CPSC 524 Assignment 4

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Environment

1. module load OpenMPI/3.1.1-iccifort-2018.3.222-GCC-7.3.0-2.3
2. module list

> [cpsc424_yl2335@grace1 yl2335_ps4_cpsc424]$ module list
>
> Currently Loaded Modules:
> 1) StdEnv (S)
> 2) GCCcore/7.3.0
> 3) binutils/2.30-GCCcore-7.3.0
> 4) icc/2018.3.222-GCC-7.3.0-2.30
> 5) ifort/2018.3.222-GCC-7.3.0-2.30
> 6) iccifort/2018.3.222-GCC-7.3.0-2.30
> 7) zlib/1.2.11-GCCcore-7.3.0
> 8) numactl/2.0.11-GCCcore-7.3.0
> 9) XZ/5.2.4-GCCcore-7.3.0
> 10) libxml2/2.9.8-GCCcore-7.3.0
> 11) libpciaccess/0.14-GCCcore-7.3.0
> 12) hwloc/1.11.10-GCCcore-7.3.0
> 13) OpenMPI/3.1.1-iccifort-2018.3.222-GCC-7.3.0-2.30
>
> Where:
> S: Module is Sticky, requires --force to unload or purge

Running the program

1. sh submit.sh
2. Output files used for this report are stored in the output directory.

Task 1: Serial Program

Size	Time 1	Time 2	Time 3	Avg Time
1000	0.1543	0.154	0.1532	0.1538
2000	1.1898	1.1802	1.181	1.1837
4000	14.3322	14.2961	14.2572	14.2952
8000	121.2531	120.754	120.826	120.9444

As can be seen from the table, running time grows almost cubically with respect to N.

Task2: Blocking MPI Parallel Program

Part A

- 3 runs for each combination of $N \in \{1000, 2000, 4000, 8000\}$ and $p \in \{1, 2, 4, 8\}$
- The results are shown below

size = 1000

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.1858	0.1849	0.1848	0.1852

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.1503	0.1486	0.15	0.1496
p1	0.1503	0.1486	0.15	0.1496

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.1016	0.1033	0.1045	0.1031
p1	0.0976	0.0992	0.1002	0.0990
p2	0.1012	0.1029	0.1039	0.1027
p3	0.1016	0.1035	0.1044	0.1032

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.077	0.0772	0.0781	0.0774
p1	0.0731	0.0733	0.0739	0.0734
p2	0.0737	0.0741	0.0742	0.0740
p3	0.0741	0.0743	0.0756	0.0747
p4	0.0753	0.0751	0.0759	0.0754
p5	0.076	0.0767	0.0774	0.0767
p6	0.0768	0.0769	0.0777	0.0771
p7	0.0771	0.0771	0.0781	0.0774

size = 2000

Process	Time 1	Time 2	Time 3	Avg Time
p0	1.3105	1.3095	1.3091	1.3097

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.9545	0.9491	0.9542	0.9526
p1	0.9586	0.953	0.9584	0.9567

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.6787	0.6802	0.6824	0.6804
p1	0.633	0.6352	0.6355	0.6346
p2	0.6807	0.6821	0.6844	0.6824
p3	0.683	0.6846	0.687	0.6849

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.4164	0.4167	0.4151	0.4161
p1	0.3777	0.3775	0.3773	0.3775
p2	0.3827	0.3831	0.383	0.3829
p3	0.4028	0.403	0.4019	0.4026
p4	0.4076	0.4091	0.4076	0.4081
p5	0.4142	0.4144	0.4132	0.4139
p6	0.4199	0.4205	0.4189	0.4198
p7	0.4211	0.4215	0.4199	0.4208

size = 4000

Process	Time 1	Time 2	Time 3	Avg Time
p0	14.7601	14.7728	14.7638	14.7656

Process	Time 1	Time 2	Time 3	Avg Time
p0	10.7895	10.7378	10.7709	10.7661
p1	10.8061	10.7546	10.7886	10.7831

Process	Time 1	Time 2	Time 3	Avg Time
p0	6.1392	6.1209	6.1357	6.1319
p1	5.475	5.4208	5.4398	5.4452
p2	6.147	6.1295	6.1437	6.1401
p3	6.1567	6.1388	6.1537	6.1497

Process	Time 1	Time 2	Time 3	Avg Time
p0	3.3554	3.3564	3.3491	3.3536
p1	2.8991	2.8898	2.8901	2.8930
p2	2.9442	2.9348	2.9343	2.9378
p3	3.1447	3.1409	3.138	3.1412
p4	3.2216	3.2206	3.2145	3.2189
p5	3.2929	3.2925	3.288	3.2911
p6	3.3684	3.3686	3.3618	3.3663
p7	3.3724	3.3736	3.3663	3.3708

size = 8000

Process	Time 1	Time 2	Time 3	Avg Time
p0	122.6584	122.701	122.7282	122.6959

Process	Time 1	Time 2	Time 3	Avg Time
p0	88.834	88.7642	88.7727	88.7903
p1	88.8998	88.8301	88.8391	88.8563

Process	Time 1	Time 2	Time 3	Avg Time
p0	62.3269	60.748	60.8343	61.3031
p1	55.7167	55.8437	55.8725	55.8110
p2	62.3536	60.7752	60.8611	61.3300
p3	62.3929	60.8145	60.9003	61.3692

Process	Time 1	Time 2	Time 3	Avg Time
p0	34.0834	34.051	34.081	34.0718
p1	29.7546	29.4313	29.8036	29.6632
p2	30.3971	29.9597	30.4395	30.2654
p3	32.388	32.2264	32.4179	32.3441
p4	33.2962	33.2684	33.3329	33.2992
p5	33.4583	33.4892	33.4889	33.4788
p6	34.1307	34.0986	34.1282	34.1192
p7	34.1498	34.1177	34.1478	34.1384

Raw Performance

To assess raw performance, one can measure the longest time taken by all the processes. In our case, the first/the last process almost always have the longest running time due to more communication/computational workload. We can see that the running time increases with as the size of the matrix increases and decreases as the number of processes increases.

Scalability

1. As N doubles, running time tends to scale by 8. This is because the time complexity of triangular matrix multiplication is approximately $O(N^3)$.
2. As p doubles, performance scales from near 0.75 to near 0.5. This is because the largest computational workload is distributed to the last process. When p goes from 1 to 2, we cut the triangle into two parts and take the larger one. After that, we always cut the last trapezoid into two parts and take the larger one, gradually the ratio becomes closer to 0.5.

Load Balance

1. The overall time of processes are close to evenly distributed.
2. Computational wordload is not evenly distributed. Higher rank processes spend more time computing and less time communicating, this is because their blocks have more non-zero elements than lower rank processes. Process 0 has the highest communication time due to both setting up and performing the actual computation.

Part B

- 3 runs for each combination of $n \in \{1, 2, 4\}$ and $p \in \{4, 8\}$ when $N = 8000$
- The results are shown below

p = 4, n = 1

Process	Avg Comm	Avg Comp	Avg Time
p0	51.6059	9.6972	61.3031
p1	26.2234	29.5876	55.8110
p2	20.9181	40.4119	61.3300
p3	16.9910	44.3783	61.3693

p = 4, n = 2

Process	Avg Comm	Avg Comp	Avg Time
p0	54.0109	7.4599	61.4707
p1	29.7520	25.9881	55.7401
p2	24.1578	37.2624	61.4202
p3	14.8595	46.6112	61.4706

p = 4, n = 4

Process	Avg Comm	Avg Comp	Avg Time
p0	50.3672	7.4629	57.8301
p1	26.1281	25.9321	52.0602
p2	20.4410	37.3382	57.7792
p3	14.8620	42.9679	57.8300

p = 8, n = 1

Process	Avg Comm	Avg Comp	Avg Time
p0	31.7115	1.7576	33.4711
p1	22.8718	4.7819	27.6461
p2	18.4120	9.8790	28.3087
p3	17.3118	13.7022	31.0150
p4	14.8909	17.4310	32.3077
p5	13.6668	19.3427	33.0190
p6	10.5030	22.9410	33.4451
p7	10.1326	23.3384	33.4710

p = 8, n = 2

Process	Avg Comm	Avg Comp	Avg Time
p0	31.7610	1.7558	33.5168
p1	22.8316	4.7950	27.6266
p2	18.3912	9.8929	28.2841
p3	17.3204	13.6725	30.9929
p4	14.8893	17.3912	32.2805
p5	13.6589	19.3344	32.9933
p6	10.4760	23.0150	33.4910
p7	10.1900	23.3268	33.5168

p = 8, n = 4

Process	Avg Comm	Avg Comp	Avg Time
p0	30.1496	1.7558	31.9054
p1	21.4063	4.7954	26.2017
p2	18.1037	8.7636	26.8673
p3	16.4451	12.9559	29.4010
p4	14.1812	16.0455	30.2267
p5	11.8814	19.2291	31.1105
p6	11.8406	20.0389	31.8794
p7	10.2558	21.6495	31.9053

Raw Performance

- For p = 4, the communication time goes up when n changes from 1 to 2 and drops when n changes from 2 to 4 . When n changes from 1 to 2, communication between nodes and within each node are both required, increasing the communication time. When n changes from 2 to 4, communication within each node are no longer required, which decreases the communication time. The computation time goes down when n changes from 1 to 2 and nearly stays the same when n changes from 2 to 4. Computation time decreases because each socket only runs one process, there's less competition for resources such as cache and bandwidth. Computation time stays the same because resources are not the bottleneck any more.
- For p = 8, the communication and computation time nearly stays the same when n changes from 1 to 2 and drops when n changes from 2 to 4. When n changes from 1 to 2, communication between nodes and within each node are both required, therefore the communication time increases slightly. There are multiple processes on one sockets, they may compete for resources such as bandwidth, L3 cache, therefore the computation time nearly stays the same. When n changes from 2 to 4, each process occupy a single socket, the computation time and communication time decreases.
- It can be seen that the change of time for p = 8 are less obvious than p = 4 because the chunk size are smaller when p = 8.

Load Balance

- As in Part A, total time is almost evenly distributed.

- As in Part A, higher rank processes spend more time computing and less time communicating. lower rank processes spend more time communicating and less time computing.

Possible improvements

1. Dividing blocks according to the number of elements rather than the number of rows/cols.
    - This would lead to a more evenly distributed workload, resulting in reduced computing time and communication time, improving the overall performance.
2. Using non-blocking communication for ring-pass communications.
    1. This can improve the performance since send/recv operations can be overlapped with computation.
    2. Load balance would basically stays the same since computational work distribution is the same. Scaling factor would also stay the same.

Task3: Non-Blocking MPI Parallel Program

Part A

- 3 runs for each combination of $N \in \{1000, 2000, 4000, 8000\}$ and $p \in \{1, 2, 4, 8\}$
- The results are shown below

size = 1000

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.1859	0.1857	0.1858	0.1858

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.153	0.1509	0.1517	0.1519
p1	0.153	0.1509	0.1517	0.1519

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.1106	0.109	0.1102	0.1099
p1	0.093	0.0912	0.0921	0.0921
p2	0.1102	0.108	0.1091	0.1091
p3	0.1107	0.1091	0.1102	0.1100

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.0743	0.0742	0.0743	0.0743
p1	0.0682	0.0683	0.0684	0.0683
p2	0.0687	0.0689	0.0686	0.0687
p3	0.0691	0.0691	0.0693	0.0692
p4	0.0707	0.0706	0.0707	0.0707
p5	0.0737	0.0735	0.0737	0.0736
p6	0.0738	0.0737	0.0739	0.0738
p7	0.0744	0.0743	0.0744	0.0744

size = 2000

Process	Time 1	Time 2	Time 3	Avg Time
p0	1.3093	1.3089	1.3087	1.3090

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.9589	0.9534	0.9542	0.9555
p1	0.9633	0.9576	0.9579	0.9596

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.6736	0.6753	0.6748	0.6746
p1	0.5591	0.562	0.5618	0.5610
p2	0.6752	0.6784	0.6779	0.6772
p3	0.678	0.6803	0.6798	0.6794

Process	Time 1	Time 2	Time 3	Avg Time
p0	0.4042	0.4039	0.4088	0.4056
p1	0.3572	0.3574	0.3613	0.3586
p2	0.3581	0.3582	0.3626	0.3596
p3	0.3594	0.3596	0.3636	0.3609
p4	0.3757	0.3734	0.3784	0.3758
p5	0.4066	0.4062	0.4112	0.4080
p6	0.4078	0.4076	0.4125	0.4093
p7	0.4087	0.4084	0.4134	0.4102

size = 4000

Process	Time 1	Time 2	Time 3	Avg Time
p0	14.7512	14.72	14.7688	14.7467

Process	Time 1	Time 2	Time 3	Avg Time
p0	10.7812	10.7975	10.7765	10.7851
p1	10.7985	10.8145	10.7934	10.8021

Process	Time 1	Time 2	Time 3	Avg Time
p0	6.1836	6.204	6.1601	6.1826
p1	5.0442	5.0772	5.0553	5.0589
p2	6.191	6.2119	6.1679	6.1903
p3	6.2005	6.2219	6.1778	6.2001

Process	Time 1	Time 2	Time 3	Avg Time
p0	3.1922	3.1952	3.2157	3.2010
p1	2.7012	2.7058	2.7256	2.7109
p2	2.7055	2.7105	2.7299	2.7153
p3	2.7105	2.715	2.7344	2.7200
p4	2.9159	2.9075	2.9358	2.9197
p5	3.2004	3.2032	3.2237	3.2091
p6	3.2043	3.2071	3.2279	3.2131
p7	3.2095	3.2125	3.2329	3.2183

size = 8000

Process	Time 1	Time 2	Time 3	Avg Time
p0	122.4523	122.5093	122.6978	122.5531

Process	Time 1	Time 2	Time 3	Avg Time
p0	88.8197	88.7429	88.8293	88.7973
p1	88.8848	88.8079	88.8954	88.8627

Process	Time 1	Time 2	Time 3	Avg Time
p0	59.533	59.6788	59.6117	59.6078
p1	46.9258	46.894	46.8683	46.8960
p2	59.56	59.7057	59.6383	59.6347
p3	59.599	59.7448	59.6773	59.6737

Process	Time 1	Time 2	Time 3	Avg Time
p0	31.3002	31.3095	31.6114	31.4070
p1	26.0875	26.0962	26.3774	26.1870
p2	26.1074	26.1159	26.3961	26.2065
p3	26.128	26.1363	26.4171	26.2271
p4	27.7984	27.7428	28.0051	27.8488
p5	31.3286	31.3378	31.6387	31.4350
p6	31.3475	31.3568	31.6579	31.4541
p7	31.3665	31.3757	31.6772	31.4731

Raw Performance

Comparing Task 3 to Task 2, the raw performance in Task 3 is slightly better when N is large and p is large. Because communication plays a bigger role with large N and p.

Scalability

Task 3 has similar characteristics to Task 2.

Load Balance

Task 3 has similar load balances to Task 2. Higher rank processes spend more time computing and less time communicating.

Part B

- 3 runs for each combination of $n \in \{1, 2, 4\}$ and $p \in \{4, 8\}$ when $N = 8000$
- The results are shown below

p = 4, n = 1

Process	Avg Comm	Avg Comp	Avg Time
p0	50.1049	9.5030	59.6078
p1	18.1434	28.7526	46.8960
p2	20.9575	38.6771	59.6346
p3	14.8510	44.8227	59.6737

p = 4, n = 2

Process	Avg Comm	Avg Comp	Avg Time
p0	53.8241	7.4506	61.2747
p1	24.0216	26.2075	50.2291
p2	23.6636	37.5605	61.2241
p3	14.8499	46.4248	61.2747

p = 4, n = 4

Process	Avg Comm	Avg Comp	Avg Time
p0	50.1650	7.4514	57.6165
p1	20.4401	25.9392	46.3793
p2	19.8513	37.7143	57.5655
p3	14.9167	42.6996	57.6163

p = 8, n = 1

Process	Avg Comm	Avg Comp	Avg Time
p0	29.3939	2.0158	31.4070
p1	18.0568	8.1428	26.1870
p2	13.7390	12.4486	26.2064
p3	9.2230	17.0287	26.2271
p4	8.7843	19.0358	27.8487
p5	9.8471	21.6298	31.4350
p6	8.9306	22.4836	31.4541
p7	7.5428	23.9797	31.4731

p = 8, n = 2

Process	Avg Comm	Avg Comp	Avg Time
p0	29.7246	1.7637	31.4883
p1	20.1438	4.8394	24.9832
p2	14.7210	10.2886	25.0097
p3	12.2976	13.8793	26.1769
p4	9.8587	17.6108	27.4695
p5	11.8918	19.5447	31.4365
p6	8.4308	23.0314	31.4622
p7	8.1426	23.3457	31.4883

p = 8, n = 4

Process	Avg Comm	Avg Comp	Avg Time
p0	28.4366	1.7494	30.1860
p1	18.4818	4.7763	23.2580
p2	14.5799	8.7031	23.2830
p3	14.1170	12.9514	27.0684
p4	11.8440	16.0761	27.9202
p5	10.9573	19.1769	30.1342
p6	10.1049	20.0552	30.1602
p7	8.5720	21.6140	30.1860

Raw Performance

- The trends when p = 4 and p = 8 are similar to task 2. When n changes from 1 to 2, communication time almost stays the same or even goes up, computation time decreases. When n changes from 2 to 4, communication time drops and computation time almost stays the same.
- The reasoning for task 2 still applies to task 3. However, since non-blocking send/recv leads to overlapping between computation and communication, the communication time decreases and the overall performance increases.

Load Balance

- The characteristics of load balance are similar to task 2.
- The workload is slightly less evenly distributed than task 2. Probably because of the overlapped send/recv time. (Before overlapping, process with lower ranks spends less time sending blocks in the ring-pass stage, which helps decrease the gap in communication time between processes. With overlapping, this proportion of time may have been overlapped with computation time, the gap in communication time may become larger between processes.)

Task 4: Load Balance

- 3 runs for each combination of $n \in \{1, 2, 4\}$ and $p \in \{4, 8\}$ when $N = 8000$
- The results are shown below

p = 4, n = 1

Process	Avg Comm	Avg Comp	Avg Time
p0	2.3823	37.3345	39.7168
p1	7.0158	32.7363	39.7521
p2	10.9595	28.8163	39.7758
p3	14.4756	25.3072	39.7829

p = 4, n = 2

Process	Avg Comm	Avg Comp	Avg Time
p0	2.4085	33.3280	35.7365
p1	4.4883	31.1879	35.6761
p2	8.7645	26.9442	35.7086
p3	10.8953	24.8412	35.7364

p = 4, n = 4

Process	Avg Comm	Avg Comp	Avg Time
p0	3.2647	33.3132	36.5779
p1	5.4125	31.1038	36.5163
p2	9.6028	26.9469	36.5497
p3	12.7095	23.8683	36.5778

p = 8, n = 1

Process	Avg Comm	Avg Comp	Avg Time
p0	2.6415	21.2666	23.8950
p1	2.9313	20.8615	23.7948
p2	4.6447	19.1557	23.8162
p3	5.9905	17.8352	23.8343
p4	7.8999	15.9342	23.8518
p5	8.8332	15.0247	23.8677
p6	9.7420	14.1421	23.8820
p7	10.8657	13.0228	23.8950

p = 8, n = 2

Process	Avg Comm	Avg Comp	Avg Time
p0	3.6021	16.7886	16.7886
p1	3.1970	17.0896	20.6916
p2	3.6359	16.6741	19.8711
p3	4.4584	15.8711	19.5069
p4	5.6005	14.7456	19.2040
p5	6.4158	13.9465	19.5470
p6	7.2472	13.1293	19.5451
p7	7.9569	12.4337	19.6809

p = 8, n = 4

Process	Avg Comm	Avg Comp	Avg Time
p0	2.9419	14.0011	16.9430
p1	2.7367	14.1020	16.8387
p2	3.4384	13.4226	16.8610
p3	3.8172	13.0641	16.8813
p4	4.7141	12.1840	16.8982
p5	5.1164	11.7983	16.9147
p6	6.0215	10.9074	16.9289
p7	6.4188	10.5241	16.9429

Raw Performance:

- Higher rank processes spend more time communicating and less time computing, which is the opposite as that in task 2 and task 3. This may be caused by the fact that higher rank processes have fewer number of rows/cols than lower rank processes in order for the total number of elements to be the same.

Load Balance

- Load balance is better than task 3. Computational work is more close to evenly distributed. However, it's still far from an even distribution, part of this may be due to different number of rows/cols each processe possesses.
- Higher rank processes spend less time computing and more time communicating.

Task 5: Generalization

- 3 runs for $n = 4$ and $p = 4$ when $N = 7633$
- The results are shown below

> Currently Loaded Modules:
> 1) StdEnv (S)
> 2) GCCcore/7.3.0
> 3) binutils/2.30-GCCcore-7.3.0
> 4) icc/2018.3.222-GCC-7.3.0-2.30
> 5) ifort/2018.3.222-GCC-7.3.0-2.30
> 6) iccifort/2018.3.222-GCC-7.3.0-2.30

```
 7) zlib/1.2.11-GCCcore-7.3.0
 8) numactl/2.0.11-GCCcore-7.3.0
 9) XZ/5.2.4-GCCcore-7.3.0
10) libxml2/2.9.8-GCCcore-7.3.0
11) libpciaccess/0.14-GCCcore-7.3.0
12) hwloc/1.11.10-GCCcore-7.3.0
13) OpenMPI/3.1.1-iccifort-2018.3.222-GCC-7.3.0-2.30

 Where:
  S:  Module is Sticky, requires --force to unload or purge


/home/cpsc424_yl2335/project/yl2335_ps4_cpsc424
c03n[09,11-12],c04n02
2
mpicc -g -O3 -xHost -fno-alias -std=c99 -I/home/cpsc424_ahs3/utils/timing -c task5.c
mpicc -o task5 -g -O3 -xHost -fno-alias -std=c99 -I/home/cpsc424_ahs3/utils/timing
task5.o matmul.o /home/cpsc424_ahs3/utils/timing/timing.o

p = 7

Matrix multiplication times:
  N        COMM (secs)        COMP (secs)        TIME (secs)       F-norm of Error
-------------------------------------------------------------------------------------
 7633         2.6383             13.9591            16.5975          0.0000000008
 7633         2.4790             14.0254            16.5044          0.0000000008
 7633         3.1477             13.3787            16.5265          0.0000000008
 7633         3.6457             12.9008            16.5465          0.0000000008
 7633         4.6544             11.9128            16.5672          0.0000000008
 7633         5.2152             11.3682            16.5835          0.0000000008
 7633         6.1581             10.4393            16.5974          0.0000000008

real 0m17.440s
user    0m32.821s
sys  0m0.933s

Matrix multiplication times:
  N        COMM (secs)        COMP (secs)        TIME (secs)       F-norm of Error
-------------------------------------------------------------------------------------
```

7633	2.6478	13.9418	16.5896	0.0000000008
7633	2.4926	14.0041	16.4967	0.0000000008
7633	3.1345	13.3844	16.5189	0.0000000008
7633	3.6560	12.8825	16.5386	0.0000000008
7633	4.7001	11.8595	16.5596	0.0000000008
7633	5.1944	11.3814	16.5758	0.0000000008
7633	6.1559	10.4337	16.5896	0.0000000008

real 0m17.378s
user 0m32.817s
sys 0m0.919s

Matrix multiplication times:

N	COMM (secs)	COMP (secs)	TIME (secs)	F-norm of Error
7633	2.5964	13.9703	16.5667	0.0000000008
7633	2.4767	13.9985	16.4752	0.0000000008
7633	3.1588	13.3382	16.4970	0.0000000008
7633	3.6311	12.8855	16.5165	0.0000000008
7633	4.7280	11.8088	16.5368	0.0000000008
7633	5.1427	11.4103	16.5530	0.0000000008
7633	6.1359	10.4306	16.5666	0.0000000008

real 0m17.319s
user 0m32.778s
sys 0m0.913s