The Mathematics of the Chinese Calendar

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1 Introduction

Chinese New Year is the main holiday of the year for more than one quarter of the world's population; very few people, however, know how to compute its date. For many years I kept asking people about the rules for the Chinese calendar, but I wasn't able to find anybody who could help me. Many of the people who were knowledgeable about science felt that the traditional Chinese calendar was backwards and superstitious, while people who cared about Chinese culture usually lacked the scientific knowledge to understand how the calendar worked. In the end I gave up and decided that I had to figure it out for myself. This paper is the result.

The rules for the Chinese calendar have changed many times over the years. This has caused a lot of confusion for people writing about it. Many sources describe the rules that were used before the last calendar reform in 1645, and some modern sources even describe the rules that were used before 104 BCE! In addition, the otherwise authoritative work of Needham ([32]) almost completely ignores the topic. For many years, the only reliable source in English was the article by Doggett in the Explanatory Supplement to the Astronomical Almanac ([12]), based on unpublished work of Liu and Stephenson ([25]). But thanks to the efforts of Dershowitz and Reingold ([11]), correct information and computer programs are now easily available. Among Chinese sources, my favorite is the book by Tang ([39]).

Even though these sources give the basic rules, the Chinese calendar is such a rich subject that there's still a lot left to study. The basic principles are fairly simple, but in some exceptional cases it can get extremely complicated. The year 2033 is such an exceptional case and until the early 1990's, all the Chinese calendars placed the leap month incorrectly for that year. This "Chinese Y2033" issue was what finally motivated me to write this paper.

In this article I first explain the rules for the Chinese calendar, leading to a discussion of the Y2033 problem. I then discuss some other mathematical issues in the Chinese calendar. Many Chinese astronomers claim that there can be no leap month after the 12th or 1st month ([39]). This is true in the sense that it hasn't happened since the last calendar reform in 1645, and that it will not happen in the 21st century. But because of the precession of the equinoxes (Section 2) it is clear that in the future there will be many such leap months. In 2262 there will be a leap month after the 1st month, and in 3358 there will be a leap month after the 12th. Given the difficulty in making accurate astronomical predictions more than 100 years ahead, these computations must obviously be taken with a grain of salt. I also believe that there was an error in the computations for 1651, and that there should have been a leap month after the 1st month that year. Instead the leap month was added after the 2nd month.

There are many different statements about what are the possible dates for Chinese New Year. In the 1000 years between 1645 (the last calendar reform) and year 2644, Chinese New Year will always fall between January 21 and February 21.

I know of at least three commonly stated, but not always correct, rules for determining the date for Chinese New Year.

1. *Chinese New Year falls on the day of the second new Moon after the December solstice (approximately December 22).* This fails whenever there's a leap month after the 11th or 12th month. In 2033 it fails for the first time since 1645 (the last calendar reform).

2. *Chinese New Year falls on the day of the new Moon closest to the minor solar term (jié qì) lì chūn* (立春)*, "beginning of spring" (approximately February 4).* This rule failed in 1985 and will fail again in 2015.

3. *Chinese New Year falls on the day of the first new Moon after the major solar term (zhōng qì) dà hán* (大寒) *(approximately January 20).* This rule failed in 1985 and will fail again in 2053.

At the end I give an outline of certain aspects of the history of the Chinese calendar related to this paper.

For the computations in this paper I used the Mathematica version of the code from the book by Dershowitz and Reingold ([11]). The conversion from Lisp to Mathematica was done by Robert C. McNally. Their astronomical functions are based on the book by Meeus ([31]). Additional functions are in my Mathematica package ChineseCalendar ([1]).

I would like to thank a number of people who have inspired me over the years. My father for making me interested in astronomy by first pointing out the equation of time to me, Professor Wu-yi Hsiang, my Ph.D. advisor in the Mathematics department of UC Berkeley, for teaching me that "first you search, then you search again, and then you re-search", and Professor Frederic E. Wakeman Jr. of the History department at UC Berkeley whose lectures on Chinese history were models in terms of teaching, scholarship and stand-up comedy.

I would also like to thank Peter Davison, Salvo De Meis, Nachum Dershowitz, Jean Meeus, Peter Meyer, Neo Peng Fu, Karl Palmen, Edward M. Reingold, E. G. Richards and Hugh Thurston for help and corrections. Thanks also to my students Chin Hei Ting, Kuan Shau Hong, Lee Tang Hwee, Teng Keat Huat and Ng Say Tiong.

2 A quick course in astronomy

In order to explain how the Chinese calendar works, we must start by recalling some basic facts from astronomy. For the purpose of reference, I will go into more detail than is strictly necessary, so the reader may skip parts of this section. For excellent introductions to spherical astronomy, see the books by Brown ([3]), Evans ([19]), Kaler ([22]) and Rogers ([35]). For technical details, I rely on the books by Meeus ([30, 31]) and the Explanatory Supplement to the Astronomical Almanac ([12]). I also recommend the web pages of the Astronomical Applications Department of the U.S. Naval Observatory ([41]).

The Earth revolves counterclockwise (when viewed from the north celestial pole) around the Sun in an elliptical orbit (Figure 1). The plane of the orbit is called the plane of the *ecliptic*. The word "ecliptic" is derived from the fact that eclipses can only occur when the Moon crosses this plane. The Earth rotates counterclockwise around an axis that is tilted approximately 23.5 degrees. Notice how astronomers make a distinction between *revolving* and *rotating*. An object rotates around an axis that passes through it, but it revolves around some outside object.

Early astronomers realized that the motion of the Sun (or the Earth, depending on your point of view) along the ecliptic was not uniform. This is a consequence of Kepler's Second Law, which says that the planets sweep

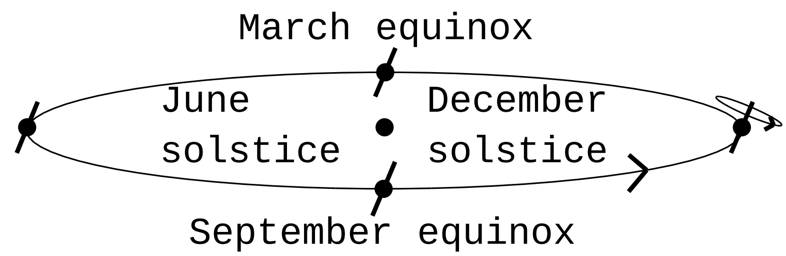

Figure 1: The ecliptic plane

out equal areas in equal time (Figure 2). This means that the Earth moves faster along the orbit near *perihelion*, the point on the orbit where the Earth is closest to the Sun.

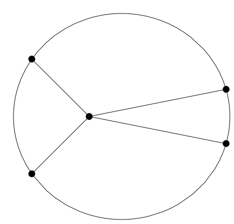

Figure 2: Kepler's Second Law

When the Earth's axis tilts towards the Sun, there is summer in Beijing. At the June and December *solstices*, also called the summer and winter solstices, the projection of the Earth's axis onto the plane of the ecliptic points directly towards the Sun (Figure 3). At the March and September *equinoxes*, also called the vernal (spring) and autumnal equinoxes, the radial line from the Sun to the Earth is perpendicular to the Earth's axis. These four points are called *seasonal markers*.

The above definition is of course not the way people in ancient civilizations determined the seasonal markers. A simple way was to look at how the rising position of the Sun changes over the course of the year. The Sun rises due east at the equinoxes, at which time day and night are equally long. The word "equinox" is derived from a Latin word meaning "equal night". Strictly speaking, the day is a bit longer at the time of the equinox, since sunrise is the time when the top of the Sun reaches the horizon, while sunset is the time when the top of the Sun goes below the horizon. In addition, refraction bends

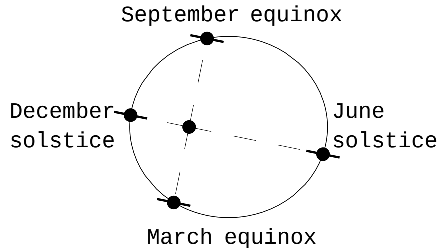

Figure 3: Solstices and equinoxes

the image of the Sun upwards near the horizon. After the March equinox, the rising position of Sun moves north until the Sun reaches its northernmost rising position at the June solstice. Here the Sun seems to stand still, and the word "solstice" is derived from the Latin word "solstitium", which means "standing Sun". The Sun is now in the zenith over the tropic of Cancer. The word "tropic" is derived from the Greek word for turning, because the Sun now turns and starts moving south, rising due east at the September equinox before it reaches its southernmost rising position at the December solstice, at which time it is in the zenith over the tropic of Capricorn. At the time when the terms tropic of Cancer and tropic of Capricorn were coined, the solstices were in the zodiac constellations of Cancer and Capricorn. Because of precession (see below) they have since moved. Figure 4 shows the path of the Sun in the sky for observers in Beijing and Singapore.

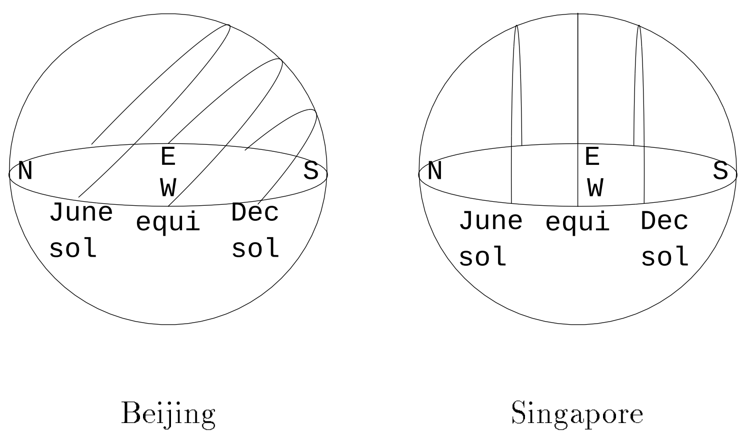

Figure 4: The path of the Sun in the sky

A more accurate method is to study the shadow of a vertical pole, called a gnomon. At the equinoxes, the path of the shadow cast by a gnomon is a straight line. The rest of the year, the path is a hyperbola. In order to understand this, think of how the line from the Sun to the tip of the gnomon forms a cone in the course of the day. The hyperbola is the conic section obtained by intersecting this cone with the plane corresponding to the Earth's surface. At the equinoxes, the cone degenerates to a cone with vertex angle equal to 180°, i.e., a plane, so the intersection is a line. On the northern hemisphere, the noon shadow is shortest at the June solstice and longest at the December solstice. Chinese astronomers were experts at using the gnomon. The famous mathematician Guō Shǒu Jìng (郭守敬) had a 13 meter gnomon built in 1276!

A common mistake is to think that the projection of the Earth's axis is tangential to the orbit at the equinoxes (Figure 3). It is an easy exercise in analytic geometry to see that the radial line from a focus is never perpendicular to the tangent line.

Another common error is to believe that the seasonal markers coincide with the vertices of the elliptic orbit, e.g., to think that the December solstice coincides with perihelion, the point on the orbit that is closest to the Sun. This was the case around 1246, but is no longer true. In order to understand this, we must explain a phenomenon called *precession*, or more formally, precession of the equinoxes. In the second century BCE, the Greek astronomer Hipparchus discovered that older star records did not match his observations. He realized that the position of the March equinox had drifted backwards along the ecliptic. The Earth's axis revolves around in a circle with a period of about 25,800 years (Figure 5). This makes the March equinox move clockwise by 50" with respect to the stars each year. The reason for this, first explained by Newton, is that the Sun's gravitation tries to "straighten" the earth.

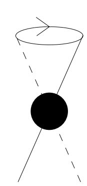

Figure 5: Precession

Some early astronomers measured the time from one December solstice to the next and called it the *tropical year* because it measured the return of the Sun to the same tropic. In Western astronomy it was customary to measure the time between two March equinoxes and call this the tropical year. We will see below that these are not the same! For both of them, the variation from the mean can be up to 10 minutes.

Most people think of a year as the time it takes the Earth to complete one revolution around the orbit. This is not true! The time it takes for the Earth to complete one revolution with respect to the stars is 365.25636 days ([11]) and is called the *sidereal year*. This is about 20 minutes longer than the tropical year. The reason for this is that in the course of the year the Earth's axis has rotated clockwise by a small amount, so the axis will be perpendicular to the radial line a bit earlier than a year ago. Hence the Earth covers one orbit minus a small piece. (If you have a hard time visualizing this, you may want to try to think of a solstice year instead.)

Since the speed of the Earth changes according to Kepler's Second Law, the time it takes to cover the extra piece depends on where in the orbit this small piece is. In particular, the time between two December solstices, currently 365.242740 days ([28]), is longer than the time between two March equinoxes, currently 365.242374 days ([28])! The tropical year was traditionally defined to be the mean time from one March equinox to the next. The modern definition is the time it takes the Sun's mean longitude to increase by 360° ([28]). It is currently 365.24219 days. This problem is ignored by most people. I will feel free to use the term "tropical year" for either the value derived from the mean longitude, the March equinox year (used in Western astronomy) or the December solstice year (used in Chinese astronomy). Notice that for all these years, the length is decreasing by about half a second each century ([34]), due to to slowing down of the Earth's rotation.

Since the seasons are tied to the seasonal markers, most calendars try to make their year an approximation of the tropical year, or the seasonal year as it is sometimes called. This is true for the Gregorian and the Chinese calendars, but not for the Indian calendars, which use the sidereal year.

In addition to precession there is another factor involved in the behavior of the equinox. The Earth's orbit rotates counterclockwise in the plane of the ecliptic with a period of about 110,000 years. This means that the orbit rotates in the opposite direction of the precession of the equinoxes. The net effect is that while it takes the March equinox 25,800 years to complete one clockwise revolution with respect to the stars, it only takes about 21,000 years to complete one clockwise revolution with respect to the orbit. The rotation of the orbit has an important consequence. The point where the Earth is closest to the Sun is called *perihelion*. The point where the Earth is farthest from the Sun is called *aphelion*. To say that the equinox rotates clockwise around the orbit is the same as saying that perihelion falls later each year, and that after 21,000 years it drifts through the calendar once.

So we can sum up by saying that the solstices and equinoxes move along the orbit with period 21,000 years, but stays fixed in the calendar, while perihelion stays fixed in the orbit, but progresses through the calendar with period 21,000 years.

It follows from this that at the present time, the Earth moves fastest during the winter in Beijing, but in 10,500 years, it will move fastest during the summer.

As always in astronomy, there are some complicating factors! First of all, the Moon deforms the orbit of the Earth. This is because the elliptical orbit is the orbit of the Earth-Moon barycenter (center of mass), while the orbit of the Earth itself is more complex. Since the Earth's orbit is almost circular, even small deformations can change the position of perihelion noticeably. The net result is that perihelion might be up to 32 hours off the expected time ([30]). The time of the solstices and equinoxes, however, are not so sensitive to these deformations.

Secondly, the Gregorian calendar is not a perfect approximation of the tropical year. The insertion of leap days and the fact that the Gregorian year is somewhat longer than the tropical year shifts the time for the seasonal markers and perihelion. Table 1 gives the extreme dates for the seasonal markers and perihelion for the period 1980 to 2020 ([29]).

	Earliest	Latest
Perihelion	January 1 22h (1989)	January 5 8h (2020)
March equinox	March 20 4h (2020)	March 21 5h (1983)
June solstice	June 20 22h (2020)	June 21 23h (1983)
September equinox	September 22 14h (2020)	September 23 15h (1983)
December solstice	December 21 10h (2020)	December 22 11h (1983)

Table 1: Extreme dates for the seasonal markers and perihelion between 1980 and 2020

Ancient civilizations used geocentric models, and from that point of view, the Sun moves along a great circle on the celestial sphere called the ecliptic (Figure 6). Since you cannot see the Sun and the stars at the same time, it is not immediately obvious how the Sun moves among the stars. But by noticing which stars become visible right after sunset near the spot where the Sun crossed the horizon, (or are visible right before the Sun rises), it is possible to chart the course of the Sun across the celestial sphere. The equinoxes are the points where the ecliptic intersects the celestial equator, and the solstices are the points where the ecliptic and the celestial equator are farthest apart.

It is important to understand that for the purpose of calendar theory, it doesn't matter whether we take a heliocentric or geocentric point of view. What matters is the quality of our tables. In fact, the first tables based on the Copernican system were worse than the old tables based on the Ptolemaic system ([19])!

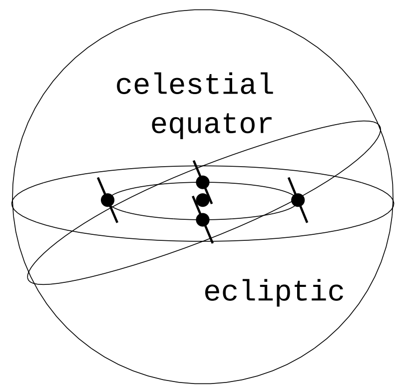

Figure 6: The celestial sphere

The motion of the Moon is very complex. The *synodic month* (or lunation) is the mean time from one new Moon (conjunction) to the next. The word "synodic" comes from the Greek word "synodos" or "meeting", referring to the Moon's conjunction with the Sun. Between 1000 BCE and 4000 CE it ranges from 29 days 6 hours and 26 minutes (29.27 days) to 29 days 20 hours 6 minutes (29.84 days) with a mean of 29 days 12 hours 44 minutes 3 seconds (29.530588853 days) ([27, 31]).

3 Basic calendrical concepts

If we consider a *lunar year* consisting of 12 mean lunar months, the length equals 354.36707 days ([12]), which is about 11 days shorter than a tropical year of 365.242374 days. This difference between the lunar year and the solar year was a fundamental problem for the ancients. They tried to overcome the problem by trying to find some longer resonance cycle. Early astronomers discovered that 235 mean lunar months is almost the same as 19 tropical years. In fact, 235 mean lunar months equals 6939.6884 days, while 19 tropical years equals 6939.6018 days. The difference is only about two hours, which accumulates to an error of one day in about 220 years. This is called the *Metonic cycle* after the Greek astronomer Meton who used it in 432 BCE, but it was known to the Babylonians by around 500 BCE and to the Chinese around 600 BCE ([5]). In China it was called the zhāng (章) cycle. Unfortunately, many ancient astronomers were so convinced of the harmony of the heavens that they assumed that this wonderful relation was exact, so for a long time this cycle was hardwired into the Chinese calendar. The Metonic cycle is used in the Jewish calendar, in the computation of Easter, and was used in the Chinese calendar before 104 BCE.

There are several ways to classify calendars. A *lunar* calendar is a calendar that ignores the Sun and the tropical year (and hence the seasons) but tries to follow the Moon and the synodic month. An example of this is the Muslim calendar. The Muslim calendar is based on first visibility of the crescent Moon. Since 12 lunar months is about 11 days shorter than the tropical year, the Islamic holidays regress through the seasons. Some sources (mostly Western) describe an arithmetical, civil, Islamic calendar (hisabi) with alternating days of 29 and 30 days and a system of leap years with one extra day in the last month. In the past this was sometimes used for constructing calendars for previous years and for making very rough predictions of lunar visibility. To the best of my knowledge, it is not currently used anywhere for any purposes.

A *solar* calendar uses days to approximate the tropical year. An example is the Gregorian calendar. A year consists of 365 days, while leap years have 366 days. Year n is a leap year if n is divisible by 4, but not by 100 or if n is divisible by 400. So 1900 is not a leap year, while 2000 is. The Gregorian calendar has a cycle of 400 years, with average length of the year equal to 365.2425. This is about 27 seconds longer than the current value of the tropical year. There are various estimates for when this will accumulate to an error of one day. Unfortunately, the shortening of the tropical year (Section 2) makes it impossible to predict how the error will accumulate. Some people, however, have suggested that we should adjust the rules by saying that years divisible by 4,000 are not leap years.

Some people argue ([38]) that since in the past the Gregorian calendar was compared to the March equinox year, we should continue to do so. According to this view, the Gregorian calendar is more accurate that people think, because the Gregorian year is closer to the current value of the March equinox year than to the tropical year. My view is that as long as astronomers defined the tropical year to be the March equinox year, it made sense to compare the Gregorian year with the March equinox year, but now that astronomers have redefined the tropical year to be the period of the mean longitude of the Sun, this is what we should compare the calendar with. The modern Gregorian calendar is determined by scientists and not by religious authorities. Leap seconds are inserted by international scientific organizations and not the by the Pope! Some people also claim that the ecclesiastical rule for computing Easter is an essential part of the calendar. However, many Eastern European countries use the Gregorian calendar for civil purposes and the Eastern Orthodox rules for the computation of Easter.

Since the Gregorian year is an approximation to the tropical year, the equinox stays almost constant. The main movement is caused by the insertion of leap days (also called bissextile days). Each normal year is a bit too short, so the equinox moves forward a quarter day in the calendar for three years in a row. The leap year then evens it out, so the equinox performs a "four step dance": Three small steps forward and one long step back. The old Julian calendar kept the rhythm, but the Gregorian calendar "misses a beat" three times every 400 years.

As explained in Section 2, the deformation of the orbit caused by the Moon changes the time of the solstices and equinoxes ([30]), as does the fact that the Gregorian year is longer than the tropical year. Table 2 shows the earliest and latest dates for the seasonal markers for the Gregorian cycle between 1800 and 2200. The reason why 1903 and 2096 are the extreme values is because 2000 is a leap year, while 1900 and 2100 are not.

	Earliest (2096)	Latest (1903)
March equinox	March 19 14h	March 21 19h
June solstice	June 20 7h	June 22 15h
September equinox	September 21 23h	September 24 6h
December solstice	December 20 21h	December 23 0h

Table 2: Extreme dates for the seasonal markers between 1800 and 2200

Lunisolar calendars use months to approximate the tropical year. Examples are the Jewish and Chinese calendars. Since 12 months are about 11 days shorter than the tropical year, a leap month (also called intercalary month) is inserted about every third year to keep the calendar in tune with the seasons. The big question is how to do this. A simple method is to just base it on nature. In ancient Israel, the religious leaders would determine the date for Passover each spring by seeing if the roads were dry enough for the pilgrims and if the lambs were ready for slaughter. If not, they would add one more month. An aboriginal tribe in Taiwan would go out to sea with lanterns near the new Moon at the beginning of spring. If the migrating flying fish appeared, there would be fish for New Year's dinner. If not, they would wait one month.

A more predictable method is to use the Metonic cycle. Recall from Section 2 that 19 tropical years is almost the same as 235 synodic months. Since $235 = 19 \times 12 + 7$, it follows that we get a fairly good lunisolar calendar if we insert 7 leap months in each 19-years period. The exact rules for this intercalation can be quite tricky, however. This is the method used in the Jewish calendar, and was used in the Chinese calendar before 104 BCE. The modern Chinese calendar is different, in that it uses the motion of the true Moon rather than the mean Moon. I will explain the exact rules for the Chinese calendar in Section 4.

Notice that the Chinese calendar is *not* a lunar calendar! The traditional Chinese name is in fact yīn yáng lì (阴阳历), which simply means lunisolar calendar. Lunisolar calendars are solar calendars that just happen to use the lunar month as the basic unit rather than the solar day.

Since the year in a lunisolar calendar is an approximation to the tropical year, the solstices and equinoxes stay relatively constant. The main movement is caused by the insertion of leap months. Each 12-month year is about 11 days too short, so the solstices and equinoxes move forward 11 (or 10 or 12) days. Each 13-month leap year (also called embolismic year) is about 19 days too long, so the solstices and equinoxes jump back 19 (or 18 or 20) days. The solstices and equinoxes performs a "three step dance": Two small steps forward and one long step back. But this dance is a bit off-beat. Two of the seven leap years in each 19-years cycle come after just one normal year, i.e., two years after the previous leap year, so in that case the solstices and equinoxes change into a two step rhythm. Notice also that the steps are much bigger than in the Gregorian four step dance.

There is also another way of classifying calendars. An *arithmetical* calendar is defined by arithmetical rules. Examples are the Gregorian and Jewish calendars. Prediction and conversion between different arithmetical calendars is in principle simple.

An *astronomical* calendar is a calendar that is defined directly in terms of astronomical events. Examples are the Islamic and Chinese calendars. Strictly speaking, there are two kinds of astronomical calendars. The modern (post 1645) Chinese calendar is modular in the sense that the rules are defined in terms of the motion of the true Sun and true Moon, but the description of this motion is considered to be a separate problem and is not specified as part of the calendar. The Jesuit missionaries who carried out the 1645 reform did not have accurate methods for computing this, but since their methods were not hardwired in the calendar, there was no need for further reforms as the computational methods improved. The computational problem is (in principle) separated from the rules for the calendar, and the calendar is (in principle) always correct. Many Indian calendars, however, use traditional formulas for approximating the true motions. In such a *semi-astronomical* calendar, errors are likely to develop, because the traditional methods are not accurate.

For completeness, let me also briefly mention the French Revolutionary calendar. To mathematicians it's interesting to know that Lagrange and Monge were in the committee that formulated the French Revolutionary calendar, while Laplace was in the committee that eventually abolished it ([34]). Its New Year fell on the day of the September equinox. The length of the year depends on which day the next September equinox falls on. For most years, the next September equinox falls on the 366th day of the year. That day is the first day of the new year, so the old year has 365 days. But after 4, or occasionally 5, such years the September equinox falls on the day after the 366th day, and we have a leap year with 366 days. This system has the advantage that it is always in tune with the tropical year, but it is hard to predict which years will be leap years, especially for years when the September equinox happens to occur near midnight. If we wanted a true astronomical solar calendar, we should of course also define the day using the motion of the true Sun rather than the mean Sun. As you can see, this calendar would not be very practical! Still it is an interesting thought experiment, because as we will see, the Chinese calendar is based on related principles.

Table 3 illustrates these classifications. Prediction and conversion involving astronomical calendars is hard, and requires knowledge about the position of the Sun and the Moon.

	Arithmetical	Astronomical
Solar	Gregorian	French Revolutionary
Lunisolar	Jewish	Chinese
Lunar	Civil Muslim	Religious Muslim

Table 3: Classification of calendars

4 The Chinese calendar

4.1 The 24 jié qì

In order to understand the rules for the Chinese calendar, we must first define the 24 solar terms or jié qì (节气). They are a generalization of the solstices and equinoxes. The seasonal markers cut the ecliptic into 4 sections of 90°

each (Figure 3). The 24 jié qì cuts the ecliptic into 24 sections of 15° each. The even ones are called major solar terms or zhōng qì (中气), while the odd ones are called minor solar terms or jié qì. Strictly speaking, the word jié qì is used in two ways. It can either refer to the 12 odd ones, or it can refer to all 24. Since the English word solar term is not well known, I will use the Chinese word jié qì since this is well known among Chinese people. Table 4 gives the names and approximate dates.

J1	Lì chūn	立春	Beginning of spring	February 4
Z1	Yǔ shuǐ	雨水	Rain water	February 19
J2	Jīng zhé	惊蛰	Waking of insects	March 6
Z2	Chūn fēn	春分	March equinox	March 21
J3	Qīng míng	清明	Pure brightness	April 5
Z3	Gǔ yǔ	谷雨	Grain rain	April 20
J4	Lì xià	立夏	Beginning of summer	May 6
Z4	Xiǎo mǎn	小满	Grain full	May 21
J5	Máng zhòng	芒种	Grain in ear	June 6
Z5	Xià zhì	夏至	June solstice	June 22
J6	Xiǎo shǔ	小暑	Slight heat	July 7
Z6	Dà shǔ	大暑	Great heat	July 23
J7	Lì qiū	立秋	Beginning of autumn	August 8
Z7	Chǔ shǔ	处暑	Limit of heat	August 23
J8	Bái lù	白露	White dew	September 8
Z8	Qiū fēn	秋分	September equinox	September 23
J9	Hán lù	寒露	Cold dew	October 8
Z9	Shuāng jiàng	霜降	Descent of frost	October 24
J10	Lì dōng	立冬	Beginning of winter	November 8
Z10	Xiǎo xuě	小雪	Slight snow	November 22
J11	Dà xuě	大雪	Great snow	December 7
Z11	Dōng zhì	冬至	December solstice	December 22
J12	Xiǎo hán	小寒	Slight cold	January 6
Z12	Dà hán	大寒	Great cold	January 20

Table 4: The 24 jié qì

The reason for the variation in the date of the jié qì is the same as the reasons for the variation of the dates of the seasonal markers discussed in Section 2. I denote the n'th (odd) jié qì by "Jn", and the n'th zhōng qì by "Zn".

The major solar terms Z2, Z5, Z8 and Z11 are simply the Western seasonal markers. The minor solar terms J1, J4, J7 and J10 start the Chinese seasons. Notice that in Western astronomy, spring begins at the March equinox, while in Chinese astronomy, spring begins midway between the December solstice and the March equinox. In Western popular culture this convention is often used. The traditional dates for the equinoxes and solstices were March 25, June 24, September 24 and December 25. Shakespeare's "A Midsummer Night's Dream" takes place on June 23, the eve of Midsummer Day on June 24. To Shakespeare, the June solstice was the middle of summer, not the beginning. Midsummer Day on June 24 is one of the four Quarter Days in the Legal Calendar in the UK. The others are Lady Day (or Annunciation Day) on March 25, Michaelmas on September 29 and Christmas on December 25. These Christian festivals are related to the seasonal markers. Lady Day on March 25 marked the beginning of the year in the UK until 1752. When the UK switched to the Gregorian calendar in 1752 and removed 11 days, they also moved the start of the civil year to January 1, but the start of the financial year was moved to April 5 (25 March plus 11 days).

The Chinese beginning of season markers also have their analogies in Western culture. Groundhog Day or Candlemas on February 2 is close to lì chūn (beginning of spring) on February 4. May Day on May 1 and Walpurgisnacht on April 30 are close to lì xià (beginning of summer) on May 6. Lammas on August 1 is close to lì qiū (beginning of autumn) on August 8. Halloween (Hallowmas) on October 31, All Saints' Day on November 1, Guy Fawkes Day on November 5 and Martinmas on November 11 are close to lì dōng (beginning of winter) on November 8. These Christian holidays are related to the Celtic holidays Imbolg, Beltane, Lughnasa and Samhain ([34]). These holidays are listed in Table 5.

Astronomical	Chinese	Western	Celtic
	lì chūn	Groundhog Day, Candlemas	Imbolg
March equinox	chūn fēn	Lady Day, Annunciation Day	
	lì xià	May Day, Walpurgisnacht	Beltane
June solstice	xià zhì	Midsummer Day	
	lì qiū	Lammas	Lughnasa
September equinox	qiū fēn	Michaelmas	
	lì dōng	Halloween, All Saints', Guy Fawkes, Martinmas	Samhain
December solstice	dōng zhì	Christmas Day	

Table 5: Holidays related to seasonal markers

Two of the jié qì's are Chinese festivals: qīng míng on April 5 and dōng zhì (December solstice) on December 22. All the other traditional Chinese holidays are lunar. This is similar to the ecclesiastical calendar, where Christmas Day and Annunciation Day on March 25 are solar holidays, while all the other holidays are tied to Easter and are therefore lunar.

Because of Kepler's second law, the speed of the (apparent) motion of the Sun across the ecliptic is not constant. This was known to the Chinese astronomers since the 7th century, but it was not until the last calendar reform in 1645 that they started using the true Sun, dìng qì (定气), in their computations of the jié qì. Before that, they had used the mean Sun, píng qì (平气).

It turns out that it is the zhōng qì's that are most important in the calendar computations. Under the mean Sun system, the length between two zhōng qì's is always about 30.44 days, which is a little bit longer than the lunar months. Hence it is possible to have two new Moons between two zhōng qì's or equivalently, a month without any zhōng qì. Under the true Sun system, the zhōng qì's are closer together during the winter. The time between two zhōng qì's ranges from 29.44 days to 31.44 days (Table 17). So under the modern system it is also possible to get a month with two zhōng qì's.

4.2 The Chinese month

Here are the rules for the Chinese calendar.

Rule 1 *Calculations are based on the meridian* 120° *East.*

Before 1929 the computations were based on the meridian in Beijing (116°25′), but in 1928 China adopted a standard time zone based on 120° East, which incidentally is close to the longitude of Nanjing (118°46′), the republican capital. Since 1929 the Institute of Astronomy in Nanjing, and since 1949 the Purple Mountain Observatory in Nanjing have been responsible for calendrical calculations in China.

Rule 2 *The day on which a new Moon occurs is the first day of the new month.*

Notice that the new Moon "takes" the whole day, no matter what time of the day conjunction occurs. So even if the new Moon takes place late in the evening, the whole day is considered to be part of the new month, and if a zhōng qì occurred in the early morning, it is considered as having fallen in the new month, even though it may have occurred almost 24 hours before the new Moon.

The length of the months are determined astronomically (Table 6). Suppose a month is 29.5 days, and starts with a new Moon at 13h on May 1. The next new Moon then takes place at 1h on May 31, so the month has 30 days. But if the new Moon occurred at 1h on May 1, then the next new Moon would be at 13h on May 30, so the new month would start one day earlier, and we would only get 29 days in the month.

New Moon	Next new Moon	Length
May 1 13h	May 31 1h	30 days
May 1 1h	May 30 13h	29 days

Table 6: Determining the length of the months

In the Gregorian calendar all the months (except for February) have the same number of days in different years. This is not the case for the Chinese calendar. A month may have 29 or 30 days in different years. Since the mean synodic month is 29.53 days, a little over half the months are "big" months, dà yuè (大月), with 30 days and a little less than half the months are "small" months, xiǎo yuè (小月), with 29 days. From a naive point of view, we would expect them to more or less alternate, with occasionally two long months, lián dà (连大), in a row. This was the method until the start of the Táng (唐) dynasty in 619, when the mean Moon, píng shuò (平朔), was abandoned in favor of the true Moon, dìng shuò (定朔). The motion of the true Moon is highly irregular, and it turns out that it is possible to have up to four big months or three small months in a row. For an example of four big months in a row, consider the sequence of new Moons given in Table 7 ([39]).

New Moon	Length
1990 Oct. 18 23h 36m	29d 17h 29m
1990 Nov. 17 17h 5m	29d 19h 17m
1990 Dec. 17 12h 22m	29d 19h 28m
1991 Jan. 16 7h 50m	29d 17h 42m
1991 Feb. 15 1h 32m	

Table 7: Four big months in a row

The next string of three short in a row will start in June 2089. In fact the occurrence of strings of four long or three short is very irregular. I have computed all such strings between 1645 and 2644, and with one exception, all of them occur in strings of such strings, with sometimes long gaps between them. Table 8 shows all such strings between 1646 and 2496. The strings of short months all occur during the summer and the strings of long months occur during the winter. This is because the Earth is moving faster in the winter, which tends to make the lunations longer. Many of the strings are about 9 years apart. This is related the fact that the Moon's perigee has a period of 8.85 years. I will write more about this in another paper.

3 short				1735	1743	1744	1752	1753	1760
4 long	1646	1700	1708						

3 short	1761	1762	1769	1770	1788	1797	1805	1806	1814
4 long									

3 short	1822								2089
4 long		1921	1929	1983	1991	2037	2045	2053	

3 short	2097	2098		2133	2142	2143	2150	2151	2152
4 long			2107						

3 short	2158	2159	2160	2167	2168	2176	2177		
4 long								2328	2336

3 short						2487	2488	2495	2496
4 long	2382	2390	2398	2444	2452				

Table 8: Strings of short and long months between 1646 and 2496

If the new Moon happens near midnight, it can be difficult to determine the beginning of the new month correctly. The Mid-Autumn Festival is celebrated on the 15th day of the 8th month. In 1978, calendars in Hong Kong and Taiwan were still based on the old imperial calendar that had the 7th month as a short month, while the modern calendar in China (put out by the Purple Mountain Observatory in Nanjing) had the 7th month as a long month. Because of this, the Mid-Autumn festival was celebrated on different days, causing a lot of confusion.

The Mid-Autumn festival is celebrated on the 15th of the month in order to make it coincide with the full Moon. But will the full moor really occur on the 15th? The motion of the Moon is very complex, and it turns out that the full Moon can fall on the 14th, 15th, 16th or 17th. Table 9 shows the day of the full Moon between 1984 and 2049. We see that the most common day is in fact the 16th day.

14th day	6
15th day	306
16th day	380
17th day	124

Table 9: Day of the full Moon between 1984 and 2049

For examples of this, the full Moon on 1995 October 8 fell on the 14th day of the Chinese month, while the full Moon on 1996 April 5 fell on the 17th day of the Chinese month.

4.3 The Chinese year

It is important to understand that the Chinese calendar is a combination of two calendars, a solar calendar and a lunisolar calendar. The solar calendar starts at the December solstice and follows the 24 jié qì. This is traditionally called the farmer's calendar (农历). The lunisolar calendar starts at Chinese New Year and consists of 12 or 13 months. This is what most people think of as the Chinese calendar, but unfortunately the term farmer's calendar has come to include the lunisolar calendar. The Chinese solar calendar follows the tropical year closely, so it is perfect for farming purposes, but the lunisolar calendar is not at all suitable for farmers.

There are therefore two different years in the Chinese calendar, the suì (岁) and the nián (年). A *suì* is the solar year from one December solstice to the next. This is similar to the tropical year (except that in Western astronomy the tropical year was traditionally measured from one March equinox to the next). A *nián* is the Chinese year from one Chinese New Year to the next. Since a Chinese year can contain 12 or 13 lunar months, and they can each have 29 or 30 days, the length of a nián can be 353, 354 or 355 in case of a normal year and 383, 384 or 385 days in case of a leap year. There are many conflicting figures for the number of days in a Chinese year. Tang ([39]) does not include 385, but there will be 385 days in 2006. Table 10 gives the distribution of the length of the years between 1911 and 2110.

In modern Chinese, the word suì is only used when talking about a person's age. Traditionally, Chinese people count their age from the December solstice, but in some parts of China they instead count from Chinese New Year or the seventh day of the new year (人日). Using the word suì when talking about a person's age is probably related to this custom.

353 days	354 days	355 days	383 days	384 days	385 days
1	84	41	5	66	3

Table 10: The length of Chinese years between 1911 and 2110

The suì can either be thought of as the exact time between to consecutive December solstices, in which case the average value is 365.242740 days, or we can think of the solar year as starting on the day of the December solstice and ending on the day before the next December solstice. In the latter case, the suì will always contain a whole number of days. The traditional Chinese way is to think of it as the exact value, but I will use whichever is convenient.

Just as we think of the Gregorian year as an approximation to the tropical year, we can think of the nián as an approximation to the suì. This again shows that the Chinese calendar is in a sense really a solar calendar that just uses lunar months rather than solar days as the basic unit.

Let me clarify some terminology. When I talk about the Chinese year 2033, I mean the nián from Chinese New Year 2033 to Chinese New Year 2034. The problem with this convention is that dates in the 11th or 12th months may fall in the following Gregorian year. For example, the 12th month of the Chinese year 2033 starts in January 2034. The suì 2033 is the suì from the December solstice in 2032 to the December solstice in 2033.

The suì can be divided into 12 whole months and about 11 days, or 11 whole months and about 40 days. Table 11 gives two examples.

365 days		
5 days	354 days (12 months)	6 days
13 days	325 days (11 months)	27 days

Table 11: Determining the number of months in a suì

When I say that 2033 is a leap year, it means that the nián 2033 contains 13 months. I will now define a leap suì.

Rule 3 *The December solstice falls in month 11. A suì is a leap suì if there are 12 complete months between the two 11th months at the beginning and end of the suì.*

If there is a new Moon on the day after the December solstice or within about 11 days, the suì is a leap suì. If there is a new Moon on the same day as the December solstice or the first new Moon after the December solstice is more than about 12 days later, it is a normal year. Notice that the leap year test applies to suì's and not to nián's. This again illustrates the fact that the Chinese calendar is primarily solar.

We will see later that in 2033, the leap month follows the 11th month. One of the rules is that the December solstice always falls in the 11th month. Hence 2033 is a leap year but *not* a leap suì, while 2034 is a leap suì but not a leap year.

If we consider the first December solstice and the first 11th month as part of the suì, but not the second December solstice and 11th month, then a leap suì contains 13 months and 12 zhōng qì's. Hence there must be at least one month without a zhōng qì. Notice that in extreme cases (Section 4.4), there may also be a month with two zhōng qì's, and hence two months without a zhōng qì.

Rule 4 *In a leap suì, the first month that doesn't contain a zhōng qì is the leap month, rùn yuè* (闰月)*. The leap month takes the same number as the previous month.*

Notice that *any* month can have a leap month. Provided there are no months with two zhōng qì's there is exactly one zhōng qì in each non-leap month, and the number of the month is the same as the number of the zhōng qì.

Let me try to illustrate this idea. I run a lot, and on one of my training runs I run up a very gentle hill with small steps that are far apart. The distance between the steps is a little bit more than the length of my running stride. On most strides I climb to the next step, but once in a while, I land near the edge, and I have to take a "resting" stride on the same level. If you think of the steps as the zhōng qì's, and my stride as the lunar months, you get a nice analogy with the leap month rule in the Chinese calendar. Another way to think of it, is to say that whenever the lunar months have gotten too far ahead of the zhōng qì's, they need to take a pause (leap month) to let the zhōng qì's catch up.

In recent years, some people have started saying that when a Gregorian calendar month contains two full Moons, then the second is called a "blue Moon". This term has an interesting history ([33]). This concept is somewhat similar to the system of Chinese leap months.

The date of Chinese New Year follows from these rules. For more details see Section 4.6

I would also like to mention that some astrological sources also use a third year running from lì chūn to lì chūn, and claim that your Chinese zodiac animal should be based on this. In 1960, Chinese New Year fell on

January 28 while lì chūn fell on February 5. If you were born on February 1, you would not be considered a rat, but a pig!

4.4 Why is 2033 an exceptional year?

Let us start by looking at the times for the zhōng qì's and new Moons during the end of 2033. This is given in Table 12. I denote the n'th month (or the n'th new Moon) by "Mn", and I denote the new Moon after Mn by Mn+ and the new Moon after that by Mn++. The reason is that before I have compared with the zhōng qì's, I cannot tell whether any of them are leap months or not. I denote a leap month after Mn by Mn-leap.

M7:	2033 7 26 16h 11m	Z7:	2033 8 23 3h 0m
M8:	2033 8 25 5h 38m	Z8:	2033 9 23 0h 50m
M9:	2033 9 23 21h 38m	Z9:	2033 10 23 10h 26m
M10:	2033 10 23 15h 27m	Z10:	2033 11 22 8h 14m
M11:	2033 11 22 9h 38m	Z11:	2033 12 21 21h 44m
M11+:	2033 12 22 2h 45m	Z12:	2034 1 20 8h 25m
M12:	2034 1 20 18h 0m		

Table 12: Times for the zhōng qì's and new Moons during the winter of 33/34

It can be seen from Table 4 that the zhōng qì's all occur between the 19th and 23rd of the month. The date of the new Moon, however, more or less regresses through the Gregorian month. If you write out the Gregorian calendar with the months as columns, and mark the new Moons and the zhōng qì's, you see that the zhōng qì's form a more or less horizontal line, while the date of the new Moon climbs upwards until it reaches the top and jumps to the bottom and starts climbing again. Leap months occur when the new Moon curve crosses the zhōng qì curve. Most of the time you get a "clean" crossing, but sometimes the curves might get intertwined is complex ways. In 1998 (Table 13) the zhōng qì's fell before the new Moon until June, in July they coincided, and from August on the zhōng qì's fell after the new Moon. This clean crossing gave a "normal" leap year. In 2033 (Table 14), however, the zhōng qì's fall before the new Moon until August, and for 7 months between September and March they either coincide, or the zhōng qì fall earlier. Not until April do the zhōng qì's fall after the new Moon.

The distribution of the zhōng qì's for the different months during the winter of 2033/34 is given in Table 15.

We see that the 9th month takes Z8, the 10th month takes Z9, and the 11th month takes Z10. But the 11th month also holds on to the December

	June	July	August
19			
20			
21	Z5		
22			M7
23		Z6 M6	Z7
24	M5-leap		

Table 13: Position of the zhōng qì's and new Moons in 1998

	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.
18							Z1		
19							M1		M3
20						Z12/M12		Z2/M2	Z3
21					Z11				
22				Z10/M11	M11-leap				
23	Z7	Z8/M9	Z9/M10						
24									
25	M8								

Table 14: Position of the zhōng qì's and new Moons in 2033/34

solstice, Z11. The fact that the 8th month doesn't have a zhōng qì is compensated for by the fact that the 11th month has two. Hence the suì 2033 has only 11 complete months, while the suì 2034 has 12 complete months. In other words, suì 2033 is not a leap suì, while suì 2034 is a leap suì. It follows that the month after the 7th month is not a leap month, because there's no room for a leap month in the suì. The 8th month is a "fake" leap month, in the sense that it doesn't contain any zhōng qì, but is not a leap month. This was an error in all Chinese calendars up until the early 1990's.

It is clear that fake leap months are closely related to months with two zhōng qì's. Table 16 shows all such months between 1800 and 2100. Notice that 1832, 1851, 1870 and 1984 are both leap years and leap suì's. This is related to the fact that the December solstice is early in the 11th month. 2033 is unique in that the December solstice is the *second* zhōng qì in the 11th month. Since there is a leap suì if M11+ falls within about 11 days of Z11, we see that this is the reason why 2034 is a leap suì while 1833, 1852, 1871 and 1985 are not. The fake leap month in 2034 is the first fake leap month in a leap suì since 1645. The next will occur in 2129. It also follows

Month	Number of zhōng qì's
2033 M7	1
2033 M8	0
2033 M9	1
2033 M10	1
2033 M11	2
2033 M11-leap	0
2033 M12	2
2034 M1	0
2034 M2	1

Table 15: Distribution of zhōng qì's during the winter of 2033/34

Year	Leap year	Leap suì	Leap month	Month with 2 zhōng qì's	Fake leap month
1832	Yes	Yes	9-leap	11	
1833	No	No			1
1851	Yes	Yes	8-leap	12	
1852	No	No			2
1870	Yes	Yes	10-leap	11	12
1984	Yes	Yes	10-leap	11	
1985	No	No			1
2033	Yes	No	11-leap	11, 12	8
2034	No	Yes			1

Table 16: Fake leap months

that Chinese New Year is the *third* new Moon after the December solstice in 2034. Notice also that 2033 contains *two* months with two zhōng qì's. It is interesting to observe that in the Indian calendar, a fake leap month is counted as a leap month, but when a month has two zhōng qì's they skip a month!

A month with two zhōng qì's will of course have three jié qì's . Sometimes there are months with three jié qì's where one is a zhōng qì and two are odd jié qì's. This happened in the 10th month in 1999. They are not so interesting since they don't affect the leap months.

A year is said to have "double spring", shuāng chūn (双春), if it contains a J1, "beginning of spring", lì chūn, at both its beginning and end. It is easy to see that this happens if and only if the year is a leap year. In the same way, a year is said to have "double spring, double rain", shuāng chūn shuāng yǔ (双春双雨), if the nián contains both a J1, "beginning of spring", lì chūn, and a Z1, "rain water", yǔ shuǐ (雨水), at both its beginning and end. This is considered significant in Chinese astrology. Between 1645 and 2644, this happens only 15 times. It happened in 1699, 1832, 1851 and 1984, and will happen again in 2033 and 2053. We see that these years are almost the same as the exceptional years we have discussed earlier.

4.5 Can any month have a leap month?

Many Chinese astronomers claim that there can be no leap month after the 12th or 1st month ([39]). This is true in the sense that it hasn't happened since the last calendar reform in 1645, and that it will not happen in the 21st century. Before 1645, the Chinese calendar used the mean Sun, and then all months had leap months with equal probability. Because of precession, the Sun will move faster during the summer in 10,500 years, so by then, there will be lots of winter leap months. But what about our current period?

Z11	29.44	Z12	29.59	Z1	29.97	Z2	30.47
Z3	30.97	Z4	31.34	Z5	31.44	Z6	31.29
Z7	30.89	Z8	30.37	Z9	29.89	Z10	29.55

Table 17: Distance between the zhōng qì's

Table 17 gives the distance between zhōng qì's. In order to have a leap month after the 11th month, M11++ must be on the same day as Z12. Let us assume that our months are 29.53 days long and that the December solstice happens right before midnight with a new Moon happening right after midnight (Table 18). We then get a leap month after month 11.

Z11	M11+	Z12	M11++
-0.01	0.01	29.43	29.54

Table 18: Leap month after the 11th month

In order to have a leap month after the 12th month, Z12 must fall on the day before M12+. The first row in Table 19 shows one attempt that fails because M12+ and Z1 fall on the same day. But by shifting M12 forward, I get a leap month after the 12th month (see the second row in Table 19). Table 20 shows a situation that almost gives a leap month after the 1st month. If M1++ fell 0.08 days earlier, we would get a leap month after the 1st month. Given the irregularity of the Moon's motion, this is not impossible.

Z11	M12	Z12	M12+	Z1	M12++
-0.01	0.01	29.43	29.54		
	0.48		30.01	59.02	59.54

Table 19: Leap month after the 12th month

Z11: -0.04	M12: 0.48
Z12: 29.40	M1: 30.01
Z1: 58.99	M1+: 59.54
Z2: 88.96	M1++: 89.07

Table 20: Possible leap month after the 1st month

Based on my computations, I believe that in 2262 there will be a leap month after the 1st month, and that in 3358 there will be a leap month after the 12th. Given the difficulty in making accurate astronomical predictions more than 100 years ahead, these computations must obviously be taken with a grain of salt. I also believe that there was an error in the computations for 1651, and I believe there should have been a leap month after the 1st month that year. Instead the leap month was added after the 2nd month.

Notice that there can only be a leap month between the December solstice and Chinese New Year if there is a new Moon very soon after (but not on the same day as) the December solstice, so the second new Moon would be around January 21 and the third around February 21. So even though a leap month after the 11th or 12th month causes Chinese New Year to fall on the third new Moon after the December solstice, it does not mean that Chinese New Year will fall in March!

I have computed all the leap months between 1645 and 2644. The most common leap month is a 5th leap month. Notice how all the months between the 9th and the 1st very rarely have leap months. The distribution is given in Table 21.

4.6 What is the date of Chinese New Year?

The exact date of Chinese New Year follows from the above rules. In this section we will try to describe more closely the variation in the date of Chinese

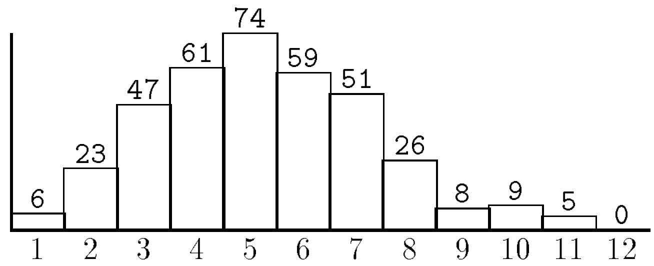

Table 21: Distribution of leap months between 1645 and 2644

New Year.

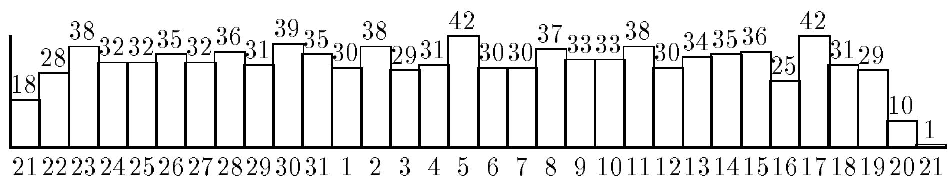

Table 22: Dates of Chinese New Year between 1645 and 2644

According to Tang ([39]), Chinese New Year always falls between January 21 and February 20. This is basically correct. Table 22 shows that the possible dates for of Chinese New Year between 1645 and 2644 are between January 21 and February 21. We see that dates between January 22 and February 19 are common, January 21 and February 20 are rare, and February 21 is extremely rare. Chinese New Year will fall on February 21 in 2319.

The Chinese New Year performs the same off-beat three step dance as the solstices and equinoxes (Section 3), but in the opposite direction. It moves backwards by 11 days (or 10 or 12) once or twice, but if a step would take us past the edge of the January 21 to February 21 period (or in exceptional cases, just near the edge), it jumps forward by 19 (or 18 or 20) days. This is illustrated in Table 23.

1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
7/2	28/1	16/2	5/2	24/1	12/2	1/2	22/1	9/2	29/1	18/2
-10	+19	-11	-12	+19	-11	-10	+18	-11	+20	

Table 23: The movement of the dates of Chinese New Year

There are also three simple and commonly stated, but not always correct rules of thumb that are often given for determining the date for Chinese New Year.

Rule of thumb 1 *Chinese New Year falls on the day of the second new Moon after the December solstice (approximately December 22).*

As explained before, this rule of thumb is correct, provided there is no leap month after the 11th or 12th month. In that case, Chinese New Year falls on the third new Moon after the December solstice. This, however, does not mean that we will be celebrating Chinese New Year in March. It follows from Section 4.5 that there can only be a leap month between the December solstice and Chinese New Year if there is a new Moon very soon after (but not on the same day as) the December solstice, so the second new Moon will then be around January 21 and the third around February 21. Notice that this rule of thumb only fails when Chinese New Year is exceptionally late.

Rule of thumb 2 *Chinese New Year falls on the day of the new Moon closest to the minor solar term J1, lì chūn* (立春)*, "beginning of spring" (approximately February 4).*

This rule of thumb is important because it explains why Chinese New Year is called the Spring Festival, chūn jié (春节). Recall that Chinese astronomers put the seasonal markers in the middle of the seasons, so the start of the seasons fall between the seasonal markers. That's why the Spring Festival can be pretty chilly in Beijing!

This rule of thumb is normally true, but since lì chūn falls around February 4 and Chinese New Year falls between January 21 and February 21 it is hard to determine which new Moon is closest if we have a very early or a very late Chinese New Year. The rule of thumb fails 31 times between 1645 and 2644. In 1985 lì chūn fell on February 4 5h 11m, Chinese New Year was on February 20 2h 42m, while the previous new Moon was on January 21 10h 28m. The distance between lì chūn and the previous new Moon was 13.78 days, while the distance between lì chūn and Chinese New Year was 15.90 days, so the rule of thumb failed in 1985. It will fail again in 2015.

At first I thought that it would be possible for this rule of thumb to fail in case of either an early or a late Chinese New Year, but when I tested on a computer, it only failed for late Chinese New Years. To see why, suppose that we have a very early Chinese New Year. In that case Z11 must fall late in the 11th month, but before the day of M12. We must also have Z12 in the month after the 11th month (which is then the 12th month), because if Z12 fell in the second month after the 11th month, we would get a leap month after the 11th month, and this would cause a late Chinese New Year. We must also have Z1 in the 1st month, because if it fell in the 12th month, we would get a leap month after the 12th month, and this would cause a late

Chinese New Year. So whenever we have an early Chinese New Year, Z11 must fall before the day of Chinese New Year, and Z1 must fall before the day of M2. Therefore the time between Chinese New Year and J1 is less than the time between Z12 and J1. This is approximately the same as the time between J1 and Z1, which is less than the time between J1 and M1+. Hence it is impossible for lì chūn to be closer to the new Moon following Chinese New Year than Chinese New Year itself (assuming that the time from Z12 to J1 is close to the time between J1 and Z1).

Notice also that if Chinese New Year is at the beginning of spring, then the middle of spring should be in the middle of the 2nd month. This explains why the Mid-Autumn Festival is celebrated on the 15th day of the 8th month.

Rule of thumb 3 *Chinese New Year falls on the day of the first new Moon after the major solar term Z12, dà hán* (大寒) *(approximately January 20).*

This can be expressed as saying that Chinese New Year is the first new Moon after the Sun has entered the Zodiac sign of Aquarius.

Since Z12 normally falls in the 12th month, this rule of thumb holds most of the time, but if we have a very late Chinese New Year, it is possible for Z12 to fall before the day of M12. The rule of thumb fails 23 times between 1645 and 2644. It failed in 1985 and will fail again in 2053.

Notice that all these rules of thumb only fail in case of exceptionally *late* Chinese New Years. These rule of thumbs help explain why the range for Chinese New Year is between January 21 and February 21. This also shows how it is usually possible to determine the approximate date of Chinese New Year if you have a calendar that indicates the phases of the Moon (for the Chinese time zone, UT +8). We have seen above that determining the date of Chinese New Year is hard if there are new Moons near the ends of the period from January 21 to February 21. But if there's a new Moon in that period that is not close to the ends, we can use either of the three rules of thumb to conclude (correctly) that it will be the date of the Chinese New Year. More specifically, if you have a new Moon between January 23 and February 19, you can conclude that it will fall on the date of Chinese New Year. But in 1985 there were new Moons on January 21 and February 20 and in 2319 on January 22 and February 21 and in both cases Chinese New Year fell on the later date.

4.7 The 19-year cycle

Because of the Metonic cycle, there is almost a 19-year cycle in the Chinese calendar. I was born on April 16, 1960. This was the 21st day in the 3rd month in the Chinese calendar. Normally my birthday falls on different days in the Chinese calendar, but my 19th birthday fell on the 20th day in the 3rd month. The same goes for my 38th and 57th birthday. So we see that the 19-years cycle is close but not exact. There are two reasons for this. First of all, the Metonic cycle is off by about two hours. But more importantly, we are now comparing the Chinese calendar not with the tropical year, but with the Gregorian calendar, which is just an approximation to the tropical year. In particular, since 19 is not a multiple of 4, different cycles contain different numbers of leap years.

The 19-years cycle is still apparent when it comes to leap years. Table 24 ([27]) shows the leap years and the leap months between 1824 and 2050. If no leap month number is indicated, it means that the leap month is the same as the previous year in the cycle. The columns contain years in the same 19-years cycle. Our old friends 1832, 1851, 1870 and 1984 (Section 4.4) again appear as exceptional cases, in that they break out of the columns corresponding to their 19-years cycles. Notice how these four exceptional years have leap months in the fall, while the other years in the cycle all have spring leap months. So the jump from 1966 to 1984 is 18 years and 7 months, and the jump from 1984 to 2004 is 19 years and 4 months. I like to think of this as an unstable situation, where the leap month "should" have fallen in the winter, but instead either slips forward or backward. We also notice that 2033 is for once reasonably well behaved, in that it stays in its proper column. But it does insist on a different leap month.

1805–7	1808–5	1811–3		1814–2	1816–6	1819–4	1822–3
1824	1827	1830–4	1832–9		1835	1838	1841
1843	1846	1849	1851–8		1854–7	1857–5	1860
1862–8	1865	1868	1870–10		1873–6	1876	1879
1881–7	1884	1887		1890–2	1892	1895	1898
1900–8	1903	1906		1909	1911	1914	1917–2
1919–7	1922	1925		1928	1930	1933	1936–3
1938	1941–6	1944		1947	1949–7	1952	1955
1957–8	1960	1963		1966–3	1968	1971	1974–4
1976	1979	1982	1984-10		1987–6	1990	1993–3
1995	1998–5	2001		2004–2	2006–7	2009	2012–4
2014–9	2017–6	2020		2023	2025–6	2028	2031–3
2033–11	2036	2039–5		2042	2044–7	2047	2050

Table 24: Leap years and leap months between 1824 and 2050

There is also a similar pattern in the date of Chinese New Year. Table 25 shows the date of Chinese New Year between 1980 and 2017. For most days there's at most a difference of a day (caused by leap years), but notice how 1985 is exceptional. From the discussion of 1984 in Section 4.4, however, this is not surprising.

1980: Feb. 16	1999: Feb. 16
1981: Feb. 5	2000: Feb. 5
1982: Jan. 25	2001: Jan. 24
1983: Feb. 13	2002: Feb. 12
1984: Feb. 2	2003: Feb. 1
1985: Feb. 20	2004: Jan. 22
1986: Feb. 9	2005: Feb. 9
1987: Jan. 29	2006: Jan. 29
1988: Feb. 17	2007: Feb. 18
1989: Feb. 6	2008: Feb. 7
1990: Jan. 27	2009: Jan. 26
1991: Feb. 15	2010: Feb. 14
1992: Feb. 4	2011: Feb. 3
1993: Jan. 23	2012: Jan. 23
1994: Feb. 10	2013: Feb. 10
1995: Jan. 31	2014: Jan. 31
1996: Feb. 19	2015: Feb. 19
1997: Feb. 7	2016: Feb. 8
1998: Jan. 28	2017: Jan. 28

Table 25: The date of Chinese New Year between 1980 and 2017

4.8 The sexagenary cycle

An important aspect of the Chinese calendar is the sexagenary cycle. This is a combination of the 10 heavenly stems, tiān gān (天干), and the 12 earthly branches, dì zhī (地支) ([32]).

To explain how this cycle works, let us denote both the stems and the branches by their numbers. We denote 1 by $(1, 1)$ or (甲, 子), 2 by $(2, 2)$ or (乙, 丑) and so on up to $(10, 10)$ or (癸, 酉). But now we have run out of stems, so we denote 11 by $(1, 11)$ or (甲, 戌) and 12 by $(2, 12)$ or (乙, 亥). Now we have run out of branches, too, so 13 becomes $(3, 1)$ or (丙, 子). We continue in this way through 6 cycles of stems and 5 cycles of branches up to

Stems	天干	tiān gān	Element	Branches	地支	dì zhī	Animal
1	甲	jiǎ	Wood	1	子	zǐ	Rat
2	乙	yǐ	Wood	2	丑	chǒu	Ox
3	丙	bǐng	Fire	3	寅	yín	Tiger
4	丁	dīng	Fire	4	卯	mǎo	Rabbit
5	戊	wù	Earth	5	辰	chén	Dragon
6	己	jǐ	Earth	6	巳	sì	Snake
7	庚	gēng	Metal	7	午	wǔ	Horse
8	辛	xīn	Metal	8	未	wèi	Goat
9	壬	rén	Water	9	申	shēn	Monkey
10	癸	guǐ	Water	10	酉	yǒu	Chicken
				11	戌	xū	Dog
				12	亥	hài	Pig

60, which is $(10, 12)$ or (癸, 亥). The next number is then $(1, 1)$ or (甲, 子), which starts a new sexagesimal cycle.

This cycle is used for keeping track of years, months, days and (double) hours in Chinese astrology. Your date and time of birth is determined by the Eight Characters (八字) formed by the pair of cyclical characters for the year, month, day and hour. The counting of the months and hours are not relevant for this paper. The 60-day cycle has been used for keeping track of days since ancient times. In 4 CE during the Hàn (汉) dynasty, the 60-year cycle was also introduced ([4]). The earliest recorded use of the 60-year cycle is from 13 CE ([13, p. 330]). In modern times, the year cycle is the only one that is in common use. Notice that each branch, or animal, occurs five times in each 60-year cycle. An animal corresponding to an odd number, will meet the stems that correspond to the odd numbers. Year 2000 is the 17th year in the current cycle (see below), so it corresponds to $(7, 5)$ $(17 = 10+7 = 12+5)$ or (庚, 辰). So we see that it is a metal dragon year, or a "Golden Dragon".

Because of my web page about the Chinese calendar [2], I get a lot of e-mail about the Chinese calendar. I once got an e-mail from a Spanish greeting cards company who needed to know which year 2000 would be in the Chinese calendar. The answer is that the Chinese do not have a continuous year count. They started counting from one again with each new emperor. However, from the Hàn dynasty, some scholars tried to reconstruct the ancient Chinese chronology, and it became customary to claim that the calendar was invented by the Yellow Emperor, Huáng Dì (黄帝), in 2637 BCE during the 61st year of his reign. However, many people prefer to start the count with the first year of his reign in 2697 BCE. Since these years are 60 years apart, it follows that 1984 was the first year of either the 78th or 79th 60-year cycle. Using this as a starting point, Chinese New Year in 2000 marks the beginning of the Chinese year 4637 or 4697. To give you an example of the level of confusion on this point, in Chapter 3 of Volume III of the translation of the Shoo King (shū jīng, 书经) by James Legge ([24]), he refers to the current year, 1863, as being in the 76th cycle, implying a starting point of 2697 BCE. However, the book has an appendix on Chinese astronomy, written by John Chalmers, where the starting point is taken to be 2637 BCE! Chalmers actually writes 2636 BCE, but that really means -2636, using the astronomical year count, where 1 BCE is year 0, 2 BCE is -1, etc. This is fairly typical of the level of confusion about the continuous year count in the Chinese calendar, and simply illustrates the fact that the continuous year count is not an integral part of the Chinese calendar. That's why I told the Spanish greeting cards company to stick with calling it the year of the Dragon!

To add to the confusion, some authors use an epoch of 2698 BCE. I believe this because they want to use a year 0 as the starting point, rather than counting 2697 BCE as year 1.

A curious question that I don't know the answer to is which day is the starting day for the 60-day count. Ideally, there should be a (甲, 子) day in a (甲, 子) month in a (甲, 子) year, but I haven't found such a date in either 2637 BCE or 2697 BCE. I should also point out, that while Chinese chronology is fairly reliable going back to 841 BCE, and oracle bones with date inscription go back to the 13th century BCE, modern scholars consider the Yellow Emperor to be a mythological figure. So this whole discussion of ancient dates is just a curiosity.

4.9 A brief history of the Chinese calendar

The calendar has always been very important in Chinese society. The Chinese emperor based his authority on being the "Son of Heaven". In that case, it was very embarrassing if the calendar was not in harmony with the heavens. Unfortunately, with a lunar or lunisolar calendar, errors are much more obvious than with a solar calendar. A solar calendar can be off by a couple of weeks without anybody noticing. The reason why the Catholic church had to reform the Julian calendar was because the rules for computing Easter had frozen the March equinox to be March 21. That meant that Easter was drifting noticeably towards summer. Otherwise, few would have cared about the drift of the March equinox. But with a lunar calendar, an error of even a couple of days is a serious problem. Every peasant could each month see that the new Moon was visible near the end of the previous month or that the old Moon was visible in the next month. Why should they pay taxes and serve in the army if the emperor didn't know the secrets of the heavens? For the same reason, prediction of eclipses has always been very important in China. If an eclipse was predicted, but did not occur, it was a sign that Heaven looked favorably upon the Emperor. But if an eclipse occurred that the Emperor's astronomers had failed to predict, it was taken as a sign that the Emperor had lost the "Mandate of Heaven".

Because of the importance the Chinese rulers placed on calendars, they were surprisingly open to incorporate foreign ideas into the making of calendars. The last three main calendar reforms have all been associated with foreign impulses.

Before 621 BCE, the Chinese determined the start of the month based on visibility of the crescent Moon ([14]). During the Zhōu (周) dynasty, the Metonic cycle was used for determining leap months and the leap months were always placed at the end of the year. After the Tài Chū (太初) calendar reform in 104 BCE, the "no zhōng qì" (无中气) rule was used for determining leap months, and the month containing the December solstice was fixed to be the 11th month. (For details about the calendars during the Hàn Dynasty, see [8, 9, 17, 18, 20, 23, 36].)

However, the difference between the zhōng qì and the 19-year cycle is not that big. Consider for example the Sì fēn lì calendar that was used between 85 CE and 263 CE during the later Hàn dynasty. The Metonic cycle was still hardwired into the calendar. The year was taken to be $365\frac{1}{4}$ days and a month was taken as $19/235 \times 365\frac{1}{4} = 29\frac{499}{940}$. The "no zhōng qì" system is then almost equivalent to the Metonic cycle. The only problem is the we have to worry about how the new Moons and zhōng qì's fall with respect to midnight. The difference would never be more than one month, though. For details, see [23].

The "no zhōng qì" system only became significant when the Táng (唐) dynasty calendar reform in 619 switched to following the true Moon. This was inspired by Indian Buddhist astronomers.

The next significant reform came in 1280 during the Yuán (元) dynasty. It was inspired by Muslim astronomers, but designed by the famous mathematician Guō Shǒu Jìng (郭守敬). It was the most accurate calendar in the world at that time.

The last calendar reform came in 1645 during the Qīng dynasty (清) and was implemented by Jesuit missionaries. It used the true Sun. In a system that uses both mean sun and mean moon, all months have leap months with the same probability, and there are no fake leap months (Section 4.4). In a system that uses mean Sun but true Moon, summer leap months are more likely to occur, but there are no fake leap months. After 1645 leap months are more likely to occur in the summer, and there's also the possibility of fake leap months.

After the 1911 Revolution, the Republican government made the Gregorian calendar the official calendar in 1912. However, the traditional Chinese calendar is still important to Chinese people all over the world when it comes to determining the traditional festivals and holidays.

4.10 The Jesuit missionaries

As explained above, the current Chinese calendar is due to Jesuit missionaries. I would like to give some more details about how it came about ([10, 15, 37]). In 1582, the first Jesuit missionary Matteo Ricci came to China. He managed to convert a leading Chinese official, Xú Guāng Qǐ (徐光启), and together they translated Euclid. At that time, the Chinese calendar was no longer accurate. Positions in the Bureau of Astronomy had become hereditary, and the astronomers no longer understood the principles behind the old calendar. When they made an error of more than half an hour in computing a solar eclipse on December 15, 1610, it caused serious embarrassment. Finally, in 1629 Xu was asked to revise the calendar, and he asked the Chinese and Muslim astronomers in the Bureau and the Jesuits to make predictions for an upcoming solar eclipse on June 21, 1629. The Jesuits had the best prediction, and when Xu was made director of the Bureau, he appointed the Italian Terrentius and another Jesuit as members. Terrentius had been a member of the Cesi Academy with Galileo, and wrote him repeatedly for help. The Pope had forbidden Galileo to promote his views, and even though Terrentius promised that he would keep any help secret, Galileo was understandably not very eager to help the Jesuits. Finally, in 1623 Terrentius wrote to Kepler. It took more than four years before Kepler received the letter! This was in the middle of the Thirty Years War, but even though Kepler was a Protestant, he did not hesitate to help the Jesuits. As a thank you, the Jesuits sent him some data about old Chinese eclipse observations.

In 1644, the German Adam Schall went to the new Qīng rulers and presented his calculations for an upcoming solar eclipse on September 1. At this time the Manchus were suspicious of the Chinese, but Schall told them that they could trust him, because he was a foreigner like them. He challenged the Chinese and the Muslim astronomers in the Bureau, and again the Jesuits' calculations were best. Schall was appointed director of the Bureau. The next year, he formulated the current rules for the Chinese calendar. He became good friends with the Shùn Zhì (顺治) emperor, and became a mandarin of the first grade, first division.

The fortunes turned for Jesuits, however, when the Shun Zhi emperor died in 1661. A Chinese official, Yáng Guāng Xiān (杨光先), had as his slogan that it was "better to have a wrong calendar than to have foreigners in China". Yáng had several complaints against the Jesuit. The new calendar had two zhōng qì's in the 11th month of 1661, something that was impossible under the old system. Both the month after the 7th month and the 12th month had no zhōng qì's. The first was a 7th leap month, but the 12th was a fake leap month. Fake leap months did not exist under the old system. In the new calendar, the 11th month had three jié qì's in 1661, something that was only possible in the new system. (In fact, the last jié qì should have been in the following month, because it occured 39 minutes after midnight, but the Jesuits made an error.) Schall had also presented the emperor with a calendar for the next 200 years, and Yáng claimed that this was improper since the emperor was blessed with infinite reign. He was also accused of having picked an inauspicious date for the funeral of the favorite son of the Shun Zhi emperor.

Yáng managed to have Schall, the Belgian Ferdinand Verbiest, and two other Jesuits arrested in 1664. A solar eclipse was coming up on January 16, 1665, and while in prison, the Jesuits predicted it would occur at 3 pm, Yáng predicted 2.15 pm, and the Muslim Wú Míng Xuǎn (吴明炫) predicted 2.30 pm. On the day of the eclipse, the Jesuits were brought into the palace in chains, and everybody watched as the eclipse occurred at 3pm sharp (14:59:54 according to computations by Salvo De Meis), exactly as the Jesuits had predicted! Unfortunately, the regents were not impressed and on April 15, the Jesuits were sentenced to death. However, the next day a strong earthquake struck Beijing. This was taken as a sign from Heaven that the sentence was unjust, and the sentence of the Jesuits was first converted to flogging and eventually to just house arrest. The death sentence for five of their Chinese assistants, however, was upheld and carried out. In 1666, Schall died while still in house arrest.

In 1668, the Kāng Xī (康熙) emperor took over from the regents. The emperor sent a copy of the calendar made by Yáng and Wú to Verbiest and asked him to comment. When Verbiest pointed out several errors, the emperor ordered Verbiest, Yáng and Wú to compute the length of the shadow of a pole on a certain day. Again the Jesuits won. Kāng Xī then asked them to compute the position of the Sun at noon on a certain day. They were to leave their instruments pointing towards the predicted spot in the emperor's garden two weeks in advance. By now Yáng had been humiliated so thoroughly that he didn't even bother to take part, and Verbiest easily beat Wú in the tests. On April 17 Verbiest was appointed director of the Bureau, while Yáng and Wú were arrested. Verbiest became personal tutor to the Kāng Xī emperor, and even learned Manchu. Jesuits remained as directors of the Bureau until 1746 and it was run by other Westerners until 1826.

There has been a lot of controversy over the contributions of the Jesuits. Because of the Catholic Church's condemnation of Copernicus, the Jesuits did not dare to introduce the Copernican theory to China. Some Chinese critics argue that this was an attempt at keeping China backwards. This is a complex issue, but it is important to realize that when it comes to calendar making, the underlying theory is not that important. Accurate observations and computational skills are more important. In fact, one of the main advantages the Jesuit had was that they knew about logarithms!

This raises an interesting point. Since the fourth century BCE, Chinese astronomers knew that the motion of Moon was not uniform, and in the sixth century CE they knew that the Sun's motion was also irregular ([6]). They had started using the true Moon in the calendars since 619, but until the time of the Jesuits, they continued to use the mean Sun. I think there are several reasons for this difference. Using the mean Moon created discrepancies that were noticeable to everybody, but the errors caused by using the mean Sun were only noticeable to astronomers. Given the computational complexities associated with using the true Sun, the Chinese astronomers choose to stay with the old method. The Jesuits, however, needed to demonstrate their superiority. Changing the calendar by using the true Sun was a great way of making themselves essential. It is therefore somewhat ironic that this backfired on them in that part of the reason why they were thrown in jail was because Yáng accused them of having made an error because their calendar did not agree with the old system!

It is of course true that the Jesuits had ulterior motives; their goal was to win converts. But in spite of this, the Jesuits made a positive contribution to China, which can serve as an example of a successful Sino-Western exchange.

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