Advanced Algorithm
Assignment 3
Load Balancing Problem

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context

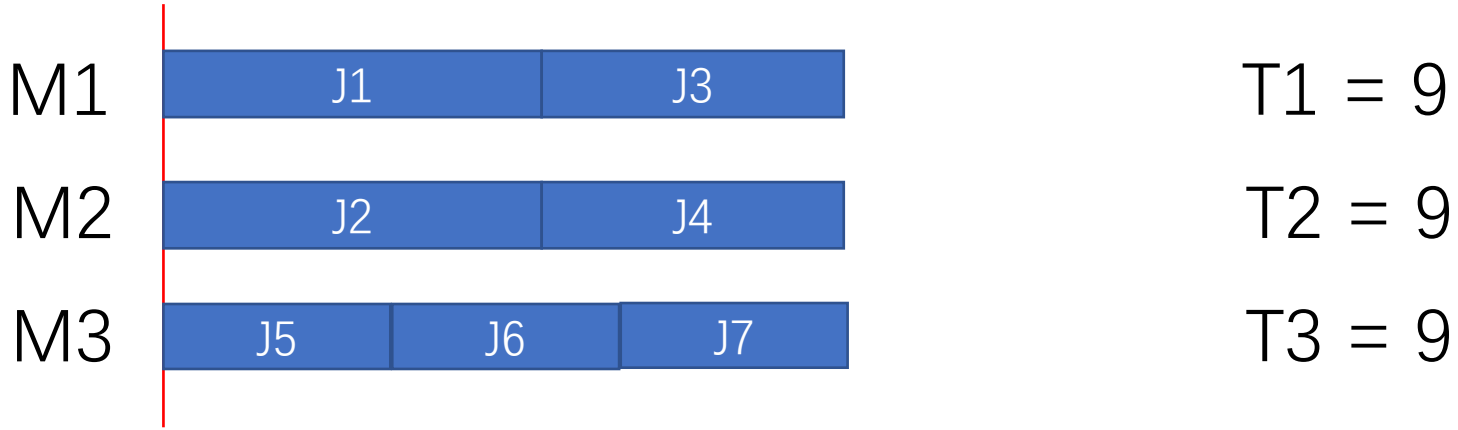
- **Exercise 2-1**
- **Exercise 2**
- **Exercise 3**

Exercise 2-1

We assume that there are three machines and seven jobs $(m = 3, n = 6)$

$$t_1 = 5,\ t_2 = 5, t_3 = 4, t_4 = 4, t_5 = 3, t_6 = 3, t_7 = 3$$

The optimal solution:



T1 = 9

T2 = 9

T3 = 9

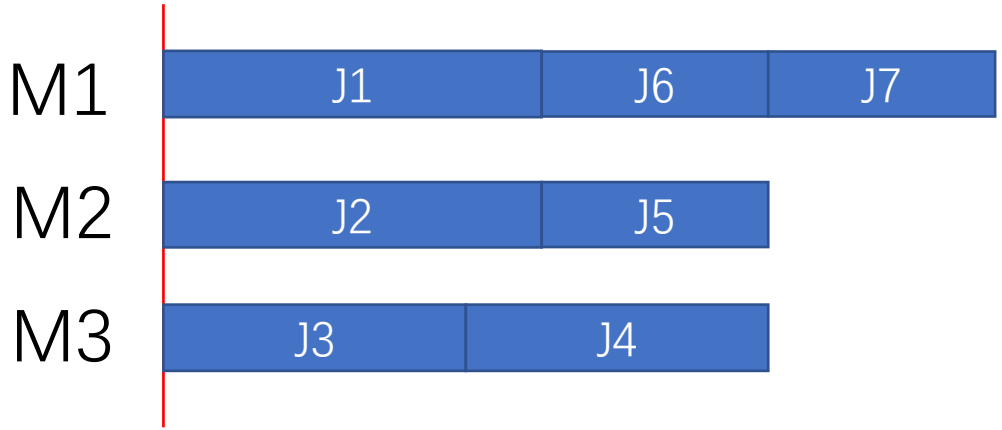
$$T^* = 9$$

Exercise 2-1

We assume that there are three machines and seven jobs $(m = 3, n = 6)$

$$t_1 = 5,\ t_2 = 5, t_3 = 4, t_4 = 4, t_5 = 3, t_6 = 3, t_7 = 3$$

The Sort Greedy Algorithm:



T1 = 11

T2 = 9

T3 = 9

$\mathrm{T}_{max} = 11$

$$\frac{\mathrm{T}_{\max}}{T^*} \approx 1.22 \Longrightarrow \mathrm{T}_{max} = 1.22T^*$$

Exercise 2-2

- Example: M_3 needs less processing times than the others.

  Three Machines:M_1, M_2, M_3

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time(t_j):2,4,6,⋯,20 on M_1 and M_2

  1,2,3,⋯,10 on M_3

  To solve this problem, we proposed the Sorted Posterior Greedy Algorithm(SPGA),the algorithm is shown as follow:

  1. We Caculate the weight according to the execution time of different machines, and sort the jobs after tradeoff.
  2. Assign jobs in descending order to the machine that can complete the job earliest.

Exercise 2-2

We define t_j^m representative the execution time of job j with machine m, and w^i denote the weigh of the execution time of machine m, with $\sum_{i=1}^{m} w^i = 1$

SPGA(m, n, $J_1, J_2, \cdots J_n$){
    $Ret^j = \sum_{i=1}^{m} w^i t_j^i$
    Sort jobs so that $Ret^1 \geq Ret^2 \geq \cdots \geq Ret^n$
    for i = 1 to m {
        $L_i \leftarrow 0$ ← load on machine i
        J(i) ← ∅ ← jobs assigned to machine I
    }
    for j = 1 to n {
        i = $argmin_k \mathrm{L}_K + t_j^k$ ← **machine i can** complete the job j earliest
        J(i) ←J(i) ∪ $\{j\}$ ← assign job j to machine i
        $L_i \leftarrow L_i + t_j^k$ ← updata load of machine i
    }
    return J(1),⋯,J(m)
}

Exercise 2-2

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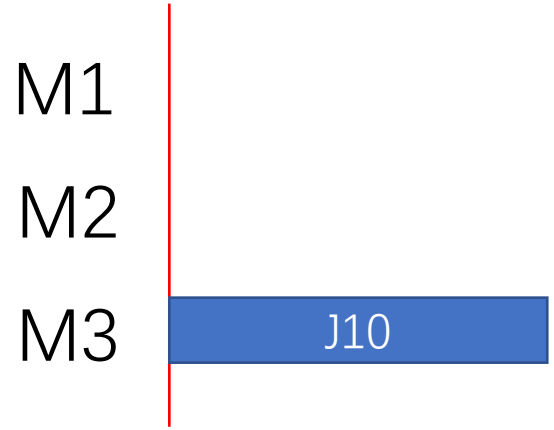
  1,2,3,⋯,10 on M_3

  Job order: $J_{10}, J_9, \cdots J_1$



T1 = (20)

T2 = (20)

T3 = 10

Exercise 2-2

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  1,2,3,⋯,10 on M_3

  Job order: $J_{10}, J_9, \cdots J_1$

T1 = 18

T2 = (18)

T3 = 10 (19)

Exercise 2-2

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  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time(t_j):2,4,6,⋯,20 on M_1 and M_2

  1,2,3,⋯,10 on M_3

  Job order: $J_{10}, J_9, \cdots J_1$

T1 = 18(34)

T2 = 16

T3 = 10(18)

Exercise 2-2

- Example: M_3 needs less processing times than the others.

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  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time(t_j):2,4,6,⋯,20 on M_1 and M_2

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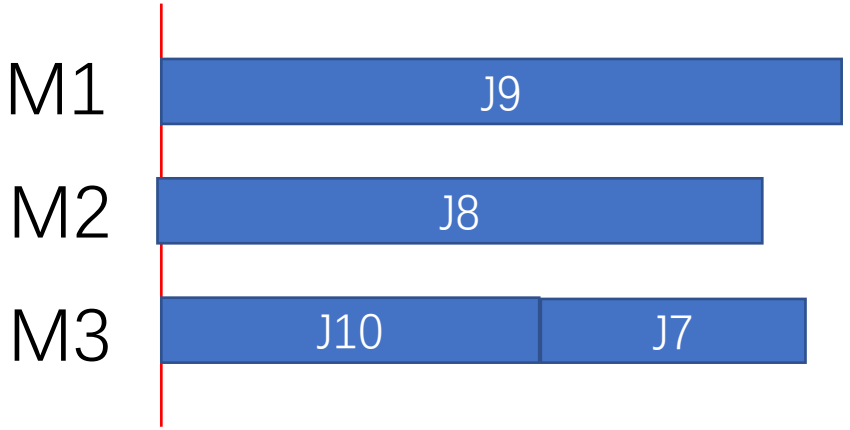

T1 = 18(32)

T2 = 16(30)

T3 = 17

Exercise 2-2

- Example: M_3 needs less processing times than the others.

  Three Machines:M_1, M_2, M_3

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time(t_j):2,4,6,⋯,20 on M_1 and M_2

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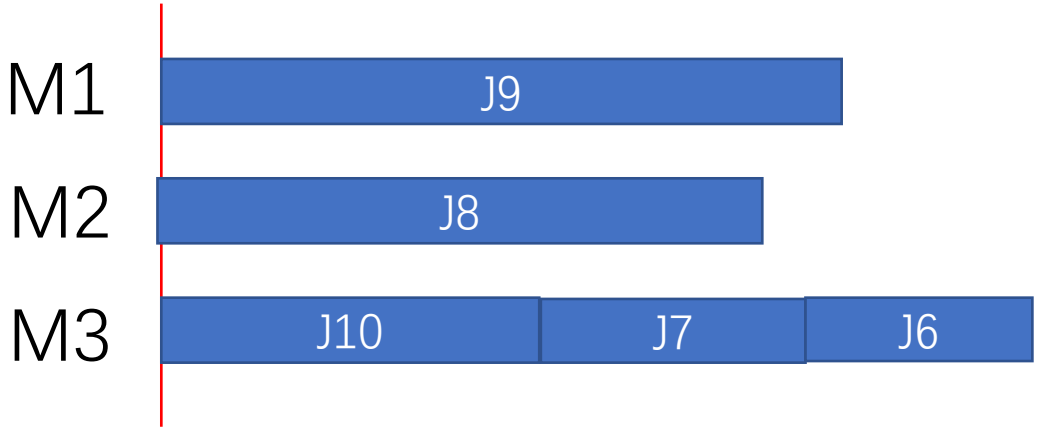


T1 = 18(30)

T2 = 16(28)

T3 = 23

Exercise 2-2

- Example: M_3 needs less processing times than the others.

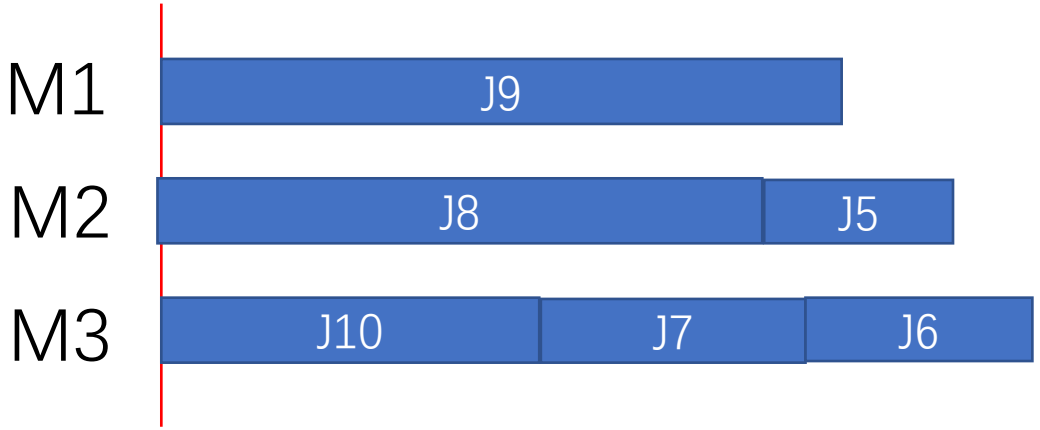
  Three Machines:M_1, M_2, M_3

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time(t_j):2,4,6,⋯,20 on M_1 and M_2

  1,2,3,⋯,10 on M_3

  Job order: $J_{10}, J_9, \cdots J_1$



T1 = 18(28)

T2 = 26

T3 = 23(28)

Exercise 2-2

- Example: M_3 needs less processing times than the others.

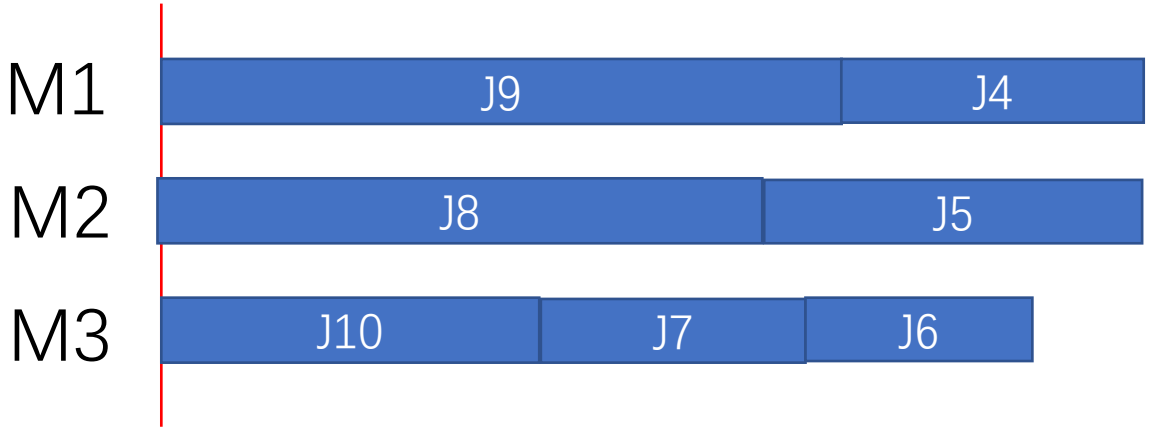
  Three Machines:M_1, M_2, M_3

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time(t_j):2,4,6,⋯,20 on M_1 and M_2

  1,2,3,⋯,10 on M_3

  Job order: $J_{10}, J_9, \cdots J_1$



T1 = 26

T2 = 26

T3 = 23(27)

Exercise 2-2

- Example: M_3 needs less processing times than the others.

  Three Machines:M_1, M_2, M_3

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time(t_j):2,4,6,⋯,20 on M_1 and M_2

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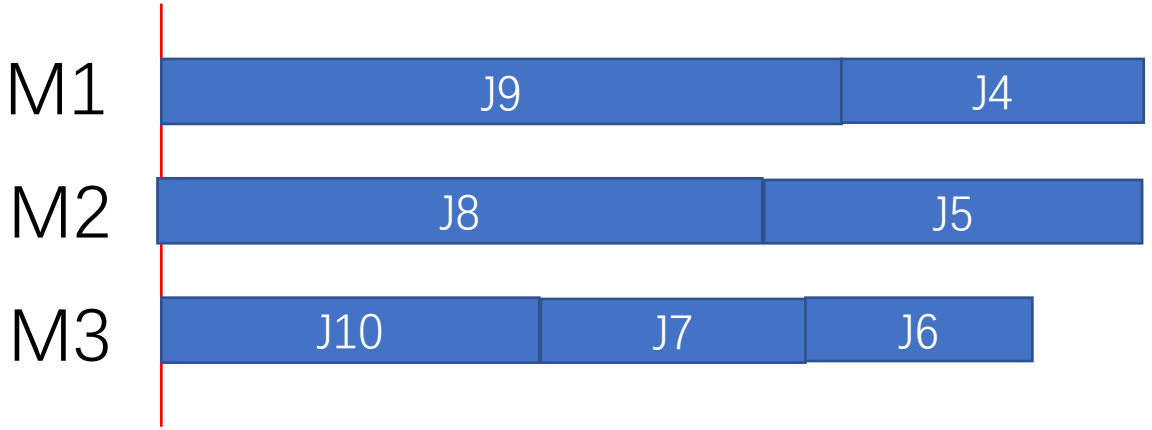
  Job order: $J_{10}, J_9, \cdots J_1$



T1 = 26

T2 = 26

T3 = 23(27)

Exercise 2-2

- Example: M_3 needs less processing times than the others.

  Three Machines:M_1, M_2, M_3

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time(t_j):2,4,6,⋯,20 on M_1 and M_2

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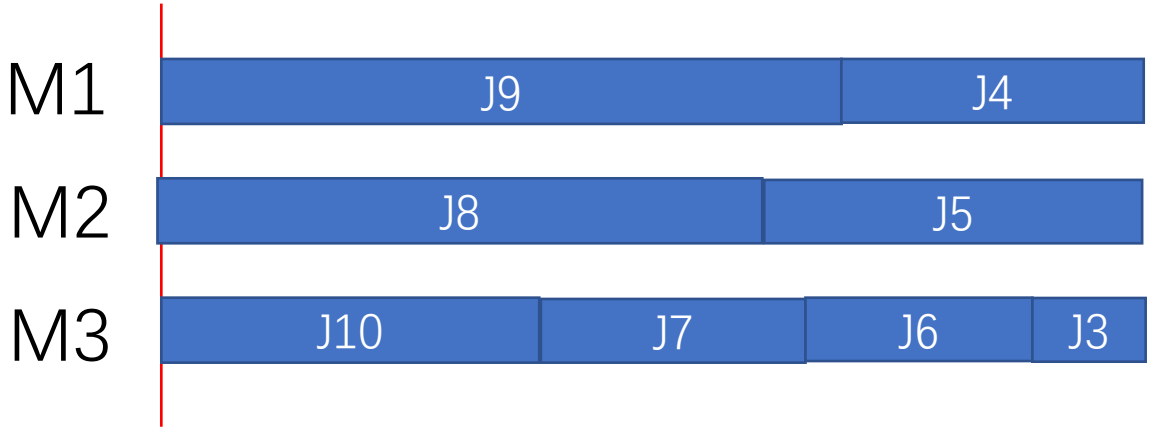


T1 = 26(32)

T2 = 26(32)

T3 = 26

Exercise 2-2

- Example: M_3 needs less processing times than the others.

  Three Machines:M_1, M_2, M_3

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time(t_j):2,4,6,⋯,20 on M_1 and M_2

  1,2,3,⋯,10 on M_3

  Job order: $J_{10}, J_9, \cdots J_1$

T1 = 26(30)

T2 = 26(30)

T3 = 28

Exercise 2-2

- Example: M_3 needs less processing times than the others.

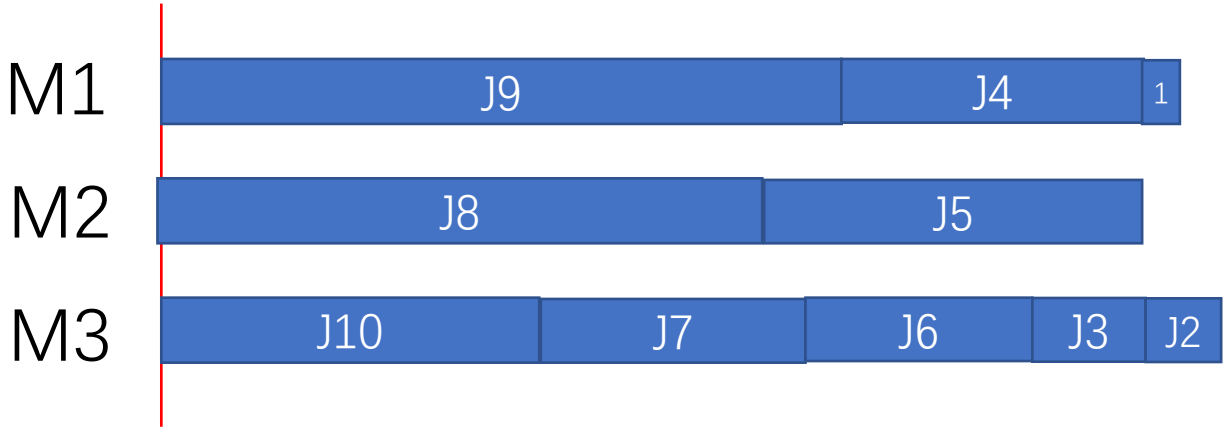
  Three Machines:M_1, M_2, M_3

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time(t_j):2,4,6,⋯,20 on M_1 and M_2

  1,2,3,⋯,10 on M_3

  Job order: $J_{10}, J_9, \cdots J_1$



T1 = 28

T2 = 26(28)

T3 = 28

In this case $T = T^* = 28$

Exercise 2-2

- Difficult Example: M_3 needs less processing times than the others.

  Three Machines:M_1, M_2, M_3

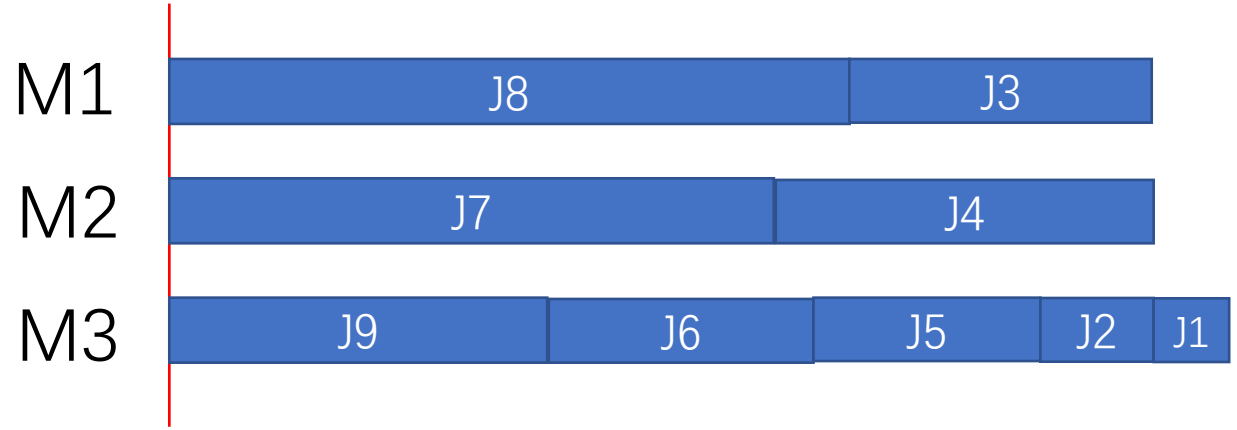
  Nine Jobs: $J_1, J_2, \cdots J_9$

  Processing time(t_j):2,4,6,⋯,18 on M_1 and M_2

  1,2,3,⋯,9 on M_3

  Job order: $J_9, J_8, \cdots J_1$

SPGA Function:



T1 = 26(30)

T2 = 26(30)

T3 = 28

T = 28

Exercise 2-2

- Difficult Example: M_3 needs less processing times than the others.

  Three Machines:M_1, M_2, M_3

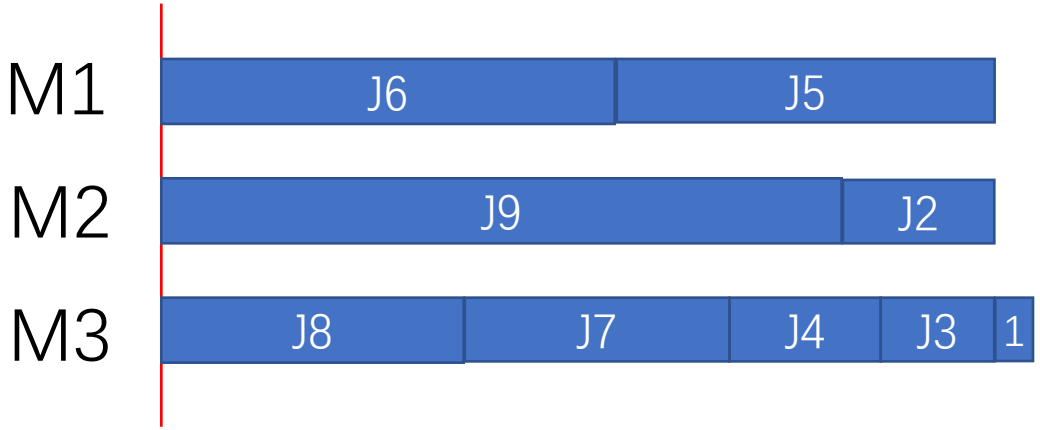
  Nine Jobs: $J_1, J_2, \cdots J_9$

  Processing time(t_j):2,4,6,⋯,18 on M_1 and M_2

  1,2,3,⋯,9 on M_3

  Job order: $J_9, J_8, \cdots J_1$

Optimal Function:



T1 = 22

T2 = 22

T3 = 23

$T^* = 23$ $\frac{T_{\max}}{T^*} = 1.22$

Thank you