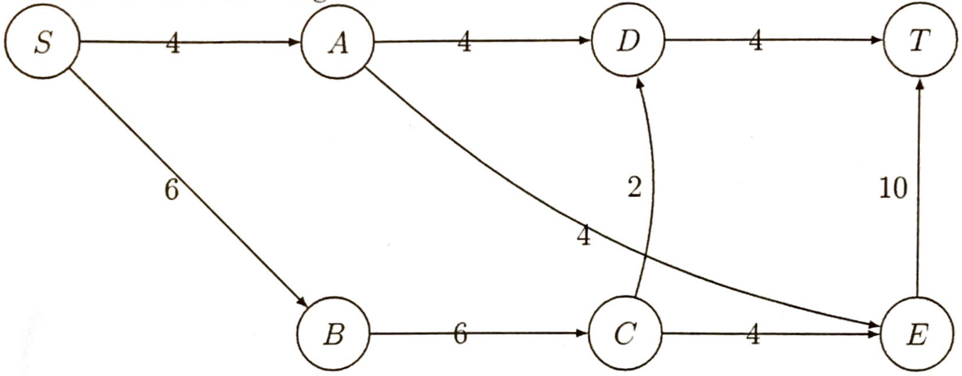
Problem 1. (Difficulty 1) Give the definition of an AA tree and explain how to insert a node in an AA Tree. Describe all the functions involved.

Problem 2. (Difficulty 2) Compute the maximum flow of the following flow network using the Edmonds-Karp algorithm, where the source is S and the sink is T. Show the residual network after each flow augmentation. Write down the maximum flow value.



Problem 3. (Difficulty 3) You are given a directed graph G with nodes $\{1, 2, \ldots, n\}$. A node i may only have edges to nodes j such that $j > i$. Give a polynomial-time algorithm to compute the longest path from 1 to n. (Note that this graph does not have cycles, and so such a path is well defined.)

C S n T

Problem 4. (Difficulty 4) A school has S students, C classes, and T teams. Each student belongs to exactly one class, but is a member of one or more teams. Each team wants to select a member to be its representative in the student council. But a student cannot represent more than one team. The school also does not want to have more than 10 students in the council coming from the same class. Describe an algorithm to determine if it is possible to form a student council subject to these constraints.

k n

Problem 5. (Difficulty 4) The solitaire game STONES is played on an $n \times k$ grid of squares (n rows and k columns). In the initial configuration each square is occupied either by a green stone, or by a red stone, or is empty. You play the game by removing stones. You win if you can remove a set of stones so that:

(1) Every row only has stones of the same color, and

(2) Every column has at least one stone.

Let STONES $= \{g : g$ is an $n \times k$ grid configuration that you can win$\}$. Reduce 3SAT to STONES. (Hint: Given a 3SAT instance ϕ with n variables and k clauses, construct a configuration where the (i, j) square depends on variable x_i and clause C_j.)

Problem 6. (Difficulty 2) Consider the following linear program on variable x and y:

Minimize $x + y$

subject to $2x + 3y = 1$ and $x \geq 0$ and $y \geq 0$.

Starting from vertex $(1/2, 0)$, solve the program using the simplex algorithm. Describe each step of the algorithm.

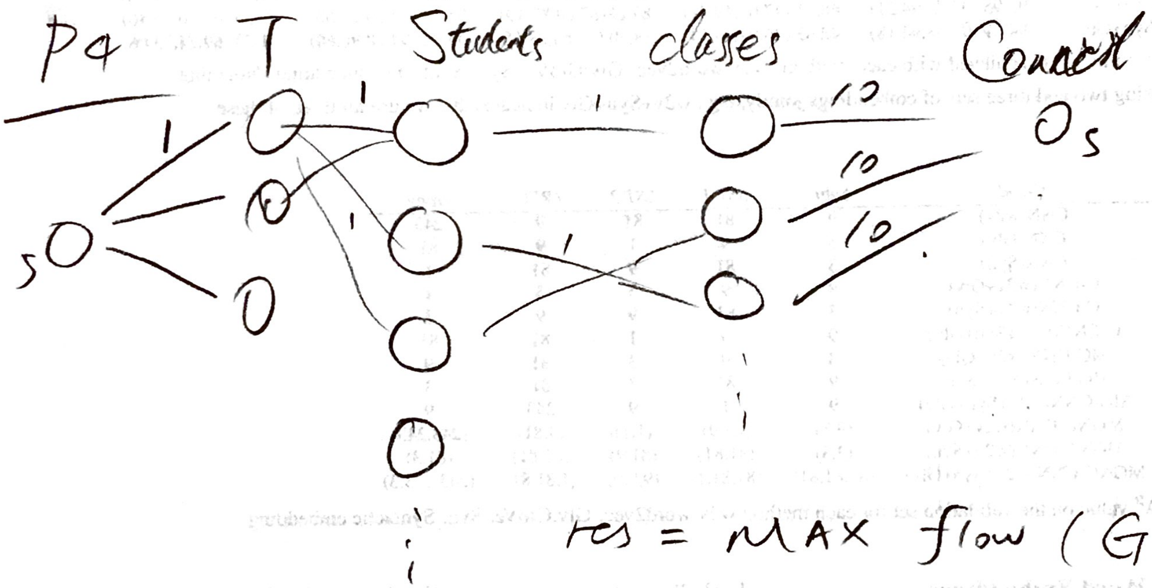
P3

$LP(i) = MAX(\ LP(x)\ \text{if}\ E(x,i)\ \text{for}\ x\ \text{in}\ \{1 \sim i\}\ )+1$

$O(n^2)$

res = $LP(j)$

P4



res = MAX flow (G) == |T|

P5 3SAT → STONES.

R(3CNF):

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Grid[|C|][|Vars|] ← empty.
i = 0
for v1,v2,v3 in 3CF:
    Grid[V1~3][i] = Green if not negated else Red.
    i++

return Grid.
```