

神經與行為模型建構 (Neural & Behavioral Modeling)

課號：Psy5352

識別碼：227U2810

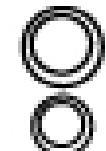
教室：普 101

時間：— 234



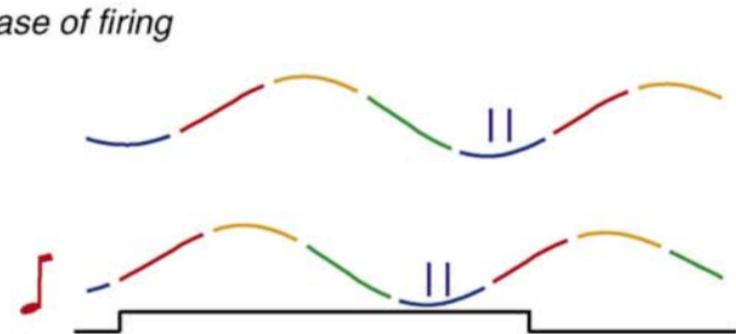
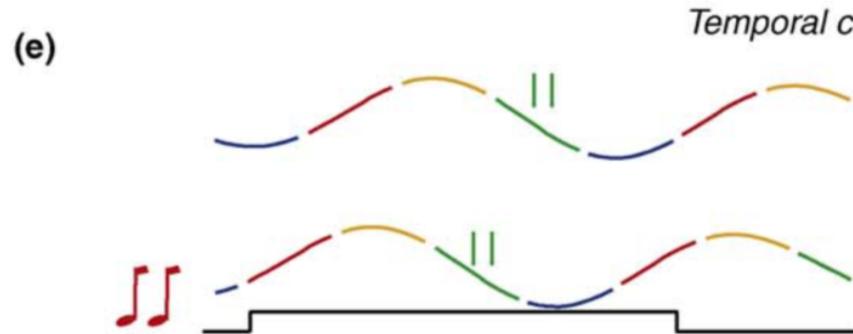
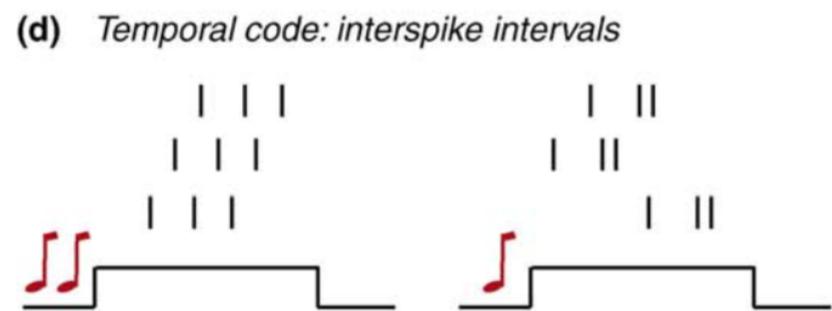
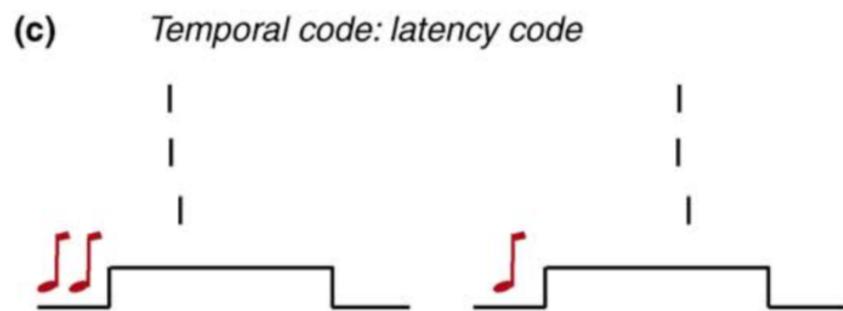
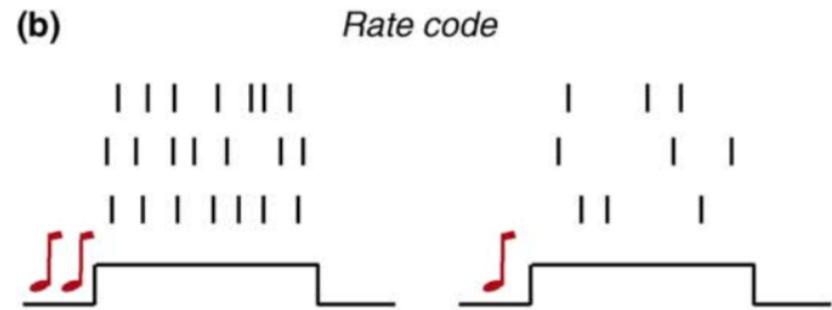
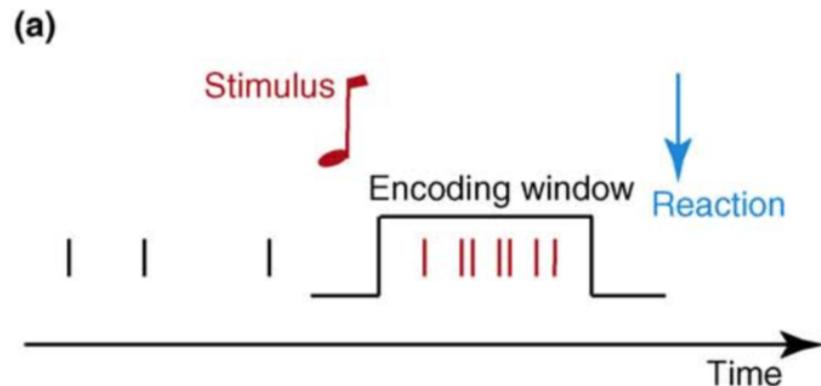


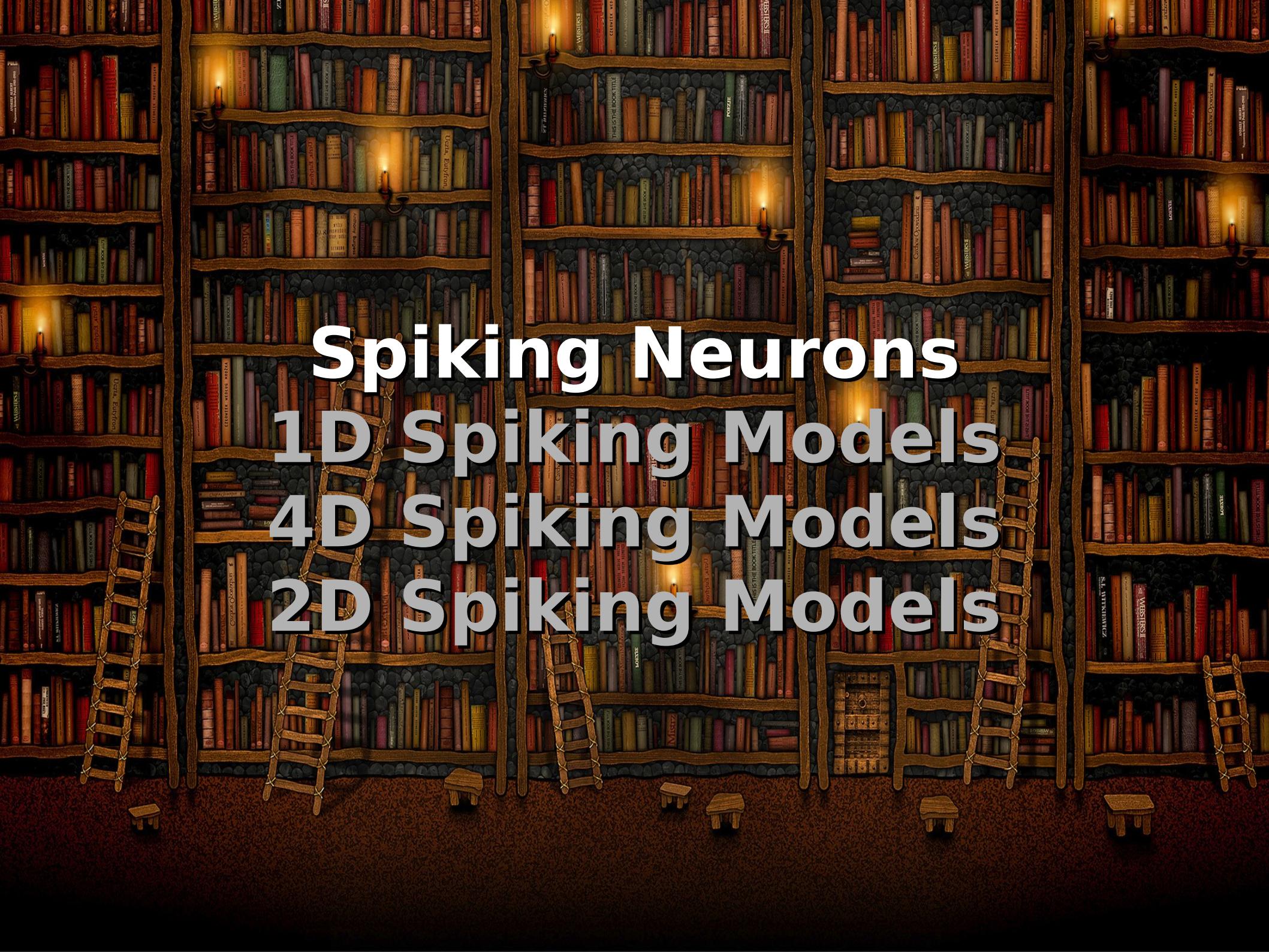
各位的 reset gate 可開了
!!



Rate Codes vs. Temporal Codes

Rate models 裡面沒有的 spike timing 可帶有資訊





Spiking Neurons

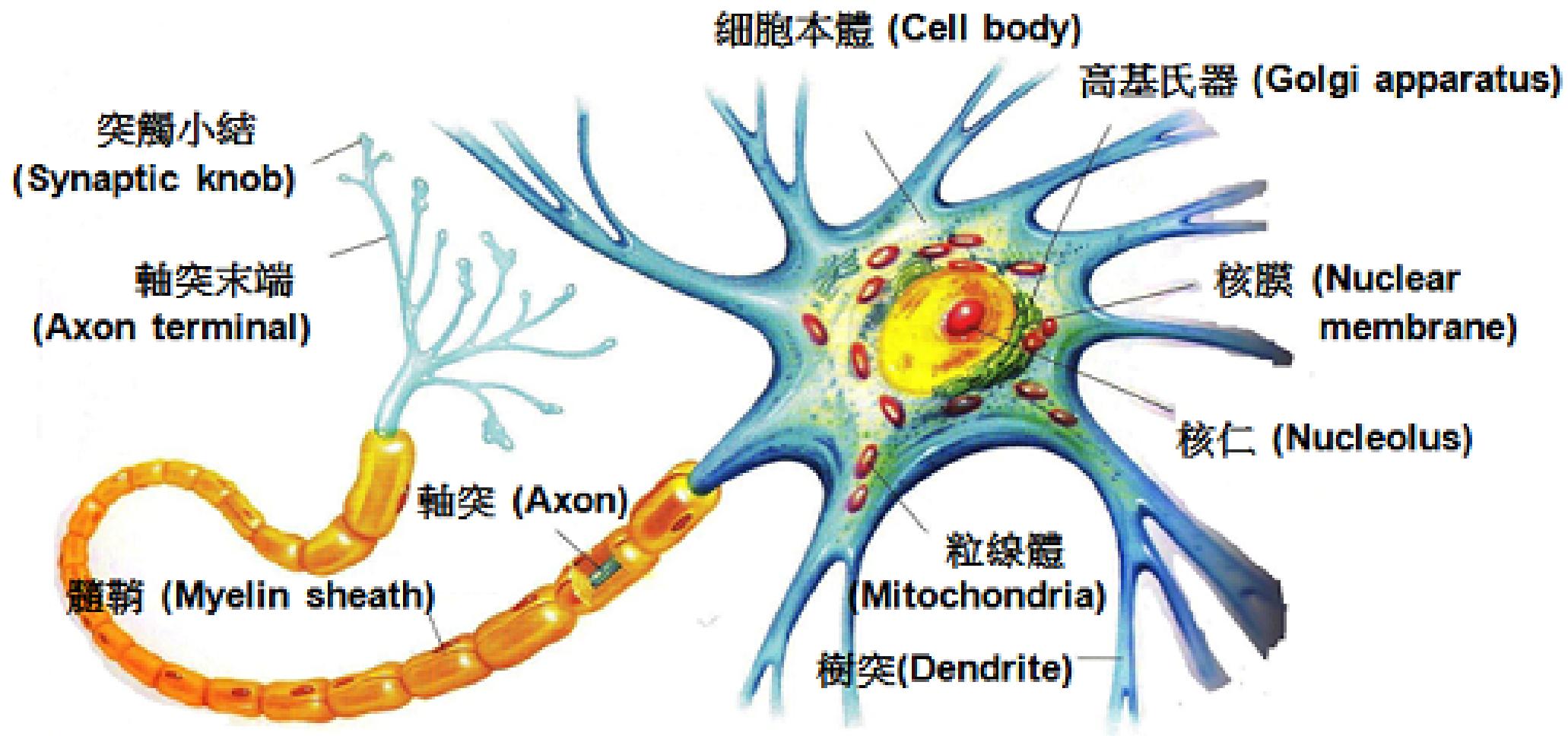
1D Spiking Models

4D Spiking Models

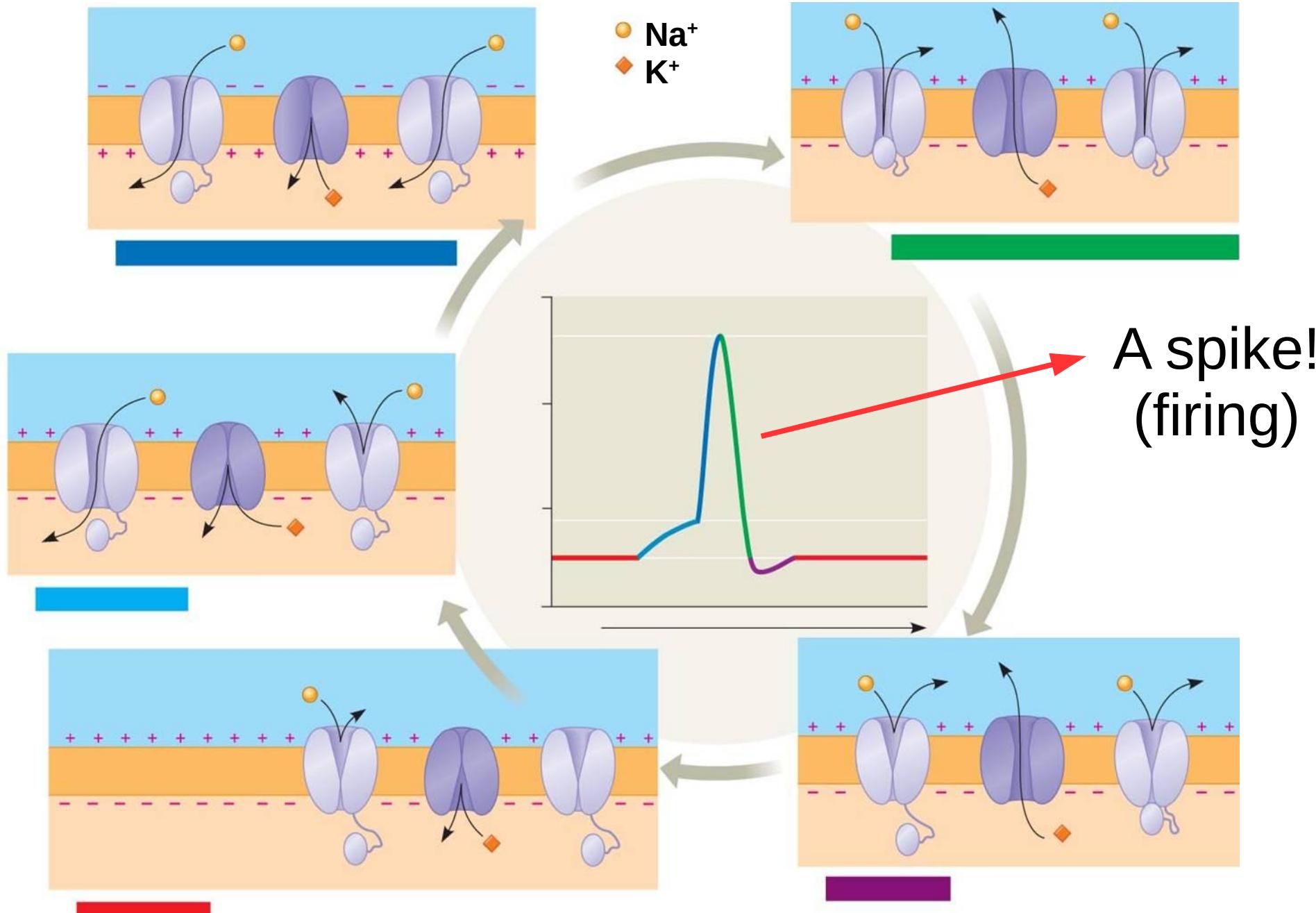
2D Spiking Models

神經元 (Neuron)

神經元可視為神經系統的“原子”



動作電位 (Action Potential): 0 與 1



單一神經元的模型

現象的尺度決定模型的尺度；夸克不在化學方程式內

區段神經元

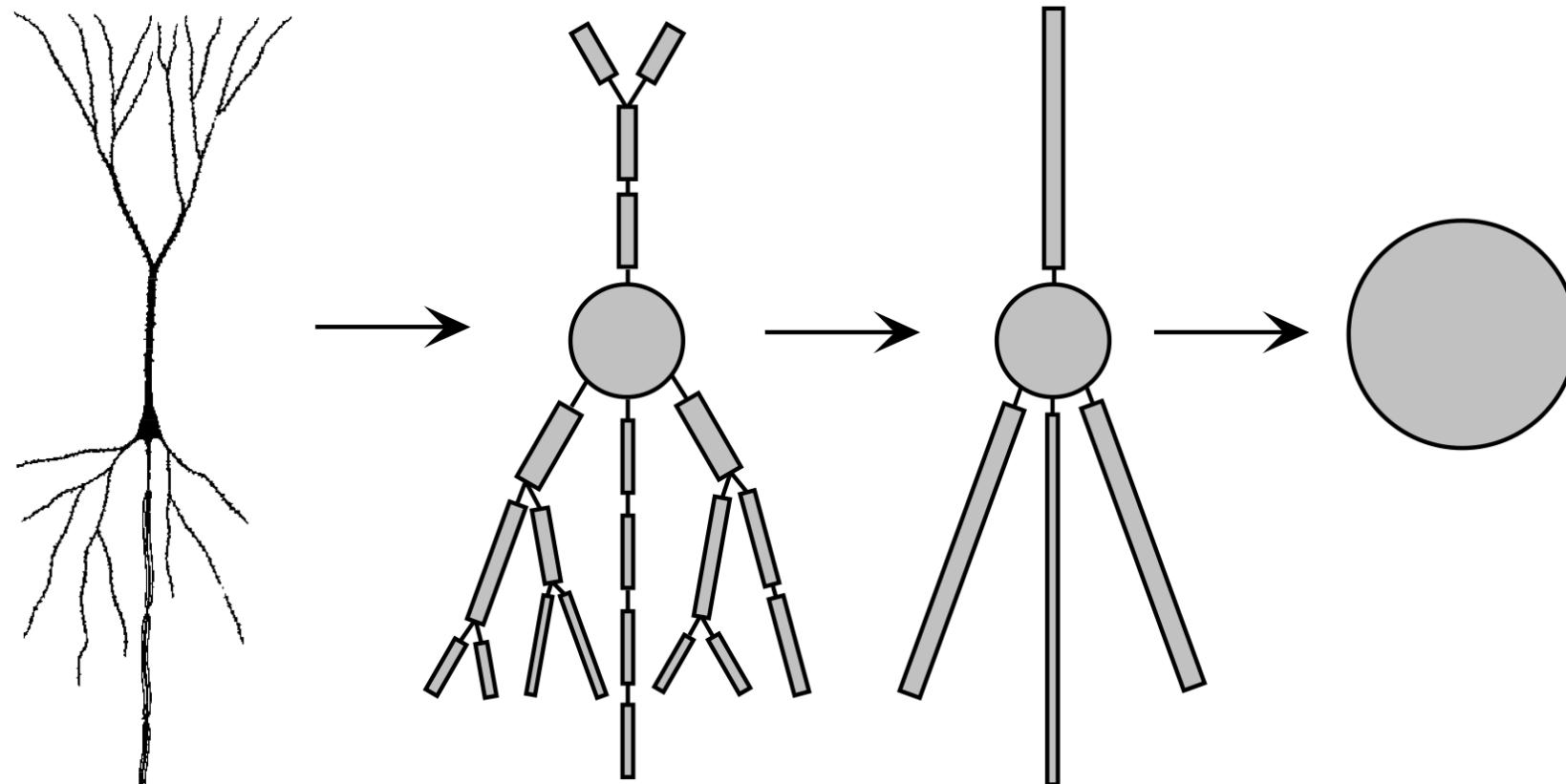
Compartmental Model

線神經元

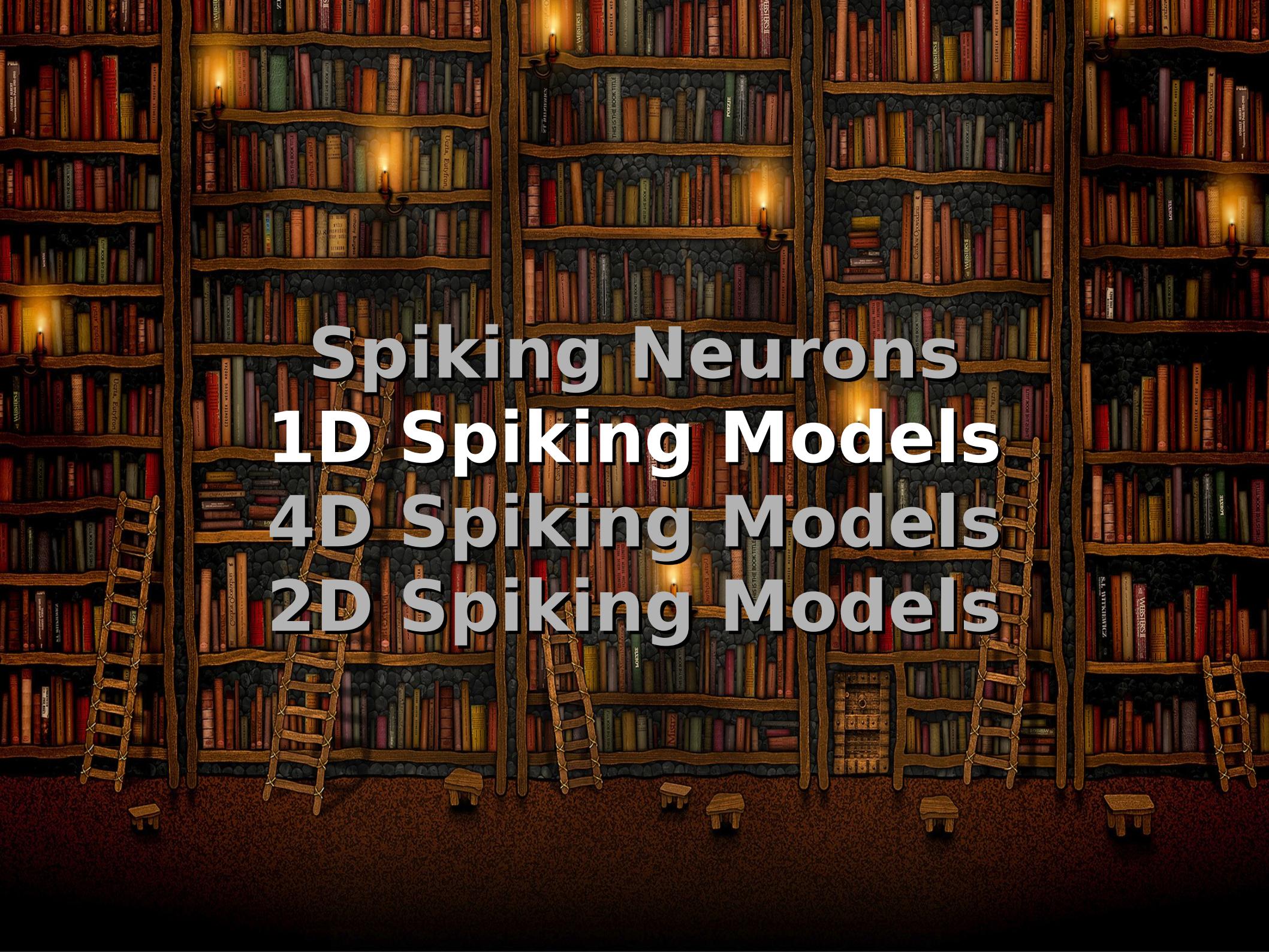
Cable Model

點神經元

HH Model



常用 Neuron(新) & GENESIS(舊) 來做此類模型



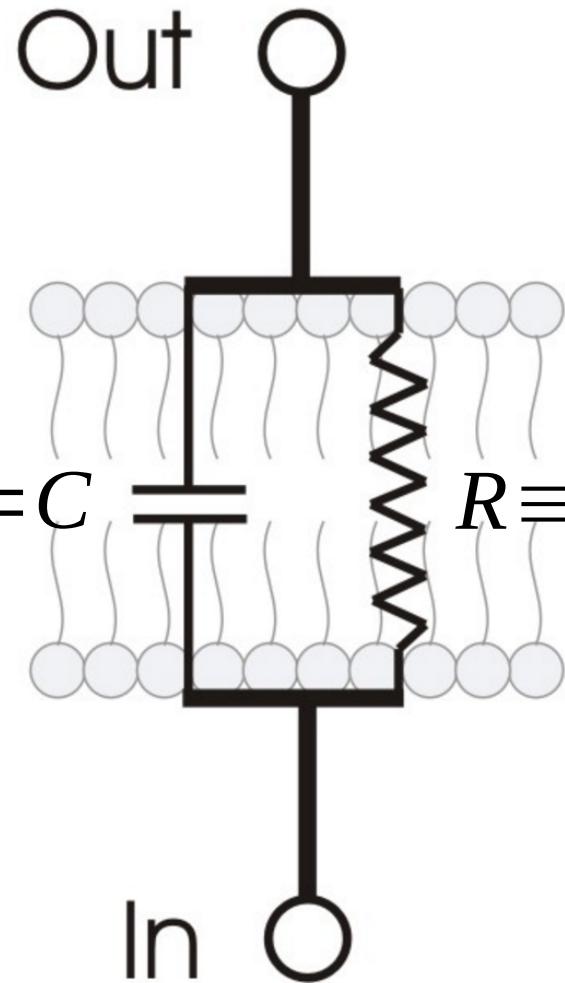
Spiking Neurons

1D Spiking Models

4D Spiking Models

2D Spiking Models

Point Neuron as a RC Circuit



C: 脂膜堆電特性
g: 離子穿越膜通道容易度
R: g 的反數 (即困難度)

V: 膜內外的電位差
I: 穿過細胞膜的電流

$$I_C = \frac{dQ}{dt} = C \frac{dV}{dt}$$
 電容器公式

$$I_g = \frac{V}{R} = gV$$
 歐姆定律

穿過細胞膜的電流 (I) 造成膜內外電位差 (V) 的變化

Integrate-and-Fire Model

考慮 $I=I_g+I_C$, 不管是往膜內還是往膜外流

$$I = I_g + I_C = gV + C \frac{dV}{dt} \Rightarrow \frac{dV}{dt} = -\frac{g}{C}V + \frac{1}{C}I$$

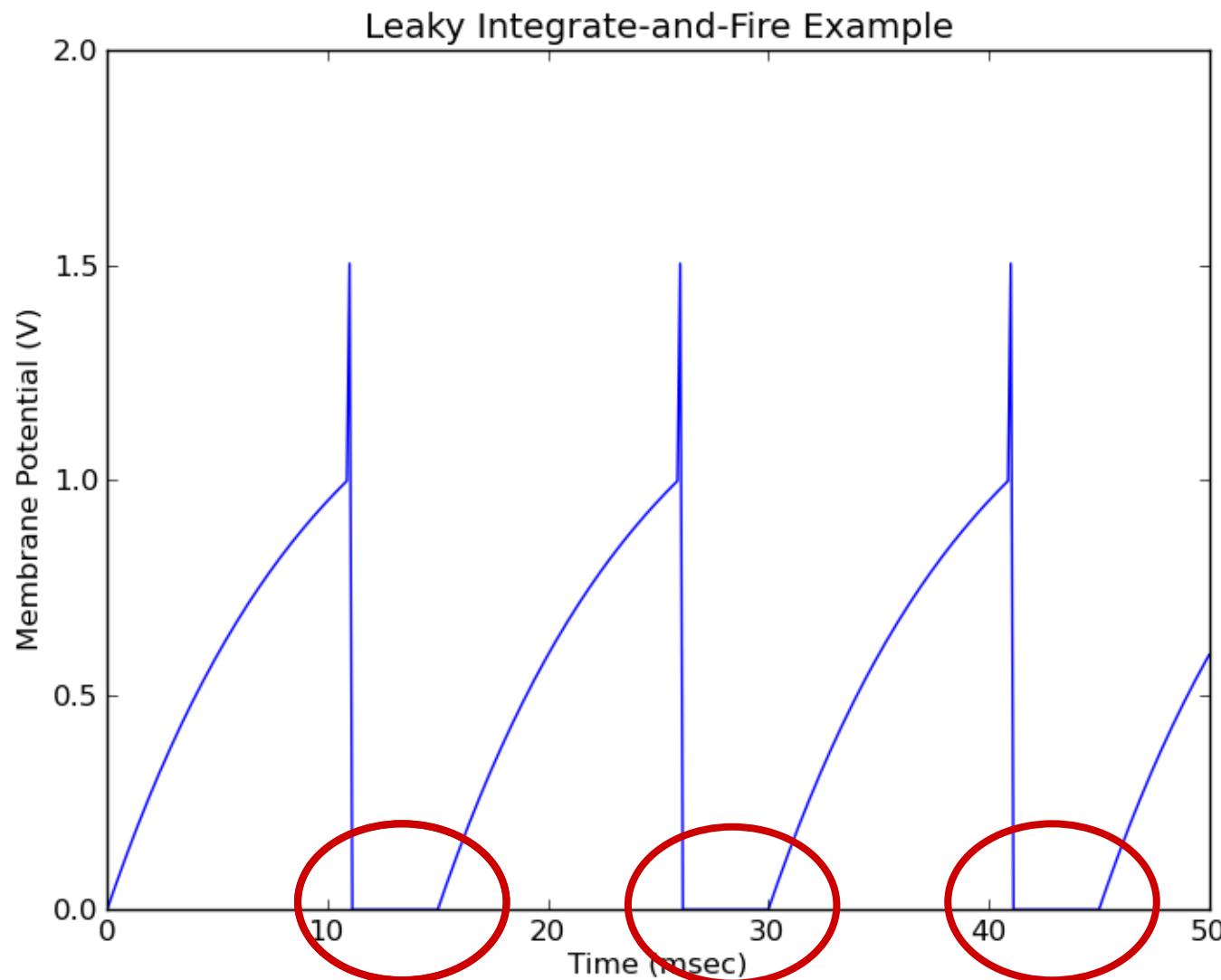
(self-decay+excitation)

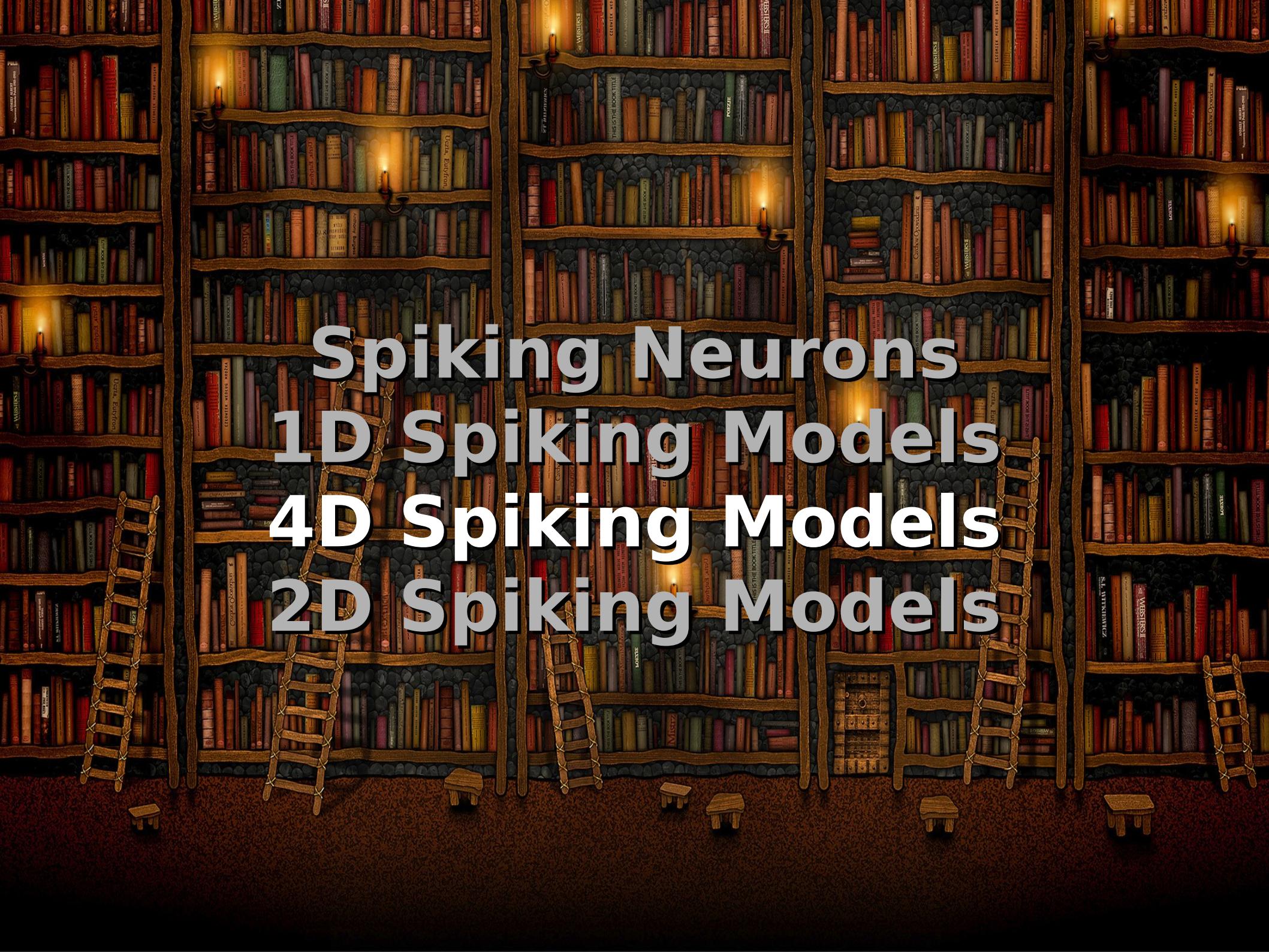
Spiking: 設個 V_{thr} , 一旦 V 超過 V_{thr} 就拉回 V_{rest}

```
I=2;g=5;C=10;dt=.1;Vrst=0;Vthr=.3;Vspk=1;V=Vrst;  
dt=.1; V=[Vrst];  
for t in np.arange(0,10,dt):  
    V1=V[-1] # the last element  
    s=V1>Vthr; # An implicit IF-THEN hack  
    V2=s*Vrst+(1-s)*(V1+dt/C*(-g*V1+I)) # Euler  
    V1=s*Vspk+(1-s)*V1 # Spiking now?  
    V[-1]=V1; V.append(V2)
```

本週作業

用一個計數器變數配合 IF-THEN 使 IF 模型有不反應期





Spiking Neurons

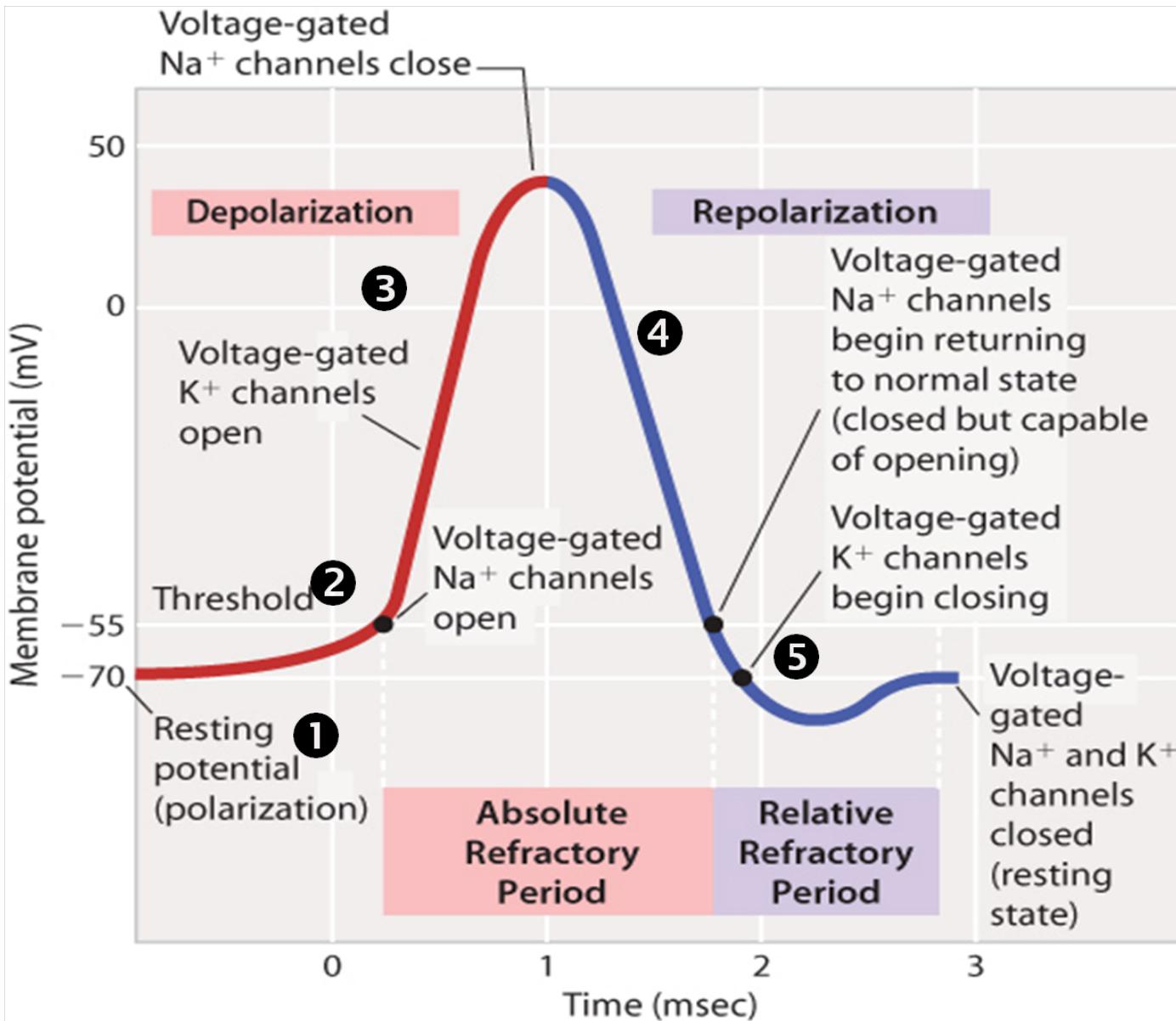
1D Spiking Models

4D Spiking Models

2D Spiking Models

Leaky IF Model 的問題

許多重要的神經性質都是由 hacks 來模擬



動作電位的產生

鈉通道要快速打開

靜止電位的回復

鈉通道要快速關閉

鉀通道要慢速打開

不反應期的存在

鉀通道要慢速關閉

Hodgkin-Huxley 模型

看起來有點複雜；需要分成三部分講解

$$I = C_m \frac{dV_m}{dt} + \bar{g}_K n^4 (V_m - V_K) + \bar{g}_{Na} m^3 h (V_m - V_{Na}) + \bar{g}_l (V_m - V_l)$$

$$\frac{dn}{dt} = \alpha_n(V_m)(1 - n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1 - m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1 - h) - \beta_h(V_m)h$$

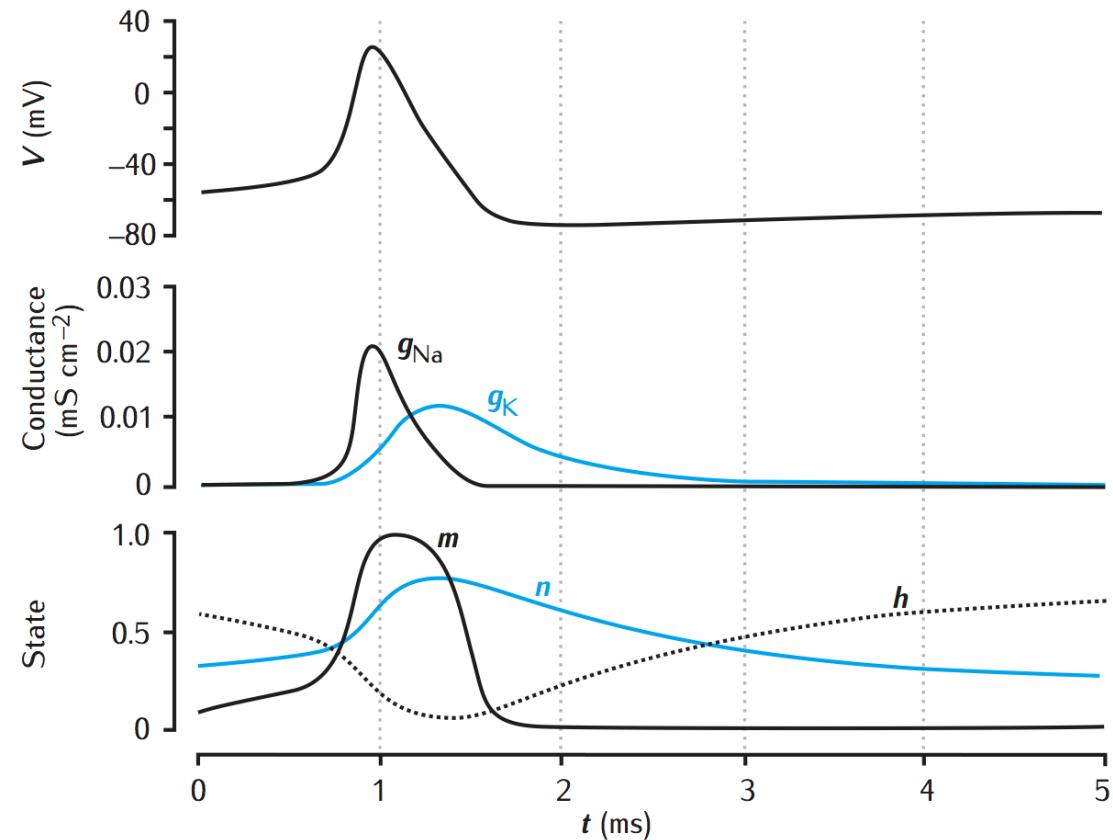
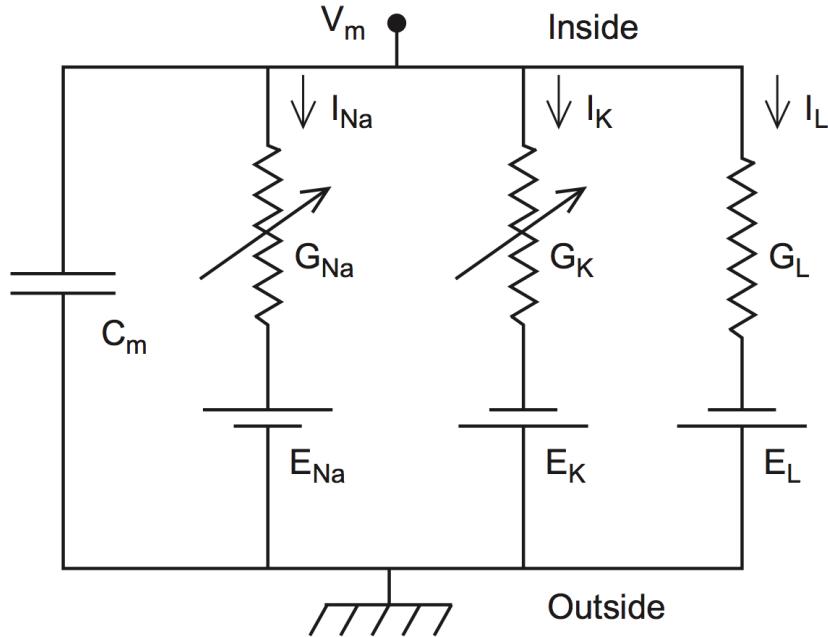
Reversal Potential
of K⁺, at which the
net current of K⁺ is 0

$$\alpha_n(V_m) = \frac{0.01(V_m - 10)}{\exp\left(\frac{V_m - 10}{10}\right) - 1} \quad \alpha_m(V_m) = \frac{0.1(V_m - 25)}{\exp\left(\frac{V_m - 25}{10}\right) - 1} \quad \alpha_h(V_m) = 0.07 \exp\left(\frac{V_m}{20}\right)$$

$$\beta_n(V_m) = 0.125 \exp\left(\frac{V_m}{80}\right) \quad \beta_m(V_m) = 4 \exp\left(\frac{V_m}{18}\right) \quad \beta_h(V_m) = \frac{1}{\exp\left(\frac{V_m - 30}{10}\right) + 1}$$

Main Hodgkin-Huxley Equation

同時考慮三個離子通道：氯（恆）、鈉（快）、鉀（慢）

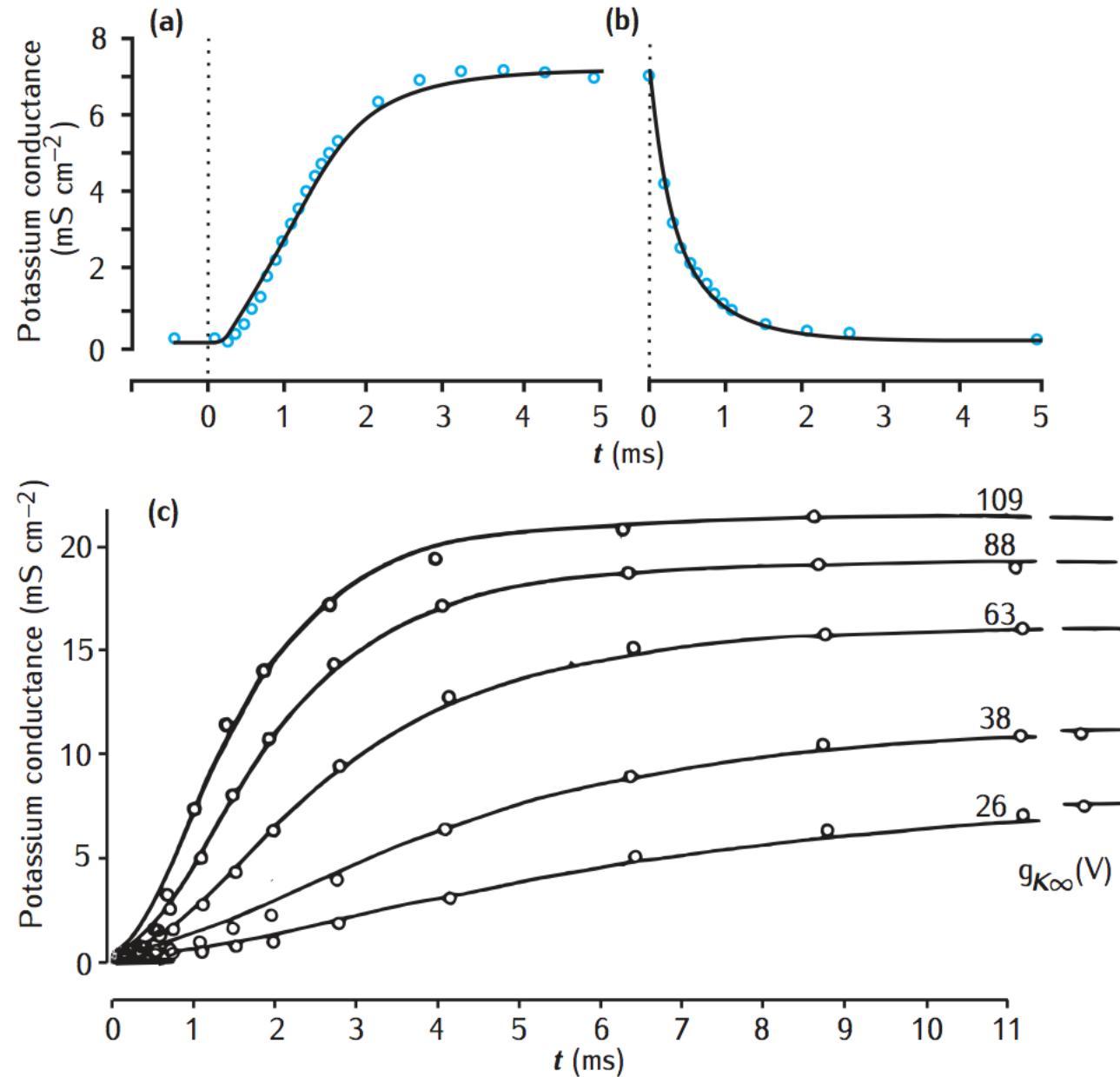


$$I = C \frac{dV}{dt} + \bar{g}_{Cl}(V - \bar{V}_{Cl}) + \bar{g}_{Na} m^3 h (V - \bar{V}_{Na}) + \bar{g}_K n^4 (V - \bar{V}_K)$$

魔鬼藏在細節 (m, h, n) 裡！

HH's Data of $g_K(t, V)$

比較第一週作業最後一題 (i.e., shunting eq.)



$g_K(t, V) = \bar{g}_K n^4$ 的由來 (1/3)

HH 說有些理論猜測，但主要是 data fitting

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A. L. HODGKIN AND A. F. HUXLEY

where

$$V = E - E_r,$$

$$V_{\text{Na}} = E_{\text{Na}} - E_r,$$

$$V_{\text{K}} = E_{\text{K}} - E_r,$$

$$V_l = E_l - E_r,$$

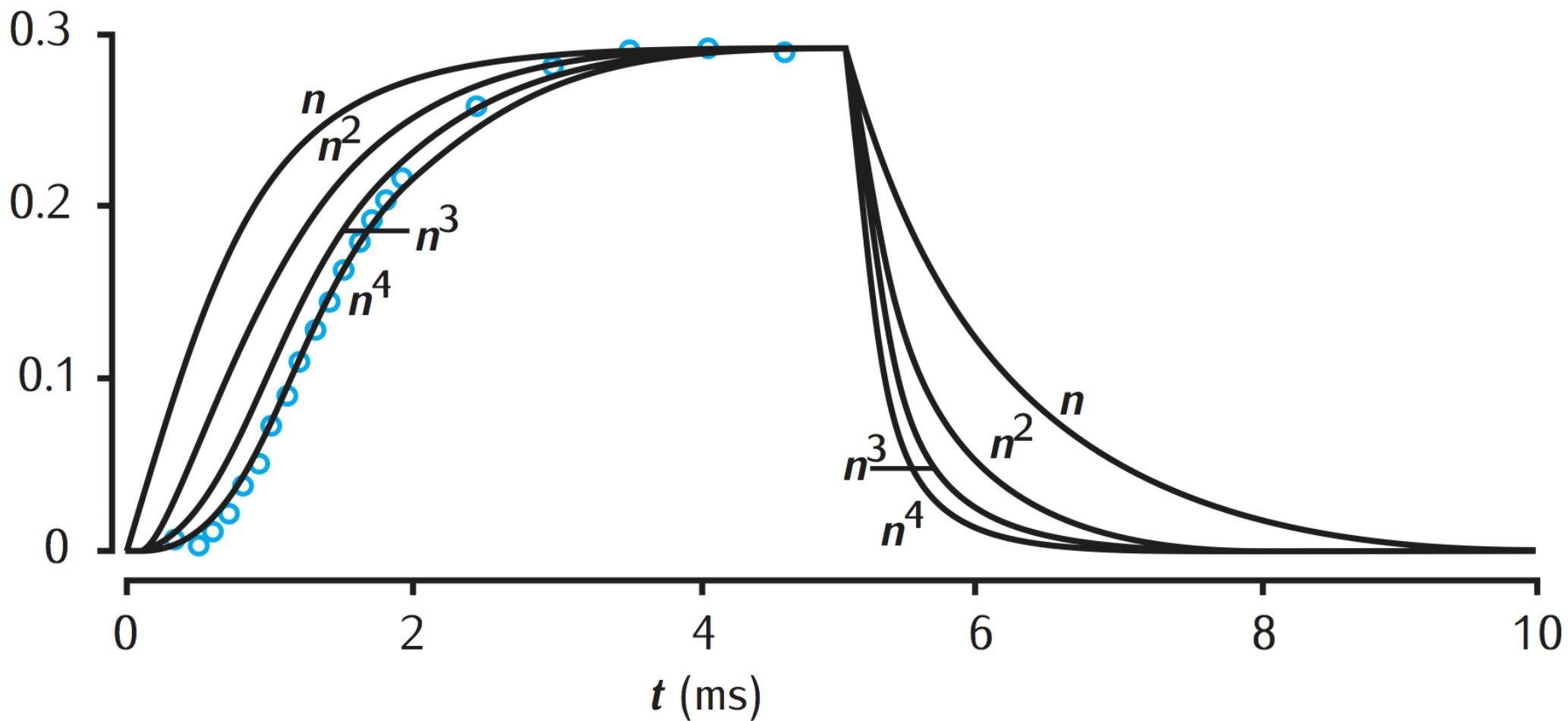
and E_r is the absolute value of the resting potential. V , V_{Na} , V_{K} and V_l can then be measured directly as displacements from the resting potential.

The ionic conductances

The discussion in Part I shows that there is little hope of calculating the time course of the sodium and potassium conductances from first principles. Our object here is to find equations which describe the conductances with reasonable accuracy and are sufficiently simple for theoretical calculation of the action potential and refractory period. For the sake of illustration we shall try to provide a physical basis for the equations, but must emphasize that the interpretation given is unlikely to provide a correct picture of the membrane.

$g_K(t, V) = \bar{g}_K n^4$ 的由來 (2/3)

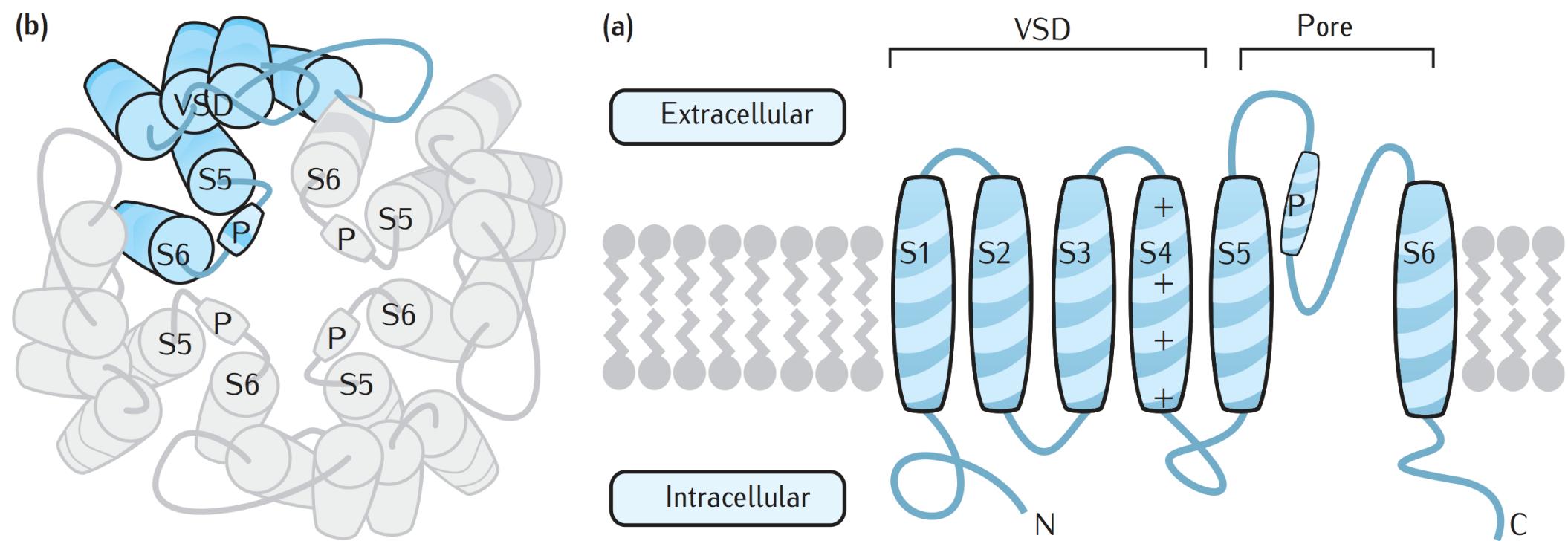
重要的觀察是 $g_K(t, V)$ 在 spike 開始時有 initial delay



HH 說 n^3 或 n^4 都可以但 n^4 較符合資料

$g_K(t, V) = \bar{g}_K n^4$ 的由來 (3/3)

n 可以理解成鉀通道機構被 (電壓) 打開的機率



鉀通道四個 subunits 同時打開的機率為 n^4

Hodgkin-Huxley 模型

看起來有點複雜；需要分成三部分講解

$$I = C_m \frac{dV_m}{dt} + \bar{g}_K n^4 (V_m - V_K) + \bar{g}_{\text{Na}} m^3 h (V_m - V_{Na}) + \bar{g}_l (V_m - V_l)$$

$$\frac{dn}{dt} = \alpha_n(V_m)(1 - n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1 - m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1 - h) - \beta_h(V_m)h$$

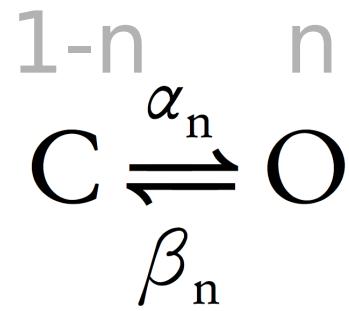
$$\alpha_n(V_m) = \frac{0.01(V_m - 10)}{\exp\left(\frac{V_m - 10}{10}\right) - 1} \quad \alpha_m(V_m) = \frac{0.1(V_m - 25)}{\exp\left(\frac{V_m - 25}{10}\right) - 1} \quad \alpha_h(V_m) = 0.07 \exp\left(\frac{V_m}{20}\right)$$

$$\beta_n(V_m) = 0.125 \exp\left(\frac{V_m}{80}\right) \quad \beta_m(V_m) = 4 \exp\left(\frac{V_m}{18}\right) \quad \beta_h(V_m) = \frac{1}{\exp\left(\frac{V_m - 30}{10}\right) + 1}$$

$g_K(t, V) = \bar{g}_K n^4$ 中的 t 與 V

$n(t, V)$ 的具體方程式為何？

可用化學反應方程式的觀點來討論開 (O) 與關 (C):



n : 鉀通道打開的比率

α_n : 鉀通道打開的係數

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$

β_n : 鉀通道關閉的係數

勒沙特列原理

另外可從 shunting equation 的觀點來看：

n 的興奮項是 α_n 而抑制項是 β_n ，且值域在 $[0,1]$ 之間

Hodgkin-Huxley 模型

看起來有點複雜；需要分成三部分講解

$$I = C_m \frac{dV_m}{dt} + \bar{g}_K n^4 (V_m - V_K) + \bar{g}_{\text{Na}} m^3 h (V_m - V_{Na}) + \bar{g}_l (V_m - V_l)$$

$$\frac{dn}{dt} = \alpha_n(V_m)(1 - n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1 - m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1 - h) - \beta_h(V_m)h$$

$$\begin{aligned}\alpha_n(V_m) &= \frac{0.01(V_m - 10)}{\exp\left(\frac{V_m - 10}{10}\right) - 1} & \alpha_m(V_m) &= \frac{0.1(V_m - 25)}{\exp\left(\frac{V_m - 25}{10}\right) - 1} & \alpha_h(V_m) &= 0.07 \exp\left(\frac{V_m}{20}\right) \\ \beta_n(V_m) &= 0.125 \exp\left(\frac{V_m}{80}\right) & \beta_m(V_m) &= 4 \exp\left(\frac{V_m}{18}\right) & \beta_h(V_m) &= \frac{1}{\exp\left(\frac{V_m - 30}{10}\right) + 1}\end{aligned}$$

$g_K(t, V) = \bar{g}_K n^4$ 中的 t 與 V (1/2)

$n(t, V)$ 的微分方程可以直接積分求解

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$

`sym.dsolve(n.diff()-a*(1-n)+b*n)`

$$n(t, V) = C_1 + C_2 e^{-(\alpha_n(V) + \beta_n(V))t}$$

帶入可量到的邊界條件 $n_0 = n(t=0)$ 與 $n_\infty = n(t=\infty)$ 得：

$$n(t, V) = n_\infty + (n_0 - n_\infty) e^{-(\alpha_n(V) + \beta_n(V))t} \equiv n_\infty + (n_0 - n_\infty) e^{-t/\tau_n(V)}$$

特徵時間 $\tau_n \equiv 1/(\alpha_n + \beta_n)$ 是可以從實驗結果推估的量

$g_K(t, V) = \bar{g}_K n^4$ 中的 t 與 V (2/2)

$$n(t, V) = n_\infty + (n_0 - n_\infty) e^{-t/\tau_n}$$

在定義特徵壓力 $\tau_n \equiv 1/(\alpha_n + \beta_n)$ 後 $n(t)$ 的微方變成：

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n \Rightarrow \frac{dn}{dt} = \alpha_n - \frac{n}{\tau_n}$$

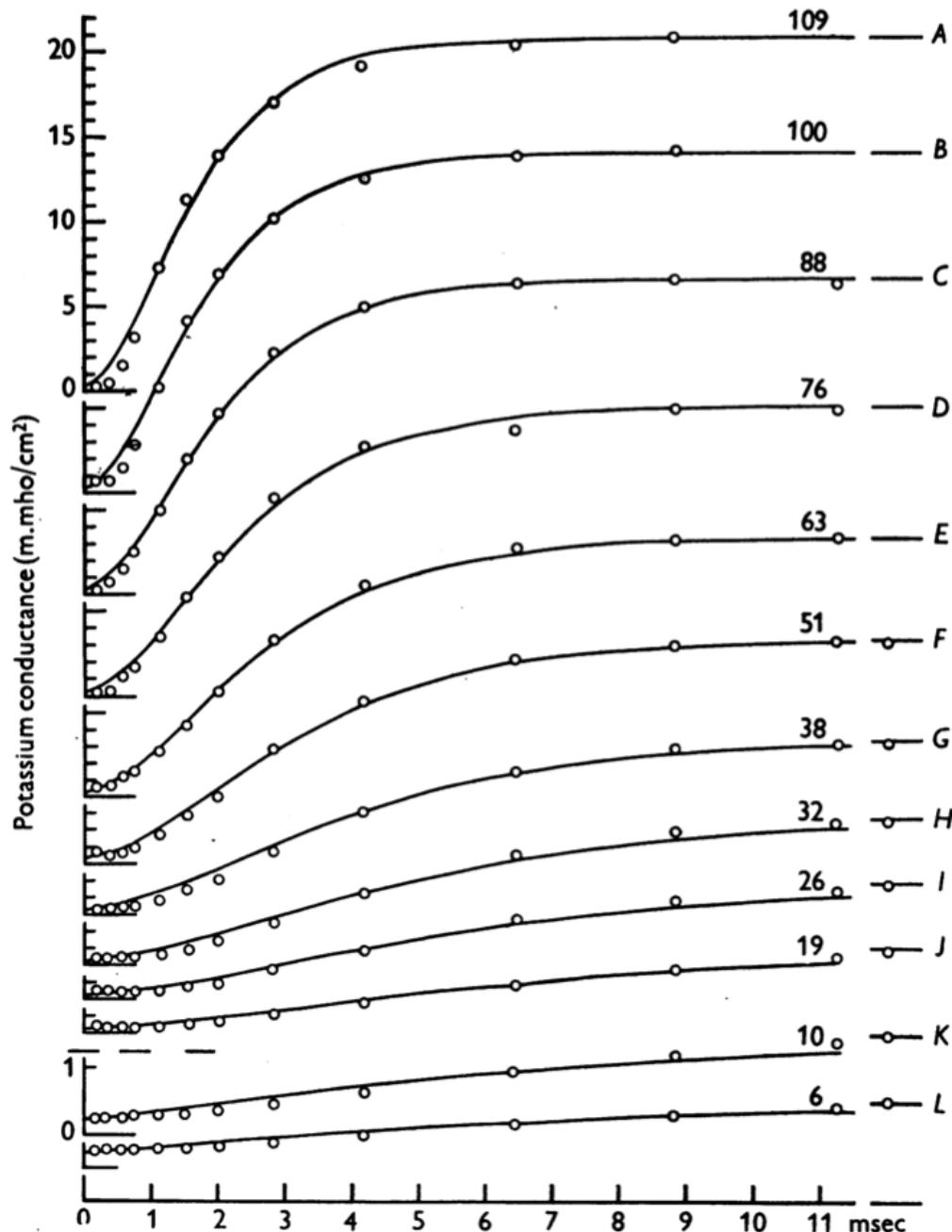
利用穩態時 $dn(t=\infty)/dt = \alpha_n - n_\infty/\tau_n = 0$ 故得：

$$n_\infty = \frac{\alpha_n}{\alpha_n + \beta_n} \quad \text{and} \quad \tau_n = \frac{1}{\alpha_n + \beta_n}$$

用可觀察到的 n_0 與 n_∞ 與 τ_n 推算不可觀察到的 α_n 與 β_n

$\tau_n(V)$ 愈小代表成長斜率愈大，而斜率受 V 影響

HH 箔制膜片電壓量到的 $g_K(t, V)$



$$n(t, V) = n_\infty + (n_0 - n_\infty) e^{-t/\tau_n}$$

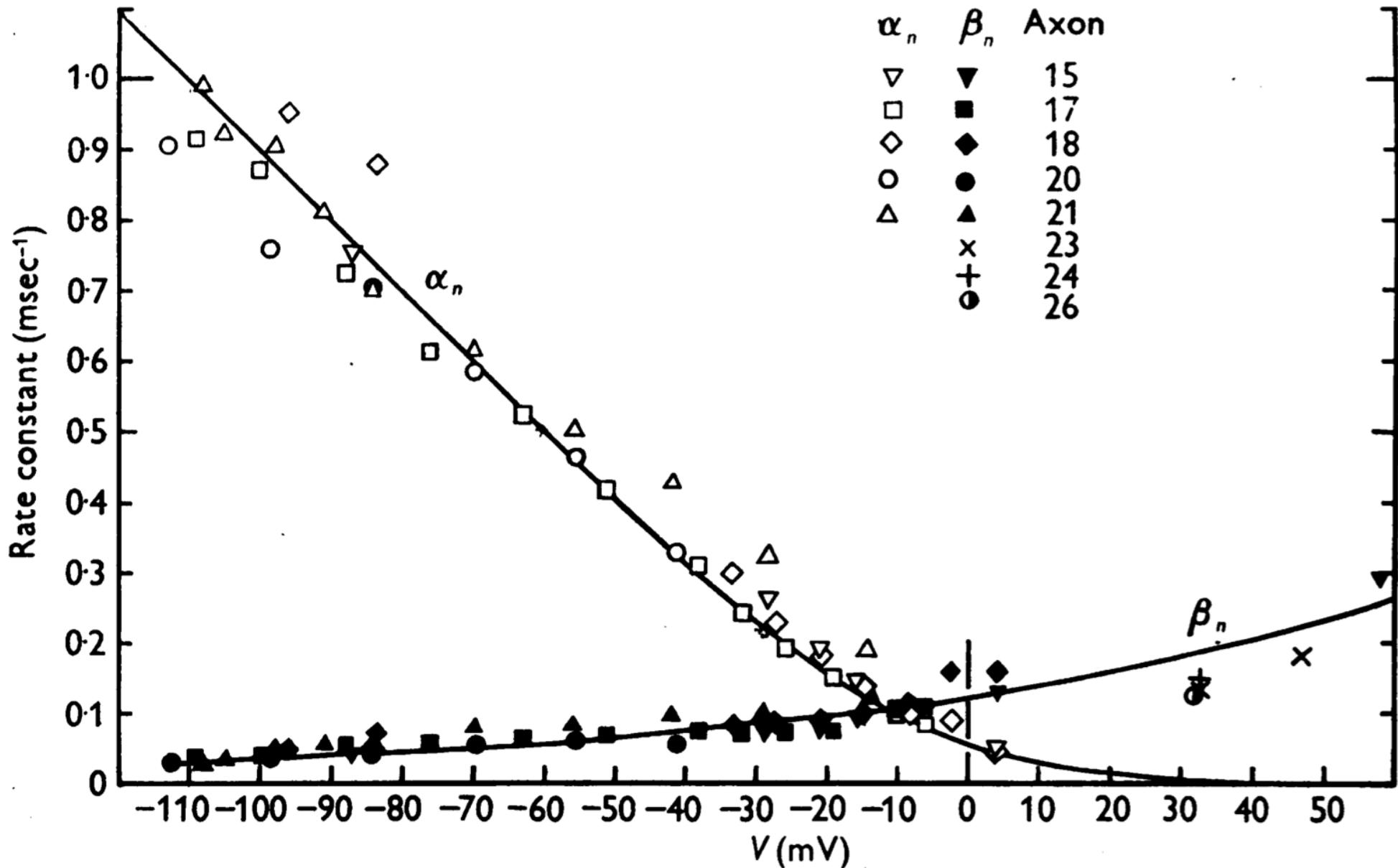
用 patch clamp 技術固定
電壓後可量到 n , n_0 與 n_∞

有 τ_n 後反推 α_n 與 β_n 數值
(即 α_n 與 β_n 的數值解)

知道數值後再找描述方程
(即 α_n 與 β_n 的解析解)

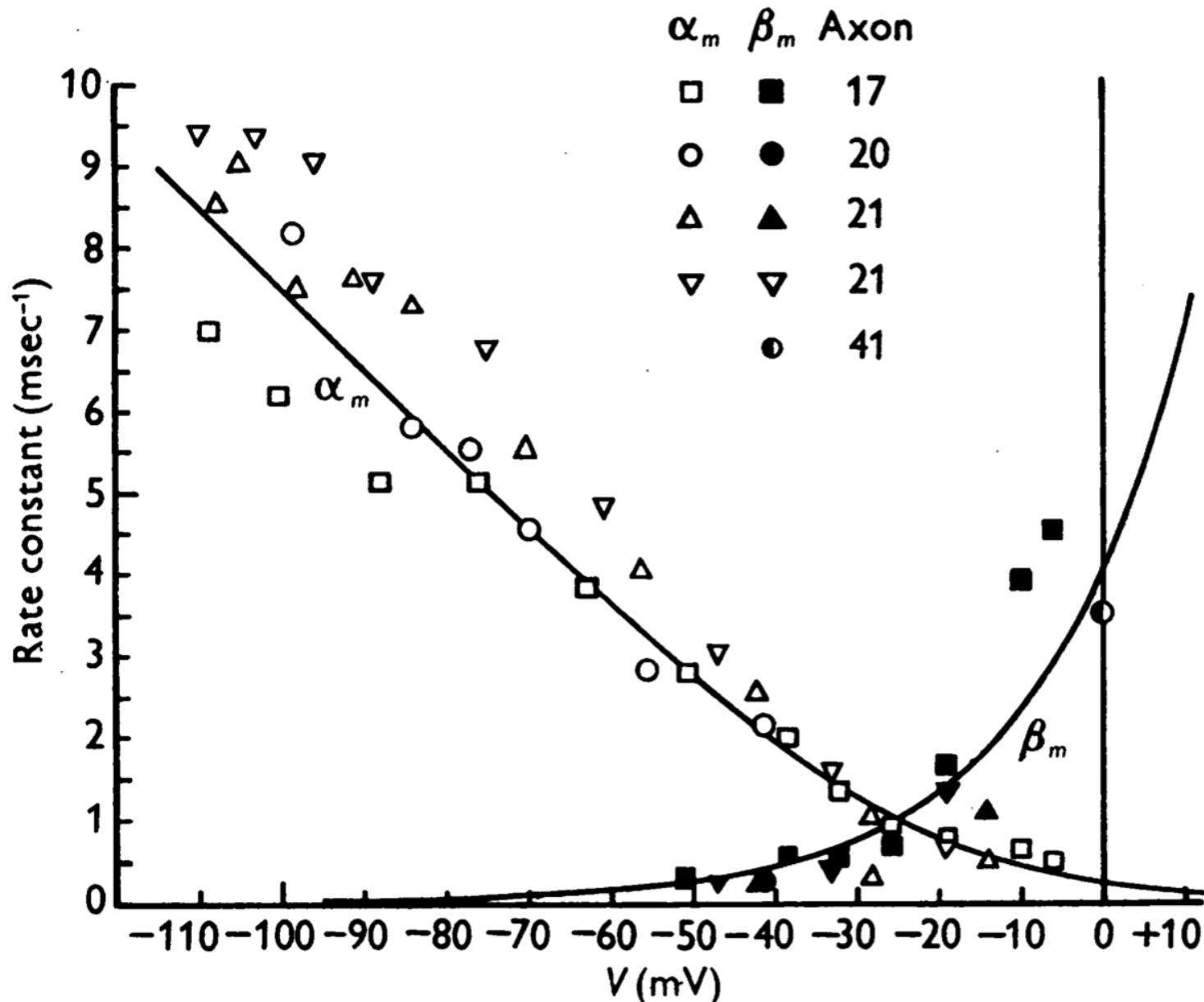
HH 算出的 $\alpha_n(V)$ 與 $\beta_n(V)$

你會寫下怎樣的方程式來描述鉀通道 α_n 與 β_n 的數值?



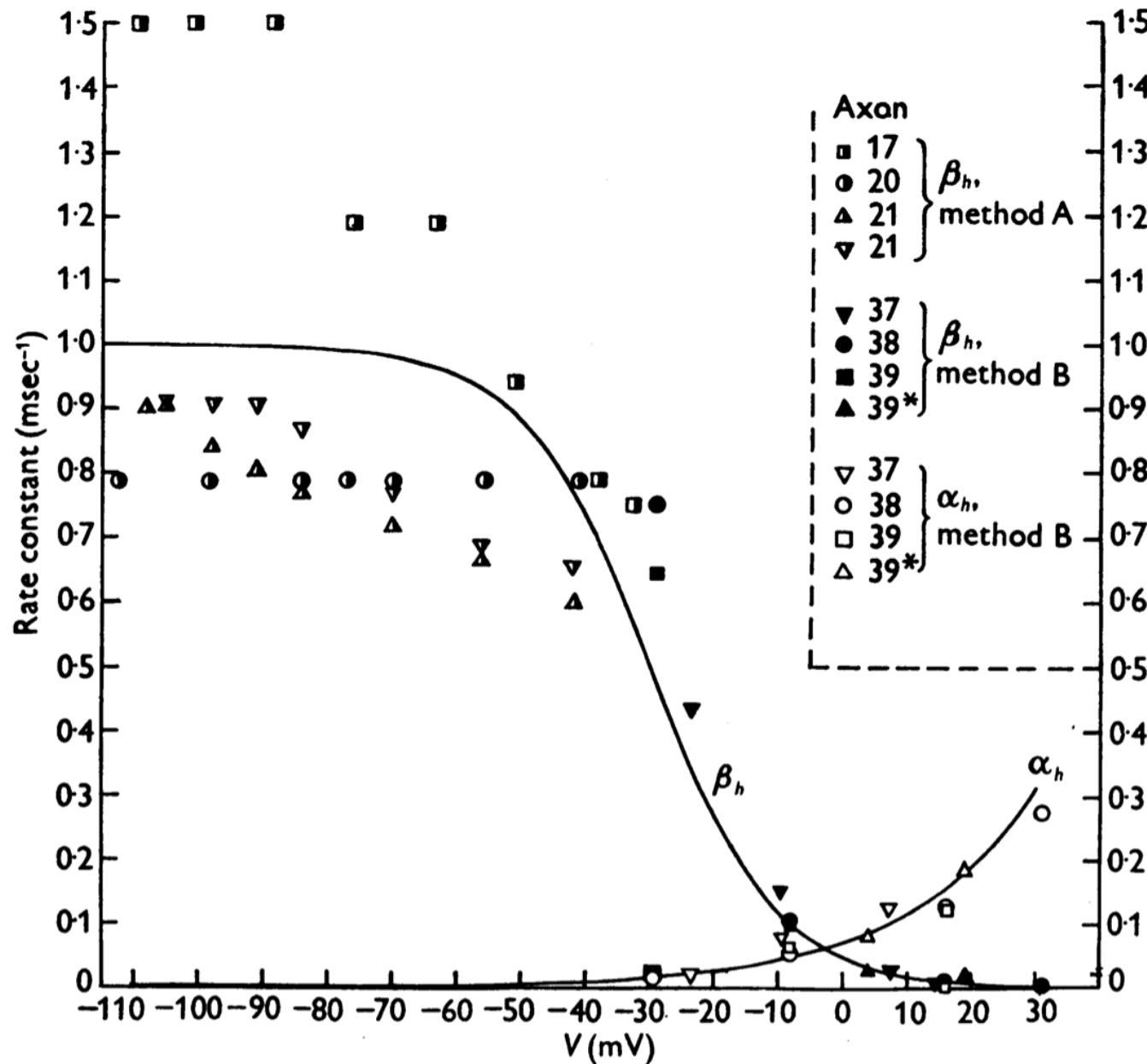
HH 算出的 $\alpha_m(V)$ 與 $\beta_m(V)$

你會寫下怎樣的方程式來描述鈉通道 α_m 與 β_m 的數值？



HH 算出的 $\alpha_h(V)$ 與 $\beta_h(V)$

你會寫下怎樣的方程式來描述鈉通道 α_h 與 β_h 的數值？



完整的 HH 方程 (depolarization -)

$$I = C_m \frac{dV_m}{dt} + \bar{g}_K n^4 (V_m - V_K) + \bar{g}_{Na} m^3 h (V_m - V_{Na}) + \bar{g}_l (V_m - V_l)$$

$$\frac{dn}{dt} = \alpha_n(V_m)(1 - n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1 - m) - \beta_m(V_m)m$$

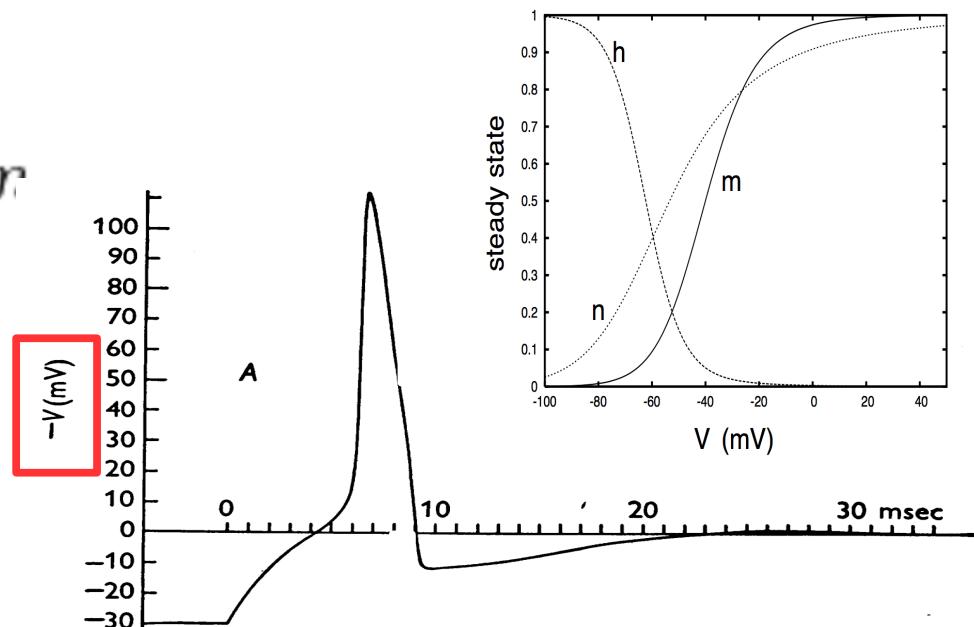
$$\frac{dh}{dt} = \alpha_h(V_m)(1 - h) - \beta_h(V_m)h$$

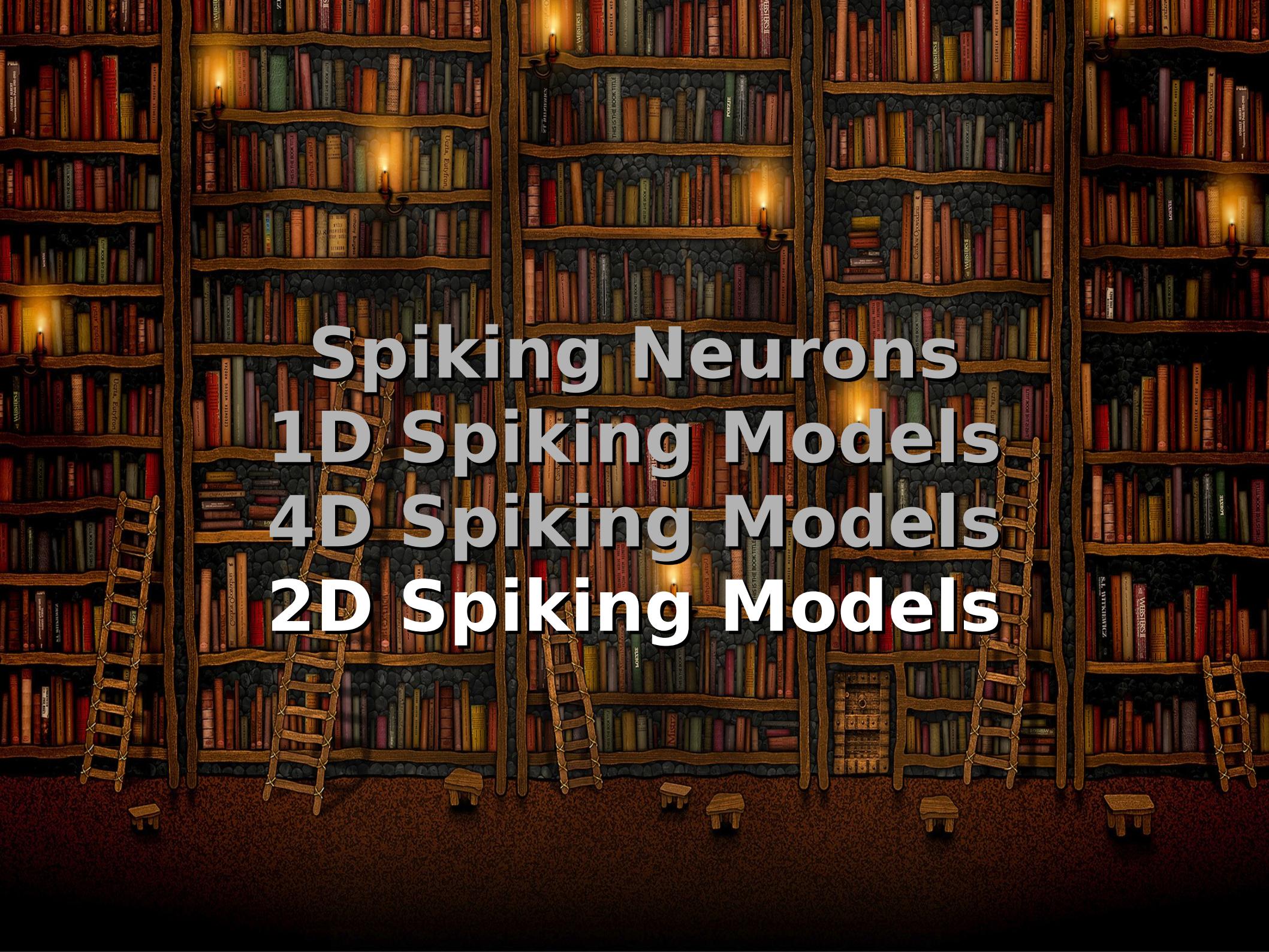
上三個方程可用一條取代：

$$\frac{dx}{dt} = (x_\infty - x)/\tau_x \quad (\text{P.24})$$

故在實作時可以向量化執行

$$\begin{aligned} \alpha_n(V_m) &= \frac{0.01(V_m - 10)}{\exp\left(\frac{V_m - 10}{10}\right) - 1} & \alpha_m(V_m) &= \frac{0.1(V_m - 25)}{\exp\left(\frac{V_m - 25}{10}\right) - 1} & \alpha_h(V_m) &= 0.07 \exp\left(\frac{V_m}{20}\right) \\ \beta_n(V_m) &= 0.125 \exp\left(\frac{V_m}{80}\right) & \beta_m(V_m) &= 4 \exp\left(\frac{V_m}{18}\right) & \beta_h(V_m) &= \frac{1}{\exp\left(\frac{V_m - 30}{10}\right) + 1} \end{aligned}$$





Spiking Neurons

1D Spiking Models

4D Spiking Models

2D Spiking Models

Morris-Lecar Model

隨時間累積的慢速煞車項（鉀）才需要特別處理：

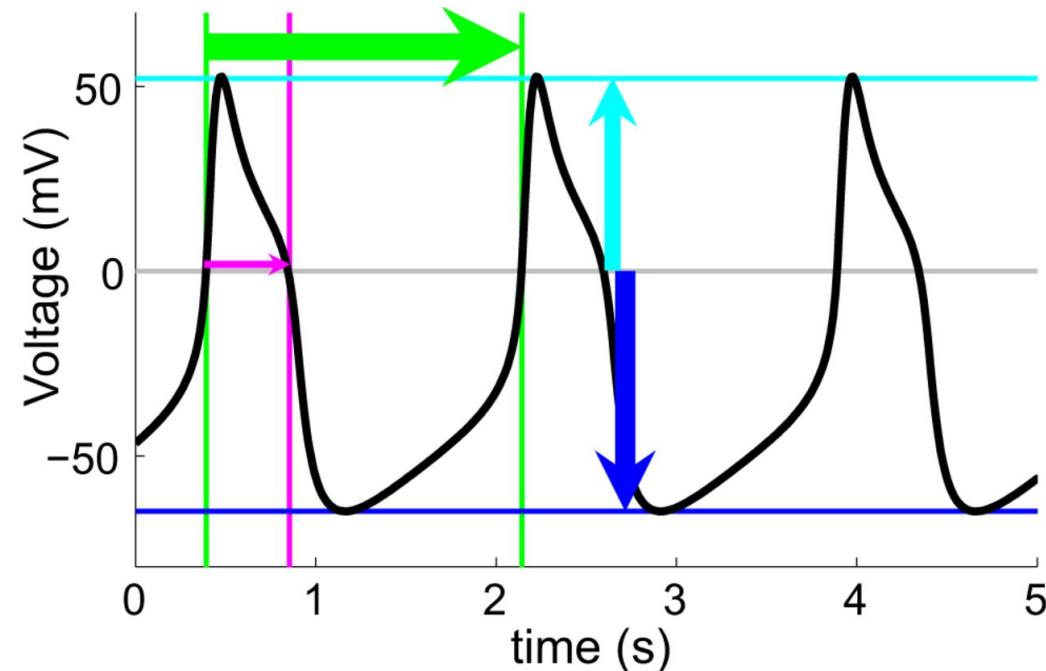
$$C \frac{dV}{dt} = I - g_L(V - V_L) - g_{\text{Ca}} M_{ss}(V - V_{\text{Ca}}) - g_K N(V - V_K)$$
$$\frac{dN}{dt} = \frac{N_{ss} - N}{\tau_N}$$

where

$$M_{ss} = \frac{1}{2} \cdot (1 + \tanh[\frac{V - V_1}{V_2}])$$

$$N_{ss} = \frac{1}{2} \cdot (1 + \tanh[\frac{V - V_3}{V_4}])$$

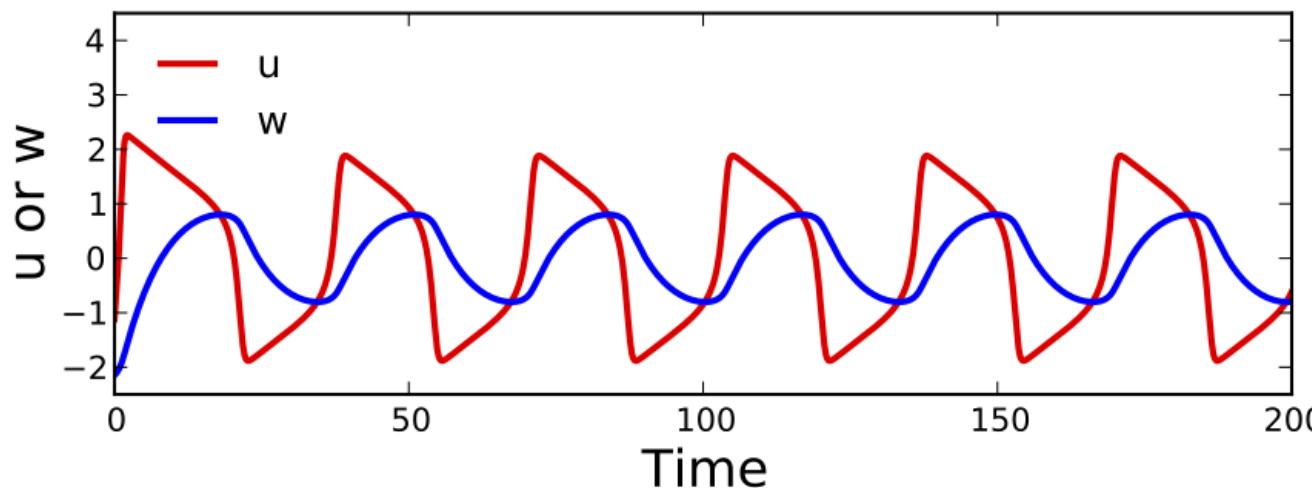
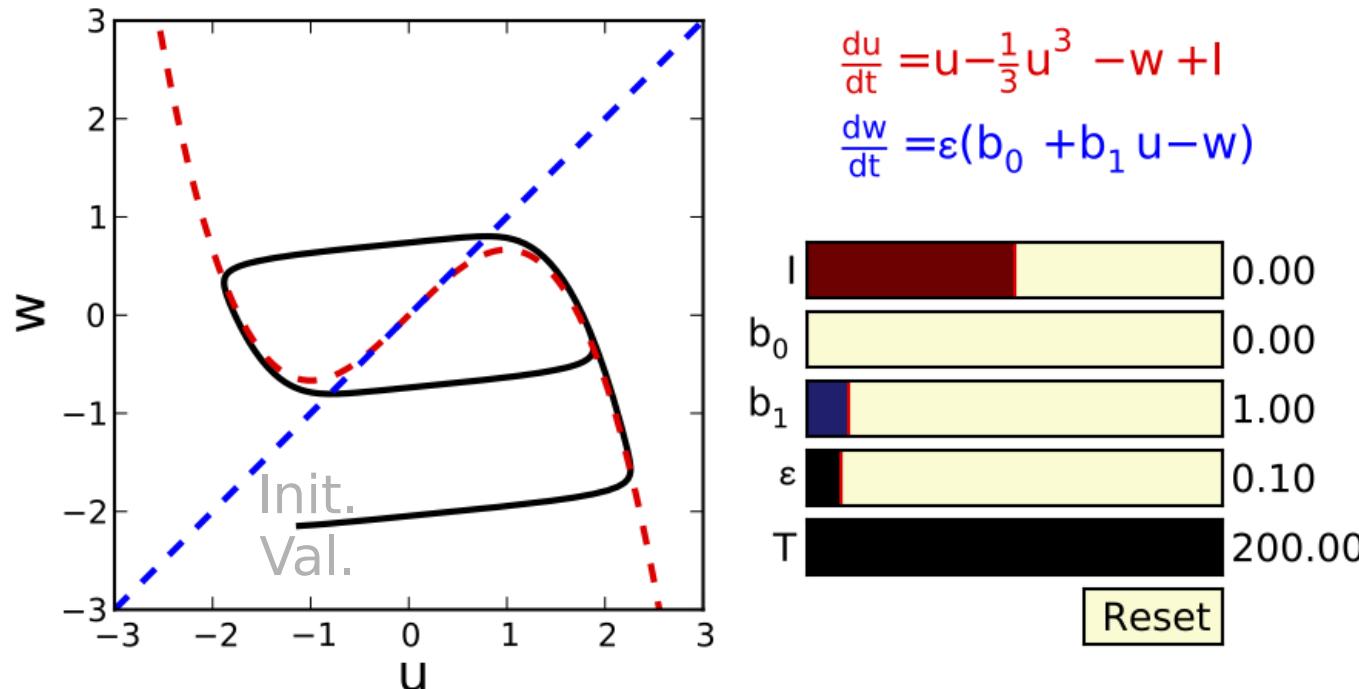
$$\tau_N = 1 / (\phi \cosh[\frac{V - V_3}{2V_4}])$$



雖四個變數化簡為兩個變數仍能描述許多現象

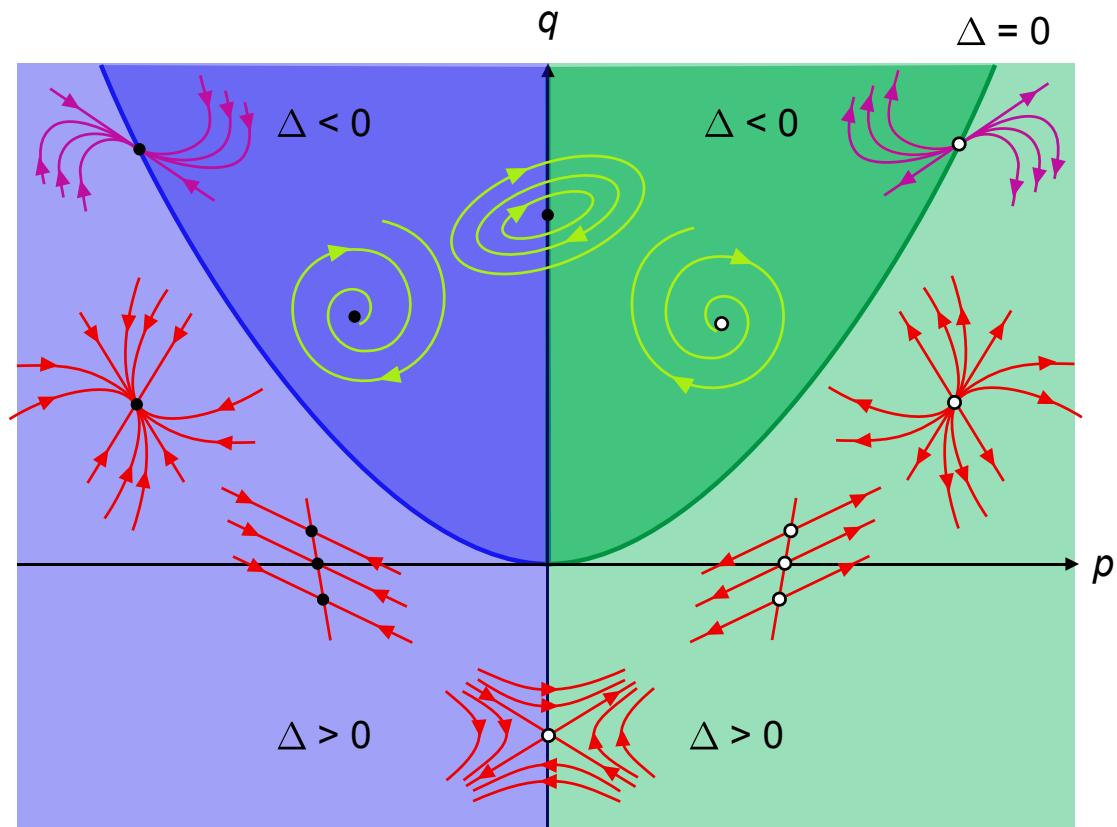
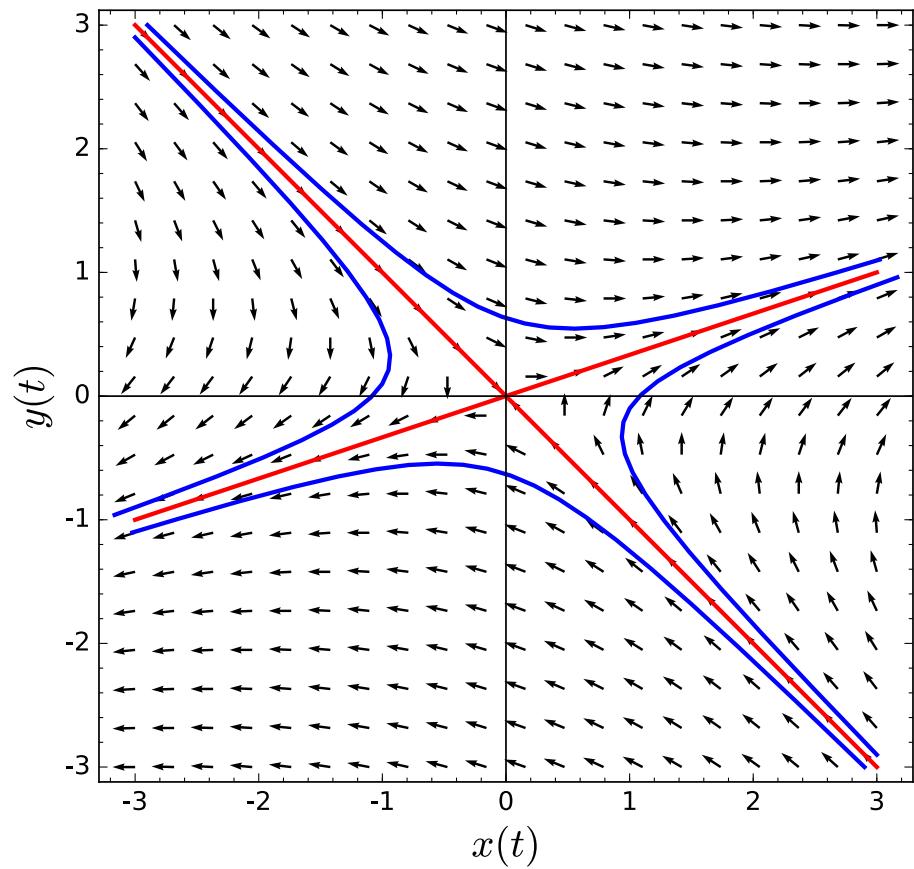
Fitzhugh-Nagumo Model

從有週期行為的Van der Pol方程改來的



Phase Plane Analysis

數值上幫助對一個動態系統有全面的了解



$$\begin{aligned}\frac{dx}{dt} &= Ax + By \\ \frac{dy}{dt} &= Cx + Dy\end{aligned}$$

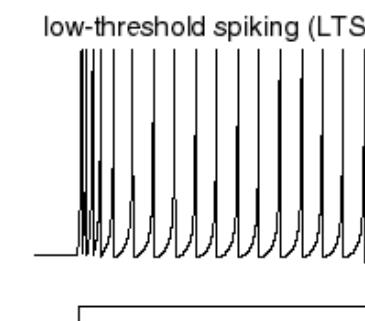
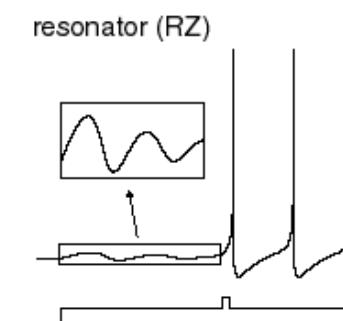
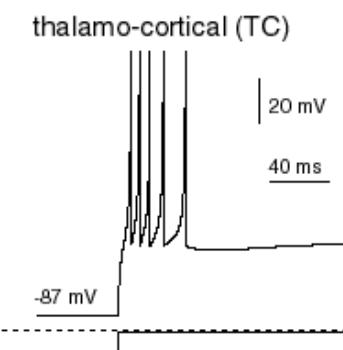
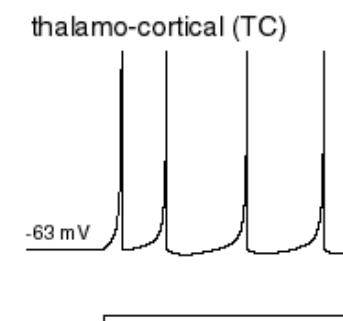
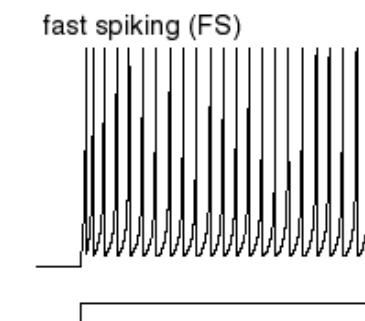
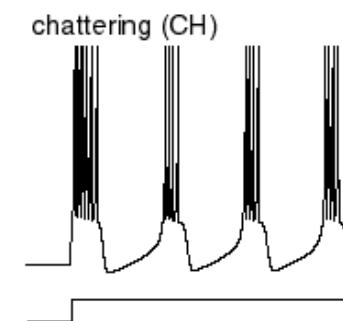
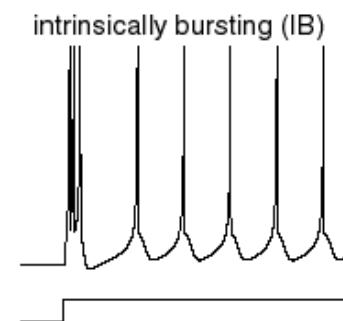
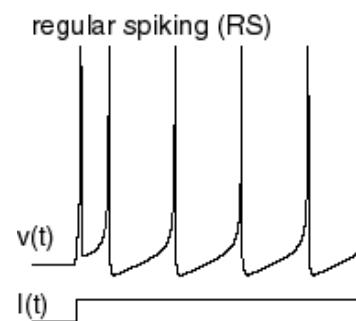
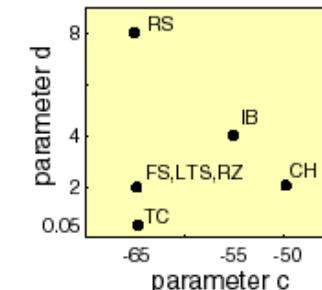
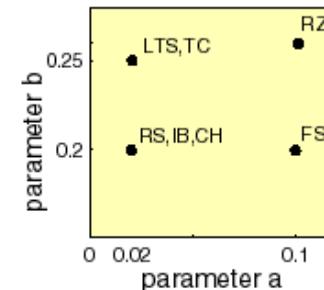
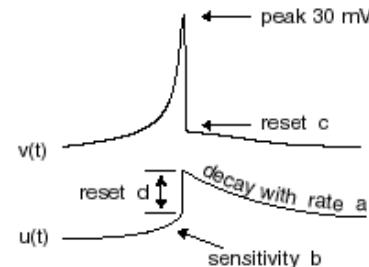
$$\begin{aligned}p &= A + D \\ q &= AD - BC \\ \Delta &= p^2 - 4q\end{aligned}$$

Izhikevich Model (1/2)

修改 quadratic IF model: $dV/dt = V^2 + I$ 而來的

$$v' = 0.04v^2 + 5v + 140 - u + I$$
$$u' = a(bv - u)$$

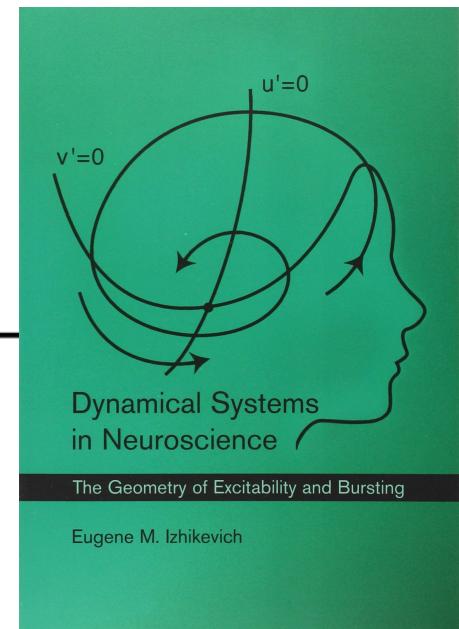
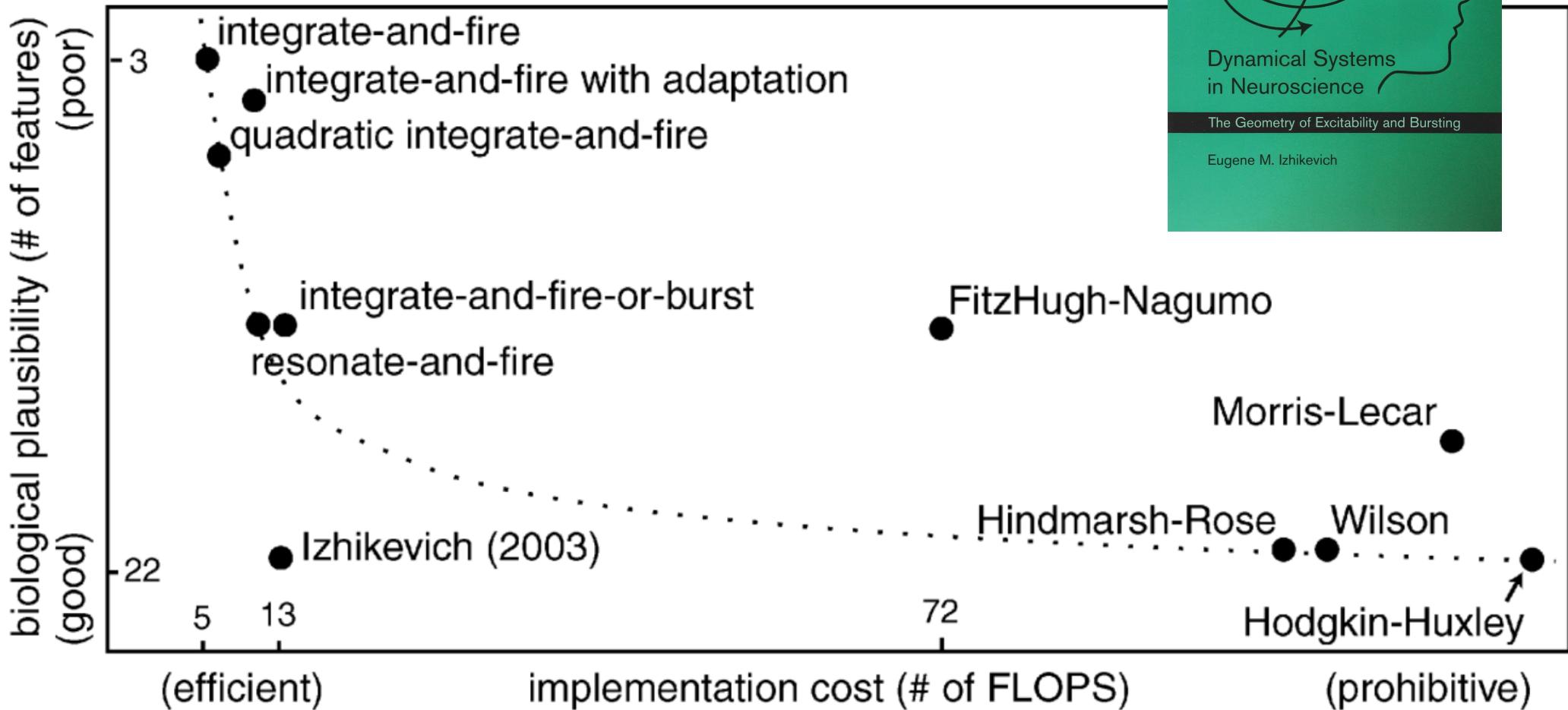
if $v = 30 \text{ mV}$,
then $v \leftarrow c, u \leftarrow u + d$



可產生多種 spiking dynamics

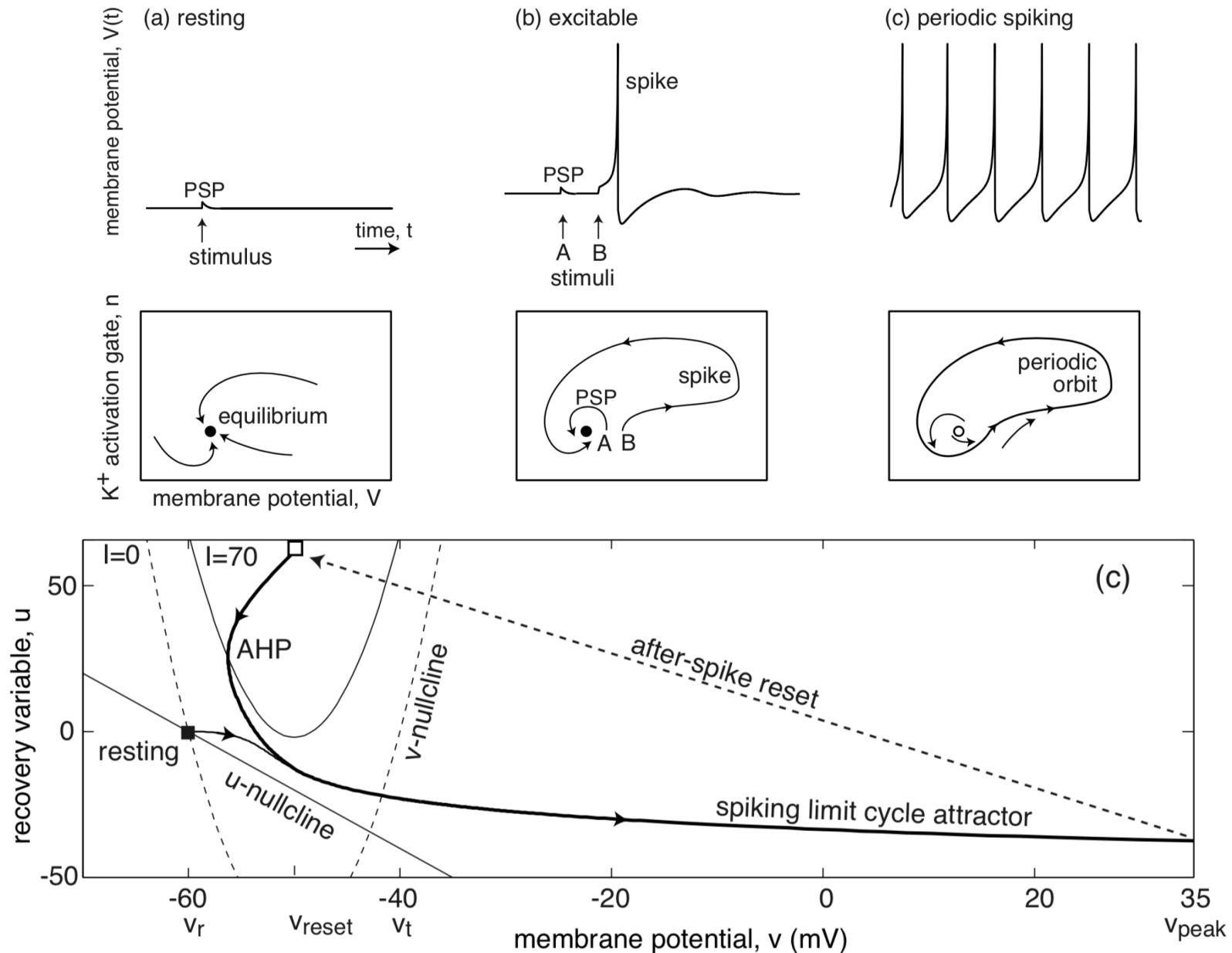
Izhikevich Model (2/2)

推薦看前面約莫 50 頁



Izhikevich 模型建構背後的道理 (1/2)

主要是利用微分方程裡的 phase plane analysis

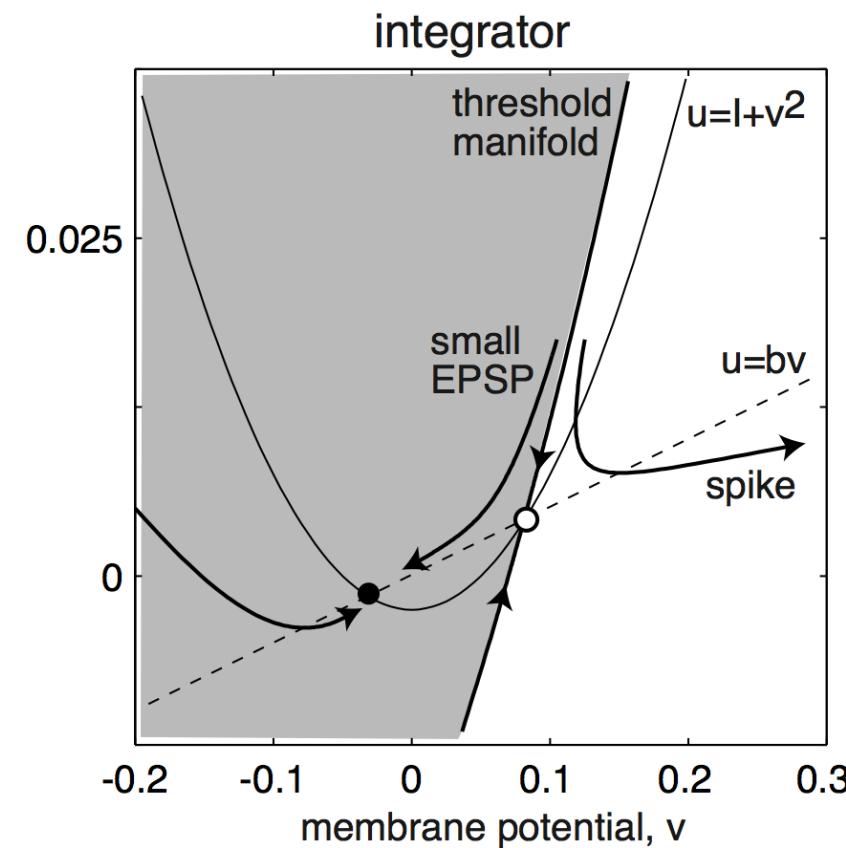


Izhikevich 模型建構背後的道理 (2/2)

兩條 nullclines因模型參數的關係可有不同的交會

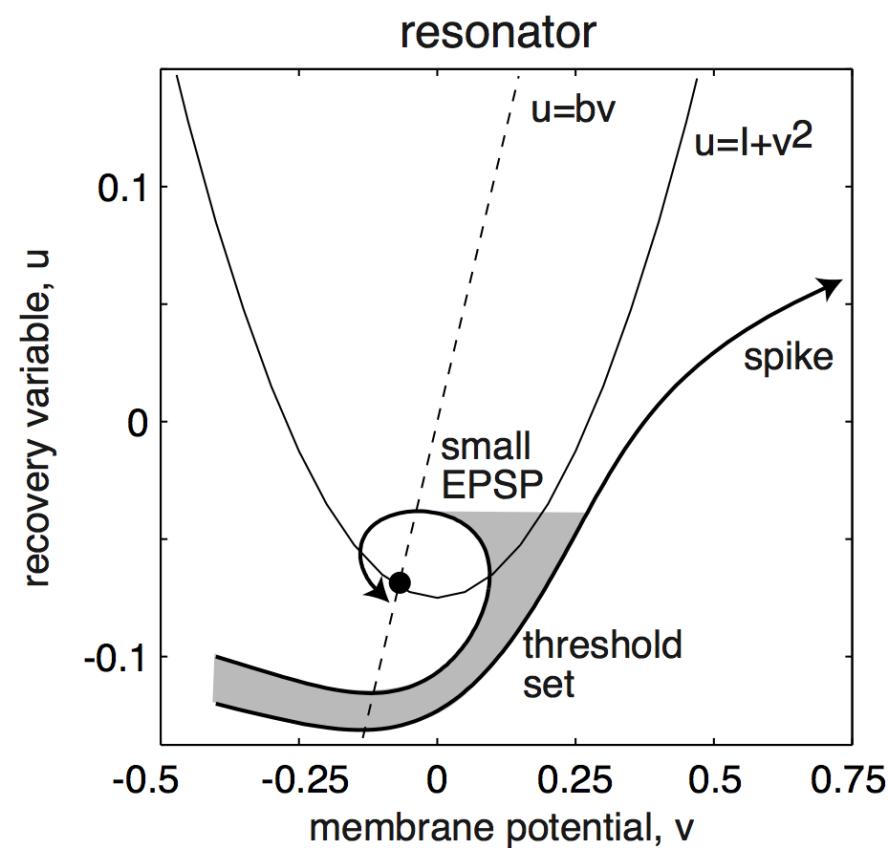
$$\dot{v} = I + v^2 - u$$

$$\dot{u} = a(bv - u)$$



if $v \geq 1$, then

$$v \leftarrow c, u \leftarrow u + d$$



因此模型會因為參數改變有不同的 dynamics

Spiking Dynamics 的分類

可純粹從數學上的 dynamics 來分四類

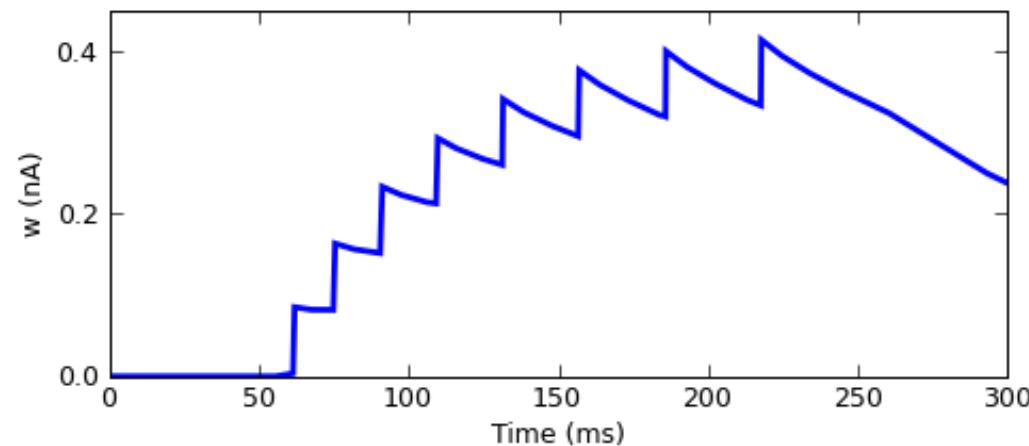
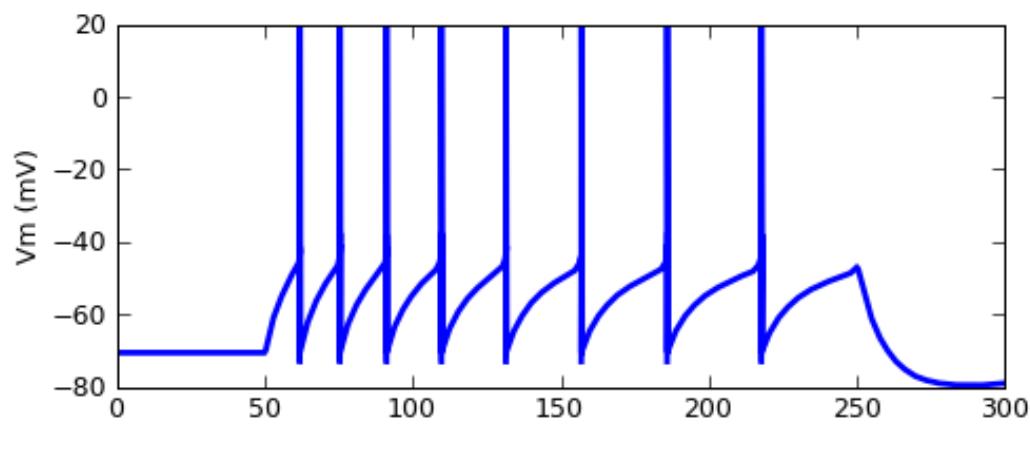
	YES (bistable)	NO (monostable)
NO (integrator)	saddle-node	saddle-node on invariant circle
YES (resonator)	subcritical Andronov-Hopf	supercritical Andronov-Hopf

Adaptive Exponential IF Model

AdEx 修正 Izhikevich 神經元但其實是同一類模型

$$C \frac{dV}{dt} = -g_L(V - E_L) + g_L \Delta_T \exp\left(\frac{V - V_T}{\Delta_T}\right) - w + I$$

$$\tau_w \frac{dw}{dt} = a(V - E_L) - w$$



This is the state-of-the-art!

Game Over

