Problem Set 7, CS 5800 Spring 2017

Due: Mon, 4/17, 11AM

Problem 1. Suppose we have one machine and a set of n tasks a_1,a_2,. . .,a_n, each of which requires time on the machine. Each task a_j requires t_j time units on the machine (its processing time), yields a profit of p_j, and has a deadline d_j. The machine can process only one task at a time, and task a_j must run without interruption for t_j consecutive time units. If we complete task a_j by its deadline d_j, we receive profit p_j, but if we complete it after its deadline, we receive no profit.

A *schedule* σ gives the start time $\sigma(j)$ for every task a_j with the following property: for all j, the interval $[\sigma(j), \sigma(j) + t_j)$ are pairwise disjoint. The profit $\pi(\sigma)$ achieve by σ is given by the following sum:

$$\sum_{j:\sigma(j)+t_j \leq d_j} p_j$$

Reduce the SUBSET-SUM problem to the problem SCHEDULING defined as follows: Given n tasks, where each task j is specified by its processing time t_j, profit p_j, and deadline d_j, and given an integer P, is there a schedule σ such that $\pi(\sigma) \geq P$?

Prove that your reduction works.

Problem 2. Reduce 3SAT to SYSTEM, where the latter is defined as follows. A *linear inequality* is an inequality involving sums of variables and constants, such as $x + y \geq z$, $x \leq -17$, and so on. A system of linear inequalities has an *integer* solution if it is possible to substitute integer values for the variables so that every inequality in the system becomes true. The language SYSTEM consists of systems of linear inequalities that have an integer solution. For example,

$$(x + y \geq z, x \leq 5, y \leq 1, z \geq 5) \in \text{SYSTEM}$$
$$(x + y \geq 2z, x \leq 5, y \leq 1, z \geq 5) \notin \text{SYSTEM}$$

Also prove that your reduction works.

Problem 3. For an integer k, k-COLOR is the problem of deciding if the nodes of a given undirected graph G can be colored using k colors in such a way that no two adjacent vertices have the same color.

Prove that 4-COLOR is hard by reducing 3-COLOR to 4-COLOR. Exhibit a reduction and prove that your reduction works.