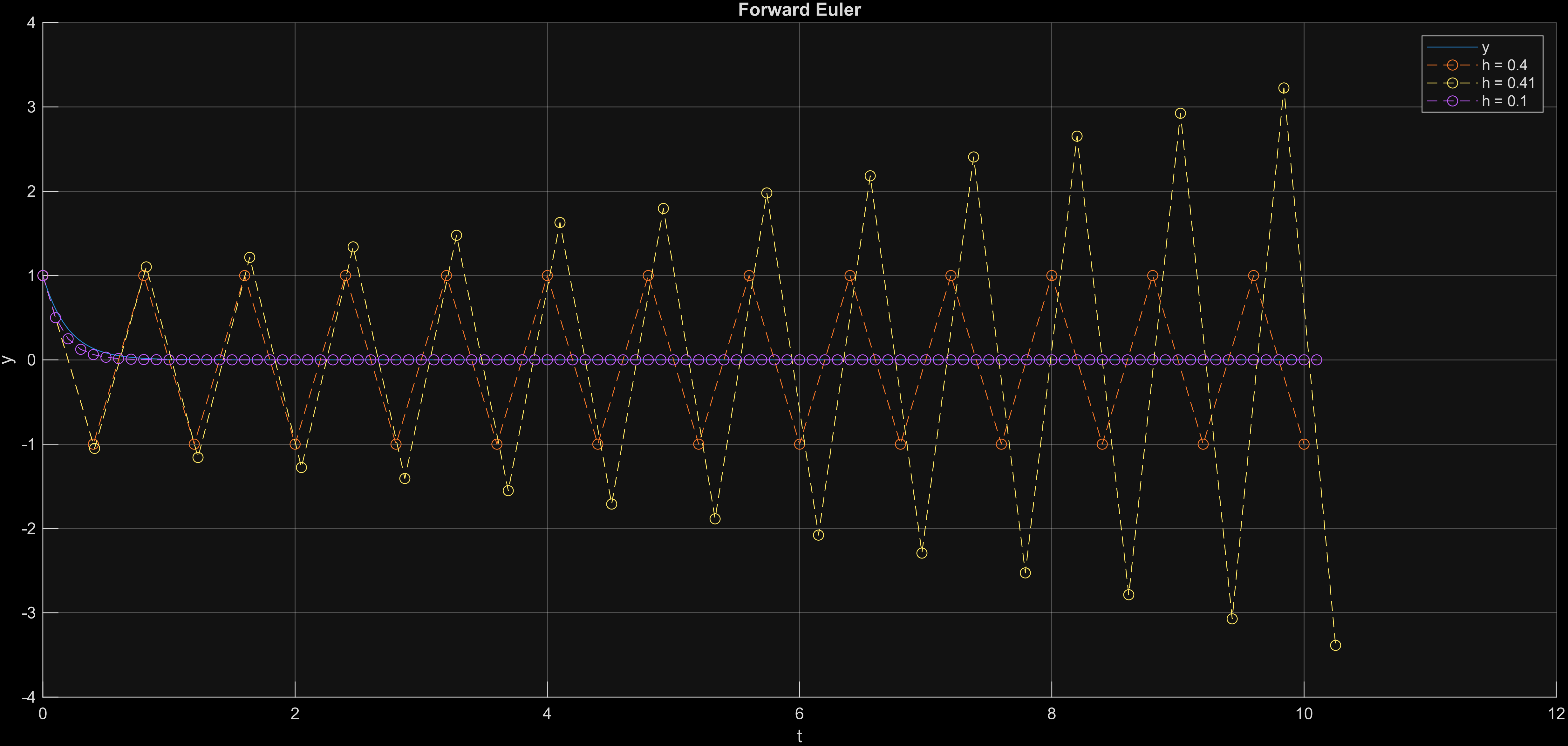


Forward Euler
y
h = 0.4
h = 0.41
h = 0.1
4
3
2
1
0
-1
-2
-3
-4
y
0
2
4
6
8
10
12
t

$y' = -5y$, $y(0) = 1$. $y = e^{-5t}$.

let $N = \lceil \frac{10}{h} \rceil$.

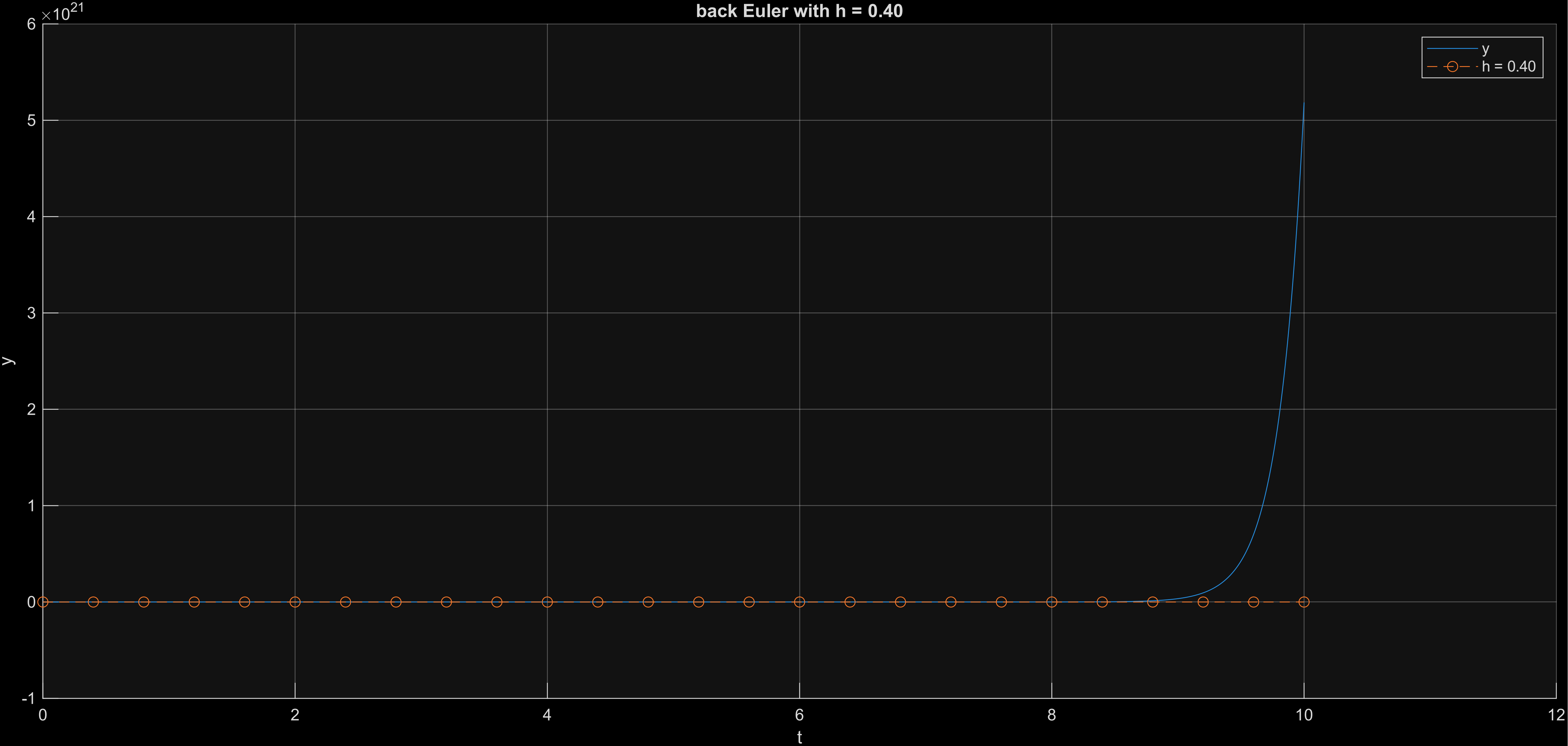
By forward Euler method, $u_{n+1} = u_n + h(-5)u_n \quad \forall n \in [0, N]$.

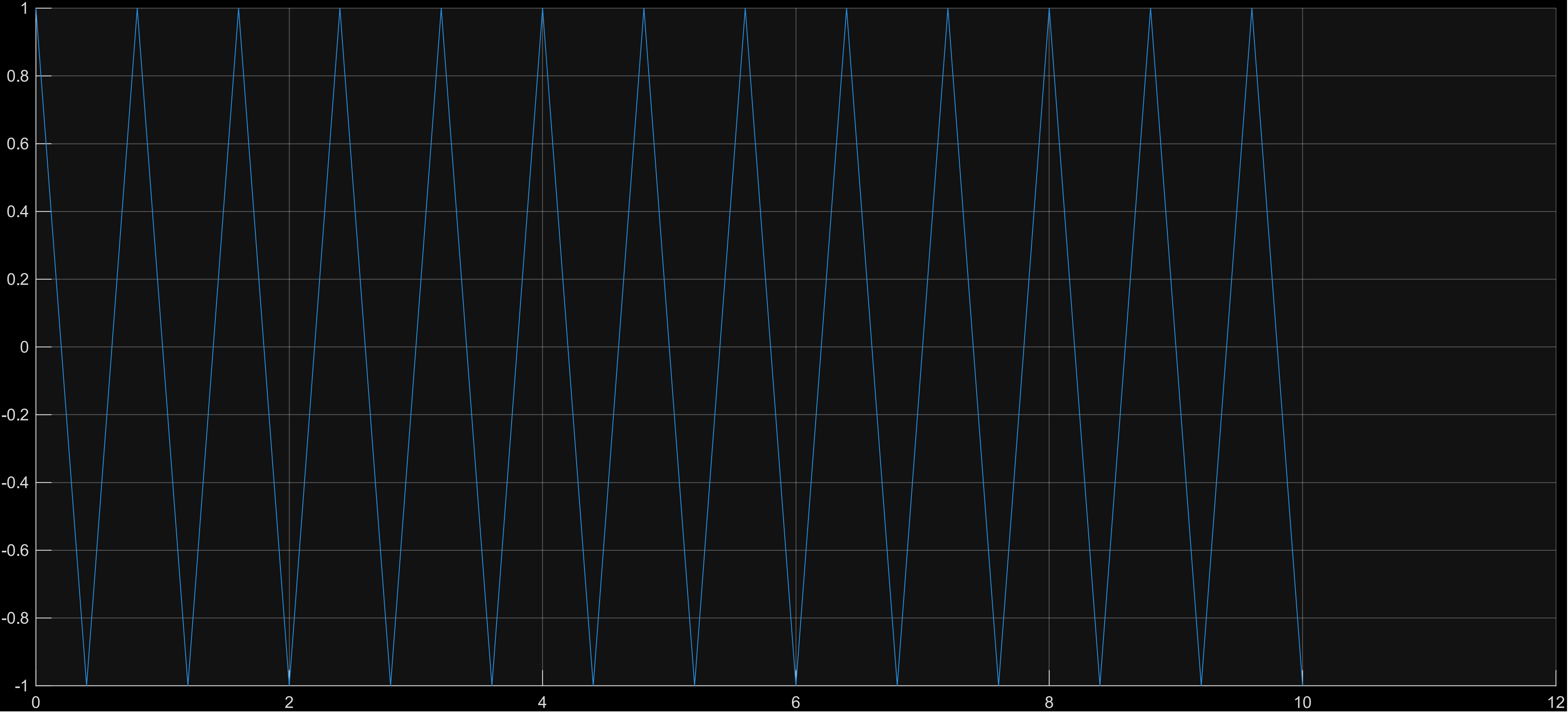
Thus $u_{n+1} = (1-5h)^n u_0 = (1-5h)^n$.

Hence when $|1-5h| < 1$ and $h > 0$, i.e., $0 < h < \frac{2}{5}$, u_{n+1} will approach y.

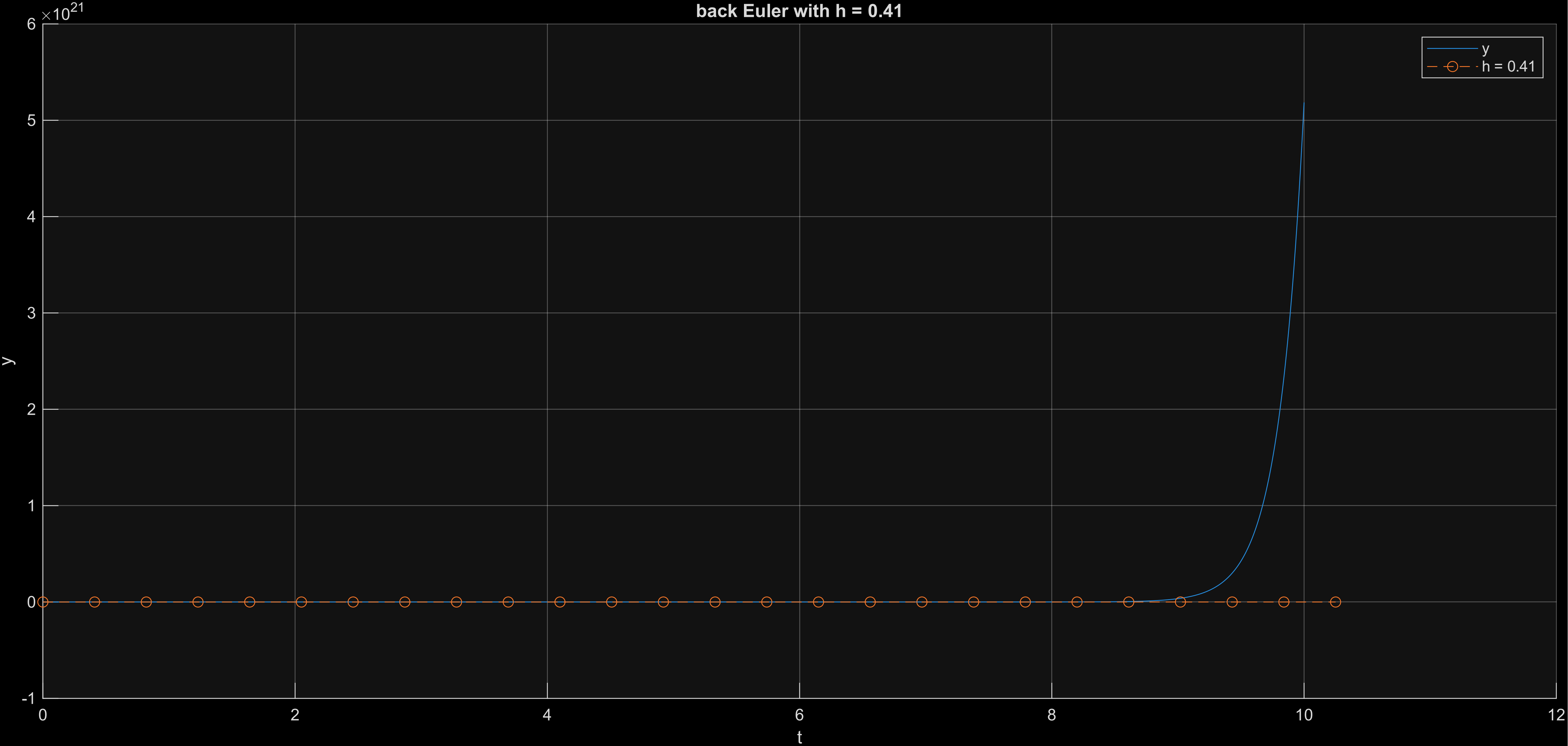
when $|1-5h| = 1$ and $h > 0$, i.e., $h = \frac{2}{5}$, $u_{n+1} = (-1)^n$.

when $|1-5h| > 1$ and $h > 0$, i.e., $h > \frac{2}{5}$, u_{n+1} diverges from y.

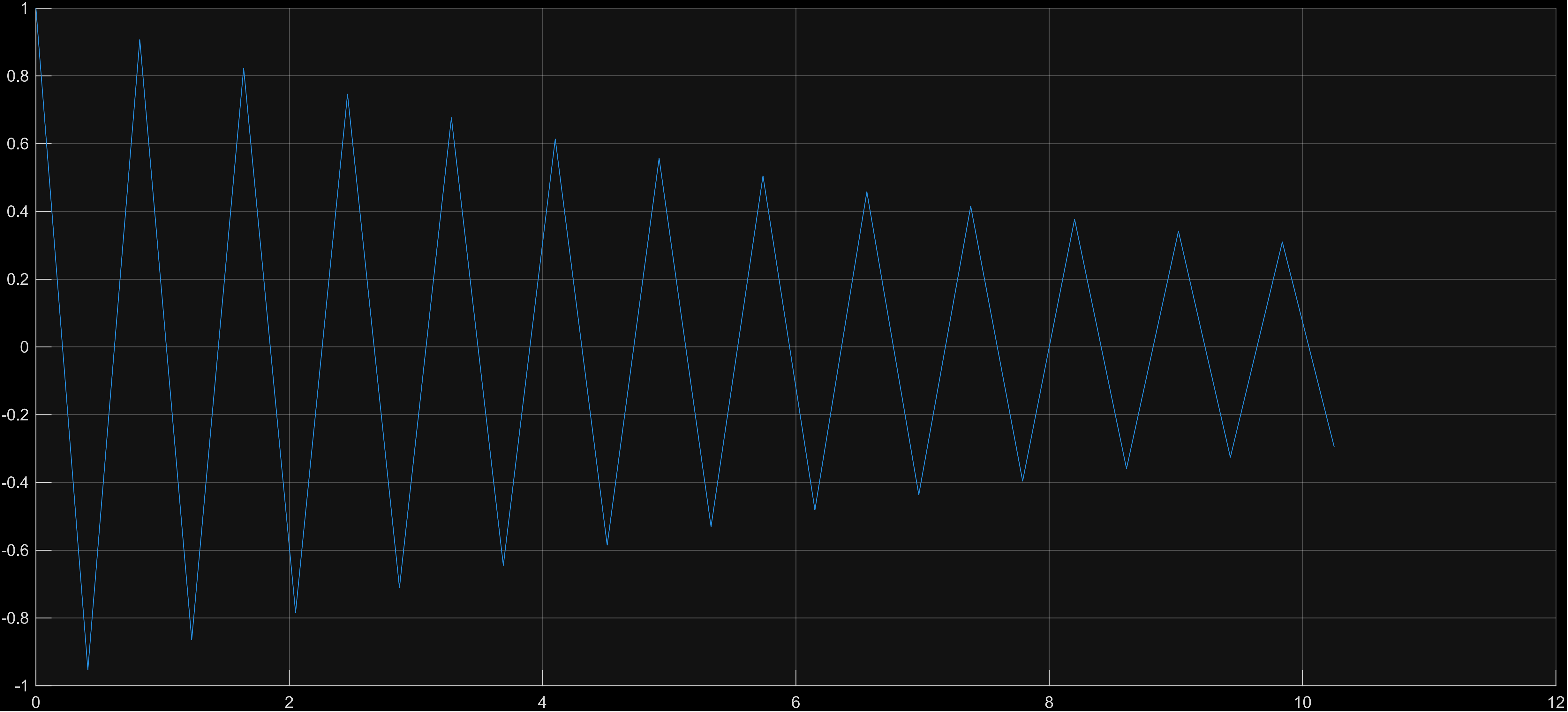

back Euler with h = 0.40
$\times 10^{21}$
6
5
4
3
2
1
0
-1
y
0
2
4
6
8
10
12
t
y
h = 0.40



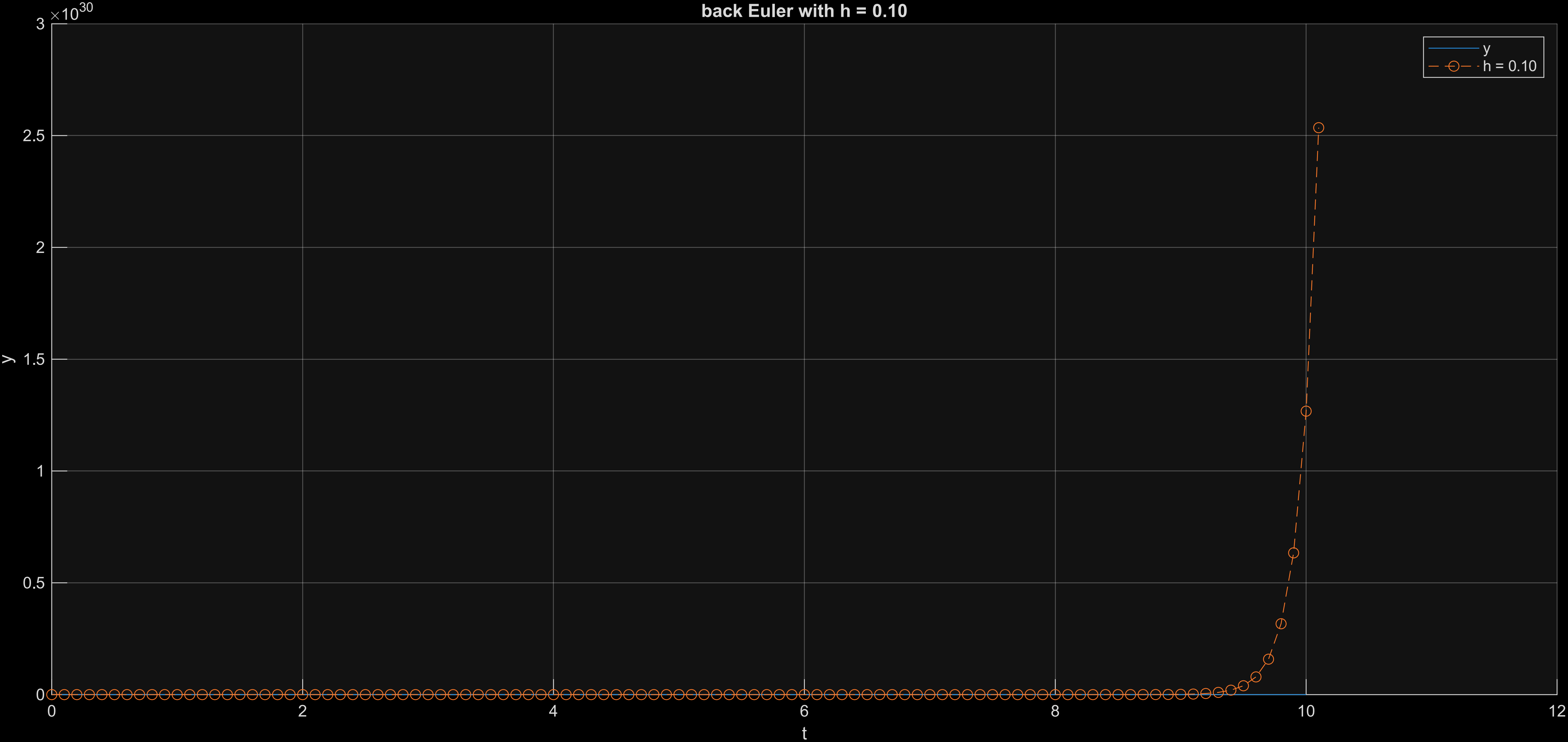
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
2
4
6
8
10
12



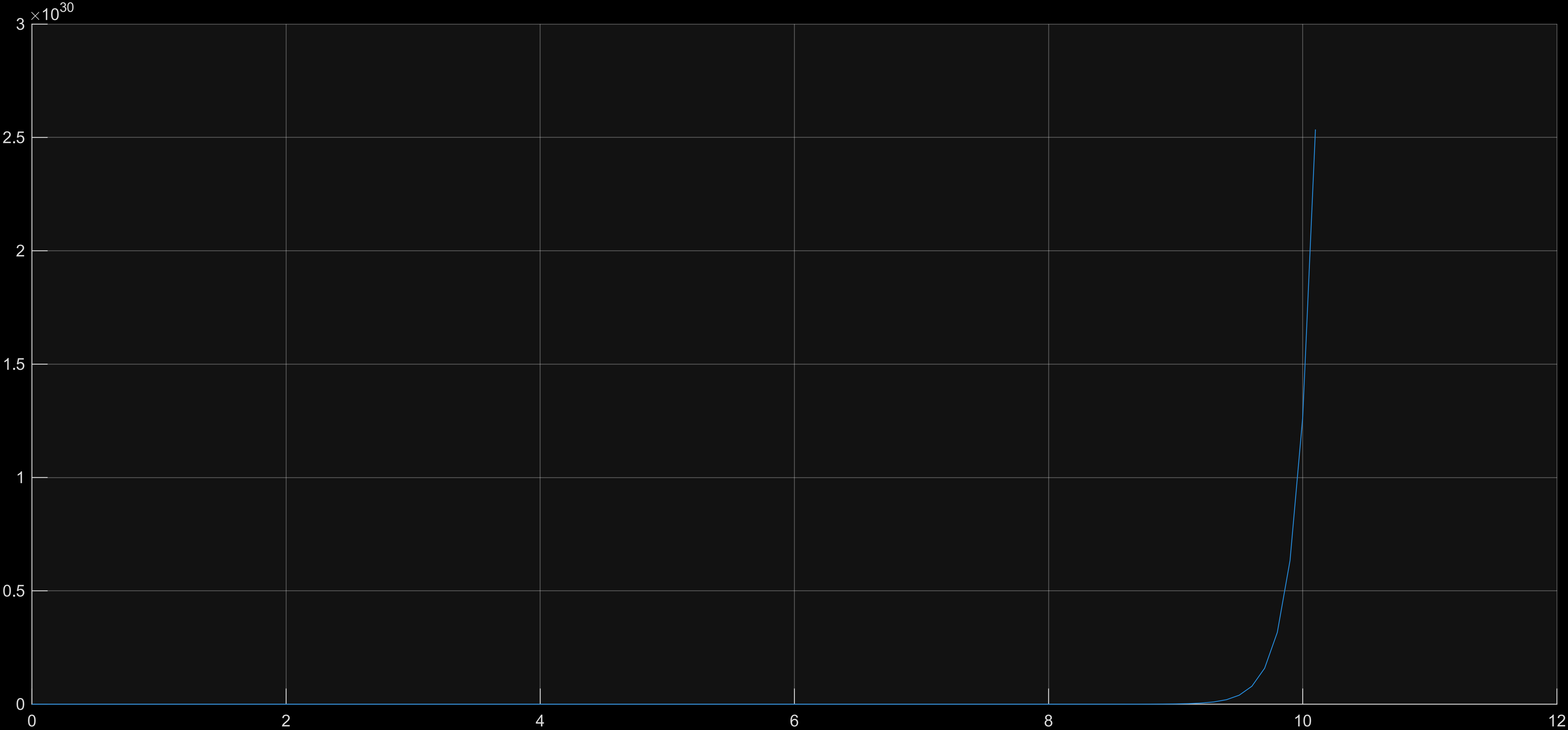
back Euler with h = 0.41
$\times 10^{21}$
6
5
4
3
2
1
0
-1
y
0
2
4
6
8
10
12
t
y
h = 0.41



1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
2
4
6
8
10
12



back Euler with h = 0.10
×10^30
3
2.5
2
1.5
1
0.5
0
y
0
2
4
6
8
10
12
t
y
h = 0.10


×10^30
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
12

$y' = 5y$, $y(0) = 1$, $y = e^{5t}$

let $N = \left\lceil \frac{10}{h} \right\rceil$.

By backward Euler method, $U_{n+1} = U_n + h(5)U_{n+1}$

$$\Rightarrow (1-5h)U_{n+1} = U_n$$

$$\Rightarrow U_{n+1} = \frac{U_n}{1-5h}$$

Thus $U_{n+1} = \frac{U_0}{(1-5h)^n} = \frac{1}{(1-5h)^n}$.

For $|1-5h| > 1$ and $h > 0$, i.e., $\frac{2}{5} < h$, $U_{n+1} = (-1)^n \cdot (5h-1)^{-n}$ oscillates and converges to 0.

For $|1-5h| = 1$ and $h > 0$, i.e., $\frac{2}{5} = h$, $U_{n+1} = (-1)^n$ oscillates between 1 and -1.

For $|1-5h| < 1$ and $h > 0$, i.e., $\frac{2}{5} > h$, $U_{n+1} = (1-5h)^{-n}$ diverges to infinity but can't approach to y since e^{5t} grows much faster than $(1-5h)^{-t}$ as $t \to \infty$.