LPV Multi-objective Robust Control of Wind Energy Conversion System

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Abstract: In light of the multi-time scale characteristic of the wind velocity, and the normalized double-frequency model of wind energy conversion system (WECS), a linear parameter varying (LPV) model is established. The PI controller is used in the low-frequency loop, and the H_∞ gain scheduling control method is adopted in the high-frequency loop. Simulation results depict that the coalition of double-frequency control compensate the controller dynamically, and the proposed method can not only reduce the sensitivity of system steady-state operation point, but also have better dynamic and static characteristics.

Key Words: Wind Energy Conversion System, LPV Model, PI Control, Gain Scheduling Control

1 Introduction

For WECS, the ob1jectives of harvesting the maximum power, improving the power quality and raising the system reliability conflict with each other. If only one of the objectives is optimized, others are bound to be interfered. So, it is quite necessary to choose a compromising control method. In [1, 2], this is realized by using linear quadratic optimal method with the weighting coefficient chosen reasonably. In [3, 4], the energy conversion efficiency and realiability are taken as the optimization objectives to design the PI controller and linear quadratic gaussian (LQG) optimal controller. In [5], it introduces a single neuron controller to replace the active stator power PI regulator, and the new controller is able to decouple the control of active and reactive power, which realize the multi-objective. These control methods have overcome the time-varying and nonlinear factors of wind turbines, which obtain certain effects. But the wind turbine is a complex multi-variable nonlinear system, the dynamic characteristic and system parameters are affected by many factors in the whole operating range, like the uncertainty and measurement error of wind velocity.

An efficient way of designing parameter dependent controller is within the framework of LPV control. Here, a controller is synthesized to satisfy a performance specification for all possible parameter values within a specified model and specified rate of parameter variation. The controller can be synthesized after solving an convex optimization problem subject to linear matrix inequalities (LMIs).

This paper is organized as follows: Section 2 gives the WECS mechanical model, and the normalized error LPV model is established on the basis of [5, 6]. In Section 3, the optimization problem is presented and the LPV controller is synthesized by $Lyapunov$ function and LMIs with rational weighting function. Section 4 contains a simulation result and followed by the conclusion in Section 5.

2 Multi-variable Model of WECS

Based on the variable-speed variable-pitch control of WECS, and the multi-scale characteristic of wind velocity, the normalized double-frequency model is built [6, 7]. The low-frequency model and the high-frequency one are given as:

$$\begin{cases} \overline{\Gamma_{wt}} & = 0.5\pi\rho R^3\overline{v}C_\Gamma(\overline{\lambda},\overline{\beta}) \\ J_l\dot{\overline{\Omega_l}} & = \overline{\Gamma_{wt}} - \dfrac{i}{\eta}\overline{\Gamma_G} \\ \dot{\overline{\beta}} & = \dfrac{1}{T_\beta}(\overline{\beta_{ref}} - \overline{\beta}) \end{cases} \quad (1)$$

$$\begin{cases} \dot{\overline{\Delta\Omega_l}}(t) & = \dfrac{1}{J_T}(\overline{\Delta\Gamma_{wt}}(t) - \overline{\Delta\Gamma_G}(t)) \\ \dot{\overline{\Delta\beta}} & = \dfrac{1}{T_\beta}(\overline{\Delta\beta_{ref}} - \overline{\Delta\beta}) \\ \dot{\overline{\Delta\Gamma_{wt}}}(t) & = (\dfrac{\gamma}{J_T} - \dfrac{1}{T_w})\overline{\Delta\Gamma_{wt}}(t) \\ & \quad +\dfrac{\gamma}{T_w}\overline{\Delta\Omega_l}(t) + (\dfrac{\zeta}{T_w} - \dfrac{\zeta}{T_\beta})\overline{\Delta\beta}(t) \\ & \quad -\dfrac{\gamma}{J_T}\overline{\Delta\Gamma_G}(t) + \dfrac{\zeta}{T_\beta}\overline{\Delta\beta_{ref}}(t) + \dfrac{2-\zeta}{T_w}e(t) \end{cases} \quad (2)$$

where T_{wt} is the wind torque produced by the wind turbine, C_p is the wind energy conversion coefficient, C_Γ is the wind torque coefficient, λ is the tip speed ratio, β is the pitch angle, R is the rotor radius of the wind turbine, η is the air density, Ω_l is the rotor speed, v is the wind velocity, i is the gear ratio, Γ_G is the electromagnetic torque, η is the gear efficiency and J_T is the inertia.

In order to describe the WECS operation status accurately, $\rho(t) = [\gamma(t), \zeta(t)]^T$ is taken as the time-varying parameter. Inside the double-frequency loop, the parameter J_T, T_w vary far more slowly than γ, ζ, so J_T and T_w can be valued as constant. From Eq.1 and Eq.2, the linear time-varying state equation is given as:

$$\dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t) + L(\rho(t))e(t) \quad (3)$$

where the state vector $x(t)$ is $[\overline{\Delta\Omega_l}(t), \overline{\Delta\beta}(t), \overline{\Delta\Gamma_{wt}}(t)]^T$, and the control input vector $u(t)$ is $[\overline{\Delta\Gamma_G}(t), \overline{\Delta\beta_{ref}}(t)]^T$. The coefficient matrix $L(\rho(t))$ is $[0, 0, (2-\gamma(t))/T_w]^T$, $A(\rho(t))$ is $\begin{bmatrix} 0 & 0 & \dfrac{1}{J_T} \\ 0 & -\dfrac{1}{T_\beta} & 0 \\ \dfrac{\gamma(t)}{T_w} & \dfrac{\zeta(t)}{T_w} - \dfrac{\zeta(t)}{T_\beta} & \dfrac{\gamma(t)}{J_T} - \dfrac{1}{T_w} \end{bmatrix}$,

This work is supported by National Natural Science Foundation (NNSF) of China(60774030); Prospective Joint Research Project of Industry, Education and Academy of Jiangsu Province(BY2012071)

$B(\rho(t))$ is $\begin{bmatrix} -\frac{1}{J_T} & 0 \\ 0 & \frac{1}{T_\beta} \\ -\frac{\gamma(t)}{J_T} & \frac{\zeta}{T_\beta} \end{bmatrix}$.

The objective of WECS can be divided into two parts: When the wind velocity is below the rated one, the objective is to reduce the error of tracking the optimal value $lambda_{opt}$ from the high-frequency component. While the velocity is above the rated one, the objective is to keep the generated power and rotor speed at the rated value. Meanwhile, the amplitude and frequency of the electromagnetic torque oscillation should be taken into consideration. From the above, the system output vector is defined as:

$$z(t) = \begin{bmatrix} \overline{\Delta\lambda}(t) \\ \overline{\Delta P_e}(t) \\ \overline{\Delta\Omega_h}(t) \end{bmatrix} = C(\rho(t))x(t) + Du(t) \tag{4}$$

where, $\overline{\Delta\lambda}(t) = \overline{\Delta\Omega_l}(t) - \frac{1}{2-\gamma}\overline{\Delta\Gamma_{wt}}(t) + \frac{\gamma}{2-\gamma}\overline{\Delta\Omega_l}(t)$, P_e is the power of generator, and $P_e = \Omega_h\Gamma_G$, $\overline{\Delta P_e}(t) = \overline{\Delta\Omega_h}(t) + \overline{\Delta\Gamma_G}(t)$, $\overline{\Delta\Omega_h}(t) = \overline{\Delta\Omega_l}(t)$. Then the coefficient matrix in Eq.4 can be deduced as: $C(\rho(t)) = \begin{bmatrix} 2/(2-\gamma(t) & 0 & 1/(2-\gamma(t)) \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$.

The Eq.3 and Eq.4 constitute the normalized error LPV model of WECS, and the coefficient matrix is affine dependent on the parameter vector $\rho(t)$.

3 Multi-variable Control of WECS

This section presents LMIs based method for designing and synthesizing of the double-frequency loop gain scheduling controller of WECS. The low-frequency loop adopts PI control while the high-frequency loop uses LPV control method as a dynamic compensation which is based on the normalize error.

3.1 PI Optimization of the Low-frequency Loop

Different objective is selected under different wind velocity: When the wind velocity is under the rated one, the pitch is kept at the optimal value. To capture the maximum power, the low-frequency component of rotor speed $\overline{\Omega_l}$ is controlled to track the reference $\overline{\Omega_{ref}} = \lambda_{opt}\overline{v}/R$ using the PI method. When the wind velocity is above the rated one, the pitch angle β and generator torque Γ_G are controlled together to keep the power at a constant.

(1) power control:

The power error is defined as: $\Delta P = P_{ref} - P$.

The first-order dynamic process of power error is:

$$\Delta\dot{P} + c_0\Delta P = 0, c_0 > 0 \tag{5}$$

Take $P_e = \Omega_h\Gamma_G$ into account, the following can be obtained from Eq.5:

$$-\dot{\Omega_h}\Gamma_G - \Omega_h\dot{\Gamma_G} + c_0\varepsilon_p = 0 \tag{6}$$

Then the torque reference can solved out from Eq.5 and Eq.6:

$$\dot{\Gamma}_{ref}(t) = \frac{1}{\Omega_h(t)}[c_0\varepsilon_p(t) - \frac{1}{J_h}(\frac{\eta}{i}\Gamma_{wt}(t)\Gamma_G(t) - \Gamma_G^2(t))] \tag{7}$$

(2) rotor speed control

The pitch reference can be calculated from the following:

$$\beta_{ref}(t) = K_{P2}\Delta(t) + K_{I2}\int\Delta\Omega(t) \tag{8}$$

where the rotor speed error is $\Delta\Omega(t) = \Omega_{ref} - \Omega_h(t)$, and the propotional and integral coefficient is K_{P2}, K_{I2}.

3.2 LPV Control of High-frequency Loop

For the high inertia of the wind turbine and the randomness of the wind velocity, the PI control of the low-frequency loop will produce large torque variation on the shaft of the motor. As a result of the limitation of the response speed of the pitch servo motor and generator, the whole mechanical subsystem will become unreliable. To make up this shortcoming, the LPV gain scheduling control is introduced to optimize the high-frequency component dynamically. As the LPV focuses more on the variation of the operation point by adjusting the controller's parameter real-time, the sensitivity of the steady state can be reduced remarkably.

From Eq.3, there exist external interference and error of the linear model. To restrain the compact of the wind velocity and the model error, the H_∞ controller $K(s)$ is designed, which is based on the LPV model. The controller is to make the H_∞ norm of the closed-loop transfer function $T_{ez}(s)$ from the external interference $e(t)$ to the control input $z(t)$ less than a given performance objective γ:

$$\|T_{ez}(s)\|_\infty < \gamma \tag{9}$$

Assuming that the open-loop LPV system from Eq.3 is dependent on the trajectory of the parameter $\rho(t)$, there exists two independent symmetric matrix parameters X, Y, as well as four independent matrix parameters $\hat{A}, \hat{B}, \hat{C}, \hat{D}$, which will make all ρ satisfy

$$\begin{bmatrix} \dot{X}+XA+\hat{B}C_2 & * & * & * \\ \hat{A^T}+A+B_2\hat{D}C_2 & -\dot{Y}+AY+B_2\hat{C}+* & * & * \\ (XB_1+\hat{B}D_{21})^T & (B_1+B_2\hat{D}D_{21})^T & -\gamma I_{n_w} & * \\ C_1+D_{12}D\hat{C}_2 & C_1Y+D_{12}\hat{C} & D_{11}+D_{12}\hat{D}D_{21} & -\gamma I_{n_z} \end{bmatrix} < 0 \tag{10}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \tag{11}$$

then there exists a controller[8, 9], such that

(1) The closed-loop is parameter dependent quadratic (PDQ) stable in the varying range of $\theta(t)$.

(2) The induced $L_2 - norm$ of the operator T_{zw} is bounded by $\gamma > 0$ (i.e. $\|T_{zw}\|_{i,2} < \gamma$).

The next step is to find a appropriate LPV controller:

$$\begin{cases} \dot{x_c}(t) &= A_c(\rho(t))x_c(t) + B_c(\rho(t))y(t) \\ u(t) &= C_c(\rho(t))x_c(t) + D_c(\rho(t))y(t) \end{cases} \tag{12}$$

and the four control matrixes are shown below

$$
\begin{aligned}
A_c(\rho) &= N^{-1}(\rho)(N(\rho)M^T(\rho) + \hat{A}(\rho) - \hat{B}(\rho)C_2(\rho)Y \\
&\quad X(A(\rho) - B_2(\rho)\hat{D}(\rho)C_2(\rho)Y - XB_2(\rho)\hat{C}(\rho)M^{-T}(\rho)) \\
B_c(\rho) &= N^{-1}(\rho)(\hat{B}(\rho) - XB_2(\rho)\hat{C}(\rho)) \\
C_c(\rho) &= (\hat{C}(\rho) - \hat{D}(\rho)C_2(\rho)Y)M^{-T}(\rho) \\
D_c(\rho) &= \hat{D}(\rho)
\end{aligned}
$$

Then the high-frequency loop controller is simplified as a mixed sensitivity problem, as illustrated in Fig.1

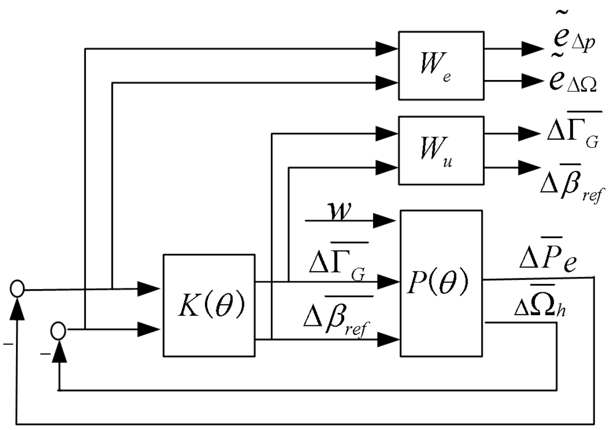

Fig. 1: H_∞ Control with Multi-variable controllers

In the figure, $\Delta\overline{P}_e, \Delta\overline{\Omega}_l$ are the controlled variables, $\overline{\Delta\Gamma_G}(t), \overline{\beta}_{ref}(t)$ are the control signals and w is the wind interference. When the rotational speed is at different operating range, the objective is different. In the synthesis model from Fig.1, the excepted output performance can be obtained by designing the appropriate sensitivity weighting function $W_e(s)$ and control weighting function $W_u(s)$.

The $W_e(s)$ emphasizes the importance of error component, and improves the tracking performance and the restraint of external disturbance. While the $W_u(s)$ weights the control effects, which guarantee the output limitation to meet the real application restriction. The selection of the weighting function directly reflects the requirement of the system performance in the H_∞ optimization, so it is necessary to choose the low-order weighting function as far as possible.

Sensitivity weighting function $W_e(s)$ can restrain the disturbance of the wind velocity and can accurately track the speed and power reference according to [10]. The high-frequency wind velocity spectrum is $0.06 - 0.7m/s$ which is quite considerable, so when the wind velocity is below the rated one, $[k_{wel1}, k_{wel2}]$ is $[1, 10]$, and when above the rated one, $[k_{weh1}, k_{weh2}]$ is $[0.05, 0.01]$. The control variable must be limited by the weighting function $W_u(s)$. If the torque and pitch frequency is too high, and the variation scope is too large, it will bring mechanical vibration and impact on the reliability of the general system. According to the actual situation, when the wind velocity is below the rated one, $[k_{wul1}, k_{wul2}]$ is $[1, 10]$, and when above rated one, $[k_{wuh1}, k_{wuh2}]$ is $[0.01, 0.3]$. In the whole control process, $k_{e1}, k_{e2}, k_{u1}, k_{u2}$ in the weighting function is affine dependent on ρ

$$
\begin{cases}
k_{e1}(\rho) &= \phi_e(\rho)k_{wel1} + (1 - \phi_e(\rho))k_{weh1} \\
k_{e2}(\rho) &= \phi_e(\rho)k_{wel2} + (1 - \phi_e(\rho))k_{weh2} \\
k_{u1}(\rho) &= \phi_u(\rho)k_{wul1} + (1 - \phi_u(\rho))k_{weu1} \\
k_{u2}(\rho) &= \phi_u(\rho)k_{wul2} + (1 - \phi_u(\rho))k_{weu2}
\end{cases}
\tag{13}
$$

And the final weighting function is shown below.

$$
W_e(s) = \begin{bmatrix} k_{e1} & 0 \\ 0 & k_{e2} \end{bmatrix} \frac{s/100 + 1}{s} \tag{14}
$$

$$
W_u(s) = \begin{bmatrix} k_{u1}\dfrac{s/(0.1\omega_1) + 1}{s/(10\omega_2) + 1} & 0 \\ 0 & k_{u2}\dfrac{s/(0.1\omega_2) + 1}{s/(10\omega_2) + 1} \end{bmatrix} \tag{15}
$$

4 Simulation

The multi-variable controller output is the steady-state component of the total control input of the WECS. The LPV dynamic compensation controller's output is the dynamic part of the normalized one. So the output is described in Eq.16 and Eq.17. Where $\overline{\Gamma_{Gref}}(t), \overline{\beta_{Gref}}(t)$ are the multi-variable control input, $\overline{\Delta\Gamma_{Gref}}(t), \overline{\Delta\beta_{Gref}}(t)$ are the LPV dynamic compensation control output. The compensation intensity of LPV control is described by the coefficient k_Γ and k_β.

$$
\Gamma_{Gref}(t) = \overline{\Gamma_{Gref}}(t) + k_\Gamma \frac{\overline{\Delta\Gamma_{Gref}}(t)}{\overline{\Gamma_{Gref}}(t)} \tag{16}
$$

$$
\beta_{Gref}(t) = \overline{\beta_{Gref}}(t) + k_\beta \frac{\overline{\Delta\beta_{Gref}}(t)}{\overline{\beta_{Gref}}(t)} \tag{17}
$$

The variation spectrum of the parameter ρ is evenly divided by the grid method and value of $[\gamma(t), \zeta(t)]^T$ is observed online. The Stability condition Eq.11 and Eq.12 form a group of LMIs, and with the help of the LMI toolbox in $Matlab$, a feasible solution can be figured out. The simulation parameters are shown in Tab.1

Table 1: Parameters' Value

Parameter	Value
R	$2.5m$
i	6.25
J_T	$0.236kgm^2$
L_t	$150m$
T_w	$10s$
P_{ref}	$6000W$
Ω_{ref}	$200rad/s$
η	0.95
T_β	0.05s
Γ_{Gmax}	40Nm
J_g	$0.01kgm^2$
J_t	$0.09kgm^2$

To see the full operation situation of WECS, simulation of duration $300s$ is conducted both with wind velocity below the rated one for the first $150s$ and the second $150s$ above the rated one. The curve is depicted in Fig.2

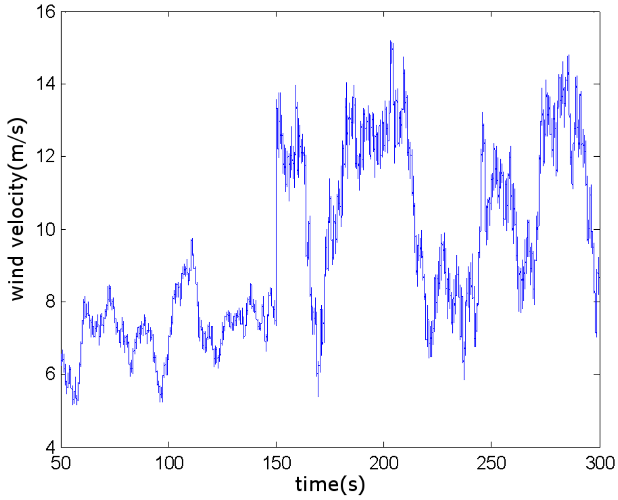

Fig. 2: Simulation Curve of Wind Velocity

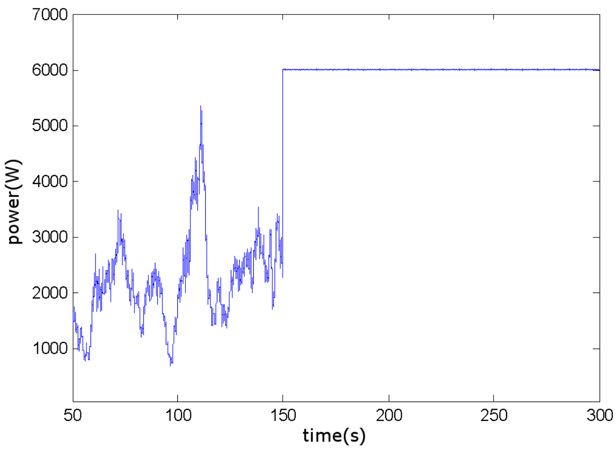

Fig. 3: Simulation Curve of Power Output

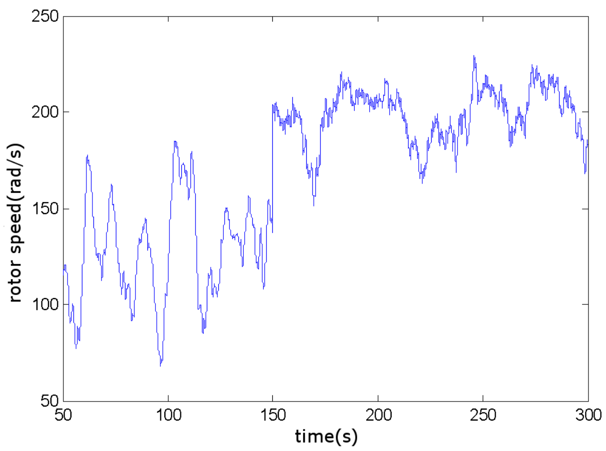

Fig. 4: Simulation Curve of Rotor Speed

The wave of wind power, rotor speed, torque and pitch angle are shown from Fig.3 to Fig.6.

The above figure demonstrate that when the wind velocity is below the rated one, the pitch angle remains constant to capture the maximum energy, and when above the rated one, the torque and pitch are controlled together to maintain the power constant and reduce the torque oscillation. The sim-

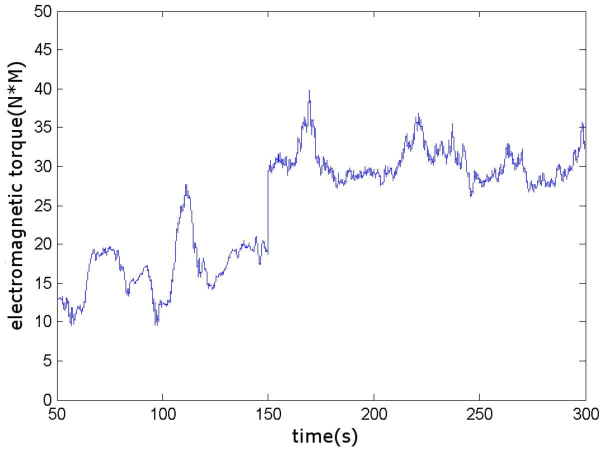

Fig. 5: Simulation Curve of Electromagnetic Torque

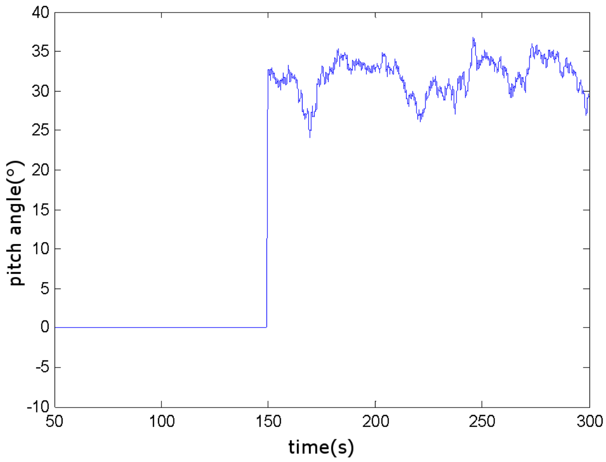

Fig. 6: Simulation Curve Pitch Angle

ulation result show that the multi-objective control based on LPV method is efficient.

5 Conclusion

This paper addresses the design of multi-objective controller based on LPV method for the double-frequency model of WECS in the full operation region. The controller handles both the parameter variations along the nominal operating trajectory and parameter variations introduced by the wind velocity turbulence.

Simulation show that the multi-variable controller is compensated by the double-frequency loop method compared with the conventional one. In general, the adopted method reduces the sensitivity of the relative steady-state remarkably, and achieves better dynamic and static characteristics.

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