part1

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1 Assignment 1

1.1 Report the underlying probabilistic model and comment on the quality of fit and prediction and model.

The probability model is:

$$\mathrm{Y}_i = \beta_0 + \beta_1 \cdot \mathrm{Channel}_1 + \beta_2 \cdot \mathrm{Channel}_2 + \cdots + \beta_{100} \cdot \mathrm{Channel}_{100} + \epsilon, \quad \epsilon \sim N(0, \sigma^2)$$

```
## The MSE of training data is: 0.005709117
## The MSE of test data is: 722.4294
```

The model performs well on the training data (MSE = 0.0057), but has a large error on the test data (MSE = 722.4294), indicating that the model cannot be generalized and there is a serious overfitting phenomenon.

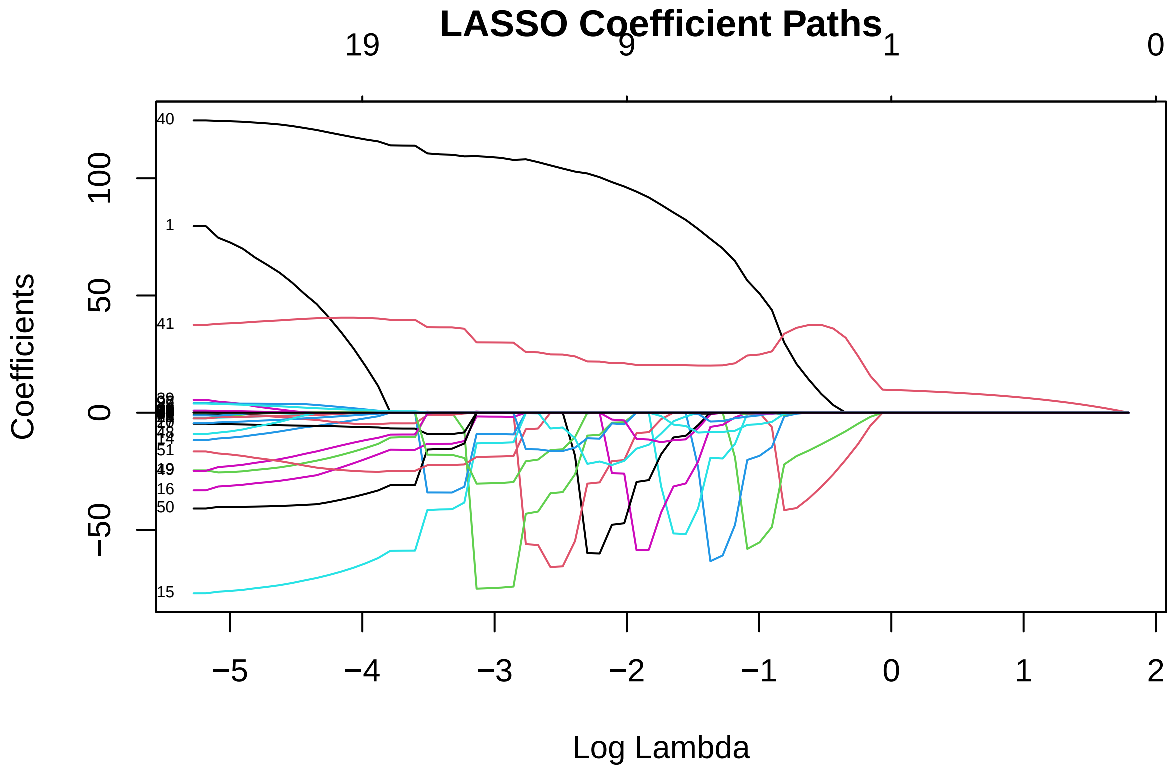
1.2 Report the cost function

The cost function for **LASSO regression** is defined as:

$J(\beta) = \frac{1}{2n} \sum_{i=1}^{n} \left(y_i - \mathbf{X}_i \cdot \beta\right)^2 + \lambda \sum_{j=1}^{p} |\beta_j|$

Where lambda controls the regularization strength.

1.3 interpret LASSO Coefficient Paths Plot



1. This graph shows the path that the regression coefficient of each feature in the **LASSO regression model** changes with the regularization parameter $\log(\lambda):$ the horizontal axis is $\log(\lambda)$, and the vertical axis is the value of the regression coefficient for each feature.
2. With the increase of λ (from right to left), the regularization force is enhanced, and the coefficients of most features are compressed to 0, leaving only a few important features retaining non-zero coefficients, which realizes feature selection.

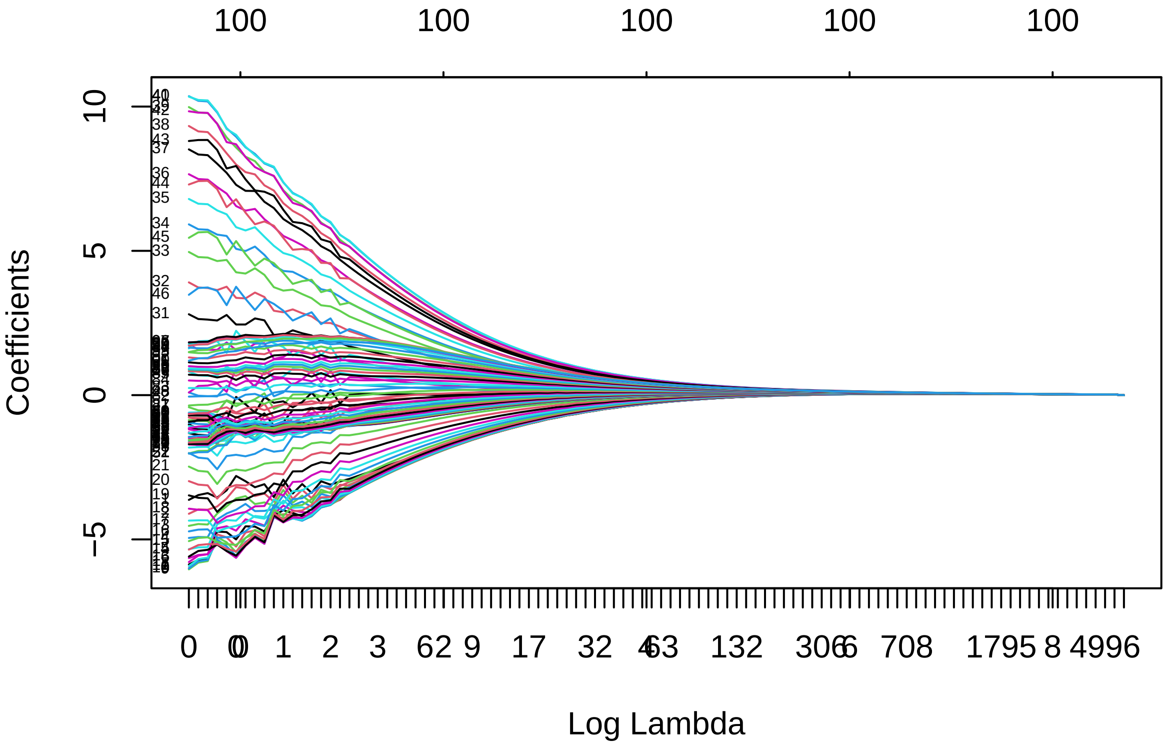
1.4 model with only three features

```
## Lambda with 3 non-zero coefficients: 0.8530452
```

```
## The remaining features are:
```

```
##  Channel6  Channel7 Channel41
## -5.580149 -1.793787 15.696612
```

1.5 Fit Ridge regression and compare the plots obtained in steps 3 and 4



LASSO coefficient path diagram:

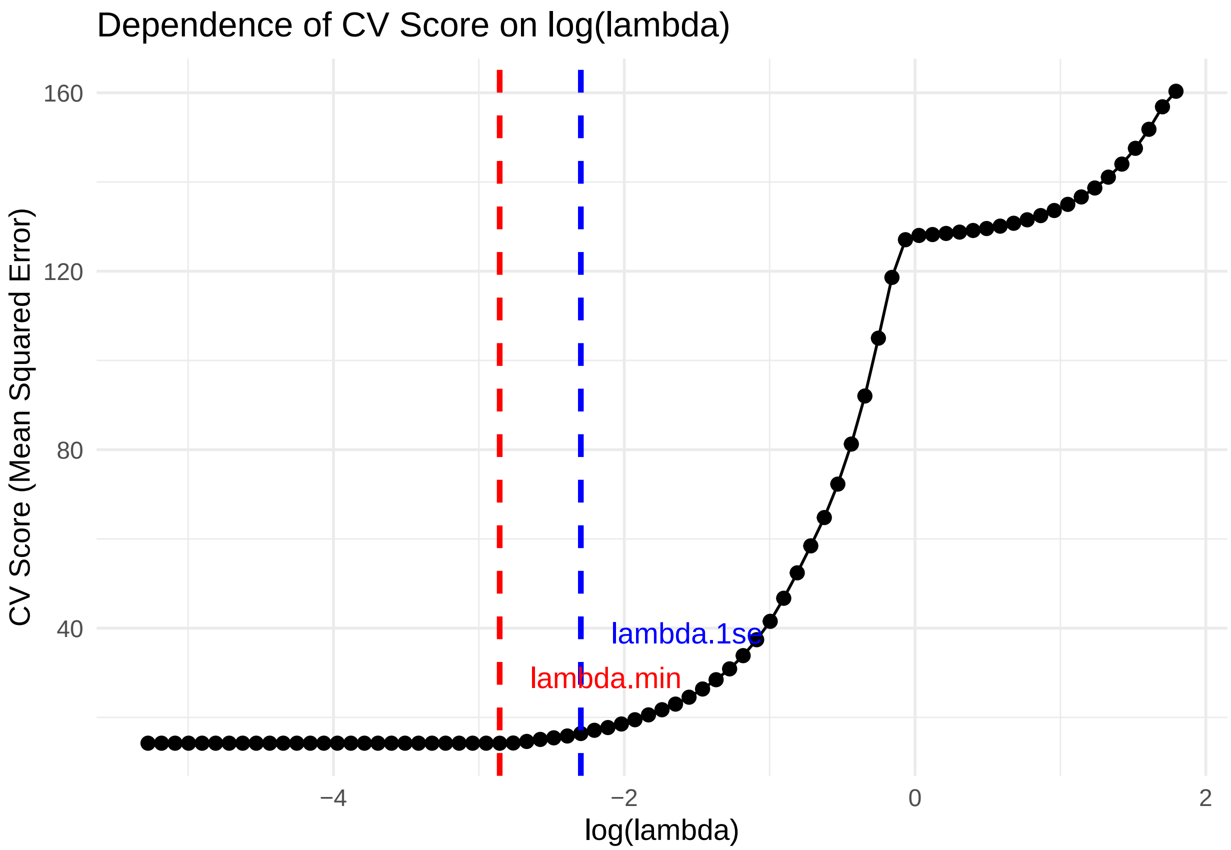
As λ increases, the coefficients of many features are compressed to 0, and the model becomes sparse.

Ridge coefficient path map:

As λ increases, the coefficients of all features gradually shrink, but they do not become zero.

1.6 Task 5

1.6.1 Dependence of the CV score on $\log(\lambda)$



Dependence of CV score on $\log(\lambda)$: As $\log(\lambda)$ increases, the CV scores increase.At the optimal $\log(\lambda)$, the cv score is the lowest, indicating the best performance. When the $\log(\lambda)$ value is large, the effect of the regularization term increases, forcing the model coefficients to shrink, leading to underfitting.

1.6.2 Optimal *lambda*

```
## the optimal lambda:  0.05744535
```

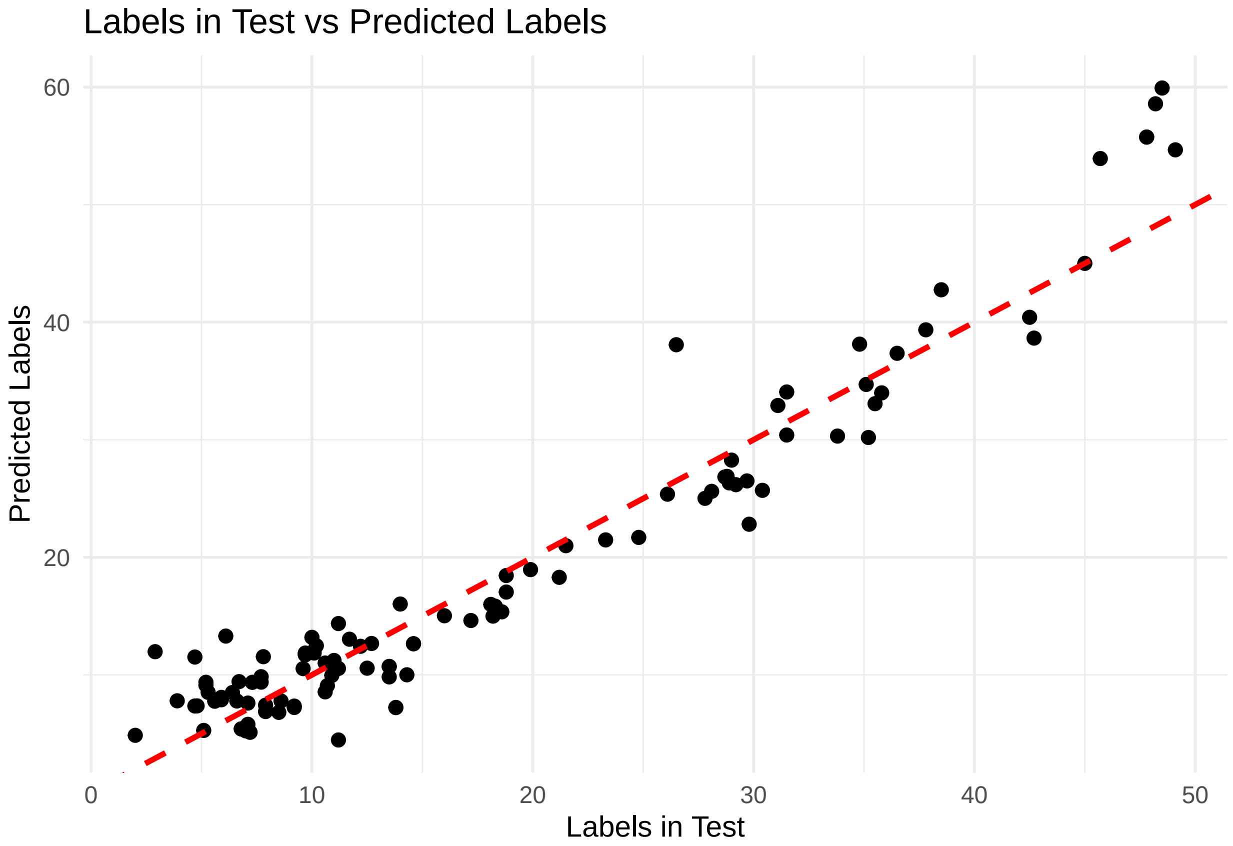
```
## the number of variables:  8
```

```
## log_lambda = -4, mse =  13.48375
```

```
## lambda =  0.05744535 mse =  13.67339
```

The difference in MSE values is very small, approximately 0.19, which is unlikely to be statistically significant.

1.6.3 Scatter plot of the original test versus predicted test values



The model performs well overall, as most predictions are close to the actual values.

The scatterplot shows that the model captures the general trend of the data, with no significant systematic errors.

But a few outliers at higher values suggest the model may struggle with extreme cases or higher variability in predictions for larger labels.

2 Assignment 3

2.1 Task 1

```
## At least  2182 components to obtain a 95% of variance in the data.
```

```
## The proportion of variance of explained by each of the two principal components:
```

```
## The first components:  0.2501699
```

```
## The second components:  0.1693597
```

2.2 Task 2

2.2.1 Trace plot of the first principle component.

2.2.2 Top 5 contributing features

```
## Top 5 features that contributed mostly to the first principle component:  medFamInc medIncome PctKid
```

medFamInc:median family income (differs from household income for non-family households

medIncome:median household income

PctKids2Par: percentage of kids in family housing with two parents

pctWInvInc:percentage of households with investment / rent income in 1989

PctPopUnderPov:percentage of people under the poverty level

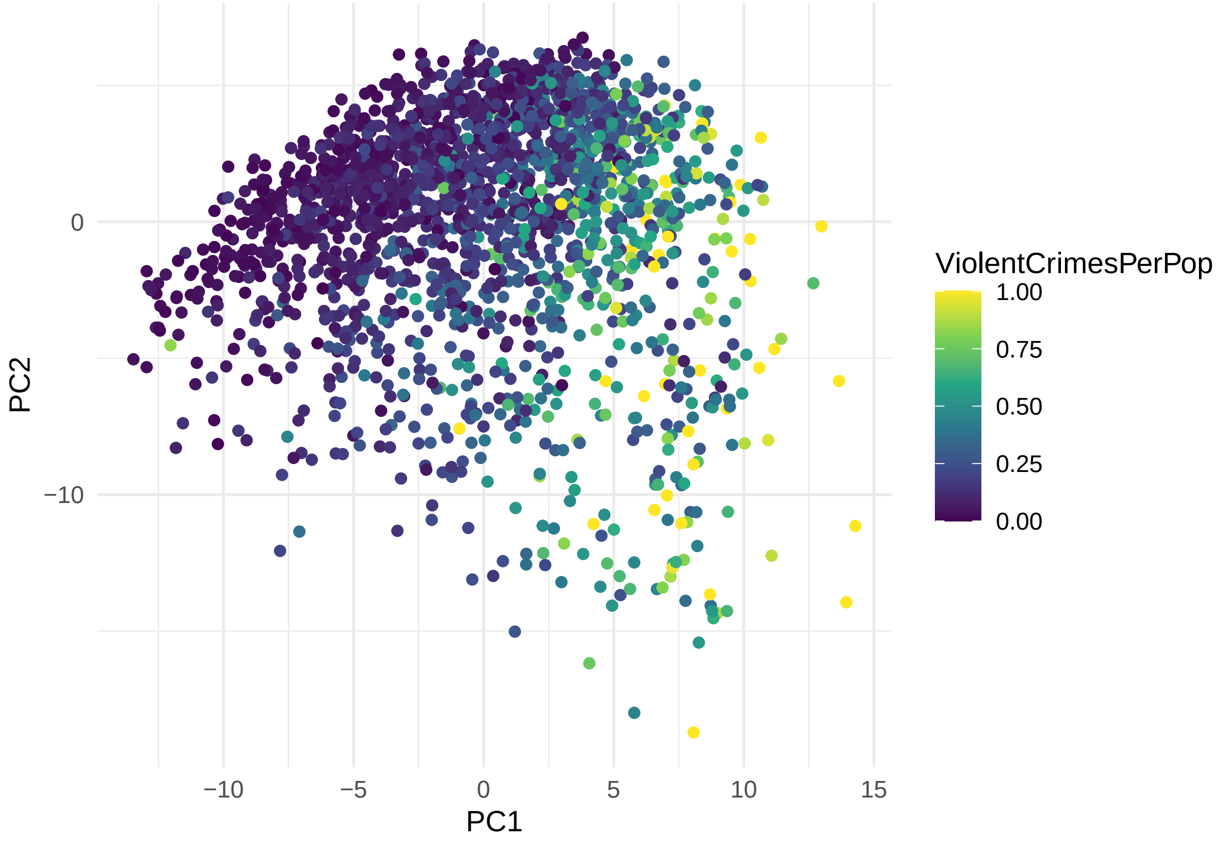
Conclusion:

These five features mainly describes the following aspects:

1.Median family income; 2.Median household income; 3.Percentage of kids in family housing with two parents; 4.Percentage of households with investment / rent income; 5.Percentage of people under the poverty level.

All five features are related to family property and household wealth levels. The logical relationship to crime levels is that families with fewer financial resources or more children (which increases financial budget) may be more likely to resort to illegal activities to obtain money. Financial stress within household may be a significant factor contributing to crime level.

2.2.3 plot of the PC scores in the coordinates (PC1, PC2)



1. The score of the data point on the first principal component (PC1) is significantly positively correlated with the violent crime rate ("ViolentCrimesPerPop"), which tends to increase when PC1 increases (yellow).
2. The second principal component (PC2) has a weak ability to distinguish the violent crime rate, and the distribution of data points in this direction is relatively symmetrical, and there is no obvious pattern.
3. The overall arc distribution of the data indicates that there may be a nonlinear relationship, and further analysis of the characteristic contribution of PC1 is needed to understand the key factors affecting the violent crime rate.

2.3 Task 3

After training the linear regression model.

```
## MSE for the training data is : 0.2752071
```

```
## MSE for the test data is : 0.4248011
```

```
## The R^2 value for the model is:  0.7245166
```

1. The MSE of the model on the training data is 0.275, and the MSE on the test data is 0.425. The test error is slightly higher but generally close, indicating that the model is stable and the degree of overfitting is low.

2. The coefficient of determination R^2 is 0.725, indicating that the model can explain 72.5% of the variance of violent crime rate, but 27.5% of the variation is still unexplained.
3. A slight increase in the test error may indicate the need to optimize features or introduce regularization methods to further improve the generalization ability of the model.

2.4 Task 4

3 Assignment 4. Theory

3.1 What are the practical approaches for reducing the expected new data error, according to the book?

To achieve minimal E_{new}, according to the decomposition $E_{new} = E_{train}$ + generalization gap, we need to have E_{train} as well as the generalization gap small.

1. Increasing the size of the training data to reduce the generalization gap and E_{new};
2. If $E_{hold-out} \approx E_{train}$ (small generalization gap; possibly underfitting), it might be beneficial to increase the model flexibility by loosening the regularization, increasing the model order (more parameters to learn), etc.
3. If E_{train} is close to zero and $E_{hold-out}$ is not (possibly overfitting), it might be beneficial to decrease the model flexibility by tightening the regularization, decreasing the order (fewer parameters to learn), etc.

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3.2 What important aspect should be considered when selecting minibatches, according to the book?

1.It's important to ensure the different mini-batches are balanced and representative for the whole dataset.If we have a few different output classes and the dataset is sorted with respect to the output, the mini-batch with the first n data points would only include one class and not give a good approximation of the gradient for the full dataset.

2.The mini-batches should be formed randomly. To ensure each mini-batch with representativeness, the training data should be shuffled randomly before diving into mini-batches. And after completing one epoch, the dataset should be reshuffled before another epoch.

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