3. Prove that $\omega_{n+1}'(x_i) = \prod_{\substack{j=0 \\ j\neq i}}^{n} (x_i - x_j)$ where ω_{n+1} is the nodal polynomial (8.6).

Then, check (8.5).

Since $\lim_{t\to x_i} \dfrac{W_{n+1}(t) - W_{n+1}(x_i)}{t - x_i} = \lim_{t\to x_i} \dfrac{\prod_{j=0}^{n} (t - x_j) - \prod_{j=0}^{n} (x_i - x_j)}{t - x_i}$

$= \lim_{t\to x_i} \dfrac{\prod_{j=0}^{n} (t - x_j)}{t - x_i} = \lim_{t\to x_i} \prod_{j=0,\, j\neq i}^{n} (t - x_j) = \prod_{j=0,\, j\neq i}^{n} (x_i - x_j)$ exists.

We have $W_{n+1}'(x_i) = \lim_{t\to x_i} \dfrac{W_{n+1}(t) - W_{n+1}(x_i)}{t - x_i} = \prod_{j=0,\, j\neq i}^{n} (x_i - x_j)$.

$$\Pi_n(x) = \sum_{i=0}^{n} y_i l_i(x) = \sum_{i=0}^{n} y_i \prod_{j=0,\, j\neq i}^{n} \frac{x - x_j}{x_i - x_j} = \sum_{i=0}^{n} y_i \frac{\prod_{j=0}^{n} (x - x_j)}{(x - x_i)} \cdot \frac{1}{\prod_{j=0,\, j\neq i}^{n} (x_i - x_j)}$$

$$= \sum_{i=0}^{n} \frac{W_{n+1}(x)}{(x - x_i)\, W_{n+1}'(x)} y_i .$$