FL. 2018 ESE 447.02 Robotics Lab

Lab Journal 17

11/13/2018

1. State –space control method for balancing the second link.
   In one of the previous labs, we derived the Lagrangian dynamics for the system, and we got the equation:
   $$\ddot{q} = m^{-1}(g)\left(\begin{bmatrix} v \\ 0 \end{bmatrix} - C(q,\dot{q})\begin{bmatrix} \dot{q1} \\ \dot{q2} \end{bmatrix}\right) - f(q) - g(q)$$
   The state for the system can be determined by the variant
   $$x = \begin{bmatrix} q1 \\ \dot{q}1 \\ q2 \\ \dot{q}2 \end{bmatrix}$$
   In this lab, we were trying to linearize the equation into a form shown below.
   $$\dot{x} = Ax + Bu$$
   Where A and B are matrices we need to find.
   We know that matrix A takes the form shown below:
   $$A = \begin{bmatrix} \frac{\partial f_{1(0,0)}}{\partial q_1} & \frac{\partial f_{1(0,0)}}{\partial \dot{q_1}} & \frac{\partial f_{1(0,0)}}{\partial q_2} & \frac{\partial f_{1(0,0)}}{\partial \dot{q_2}} \\ \frac{\partial f_{2(0,0)}}{\partial q_1} & \frac{\partial f_{2(0,0)}}{\partial \dot{q_1}} & \frac{\partial f_{2(0,0)}}{\partial q_2} & \frac{\partial f_{2(0,0)}}{\partial \dot{q_2}} \\ \frac{\partial f_{3(0,0)}}{\partial q_1} & \frac{\partial f_{3(0,0)}}{\partial \dot{q_1}} & \frac{\partial f_{3(0,0)}}{\partial q_2} & \frac{\partial f_{3(0,0)}}{\partial \dot{q_2}} \\ \frac{\partial f_{4(0,0)}}{\partial q_1} & \frac{\partial f_{4(0,0)}}{\partial \dot{q_1}} & \frac{\partial f_{4(0,0)}}{\partial q_2} & \frac{\partial f_{4(0,0)}}{\partial \dot{q_2}} \end{bmatrix}$$
   Where
   $$f_1 = \dot{q_1} = \dot{q_1}$$
   $$f_2 = \ddot{q_1}$$
   $$f_3 = \dot{q_2} = \dot{q_1}$$
   $$f_4 = \ddot{q_2}$$
   We first need to derive the symbolic functions for all elements in A.
   It could be tell from first sight that A_{11}, A_{13}, A_{14}, A_{21}, A_{31}, A_{32}, A_{33} and A_{41} are zeros according to whether the function there is a function of the element that we are taking derivatives with respect to. Besides, we could tell that A_{21} and A_{31} are both 1 by taking derivatives with respect to the elements themselves.
   Similarly, we know that matrix B is in a form shown below:

$$B = \begin{bmatrix} \frac{\partial f_{1(0,0)}}{\partial u_1} \\ \frac{\partial f_{2(0,0)}}{\partial u_1} \\ \frac{\partial f_{3(0,0)}}{\partial u_1} \\ \frac{\partial f_{4(0,0)}}{\partial u_1} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\partial f_{2(0,0)}}{\partial u_1} \\ 0 \\ \frac{\partial f_{4(0,0)}}{\partial u_1} \end{bmatrix}$$

Where u1 is the voltage v we input into the first link motor.

2. Symbolic derivations for A and B and substitution of theta values.
   Before we plug the values in, we had to derivate the formulas for the elements in the matrices symbolically since we could not take derivatives of numbers. We used the equations we used for the dynamics exercise for getting the state variables. Then we substituted the theta values for the real system we got from the previous Hamilton exercise. The Matlab code is shown below:

```
syms theta1 theta2 theta3 theta4 theta5 theta6;
syms q1 q1_dot q2 q2_dot;
syms v

V=[v;0]

A = [theta1+theta2*(sin(q2)^2), theta3*cos(q2) ; theta3*cos(q2), theta2];
B = [2*theta2*sin(q2)*cos(q2)*q2_dot, (-1)*theta3*sin(q2)*q2_dot; (-theta2*sin(q2)*cos(q2)*q1_dot,0];
C = [theta5*q1_dot ; theta6*q2_dot];
D = [0;-theta4*9.8*sin(q2)];

q_dd = (A^(-1))*((V - B*[q1_dot ; q2_dot] - C - D)) ;
q1_dd = q_dd(1)
q2_dd = q_dd(2)

partial22 = diff(q1_dd,q1_dot);
p22 = subs(partial22,[q1,q1_dot,q2,q2_dot],[0,0,0,0]);
p22_val = subs(p22,[theta1,theta2,theta3,theta4,theta5,theta6],[0.0785, 0.0272,0.0238, 0.1145, 0.5436,
0.0107]);

partial23= diff(q1_dd,q2);
p23 =subs(partial23,[q1,q1_dot,q2,q2_dot],[0,0,0,0]);
p23_val = subs(p23,[theta1,theta2,theta3,theta4,theta5,theta6],[0.0785, 0.0272,0.0238, 0.1145, 0.5436,
0.0107]);

partial24= diff(q1_dd,q2_dot);
p24 = subs(partial24,[q1,q1_dot,q2,q2_dot],[0,0,0,0]);
p24_val = subs(p24,[theta1,theta2,theta3,theta4,theta5,theta6],[0.0785, 0.0272,0.0238, 0.1145, 0.5436,
0.0107]);

partial42 = diff(q2_dd,q1_dot);
p42 = subs(partial42,[q1,q1_dot,q2,q2_dot],[0,0,0,0]);
p42_val = subs(p42,[theta1,theta2,theta3,theta4,theta5,theta6],[0.0785, 0.0272,0.0238, 0.1145, 0.5436,
0.0107]);

partial43 = diff(q2_dd,q2);
p43  = subs(partial43,[q1,q1_dot,q2,q2_dot],[0,0,0,0]);
p43_val = subs(p43,[theta1,theta2,theta3,theta4,theta5,theta6],[0.0785, 0.0272,0.0238, 0.1145, 0.5436,
0.0107]);

partial44 = diff(q2_dd,q2_dot);
p44 = subs(partial44,[q1,q1_dot,q2,q2_dot],[0,0,0,0]);
p44_val = subs(p44,[theta1,theta2,theta3,theta4,theta5,theta6],[0.0785, 0.0272,0.0238, 0.1145, 0.5436,
0.0107]);

P = [ 0 1 0 0; 0 p22 p23 p24;0 0 0 1;0 p42 p43 p44]
P_val = [0 1 0 0; 0 p22_val p23_val p24_val;0 0 0 1;0 p42_val p43_val p44_val]

B_partial2 = diff(q1_dd,v);
b2 = subs(B_partial2,[q1,q1_dot,q2,q2_dot],[0,0,0,0]);
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b2_val = subs(b2,[theta1,theta2,theta3,theta4,theta5,theta6],[0.0785, 0.0272,0.0238, 0.1145, 0.5436,
0.0107]);

B_partial4 = diff(q2_dd,v);
b4 = subs(B_partial4,[q1,q1_dot,q2,q2_dot],[0,0,0,0]);
b4_val = subs(b4,[theta1,theta2,theta3,theta4,theta5,theta6],[0.0785, 0.0272,0.0238, 0.1145, 0.5436,
0.0107]);
Q = [0;b2;0;b4]
Q_val = [0;b2_val;0;b4_val]
poles = [-5,-5.1,-5.2,-5.3];
k = place(P_val,Q_val,poles)
```

The A, B, C, and D matrices were already defined for the $\ddot{q}$ formula above, so we used P and Q to represent the A and B matrices in the linearized formula. As shown in the code, we took the derivatives of the q1_dd and q2_dd with respect to q1, q1_dot, q2 and q2_dot to get the elements in the P and Q matrices. The P_val and Q_val are these two matrices after substituting the theta values. The poles were given and we calculated the constant k by using the “place” function in Matlab.