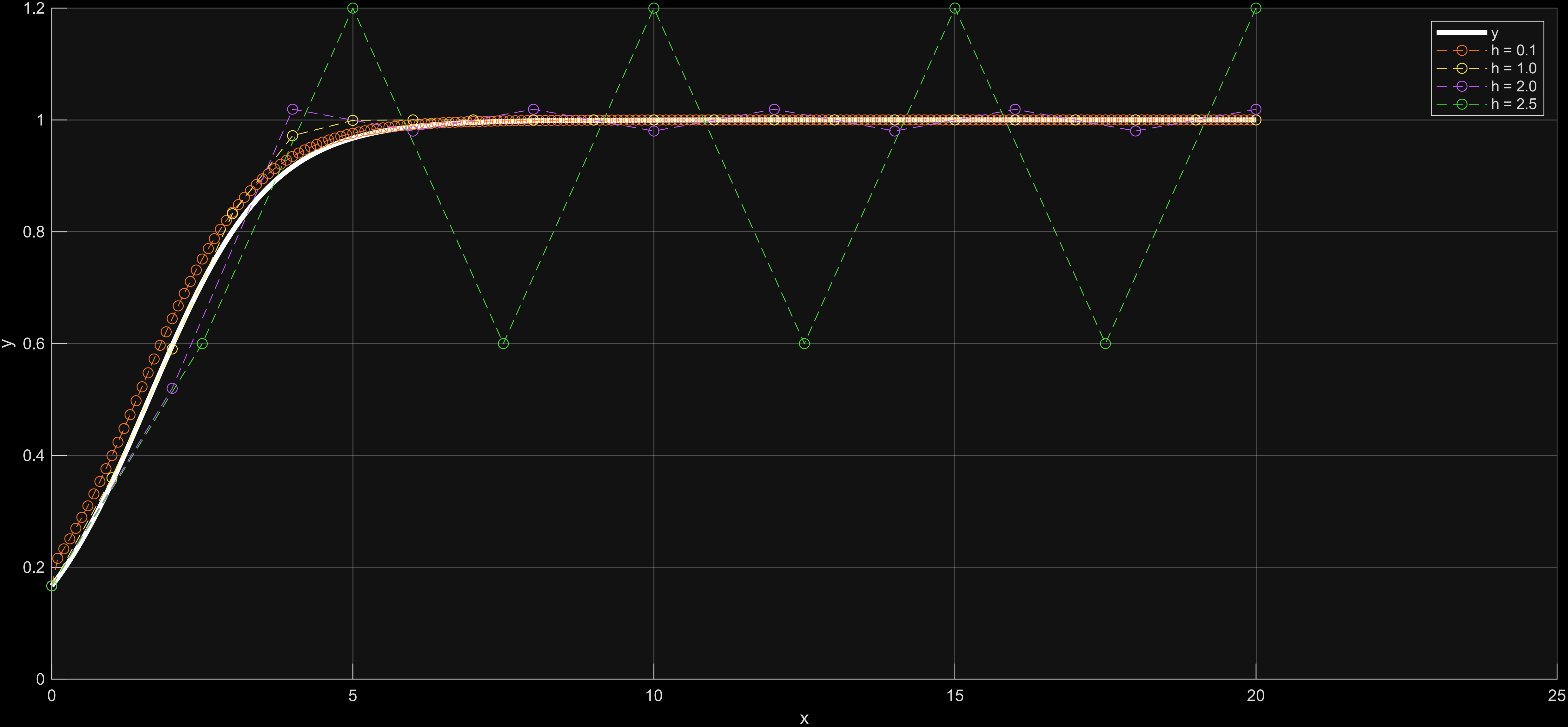

y
h = 0.1
h = 1.0
h = 2.0
h = 2.5
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
x
y

$y' = y(1-y)$, $y(0) = y_0$, $0 < y_0 < 1$

Forward Euler method : $U_{n+1} = U_n + h U_n (1 - U_n)$.

Equilibrium point : $U = U + hU(1-U) \Rightarrow U = 0$ or 1.

Since $y' > 0$ and $y_0 > 0$, the equilibrium point is 1.

We need to check that : ① $0 < U_{n+1} < 1$ for all n

② $U_{n+1} > U_n$ for all n

③ $U_n \to 1$

① $U_{n+1} = U_n + h(U_n)(1-U_n) < 1$

$\Rightarrow h(U_n)(1-U_n) < 1 - U_n$

$\Rightarrow h U_n < 1$

$\Rightarrow h < \frac{1}{U_n} \Rightarrow h < 1$

② $U_{n+1} = U_n + h(U_n)(1-U_n)$

Since $h > 0$ and $(U_n)(1-U_n) > 0$ when $0 < U_n < 1$, $U_{n+1} > U_n$.

③ Since $\{U_n\}$ is bounded above by 1 and is monotone increasing, it will converge to 1.