

Energy-Efficient Power Allocation for Non-Orthogonal Multiple Access with Imperfect Successive Interference Cancellation

Hong Wang^{1,2}, Zhaoyang Zhang^{1,2,†}, and Xiaoming Chen^{1,2}

¹College of Information Science and Electronic Engineering, Zhejiang University, Hangzhou, China

²Zhejiang Provincial Key Laboratory of Information Processing, Communication and Networking, Zhejiang, China

E-mail: {whzju, [†]ning_ming, chen_xiaoming}@zju.edu.cn

Abstract—As a promising technology for 5G, non-orthogonal multiple access (NOMA) has attracted much attention from academia and industry due to its superior spectral efficiency and user fairness, but the error propagation in successive interference cancellation decoding (SIC) might seriously restrict its performance. In this paper, we propose to utilize power allocation to mitigate the impact of *imperfect SIC* for a multiuser downlink NOMA system. In particular, we formulate a non-convex fractional programming problem with maximizing the system energy efficiency subject to a minimum data rate constraint for each user. To solve the intractable problem, we develop an iterative algorithm with a fast convergence speed. Simulation results validate that the proposed scheme can effectively alleviate the impact of imperfect SIC and achieve an obvious performance gain over two conventional baseline schemes.

Index Terms—NOMA, imperfect SIC, energy efficiency, power allocation.

I. INTRODUCTION

With the explosive growth of the mobile devices and mobile applications, it is very difficult for the traditional orthogonal multiple access (OMA) technologies to support massive connectivity due to limited radio resources and high signalling overhead [1], [2]. In this context, non-orthogonal multiple access (NOMA) has recently emerged as an enabling solution for massive access in the fifth generation (5G) systems [3]. As expected, NOMA can achieve a higher spectral efficiency through power-domain superposition coding (SC) at the transmitter and successive interference cancellation decoding (SIC) at the receiver [4], [5].

The existing literatures on NOMA have mainly focused on the analysis and optimization of the system sum rate. For instance, asymptotic expressions for the ergodic sum rate and outage probability were derived for a downlink NOMA scenario with randomly deployed users in [6]. The performance of two feasible power allocation schemes, i.e., fixed and cognitive-radio-inspired power allocation, was characterized for a two-user NOMA system [7]. In order to further improve the throughput performance, the relay technique and the multiple-input

multiple-output (MIMO) technique were applied to the NOMA systems in [8] and [9], respectively. In addition to system rate, energy efficiency (EE) has recently drawn significant attention due to the green communication idea [10]. Currently, only a few works have studied NOMA systems from the perspective of EE. In [11], an energy-efficient power allocation scheme was proposed for maximizing the EE of a two-user MIMO-NOMA system. The EE of a multi-user downlink NOMA system was further investigated in [12].

A common point of these previous works on NOMA is that they assume that SIC at the receiver is perfect, and thus the interference from the users with weaker channel gains can be completely cancelled. However, in practical systems, SIC is not a trivial task. Especially in the massive access scenario, wireless devices have distinct detection capabilities. As for some simple devices, the decoding error might be inevitable during the procedure of SIC, and then results in severe performance degradation [1]. As pointed out in [5], there is no prominent research that provides a mathematical understanding of the impact of imperfect SIC on NOMA systems. Besides, most of the prior works only consider a two-user scenario which can greatly restrict the application of NOMA. To solve these practical problems, this paper aims to develop a novel power allocation algorithm to alleviate the effect of imperfect SIC for a multiuser downlink NOMA system. The contributions of this paper are summarized as:

- 1) To the best of the authors' knowledge, we are the first to quantitatively analyze the practical impact of imperfect SIC on the energy efficiency of a multiuser downlink NOMA system.
- 2) We formulate a non-convex optimization problem in the presence of imperfect SIC with maximizing the system energy efficiency subject to the required communication quality for each user. To deal with it, an energy-efficient power allocation algorithm are designed, whose convergent behavior and com-

where p_c is the constant circuit power consumption of the BS antenna, including the power dissipations in the transmit filter, mixer, frequency synthesizer, digital-to-analog converter and analog-to-digital converter which are independent of the actual transmit power and are assumed to occur all the time, and p_0 is the basic power independent of the BS antenna [14]. Thereby, energy-efficient power allocation can be described as the following optimization problem

$$\begin{aligned} \max_{p_j} \quad & \eta \\ \text{s.t.} \quad & \begin{cases} p_j \geq 0, \forall j \\ R_j \geq R_j^{\text{LB}}, \\ \sum_{j=1}^J p_j \leq P_{\text{tot}}, \end{cases} \end{aligned} \quad (4)$$

where P_{tot} is the maximum transmit power budget at the BS, and R_j^{LB} is the minimum data rate requirement of the user j , with $R_j^{\text{LB}} = \log_2(1 + \gamma_j^{\text{LB}})$ and γ_j^{LB} being the corresponding SINR requirement of the user j .

III. ENERGY-EFFICIENT POWER ALLOCATION

In this section, we focus on the design of an energy-efficient power allocation algorithm to solve the optimization problem (4). However, due to the fractional expression of the objective function η in the optimization variable p_j , the problem (4) is non-convex, and thus it is challenging to find a globally optimal solution within a polynomial time. As is well known, for solving the energy-efficient optimization problem, fractional programming is a feasible tool by transforming the fraction into a subtractive form, i.e., a difference of the numerator and the denominator [15]. In what follows, according to the characteristic of NOMA with imperfect SIC, we develop a solvable algorithm by using the fractional programming theory.

A. Problem Transformation

From [15], it has been proved that fractional programming is usable to solve the ratio maximization problem only if the numerator is a concave function, the denominator is a convex function, and the constraints are convex. However, checking the expression of energy efficiency in (3), the numerator in η is not concave with respect to the variable p_j . To make it tractable, we resort to the sequential convex approximation (SCA) method [16]. Specifically, the SCA is able to find local optima of an intractable problem with maximizing the objective function f , by solving a sequence of tractable problems with objectives $\{f_l\}_l$. In the generic l -th step of the sequence, the following three properties need to be satisfied:

$$\begin{cases} f_l(x) \leq f(x), \forall x \\ f_l(x^{(l-1)}) = f(x^{(l-1)}), \\ \nabla f_l(x)|_{x^{(l-1)}} = \nabla f(x)|_{x^{(l-1)}}, \end{cases} \quad (5)$$

where $x^{(l-1)}$ denotes the maximizer of $f_{l-1}(\cdot)$, and $f(\cdot)|_{x^{(l-1)}}$ is the value of the function $f(\cdot)$ at $x^{(l-1)}$. Especially, this method is guaranteed to converge to a Karush-Kuhn-Tucker (KKT) point of the original problem with maximizing f .

The key of SCA is to find a suitable approximation $\{f_l\}_l$, which can satisfy the requirements in (5). For the focused problem (4), we leverage the following lower-bound on the logarithmic function to accomplish the approximation:

$$\log_2(1+\gamma) \geq a \log_2 \gamma + b, \quad (6)$$

where

$$a = \frac{\tilde{\gamma}}{1 + \tilde{\gamma}}, b = \log_2(1 + \tilde{\gamma}) - \frac{\tilde{\gamma}}{1 + \tilde{\gamma}} \log_2 \tilde{\gamma}, \tilde{\gamma} \geq 0. \quad (7)$$

It is easy to prove that the approximation (6) satisfies the properties (5) of SCA at $\gamma = \tilde{\gamma}$. Hence, with the approximation, we can get the following inequality:

$$\begin{aligned} \eta & \geq \frac{\sum_{j=1}^J (a_j \log_2 \gamma_j + b_j)}{\sum_{j=1}^J p_j + p_c + p_0} = \frac{\sum_{j=1}^J [a_j \log_2 (|h_j|^2 p_j) + b_j]}{\sum_{j=1}^J p_j + p_c + p_0} \\ & = \frac{\sum_{j=1}^J \left[a_j \log_2 \left(|h_j|^2 \sum_{i=j+1}^J \varepsilon_{j,i} p_i + |h_j|^2 \sum_{k=1}^{j-1} p_k + \sigma^2 \right) \right]}{\sum_{j=1}^J p_j + p_c + p_0} \\ & \triangleq \underline{\eta}, \end{aligned} \quad (8)$$

where $\underline{\eta}$ is a low bound on the original objective function η .

Letting $p_j = 2^{q_j}$ where q_j is an auxiliary variable, $\underline{\eta}$ can be rewritten as

$$\begin{aligned} \underline{\eta} & = \frac{\sum_{j=1}^J [a_j \log_2 (|h_j|^2) + a_j q_j + b_j]}{\sum_{j=1}^J 2^{q_j} + p_c + p_0} \\ & = \frac{\sum_{j=1}^J \left[a_j \log_2 \left(|h_j|^2 \sum_{i=j+1}^J \varepsilon_{j,i} 2^{q_i} + |h_j|^2 \sum_{k=1}^{j-1} 2^{q_k} + \sigma^2 \right) \right]}{\sum_{j=1}^J 2^{q_j} + p_c + p_0} \\ & \triangleq \frac{f(\mathbf{q})}{g(\mathbf{q})}, \end{aligned} \quad (9)$$

where $\mathbf{q} = [q_1, q_2, \dots, q_j, \dots, q_J]^T$. It is clear that $g(\mathbf{q})$ is a convex function, and $f(\mathbf{q})$ is a concave function due to the fact that the log-sum-exp function is convex [17].

Meanwhile, the minimum data rate constraint $R_j \geq R_j^{\text{LB}}$ can be written as

$$|h_j|^2 2^{q_j} \geq \gamma_j^{\text{LB}} \left(|h_j|^2 \sum_{i=j+1}^J \varepsilon_{j,i} 2^{q_i} + |h_j|^2 \sum_{k=1}^{j-1} 2^{q_k} + \sigma^2 \right). \quad (10)$$

By applying the logarithm operation to both sides of the inequality (10), we can get

$$q_j - \log_2 \left(|h_j|^2 \sum_{i=j+1}^J \varepsilon_{j,i} 2^{q_i} + |h_j|^2 \sum_{k=1}^{j-1} 2^{q_k} + \sigma^2 \right) + \log_2 \left(\frac{|h_j|^2}{\gamma_j^{\text{LB}}} \right) \geq 0, \quad (11)$$

which forms a convex set with respect to the variable \mathbf{q} .

Thus, to solve the original optimization problem (4), we can turn to iteratively deal with the following approximation problem as

$$\max_{\mathbf{q}} \eta = \frac{f(\mathbf{q})}{g(\mathbf{q})} \quad (12)$$

$$s.t. \begin{cases} 2^{q_j} \geq 0, \\ \text{constraint (11)}, \\ \sum_{j=1}^J 2^{q_j} \leq P_{\text{tot}}. \end{cases}$$

It is provable that (12) is a standard fractional programming problem with a concave numerator $f(\mathbf{q})$ and a convex denominator $g(\mathbf{q})$ for the objective function η , and convex constraints [15].

B. Algorithm Development

As analyzed in the last subsection, in each outer iteration, given a_j and b_j , it is able to obtain the optimal solution \mathbf{q}^* of the approximated optimization problem (12) by transforming the fraction $\frac{f(\mathbf{q})}{g(\mathbf{q})}$ into an equivalently subtractive form, i.e., $f(\mathbf{q}) - \lambda^{(m)} g(\mathbf{q})$. Here $\lambda^{(m)}$ is an auxiliary variable denoting the maximum energy efficiency in the m -th inner iteration. Especially, $\lambda^{(0)} = 0$ [15]. By iteratively updating the parameters a_j and b_j , we can finally get a feasible solution to the original problem (4). A detailed procedure for energy-efficient power allocation is summarized in **Algorithm 1**, whose convergence is proved in Appendix A.

C. Computational Complexity Analysis

Now we analyze the computational complexity of the proposed Algorithm 1. Generally speaking, the total complexity depends on the number of outer iterations required to reach convergence and the complexity of each outer iteration. Unfortunately, it is difficult to obtain the number of outer iterations theoretically. However, from the numerical results later, it is found that the algorithm has a fast convergence speed and the number of outer iterations is limited. Hence, we can mainly focus on the computational complexity in each iteration. From [15], it is known that the auxiliary variable sequence $\{\lambda^{(m)}\}$ converges with a super-linear rate, which is not related to the complexity required to compute \mathbf{q}^* . The problem $\arg \max_{\mathbf{q}} f(\mathbf{q}) - \lambda^{(m)} g(\mathbf{q})$ for obtaining \mathbf{q}^* is convex, and thus can be solved with polynomial

Algorithm 1 Energy-efficient power allocation for downlink NOMA system with imperfect SIC

- 1: Initialize the iteration index $l = 0$ and transmit power $\mathbf{p}^{(0)}$;
- 2: Set $\tilde{\gamma}_j^{(0)} = \gamma_j^{(0)}(\mathbf{p}^{(0)})$, and compute $a_j^{(0)}$, $b_j^{(0)}$ according to (7);
- 3: **repeat**
- 4: Set $m = 0$, $\lambda^{(0)} = 0$, and a required search precision δ ;
- 5: **repeat**
- 6: Solve the convex problem $\mathbf{q}^* = \arg \max_{\mathbf{q} \in \mathcal{F}} f(\mathbf{q}) - \lambda^{(m)} g(\mathbf{q})$ with the Lagrangian dual method, where \mathcal{F} is the feasible set of (4);
- 7: Set $F = f(\mathbf{q}^*) - \lambda^{(m)} g(\mathbf{q}^*)$;
- 8: Set $\lambda^{(m+1)} = F / g(\mathbf{q}^*)$;
- 9: $m = m + 1$;
- 10: **until** $F \leq \delta$;
- 11: Update $\mathbf{p}^{(l+1)} = 2^{\mathbf{q}^*}$, $\tilde{\gamma}_j^{(l+1)} = \gamma_j^{(l+1)}(\mathbf{p}^{(l+1)})$, and compute $a_j^{(l+1)}$, $b_j^{(l+1)}$ as in (7);
- 12: $l = l + 1$;
- 13: **until** convergence.

complexity, which depends on the number of variables J and the number of constraints $(1 + J)$. Combining the computations of $\{\lambda^{(m)}\}$ and \mathbf{q}^* , the total computational complexity of the proposed Algorithm 1 is polynomial in each iteration.

IV. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed algorithm for the downlink NOMA systems. Similar to [9], the channel variances for users are with equal interval of 10 dB, and the minimum variance is $\sigma_J^2 = 0$ dB. For simplicity, all $\varepsilon_{j,i}$ are equal to ε [13], and the power consumption $(p_c + p_0)$ is 1 W. The noise variance σ^2 is normalized as 1, and the search precision δ is 0.001. The threshold R_j^{LB} is 0.2 bps/Hz, $\forall j$.

We first show the convergence of the Algorithm 1 with $P_{\text{tot}} = 4$ W in three different scenarios, i.e., “ $\varepsilon = 0.00, J = 2$ ”, “ $\varepsilon = 0.02, J = 2$ ”, and “ $\varepsilon = 0.02, J = 6$ ”. As seen in Fig. 2, Algorithm 1 has a fast convergence speed, and it reaches the optimal solution within about 3 iterations. It is found that a slight residual interference factor, i.e., $\varepsilon = 0.02$, results in an obvious performance loss with respect to the ideal case of $\varepsilon = 0$. Besides, increasing the number of users leads to a higher EE due to multi-user diversity.

Next, we compare the performance of Algorithm 1 and two conventional baseline schemes labeled as “MaxEE w/o RI” and “MaxSR w/o RI”. As the names imply, “MaxEE w/o RI” and “MaxSR w/o RI” allocate the transmit power from the perspective of maximizing the energy efficiency and the sum rate without considering

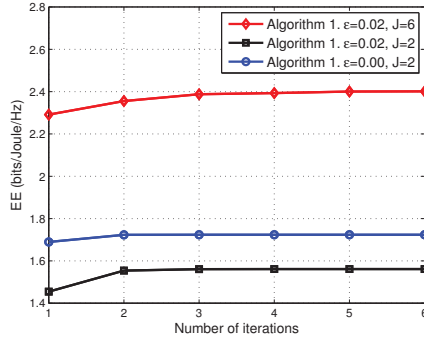


Fig. 2: The convergence of the proposed Algorithm 1 with $P_{\text{tot}} = 4$ W.

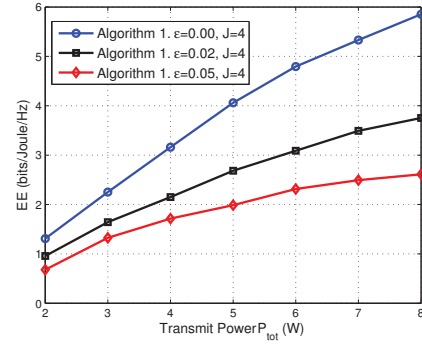


Fig. 4: The impact of imperfect SIC on the performance of the proposed power allocation scheme.

SIC residual interference, respectively. As shown in Fig. 3, Algorithm 1 performs much better than the baseline scheme “MaxEE w/o RI” in the whole region of P_{tot} . Especially, as P_{tot} increases, the performance gap becomes larger. It is reconfirmed that the imperfect SIC has a great impact on the system performance, and the proposed algorithm can effectively alleviate the performance loss caused by imperfect SIC. Moreover, it is noted that when P_{tot} is larger, the EE achieved by the baseline scheme “MaxSR w/o RI” decreases. This is because the baseline scheme makes use of the excess transmit power to maximize the sum rate, but the EE performance gain with adding transmit power is not obvious due to interference limited at the high P_{tot} region. This phenomenon illustrates that using a larger power is not optimal in the sense of maximizing the EE.

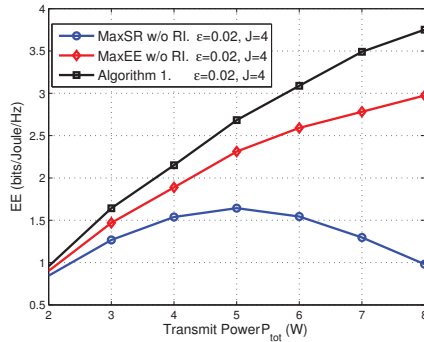


Fig. 3: The impact of SIC residual interference on the performance of different power allocation schemes.

Finally, we depict the impact of residual interference factor ε on the EE of the proposed power allocation algorithm with $J = 4$. As seen in Fig. 4, with respect to perfect SIC of $\varepsilon = 0$, imperfect SIC even with a small $\varepsilon = 0.02$ would lead to an obvious performance degradation. For instance, at the total transmit power $P_{\text{tot}} = 6$ W, there is nearly 2 bits/Joule/Hz performance

loss. This is because a part of transmit power not only improves the rate, but also raise the SIC residual interference. In addition, it is found that with the growth of ε , the performance loss caused by imperfect SIC decreases. For example, at the total transmit power $P_{\text{tot}} = 6$ W, the performance loss is nearly 2 bits/Joule/Hz from $\varepsilon = 0$ to $\varepsilon = 0.02$, but it is less than 1 bits/Joule/Hz from $\varepsilon = 0.02$ to $\varepsilon = 0.05$. Therefore, the proposed power allocation scheme can effectively mitigate the effect of SIC residual interference.

V. CONCLUSIONS

In this paper, a practical problem of imperfect SIC in multiuser downlink NOMA systems has been investigated. This paper first quantitatively analyzes the SIC residual interference, and then proposes an energy-efficient power allocation algorithm to alleviate the impact of imperfect SIC. As shown by theoretical analysis and numerical simulations, the proposed algorithm has a fast convergence speed, and achieves a much better EE performance as compared to two conventional baseline schemes.

APPENDIX A

As described in the proposed Algorithm 1, in the l -th iteration, we need to solve the transformed convex optimization problem (12) with the Lagrangian dual method, and then obtain the corresponding power allocation vector $\mathbf{p}^{(l)}$, which maximizes the lower-bound η_l shown in (8). With the approximation properties in (5), we can finally get the following inequalities:

$$\eta(\mathbf{p}^{(l)}) \stackrel{(a)}{\geq} \underline{\eta}_l(\mathbf{p}^{(l)}) \stackrel{(b)}{\geq} \underline{\eta}_l(\mathbf{p}^{(l-1)}) \stackrel{(c)}{=} \eta(\mathbf{p}^{(l-1)}), \quad (13)$$

where (a) holds due to the fact that $\underline{\eta}_l$ is a lower-bound of η , (b) holds because $\mathbf{p}^{(l)}$ is the maximizer of $\underline{\eta}_l$, and (c) follows from the second property that $f_l(x^{(l-1)}) = f(x^{(l-1)})$, i.e., the parameters $a_j^{(l)}$ and $b_j^{(l)}$ in $\underline{\eta}_l$ make the bound strictly tight.

As a consequence, according to (13), the objective value of η increases with the iteration. Since the total power budget is limited, the Algorithm 1 must converge.

ACKNOWLEDGEMENT

This work was supported in part by National Key Basic Research Program of China (No. 2012CB316104), National Hi-Tech R&D Program of China (No. 2014AA01A702), and National Natural Science Foundation of China (Nos. 61371094, 61401391).

REFERENCES

- [1] A. Benjebbour, Y. Saito, Y. Kishiyama, A. Li, A. Harada, and T. Nakamura, "Concept and practical considerations of non-orthogonal multiple access (NOMA) for future radio access," in *Proc. IEEE Int. Symp. Intell. Signal Process. Commun. Syst. (ISPACS)*, 2013, pp. 770-774.
- [2] Y. Yuan, Z. Yuan, G. Yu, C-H. Hwang, P-K. Liao, A. Li, and K. Takeda, "Non-orthogonal transmission technology in LTE evolution," *IEEE Commun. Mag.*, vol. 54, no. 7, pp. 68-74, 2016.
- [3] L. Dai, B. Wang, Y. Yuan, S. Han, C-L. I, and Z. Wang, "Non-orthogonal multiple access for 5G: solutions, challenges, opportunities, and future research trends," *IEEE Commun. Mag.*, vol. 53, no. 9, pp. 74-81, 2015.
- [4] D. Tse and P. Viswanath, *Fundamentals of wireless communication*. New York: Cambridge university press, 2005.
- [5] S. M. R. Islam, N. Avazov, O. A. Dobre, and K-S. Kwak, "Power domain non-orthogonal multiple access (NOMA) in 5G: potentials and challenges," *IEEE Commun. Surv. Tutorials*, vol. PP, no. 99, pp. 1-41, 2017.
- [6] Z. Ding, Z. Yang, P. Fan, and H. V. Poor, "On the performance of non-orthogonal multiple access in 5G systems with randomly deployed users," *IEEE Signal Process. Lett.*, vol. 21, no. 12, pp. 1501-1505, 2014.
- [7] Z. Ding, P. Fan, and H. V. Poor, "Impact of user pairing on 5G non-orthogonal multiple-access downlink transmissions," *IEEE Trans. Veh. Tech.*, vol. 65, no. 8, pp. 6010-6023, 2016.
- [8] Z. Ding, M. Peng, and H. V. Poor, "Cooperative non-orthogonal multiple access in 5G systems," *IEEE Commun. Lett.*, vol. 19, no. 8, pp. 1462-1465, 2015.
- [9] Q. Sun, S. Han, C-L. I, and Z. Pan, "On the ergodic capacity of MIMO NOMA systems," *IEEE Wireless Commun. Lett.*, vol. 4, no. 4, pp. 405-408, 2015.
- [10] F. Fang, H. Zhang, J. Cheng, and V. C. M. Leung, "Energy efficiency of resource scheduling for non-orthogonal multiple access (NOMA) wireless network," in *Proc. IEEE Int. Conf. Commun. (ICC)*, 2016, pp. 1-5.
- [11] Q. Sun, S. Han, C. L. I, and Z. Pan, "Energy efficiency optimization for fading MIMO non-orthogonal multiple access systems," in *Proc. IEEE Int. Conf. Commun. (ICC)*, 2015, pp. 2668-2673.
- [12] Y. Zhang, H. M. Wang, T. X. Zheng, and Q. Yang, "Energy-efficient transmission design in non-orthogonal multiple access," *IEEE Trans. Veh. Technol.*, vol. 66, no. 3, pp. 2852-2857, 2016.
- [13] A. Agrawal, J. G. Andrews, J. Cioffi, and T. Meng, "Iterative power control for imperfect successive interference cancellation," *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 878-884, 2005.
- [14] X. Chen, X. Wang, and X. Chen, "Energy-efficient optimization for wireless information and power transfer in large-scale MIMO systems employing energy beamforming," *IEEE Wireless Commun. Lett.*, vol. 2, no. 6, pp. 667-670, 2013.
- [15] A. Zappone and E. Jorswieck, "Energy efficiency in wireless networks via fractional programming theory," *Found. Trends[®] in Commun. Inf. Theory*, vol. 11, no. 3-4, pp. 185-396, 2015.
- [16] B. R. Marks and G. P. Wright, "A general inner approximation algorithm for nonconvex mathematical programs," *Oper. Res.*, vol. 26, no. 4, pp. 681-683, 1978.
- [17] S. Boyd and L. Vandenberghe, *Convex optimization*. New York: Cambridge university press, 2004.