# Advanced Algorithm
# Assignment 3
# Load Balancing Problem

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# context

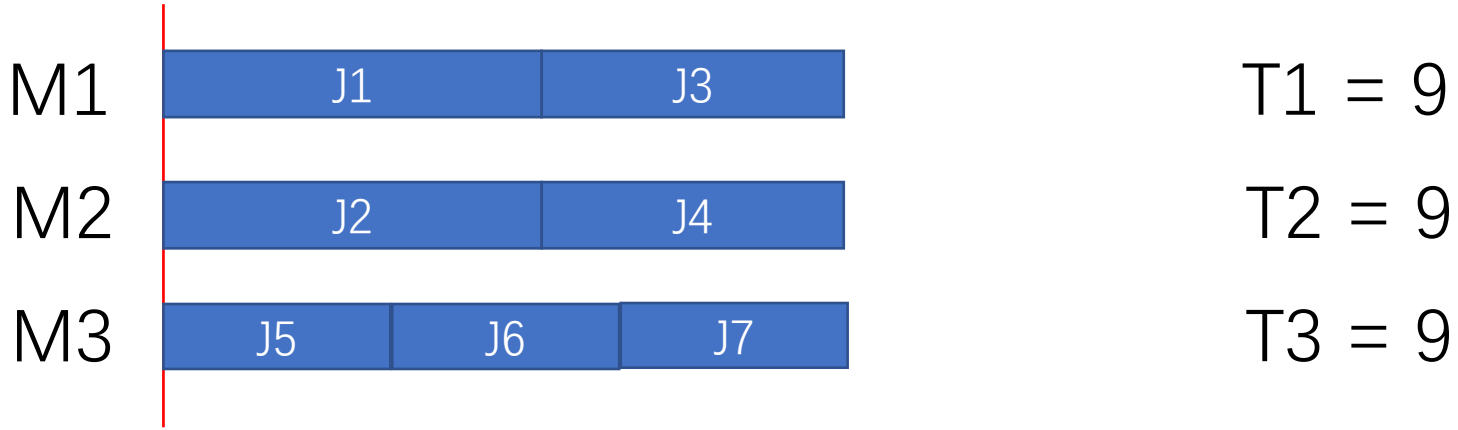
- **Exercise 2-1**
- **Exercise 2**
- **Exercise 3**

# Exercise 2-1

We assume that there are three machines and seven jobs $(m = 3, n = 6)$

$$t_1 = 5,\ t_2 = 5, t_3 = 4, t_4 = 4, t_5 = 3, t_6 = 3, t_7 = 3$$

**The optimal solution:**




T1 = 9

T2 = 9

T3 = 9

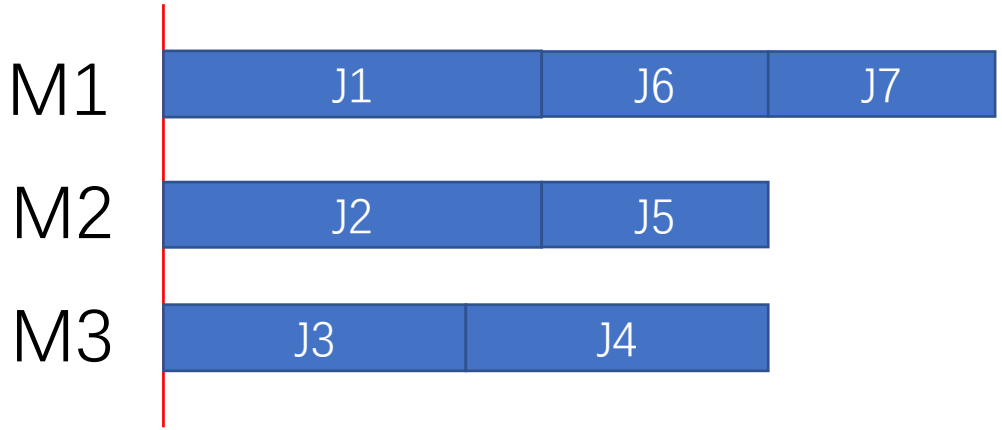
$$T^* = 9$$

# Exercise 2-1

We assume that there are three machines and seven jobs $(m = 3, n = 6)$

$$t_1 = 5,\ t_2 = 5, t_3 = 4, t_4 = 4, t_5 = 3, t_6 = 3, t_7 = 3$$

**The Sort Greedy Algorithm:**




T1 = 11

T2 = 9

T3 = 9

$\mathrm{T}_{max} = 11$

$$\frac{\mathrm{T}_{\max}}{T^*} \approx 1.22 \Longrightarrow \mathrm{T}_{max} = 1.22T^*$$

# Exercise 2-2

- Example: $M_3$ needs less processing times than the others.

  Three Machines:$M_1$, $M_2$, $M_3$

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time($t_j$):2,4,6,⋯,20 on $M_1$ and $M_2$

  1,2,3,⋯,10 on $M_3$

  To solve this problem, we proposed the Sorted Posterior Greedy Algorithm(SPGA),the algorithm is shown as follow:

  1. We Caculate the weight according to the execution time of different machines, and sort the jobs after tradeoff.
  2. Assign jobs in descending order to the machine that can complete the job earliest.

# Exercise 2-2

We define $t_j^m$ representative the execution time of job j with machine m, and $\mathrm{w}^i$ denote the weigh of the execution time of machine m, with $\sum_{i=1}^{m} w^i = 1$

SPGA(m, n, $J_1, J_2, \cdots J_n$){
    $Ret^j = \sum_{i=1}^{m} w^i t_j^i$
    Sort jobs so that $Ret^1 \geq Ret^2 \geq \cdots \geq Ret^n$
    for i = 1 to m {
        $L_i \leftarrow 0$ ← load on machine i
        J(i) ← ∅ ← jobs assigned to machine I
    }
    for j = 1 to n {
        i = $argmin_k \mathrm{L}_K + t_j^k$ ← **machine i can** complete the job j earliest
        J(i) ←J(i) ∪ $\{j\}$ ← assign job j to machine i
        $L_i \leftarrow L_i + t_j^k$ ← updata load of machine i
    }
    return J(1),⋯,J(m)
}

# Exercise 2-2

- Example: $M_3$ needs less processing times than the others.

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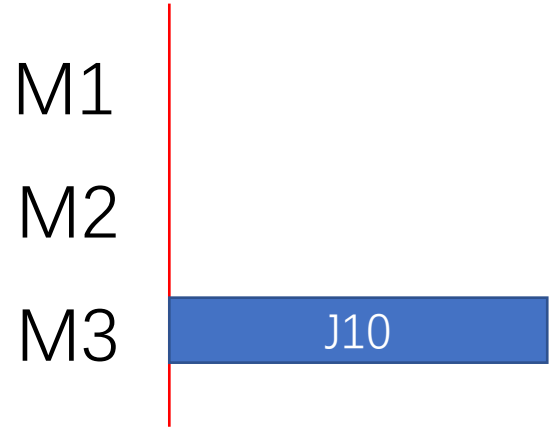
  1,2,3,⋯,10 on $M_3$

  Job order: $J_{10}, J_9, \cdots J_1$




T1 = (20)

T2 = (20)

T3 = 10

# Exercise 2-2

- Example: $M_3$ needs less processing times than the others.

  Three Machines:$M_1$, $M_2$, $M_3$

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time($t_j$):2,4,6,⋯,20 on $M_1$ and $M_2$

  1,2,3,⋯,10 on $M_3$

  Job order: $J_{10}, J_9, \cdots J_1$


T1 = 18

T2 = (18)

T3 = 10 (19)

# Exercise 2-2

- Example: $M_3$ needs less processing times than the others.

  Three Machines:$M_1$, $M_2$, $M_3$

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time($t_j$):2,4,6,⋯,20 on $M_1$ and $M_2$

  1,2,3,⋯,10 on $M_3$

  Job order: $J_{10}, J_9, \cdots J_1$


T1 = 18(34)

T2 = 16

T3 = 10(18)

# Exercise 2-2

- Example: $M_3$ needs less processing times than the others.

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  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time($t_j$):2,4,6,⋯,20 on $M_1$ and $M_2$

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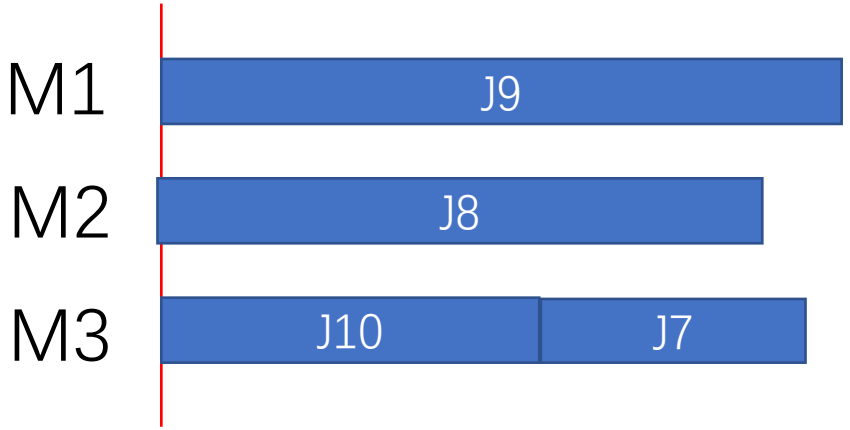


T1 = 18(32)

T2 = 16(30)

T3 = 17

# Exercise 2-2

- Example: $M_3$ needs less processing times than the others.

  Three Machines:$M_1$, $M_2$, $M_3$

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time($t_j$):2,4,6,⋯,20 on $M_1$ and $M_2$

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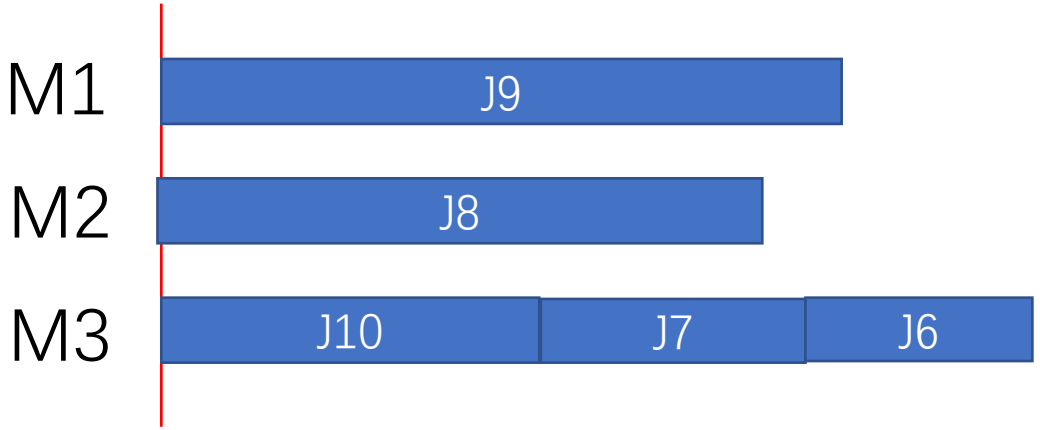
  Job order: $J_{10}, J_9, \cdots J_1$




T1 = 18(30)

T2 = 16(28)

T3 = 23

# Exercise 2-2

- Example: $M_3$ needs less processing times than the others.

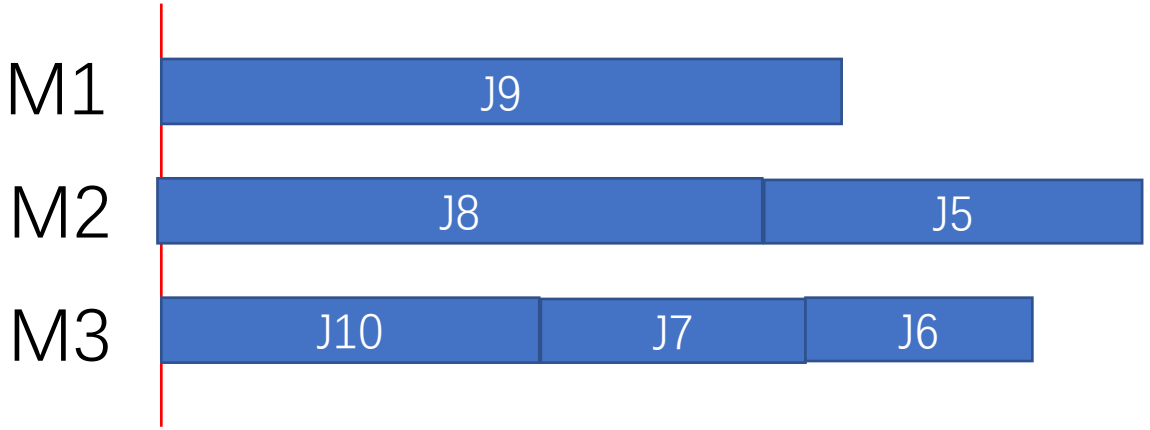
  Three Machines:$M_1$, $M_2$, $M_3$

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time($t_j$):2,4,6,⋯,20 on $M_1$ and $M_2$

  1,2,3,⋯,10 on $M_3$

  Job order: $J_{10}, J_9, \cdots J_1$




T1 = 18(28)

T2 = 26

T3 = 23(28)

# Exercise 2-2

- Example: $M_3$ needs less processing times than the others.

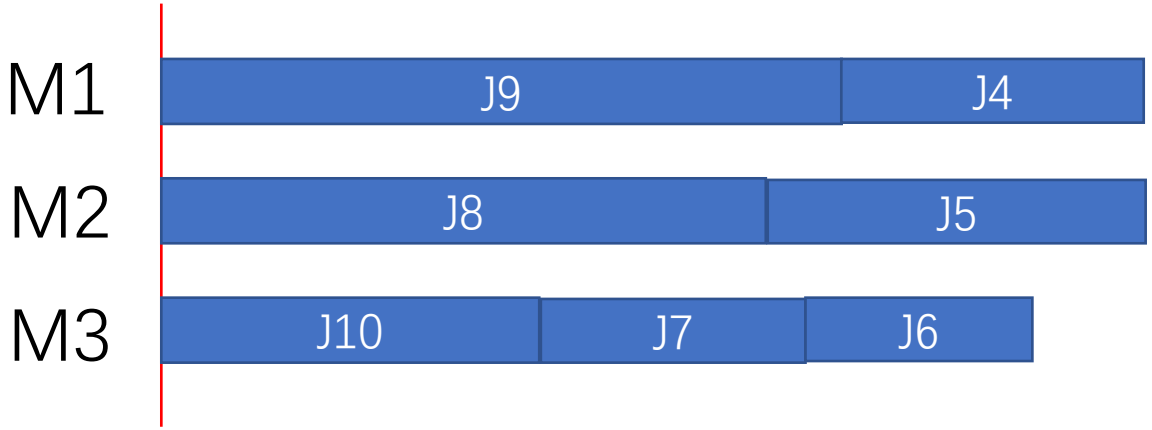
  Three Machines:$M_1$, $M_2$, $M_3$

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time($t_j$):2,4,6,⋯,20 on $M_1$ and $M_2$

  1,2,3,⋯,10 on $M_3$

  Job order: $J_{10}, J_9, \cdots J_1$




T1 = 26

T2 = 26

T3 = 23(27)

# Exercise 2-2

- Example: $M_3$ needs less processing times than the others.

  Three Machines:$M_1$, $M_2$, $M_3$

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time($t_j$):2,4,6,⋯,20 on $M_1$ and $M_2$

  1,2,3,⋯,10 on $M_3$

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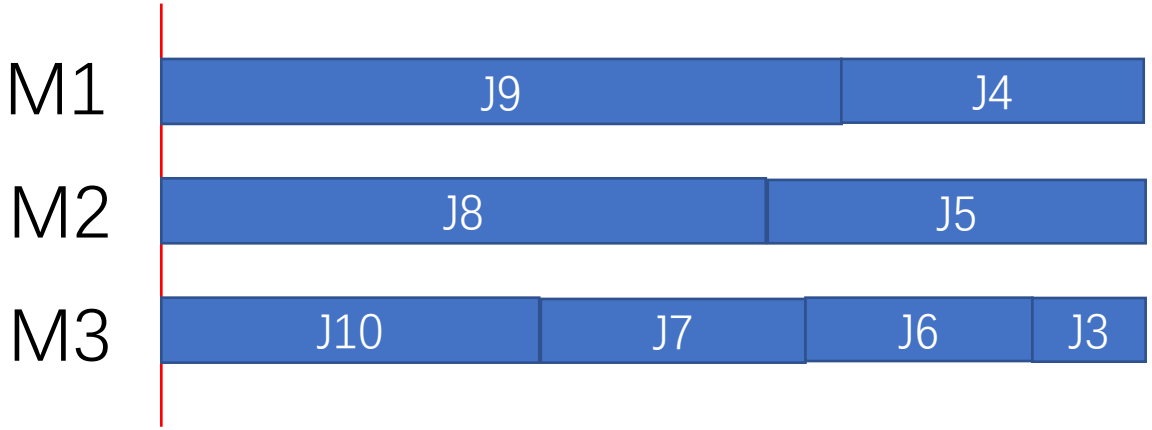



T1 = 26(32)

T2 = 26(32)

T3 = 26

# Exercise 2-2

- Example: $M_3$ needs less processing times than the others.

  Three Machines:$M_1$, $M_2$, $M_3$

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time($t_j$):2,4,6,⋯,20 on $M_1$ and $M_2$

  1,2,3,⋯,10 on $M_3$

  Job order: $J_{10}, J_9, \cdots J_1$


T1 = 26(30)

T2 = 26(30)

T3 = 28

# Exercise 2-2

- Example: $M_3$ needs less processing times than the others.

  Three Machines:$M_1$, $M_2$, $M_3$

  Ten Jobs: $J_1, J_2, \cdots J_{10}$

  Processing time($t_j$):2,4,6,⋯,20 on $M_1$ and $M_2$

  1,2,3,⋯,10 on $M_3$

  Job order: $J_{10}, J_9, \cdots J_1$


T1 = 28

T2 = 26(28)

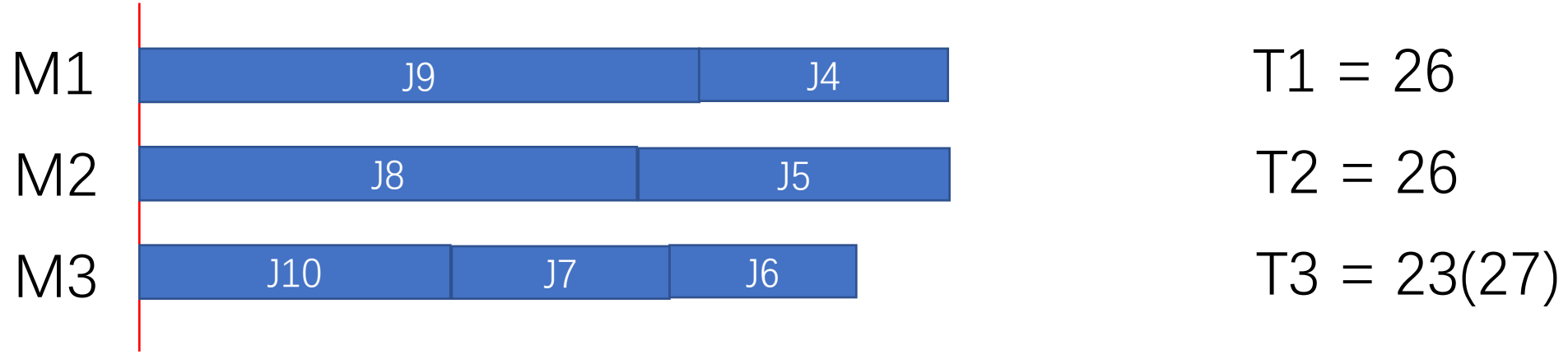
T3 = 28

In this case $T = T^* = 28$

# Exercise 2-2

- Difficult Example: $M_3$ needs less processing times than the others.
  - Three Machines:$M_1$, $M_2$, $M_3$
  - Seven Jobs: $J_4, J_5, \cdots J_{10}$
  - Processing time($t_j$):8,10,12$\cdots$,20 on $M_1$ and $M_2$
    4,5,6,$\cdots$,10 on $M_3$
  - Job order: $J_{10}, J_9, \cdots J_4$

SPGA Function:




T1 = 26

T2 = 26

T3 = 23(27)

T = 26

# Exercise 2-2

- Difficult Example: $M_3$ needs less processing times than the others.

  Three Machines:$M_1$, $M_2$, $M_3$

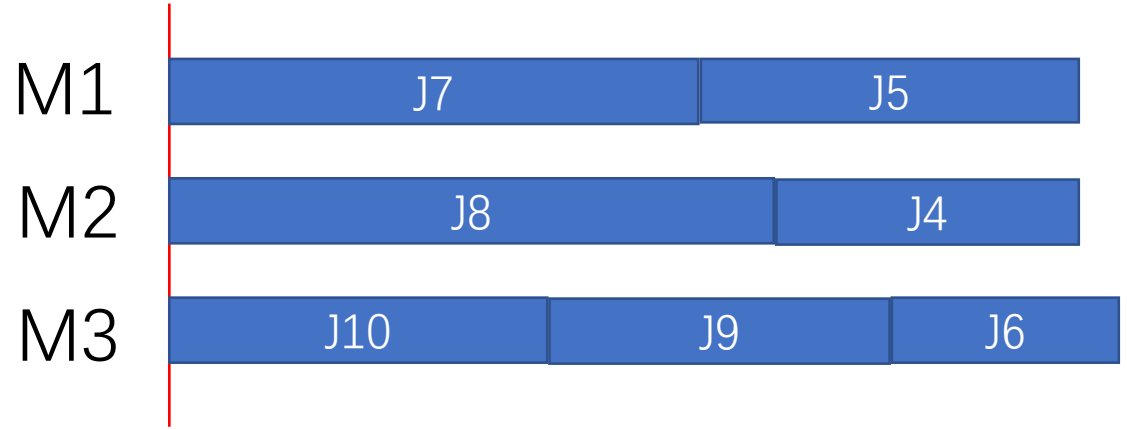
  Seven Jobs: $J_4, J_5, \cdots J_{10}$

  Processing time($t_j$):8,10,12$\cdots$,20 on $M_1$ and $M_2$

  4,5,6,$\cdots$,10 on $M_3$

  Job order: $J_{10}, J_9, \cdots J_4$

Optimal Function:




T1 = 24

T2 = 24

T3 = 25

$T^* = 25$ $\frac{T_{\max}}{T^*} = 1.04$

# Thank you