

Xueguang Lu

## Problem Set 1, CS 5800 Spring 2017

Due: Tuesday, 1/17, 11AM

**Problem 1.** For each of the following statement, say if it is true or false. If true, give a value for the constant  $c$  in the definition of Big-Oh such that the definition is satisfied. Recall that we write  $x^{y^z}$  for  $x^{(y^z)}$ .

1.  $\log(n^2) = O(\log n)$ . true,  $c=3$
2.  $n^2 = O(\binom{n}{2})$ . true,  $c=1$
3.  $n = O(2^{\sqrt{\log n}})$ . false
4.  $\log(n \cdot \log n) = O(\log n)$ . true,  $c=2$
5.  $2^{2^{100} \log n} = n^{O(1)}$ . true,  $c=2^{100}$
6.  $2^{n^2} = 2^{O(n)}$ . false
7.  $(2^n)^2 = 2^{O(n)}$ . true,  $c=3$

**Problem 2.** Return a list of the following functions separated by the symbol  $\equiv$  or  $\ll$ , where  $f \equiv g$  means  $f = \Theta(g)$  and  $f \ll g$  means  $f = O(g)$ . You do not need to justify your answer. For example, if the functions are  $\log n, n, 5n, 2^n$  a correct answer is  $\log n \ll n \equiv 5n \ll 2^n$ . All logarithms are in base 2. One point for each correct symbol.

- |                          |                        |
|--------------------------|------------------------|
| 1. $n$                   | 11. $(n + 1000)^2$     |
| 2. $n^4$                 | 12. $2^{n+\log n}$     |
| 3. $1/n$                 | 13. $\log^2 n$         |
| 4. $n \log n$            | 14. $\log^2(n \log n)$ |
| 5. 1                     | 15. $13^{\log n}$      |
| 6. $n^{1/\sqrt{\log n}}$ | 16. $\log \log n$      |
| 7. $\log n!$             | 17. $n^2$              |
| 8. $2^{n+10}$            | 18. $(1.01)^n$         |
| 9. $n^{1/\log n}$        | 19. $n^{\log n}$       |
| 10. $\log \log^2 n$      | 20. $2^n$              |

$\frac{1}{n} \ll n^{1/\log n} \equiv 1 \ll \log \log n \ll \log \log^2 n \ll \log^2 n \ll$   
 $\log^2(n \log n) \ll n^{1/\sqrt{\log n}} \ll 1 \ll n \ll n \log n \ll \log n!$   
 $\ll n^2 \equiv (n+1000)^2 \ll 13^{\log n} \ll n^4 \ll n^{\log n} \ll 1.01^n$   
 $\ll 2^n \equiv 2^{n+10} \ll 2^{n+\log n}$