1.

1\. Strong form problem:

$$(S)\begin{cases} \text{Given } f_i : \Omega \to \mathbb{R},\ g_i : \Gamma_{g_i} \to \mathbb{R}, \text{ and } h_i : \Gamma_{h_i} \to \mathbb{R}, \\ \text{find } u_i : \bar{\Omega} \to \mathbb{R} \text{ such that} \\ \quad \sigma_{ij,j} + f_i = 0 \quad \text{in } \Omega \\ \quad u_i = g_i \quad \text{on } \Gamma_{g_i} \\ \quad \sigma_{ij} n_j = h_i \quad \text{on } \Gamma_{h_i} \\ \quad \epsilon_{ij} = u_{(i,j)} = \frac{1}{2}(u_{i,j} + u_{j,i}),\ \sigma_{ij} = C_{ijkl}\epsilon_{kl} \end{cases}$$

Weak form problem:

$$\mathcal{S}_i = \{u_i : u_i \in H^1,\ u_i = g_i \text{ on } \Gamma_{g_i}\}$$

$$\mathcal{V}_i = \{w_i : w_i \in H^1,\ w_i = 0 \text{ on } \Gamma_{g_i}\}$$

$$(W)\begin{cases} \text{Given } f_i : \Omega \to \mathbb{R},\ g_i : \Gamma_{g_i} \to \mathbb{R}, \text{ and } h_i\ \Gamma_{h_i} \to \mathbb{R}, \\ \text{find } u_i \in \mathcal{S}_i \text{ such that for all } w_i \in \mathcal{V}_i \\ \quad \int_\Omega w_{(i,j)}\sigma_{ij}\, d\Omega = \int_\Omega w_i f_i\, d\Omega + \sum_{i=1}^{n_{sd}} \left(\int_{\Gamma_{h_i}} w_i h_i\, d\Gamma \right) \\ \epsilon_{ij} = u_{(i,j)} = \frac{1}{2}(u_{i,j} + u_{j,i}),\ \sigma_{ij} = C_{ijkl}\epsilon_{kl} \end{cases}$$

$$a(\vec{w},\vec{u}) = \int_\Omega w_{(i,j)} C_{ijkl} u_{(k,l)}\, d\Omega, \quad (\vec{w},\vec{f}) = \int_\Omega w_i f_i\, d\Omega$$

Galerkin: $(\vec{w},\vec{h})_\Gamma = \sum_{i=1}^{n_{sd}} \int_{\Gamma_{h_i}} w_i h_i\, d\Gamma$ $\quad \vec{v}^h \in \mathcal{V}^h$

$$(G)\begin{cases} \text{Given } \vec{f},\ \vec{g} \text{ and } \vec{h} \text{ (as in (W)), find } \vec{u}^h = \vec{v}^h + \vec{g}^h \in S^h \text{ such that} \\ \text{for all } w^h \in \mathcal{V} \\ \quad a(\vec{w}^h,\vec{v}^h) = (\vec{w}^h,\vec{f}) + (\vec{w}^h,\vec{h})_\Gamma - a(\vec{w}^h,\vec{g}^h) \end{cases}$$

2.

2、My Chosen implementation of the element stiffness

$$B = [B_1, \ldots, B_{n_{en}}]$$

$$B_a = \begin{bmatrix} N_{a,x} & 0 \\ 0 & N_{a,y} \\ N_{a,y} & N_{a,x} \end{bmatrix} \quad 1 \le a \le n_{en}$$

$$D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ & D_{22} & 0 \\ \text{symmetric} & & D_{33} \end{bmatrix}$$

$$k^e_{ab} = \int_{\Omega^e} B_a^T D B_b \, d\Omega$$

$$K_{PQ} = \vec{e}_i^T \int_{\Omega} B_A^T D B_B \, d\Omega \, \vec{e}_j$$

3. Manufactured solution for verification

很遗憾，代码并没有调试出正确的结果，因此也无法进行误差分析

4. Infinite plate with stress-free circular hole under constant far-field in-plane tension check

实现了在边界上施加解析解的应力条件，但优于求解器并不能计算正确的结果，所以没有结果对比也没有误差分析。

5. Elastic plate with a circular hole

实现了几何结构，边界的条件的定义，但求解器不能给出正确的结果。

代码已完成的部分：

Gmesh 文件的导入，创建 IEN、ID，正确识别边界并对边界条件进行定义

未完成部分：

弹性力学求解器的编写完成但未能给出正确结果，误差分析与可视化一点没写