = \phi(z_1) - 1 + \phi(-z_2) = 0{,}4472$$

2) X = "RESISTENZA AL CARICO DEI SACCHETTI"

$X \sim N(3{,}2,\ 25)$

$$P(\underset{y}{2{,}8 \le X \le 3{,}4}) = \phi\left(\frac{3{,}4-3{,}2}{25}\right) - 1 + \phi\left(\left|\frac{2{,}8-3{,}2}{25}\right|\right) = 0{,}0478$$

$X_1, \dots, X_{30}$ CAMPIONE CASUALE

$$\bar{X}_{20} \sim N\left(3{,}2,\ \frac{25}{\sqrt{30}}\right)$$

$$P(y) = \phi\left(\frac{3{,}4-3{,}2}{\frac{25}{\sqrt{30}}}\right) - 1 + \phi\left(\left|\frac{2{,}8-3{,}2}{\frac{25}{\sqrt{30}}}\right|\right) = 0{,}257$$

3) X = "DURATA IN ORE DELLE PILE"

$X \sim N(\mu = 100, \sigma^2 = ?)$

$P(\overline{X}_{10} > 88,4) = 0,95$ CON $X_1, \dots, X_{10}$ E $\overline{X}_{10} \sim N(\mu, \frac{\sigma^2}{10})$

$= 1 - P(\overline{X}_{10} \leq 88,4) = 0,95 \Rightarrow P(\overline{X}_{10} \leq 88,4) = 0,05$

$\Rightarrow \Phi\left(\frac{88,4-100}{\frac{\sigma}{\sqrt{10}}}\right) = 0,05 \Rightarrow \Phi\left(-\frac{11,6}{\frac{\sigma}{\sqrt{10}}}\right) = 0,05 \Rightarrow$

$\Rightarrow 1 - \Phi\left(\frac{11,6}{\frac{\sigma}{\sqrt{10}}}\right) = 0,05 \Rightarrow \Phi\left(\frac{11,6}{\frac{\sigma}{\sqrt{10}}}\right) = 0,95 \Rightarrow$

$\Rightarrow \frac{11,6}{\frac{\sigma}{\sqrt{10}}} = 1,64 \Rightarrow \sigma = \frac{11,6\sqrt{10}}{1,64} = 21,21$

4) $X_A \sim N(6,5,\ 0,9^2)$ CON $X \,\&\, Y$ = "PESO DI VITE DI PEZZO MECCANICO"

$X_B \sim N(6,\ 0,8^2)$

$P((\overline{X}_{A_{36}} - \overline{X}_{B_{49}}) \geq 1)$

$\overline{X}_{A_{36}} \sim N(6,5,\ \frac{0,9^2}{36})$

$-\overline{X}_{B_{49}} \sim N(-6,\ \frac{0,8^2}{49})$

$y = \overline{X}_A + (-\overline{X}_B) \sim N\left(\mu_A + \mu_B,\ \frac{\sigma_A^2}{36} + \frac{\sigma_B^2}{49}\right) \sim N\left(0,5,\ \left(\frac{0,9}{6}\right)^2 + \left(\frac{0,8}{49}\right)^2\right)$

$P(y \geq 1) = 1 - P(y < 1) = 1 - \Phi\left(\frac{1-0,5}{\left(\frac{0,9}{6}\right)^2 + \left(\frac{0,8}{7}\right)^2}\right) = 0,006$

6) X = "PESO DI CONFEZIONI"

$X \sim N(\mu,\ \sigma^2 = 2,5^2)$ $\quad m = ?$

$\overline{X}_m = \sum_{i=1}^{m} \frac{x_i}{m}$

$P(|\overline{X}_m - \mu| < 0,5) \geq 0,95$

|

$= P\left(\frac{-0,5}{\sigma_{\bar{x}_m}} \le \frac{\bar{x}_m - \mu}{\sigma_{\bar{x}_m}} \le \frac{+0,5}{\sigma_{\bar{x}_m}}\right) = P\left(\frac{-0,5}{\sigma_{\bar{x}_m}} \le z \le \frac{0,5}{\sigma_{\bar{x}_m}}\right) =$

$= 2\phi\left(\frac{0,5}{\sigma_{\bar{x}_m}}\right) - 1 \ge 0,95$

$z = 1,96$

$\Rightarrow \phi\left(\frac{0,5}{\sigma_{\bar{x}_m}}\right) \ge \frac{1,95}{2} = 0,975$

$\frac{0,5}{\sigma_{\bar{x}_m}} = 1,96 \Rightarrow \frac{0,5}{\frac{5}{\sqrt{m}}} = 1,96 \Rightarrow \frac{0,5\sqrt{m}}{2,5} = 1,96 \Rightarrow$

$\Rightarrow \sqrt{m} = \frac{1,96 \cdot 2,5}{0,5} \Rightarrow \sqrt{m} = 1,96 \cdot 5 \Rightarrow m = (1,96 \cdot 5)^2 = 96,04$

$m > 96$

8) $X_1, \dots, X_{80} \sim f(x) = 3x^2 \in (0,1) \quad S_{80} = \sum_{i=1}^{80} x_i$

$P(S_{80} \le 65) \overset{\text{NORMALIZZO *}}{=} \phi\left(\frac{65 - 80\,3/4}{\cancel{\sqrt{80}}\frac{\sqrt{3}}{\cancel{\sqrt{80}}}}\right) = \phi\left(\frac{65-60}{\sqrt{3}}\right) = \phi\left(\frac{5}{\sqrt{3}}\right) = \phi(2,87) = 0,980$

* $\frac{S_{80} - m\mu}{\sigma\sqrt{m}} \sim N(0,1)$ N.B STESSA COSA DI $\bar{x}_{80} = \sum \frac{1}{80} x_i$ SOLO SENZA $\frac{1}{80}$

$E[X_i] = \int_0^1 x f(x)\,dx = \int_0^1 x \cdot 3x^2\,dx = \int_0^1 3x^3\,dx = 3\int_0^1 x^3\,dx =$

$= 3\left[x^4/4\right]_0^1 = 3/4$

$E[X_i^2] = \int_0^1 x^2 f(x)\,dx = \int_0^1 x^2 3x^2\,dx = \int_0^1 3x^4\,dx = 3\int_0^1 x^4\,dx =$

$= 3\left[x^5/5\right]_0^1 = 1/5 = 3/5$

$VAR[X_i] = E[x^2] - E[x]^2 = 3/5 - 3/4^2 = 3/80$