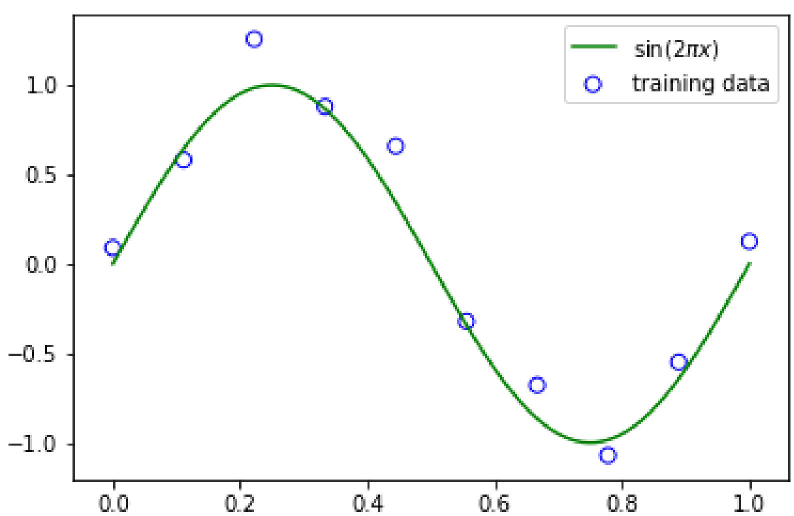
HW1_programQuestion

September 9, 2019

```
In [ ]: import numpy as np
        import matplotlib.pyplot as plt
        %matplotlib inline
```

```
In [ ]: def create_toy_data(func, sample_size, std):
            x = np.linspace(0, 1, sample_size)
            t = func(x) + np.random.normal(scale=std, size=x.shape)
            return x, t

        def func(x):
            return np.sin(2 * np.pi * x)

        x_train, y_train = create_toy_data(func, 10, 0.25)
        x_test = np.linspace(0, 1, 100)
        y_test = func(x_test)
```

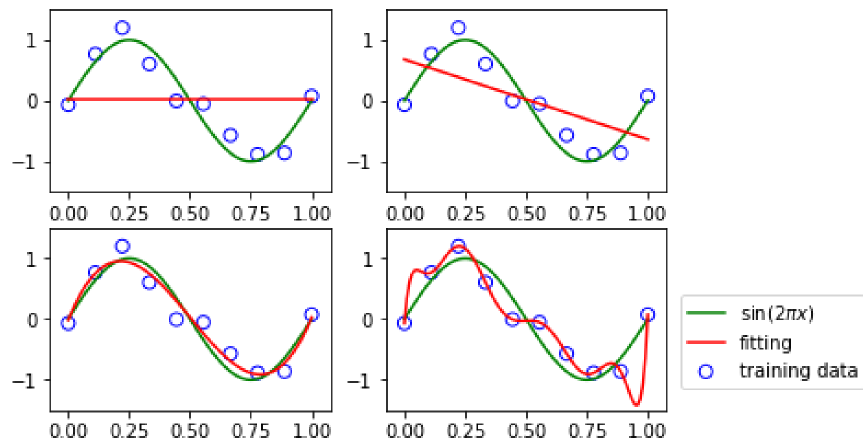
(a) Plot the graph with given code, the result should like below.



`x_train` and `y_train` are the datas you need to create, `sample_size` is 10 and `std` is 0.25.

```
In [ ]: # Write you codes here.
```

(b) On the basis of the results, you should try 0^{th} order polynomial, 1^{st} order polynomial, 3^{rd} order polynomial and some other order polynomial, show the results include fitting and



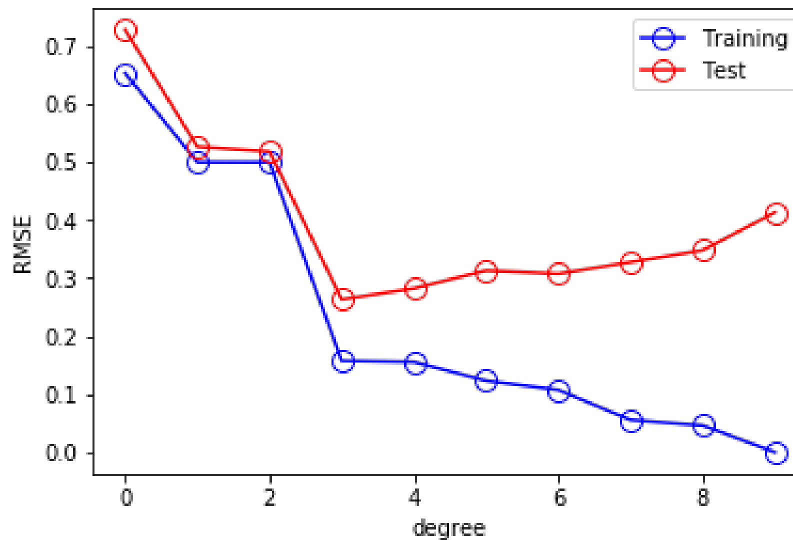
over-fitting.

```
In [ ]: class PolynomialFeature(object):
            """
            polynomial features

            transforms input array with polynomial features

            Example
            =======
            x =
            [[a, b],
            [c, d]]

            y = PolynomialFeatures(degree=2).transform(x)
            y =
            [[1, a, b, a^2, a * b, b^2],
            [1, c, d, c^2, c * d, d^2]]
            """

            def __init__(self, degree=2):
                """
                construct polynomial features

                Parameters
                ----------
                degree : int
                    degree of polynomial
                """
                assert isinstance(degree, int)
                self.degree = degree

            def transform(self, x):
                """
                transforms input array with polynomial features
```

```
        Parameters
        ----------
        x : (sample_size, n) ndarray
            input array

        Returns
        -------
        output : (sample_size, 1 + nC1 + ... + nCd) ndarray
            polynomial features
        """
        if x.ndim == 1:
            x = x[:, None]
        x_t = x.transpose()
        features = [np.ones(len(x))]
        for degree in range(1, self.degree + 1):
            for items in itertools.combinations_with_replacement(x_t, degree):
                features.append(functools.reduce(lambda x, y: x * y, items))
        return np.asarray(features).transpose()

class Regression(object):
    """
    Base class for regressors
    """
    pass

class LinearRegression(Regression):
    """
    Linear regression model
    y = X @ w
    t ~ N(t|X @ w, var)
    """

    def fit(self, X:np.ndarray, t:np.ndarray):
        """
        perform least squares fitting

        Parameters
        ----------
        X : (N, D) np.ndarray
            training independent variable
        t : (N,) np.ndarray
            training dependent variable
        """
        self.w = np.linalg.pinv(X) @ t
        self.var = np.mean(np.square(X @ self.w - t))

    def predict(self, X:np.ndarray, return_std:bool=False):
```

```
        """
        make prediction given input

        Parameters
        ----------
        X : (N, D) np.ndarray
            samples to predict their output
        return_std : bool, optional
            returns standard deviation of each predition if True

        Returns
        -------
        y : (N,) np.ndarray
            prediction of each sample
        y_std : (N,) np.ndarray
            standard deviation of each predition
        """
        y = X @ self.w
        if return_std:
            y_std = np.sqrt(self.var) + np.zeros_like(y)
            return y, y_std
        return y
```

In [ ]:
```
# Write your codes here.
```

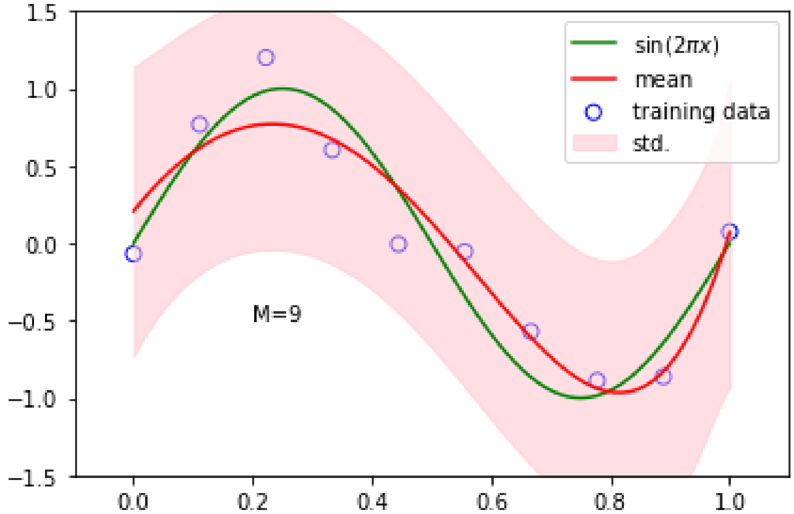


(c) Plot the graph of the root-mean-square error.

In [ ]:
```
def rmse(a, b):
    # Complete this function
```

In [ ]:
```
# Write your codes here.
training_erroes = []
test_errors = []
```

(d) Plot the graph of the predictive distribution resulting from a Bayesian treatment of polynomial curve fitting using an M=9 polynomial, with the fixed parameters $\alpha = 5 \times 10^{-3}$ and $\beta = 11.1$(corresponding to the known noise variance).




```
In [ ]: class BayesianRegression(Regression):
            """
            Bayesian regression model

            w ~ N(w|0, alpha^(-1)I)
            y = X @ w
            t ~ N(t|X @ w, beta^(-1))
            """

            def __init__(self, alpha:float=1., beta:float=1.):
                self.alpha = alpha
                self.beta = beta
                self.w_mean = None
                self.w_precision = None

            def _is_prior_defined(self) -> bool:
                return self.w_mean is not None and self.w_precision is not None

            def _get_prior(self, ndim:int) -> tuple:
                if self._is_prior_defined():
                    return self.w_mean, self.w_precision
                else:
                    return np.zeros(ndim), self.alpha * np.eye(ndim)

            def fit(self, X:np.ndarray, t:np.ndarray):
                """
                bayesian update of parameters given training dataset

                Parameters
```

```python
        ----------
        X : (N, n_features) np.ndarray
            training data independent variable
        t : (N,) np.ndarray
            training data dependent variable
        """

        mean_prev, precision_prev = self._get_prior(np.size(X, 1))

        w_precision = precision_prev + self.beta * X.T @ X
        w_mean = np.linalg.solve(
            w_precision,
            precision_prev @ mean_prev + self.beta * X.T @ t
        )
        self.w_mean = w_mean
        self.w_precision = w_precision
        self.w_cov = np.linalg.inv(self.w_precision)

    def predict(self, X:np.ndarray, return_std:bool=False, sample_size:int=None):
        """
        return mean (and standard deviation) of predictive distribution

        Parameters
        ----------
        X : (N, n_features) np.ndarray
            independent variable
        return_std : bool, optional
            flag to return standard deviation (the default is False)
        sample_size : int, optional
            number of samples to draw from the predictive distribution
            (the default is None, no sampling from the distribution)

        Returns
        -------
        y : (N,) np.ndarray
            mean of the predictive distribution
        y_std : (N,) np.ndarray
            standard deviation of the predictive distribution
        y_sample : (N, sample_size) np.ndarray
            samples from the predictive distribution
        """

        if sample_size is not None:
            w_sample = np.random.multivariate_normal(
                self.w_mean, self.w_cov, size=sample_size
            )
            y_sample = X @ w_sample.T
            return y_sample
```

```python
        y = X @ self.w_mean
        if return_std:
            y_var = 1 / self.beta + np.sum(X @ self.w_cov * X, axis=1)
            y_std = np.sqrt(y_var)
            return y, y_std
        return y
```

In [ ]:
```python
# Write your codes here.
```

(e) Change the *sample_size* to 2, 3 or 10 times than before, explain the change of M.

In [ ]:
```python
# Write your codes here.
```