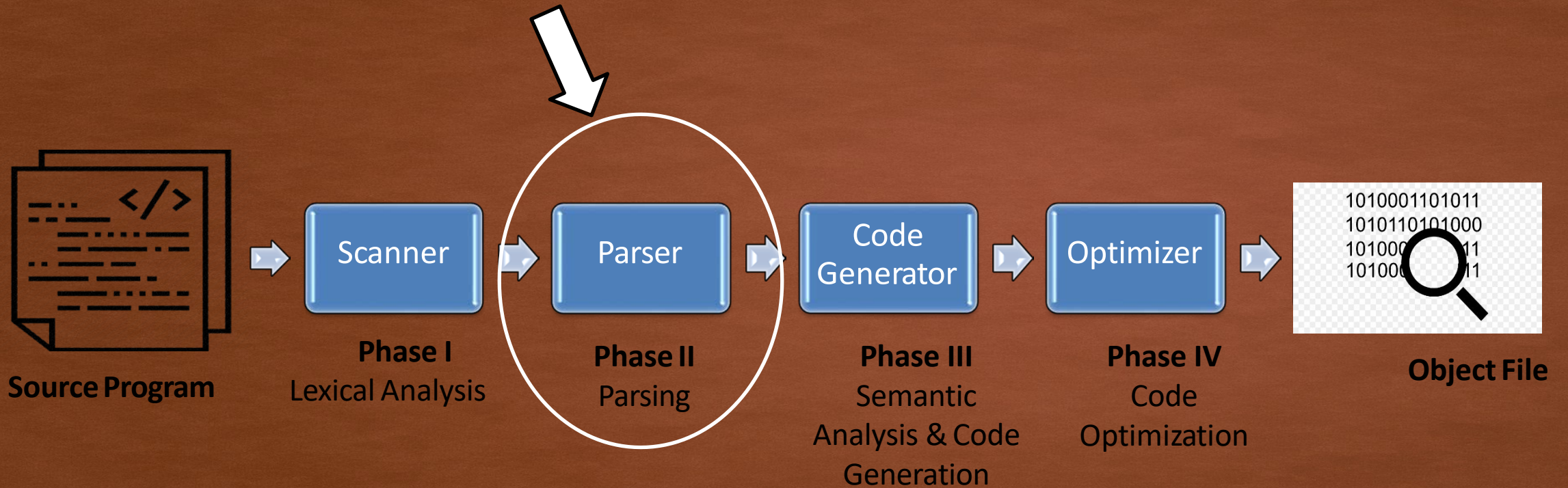


Phase II : Parsing: First Principles



PARSING- RECAP

The parse tree has as its leaves the terminals (tokens returned by the lexer).

The parser uses the rules of the grammar to combine terminals/non-terminals into other non-terminals, or transform non-terminals into other non-terminals.

Each such step of combining/transforming terminals/non-terminals into other non-terminals is called a *production*.

Eventually, there should only one non-terminal left: the *root* representing the *goal symbol*

If the parser cannot build such a tree, it concludes that the code has a *syntax error*.

Parsing Example

a1 = x + y

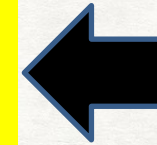
Aim: Check whether this list of tokens gives a *syntactically correct statement* (aka the *goal symbol*).

We will assume here that we are using a BNF grammar.

Our *goal symbol* in the example here is a single assignment statement.

Building a Parse Tree : An Example

```
<variable> ::= <symbol>
<operator> ::= +|-|*|/
<term> ::= <number>|<variable>
<expression> ::= <term>|<expression><operator><expression>
<assignment> ::= <variable>=<expression>
```



Grammar of our
Language in BNF

a1 = x + y



Input Program : A simple
assignment statement

<symbol> = <symbol> + <symbol>



Tokens returned by the lexer

Building a Parse Tree

<variable> ::= <symbol>

<operator> ::= + | - | * | /

<term> ::= <number> | <variable>

<expression> ::= <term> | <expression> <operator> <expression>

<assignment> ::= <variable> = <expression>

This rule enables productions that recognise a symbol as a variable

Building a Parse Tree

```
<variable> ::= <symbol>
<operator> ::= +|-|*|/
<term> ::= <number>|<variable>
<expression> ::= <term>|<expression><operator><expression>
<assignment> ::= <variable>=<expression>
```

The diagram illustrates a recursive definition for the category <expression>. A yellow box contains five grammar rules. The rule for <expression> is circled in red. Two red arrows originate from this rule: one points from the <expression> on the left-hand side (LHS) down to the text below, and the other points from the <expression> on the right-hand side (RHS) down to the text below. This visualizes how the category <expression> is defined in terms of itself.


This rule is an example of a **recursive definition**, where the category <expression> is defined in terms of its own self.

Hence the symbol <expression> appears on both the LHS and RHS of the rule.

It enables productions that combine two expressions and an operator between them into an expression.

Building a Parse Tree

```
<variable> ::= <symbol>
<operator> ::= +|-|*|/
<term> ::= <number>|<variable>
<expression> ::= <term>|<expression><operator><expression>
<assignment> ::= <variable>=<expression>
```



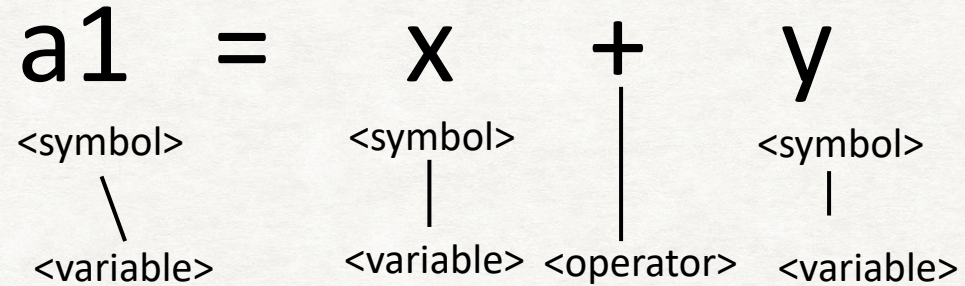
Goal symbol (we want this to be at the root of the tree we will build)

Building a Parse Tree : An Example

a1 = x + y
<symbol> <symbol> <symbol>

<variable> ::= <symbol>
<operator> ::= + | - | * | /
<term> ::= <number> | <variable>
<expression> ::= <term> | <expression><operator><expression>
<assignment> ::= <variable>=<expression>

Building a Parse Tree : An Example



`<variable> ::= <symbol>`

`<operator> ::= + | - | * | /`

`<term> ::= <number> | <variable>`

`<expression> ::= <term> | <expression> <operator> <expression>`

`<assignment> ::= <variable> = <expression>`

Building a Parse Tree : An Example

a1 = x + y

<variable> <variable><operator><variable>

<variable> ::= <symbol>

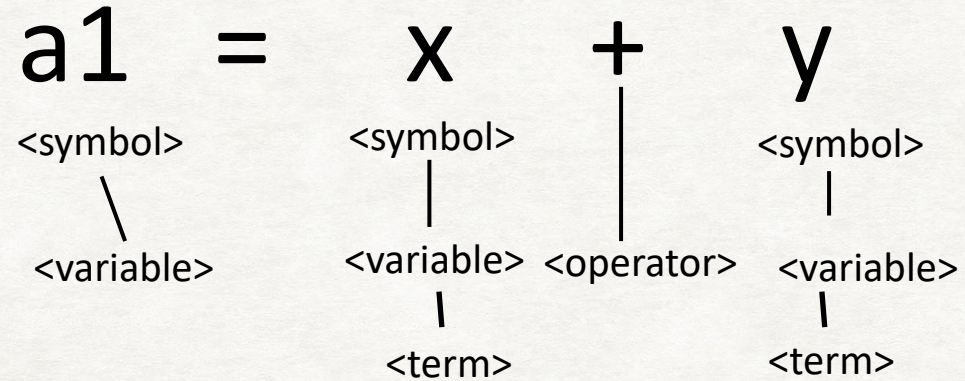
<operator> ::= + | - | * | /

<term> ::= <number> | <variable>

<expression> ::= <term> | <expression><operator><expression>

<assignment> ::= <variable> = <expression>

Building a Parse Tree : An Example



<variable> ::= <symbol>

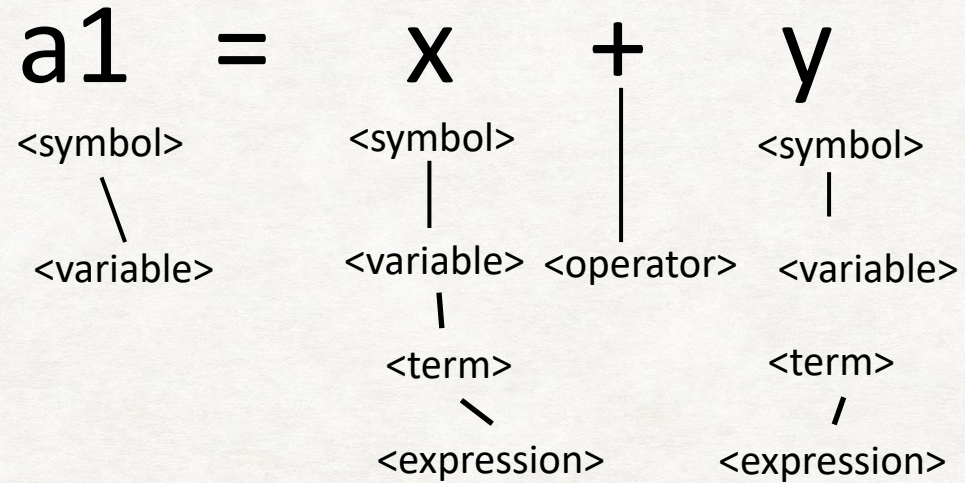
<operator> ::= + | - | * | /

<term> ::= <number> | <variable>

<expression> ::= <term> | <expression><operator><expression>

<assignment> ::= <variable> = <expression>

Building a Parse Tree : An Example



<variable> ::= <symbol>

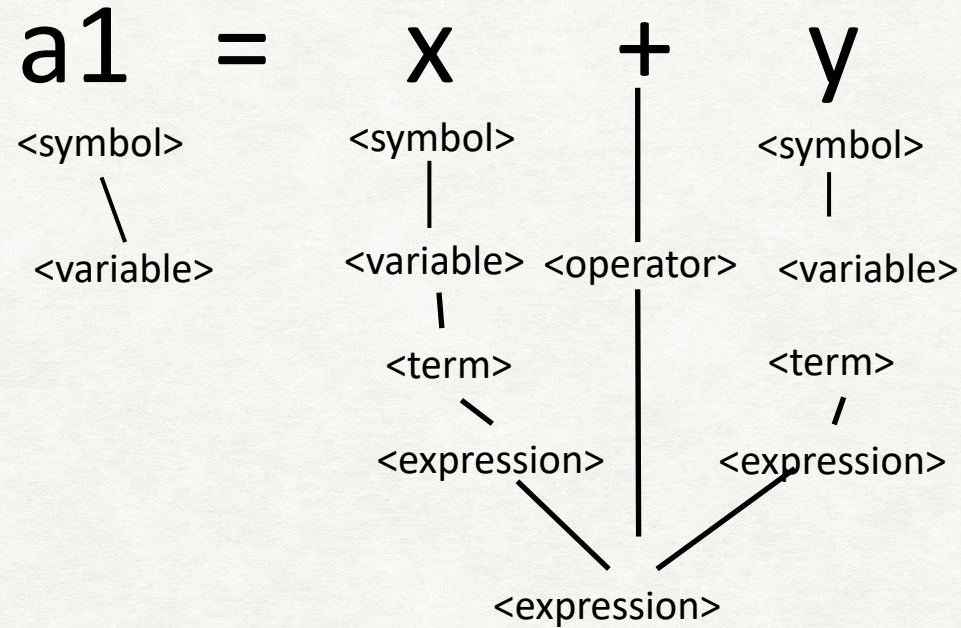
<operator> ::= + | - | * | /

<term> ::= <number> | <variable>

<expression> ::= <term> | <expression><operator><expression>

<assignment> ::= <variable>=<expression>

Building a Parse Tree : An Example



`<variable> ::= <symbol>`

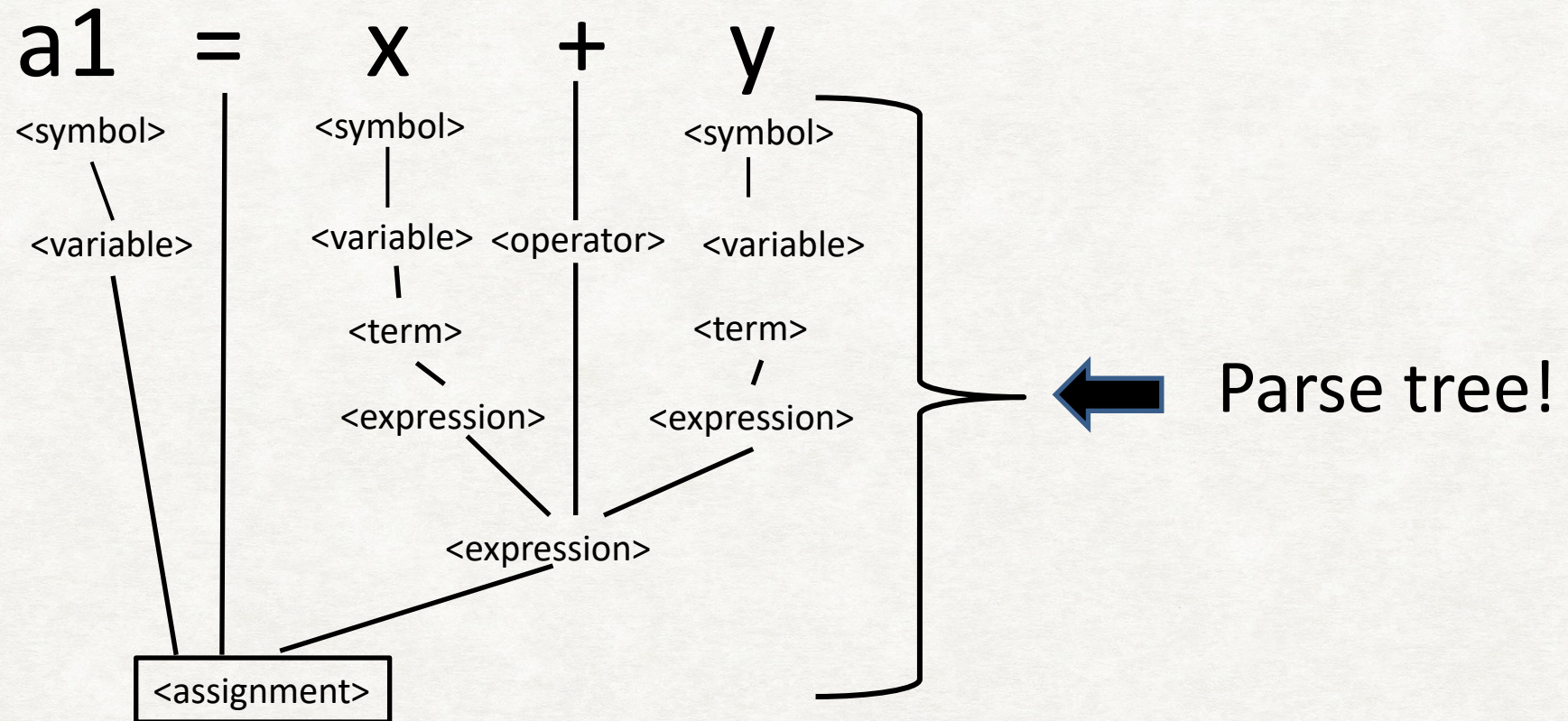
`<operator> ::= + | - | * | /`

`<term> ::= <number> | <variable>`

`<expression> ::= <term> | <expression><operator><expression>`

`<assignment> ::= <variable> = <expression>`

Building a Parse Tree : An Example



$\langle \text{variable} \rangle ::= \langle \text{symbol} \rangle$

$\langle \text{operator} \rangle ::= + | - | * | /$

$\langle \text{term} \rangle ::= \langle \text{number} \rangle | \langle \text{variable} \rangle$

$\langle \text{expression} \rangle ::= \langle \text{term} \rangle | \langle \text{expression} \rangle \langle \text{operator} \rangle \langle \text{expression} \rangle$

$\langle \text{assignment} \rangle ::= \langle \text{variable} \rangle = \langle \text{expression} \rangle$

PRACTICE EXERCISE

Given the grammar for an if-else statement, check whether the following if statement is syntactically correct or not.

```
if ( x == y) x = z; else x = y;
```

Number	Rule
1	$\langle \text{if statement} \rangle ::= \text{if} (\langle \text{Boolean expression} \rangle) \langle \text{assignment statement} \rangle ;$ $\qquad \qquad \qquad \langle \text{else clause} \rangle$
2	$\langle \text{Boolean expression} \rangle ::= \langle \text{variable} \rangle \mid \langle \text{variable} \rangle \langle \text{relational} \rangle \langle \text{variable} \rangle$
3	$\langle \text{relational} \rangle ::= == \mid < \mid >$
4	$\langle \text{variable} \rangle ::= x \mid y \mid z$
5	$\langle \text{else clause} \rangle ::= \text{else} \langle \text{assignment statement} \rangle ; \mid \Lambda$
6	$\langle \text{assignment statement} \rangle ::= \langle \text{variable} \rangle = \langle \text{expression} \rangle$
7	$\langle \text{expression} \rangle ::= \langle \text{variable} \rangle \mid \langle \text{expression} \rangle + \langle \text{variable} \rangle$

Grammar for a simplified version of an *if-else* statement

if (x == y) x = z ; else x = y ;
 <variable> <relop> <variable> <variable> <variable> <variable> <variable>

<if statement> ::= if(< Boolean expression>)< assignment>;<else clause>
 <Boolean expression> ::= <variable> | <variable> <relop> <variable>
 < relop> ::= == | < | > |
 <variable> ::= x | y | z
 < else clause> ::= else< assignment>; | \wedge
 <assignment> ::= <variable> = <expression>
 <expression> ::= < variable> | <expression> + < variable>

$$\text{if } (\quad x \quad == \quad y \quad) \quad x = z \quad ; \text{ else } x = y \quad ;$$

$$\begin{array}{ccccccc}
\text{<variable>} & \text{<relop>} & \text{<variable>} & \text{<variable>} & \text{<variable>} & \text{<variable>} & \text{<variable>} \\
& & & & / & & / \\
& & & & \text{<expression>} & & \text{<expression>}
\end{array}$$

If the parser can **look ahead**
it can often make the right
choice.

$$\begin{aligned}
\text{<if statement>} &::= \text{if}(\text{< Boolean expression>})\text{< assignment>;<else clause>} \\
\text{<Boolean expression>} &::= \text{<variable> | <variable> <relop><variable>} \\
\text{< relop>} &::= == | < | > | \\
\text{<variable>} &::= x | y | z \\
\text{< else clause>} &::= \text{else< assignment>;} \mid \Lambda \\
\text{<assignment>} &::= \text{<variable>=<expression>} \\
\text{<expression>} &::= \text{< variable> | <expression>+< variable>}
\end{aligned}$$

if (x == y) x = z ; else x = y ;

<variable> <relop> <variable>

<variable> <variable>

<expression>

<assignment>

<variable> <variable>

<expression>

<assignment>

<if statement> ::= if(< Boolean expression>)< assignment>;<else clause>

<Boolean expression> ::= <variable> | <variable> <relop> <variable>

< relop> ::= == | < | > |

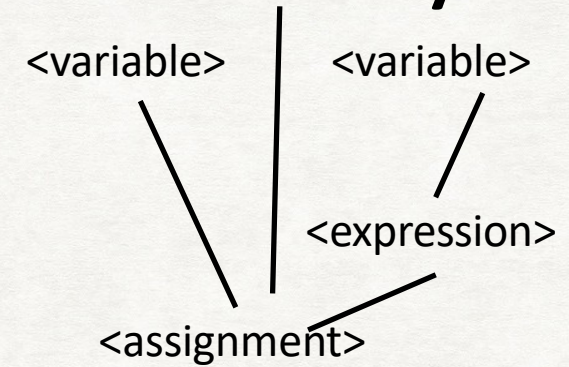
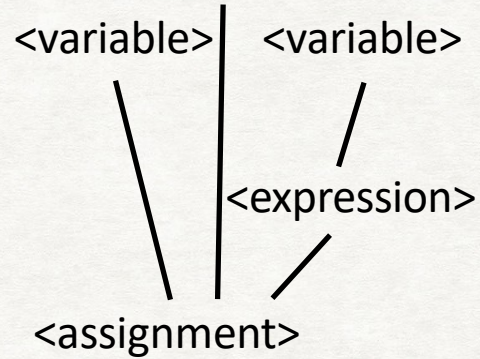
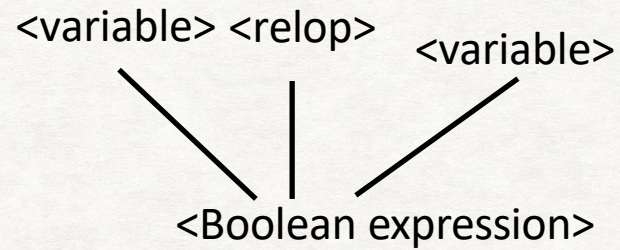
<variable> ::= x | y | z

< else clause> ::= else< assignment>; | \wedge

<assignment> ::= <variable> = <expression>

<expression> ::= < variable> | <expression> + < variable>

if (x == y) x = z ; else x = y ;



<if statement> ::= if(< Boolean expression>)< assignment>;<else clause>

<Boolean expression> ::= <variable> | <variable> <relop> <variable>

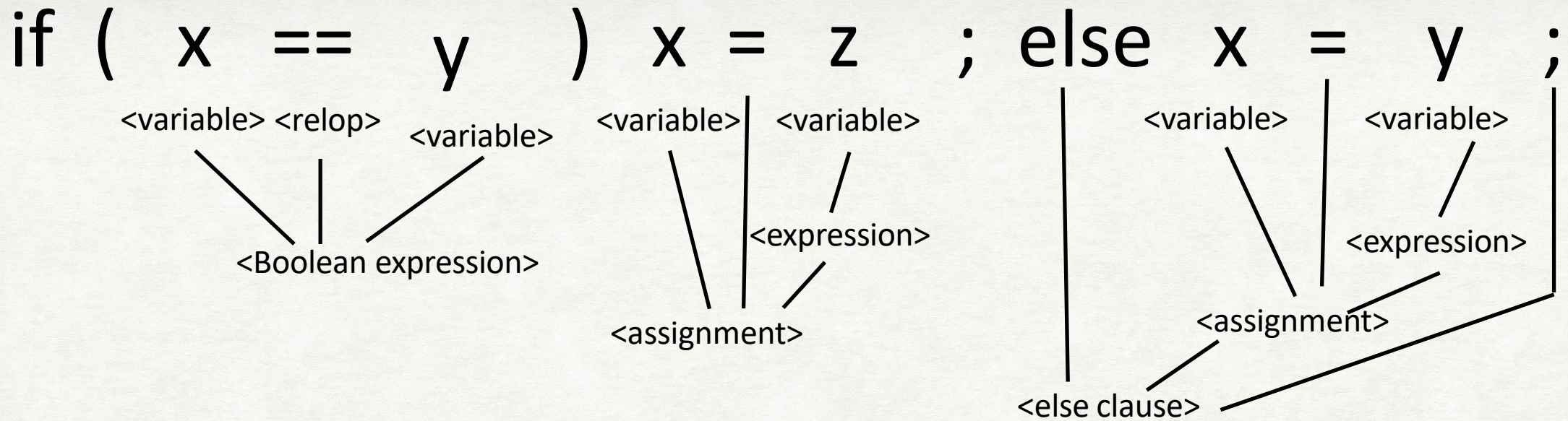
< relop> ::= == | < | > |

<variable> ::= x | y | z

< else clause> ::= else< assignment>; | ^

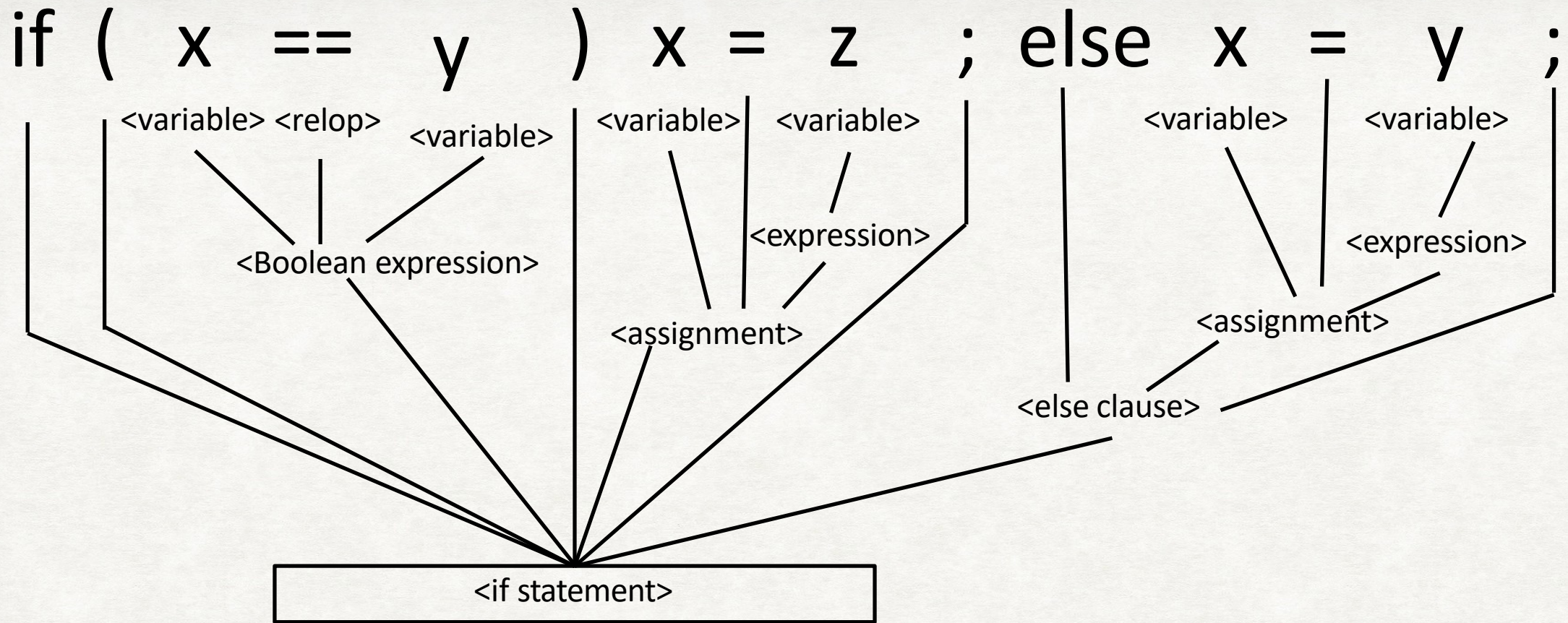
<assignment> ::= <variable> = <expression>

<expression> ::= < variable> | <expression> + < variable>



```

<if statement> ::= if(< Boolean expression>)< assignment>;<else clause>
<Boolean expression> ::= <variable>|<variable> <relop><variable>
< relop> ::= ==|<|>|
<variable> ::= x|y|z
< else clause> ::= else< assignment>;| ^
<assignment> ::= <variable>=<expression>
<expression> ::= < variable>|<expression>+< variable>
  
```

<if statement> ::= if(< Boolean expression>)< assignment>;<else clause>

<Boolean expression> ::= <variable> | <variable> <relop> <variable>

< relop> ::= == | < | > |

<variable> ::= x | y | z

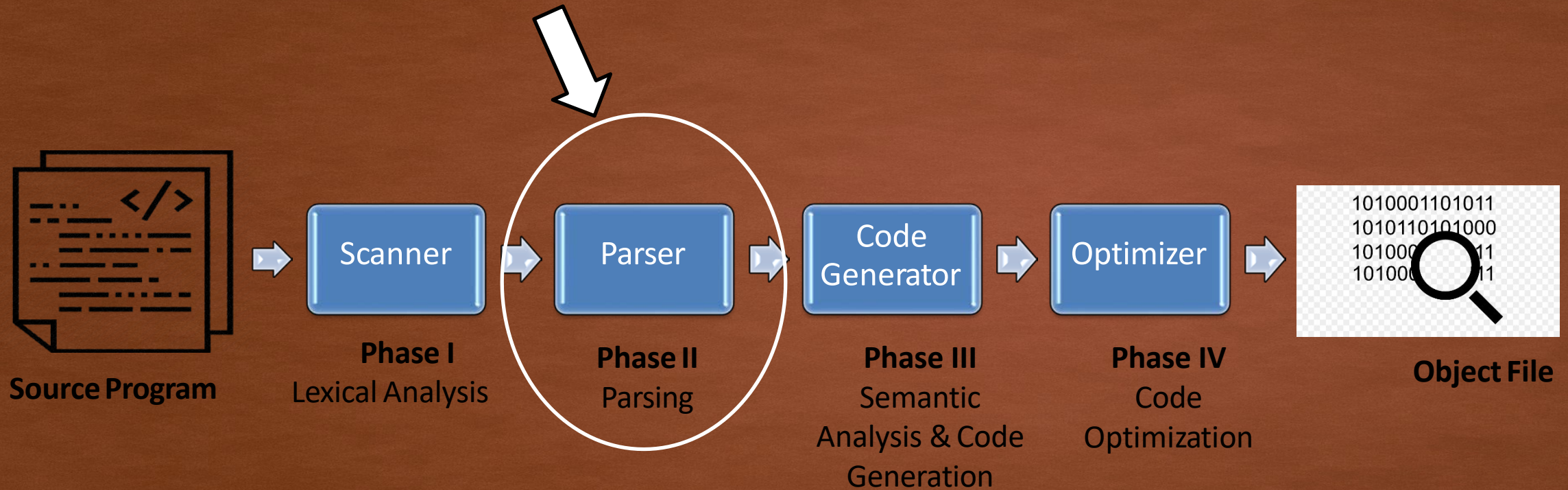
< else clause> ::= else< assignment>; | ^

<assignment> ::= <variable> = <expression>

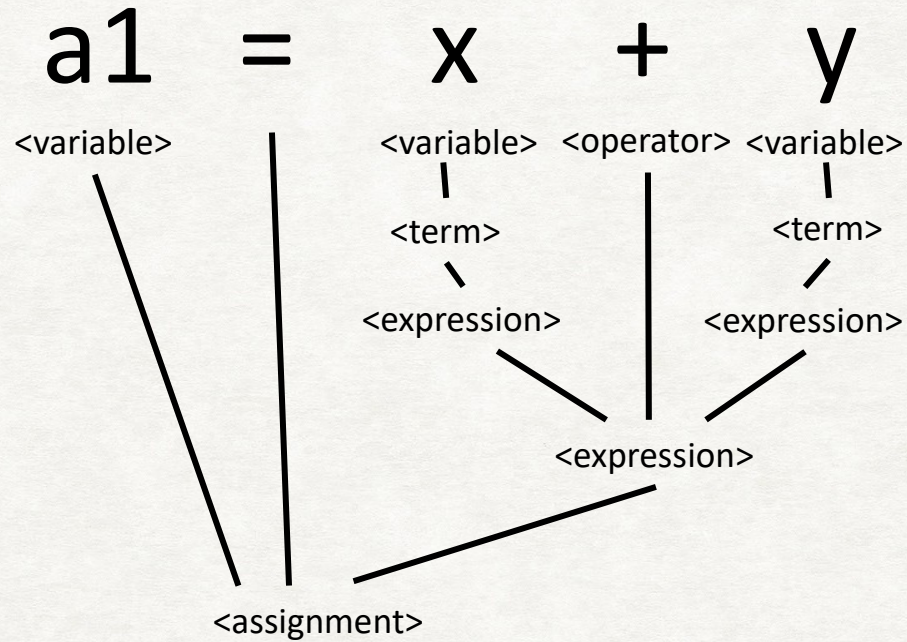
<expression> ::= < variable> | <expression> + < variable>

Phase II : Parsing

Priority and Ambiguous Grammars



Recall the previous example: Simple assignment statement



$\langle \text{variable} \rangle ::= \langle \text{symbol} \rangle$

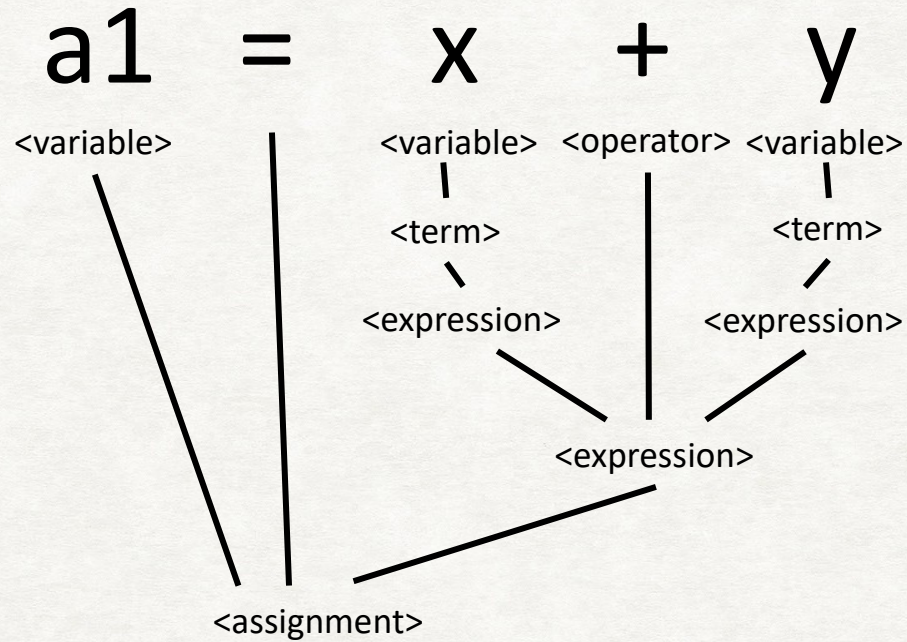
$\langle \text{operator} \rangle ::= + \mid - \mid * \mid /$

$\langle \text{term} \rangle ::= \langle \text{number} \rangle \mid \langle \text{variable} \rangle$

$\langle \text{expression} \rangle ::= \langle \text{term} \rangle \mid \langle \text{expression} \rangle \langle \text{operator} \rangle \langle \text{expression} \rangle$

$\langle \text{assignment} \rangle ::= \langle \text{variable} \rangle = \langle \text{expression} \rangle$

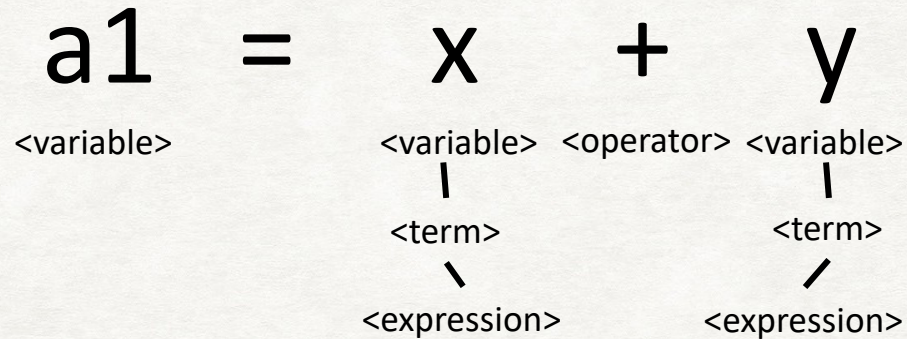
Example: Simple assignment statement



Could we have done this differently?

```
<variable> ::= <symbol>
<operator> ::= + | - | * | /
<term> ::= <number> | <variable>
<expression> ::= <term> | <expression> <operator> <expression>
<assignment> ::= <variable> = <expression>
```


Try again: Simple assignment statement

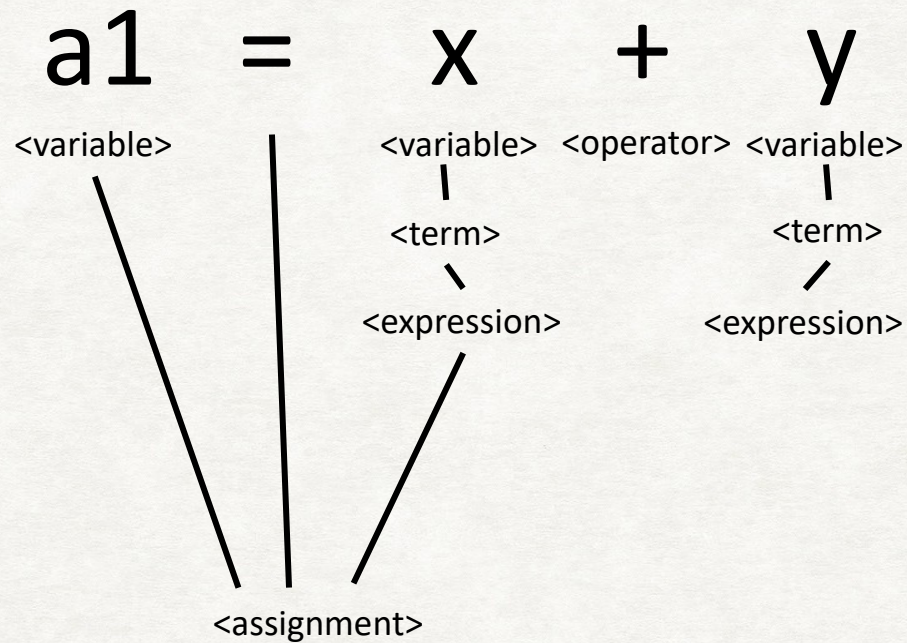


Could we have done this differently?

```
<variable> ::= <symbol>
<operator> ::= + | - | * | /
<term> ::= <number> | <variable>
<expression> ::= <term> | <expression> <operator> <expression>
<assignment> ::= <variable> = <expression>
```


Try again: Simple assignment statement

Yes we could have!



$\langle \text{variable} \rangle ::= \langle \text{symbol} \rangle$

$\langle \text{operator} \rangle ::= + \mid - \mid * \mid /$

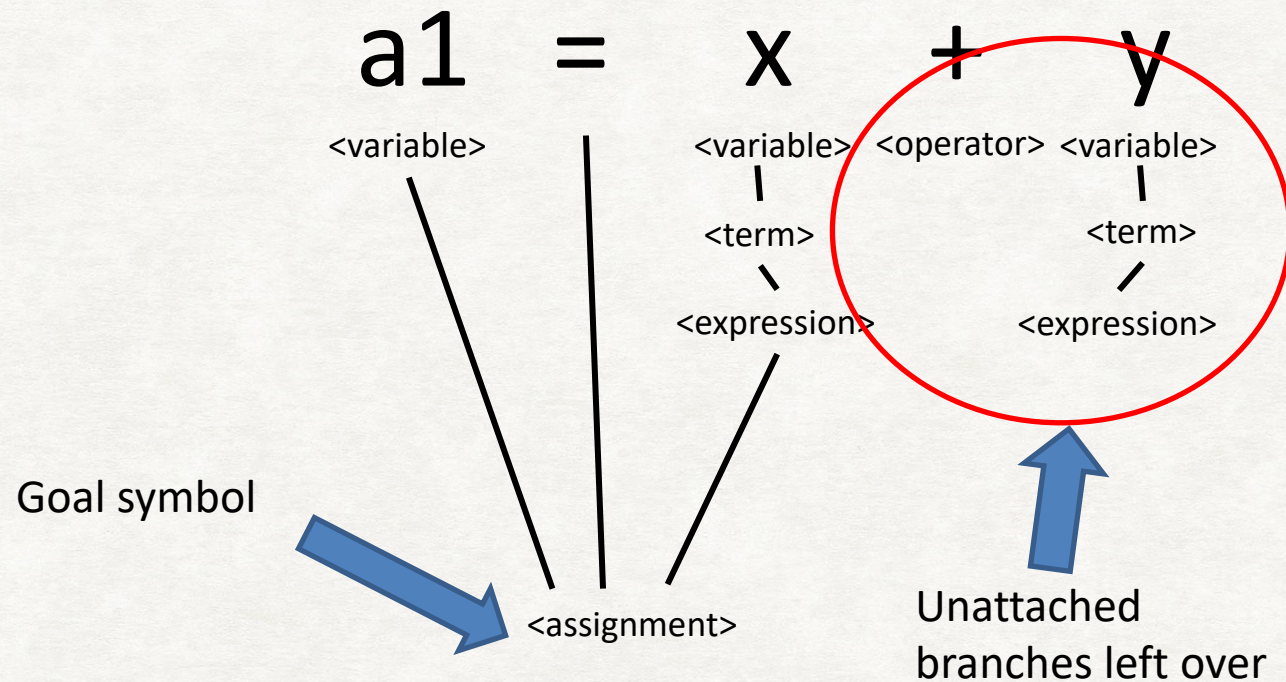
$\langle \text{term} \rangle ::= \langle \text{number} \rangle \mid \langle \text{variable} \rangle$

$\langle \text{expression} \rangle ::= \langle \text{term} \rangle \mid \langle \text{expression} \rangle \langle \text{operator} \rangle \langle \text{expression} \rangle$

$\langle \text{assignment} \rangle ::= \langle \text{variable} \rangle = \langle \text{expression} \rangle$

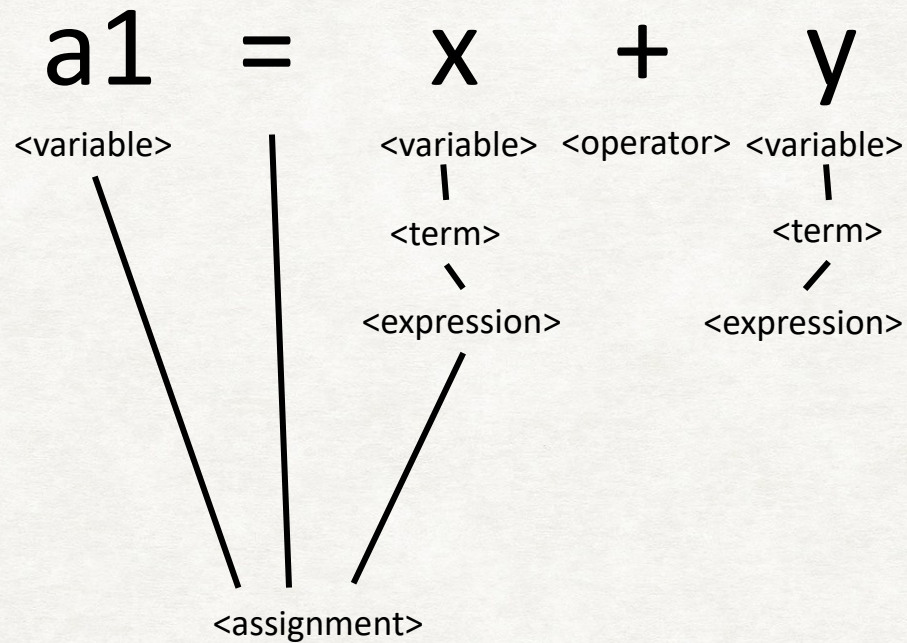
Try again: Simple assignment statement

But it doesn't work out!



```
<variable> ::= <symbol>
<operator> ::= + | - | * | /
<term> ::= <number> | <variable>
<expression> ::= <term> | <expression> <operator> <expression>
<assignment> ::= <variable> = <expression>
```


Try again: Simple assignment statement



The parser can detect this condition.

What does it do?

The parser will **backtrack** (undo the production) and try to match another grammar rule.

```
<variable> ::= <symbol>
<operator> ::= + | - | * | /
<term> ::= <number> | <variable>
<expression> ::= <term> | <expression> <operator> <expression>
<assignment> ::= <variable> = <expression>
```


PARSING EXAMPLE 2

A more complex assignment statement

a1 = x + y * z

<variable> ::= <symbol>

<operator> ::= + | - | * | /

<term> ::= <number> | <variable>

<expression> ::= <term> | <expression><operator><expression>

<assignment> ::= <variable>=<expression>

A more complex assignment statement

a1	=	x	+	y	*	z
<variable>		<variable>	<operator>	<variable>	<operator>	<variable>

<variable> ::= <symbol>

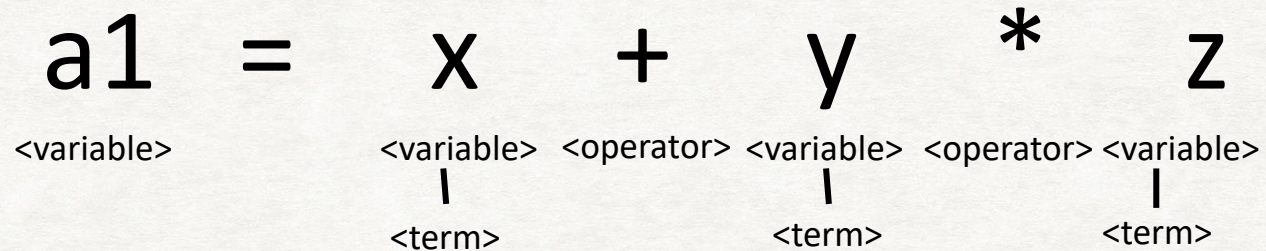
<operator> ::= + | - | * | /

<term> ::= <number> | <variable>

<expression> ::= <term> | <expression><operator><expression>

<assignment> ::= <variable> = <expression>

A more complex assignment statement



`<variable> ::= <symbol>`

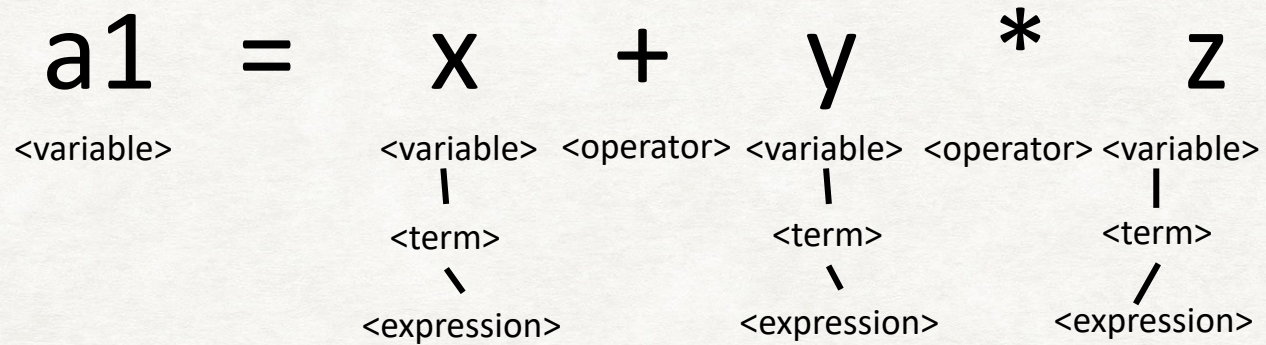
`<operator> ::= + | - | * | /`

`<term> ::= <number> | <variable>`

`<expression> ::= <term> | <expression> <operator> <expression>`

`<assignment> ::= <variable> = <expression>`

A more complex assignment statement



`<variable> ::= <symbol>`

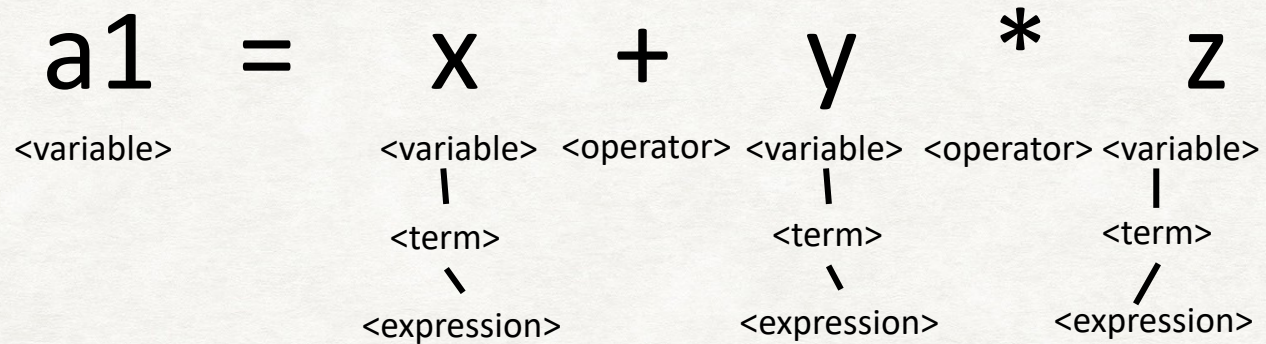
`<operator> ::= + | - | * | /`

`<term> ::= <number> | <variable>`

`<expression> ::= <term> | <expression> <operator> <expression>`

`<assignment> ::= <variable> = <expression>`

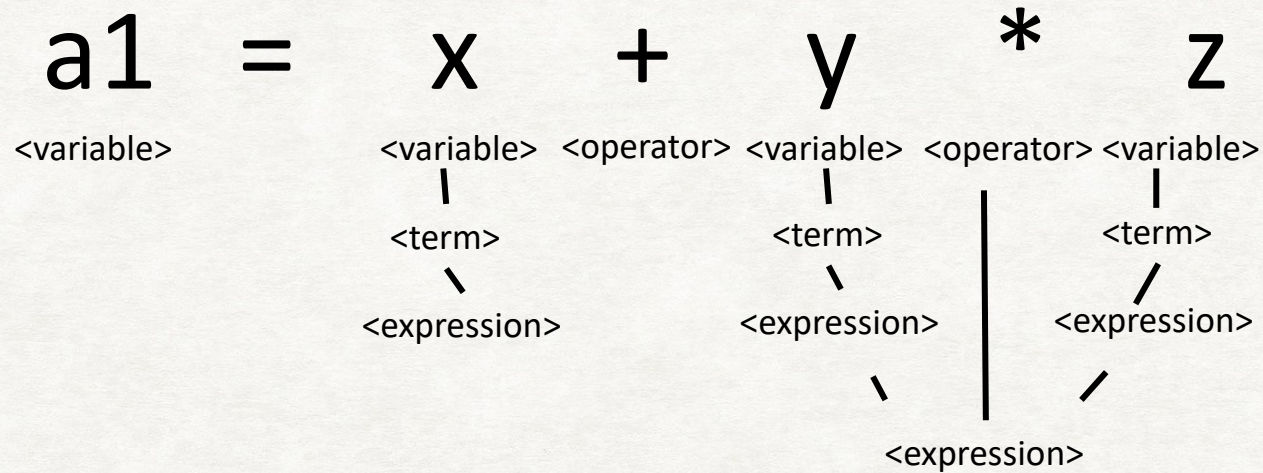
A more complex assignment statement



We now have
two ways in
which we can
proceed:

```
<variable> ::= <symbol>
<operator> ::= + | - | * | /
<term> ::= <number> | <variable>
<expression> ::= <term> | <expression> <operator> <expression>
<assignment> ::= <variable> = <expression>
```


A more complex assignment statement



We now have
two ways in
which we can
proceed:
1. Right to Left

<variable> ::= <symbol>

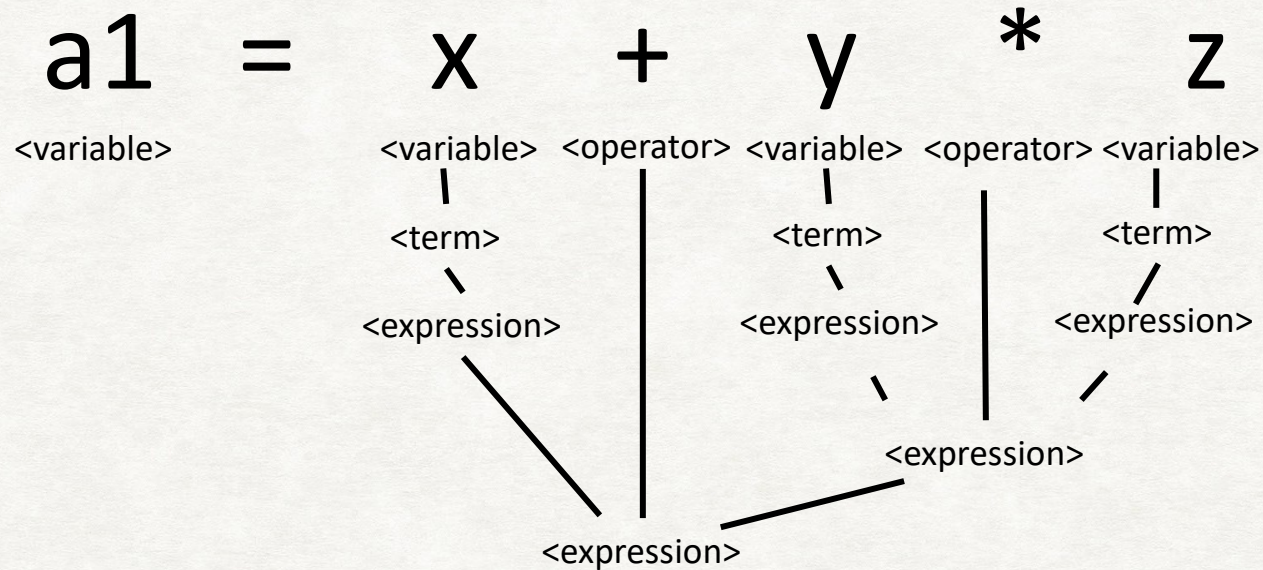
<operator> ::= + | - | * | /

<term> ::= <number> | <variable>

<expression> ::= <term> | <expression> <operator> <expression>

<assignment> ::= <variable> = <expression>

A more complex assignment statement



We now have
two ways in
which we can
proceed:
1. Right to Left

`<variable> ::= <symbol>`

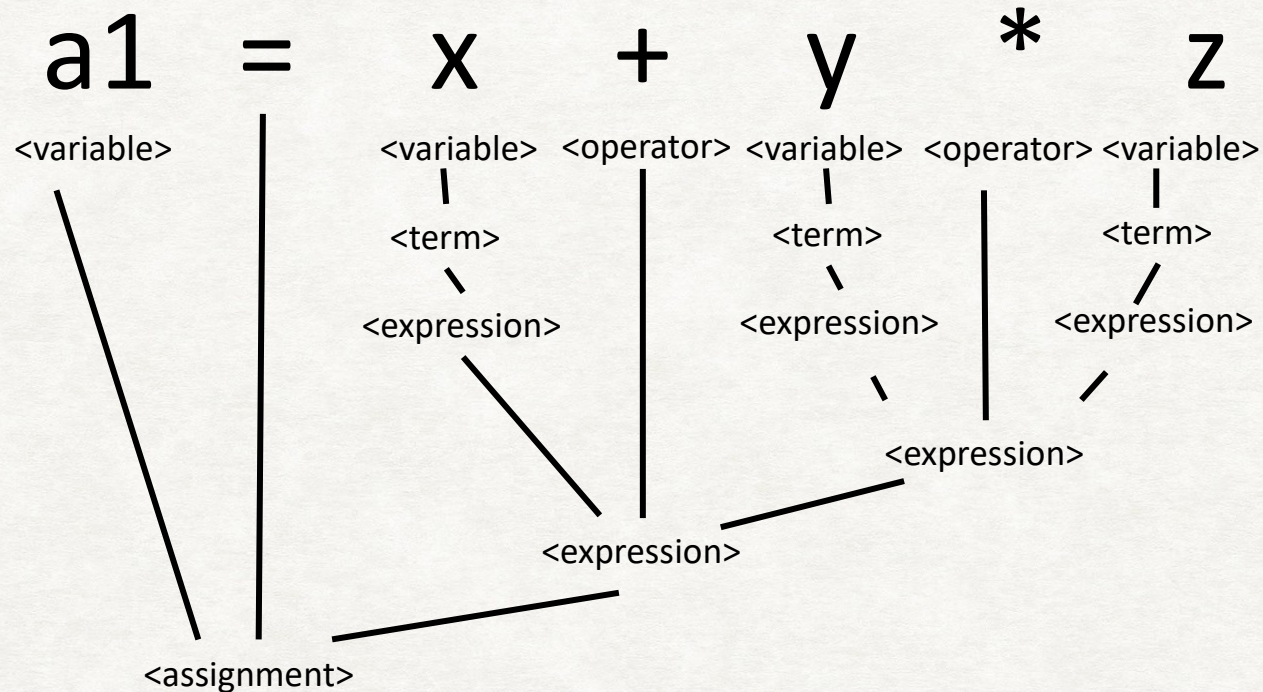
`<operator> ::= + | - | * | /`

`<term> ::= <number> | <variable>`

`<expression> ::= <term> | <expression><operator><expression>`

`<assignment> ::= <variable> = <expression>`

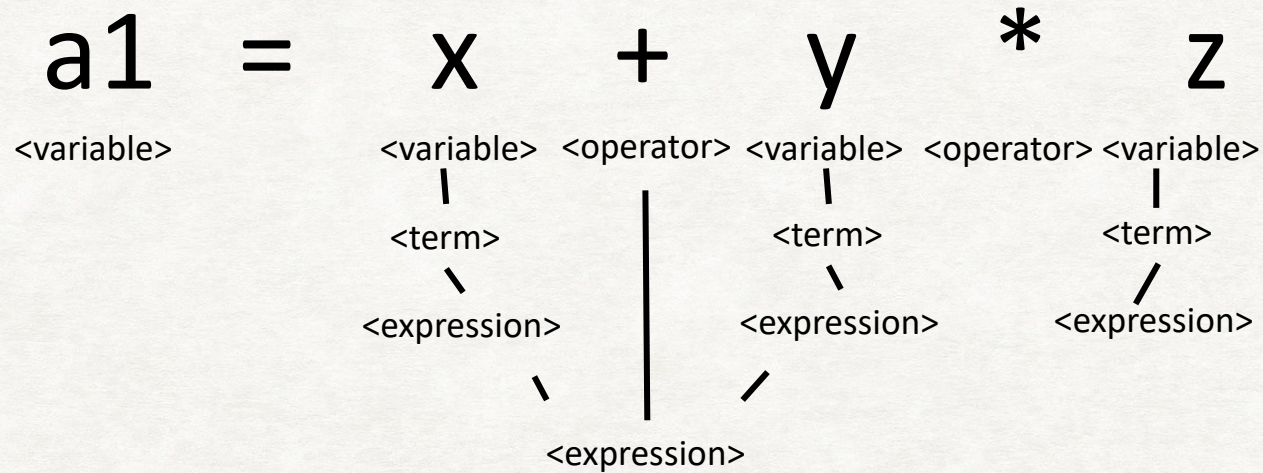
A more complex assignment statement



We now have
two ways in
which we can
proceed:
1. Right to Left

```
<variable> ::= <symbol>
<operator> ::= + | - | * | /
<term> ::= <number> | <variable>
<expression> ::= <term> | <expression> <operator> <expression>
<assignment> ::= <variable> = <expression>
```


A more complex assignment statement



We now have
two ways in
which we can
proceed:
2. Left to Right

<variable> ::= <symbol>

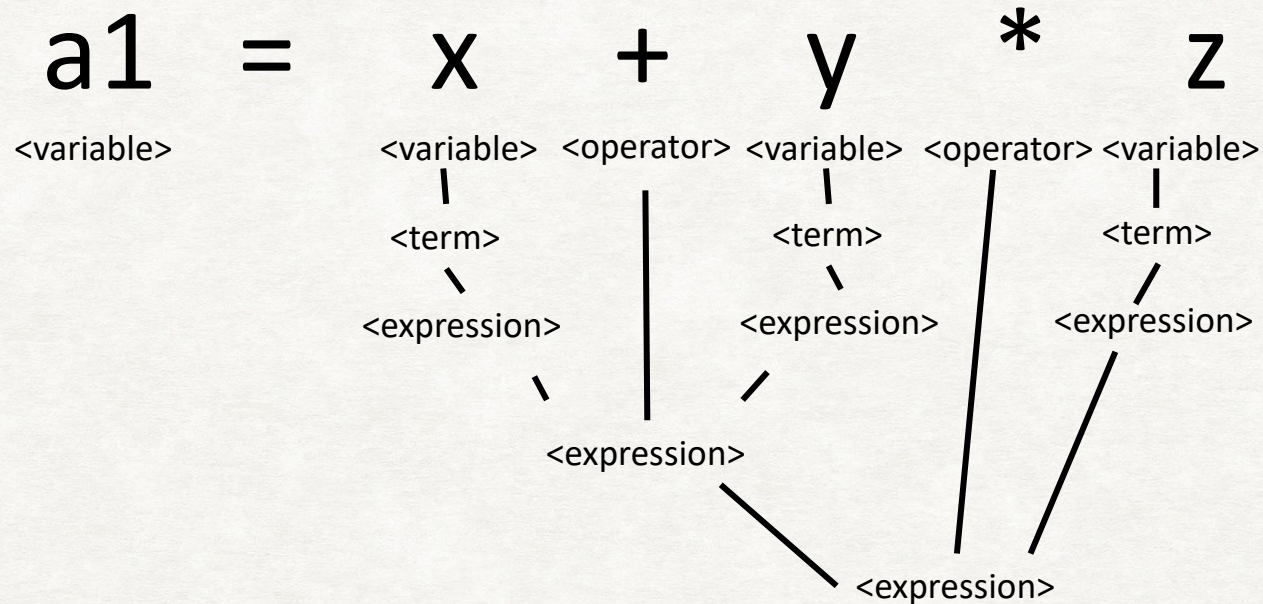
<operator> ::= + | - | * | /

<term> ::= <number> | <variable>

<expression> ::= <term> | <expression> <operator> <expression>

<assignment> ::= <variable> = <expression>

A more complex assignment statement



We now have
two ways in
which we can
proceed:
2. Left to Right

<variable> ::= <symbol>

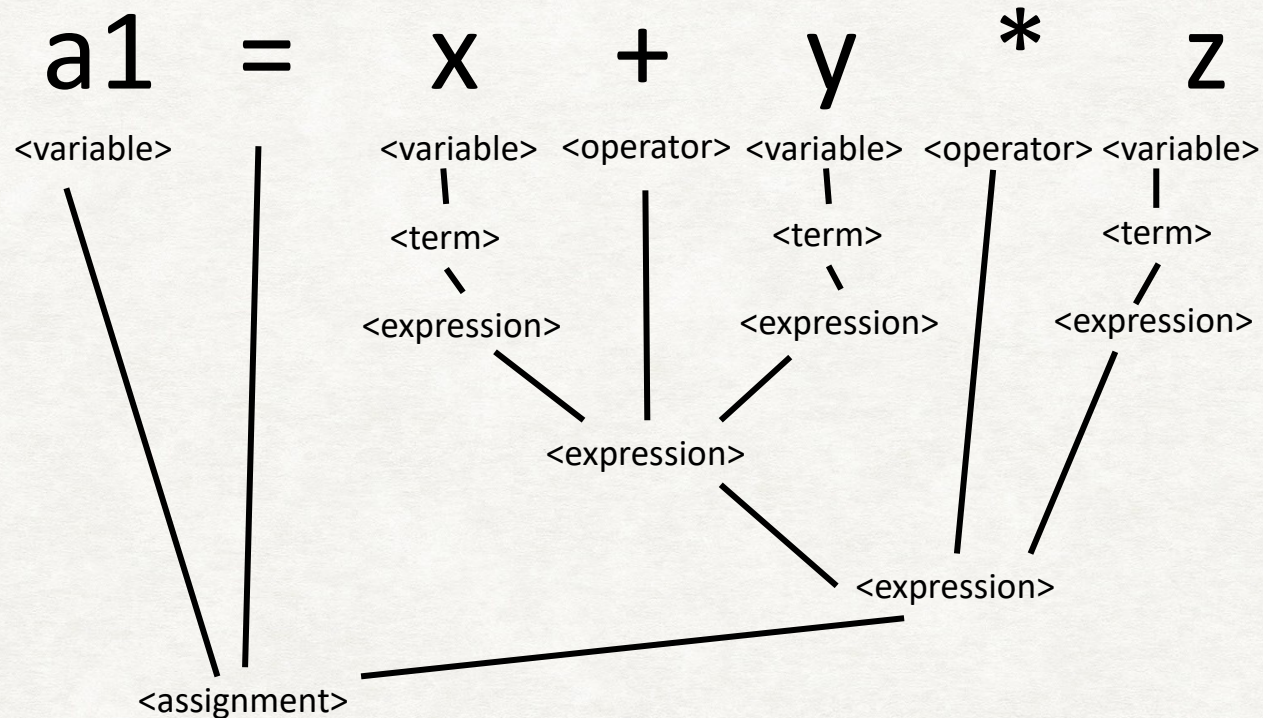
<operator> ::= + | - | * | /

<term> ::= <number> | <variable>

<expression> ::= <term> | <expression> <operator> <expression>

<assignment> ::= <variable> = <expression>

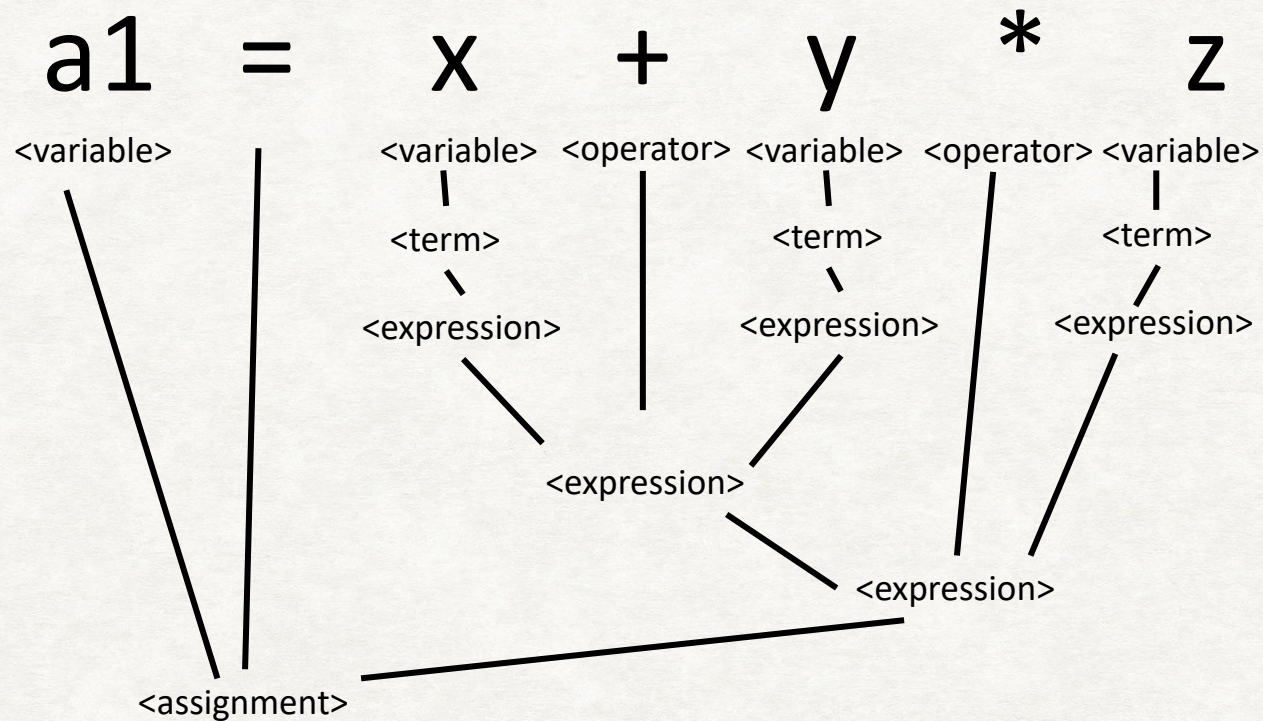
A more complex assignment statement



We now have
two ways in
which we can
proceed:
2. Left to Right

`<variable> ::= <symbol>`
`<operator> ::= + | - | * | /`
`<term> ::= <number> | <variable>`
`<expression> ::= <term> | <expression> <operator> <expression>`
`<assignment> ::= <variable> = <expression>`

A more complex assignment statement



But Option 2
gives the
wrong result!

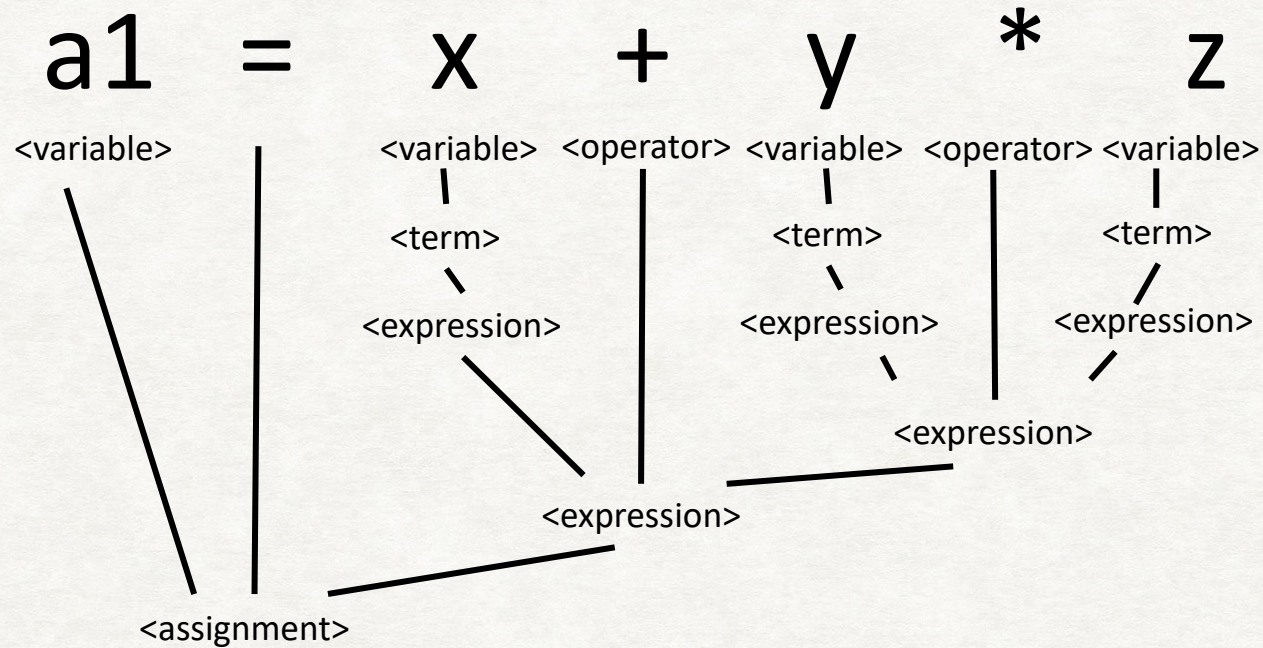
It's equivalent
to $(x+y)^*z$.

```

<variable> ::= <symbol>
<operator> ::= + | - | * | /
<term> ::= <number> | <variable>
<expression> ::= <term> | <expression> <operator> <expression>
<assignment> ::= <variable> = <expression>

```


A more complex assignment statement



Option 1 does what we want.

But our grammar gives no preference to either option 1 or 2.

It is *ambiguous*.

```
<variable> ::= <symbol>
<operator> ::= + | - | * | /
<term> ::= <number> | <variable>
<expression> ::= <term> | <expression> <operator> <expression>
<assignment> ::= <variable> = <expression>
```

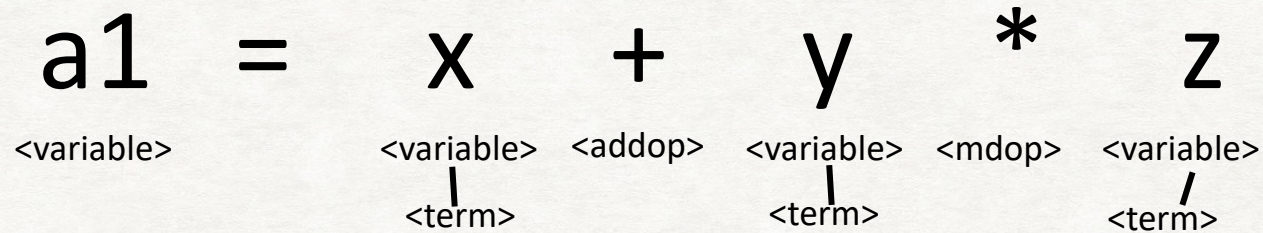

A grammar for multiplication before addition

a1 = x + y * z

The problem
can be fixed by
changing the
grammar to
make it
unambiguous

```
<variable> ::= <symbol>
<addop> ::= +|-
<mdop> ::= *|/
<term> ::= <number>|<variable>
<factor> ::= <term>|<product>|(<sum>)
<product> ::= <factor><mdop><factor>
<summand> ::= <term>|<product>|<sum>
<sum> ::= <summand><addop><summand>
<expression> ::= <term>|<sum>|<product>
<assignment> ::= <variable>=<expression>
```


A grammar for multiplication before addition



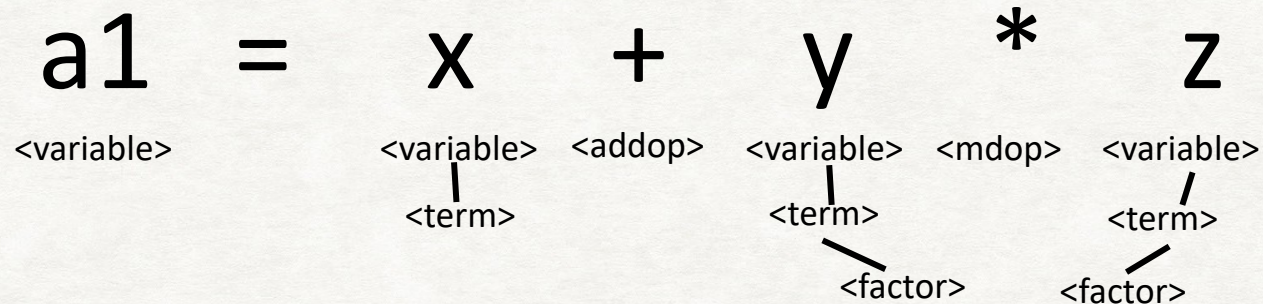
The problem
can be fixed by
changing the
grammar to
make it
unambiguous

```

<variable> ::= <symbol>
<addop> ::= +|-
<mdop> ::= *|/
<term> ::= <number>|<variable>
<factor> ::= <term>|<product>|(<sum>)
<product> ::= <factor><mdop><factor>
<summand> ::= <term>|<product>|<sum>
<sum> ::= <summand><addop><summand>
<expression> ::= <term>|<sum>|<product>
<assignment> ::= <variable>=<expression>

```


A grammar for multiplication before addition



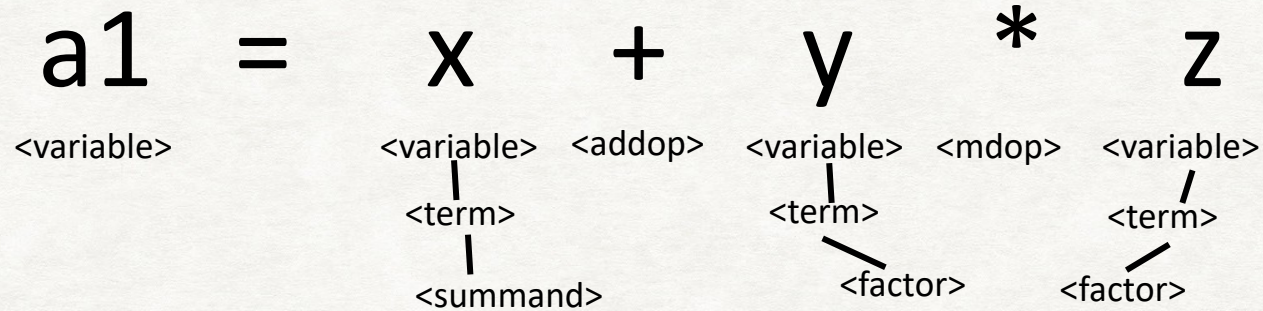
The problem
can be fixed by
changing the
grammar to
make it
unambiguous

```

<variable> ::= <symbol>
<addop> ::= +|-
<mdop> ::= *|/
<term> ::= <number>|<variable>
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<product> ::= <factor><mdop><factor>
<summand> ::= <term>|<product>|<sum>
<sum> ::= <summand><addop><summand>
<expression> ::= <term>|<sum>|<product>
<assignment> ::= <variable>=<expression>

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A grammar for multiplication before addition



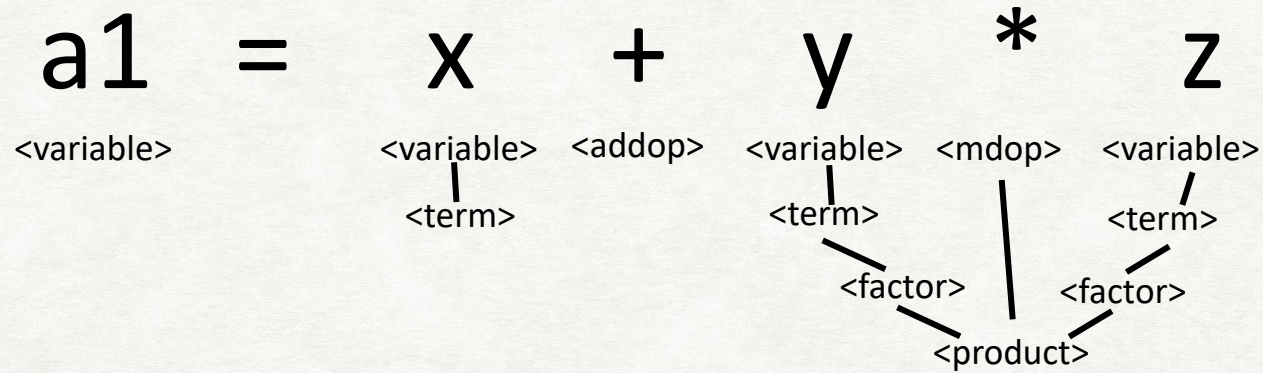
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A grammar for multiplication before addition



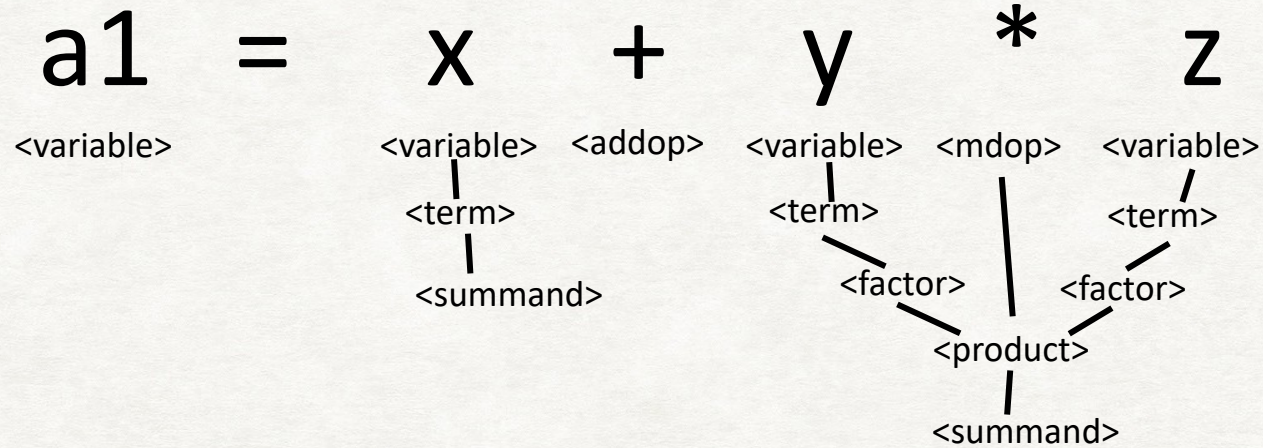
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A grammar for multiplication before addition



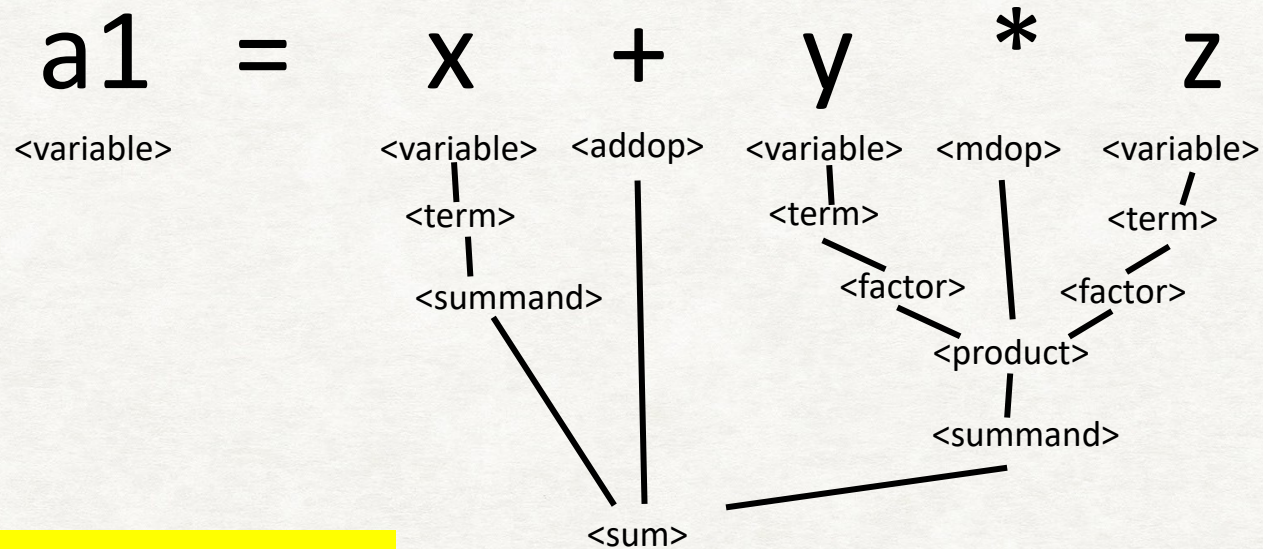
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A grammar for multiplication before addition

[illegible]

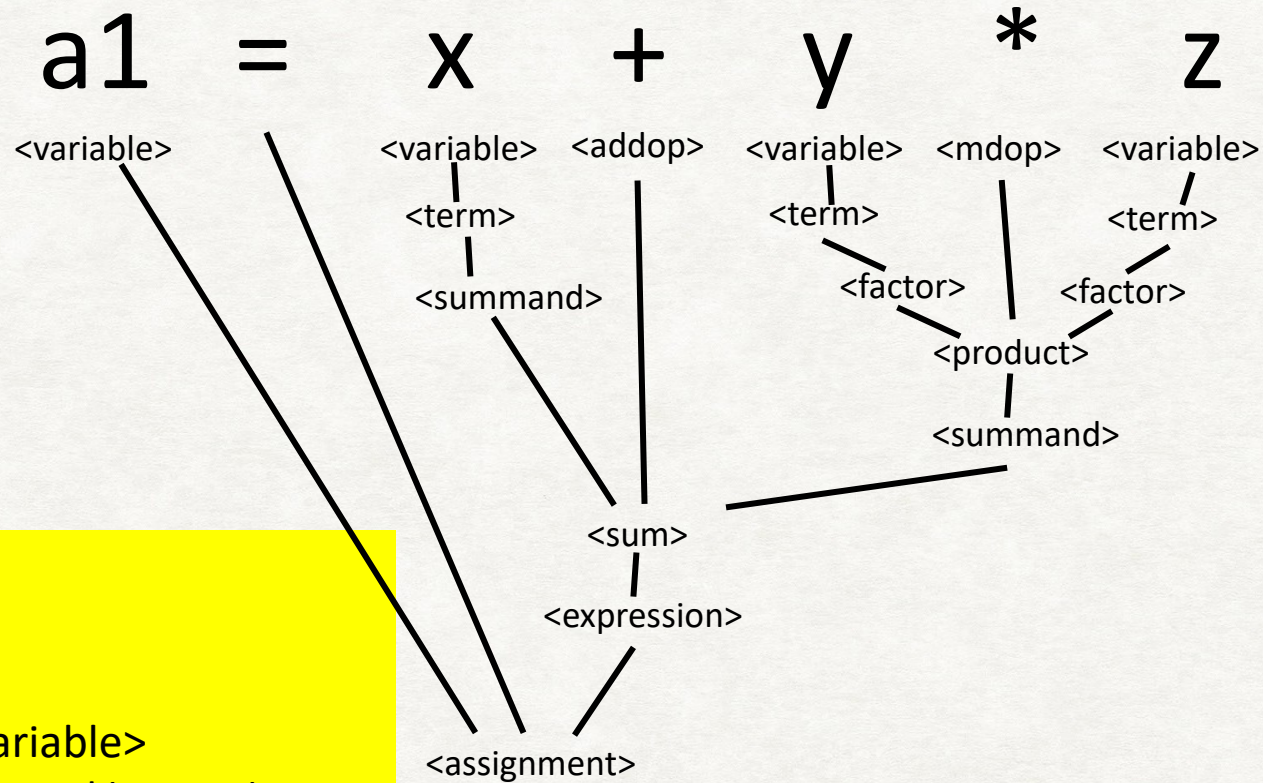
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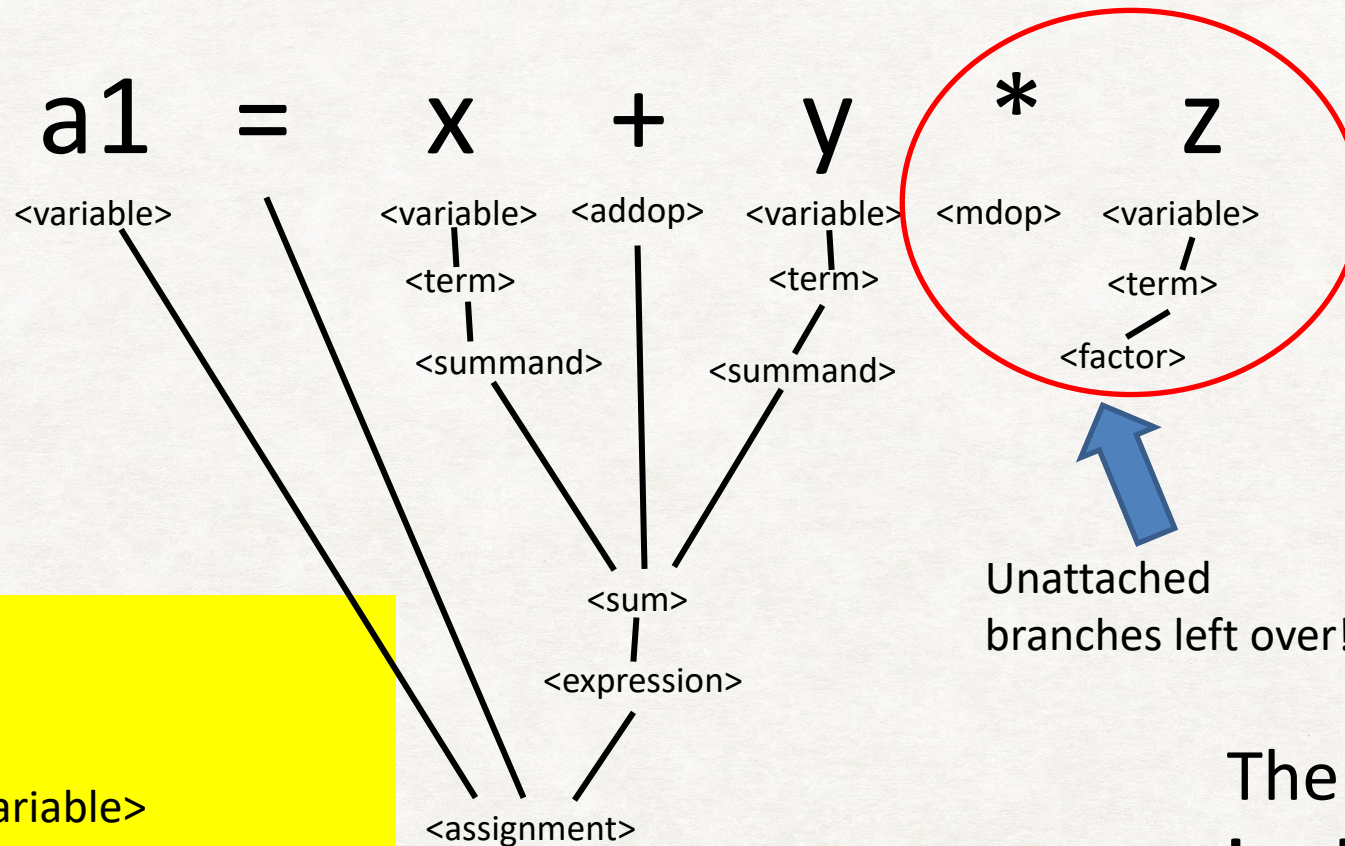

A grammar for multiplication before addition



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A grammar for multiplication before addition



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<addop> ::= + | -
<mdop> ::= * | /
<term> ::= <number> | <variable>
<factor> ::= <term> | <product> | (<sum>)
<product> ::= <factor> <mdop> <factor>
<summand> ::= <term> | <product> | <sum>
<sum> ::= <summand> <addop> <summand>
<expression> ::= <term> | <sum> | <product>
<assignment> ::= <variable> = <expression>
```

Unattached
branches left over!

The parser will **backtrack** (undo the last production) and try to match another grammar rule.

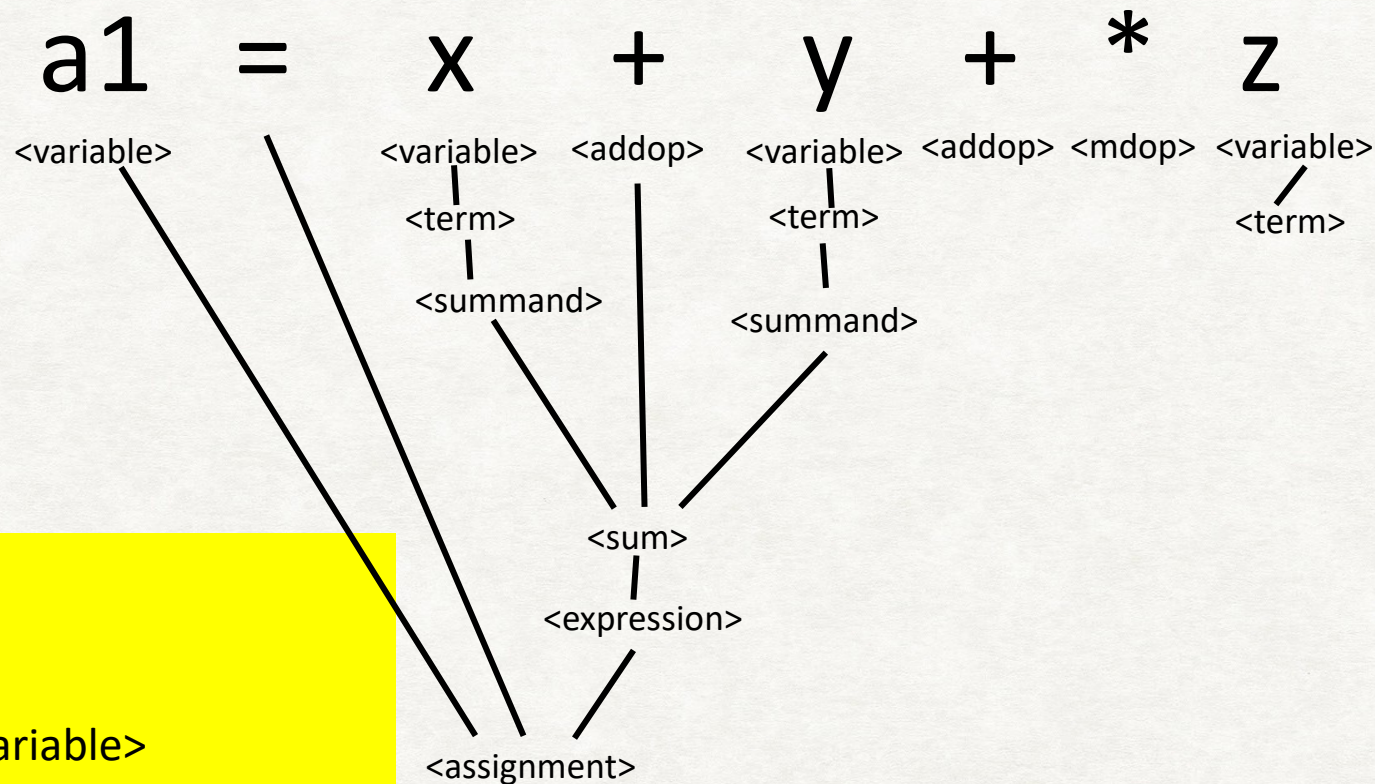
Let's try parsing another statement.

a1 = x + y + * z

Can't parse this: No rule in this grammar that allows for any component of an assignment statement to have a + and a * operator next to each other.

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<variable> ::= <symbol>
<addop> ::= +|-
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```


Let's try parsing another statement.



Can't parse this: No rule in this grammar that allows for any component of an assignment statement to have a + and a * operator next to each other.

Note: In a different grammar, there could be such a rule.

E.g., the statement above is syntactically correct in C language!

```
<variable> ::= <symbol>
<addop> ::= + | -
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<term> ::= <number> | <variable>
<factor> ::= <term> | <product> | (<sum>)
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<sum> ::= <summand> <addop> <summand>
<expression> ::= <term> | <sum> | <product>
<assignment> ::= <variable> = <expression>
```


What have we learnt?

If at first we don't succeed in building a parse tree, backtrack and try again with a different combination of rules from the grammar.

If a parser cannot build a parse tree for a given input and grammar, we have a syntax error.

Grammars *can* be ambiguous – different parse trees are possible depending on the parser implementation!

In programming languages, grammars *must not* be ambiguous!

Things worth knowing:

Not all grammars use BNF. Some variations in the notation exist. You can find the grammars for a lot of modern high level programming languages in BNF or similar specifications.

For example:

Python: <https://docs.python.org/3/reference/grammar.html>

Java: <https://docs.oracle.com/javase/specs/jls/se11/html/jls-19.html>

C#: <https://docs.microsoft.com/en-us/dotnet/csharp/language-reference/language-specification/lexical-structure>

C++ (gcc): <http://www.nongnu.org/hcb/>

[Not a programming language but still worth having a look at.](#)

PHP: https://github.com/php/php-src/blob/master/Zend/zend_language_parser.y

Ruby: <https://ruby-doc.org/docs/ruby-doc-bundle/Manual/man-1.4/yacc.html>

A quick note on grammars

Just because our input is grammatically correct in syntax, it doesn't mean that it makes sense.

Think English: "Students cannot melt out of musical wiring watermelons" is grammatical but makes absolutely no sense.

In programming languages, we may get code that is grammatically correct, but still wrong:

- Variable names refer to variables that do not exist.

- Function / method names are correct but the number / type of parameters don't match the function / method declaration.

- Variables are of the wrong type. E.g., we try to multiply with a string of letters.

These problems cannot be addressed with a grammar.