Report

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1 Introduction

Introduction

2 Model

We model this problem as a Vehicle Routing Problem with Time Windows and Capacity Constraints (VRPTWCC)(Schneider 2016). Consider a set of customers and a distribution center $\mathcal{N} = \{0, 1, 2, ..., N\}$, in which 0 represents the distribution center and $1, 2, ...N$ represent customer locations. Deliveries happen on a graph $G(V, E)$, where $V = \{i : i \in \mathcal{N}\}$ denotes customer locations and the distribution center, $E = \{(i, j) : i, j \in \mathcal{N}\}$ denotes the road between location i and j. Each customer i has a package of volume v_i and weight w_i. A truck is chosen from a set of trucks $\mathcal{K} = \{1, 2, ...K\}$ to load packages of several customers and deliver with the route that minimizes the cost. Here we assume that the number of trucks is given in advance. This assumption is relaxed later in simulation optimization. There is a fixed cost c_f associated with each truck out for delivery and a travel cost c_t accumulates per hour for each truck. Every hour that a customer waits for the delivery, a waiting cost c_w will be generated to model the impatience of that customer. Trucks are assumed to travel in a constant speed, thus travel time between location i and j is also constant and already given. We use d_{ij} and t_{ij} to denote distance and travel time between location i and j. When trucks are loading packages, the total volume and weight of packages should not exceed the maximal volume and weight, V_{max} and W_{Max}, respectively. Powered by electricity, trucks have a travel limit D_{max} before recharging. We assume that delivery starts at 9:00 am and customers expect to receive packages before 9:00 pm, which forms a delivery window of $T_{max} = 12$ hours.

Route of a truck is chosen by a binary variable x_{ij}^k. If truck k chooses to travel from i to j, x_{ij}^k is 1; otherwise it is 0. For the time windows, another variable y_i is defined to record the time that customer i is served. Note that in the model we choose the start time at 0, so that y_0 will always be 0. Objective is to minimize the total cost of operation and of customer waiting, which consists of three parts: fixed cost for each trucks, travel cost of each truck measured by time units and customer waiting cost also measured by time units. The problem can be formulated as a Mixed Integer Program.

$$\begin{aligned}
&\textit{Minimize} \quad && \sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{N}} c_f x_{0j}^k + \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\setminus\{i\}} c_t t_{ij} x_{ij}^k + \sum_{i\in\mathcal{N}\setminus\{0\}} c_w y_i, && && (2.1)\\
&s.t. && \sum_{j\in\mathcal{N}\setminus\{0\}} x_{0j}^k = 1 && \forall\ k\in\mathcal{K}; && (2.2)\\
& && \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{N}\setminus\{0,j\}} x_{ij}^k = 1 && \forall\ j\in\mathcal{N}; && (2.3)\\
& && \sum_{i\in\mathcal{N}\setminus\{j\}} x_{ij}^k = \sum_{h\in\mathcal{N}\setminus\{j\}} x_{jh}^k && \forall\ k\in\mathcal{K}, j\in\mathcal{N}; && (2.4)\\
& && \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\setminus\{i\}} d_{ij} x_{ij}^k \le D_{max} && \forall\ k\in\mathcal{K}; && (2.5)\\
& && \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\setminus\{0,i\}} w_j x_{ij}^k \le W_{max} && \forall\ k\in\mathcal{K}; && (2.6)\\
& && \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\setminus\{0,i\}} v_j x_{ij}^k \le V_{max} && \forall\ k\in\mathcal{K}; && (2.7)\\
& && M\cdot(1-x_{ij}^k) + y_j \ge y_i + t_{un} + t_j && \forall\ k\in\mathcal{K}, i,j\in\mathcal{N}; && (2.8)\\
& && y_i + t_{un} + t_{i0} \le T_{max} && \forall\ i\in\mathcal{N} && (2.9)\\
& && x_{ij}^k \in \{0,1\} && \forall\ k\in\mathcal{K}, i,j\in\mathcal{N}; && (2.10)\\
& && y_i \ge 0 && \forall\ i\in\mathcal{N}. && (2.11)
\end{aligned}$$

3 Conclusion

Conclusion

References

Schneider, Michael (2016). “The vehicle-routing problem with time windows and driver-specific times”. *European Journal of Operational Research* 250, pp. 101–119.