Related Topic: Fuzzy Clustering

Input: n sites: $S = \{s_1, s_2, ..., s_n\}$, and a constant m ($m > 1$)

Output: Locations of k centers: $C = \{c_1, c_2, ..., c_k\}$

Membership of s_i to c_j: μ_{ij} ($i = 1, ..., n$; $j = 1, ..., k$)

Objective: Minimize the weighted total distance from each site to each center.

$$\text{Minimize} \quad \sum_{i=1}^{n} \sum_{j=1}^{k} (\mu_{ij})^m dist(s_i, c_j)^2$$

where

$$0 \le \mu_{ij} \le 1, \; i = 1, 2, ..., n; \; j = 1, 2, ..., k$$

$$\sum_{j=1}^{k} \mu_{ij} = 1, \; i = 1, 2, ..., n$$

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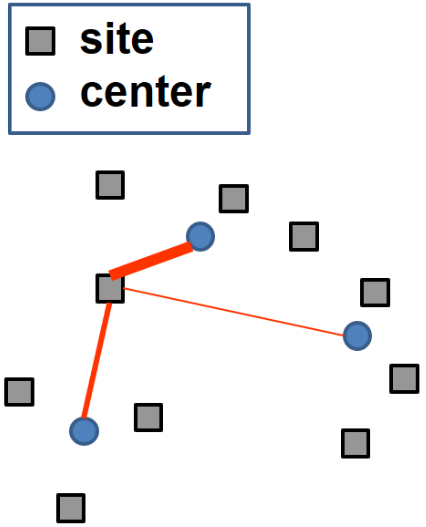


fuzzy *c*-means

$$\text{Minimize } \sum_{i=1}^{n}\sum_{j=1}^{k}(\mu_{ij})^{m} dist(s_i, c_j)^2$$

$$0 \le \mu_{ij} \le 1,\ i = 1, 2, \ldots, n;\ \ j = 1, 2, \ldots, k$$

$$\sum_{j=1}^{k}\mu_{ij} = 1,\ \ i = 1, 2, \ldots, n$$

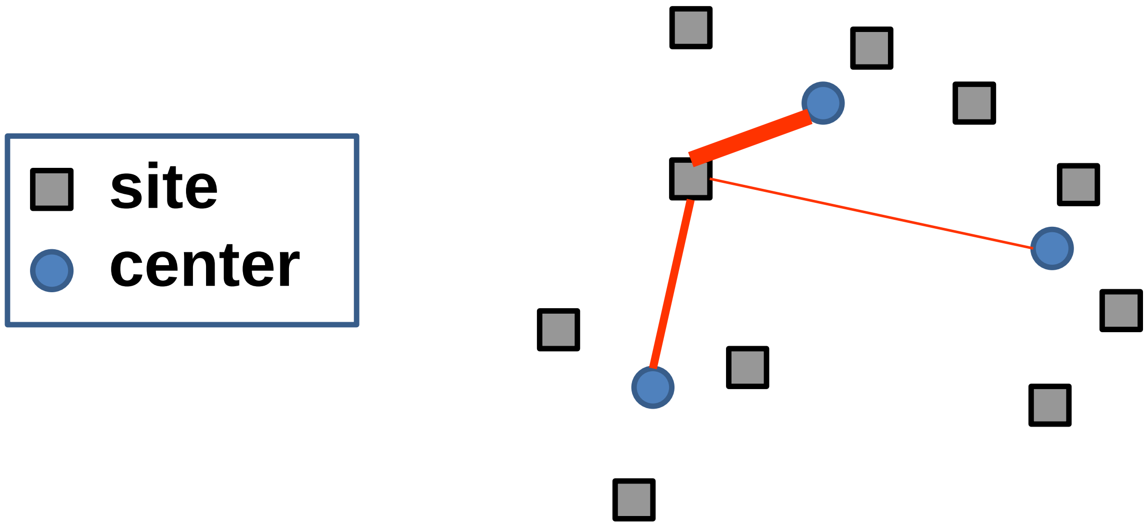
k-means

$$\text{Minimize } \sum_{i=1}^{n}\sum_{j=1}^{k}(\mu_{ij})^{1} dist(s_i, c_j)^2$$

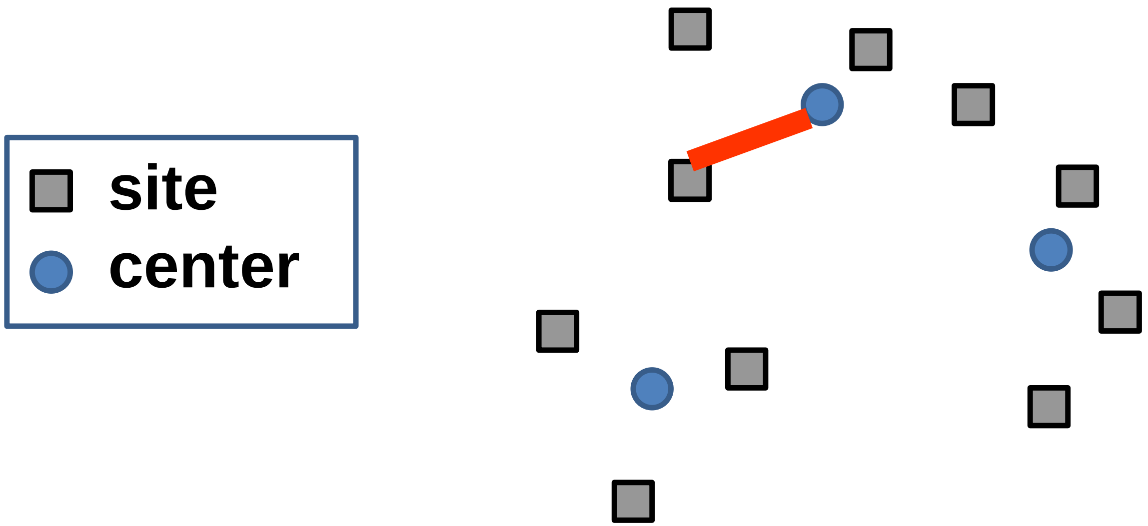
$$\boxed{\mu_{ij} = 0 \text{ or } 1,\ i = 1, 2, \ldots, n;\ \ j = 1, 2, \ldots, k}$$

$$\sum_{j=1}^{k}\mu_{ij} = 1,\ \ i = 1, 2, \ldots, n$$

fuzzy *c*-means



k-means



Fuzzy *c*-means Algorithm: **Iterate the following two steps from randomly specified values of μ_{ij}**

(i) $$c_j = \frac{\sum_{i=1}^{n} (\mu_{ij})^m s_i}{\sum_{i=1}^{n} (\mu_{ij})^m}, \quad j = 1, 2, \ldots, k$$

(ii) $$\mu_{ij} = \left[\sum_{h=1}^{k} \left(\frac{dist(s_i, c_j)}{dist(s_i, c_h)} \right)^{\frac{2}{m-1}} \right]^{-1} \quad \text{for all } i \text{ and } j$$

$$\mu_{ij} = \left[\sum_{h=1}^{k} \left(\frac{dist(s_i, c_j)}{dist(s_i, c_h)} \right)^{\frac{2}{m-1}} \right]^{-1} \quad \text{for all } i \text{ and } j$$

When m ==> ∞

$$\mu_{ij} \Longrightarrow \left[\sum_{h=1}^{k} \left(\frac{dist(s_i, c_j)}{dist(s_i, c_h)} \right)^{0} \right]^{-1} = 1/k$$

When m ==> 1

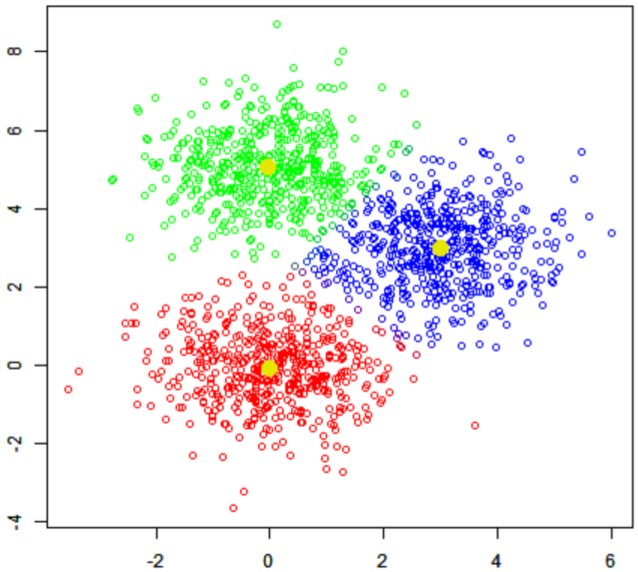
$$\mu_{ij} \Longrightarrow \left[\sum_{h=1}^{k} \left(\frac{dist(s_i, c_j)}{dist(s_i, c_h)} \right)^{\infty} \right]^{-1} = 0 \text{ or } 1$$

Exercise 4-1:

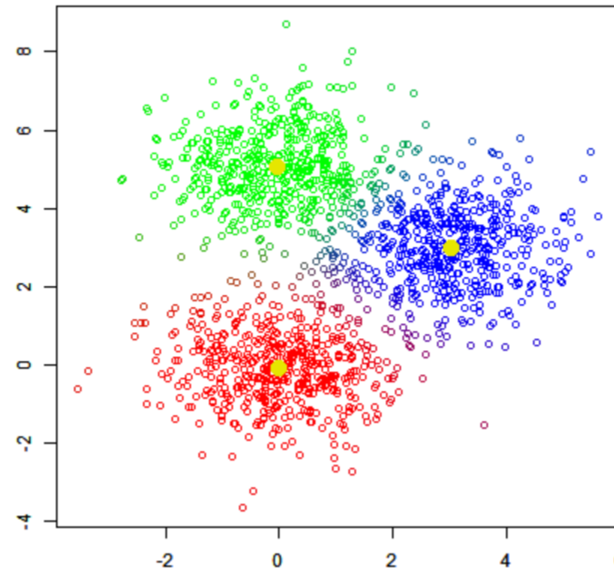
Clearly demonstrate the effects of *m* on the clustering results by the fuzzy c-means algorithm through computational experiments on a test data set (i.e., create a test data set which can be used for clearly demonstrating the effects of *m*).

Exercise 4-2:

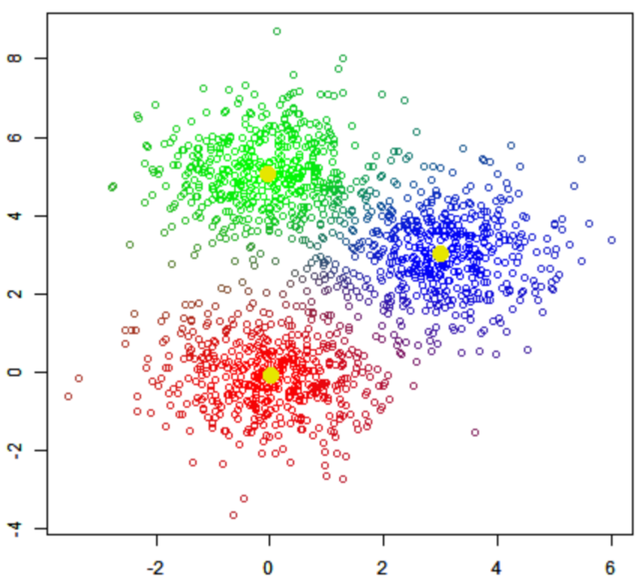
Clearly demonstrate the difference between the *k*-means algorithm and the fuzzy c-means algorithm through computational experiments on a test data set (i.e., create a test data set which can be used for clearly demonstrating the difference between the *k*-means algorithm and the fuzzy c-means algorithm).



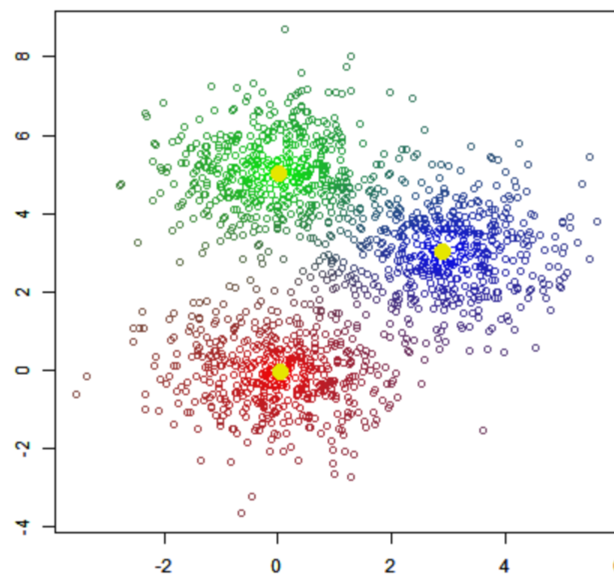
m = 1.1



m = 1.5

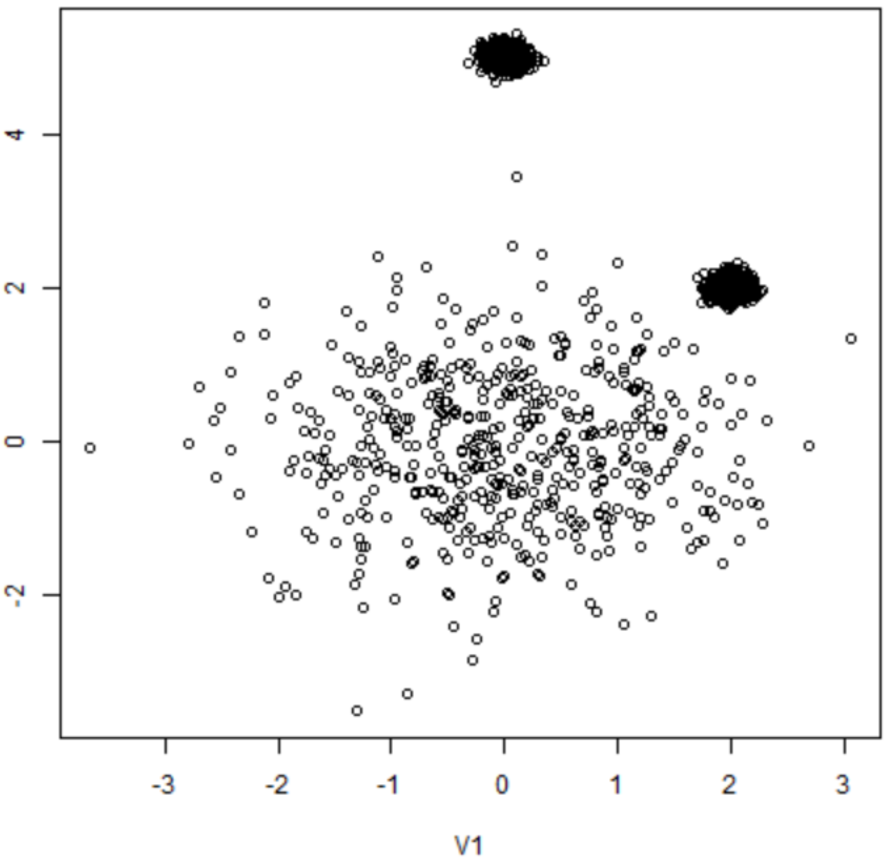


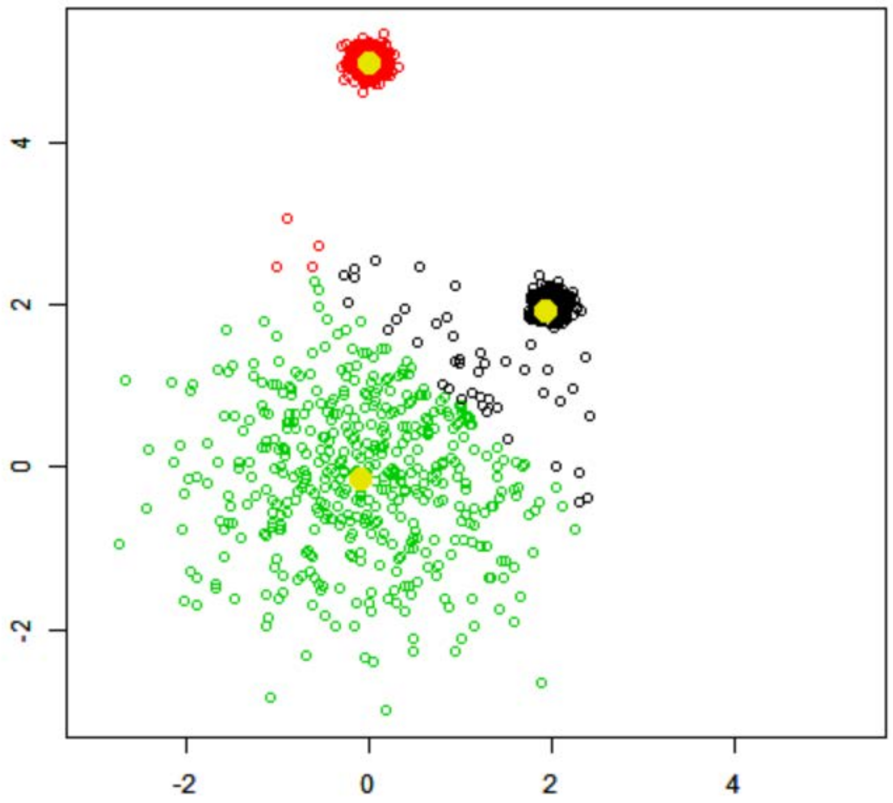
m = 2

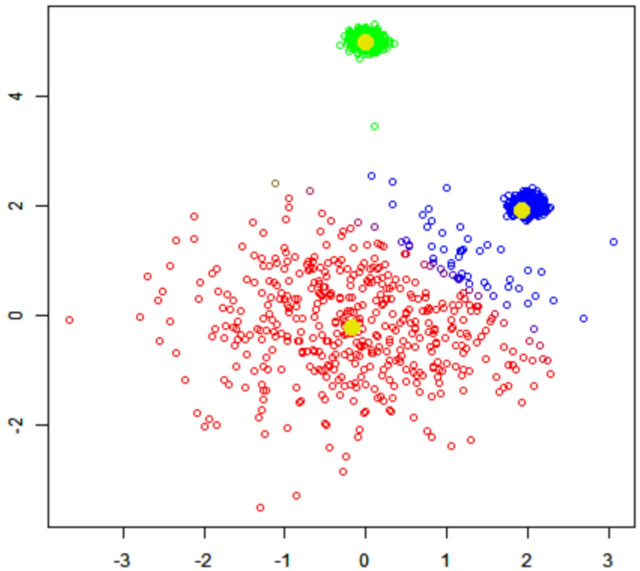


m = 3

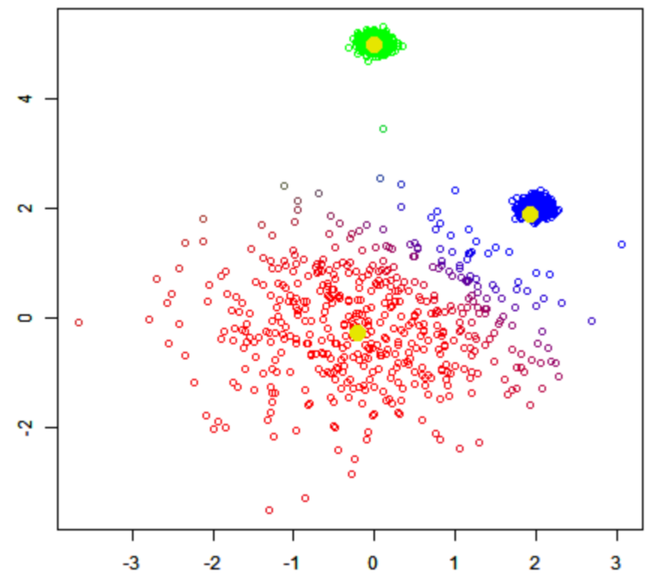
k-means



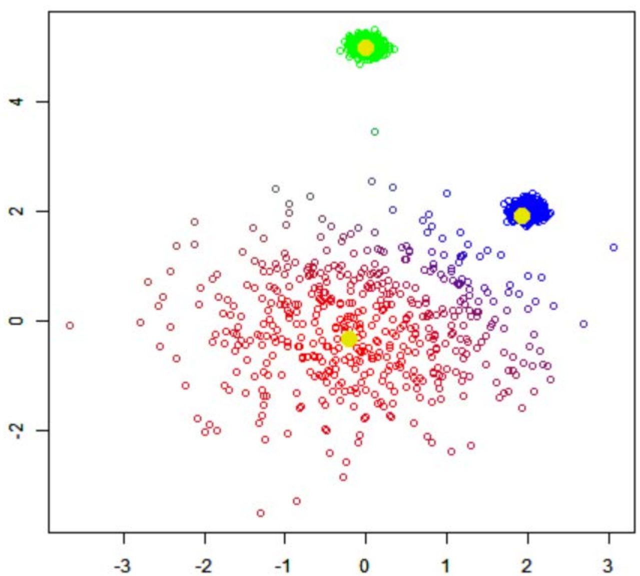




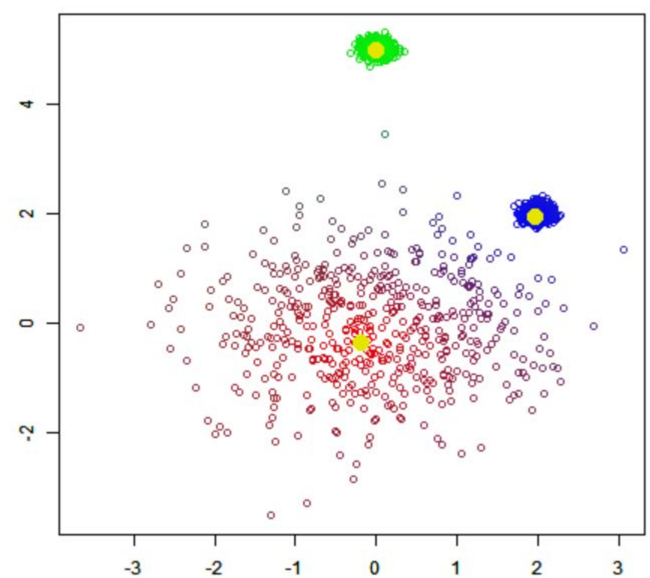
m = 1.1



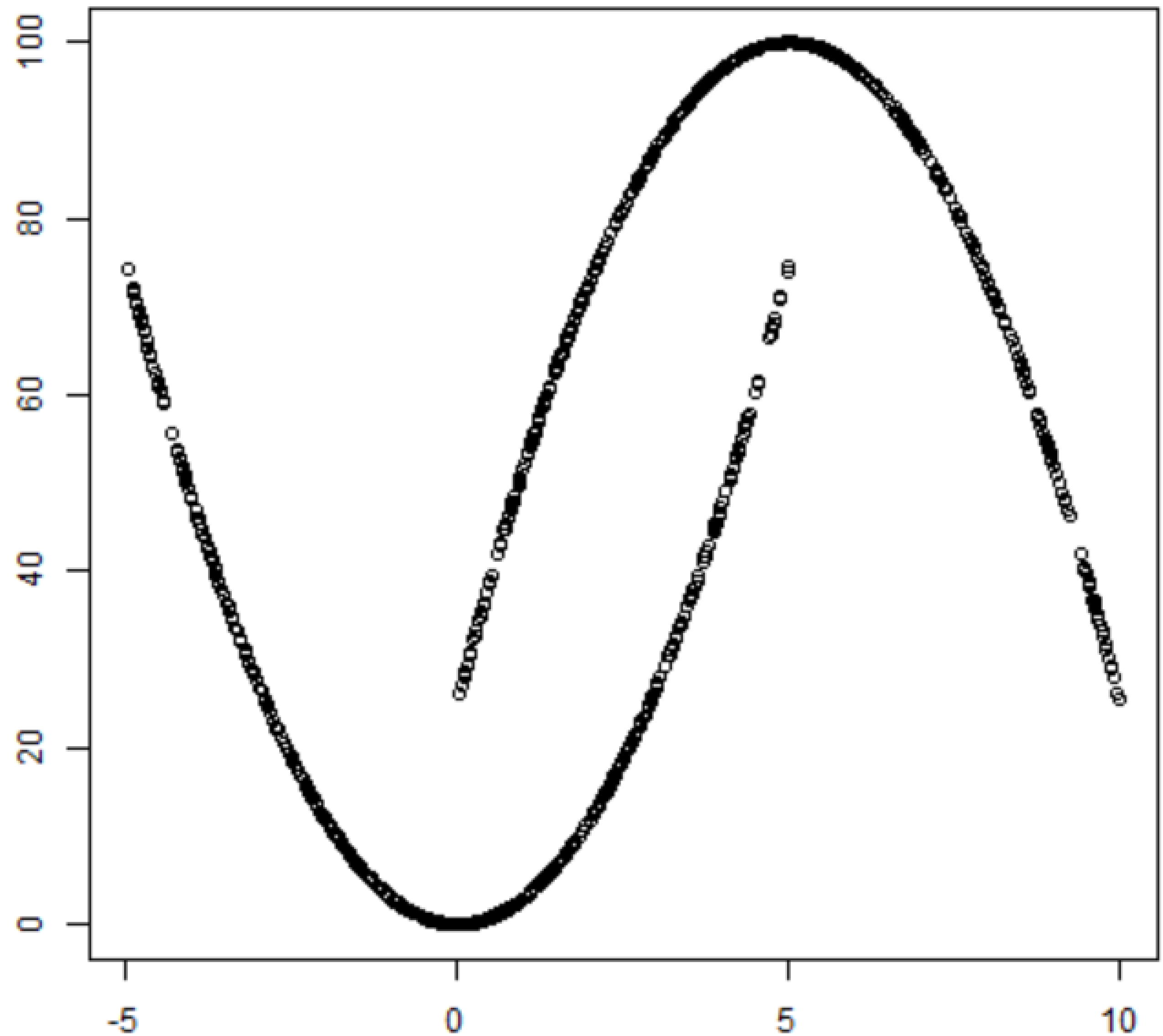
m = 1.5



m = 2



m = 3


100
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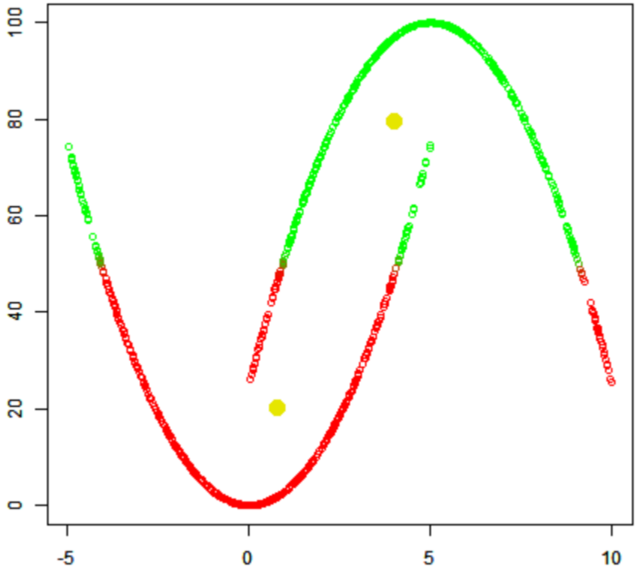
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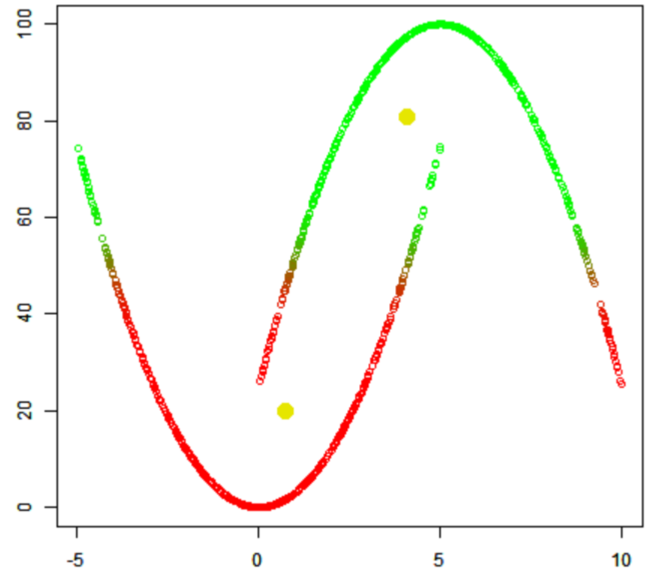
m = 1.1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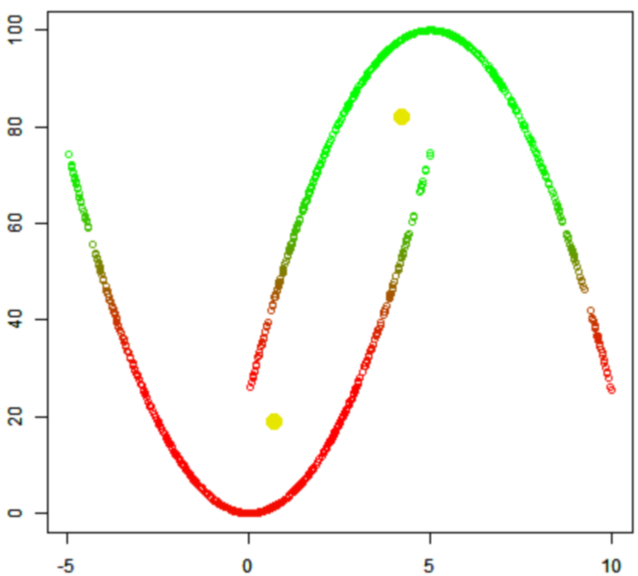
m = 1.5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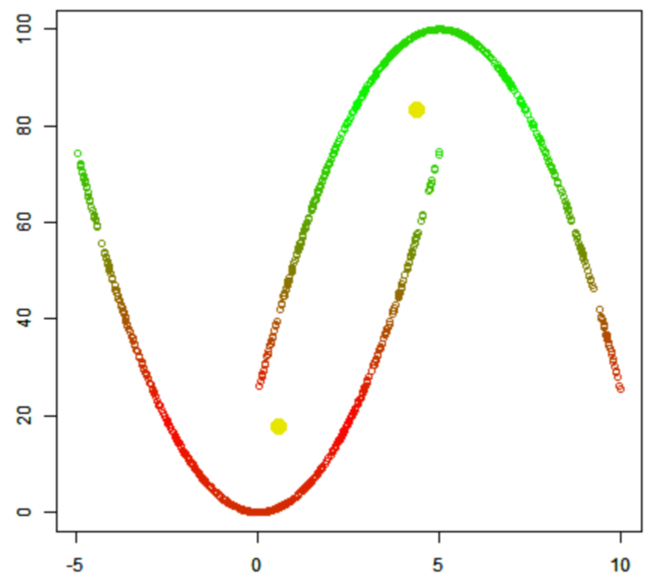
m = 2


100
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-5
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5
10

m = 3

Use of a user-defined hyper parameter (*m* in fuzzy c-means)

Positive Aspects:

A more desirable result can be obtained by appropriately specifying the value of *m* (than the case of the fixed value of m)

Different results can be obtained by examining different values of *m* (we can choose one of them based on our preference).

Negative Aspects:

It is not always easy to appropriately specify the value of *m*.

Undesirable results can be obtained when the value of *m* is inappropriate.

Example: **If *m* is specified as *m* = 1, the algorithm does not work.**

(i) $$c_j = \frac{\sum_{i=1}^{n} (\mu_{ij})^m s_i}{\sum_{i=1}^{n} (\mu_{ij})^m}, \quad j = 1, 2, \ldots, k$$

(ii) $$\mu_{ij} = \left[\sum_{h=1}^{k} \left(\frac{dist(s_i, c_j)}{dist(s_i, c_h)} \right)^{\frac{2}{m-1}} \right]^{-1} \quad \text{for all } i \text{ and } j$$