

# An Improved PSO-Based Power Allocation Algorithm for the Optimal EE and SE Tradeoff in Downlink NOMA Systems

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**Abstract**—Non-orthogonal multiple access (NOMA) technique can substantially improve the spectral efficiency (SE) and increase the throughput of system. As NOMA has these advantages, it has been investigated as a candidate radio access technology for the fifth-generation (5G) mobile communication. In this paper, we study the benefit of NOMA in enhancing energy efficiency (EE) for downlink systems by introducing a certain power allocation scheme. The expected goal is to maximize the EE of the systems while ensuring the SE is guaranteed. The most common method for achieving the tradeoff of EE and SE is the convex optimization. Because the EE of NOMA systems is a strictly quasi-concave function, it is difficult to apply the convex optimization directly. We propose to adopt the improved particle swarm optimization (PSO) algorithm. Furthermore, in order to avoid the premature convergence, *cycle strategy rotation* is used in the algorithm. Numerical results and analysis turn out that the improved PSO algorithm achieves higher EE than the exiting power allocation scheme under the same SE. Moreover, the improved PSO algorithm shows fast convergence and it converges within 9 iterations, which is 35% less than the original PSO algorithm under selected parameters.

**Index Terms**—Non-orthogonal multiple access (NOMA), energy efficiency (EE), spectral efficiency (SE), particle swarm optimization (PSO).

## I. INTRODUCTION

With the development of communication techniques and services, it's necessary to explore new radio for the fifth-generation (5G) wireless communication networks [1]. Many novel techniques have been proposed, such as sparse code multiple access (SCMA), interleaved division multiple access (IDMA), and Non-orthogonal multiple access (NOMA) [2]. We are interested in the NOMA technology, because of its simple implementation and high SE. It can simultaneously improve cell throughput and cell edge throughput. Early researches mainly focused on improving SE of NOMA systems [3]–[5].

In recent years, the exponential increment of traffic volume in wireless networks has triggered booming energy consumption, which contributes to carbon footprint, electromagnetic pollution and high operating costs.

According to the statistics, information and communication technology infrastructures consume over 3% of the world-wide energy, out of which about 60% is consumed by base stations (BSs) [5]. Accordingly, green radio (GR) dedicated to improving EE has become the research hotspot in both industry and academia. In [6], the bisection method was used to maximize EE in downlink NOMA systems. The method solved a non-convex multivariate optimization problem. As the problem is quasi-concave, the solution is slightly complex.

In this paper, we study on the problem of tradeoff between EE and SE in downlink NOMA systems. SE is defined as system throughput per unit of bandwidth, and EE is defined as the number of bits which can be transmitted per Joule of energy. On the premise of SE guarantee, EE improvement problem is a typical expectation maximization (EM). Many classic solutions have been proposed such as bisection method (BM), evolutionary algorithms (EA), simulated annealing (SA) and particle swarm optimization (PSO) [7].

In 2018, an optimal approach based on PSO was proposed to solve the EE maximization problem in wireless powered sensor network [8]. To achieve EE maximization, PSO algorithm was used to allocate energy from a hybrid access point (H-AP) to each sensor. Note that the objective function of wireless sensor network is within the scope of expectation maximization function, as the same as that of NOMA systems. As discussed above, we propose applying the PSO algorithm in the NOMA systems [9]. More importantly, *cycle strategy rotation* is used to the iteration of each particle, which takes both the deep search and breadth search into account. The improved PSO algorithm can enhance the population diversity and avoid the premature convergence [10]. The power allocation coefficient of NOMA systems is considered as a particle in the algorithm. A particle is iterated through the algorithm to derive the optimal power allocation coefficient. The EE of NOMA systems is taken as the fitness value of particles. We get the optimal SE and EE tradeoff by iteratively searching the best power allocation coefficient.

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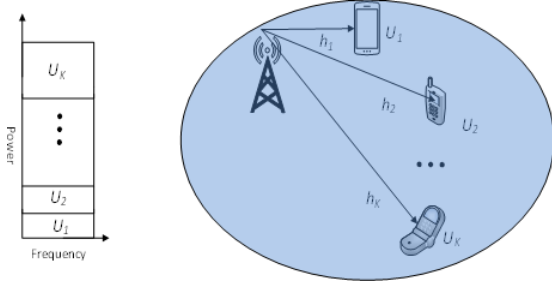


Fig. 1. System model of single-cell NOMA.

The rest of this paper is organized as follows. Section II presents the system model and the optimization problem. Then, the improved PSO algorithm is applied to solve the formulated problem in section III. The performance comparisons among our proposed scheme and other schemes are presented and analyzed in section IV. Finally, section V concludes the paper.

## II. SYSTEM MODEL

### A. System Model of Downlink NOMA

A downlink single-cell is considered with a BS simultaneously serving  $K$  users, as shown in Fig. 1. The received signal of the  $i$ -th user  $U_i$  is given by

$$y_i = h_i x + n_i, i = 1, 2, \dots, K \quad (1)$$

where the channel coefficient  $h_i$  between the BS and  $U_i$  is modeled as  $h_i = g_i P_i L^{-1}(d_i)$ .  $g_i$  is the Rayleigh fading coefficient.  $P_i$  is the transmission power allocated to  $U_i$ .  $L^{-1}(d_i)$  is the path loss function between  $U_i$  and BS where  $d_i$  denotes the distance.  $x$  is the signal transmitted by the BS containing  $s_i$ , where  $s_i$  is the data intended for  $U_i$ .  $s_i$  is assumed to be normalized, that is,  $E\{s_i^2\} = 1$ .  $n_i$  is the additive white Gaussian noise with zero mean and variance  $\sigma^2$ .

In NOMA systems, the transmitted signal is superposed as

$$x = \sum_{i=1}^K \sqrt{P_i} s_i \quad (2)$$

Without loss of generality, the user channel gains are sorted as  $|h_1|^2 \geq |h_2|^2 \geq \dots \geq |h_K|^2$ . According to the principle of successive interference cancellation (SIC), the  $U_i$  can decode the signal of  $U_j$  ( $j < i$ ) and remove this signal from its received signal and consider the remaining users' signals as noise.

Then the achievable rate of the central user  $U_i$  can be formulated as

$$R_1 = W \log \left( 1 + \frac{h_1^2 P_1}{N_0} \right) \quad (3)$$

where  $W$  is the total available bandwidth at the BS. The data rate of other users can be formulated as

$$R_i = W \log \left( 1 + \frac{h_i^2 P_i}{\sum_{j=i+1}^K h_j^2 P_j + N_0} \right) \quad (4)$$

Then the sum rate of the NOMA systems can be formulated as

$$R_{tot} = W \log \left( 1 + \frac{h_1^2 P_1}{N_0} \right) + \sum_{i=2}^K W \log \left( 1 + \frac{h_i^2 P_i}{\sum_{j=i+1}^K h_j^2 P_j + N_0} \right) \quad (5)$$

### B. Problem Formulation of EE and SE Tradeoff for NOMA Systems

This section formulates the problem of tradeoff between EE and SE in the downlink NOMA systems. The total power consumption at the BS is  $P = \sum_{i=1}^K P_i$ . The EE and SE of the NOMA systems are defined as

$$\eta_{EE} = \frac{\sum_{i=1}^K R_i}{W \sum_{i=1}^K P_i} \quad (6)$$

$$\eta_{SE} = \frac{\sum_{i=1}^K R_i}{W} \quad (7)$$

Limited by transmit power of BS and minimum rate of user, this paper wants to optimize the problem of tradeoff between SE and EE by adjusting power allocation. The tradeoff problem is formulated as an optimization problem in which the basic objective is to maximize EE  $\eta_{EE}$  under a satisfying SE  $\eta_{SE}$  requirement.

Generally,  $P = [P_1, P_2, \dots, P_K]^T$  is adopted to denote the power allocation vector.  $e_i = \frac{P_i}{P}$ , ( $i = 1, 2, \dots, K$ ) is used to represent the  $i$ -th power allocation factor. Then the optimization problem can be expressed with  $e_i$  as

$$\begin{aligned} \eta_{EE} &= \frac{1}{P} \log \left( 1 + \frac{h_1^2 e_1 P}{N_0} \right) \\ &+ \frac{1}{P} \sum_{i=2}^K \log \left( 1 + \frac{h_i^2 e_i P}{\sum_{j=i+1}^K h_j^2 e_j P + N_0} \right) \\ \text{s.t. } &\sum_{i=1}^K e_i = 1, i = 1, 2, \dots, K \\ &e_i > 0, i = 1, 2, \dots, K \end{aligned} \quad (8)$$

Accordingly, the problem can be formulated as

$$\begin{cases} \max_{T,P} & \eta_{EE} \\ \text{s.t.} & C_1 : 0 < P_i < P \\ & C_2 : R_i \geq R_i^{\min}, i = 1, 2, \dots, K \\ & C_3 : 0 < e_1 \leq e_2 \leq \dots \leq e_K < 1 \end{cases} \quad (9)$$

where  $R_i^{\min}$  is the minimum data rate required by user  $U_i$ . Constraint  $C_2$  guarantees the minimum data rates of scheduled users. Constraint  $C_3$  ensures that the transmit power of each cell-edge user is greater than that of each cell-center user.

The derivative of  $\eta_{EE}$  with respect to  $P$  is represented by

$$\frac{d^2 \eta_{EE}}{d^2 P} < 0, \quad \frac{d \eta_{EE}}{d P} > 0 \quad (10)$$

*Proof.* See Appendix A

It is proved that the optimization problem in this paper is strictly quasi-concave. In addition, the convex optimization can't converge to the optimal value in finite time because of the fixed search path. In light of the above reasons, PSO algorithm is apply to this problem.

### III. IMPROVED PSO ALGORITHM FOR NOMA SYSTEMS

For the original PSO, each particle represents a potential approach to the optimization problem. In order to find the fitness value, each particle adjusts its flight based on not only its own flight experience, but also the flight experience of others. Assuming that the search space is  $D$ -dimensional, and  $K$  particles compose a particle swarm.

In PSO, the position of particles are represented by a vector  $X$ . According to the moving experience of that particle and others, its velocity adjusts the path of each particle in the search space. The velocity of particles are represented by a vector  $V$ . The  $i$ -th particle in the population is represented by the position and velocity at the  $t$ -th iteration. Position vector can be expressed as

$$X_i(t) = [x_i^1(t), x_i^2(t), \dots, x_i^D(t)]^T \quad (11)$$

The speed vector can be expressed as

$$V_i(t) = [v_i^1(t), v_i^2(t), \dots, v_i^D(t)]^T \quad (12)$$

As of the  $t$ -th iteration, the best position searched by particle  $i$  is recorded as

$$Pbest_i(t) = [p_i^1(t), p_i^2(t), \dots, p_i^D(t)]^T \quad (13)$$

$Pbest_i(t)$  is also called the local history optimal position. The global optimal position experienced by all particles in a particle swarm is denoted as

$$Gbest(t) = \min\{pbest_1(t), pbest_2(t), \dots, pbest_K(t)\} \quad (14)$$

When entering the next iteration, the position and the velocity can be updated according to Eq.15 and Eq.16.

$$v_i^j(t+1) = \omega \cdot v_i^j(t) + c_1 \cdot r \cdot (pbest_i^j(t) - x_i^j(t)) + c_2 \cdot r \cdot (gbest^j(t) - x_i^j(t)) \quad (15)$$

$$x_i^j(t+1) = x_i^j(t) + v_i^j(t+1) \quad (16)$$

where  $\omega$  is the inertia weight,  $r$  is a random functions in the range  $[0, 1]$ ,  $c_1$  is used to adjust the moving step of the particle to the historical optimal position, and  $c_2$  is used to control the moving step of the particle to the global optimal position in the particle swarm.

The iteration formula of the particle velocity includes three components in the original PSO algorithm. The first part  $\omega \cdot v_i^j(t)$  is mainly used to balance deep search and breadth search. The second part  $c_1 \cdot r \cdot (pbest_i^j(t) - x_i^j(t))$  emphasizes the abilities of particles to search in the local area. The third part  $c_2 \cdot r \cdot (gbest^j(t) - x_i^j(t))$  emphasizes the global search capabilities of particles.

The termination condition of the iteration is generally that the optimal position searched so far satisfies the predetermined minimum adaptive threshold. PSO has the advantages of simple structure, few control parameters and owns a strong global optimization ability. However, the convergence analyses is still needed to investigate and it often trapped the particles into the local optimum when solving complex multimodal problems.

In order to solve the above problem, we introduce *cycle strategy rotation* in the iteration of the particle velocity. The global optimal is essentially a kind of deep search, and the local optimal is essentially a kind of breadth search. The breadth search here refers to a particle that deviates from the original search track and enters a search for a new solution in other locations. The deep search concept means that the current particle performs a local fine search near the current solution to a large extent. Therefore, the improved PSO algorithm strengthens the diversity of the population and avoids the premature convergence. The key process is briefly introduced as follows.

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**Algorithm 1:** *Cycle strategy rotation* in the iteration for the particle velocity of the improved PSO algorithm

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1: For  $i = 1$  to  $K$  do
2:   For  $j = 1$  to  $D$  do
3:     if  $i$  is odd
4:        $v_i^j(t+1) = \omega v_i^j(t) + c_1 \cdot r \cdot (Pbest_i^j(t) - x_i^j(t))$ 
5:        $x_i^j(t+1) = x_i^j(t) + v_i^j(t+1)$ 
6:     else
7:        $v_i^j(t+1) = \omega v_i^j(t) + c_1 \cdot r \cdot (Gbest_i^j(t) - x_i^j(t))$ 
8:        $x_i^j(t+1) = x_i^j(t) + v_i^j(t+1)$ 

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When we use the improved PSO algorithm in the NOMA systems, the specific settings are as follows.

Firstly, the power allocation coefficients  $e_i$  are used as particle's position in the particle swarm.

$$[x_1, x_2, \dots, x_K] = [e_1, e_2, \dots, e_K] \quad (17)$$

Secondly, we set the power allocation coefficient range  $(0, 1)$  as the flight range of particles. Thirdly, the EE of the NOMA systems is considered as the fitness value of the particle swarm, that is  $f = \eta_{EE}$ . The pre-defined fitness function is used to measure the performance of each particle.

According to current speed  $v_i^j(t)$ , the distance between the current position  $e_i^j(t)$  and own optimal position  $Pbest_i$ , as well as the distance between the current position and the best position of the swarm  $Gbest_i$ , cycle strategy rotation is used to calculate the new speed. The overall algorithm is presented in Algorithm 2.

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**Algorithm 2:** The Improved PSO algorithm

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**Input:** Population:  $K$ ; Iteration number:  $T$ ; Dimension:  $D$   
**Output:** the EE of NOMA  $\eta_{EE}$  and the corresponding power allocation coefficient  $e_i(t)$

- 1: **For**  $i = 1$  to  $K$  **do**
- 2:   initialize the position  $e_i^j(t)$  and velocity  $v_i^j(t)$ ;
- 3: **While**  $((|\eta_{EE}(e_i(t))| \geq \varepsilon) \text{ or } (t \leq T))$  **do**
- 4:   **For**  $i = 1$  to  $K$  **do**
- 5:      $\eta_{EE}(e_i(t)) \leftarrow$  Evaluate Fitness of Particle  $e_i(t)$
- 6:     Update the  $Pbest_i$  position of particle  $i$ ;
- 7:     Update the  $Gbest_i$  position in the  $t$ -th iteration
- 8:     repeat **Algorithm 1**
- 9:     **Until** converge
- 10: **Return** the EE of NOMA  $\eta_{EE}$  and the corresponding power allocation coefficient  $e_i(t)$

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#### IV. SIMULATIONS RESULTS

In this section, we present the numerical results of the proposed algorithm for NOMA systems. For simplicity, we set  $K = 2$ . When using the original PSO algorithm and the improved PSO algorithm, the specific setting are as follows  $T = 40$ ,  $C_1 = 2$  and  $C_2 = 2$ . In our simulations, it is assumed that  $U_2$  is located at the edge of the cell and  $U_1$  is located in the center of the cell. The radius of the cell is 500 m. The bandwidth of NOMA systems is 50 MHz and the small-scale fading gain as Rayleigh distributed with  $\sigma_h^2 = 1$ . The transmit power range of BS is set from 1 watt to 30 watt.

In Fig. 2, we compare the EE and SE performance of the improved PSO-based NOMA systems with NOMA systems employing convex optimization, the original PSO, as well as orthogonal multiple access (OMA) systems. It illustrates that  $\eta_{EE}$  slowly decreases as  $\eta_{SE}$  increases, and the performance of the improved PSO algorithm is superior to other algorithms. The reason is that the convex optimization algorithm adopts a fixed search path. The original PSO algorithm and the improved PSO algorithm have global search capability. The improved algorithm combines deep search and breadth search, and it returns a better global optimal position. It is indicated

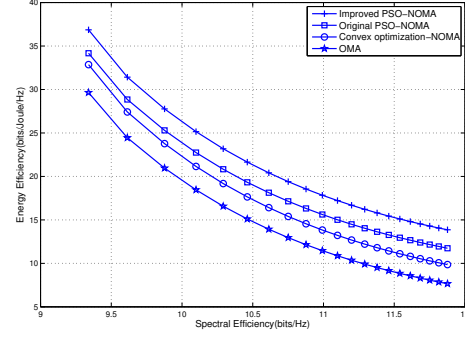


Fig. 2.  $\eta_{EE}$  versus  $\eta_{SE}$  performance comparison for NOMA systems.

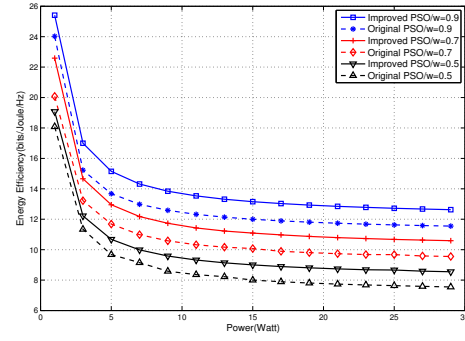


Fig. 3. EE performance versus the power of BS for NOMA systems.

that the optimal allocation coefficient is achieved by the improved PSO algorithm.

In Fig. 3, it compares the EE performance of the improved PSO algorithm with the original PSO algorithm under different power conditions. A sharp drop in  $\eta_{EE}$  appears at the power range of 1 watt to 5 watt. In the case of SE=10 bits/Hz, the improved PSO algorithm can obtain 2 bits/Joule/Hz higher than the original PSO algorithm under the same power condition. Moreover, when the inertia weight  $\omega$  is 0.9, the improved PSO algorithm achieves the highest energy efficiency in the observation range.

In Fig. 4, we can see the EE performance of different inertia weight  $\omega$  with iteration numbers. From Fig. 4, we can see that the EE obtained by this algorithm keeps growing with the number of iterations increasing. When  $\omega$  is 0.9, 0.7 and 0.5, the number iterations of the improved PSO algorithm is 30, 15 and 9, respectively. The iterations of the improved PSO algorithm is reduced by approximately 25%, 27% and 35%, respectively, compared with the original PSO algorithm. Moreover, when the inertia weight  $\omega$  is 0.5, the improved PSO algorithm achieves the fastest convergence speed within the observed range.

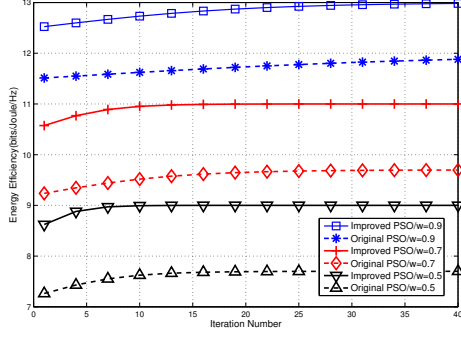


Fig. 4. Convergence performance with different  $\omega$  for PSO algorithm.

## V. CONCLUSION

In this paper, we introduce the improved PSO algorithm to solve the problem of tradeoff between EE and SE in downlink NOMA systems. We prove that the  $\eta_{EE}$  of NOMA systems is strictly quasi-concave. By combining deep search and breadth search, the improved algorithm shows better search effects and higher convergence speed than the original PSO algorithm. Simulation results show that the improved algorithm achieves better EE performance than typical schemes, such as convex optimization and the original PSO algorithm for NOMA systems. Moreover, when the inertia weight  $\omega$  is higher, the algorithm has the better breadth search capability. When it is lower, the algorithm can get faster convergence speed. The balance between the search ability and the convergence speed would be a topic for future work.

## APPENDIX A

### PROOF OF Eq.10

In NOMA systems, the superposition coding of user  $U_1$  and user  $U_2$  are as

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 \quad (18)$$

Without loss of generality, it's assumed that  $\alpha \leq \frac{1}{2}$ . We define the equivalent gain of user  $U_k$  as  $H_k = |h_k|^2 / \sigma^2$  in this paper. The data rate of cell-center user  $U_1$  can be formulated as

$$R_1 = W \log(1 + \alpha H_1 P) \quad (19)$$

The data rate of cell-edge user  $U_2$  can be formulated as

$$R_2 = W \log\left(1 + \frac{(1-\alpha)H_2 P}{1 + \alpha H_1 P}\right) \quad (20)$$

The system EE can be rewritten as

$$\eta_{EE} = \frac{R}{WP} = \frac{f_1(P) + f_2(P)}{WP} = \frac{f_3(P)}{P} \quad (21)$$

where

$$\begin{aligned} f_1(P) &= \log(1 + H_1 P), \\ f_2(P) &= \log\left(1 + \frac{H_2 P(1-\alpha)}{1 + H_1 P \alpha}\right) \end{aligned}$$

and  $f_3(P) = f_1(P) + f_2(P)$ . The derivative  $\frac{df_1(P)}{dP}$  and second derivative  $\frac{d^2 f_1(P)}{dP^2}$  can be calculated as follows,

$$\frac{df_1(P)}{dP} = \frac{1}{\ln 2} \frac{\alpha H_1}{1 + \alpha H_1 P} > 0 \quad (22)$$

$$\frac{d^2 f_1(P)}{dP^2} = \frac{-1}{\ln 2} \frac{\alpha^2 H_1 H_2}{(1 + \alpha H_1 P)^2} < 0 \quad (23)$$

At the same time,  $\frac{df_2(P)}{dP}$  and  $\frac{d^2 f_2(P)}{dP^2}$  can be calculated as follows:

$$\frac{df_2(P)}{dP} = \frac{1}{\ln 2} \frac{(1-\alpha)H_2}{1 + (\alpha H_1 + H_2 - \alpha H_2)P} > 0 \quad (24)$$

$$\frac{d^2 f_2(P)}{dP^2} = \frac{-1}{\ln 2} \frac{[(1-\alpha)H_2][1 + \alpha H_1 + (1-\alpha)H_2]}{[1 + (\alpha H_1 + H_2 - \alpha H_2)P]^2} < 0 \quad (25)$$

Since  $\frac{df_1(P)}{dP} > 0$ ,  $\frac{df_2(P)}{dP} > 0$ ,  $\frac{d^2 f_1(P)}{dP^2} < 0$  and  $\frac{d^2 f_2(P)}{dP^2} < 0$ , we have  $\frac{df_3(P)}{dP} > 0$  and  $\frac{d^2 f_3(P)}{dP^2} < 0$ . Therefore  $f_3(P)$  is a strictly concave function of  $P$ . In addition,  $P$  is affine. Then we can conclude that  $\eta_{EE}$  is strictly quasi-concave of  $P$ . The proof is completed.

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## REFERENCES

- [1] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. Soong, and J. C. Zhang, "What will 5g be?" *IEEE Journal on selected areas in communications*, vol. 32, no. 6, pp. 1065–1082, 2014.
- [2] M. Moltafet, N. M. Yamchi, M. R. Javan, and P. Azmi, "Comparison study between pd-noma and scma," *IEEE Transactions on Vehicular Technology*, no. 99, pp. 1–1, 2017.
- [3] N. Nonaka, Y. Kishiyama, and K. Higuchi, "Non-orthogonal multiple access using intra-beam superposition coding and sic in base station cooperative mimo cellular downlink," *IEICE Transactions on Communications*, pp. 1–5, 2015.
- [4] L. Lei, D. Yuan, C. K. Ho, and S. Sun, "Power and channel allocation for non-orthogonal multiple access in 5g systems: Tractability and computation," *IEEE Transactions on Wireless Communications*, vol. 15, no. 12, pp. 8580–8594, Dec 2016.
- [5] A. Zappone, E. Jorswieck *et al.*, *Energy Efficiency in Wireless Networks via Fractional Programming Theory*. Now Publishers, Inc., 2015.
- [6] Y. Zhang, H.-M. Wang, T.-X. Zheng, and Q. Yang, "Energy-efficient transmission design in non-orthogonal multiple access," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 3, pp. 2852–2857, 2017.
- [7] S. Hui, "Multi-objective optimization for hydraulic hybrid vehicle based on adaptive simulated annealing genetic algorithm," *Engineering Applications of Artificial Intelligence*, vol. 23, no. 1, pp. 27–33, 2010.
- [8] M. Song and M. Zheng, "Energy efficiency optimization for wireless powered sensor networks with non-orthogonal multiple access," *IEEE Sensors Letters*, no. 99, pp. 1–1, 2018.
- [9] J. Robinson and Y. Rahmat-Samii, "Particle swarm optimization in electromagnetics," *IEEE transactions on antennas and propagation*, vol. 52, no. 2, pp. 397–407, 2004.
- [10] X. Chen and Y. Li, "A modified pso structure resulting in high exploration ability with convergence guaranteed," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 37, no. 5, pp. 1271–1289, Oct 2007.