P71 Exercise 2.

$$\lambda u - q_i n_i = h \quad \text{on } \Gamma_h. \qquad (2.5.17)$$

$$\int_\Omega (q_{i,i} - f) w \, d\Omega = 0$$

$$-\int_\Omega q_i w_{,i} \, d\Omega + \int_\Gamma w q_i n_i \, d\Gamma - \int_\Omega w f \, d\Omega = 0$$

代入 (2.5.17)

$$-\int_\Omega q_i w_{,i} \, d\Omega + \int_{\Gamma_h} w(\lambda u - h) \, d\Gamma - \int_\Omega w f \, d\Omega = 0$$

$$-\int_\Omega w_{,i} q_i \, d\Omega = \int_\Omega w f \, d\Omega + \int_{\Gamma_h} w(h - \lambda u) \, d\Gamma$$

Which is the weak form formulation.

$$\int_\Omega w_{,i} \kappa_{ij} u_{,j} \, d\Omega = \int_\Omega w f \, d\Omega + \int_{\Gamma_h} w(h - \lambda u) \, d\Gamma.$$

$$a(w, u) = (w, f) + (w, h - \lambda u)_\Gamma$$

Galerkin approximation

$$a(w^h, v^h) + \lambda(w^h, v^h) = (w^h, f) + (w^h, h)_\Gamma - a(w^h, g^h) - \lambda(w^h, g^h)$$

L.H.S of the equation becomes

$$\sum_{B \in \eta - \eta_g} [a(N_A, N_B) + \lambda(N_A, N_B)] d_B.$$

$$k^e_{ab} = \int_{\Omega^e} (\nabla N_a^T \kappa \nabla N_b + \lambda N_a N_b) \, d\Omega.$$

P75 Exercise 1

IEN array:

IEN(a, e) = A

a \ e	1	2	3	4	5
1	1	3	9	10	12
2	2	4	3	9	11
3	3	5	7	8	10
4	4	6	5	7	9

ID array:

A	1	2	3	4	5	6	7	8	9	10	11	12
P	1	2	3	4	0	0	0	0	5	6	7	8

ID(A) = P

P82 Exercise 4

$n_{sd}=2$

$$\vec{\sigma} = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11kl}\varepsilon_{kl} \\ C_{22kl}\varepsilon_{kl} \\ C_{12kl}\varepsilon_{kl} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} C_{11kl}(u_{k,l}+u_{l,k}) \\ \frac{1}{2} C_{22kl}(u_{k,l}+u_{l,k}) \\ \frac{1}{2} C_{12kl}(u_{k,l}+u_{l,k}) \end{bmatrix}$$

$$= \begin{bmatrix} C_{1111}u_{1,1} + C_{1122}u_{2,2} + C_{1112}(u_{1,2}+u_{2,1}) \\ C_{2211}u_{1,1} + C_{2222}u_{22} + C_{2212}(u_{1,2}+u_{2,1}) \\ C_{1211}u_{1,1} + C_{1222}u_{22} + C_{1212}(u_{1,2}+u_{2,1}) \end{bmatrix}$$

$$\overline{\overline{D}}\vec{\varepsilon}(u) = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ & D_{22} & D_{23} \\ \text{symmetric} & & D_{33} \end{bmatrix} \begin{bmatrix} u_{1,1} \\ u_{2,2} \\ u_{1,2}+u_{2,1} \end{bmatrix}$$

$$= \begin{bmatrix} D_{11}u_{1,1} + D_{12}u_{2,2} + D_{13}(u_{1,2}+u_{2,1}) \\ D_{12}u_{1,1} + D_{22}u_{2,2} + D_{23}(u_{1,2}+u_{2,1}) \\ D_{13}u_{1,1} + D_{23}u_{2,2} + D_{33}(u_{1,2}+u_{2,1}) \end{bmatrix}$$

$$= \begin{bmatrix} C_{1111}u_{1,1} + C_{1122}u_{2,2} + C_{1112}(u_{1,2}+u_{2,1}) \\ C_{1122}u_{1,1} + C_{2222}u_{2,2} + C_{2212}(u_{1,2}+u_{2,1}) \\ C_{1112}u_{1,1} + C_{2212}u_{2,2} + C_{1212}(u_{1,2}+u_{2,1}) \end{bmatrix}$$

$$C_{ijkl} = C_{klij} \Rightarrow \vec{\sigma} = \overline{\overline{D}}\,\vec{\varepsilon}(u).$$

$n_{sd}=3$

$$\vec{\sigma} = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix}, \quad \cancel{\Rightarrow D} \quad \cancel{C_{1111}\ C_{1112}\ C_{12}\ C_{2222}\ C_{3333}\ C_{2323}\ C_{1313}\ C_{1212}}$$

$$D = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1123} & C_{1113} & C_{1112} \\ & C_{2222} & C_{2233} & C_{2223} & C_{2213} & C_{2212} \\ & & C_{3333} & C_{3323} & C_{3313} & C_{3312} \\ & \text{symmetric} & & C_{2323} & C_{2313} & C_{2312} \\ & & & & C_{1313} & C_{1312} \\ & & & & & C_{1212} \end{bmatrix}$$

P83 Exercise 5

$n_{sd} = 3,$

$$D = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ & & \lambda+2\mu & 0 & 0 & 0 \\ & \text{symmetric} & & \mu & 0 & 0 \\ & & & & \mu & 0 \\ & & & & & \mu \end{bmatrix}$$

P98 Exercise 1

$A = IEN(a,e)$

IEN array:

a \ e	1	2	3	4
1	1	5	3	1
2	2	6	4	2
3	7	7	5	3
4	8	8	6	4

$P = ID(A)$

ID array

i \ A	1	2	3	4	5	6	7	8
1	1	2	0	3	4	5	0	6
2	1	2	0	3	4	5	6	7

$$P = LM(a,e) = ID(IEN(a,e))$$