

## Topic 2: Center Selection Problem

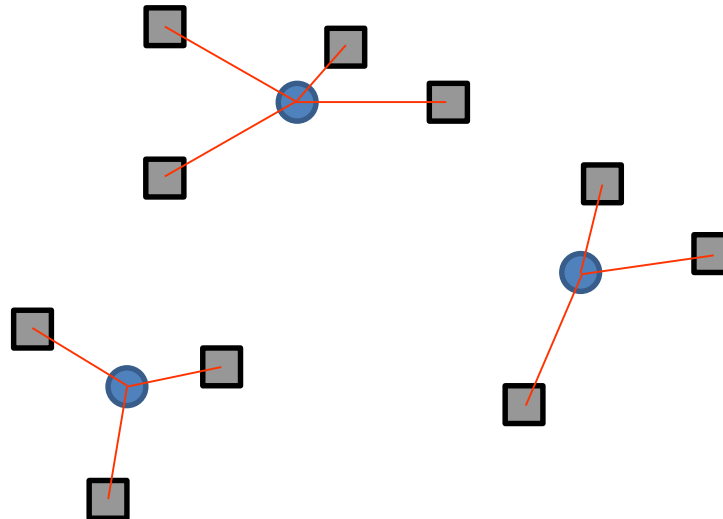
**Input:**  $n$  sites:  $S = \{s_1, s_2, \dots, s_n\}$

**Output:** Locations of  $k$  centers:  $C = \{c_1, c_2, \dots, c_k\}$

**Objective:** Minimize the maximum distance from each site to the nearest center (minimize  $r$ ).

$$\text{dist}(s, C) = \text{Min}_{c \in C} \{\text{dist}(s, c)\} \leq r, \forall s \in S$$

■ site  
● center



Optimal solution:  $C^*$

Optimal value:  $r = r(C^*)$

## Virtual Center Selection Algorithm

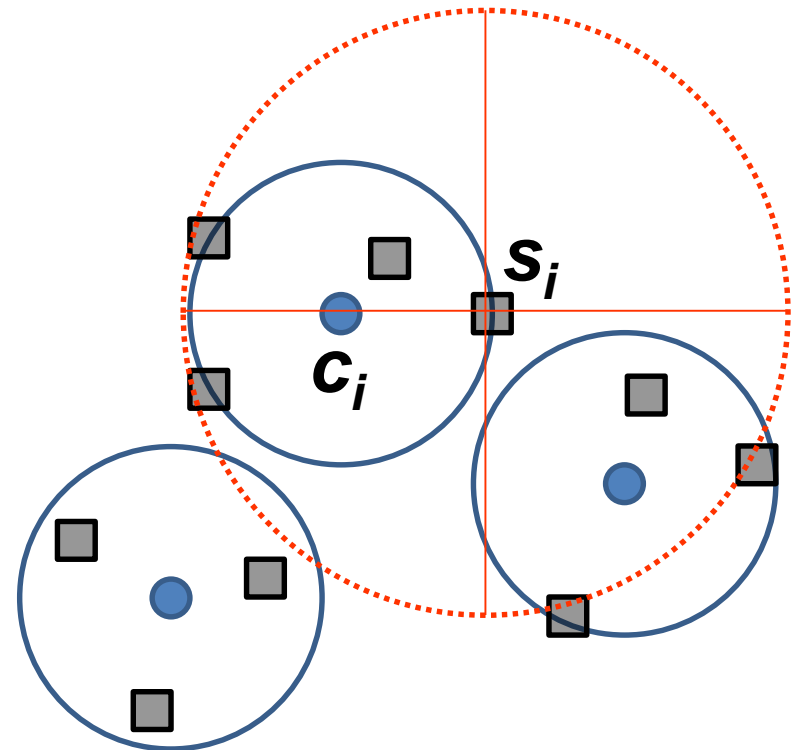
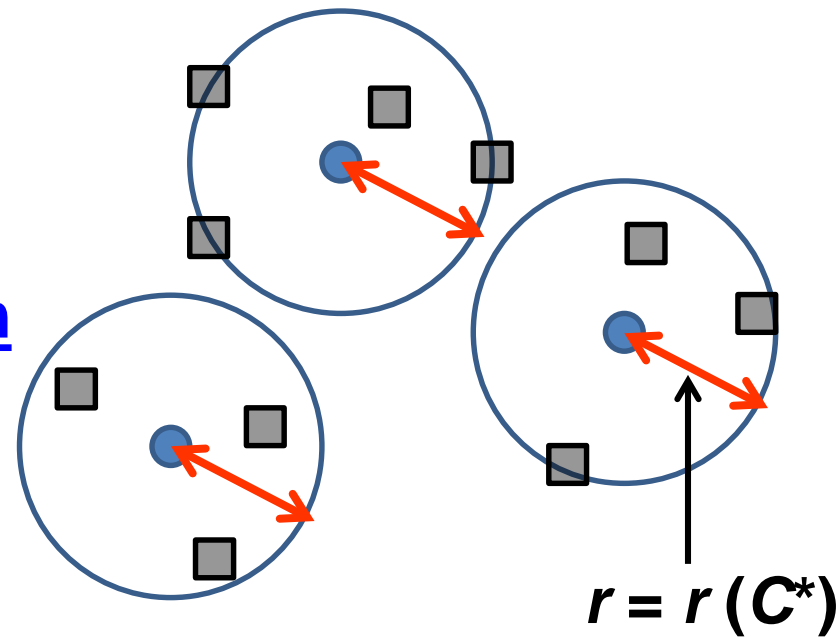
Iteration of the following:

- (i) Select a site  $s$ .
- (ii) Remove all sites covered by  $s$  within distance  $2r$

### Idea behind this algorithm

All sites covered by the center  $c_i$  within distance  $r$  in the optimal selection are always covered by a covered site  $s_i$  within distance  $2r$ .

2-approximation algorithm.



Assuming we know  $r$ :

**procedure** CENTER-SELECT-1

//  $S'$  = sites still needing to be covered

Init  $S' = S$ ,  $C = \emptyset$

**while**  $S' \neq \emptyset$  **do**

    Select any  $s \in S'$  and add  $s$  to  $C$

    Delete all  $t \in S'$  where  $\text{dist}(t, s) \leq 2r$

**end while**

**if**  $|C| \leq k$  **then**

    Return  $C$  as the selected set of sites

**else**

    Claim there is no set of  $k$  centers with covering radius at most  $r$

**end if**

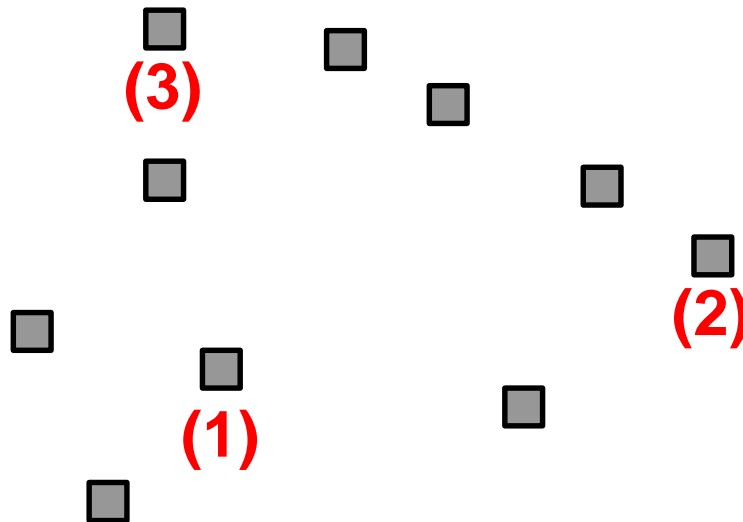
**end procedure**

## Center Selection Algorithm (2-approximation)

- (i) Select a site.
- (ii) Iterate the following: Select a site with the largest distance from the selected sites.

### Idea behind this algorithm

If the largest distance from the selected sites is larger than  $2r$ , the selection of the site can be viewed as being the same as the virtual algorithm (thus 2-approximation). If it is not larger than  $2r$ , all sites have already been covered by the selected sites within  $2r$  (thus 2-approximation).



```

Greedy-Center-Selection( $k, n, s_1, s_2, \dots, s_n$ ) {
     $C = \phi$ 
    repeat  $k$  times {
        Select a site  $s_i$  with maximum  $\text{dist}(s_i, C)$ 
        Add  $s_i$  to  $C$ 
    }
    return  $C$ 
}

```

↑  
site farthest from any center

Q: How to select an initial site ?

**procedure** CENTER-SELECT

Assume  $k \leq |S|$  (else define  $C = S$ )

Select any site  $s$  and let  $C = \{s\}$

**while**  $|C| < k$  **do**

Select a site  $s \in S$  that maximizes  $\text{dist}(s, C)$

Add  $s$  to  $C$

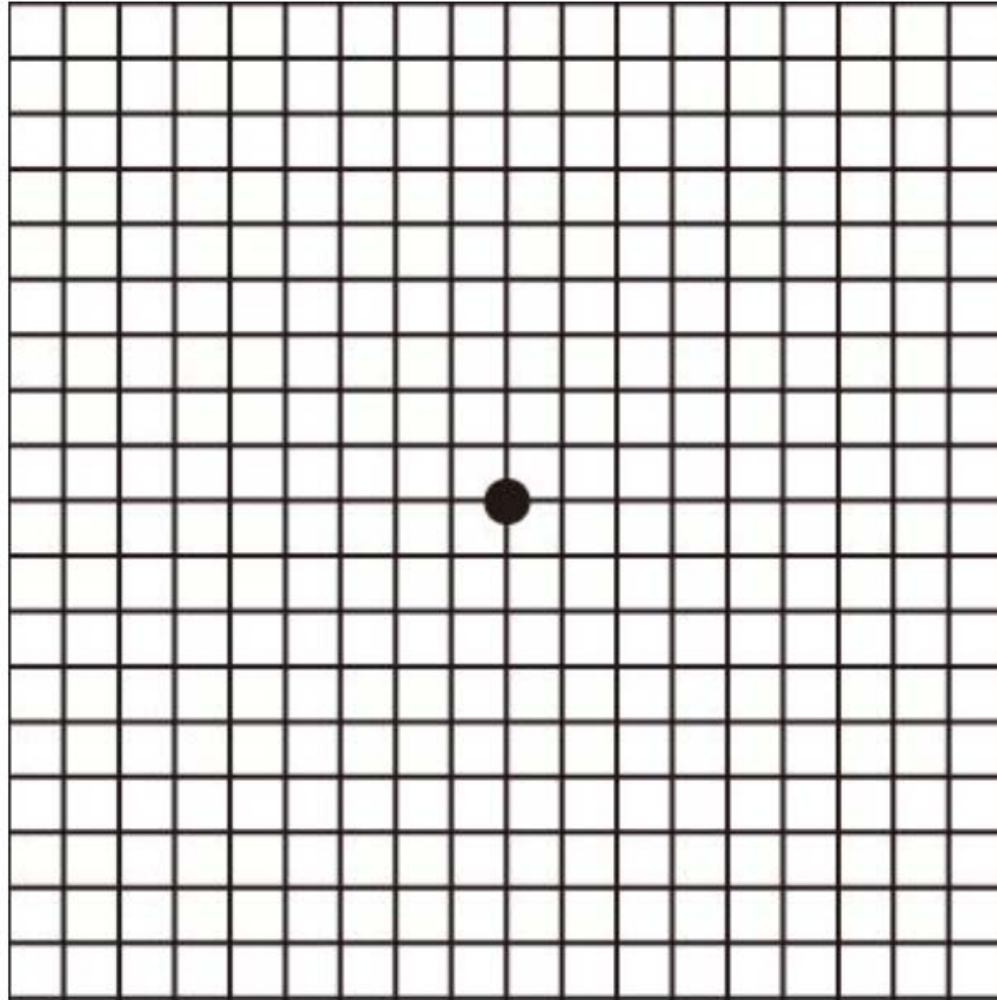
**end while**

Return  $C$  as the selected set of sites

**end procedure**

### Exercise 3-1:

Create an example where the obtained value  $r(C)$  by the algorithm is close to  $2r(C^*)$ . Create another example where the obtained value  $r(C)$  by the algorithm is close to  $r(C^*)$ .



### **Exercise 3-2:**

**Design a method to select the first site in the center selection algorithm (instead of random selection).**