

Q1. (2 points) Using propositional resolution, show the following propositional sentence is unsatisfiable.

$$(p \mid q \mid \neg r) \ \& \ ((\neg r \mid q \mid p) \rightarrow ((r \mid q) \ \& \ \neg q \ \& \ \neg p))$$

To do this, convert this sentence to clausal form and derive the empty clause using resolution.

Conversion to clausal form

I        ...  
N        ...  
D         $(p \mid q \mid \neg r) \ \& \ ((r \ \& \ \neg q \ \& \ \neg p) \mid ((r \mid q) \ \& \ \neg q \ \& \ \neg p))$   
O         $(p \mid q \mid \neg r) \ \&$   
           $((r \ \& \ \neg q \ \& \ \neg p) \mid (r \mid q)) \ \&$   
           $((r \ \& \ \neg q \ \& \ \neg p) \mid \neg q) \ \&$   
           $((r \ \& \ \neg q \ \& \ \neg p) \mid \neg p)$   
          ...  
          {p, q, -r}  
          {r, q}  
          {-q, r, q}  
          {-p, r, q}  
          {r, -q}  
          {-q}  
          {-p, -q}  
          {r, -p}  
          {-q, -p}  
          {-p}

Now do the resolution steps.

Q2. (4 points) Every horse can outrun every dog. Some greyhounds can outrun every rabbit. Show that every horse can outrun every rabbit.

$\forall x. \forall y. (\text{Horse}(x) \wedge \text{Dog}(y) \Rightarrow \text{Faster}(x, y))$

$\exists y. (\text{Greyhound}(y) \wedge \forall z. (\text{Rabbit}(z) \Rightarrow \text{Faster}(y, z)))$

$\forall y. (\text{Greyhound}(y) \Rightarrow \text{Dog}(y))$  (*background knowledge*)

$\forall x. \forall y. \forall z. (\text{Faster}(x, y) \wedge \text{Faster}(y, z) \Rightarrow \text{Faster}(x, z))$  (*background knowledge*)

$\neg \forall x. \forall y. (\text{Horse}(x) \wedge \text{Rabbit}(y) \Rightarrow \text{Faster}(x, y))$  negated conclusion

Now turn them to clausal form and do the resolution steps.

E.g. the second one becomes

$\{\text{Greyhound}(\text{Rocky})\}$

$\{\neg \text{Rabbit}(z), \text{Faster}(\text{Rocky}, z)\}$

where “Rocky” is just a skolemization constant (any name would do).

Q5. Redo the probability calculation for pits in [1,3], [2,2], [3,1], assuming that each square contains a pit with probability 0.01, independent of the other squares. What can you say about the relative performance of a logical versus a probabilistic agent in this case?

$P(p_{13}|b_{12}, b_{21})$  ends up to be quite low (you should find the exact value)

$P(p_{31}|b_{12}, b_{21})$  ends up to be quite low (you should find the exact value)

$P(p_{22}|b_{12}, b_{21})$  ends up to be quite high (you should find the exact value)

Going to [2,2] is almost certain death. So, a probabilistic agent will never choose to go to [2,2]. On the other hand, to a logical agent, squares [1,3], [2,2], [3,1] look the same. So, the logical agent would choose either one with equal chance (1/3). By doing that, the agent will die with a chance of about 1/3.

Q6. [Adapted from a CMU machine learning assignment]

As part of a comprehensive study of the role of CMU 10-601 (Machine Learning) on people's happiness, CMU has been collecting data from graduating students. In an optional survey, the following questions were asked:

- Do you party frequently [Party: Yes/No]?
- Are you wicked smart [Smart: Yes/No]?
- Are you very creative [Creative: Yes/No]? (Please only answer Yes or No)
- Did you do well on all your homework assignments? [HW: Yes/No]
- Do you use a Mac? [Mac: Yes/No]
- Did your course project succeed? [Project: Yes/No]
- Did you succeed in your most important class (which is 10-601)? [Success: Yes/No]
- Are you currently Happy? [Happy: Yes/No]

You can obtain the comma-separated survey results from the accompanying file. Each row in *students.csv* corresponds to the responses of a separate student. The columns in *students.csv* correspond to each question (random variable) in the order Party, Smart, Creative, HW, Mac, Project, Success, and Happy. The entries are either zero, corresponding to a No response, or one, corresponding to a Yes response. After consulting a behavioral psychologist, they obtained the following complete set of conditional relationships:

- HW depends only on Party and Smart
- Mac depends only on Smart and Creative
- Project depends only on Smart and Creative
- Success depends only on HW and Project
- Happy depends only on Party, Mac, and Success

1. (1 pt) Draw the Bayesian network.
2. (2 pt) Estimate the probabilities of the conditional probability tables using the data provided (you can use Excel pivot tables for counting).

Please see the Excel examples posted with the assignment. If you are not comfortable with Excel, you could create a small program in the language of your choice to compute the probabilities (fractions) from the data. Or, you can use an SQL database and derive the counts using queries with GROUP BY.

3. (2 pts) What is the probability of being happy given that you party often, are wicked smart, but not very creative? Show details of computation.

All you need to do here is specify the sums and conditional probabilities at a high level as follows.

$P(h | p, s, -c)$   
 $= \alpha * \sum_{\text{specify the hidden vars}} (\text{specify the conditional probabilities, e.g. } P(-c)P(s)P(p)P(pr|-c,s) \dots)$   
 $= \alpha * (\dots)$

$P(-h | p, s, -c)$   
 $= \alpha * \sum_{\text{specify the hidden vars}} (\text{specify the conditional probabilities})$   
 $= \alpha * (\dots)$

$\alpha = 1 / (P(h | p, s, -c) + P(-h | p, s, -c))$

No need to plug in the numbers and compute the sums and alpha. That would be too tedious.

$P(\text{happy} = T | \text{party} = T, \text{smart} = T, \text{creative} = F) = 0.6922$  (using the tool)

4. (2 pts) What is the probability of being happy given that you are wicked smart and very creative? No details required. Use the AIspace tool.
5. (0.5 pts) What is the probability of being happy given you do not party, and do well on all your homework and class project? No details required. Use the AIspace tool.
6. (0.5 pts) What is the probability of being happy given you own a mac? No details required. Use the AIspace tool.
7. (0.5 pts) What is the probability that you party often given you are wicked smart? No details required. Use the AIspace tool.
8. (0.5 pts) What is the probability that you party often given you are wicked smart and happy? No details required. Use the AIspace tool.