FL. 2018 ESE 447.02 Robotics Lab

Lab Journal 9

10/2/2018

1. Derive the Lagrangian Dynamics with friction and actuator effects in MATRIX form.

   The Lagrangian dynamics equation is as follows:

   $$[\theta_1 + \theta_2 \sin^2(q_2)]\ddot{q}_1 + \theta_3 \cos(q_2)\ddot{q}_2 + 2\theta_2 \sin(q_2)\cos(q_2)\dot{q}_1\dot{q}_2 - \theta_3 \sin(q_2)\dot{q}_2^2 + \theta_5\dot{q}_1 = v$$

   $$\theta_3 \cos(q_2)\ddot{q}_1 + \theta_2\ddot{q}_2 - \theta_2 \sin(q_2)\cos(q_2)\dot{q}_1^2 - \theta_4 g \sin(q_2) + \theta_6\dot{q}_2 = 0$$

   The equations are arranged in the matrix form shown below where v represents voltage and 'q' represents the generalized coordinate system (joint variables).

   $$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} m_{11}(q) & m_{12}(q) \\ m_{21}(q) & m_{22}(q) \end{bmatrix} * \begin{bmatrix} \ddot{q_1} \\ \ddot{q_2} \end{bmatrix} + \begin{bmatrix} c_{11}(q,\dot{q}) & c_{12}(q,\dot{q}) \\ c_{21}(q,\dot{q}) & c_{22}(q,\dot{q}) \end{bmatrix} * \begin{bmatrix} \dot{q_1} \\ \dot{q_2} \end{bmatrix} + \begin{bmatrix} f_1(\dot{q}_1) \\ f_2(\dot{q}_2) \end{bmatrix} + \begin{bmatrix} g_1(q_1) \\ g_2(q_2) \end{bmatrix}$$

   If we just expand the matrix multiplication and compare the terms in the results with that in the equations, we can decide the elements in M, C, F, and G matrices.

   The results are as follows:

   $$M=\begin{bmatrix} \theta_1 + \theta_2 sin^2(q2) & \theta_3\cos(q2) \\ \theta_3\cos(q2) & \theta_2 \end{bmatrix}$$

   $$C=\begin{bmatrix} 2\theta_2\sin(q2)\cos(q2)\dot{q}2 & -\theta_3\sin(q2)\dot{q}2 \\ -\theta_2\sin(q2)\cos(q2)\dot{q}1 & 0 \end{bmatrix}$$

   $$F=\begin{bmatrix} \theta_5\dot{q}1 \\ \theta_6\dot{q}2 \end{bmatrix}$$

   $$G=\begin{bmatrix} 0 \\ -\theta_4 g sin(\dot{q}2) \end{bmatrix}$$

2. Calculate θ values
   The provided parameters are as follows.

J_1=0.0012	m_2=0.127	(l_1+l_2')=0.2
l_2=0.3	l_{c2}=0.15	β_1=0.015
β_2=0.002	R_a=2.6	k_t=k_v=0.00768
k_r=70		

   The θ' s are calculated using the following equations.

   $$\theta'_1 = J_1 + m_2(l_1 + l'_2)^2$$

   $$\theta'_2 = \frac{1}{3}m_2(l_2)^2$$

   $$\theta'_3 = \frac{1}{2}m_2(l_1 + l'_2)l_2$$

The results are as follows.

$$\theta_1' = 0.00628$$

$$\theta_2' = 0.00381$$

$$\theta_3' = 0.00381$$

$$\theta_4' = 0.01905$$

Then the θs can be calculated using the following equations.

$$\begin{aligned}
\theta_i &= \theta_i' \frac{R_a}{k_r k_t} \quad i = 1, \ldots, 4\,, && (1)\\
\theta_5 &= \beta_1 \frac{R_a}{k_r k_t} + k_r k_v, && (2)\\
\theta_6 &= \beta_2 \frac{R_a}{k_r k_t}. && (3)
\end{aligned}$$

The results are as follows.

$$\theta_1 = 0.03037$$

$$\theta_2 = 0.01843$$

$$\theta_3 = 0.01843$$

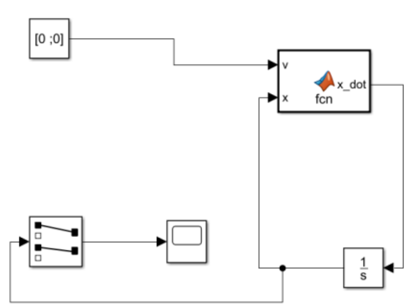
$$\theta_4 = 0.09213$$

$$\theta_5 = 0.6101$$

$$\theta_6 = 0.00967$$

3. Build a Simulink model and write the Matlab function needed to compute the status parameters.

   The Simulink model looks like follows.



Where the upper left block is the voltage input and the x input is the integrations of $\dot{x}$.

Note that:

$$x = \begin{bmatrix} q1 \\ \dot{q1} \\ q2 \\ \dot{q2} \end{bmatrix}$$

And that:

$$\dot{x} = \begin{bmatrix} \dot{q1} \\ \ddot{q1} \\ \dot{q2} \\ \ddot{q2} \end{bmatrix}$$

The Matlab function code is as follows:

```
function x_dot = fcn(v,x)
%#codegen

theta1= 0.03037
theta2 = 0.018143
theta3 = 0.18143
theta4 = 0.09213
theta5 = 0.6101
theta6 = 0.009673

q1 = x(1);
q1_dot = x(2);
q2 = x(3);
q2_dot = x(4);

%v = [0 ; 0]
A = [theta1 + theta2*(sin(q2)^2) theta3*cos(q2) ; theta3*cos(q2) theta2]
B = [2*theta2*sin(q2)*cos(q2)*q2_dot (-1)*theta3*sin(q2)*q2_dot; (-1)*theta2*sin(q2)*cos(q2)*q1_dot  0];
C = [theta5*q1_dot ; theta6*q2_dot]
D = [0 ;(-1)*theta4*9.8*sin(q2)]

q_dd = [0;0];
q_dd = (A^(-1))*(v - B*[q1_dot ; q2_dot] - C - D) ;
q1_dd = q_dd(1);
q2_dd = q_dd(2);
x_dot = [q1_dot ;q1_dd ; q2_dot; q2_dd ]

end
```

The function block in the Simulink model outputs the $\dot{x}$ and then it is integrated by the integrator block. The integrated result is then x and fed back into the function clock through the x input. The function will use the parameters in x to repeat the calculations, forming a closed loop.