Energy Efficient Resource Allocation in Multi-user Downlink Non-Orthogonal Multiple Access Systems

Qian Liu, Hui Gao, Fangqing Tan, Tiejun Lv and Yueming Lu
School of Information and Communication Engineering
Beijing University of Posts and Telecommunications, Beijing, China 100876
Email: {liuqian227,huigao,tfqing,lvtiejun,ymlu}@bupt.edu.cn

***Abstract*—Non-orthogonal multiple access (NOMA) has been investigated recently as a candidate radio access technology for the fifth generation (5G) networks due to its high spectrum efficiency (SE). As green radio which focuses on energy efficiency (EE) becomes an inevitable trend, energy efficient design is becoming more and more important. In this paper, we focus on energy efficient resource allocation problem in multi-user downlink NOMA system with the aim to optimize subchannel assignment and power allocation to maximize the system EE. We propose a novel low-complexity suboptimal subchannel assignment algorithm and obtain the optimal power allocation coefficients among subchannel multiplexed users. To further improve the system EE, unequal power allocation across subchannels (UPAAS) scheme including an optimal solution and a suboptimal Dinkelbach-like algorithm is studied. Simulation results show the effectiveness of our proposed resource allocation algorithms.**

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) has been investigated recently as a candidate radio access technology for the fifth generation (5G) networks due to its high spectrum efficiency (SE). By applying superposition coding at the transmitter side and successive interference cancellation (SIC) at the receiver side, multiple users can be multiplexed on the same subchannel in NOMA systems. Recently, different aspects of NOMA have been studied. System-level performance of downlink NOMA systems was investigated in [1], and simulation results showed that NOMA has superior SE performance as compared to tranditional orthogonal multiple access, such as orthogonal frequency multiple access (OFDMA). To maximize system throughput, power allocation schemes for downlink NOMA systems havw been studied in [2], [3]. In [4], a cooperative NOMA transmission scheme was investigated considering the fact that some users in NOMA systems had prior information about the others' messages. In [5], the authors studied the fairness problem for NOMA systems. In [6], [7], several user-subchannel matching algorithms were introduced to subchannel assignment problem in NOMA systems.

In recent years, the exponential increment of traffic volume in wireless networks has triggered booming energy consumption, which contributes to carbon footprint, electromagnetic pollution and high operating costs. Today, information and communication technology infrastructures consume more than 3% of the world-wide energy, out of which about 60% is consumed by base stations BSs [8]. Green radio (GR) that focuses on energy efficiency (EE) has become an inevitable trend in both academic and industrial worlds. Energy efficient resource allocation for NOMA systems is of great importance. EE optimization problems have been studied in [6], [9], [10] for NOMA systems. In [9], the authors studied the EE optimization problem in multiple-input multiple-output (MIMO) NOMA system. In [10], the authors studied the EE-optimal power allocation scheme for a set of users in a single-input single-output (SISO) NOMA system where one BS simultaneously served multiple users. In [6], energy efficient resource allocation for downlink NOMA system was studied, a greedy user-subchannel matching algorithm was adopted for subchannel assignment, and difference of convex programming approach was exploited to yield suboptimal power allocation schemes.

In this paper, we study energy efficient resource allocation problem in multi-user downlink NOMA systems motivated by [6]. We aim to optimize subchannel assignment, power allocation coefficients among subchannel multiplexed users, and power allocation across subchannels to further improve the system EE. Maximum transmit power constraint at the BS and the quality of service (QoS) for each scheduled user are taken into consideration. We propose a novel low-complexity suboptimal subchannel assignment algorithm that has almost the same performance as existing user-subchannel matching algorithms while the time overhead is lower. The optimal power allocation coefficients among subchannel multiplexed users are derived. To further improve the system EE performance, we study unequal power allocation across subchannels (UPAAS) for subchannel power allocation. For each subchannel, it is proved that the EE over it is pseudo-concave in transmit power on it. Based on this quasiconcavity, the optimal transmit power on each subchannel is obtained. Obviously, if the sum of optimal transmit power on each subchannel is no more than the peak transmit power at BS, optimal subchannel power solution can be obtained. Otherwise, the subchannel power allocation problem is formulated as a nonlinear non-convex sum of ratios problem (SORP) and a Dinkelbach-like algorithm is proposed to obtain a suboptimal solution.

The rest of this paper is organized as follows: Section

This work was supported by the National Natural Science Foundation of China (NSFC) under Grants 61401041 and 61671072, and the National Key Research and Development Program of China 2016YFB0800302.

II introduces the NOMA system model and formulates the EE optimization problem. The proposed subchannel assignment algorithm and power allocation schemes are presented in Section III. Section IV gives the simulation results and conclusions are drawn in Section V.

II. System Model and Problem Formulation

A. System Model

Consider a downlink single-cell NOMA system where a BS transmits signals to K users through N subchannels. The index sets of users and subchannels are denoted as $\mathcal{K} = \{1, \cdots, K\}$ and $\mathcal{N} = \{1, \cdots, N\}$, respectively. The kth user in the cell is denoted as U_k and the nth subchannel is denoted as SC_n. The total available transmit power at the BS is P_s and the allocated transmit power on subchannel SC_n is P_n. BS equally divides the total available bandwidth BW into N subchannels, and the bandwidth of each subchannel is $B_{sc} = BW/N$. It is assumed that perfect knowledge of the channel state information is available at the BS. We consider a block fading channel for which the channel fading remains constant within a time-slot, but it varies independently from one to another. The set of active users over subchannel SC_n is denoted as $\mathcal{S}_n$ and $M_n = \left|\mathcal{S}_n\right|$ is the number of users multiplexed on subchannel SC_n. The channel coefficient of SC_n between user U_k and the BS is modeled as $h_{n,k} = g_{n,k} PL\left(d_k\right)$, where $g_{n,k}$ is the Rayleigh fading coefficient, d_k is the distance between BS and user U_k and $PL\left(\cdot\right)$ is the path loss function. Considering M_n users are multiplexed on subchannel SC_n, the symbol transmitted by the BS on subchannel SC_n is given by a superposition of the modulation symbols, i.e.,

$$x_n = \sum_{i \in \mathcal{S}_n} \sqrt{p_{n,i}} s_i, \tag{1}$$

where s_i is the modulation symbol of user U_i with $E\left[\left|s_i\right|^2\right] = 1$, and $p_{n,i}$ is the power allocated to user U_i over subchannel SC_n. $P_n = \sum_{i \in \mathcal{S}_n} p_{n,i}$ is the transmit power on subchannel SC_n.

It is assumed that both BS and user are equipped with single antenna, then the received signal at user U_k over subchannel SC_n can be written as

$$y_k = h_{n,k} x_n + w_k, \tag{2}$$

where $w_k \sim \mathcal{CN}\left(0, \sigma^2\right)$ is the additive white Gaussian noise (AWGN) with zero mean and variance σ^2. We define the equivalent channel gain (ECG) of user U_k on subchannel SC_n as $H_{n,k} = |h_{n,k}|^2/\sigma^2$. In NOMA systems, the power allocated to weak users should be greater than that of strong users [1], [2], i.e., for any two users in $\mathcal{S}_n$ we always set $p_{n,i} < p_{n,j}$ if their ECGs satisfy $H_{n,i} > H_{n,j}$.

At the receivers, successive interference cancellation (SIC) technique is used to eliminate inter-user interference. The receiver of user $U_k, k \in \mathcal{S}_n$ can cancel the interference signals from any other user $U_l, l \in \mathcal{S}_n$ with a lower ECG $H_{n,l} < H_{n,k}$. Then it treats the signals of those users in $\mathcal{S}_n$ with higher ECGs as noise. Then, with SIC in receiver, the signal to interference plus noise ratio (SINR) of user U_k on subchannel SC_n is given by

$$\mathrm{SINR}_{n,k} = \frac{H_{n,k} p_{n,k}}{1 + H_{n,k} \sum_{i \in \left\{\mathcal{S}_n \left| H_{n,i} > H_{n,k}\right.\right\}} p_{n,i}}. \tag{3}$$

Therefore, the data rate of user U_k on subchannel SC_n can be formulated as

$$R_{n,k} = B_{sc} \log\left(1 + \mathrm{SINR}_{n,k}\right) \tag{4}$$

and the sum rate of subchannel SC_n is given by

$$R_n = B_{sc} \sum_{i \in \mathcal{S}_n} \log\left(1 + \mathrm{SINR}_{n,i}\right), \tag{5}$$

The EE over subchannel SC_n is defined as

$$\eta_n = \frac{R_n}{P_n + P_{n,c}}, \tag{6}$$

where $P_{n,c}$ is the constant circuit power consumption. Following [6], we also define the total system EE of the NOMA system as

$$\eta = \sum_{n=1}^{N} \eta_n. \tag{7}$$

B. Problem Formulation

In this subsection, we formulate the subchannel assignment and power allocation as an EE optimization problem in the multi-user downlink NOMA systems. Specially, in order to reduce the complexity of the SIC receiver while guarantee the number of scheduled users, we assume that each subchannel is assigned to two users at the same time and $K = 2N$. Without loss of generality, we assume subchannel SC_n is assigned to user U_1 and U_2 and their ECGs satisfy $H_{n,1} > H_{n,2}$. Then the sum rate of subchannel SC_n can be formulated as

$$R_n = \log\left(1 + H_{n,2} P_n\right) + \log\left(\frac{1 + \alpha_n H_{n,1} P_n}{1 + \alpha_n H_{n,2} P_n}\right), \tag{8}$$

where α_n is the power allocation factor between $p_{n,1}$ and $p_{n,2}$, i.e., $p_{n,1} = \alpha_n P_n$ and $p_{n,2} = \left(1 - \alpha_n\right) P_n$. To guarantee the power allocated to weak user U_2 is greater than that of strong user U_1, we always set $\alpha_n \le \hat{\alpha}$, where $\hat{\alpha} < \frac{1}{2}$ is a constant.

To better illustrate the assignment of subchannel to users, we introduce an $N \times K$ matrix $\mathit{\Gamma}$ in which the binary element $\gamma_{n,k}$ shows whether subchannel SC_n is assigned to the user U_k. If subchannel SC_n is allocated to user U_k, $\gamma_{n,k} = 1$, otherwise $\gamma_{n,k=0}$.

Remark 1 We do not impose $p_{n,k} > 0$ if and only if $\gamma_{n,k} = 1$. This is because setting $p_{n,k} > 0$ when $\gamma_{n,k} = 0$ is obviously not energy efficient.

In this paper, we focus on maximizing the total system EE of the NOMA system, and the EE optimization problem is formulated as

$$\max_{P_n,\alpha_n,\gamma_{n,k}} \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{\gamma_{n,k}\log\left(1+\mathrm{SINR}_{n,k}\right)}{P_n+P_{n,c}} \tag{9a}$$

$$s.t. \sum_{n=1}^{N} P_n \leq P_s, \tag{9b}$$

$$R_{n,k} \geqslant R_{min}, \forall n \in \mathcal{N}, \forall k \in \mathcal{S}_n, \tag{9c}$$

$$0 < \alpha_n \leq \hat{\alpha}, \forall n \in \mathcal{N}, \tag{9d}$$

$$\gamma_{n,k} \in \{0,1\}, \forall n \in \mathcal{N}, \forall k \in \mathcal{K}, \tag{9e}$$

$$\sum_{k=1}^{K} \gamma_{n,k} = 2, \forall n \in \mathcal{N}. \tag{9f}$$

Constraint (9b) guarantees the maximum transmit power constraint at the BS. Constraint (9c) guarantees the minimum data rate rates of those scheduled users. In this paper, we assume all scheduled users have the same minimum data rate requirement R_{min}. Constraint (9d) is introduced to guarantee that the allocated power to weak users must be greater than that of strong users in NOMA systems.

Note that this optimization problem is non-convex optimization problem due to the constraint (9e) and the interference term in objective function (9a) [11]. To this end, it is hard to find the global optimal solution within polynomial time. Therefore we separately solve the subchannel assignment and power allocation problem in the next section.

III. SUBCHANNEL ASSIGNMENT AND POWER ALLOCATION

A. Subchannel Assignment Scheme

In this subsection, we study how to assign subchannel to users appropriately in order to maximize the system EE. We first assume the power allocation coefficients among subchannel multiplexed users are optimal, then we propose a low-complexity suboptimal subchannel assignment algorithm.

Lemma 1. *Assuming EE-optimal power allocation coefficients for subchannnel multiplexed users can be obtained, for a given subchannel SC_n, to maximize the EE over this subchannel, the optimal subchannel assignment strategy is to assign this subchannel to two users with best ECGs on subchannel SC_n.*

Proof: Please refer to Appendix A. ■

Based on Lemma 1 we propose a suboptimal subchannel assignment algorithm as shown in Algorithm 1. In Algorithm 1, we creat an $N \times K$ ECGs matrix H in which the element $H_{i,j}$ denotes the ECG of user U_j on subchannel SC_i. We are interested in finding the maximum element $H_{n,k}$ in H and willing to allocated subchannel SC_n to user U_k.

In optimal exhaustive searching algorithm, we need to search $\frac{(2N)!}{2^N}$ combinations. The logarithm complexity of exhaustive searching algorithm is $\mathcal{O}\left(N\ln N\right)$. In matching based SOMSA algorithm in [6], the complexity comes from two steps of the algorithm, i.e., the sorting phase and the matching phase. In the sorting phase the complexity is $\mathcal{O}\left(N^3\right)$, and in the matching phase the complexity is $\mathcal{O}\left(N^2\right)$, the logarithm complexity of SOMSA is $\mathcal{O}\left(\ln N\right)$. In our proposed subchannel assignment algorithm, the complexity comes from the repeated procedures of finding the maximum element in matrix $\mathrm{H}_{N\times K}$, and the logarithm complexity is $\mathcal{O}\left(\ln N\right)$.

It is noted that power allocation factors determination is not required in our proposed subchannel assignment algorithm. However in SOMSA algorithm it requires to determine the power allocation factors and then compare the performance of different user sets whenever a user sends matching request to its most preferred subchannel, and the complexity is $\mathcal{O}\left(N^2\right)$.

Algorithm 1 Suboptimal subchannel assignment algorithm.

1) Initialization
   a) Initialize an $N \times K$ ECGs matrix H.
   b) Initialize $\mathcal{S}_{un}$ to record users who has not been allocated to any subchannel.
2) While $\mathcal{S}_{un}$ is not empty
   a) Find the maximum element $H_{n,k}$ in H.
   b) Assign subchannel SC_n to user U_k and remove user U_k from $\mathcal{S}_{un}$.
   c) Set all the elements in the kth column of matrix H to zero.
   d) If $\left|\mathcal{S}_n\right| = 2$, set all the elements in the nth row of matrix H to zero.
3) End of the algorithm.

B. Power Allocation Between Multiplexed Users on Each Subchannel

In this subsection we study the power allocation problem between subchannel multiplexed users and seek the optimal power allocation factor α_n^*. We rewrite the EE over subchannel SC_n as

$$\eta_n = \frac{f_1\left(P_n\right)+f_2\left(P_n\right)}{P_n+P_{n,c}}, \tag{10}$$

where $f_1\left(P_n\right) = \log\left(1+H_{n,2}P_n\right)$ and $f_2\left(P_n\right) = \log\left(\frac{1+\alpha_n H_{n,1}P_n}{1+\alpha_n H_{n,2}P_n}\right)$.

For any given transmit power P_n and multiplexed users, maximize EE over subchannel SC_n is equivalent to maximize $\phi\left(\alpha_n\right) = \frac{1+\alpha_n H_{n,1}P_n}{1+\alpha_n H_{n,2}P_n}$. Since $\frac{d\phi(\alpha_n)}{d\alpha_n} > 0$, maximize $\phi\left(\alpha_n\right)$ is equivalent to determine the upper bound of α_n. The data rate of user U_1 can be formulated as

$$R_{n,1} = B_{sc}\log\left(1+\alpha_n H_{n,1}P_n\right).$$

To guarantee the minimum data rate requirement of user U_1, i.e., $R_{n,1} \geq R_{min}$, we have $\alpha_n \geq \frac{A}{H_{n,1}P_n}$, where $A = 2^{\frac{R_{min}}{B_{sc}}} - 1$. The lower bound of α_n is denoted as $\alpha_n^l = \frac{A}{H_{n,1}P_n}$. The data rate of user U_2 is

$$R_{n,2} = B_{sc}\log\left(1+\frac{\left(1-\alpha_n\right)H_{n,2}P_n}{1+\alpha_n H_{n,2}P_n}\right).$$

To guarantee the minimum data rate requirement of user U_2, i.e., $R_{n,2} \geq R_{min}$, we have $\alpha_n \leq \frac{H_{n,2}P_n - A}{(1+A)H_{n,2}P_n}$. And we denote $\alpha_n^u = \frac{H_{n,2}P_n - A}{(1+A)H_{n,2}P_n}$. α_n^u is a monotone increasing function of P_n and the horizontal asymptote is $\alpha_n^u = \frac{1}{1+A}$.

Subsequently, given the subchannel multiplexed users and transmit power on each subchannel, the EE maximization problem can be equivalently formulated as

$$\max_{1 \leq n \leq N} \alpha_n \tag{11a}$$

$$s.t. \alpha_n \geq \alpha_n^l, \tag{11b}$$

$$\alpha_n \leq \alpha_n^u, \tag{11c}$$

$$0 < \alpha_n \leq \hat{\alpha}. \tag{11d}$$

If $\hat{\alpha} \geq \frac{1}{1+A}$, the optimal power allocation factor is $\alpha_n^* = \alpha_n^u$. Else if $\hat{\alpha} < \frac{1}{1+A}$, when $\alpha_n^u \leq \hat{\alpha}$, the optimal power allocation factor is $\alpha_n^* = \alpha_n^u$. When $\alpha_n^u > \hat{\alpha}$, the optimal power allocation factor is $\alpha_n^* = \hat{\alpha}$. Therefore the optimal power allocation factor α_n^* is given by

$$\alpha_n^* = \begin{cases} \alpha_n^u, & \hat{\alpha} \geq \frac{1}{1+A}, \\ \min\left(\alpha_n^u, \hat{\alpha}\right) & \hat{\alpha} < \frac{1}{1+A}. \end{cases}$$

Due to the minimum data rate requirements of all scheduled users, the transmit power on each subchannel must be sufficiently large. And there must exist a minimum transmit power $\check{P}_n$ that satisfies each user's minimum data rate requirement.

Lemma 2. *The minimum required transmit power $\check{P}_n$ on subchannel SC_n is given by*

$$\check{P}_n = \begin{cases} \frac{A(1+A)}{H_{n,1}} + \frac{A}{H_{n,2}}, & \hat{\alpha} \geq \frac{1}{1+A}, \\ \max\left(\frac{A(1+A)}{H_{n,1}} + \frac{A}{H_{n,2}}, \frac{A}{H_{n,1}\hat{\alpha}}, \right) & \hat{\alpha} < \frac{1}{1+A}. \end{cases}$$

Proof: If $\hat{\alpha} \geq \frac{1}{1+A}$, we always have $\alpha_n^u \leq \hat{\alpha}$. Since α_n^l strictly decreases with P_n and α_n^u strictly increases with P_n, the minimum required transmit power $\check{P}_n$ can be calculated by the condition $\alpha_n^l = \alpha_n^u$ as $\check{P}_n = \frac{A(1+A)}{H_{n,1}} + \frac{A}{H_{n,2}}$.

If $\hat{\alpha} < \frac{1}{1+A}$, we calculate $\check{P}_n^1 = \frac{A(1+A)}{H_{n,1}} + \frac{A}{H_{n,2}}$ by the condition $\alpha_n^l = \alpha_n^u$ and $\check{P}_n^2 = \frac{A}{H_{n,1}\hat{\alpha}}$ by the condition $\alpha_n^l = \hat{\alpha}$. It is obviously that the minimum required transmit power is $\check{P}_n = \max\left(\check{P}_n^1, \check{P}_n^2\right)$. ■

C. Energy Efficient Power Allocation Across Subchannels

In this subsection, we study UPAAS instead of equal power allocation across subchannels to further improve the system EE. As equal power allocation across subchannel (EPAAS) is not energy efficient and it is not always energy efficient to use the maximum available transmit power for transmission.

Lemma 3. *For each subchannel, the EE over this subchannel is pseudo-concave in transmit power P_n for $P_n \in [0, \infty)$.*

Proof: See refer to Appendix B. ■

As a result of this quasiconcavity, there exist three cases of η_n versus P_n for $P_n \in \left[\check{P}_n, \hat{P}_n\right]$, where $\hat{P}_n$ is the maximum available transmit power on subchannel SC_n, and it is always assumed that $\check{P}_n \leq \hat{P}_n$.

Case 1: η_n strictly decreases with P_n for $P_n \in \left[\check{P}_n, \hat{P}_n\right]$ if $\left.\frac{d\eta_n}{dP_n}\right|_{P_n = \check{P}_n} \leq 0$, and the optimal transmit power on subchannel SC_n is $P_n^* = \check{P}_n$.

Case 2: η_n strictly increases with P_n for $P_n \in \left[\check{P}_n, \hat{P}_n\right]$ if $\left.\frac{d\eta_n}{dP_n}\right|_{P_n = \hat{P}_n} \geq 0$, and the optimal transmit power on subchannel SC_n is $P_n^* = \hat{P}_n$.

Case 3: η_n first strictly increases with P_n and then strictly decreases with P_n for $P_n \in \left[\check{P}_n, \hat{P}_n\right]$ if $\left.\frac{d\eta_n}{dP_n}\right|_{P_n = \check{P}_n} > 0$ and $\left.\frac{d\eta_n}{dP_n}\right|_{P_n = \hat{P}_n} < 0$, and the optimal transmit power on subchannel SC_n can be obtained through solving $\frac{d\eta_n}{dP_n} = 0$. The solution of equation $\frac{d\eta_n}{dP_n} = 0$ can be obtained by the numerical methods, such as bisection method or Dinkelbach algotithm [12], and the solution is denoted as $P_n^* = P_n^{max}$.

Without considering the maximum transmit power constraint on each subchannel, the optimal transmit power on subchannel SC_n can be expressed as $P_n^* = \max\left(\check{P}_n, P_n^{max}\right)$. If $\sum_{n=1}^N P_n^* \leq P_s$, the transmit power on subchannel SC_n can be directly set to P_n^*. Otherwise, this optimization problem can be formulated as a nonlinear SoRP and solved by a Dinkelbach-like algorithm as shown in Algorithm 2. Before Algorithm 2, we first set the transmit power on those subchannels satisfying $\check{P}_n \geq P_n^{max}$ as $\check{P}_n$ from the perspective of both global optimality and time overhead. Then we denote the index set of subchannels satisfying $\check{P}_n < P_n^{max}$ as $\mathcal{N}'$, $N1 = \left|\mathcal{N}'\right|$.

Algorithm 2 Dinkelbach-like algorithm for SoRP

1) Initialize $\varepsilon > 0$, $l = 0$, $\{\lambda_{n,l}\}_{n=1}^{N1} = 0$.
2) While $F\left(\{\lambda_{n,l}\}_{n=1}^{N1}\right) > \varepsilon$ do
   a) $\boldsymbol{P}_l^* = \arg\max_{\boldsymbol{P} \in \mathcal{S}'} \sum_{n=1}^{N1} \{R_n - \lambda_{n,l} g_n(P_n)\}$;
   b) $F\left(\{\lambda_{n,l}\}_{n=1}^{N1}\right) = \max_{\boldsymbol{P} \in \mathcal{S}'} \sum_{n=1}^{N1} \{R_n - \lambda_{n,l} g_n(P_n)\}$;
   c) $\lambda_{n,l+1} = \frac{R_n(\boldsymbol{P}_l^*)}{g_n(\boldsymbol{P}_l^*)}, n = 1, 2, \cdots, N1$;
   d) $l = l + 1$;
3) end while

Given the subchannel assignment scheme and optimal power allocation factors, the power allocation problem for subchannels whose indexex in $\mathcal{N}'$ is formulated as

$$\max_{\boldsymbol{P}} \sum_{n=1}^{N1} \frac{R_n}{g_n(P_n)} \tag{12a}$$

$$s.t. P_n \geq \check{P}_n, \forall n \in \mathcal{N}', \tag{12b}$$

$$\left\| \boldsymbol{P} \right\|_1 = P_s - \sum_{i \in \left\{ \mathcal{N} \middle| \check{P}_n \geq P_n^{max} \right\}} \check{P}_i, \tag{12c}$$

where $g_n(P_n) = P_n + P_{n,c}$ is an affine function, as well as a convex function. $\boldsymbol{P} = [P_1, P_2, \cdots, P_{N1}]$ is the subchannel power allocation vector. Problem (12) is a typical nonlinear SoRP with concave-convex ratios and convex constraints. To the best of our knowledge, there is no algorithms that can globally solve this type of optimization problem within polynomial time and finding the global solution of an SoRP with affordable complexity is still an open problem. To this end, a Dinkelbach-like algorithm as shown in algorithm 2 is proposed to obtain suboptimal solution at affordable cost. In Algorithm 2 the original non-convex problem is converted to a sequence of convex problems and solved iteratively.

IV. SIMULATION RESULTS

In this section, simulation results are presented to evaluate the performance of our proposed resource allocation algorithms in downlink NOMA systems. All simulation results are generated by averaging the results over 10^4 channel realizations. Users are assumed to be uniformly distributed in a circular area with radius of 400 m. The peak transmit power at the BS is set to 46dBm, and it is equally allocated to all subchannels when EPAAS is adopted for subchannel power allocation. Detailed simulation parameters are shown in Table I. In particular, when the peak transmit power at the BS is not large enough to guarantee all users' minimum data rate requirements, the system will not work.

Table I
SIMULATION PARAMETERS

Cell radius	400 m
Peak transmit power at the BS	46 dBm
Total bandwidth	5 MHz
Constant circuit power $P_{n,c}$	20 dBm
Drain efficiency of the power amplifier at the BS	0.38
Thermal noise density	-174 dBm
Minimum distance between user and the BS	50 m
$\breve{\alpha}$	0.3
Path loss exponent	3

Fig. 1 shows the total system EE versus the number of users under different minimum data rate requirements. First, it can be seen that our proposed subchannel assignment algorithm has almost the same EE performance as the SOMSA algorithm. It can also be seen that our proposed subchannel power allocation algorithms achieve large performance gain than EPAAS scheme. For example, when the number of users is 30 and $R_{min} = 1Mbps$, the system EE of our proposed UPAAS scheme is 42.4 times of that of EPAAS scheme. Besides, the system EE decreases with the number of users K after K achieves a certain threshold. Therefore, it is necessary to limit the number of scheduled users in an energy efficient NOMA system with fixed QoS requirements.

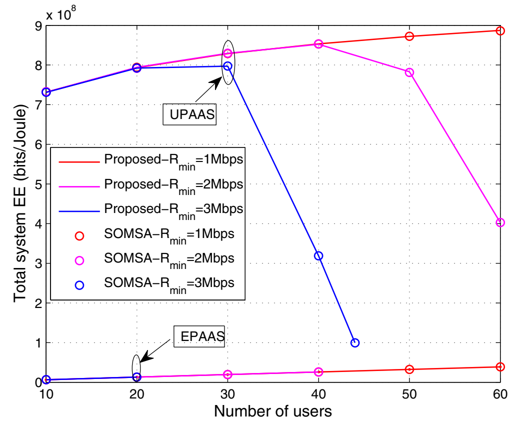

Fig. 1. Total system EE versus the number of users

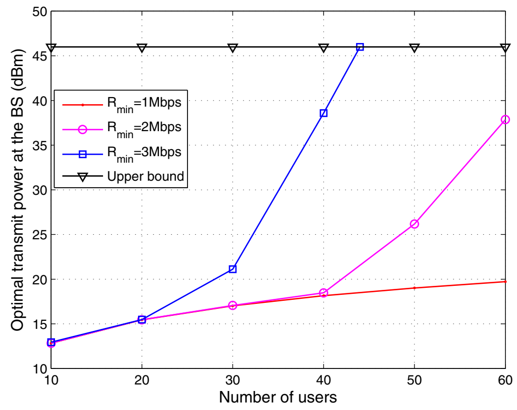

Fig. 2. Optimal transmit power at the BS versus the number of users.

Fig. 2 shows the optimal transmit power at the BS versus the number of users under our proposed UPAAS scheme. From Section III C, we know that the subchannel power allocation scheme is optimal if the optimal transmit power is less than 46 dBm. It can be seen that energy saving is a strong advantage of our proposed UPAAS. For example, we save about 57% electric energy when $K = 60$ and $R_{min} = 1Mbps$ compared to EPAAS.

Fig. 3 shows the system EE versus the minimum data rate requirement R_{min}. It can be seen that for a given number of users K, the system EE begins to deteriorate after R_{min} achieves a certain threshold. This is because the increase of R_{min} requires the system to allocated more power to users with worse ECGs, which contributes to the deterioration of system EE. It can be also seen that a flexible tradeoff between K and R_{min} can be done to keep high system EE.

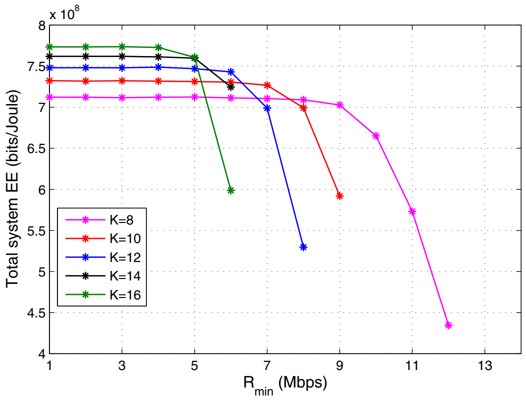

Fig. 3. Total system EE versus the minimum data rate R_{min}.

V. CONCLUSION

In this paper, we studied energy efficient resource allocation in multi-user downlink NOMA systems. We proposed a low complexity subchannel assignment algorithm to maximize the system EE, simulation results showed that our proposed subchannel assignment algorithm has almost the same EE performance as existing SOMSA algorithm. The optimal power allocation factors for subchannel multiplexed users were determined. To further improve the system EE, UPAAS was considered in this paper. Simulation results showed that our proposed UPAAS schemes has large performance gain than EPAAS scheme.

APPENDIX A
PROOF OF LEMMA 1

Proof: We prove Lemma 1 by contradiction. Assuming user U_1 and U_2 are the two users multiplexed on subchannel SC_n and their ECGs satisfy $H_{n,1} > H_{n,2}$, then the corresponding EE over subchannel SC_n can be formulated as

$$\eta_n = \frac{B_{sc}}{P_c + P_n} \log\left(1 + \alpha_n H_{n,1} P_n\right) + \frac{B_{sc}}{P_c + P_n} \log\left(1 + \frac{(1-\alpha_n) P_n}{\frac{1}{H_{n,2}} + \alpha_n P_n}\right)$$

Subchannel SC_n has not been assigned to user U_k. However user U_k has better ECG than user U_2 or U_1 on subchannel SC_n. It is obvious that if we replace user U_2 with U_k and remain the power allocation factor α_n and transmit power P_n unchanged, there will be EE gain over subchannel SC_n. Therefore it is not optimal to allocate subchannel SC_n to user U_1 and U_2. ■

APPENDIX B
PROOF OF LEMMA 3

Proof: To facilitate the prove of this Lemma, we introduce two auxiliary functions $f_{21}(P_n) = \log\left(\frac{1+\alpha_n^u H_{n,1} P_n}{1+\alpha_n^u H_{n,2} P_n}\right)$ and $f_{22}(P_n) = \log\left(\frac{1+\hat{\alpha} H_{n,1} P_n}{1+\hat{\alpha} H_{n,2} P_n}\right)$. The first derivative of f_{21} with respect to P_n is

$$\frac{df_{21}}{dP_n} = \frac{1}{\ln 2} \frac{H_{n,1} - H_{n,2}}{(1+A)\left(1+\alpha_n^u H_{n,1} P_n\right)\left(1+\alpha_n^u H_{n,2} P_n\right)}.$$

Since $\frac{df_{21}}{dP_n}$ is a monotony decrease function of P_n, the second order derivative of f_{21} with respect to P_n is negative. Therefore f_{21} is a concave function of P_n. The first derivative of f_{22} with respect to P_n is

$$\frac{df_{22}}{dP_n} = \frac{1}{\ln 2} \frac{\hat{\alpha}\left(H_{n,1} - H_{n,2}\right)}{\left(1+\hat{\alpha} H_{n,1} P_n\right)\left(1+\hat{\alpha} H_{n,2} P_n\right)}.$$

Since $\frac{df_{22}}{dP_n}$ is a monotony decrease function of P_n, the second order derivative of f_{22} with respect to P_n is negative. Therefore f_{22} is also a concave function of P_n. Besides it is easy to see that f_1 is a concave function of P_n.

If $\hat{\alpha} \geq \frac{1}{1+A}$, we have $\alpha_n^* = \alpha_n^u$ and $f_2 = f_{21}$. Since both f_1 and f_2 are concave function of P_n, $f_1 + f_2$ is a concave function of P_n. Else if $\hat{\alpha} < \frac{1}{1+A}$, f_2 can be expressed as $f_2 = \min\left(f_{21}, f_{22}\right)$. Therefore f_2 is concave this is because the minimum of concave functions is concave [13]. Similarly, $f_1 + f_2$ is a concave function of P_n.

Therefore η_n is pseudo-concave in P_n since it has a concave numerator and an affine denominator [10]. ■

REFERENCES

[1] A. Benjebbovu, A. Li, Y. Saito, Y. Kishiyama, A. Harada, and T. Nakamura, "System-level performance of downlink NOMA for future LTE enhancements," *Proc. IEEE Globecom Workshops*, pp. 66–70, Atlanta, GA, Dec 2013.

[2] C. L. Wang, J. Y. Chen, and Y. J. Chen, "Power allocation for a downlink non-orthogonal multiple access system," *IEEE Wireless Commun. Lett.*, vol. PP, no. 99, pp. 1–1, 2016.

[3] P. Parida and S. S. Das, "Power allocation in OFDM based NOMA systems: A DC programming approach," *Proc. IEEE Globecom Workshops*, pp. 1026–1031, Dec 2014.

[4] Z. Ding, M. Peng, and H. V. Poor, "Cooperative non-orthogonal multiple access in 5G systems," *IEEE Commun. Lett.*, vol. 19, no. 8, pp. 1462–1465, Aug 2015.

[5] S. Timotheou and I. Krikidis, "Fairness for non-orthogonal multiple access in 5G systems," *IEEE Signal Process. Lett.*, vol. 22, no. 10, pp. 1647–1651, Oct 2015.

[6] F. Fang, H. Zhang, J. Cheng, and V. C. M. Leung, "Energy-efficient resource allocation for downlink non-orthogonal multiple access network," *IEEE Trans. on Commun.*, vol. 64, no. 9, pp. 3722–3732, Sept 2016.

[7] B. Di, L. Song, and Y. Li, "Sub-channel assignment, power allocation, and user scheduling for non-orthogonal multiple access networks," *IEEE Tran. on Wireless Commun.*, vol. 15, no. 11, pp. 7686–7698, Nov 2016.

[8] A. Zappone and E. Jorswieck, "Energy efficiency in wireless networks via fractional programming theory," *Foundations and Trends in Commun. and Inform. Theory*, vol. 11, no. 3-4, pp. 185–396, 2015.

[9] Q. Sun, S. Han, C. L. I, and Z. Pan, "Energy efficiency optimization for fading mimo non-orthogonal multiple access systems," *Proc. IEEE International Conference on Communications (ICC)*, pp. 2668–2673, London, UK, June 2015.

[10] Y. Zhang, H. M. Wang, T. X. Zheng, and Q. Yang, "Energy-efficient transmission design in non-orthogonal multiple access," *IEEE Trans. on Veh. Technol.*, vol. PP, no. 99, pp. 1–1, 2016.

[11] L. A. Wolsey and G. L. Nemhauser, *Integer and Combinatorial Optimization*. John Wiley & Sons, 2014.

[12] F. Tan, T. Lv, and S. Yang, "Power allocation optimization for energy-efficient massive MIMO aided multi-pair decode-and-forward relay systems," *IEEE Trans. on Commun.*, vol. PP, no. 99, pp. 1–1, 2017.

[13] B. Matthiesen, A. Zappone, and E. A. Jorswieck, "Resource allocation for energy-efficient 3-way relay channels," *IEEE Tran. on Wireless Commun.*, vol. 14, no. 8, pp. 4454–4468, Aug 2015.