

# Fault Diagnosis of Variable Pitch for Wind Turbines Based on Multi-innovation Forgetting Gradient Identification Algorithm

Dinghui Wu · Yiyang Li

**Abstract** This paper presents the fault diagnosis algorithm of the variable pitch system for wind turbines. The considered variable pitch system model is characterized by a second order differential equation, and is transformed into the discretization equation and the difference equation. Then the fault diagnosis problem is transformed into a parameter estimation issue, and the multi-innovation forgetting gradient (MIFG) identification algorithm is adopted. As the MIFG algorithm uses not only current data but also the past data at each iteration, the parameter estimation accuracy can be improved. The validity of fault diagnosis using MIFG algorithm for pitch system is verified by simulation examples.

**Keywords** Wind turbine · System identification · Pitch system · Fault diagnosis · MIFG

## 1 Introduction

Wind energy has contributed to a large part of world power production[1], and variable pitch system has played a important role in wind turbines. Since most wind turbines are located in remote areas, they are expected to produce energy with reliability and stability[2]. An effective way to ensure this is to adopt the advanced fault diagnosis technology, even though it may result in less power production in some cases[3].

The wind turbine operates in four operational zones governed by the mean wind velocity, the four zones are depicted in Fig.1. In zone 1, the turbine is standstill. In zone 2, the turbine is optimized to capture the maximum wind power. In zone 3, the pitch system works to keep constant power production. In zone 4, the wind velocity is too high so that the wind turbine is kept shutdown[4]. The hydraulic pitch system fault directly affects the production stability when the turbine is in zone 3. The fault diagnosis technology has a wide range of applications in industry. Wang et al. adopt an adaptive fault diagnosis observer design method combined with the switching technique to estimate the effect of bias faults and gain faults of the actuator[5]. Xu et al. design a fuzzy state observer to estimate the faults, which are modeled as both loss of effectiveness and lock-in-place[6]. At present, fault diagnosis researches of hydraulic variable pitch system for wind turbine are mainly as follows. Wang et al. have studied to carry on fault diagnosis technology of pressure with integration BP neural network theory[7]. Wang's method is

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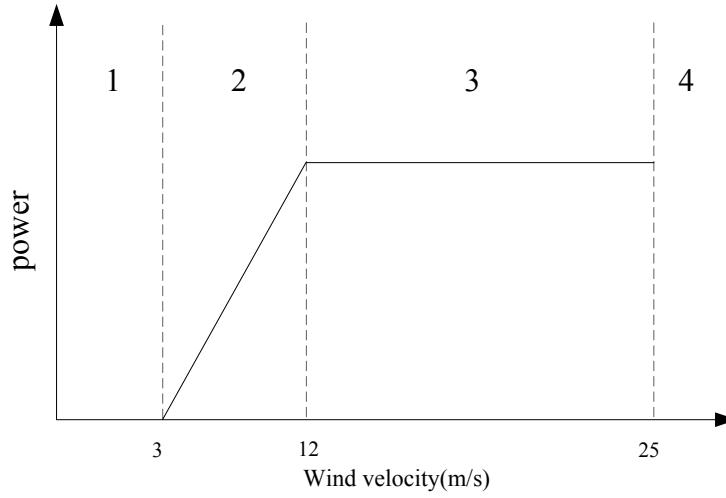
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**Fig. 1** Four Operational Zones of Wind Turbine

effective for the non-linear dynamic process in hydraulic system, but BP neural network theory still lacks proof and is difficult to obtain the estimation error bound. Goharrizi et al. analyze the pressure signal at one side of the actuator in response to periodic step inputs to the control valve[8]. It is shown that with the discrete wavelet transform (WT), the detail information of pressure signal can be obtained to detect the occurrence of the internal leakage and its severity. The similar methods can be found in [9] and [10], they both use Hilbert Huang transformation method to process the data, and analyze in the frequency domain. However, the disadvantage is that these method can only work offline, which limits the use. Sloth et al. use the extended Kalman filter to estimate the tower acceleration caused by pitch system[11], by comparing the estimated acceleration value with true tower acceleration sensor value, Sloth's study is able to determine which pitch is uncontrollable and the severity of the fault. But Sloth's method relies on the past data, the calculation amount of the built planning model is increased. This method is also dependent on the tower acceleration sensor, which is not available in some wind turbines.

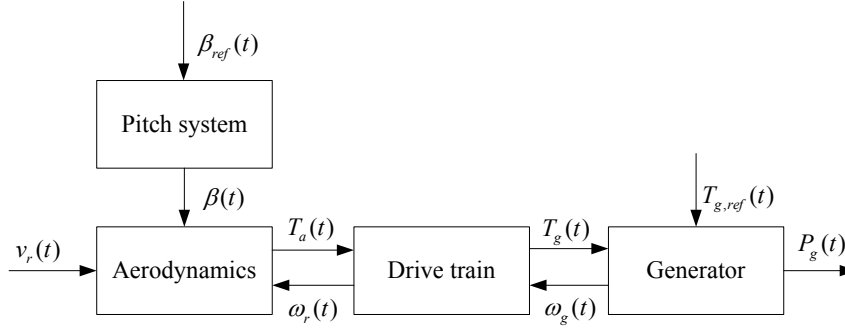
System identification is one of the major research fields of modern control theory[12]. A lot of identification methods have been proved to have minimum estimation error[13], identifiability[14] and be identified online[15]. In this paper, we try to convert the fault diagnosis problem into a system identification issue. The following faults may happen to pitch system, they are: pump wear, hydraulic leakage, high air content in the hydraulic oil[11]. These faults change the dynamics of the pitch system and make the system uncontrollable. In system identification view, the pitch system with fault can be modeled as a time-varying system. An effective way to estimate the time-varying system is within the framework of the identification algorithm with a forgetting factor  $\lambda$ .

Some work has been done to deal with the parameter estimation of time-varying system. In 1995, Guo and Ljung studied the estimation error of recursive least squares (RLS) with a forgetting factor (RFFLS for short)[16], they assumed that the error  $v(t)$  and parameter drift  $w(t)$  can be modeled as white noise. In [17][18], Ding studied the detail of RFFLS's estimation error bounds and prove only in deterministic systems, the algorithm is exponentially convergent. Zeng et al. use the locally weighted technique to identify the linear parameter varying system. While the data is close to the current time point, it is given large weight to measurement, while it is far from the current time point, small weight is given to measurement[19].

The rest of the paper is organized as follows. Section 2 gives the wind turbine model for simulation and the differential equation model of pitch system. And then a new way is utilized to convert it into a difference equation model. In Section 3, the algorithm's procedure and analysis of MIFG is given, which mainly focuses on how to choose the forgetting factor  $\lambda$ . Section 4 contains a simulation result and followed by the conclusion in Section 5.

## 2 Wind Turbine Model

It is depicted in Fig.2 that how the sub-models of the wind turbines are connected. The detailed introduction is described in the Appendix. The system is sampled at a rate of  $T = 100Hz$ .



**Fig. 2** Structure of Pitch System

Since the algorithm will finally be ported to a computer or some embedded processors, it is necessary to change the Eq.(31) into the difference one. There are many conventional ways to transform the Laplace equation, like bilinear transform and Euler transform, but these methods will lose the accuracy during transformation, and are difficult to meet the industrial requirements.

From the given plant  $G(s)$  and sampling period  $T$ , we can obtain the only linearization model  $G(z)$  by applying the impulse invariance transformation method[23]. This method can guarantee the accuracy during transformation from the continuous linear model to the discrete model.

First, we denote

$$a := -2\zeta\omega_n + \frac{\sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2},$$

$$b := -2\zeta\omega_n - \frac{\sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2},$$

$$G(s) := \frac{\beta(s)}{\beta_{ref}(s)} = \frac{ab}{s^2 + (a+b)s + ab},$$

then, by applying the impulse invariance transformation method, the discrete equation is

$$\begin{aligned} G(z) &= \frac{1}{2\pi j} \oint_c G(s) \frac{z}{z - e^{Ts}} ds \\ &= \frac{1}{2\pi j} \oint_c \frac{ab}{(s+a)(s+b)} \frac{1}{1 - e^{Ts}z^{-1}} ds \\ &= \frac{1}{b-a} \frac{ab(e^{-aT} - e^{-bT})z^{-1}}{1 - (e^{-aT} + e^{-bT})z^{-1} + e^{-(a+b)T}z^{-2}}, \end{aligned} \quad (1)$$

then, the difference equation can be obtained as

$$\begin{aligned} G(z) &= y(t)/u(t) \\ (b-a)[1 - (e^{-aT} + e^{-bT})z^{-1} + e^{-(a+b)T}z^{-2}]y(t) \\ &= [ab(e^{-aT} - e^{-bT})z^{-1}]u(t), \end{aligned} \quad (2)$$

finally,

$$\begin{aligned} y(t) - (e^{-aT} + e^{-bT})y(t-1) + e^{-(a+b)T}y(t-2) \\ = \frac{ab}{b-a}(e^{-aT} - e^{-bT})u(t-1). \end{aligned} \quad (3)$$

the identification model can be expressed as

$$y(t) = \varphi^T \theta(t) + v(t), \quad (4)$$

where,  $\theta(t) := [- (e^{-aT} + e^{-bT}), e^{-(a+b)T}, \frac{ab}{b-a}(e^{-aT} - e^{-bT})]^T$ ,  
 $\varphi(t) := [y(t-1), y(t-2), u(t-1)]^T$ ,  $v(t)$  is the zero mean white noise.

The faults considered for pitch system are: pump wear, hydraulic leakage, high air content. They can be modeled as

$$\begin{aligned}\tilde{\zeta}(t) &= (1 - \alpha_{pw}(t))\zeta + \alpha_{pw}(t)\zeta_{pw}, \\ \tilde{\omega}_n(t) &= (1 - \alpha_{pw}(t))\omega_n + \alpha_{pw}(t)\omega_{n,pw},\end{aligned}\tag{5}$$

$$\begin{aligned}\tilde{\zeta}(t) &= (1 - \alpha_{hl}(t))\zeta + \alpha_{hl}(t)\zeta_{hl}, \\ \tilde{\omega}_n(t) &= (1 - \alpha_{hl}(t))\omega_n + \alpha_{hl}(t)\omega_{n,hl},\end{aligned}\tag{6}$$

$$\begin{aligned}\tilde{\zeta}(t) &= (1 - \alpha_{ha}(t))\zeta + \alpha_{ha}(t)\zeta_{ha}, \\ \tilde{\omega}_n(t) &= (1 - \alpha_{ha}(t))\omega_n + \alpha_{ha}(t)\omega_{n,ha},\end{aligned}\tag{7}$$

where,  $\alpha_{pw}$  is an indicator for the pump wear,  $\alpha_{hl}$  is an indicator for the hydraulic leakage,  $\alpha_{ha}$  is an indicator for the high air content in oil.

The parameters for the above three faults are shown in the table.

**Table 1** Parameters' Value

Fault	Parameters
No Fault	$\omega_n = 11.11rad/s$ , $\zeta = 0.6$
Pump Wear	$\omega_{n,pw} = 7.27rad/s$ , $\zeta_{pw} = 0.75$
Hydraulic Leakage	$\omega_{n,hl} = 3.42rad/s$ , $\zeta_{n,hl} = 0.9$
High Air Content in the Oil	$\omega_{n,ha} = 5.73rad/s$ , $\zeta_{n,ha} = 0.45$

### 3 Description of Fault Diagnosis algorithm

In this section, the procedure to implement the fault diagnosis system is given, and a short discussion is presented on how to choose the forgetting factor  $\lambda$  to obtain the minimum estimation error.

#### 3.1 structure of Fault Diagnosis algorithm

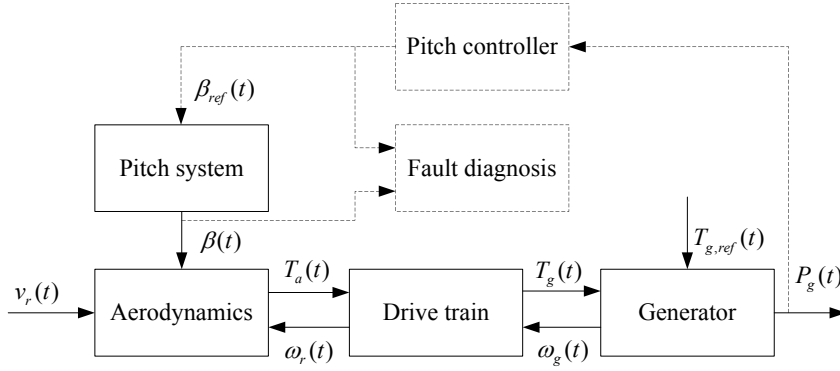
Fig.2 shows the structure of the wind turbine, the fault diagnosis based on the system identification theory needs the input data and the output data of the pitch system. We take the pitch controller's output  $\beta_{ref}(t)$  as the input, and the actual pitch angle  $\beta(t)$  as the output. The reformed structure is shown below.

As the wind turbine in this paper operates in zone 3, the pitch controller is designed by taking the electric power as the input and pitch reference  $\beta_{ref}$  as the output, which will maintain the generated power constant despite of the disturbance input  $v_r(t)$ .

#### 3.2 Analysis of MIFG

The innovation is defined as useful information which can improve parameter estimation accuracy[24]. Consider the least square and gradient stochastic algorithms, the identification system is

$$y(t) = \varphi^T(t)\theta + v(t),\tag{8}$$



**Fig. 3** Simulation Structure

where  $y(t) \in R$  is the output,  $\theta(t) \in R^n$  is the parameter vector to be identified,  $\varphi \in R^n$  is the information vector consisting of the system input-output data,  $v(t)$  is the stochastic noise with zero mean. The general identification algorithm is

$$\hat{\theta}(t) = \hat{\theta}(t-1) + L(t)e(t), \quad (9)$$

where  $L(t) \in R^n$  is the gain vector,  $e(t) := y(t) - \varphi^T(t)\hat{\theta}(t-1) \in R$  is the scalar innovation, that is the single innovation.

We extend the single innovation to multi-innovation, which means that

$$E(p, t) = \begin{bmatrix} e(t) \\ e(t-1) \\ \vdots \\ e(t-p+1) \end{bmatrix} \in R^p, \quad p \text{ is the innovation length.}$$

By defining the information matrix  $\Phi(p, t)$  and stacked output vector  $Y(p, t)$  as  $\Phi(p, t) = [\varphi(t), \varphi(t-1), \dots, \varphi(t-p+1)] \in R^{n \times p}$ ,  $Y(p, t) = [y(t), y(t-1), \dots, y(t-p+1)]^T \in R^p$ , the innovation vector  $E(p, t)$  may be expressed as  $E(p, t) = Y(p, t) - \Phi^T(p, t)\hat{\theta}(t-1)$ .

The faults are reflected in the parameters. When the pitch system operates normally, the identification algorithm is able to figure out the parameter vector. And the parameter of the pitch system changes correspondingly while the faults happen, the algorithm with a forgetting factor is able to track the dynamic parameters. So, the MIFG algorithm applied in the pitch system can detect the various fault situations.

The least square identification algorithm converges fast, but it needs huge computation due to covariance matrix. The gradient stochastic identification algorithm has a small amount of computation, but converges slow. Both algorithms are not suitable in wind turbine system. By applying the multi-innovation into the gradient stochastic algorithm, the new algorithm can compute quickly and converge fast. In order to track a time-varying system, we also apply a forgetting factor  $\lambda$ , that is MIFG algorithm.

### 3.3 How to Choose Forgetting Factor and Innovation Length

The purpose of this subsection is to give short discussion on how to choose  $\lambda$  and  $p$ . Consider the time-varying system 4, where  $\theta(t) \in R^n$  is the time-varying parameter vector to be identified.

The MIFG algorithm of estimating  $\theta(t)$  may be expressed as

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\Phi(t)}{r(t)} [Y(t) - \Phi^T \hat{\theta}(t-1)], \quad (10)$$

$$r(t) = \lambda r(t-1) + \|\varphi(t)\|^2, \quad 0 < \lambda < 1, r(0) > 0, \quad (11)$$

$$\Phi(p, t) = [\varphi(t), \varphi(t-1), \dots, \varphi(t-p+1)] \in R^{n \times p}, \quad (12)$$

$$Y(p, t) = [y(t), y(t-1), \dots, y(t-p+1)]^T \in R^p. \quad (13)$$

In engineering, the parameter estimation accuracy is measured by  $\delta_a := \|\hat{\theta}(t) - \theta(t)\|^2$ , but as the parameter vector  $\theta(t)$  is unknown, it is impossible to obtain  $\delta_a$ . It is necessary for us to analyze the upper bound of estimation error, and can be simplified as Theorem 1.

Here, we assume that the information vector  $\varphi(t)$  is persistently exciting, that is, there exist constants  $0 < \alpha \leq \beta < \infty$  and an integer  $N \geq n$  such that the following persistent excitation condition holds[12]

$$\alpha I \leq \frac{1}{N} \sum_{i=0}^{N-1} \varphi(t+i) \varphi^T(t+i) \leq \beta I, t > 0. \quad (14)$$

For the system Eq.4 and the MIFG algorithm in Eq.10-Eq.13,  $r(0)$  is chosen by  $\frac{\alpha}{1-\lambda} \leq r(0) \leq \frac{nN\beta}{1-\lambda}$ , let the innovation length  $p = N$  and  $E[\|\hat{\theta}(0) - \theta(0)\|^2] = \delta_0 < \infty$ , the observation noise  $\{v(t)\}$  and the parameter changing rate  $w(t) := \theta(t) - \theta(t-1)$  are stochastic sequences with zero mean, and the sequences  $\{v(t)\}$  and  $\{w(t)\}$  satisfy

$$E[v(t)] = 0, E[w(t)] = 0, \quad (15)$$

$$E[v^2(t)] \leq \sigma_v^2 < \infty, E[\|w(t)\|^2] \leq \sigma_w^2 < \infty. \quad (16)$$

Next, we define the noise vectors as following:  $w(t) = \theta(t) - \theta(t-1)$ , we can define the noise vector,

$$V(p, t) := \begin{bmatrix} v(t) \\ v(t-1) \\ \vdots \\ v(t-p+1) \end{bmatrix} \in R^p,$$

$$W(p, t) := \begin{bmatrix} 0 \\ \varphi^T(t-1)w(t-1) \\ \varphi^T(t-2)w(t-2) \\ \vdots \\ \varphi^T(t-p+1)w(t-p+1) \end{bmatrix} \in R^p$$

and the parameter estimation error vector  $\hat{\theta}(t) - \theta(t)$  is denoted as  $\tilde{\theta}(t)$ , and by applying Eq.(10),

$$\begin{aligned} \tilde{\theta}(t) &= \hat{\theta}(t) - [\theta(t-1) + w(t)] \\ &= [I - \frac{\Phi(p, t)\Phi^T}{r(t)}] \tilde{\theta}(t-1) \\ &\quad + \frac{\Phi(p, t)[-W(p, t) + V(p, t)]}{r(t)} - w(t). \end{aligned} \quad (17)$$

as the expectation

$$\begin{aligned} E[\|\Phi(p, t)V(p, t)\|^2] &\leq p^2 \beta \sigma_v^2, \\ &= N^2 \beta \sigma_v^2 \leq \frac{N^2 \beta (1-\lambda)^2 \sigma_v^2}{\alpha^2} \\ E[\|\Phi(p, t)W(p, t)\|^2] &\leq \frac{(p-1)p^3 \beta^2 \sigma_w^2}{2} \leq \frac{p^4 \beta^2 \sigma_w^2}{2} \\ &= \frac{N^4 \beta^2 \sigma_w^2}{2} \leq \frac{N^4 \beta^2 (1-\lambda)^2 \sigma_w^2}{2\alpha^2} \end{aligned} \quad (18)$$

and

$$I - \frac{\Phi(p, t)\Phi^T(p, t)}{r(t)} \leq [1 - \frac{\alpha(1 - \lambda)}{n\beta}]I$$

$$= (1 - \rho)I \quad (19)$$

By taking norm and expectation of both side of Eq.17 and using the inequality  $\|x + y\|^2 \leq (1 + a)\|x\|^2 + (1 + a^{-1})\|y\|^2$ , ( $a > 0$ ), we can obtain:

$$E\|\tilde{\theta}(t)\|^2 \leq (1 + a)(1 - \rho)E[\|\hat{\theta}(t - 1)\|^2] +$$

$$3(1 + a^{-1})[\frac{N^4\beta^2(1 - \lambda)^2\sigma_w^2}{2\alpha^2} +$$

$$\frac{N^2\beta(1 - \lambda^2)\sigma_v^2}{\alpha^2} + \sigma_w^2] \quad (20)$$

We take  $a$  to satisfy  $0 < a < \frac{\rho}{1 - \rho}$ , that is  $0 < (1 + a)(1 - \rho) < 1$ , then the expectation Eq.20 turns into

$$E[\|\tilde{\theta}(t)\|^2] \leq [(1 + a)(1 - \rho)]^t E[\|\tilde{\theta}(0)\|^2] +$$

$$\frac{3(1 + a^{-1})}{1 - (1 + a)(1 - \rho)} [\frac{N^4\beta^2(1 - \lambda)^2\sigma_w^2}{2\alpha^2} +$$

$$\frac{N^2\beta(1 - \lambda^2)\sigma_v^2}{\alpha^2} + \sigma_w^2]$$

$$\leq [(1 + a)(1 - \rho)]^t \sigma_0 +$$

$$\frac{3(1 + a^{-1})}{1 - (1 + a)(1 - \rho)} [\frac{N^4\beta^2(1 - \lambda)^2\sigma_w^2}{2\alpha^2} +$$

$$\frac{N^2\beta(1 - \lambda^2)\sigma_v^2}{\alpha^2} + \sigma_w^2] \quad (21)$$

and we can denote

$$[\frac{N^4\beta^2(1 - \lambda)^2\sigma_w^2}{2\alpha^2} + \frac{N^2\beta(1 - \lambda^2)\sigma_v^2}{\alpha^2} + \sigma_w^2] := f(\lambda)$$

$$\frac{3(1 + a^{-1})}{1 - (1 + a)(1 - \rho)} := g(a) \quad (22)$$

To get the upper bound of the estimation error, we must minimize the right hand side of the Eq.21, let

$$\frac{dg(a)}{da} = 0 \quad (23)$$

and as we define  $a$  to be positive, we can obtain the best value  $a_0 = \frac{1}{\sqrt{1 - \rho}} - 1$ , and the correspond minimum  $g_{min}$  is

$$\frac{3}{(1 - \sqrt{1 - \rho})^2}$$

and the minimum upper bound can be expressed as

$$[\sqrt{1 - \rho}]^t \delta_0 + g_{min} f(\lambda)$$

Since  $[\sqrt{1 - \rho}]^t \sigma_0$  is very small, we can omit it. To get the minimum  $f_{min}$ , just let

$$\frac{df(\lambda)}{d\lambda} = 0 \quad (24)$$

Eq.24 is a four-order equation, and generally has four solution to get the best forgetting factor.

From the above analysis, we conclude the following guide to choose the best forgetting factor:

- if parameter  $\rho$  is small, it will generate small estimation error upper bound, which means  $\alpha$  and  $\beta$  should be close enough,
- from Eq.18, the system noise should be as small as possible to get a small estimation error bound,
- also from Eq.24, a small innovation length  $N$  will produce small estimation error upper bound.

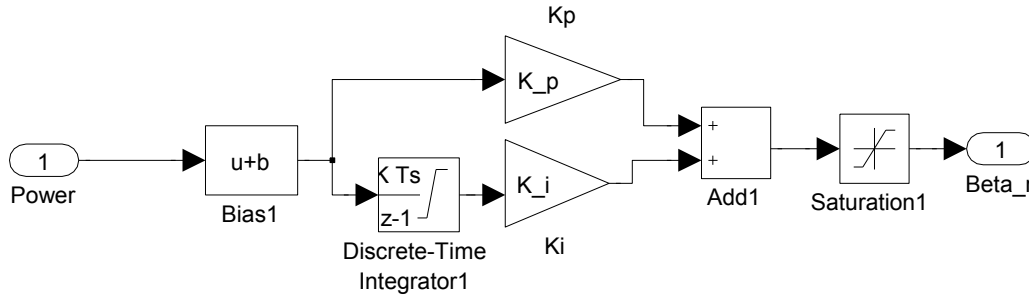
#### 4 Simulation

In this section, we test the MIFG algorithm in *Matlab/Simulink* with the model constructed in Section 2 and algorithm described in Section 3. Our identification model is Eq.(4). The wind turbine operational parameters are shown in the table.

**Table 2** Parameter Value

Parameter	Value	Parameter	Value
$N_g$	95	$B_r$	$27.8Nm/(rad/s)$
$A$	$10387m^2$	$B_{dt}$	$945kNm/(rad/s)$
$\rho$	$1.225kg/m^3$	$B_g$	$3.034Nm/(rad/s)$
$J_r$	$55 \times 10^6 kgm^2$	$J_g$	$390kgm^2$

The pitch controller described in Fig.3 is implemented as a PI controller, the detailed simulation structure is shown below. The electric power  $P_g$  is the controller input, and the  $\beta_{ref}$  is the controller output. Although there is more complex and better controller, but for simulation convenience, we choose the simplest PI controller, where  $K_p = 4, K_i = 1$ .



**Fig. 4** Simulation Model of PI Controller

The wind data is downloaded from the kk-electronics project[25], and is depicted in Fig.5. Simulation results are shown in Fig.6 to Fig.8 where the fault diagnosis result of pump wear fault, hydraulic leakage fault and high air content fault. The forgetting factor is chosen to be 0.97 according to Eq.24, and fault indicator of three situations are the same:

$$\alpha_{pw}, \alpha_{hl}, \alpha_{ha} = \begin{cases} 0, & 0 \leq t \leq 1000, 2000 \leq t \leq 3000 \\ 1, & 1000 < t < 2000, 3000 < t < 4500 \end{cases} \quad (25)$$

Results of Fig.6 to Fig.8 show that when the innovation length gets larger, the algorithm will have a faster convergence. And when the simulation begins, the  $p = 1$  algorithm fails to track the parameter.

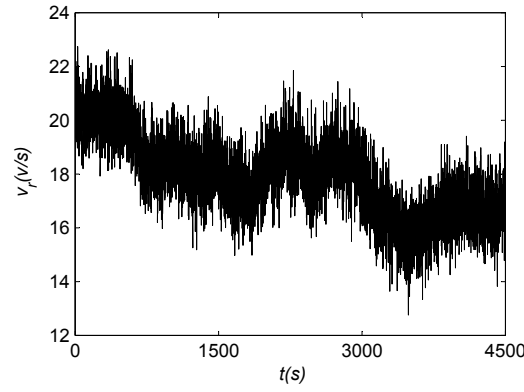
We also test the real simulation time, the time is shown in Tab.3. As the innovation length gets larger, it takes more computation time as the matrix becomes more complex. As the Tab.3 suggests, innovation length  $p = 8$  is better.

#### 5 Appendix

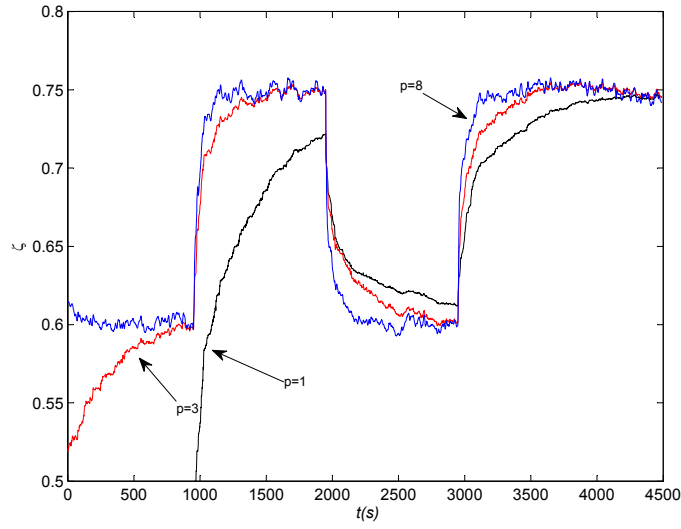
In this appendix, we shall give the simulation model of the wind turbine. The turbine is composed of aerodynamics, pitch, drive train and generator convertor[20].

The pitch converts the energy from wind to rotor shaft, the shaft rotates at the speed  $\omega_r(t)$ , the wind power is dependent on wind velocity  $v_r(t)$ , air density  $\rho$  and the rotate area  $A$ . The





**Fig. 5** Wind Velocity



**Fig. 6**  $\zeta$  with Pump Wear Fault

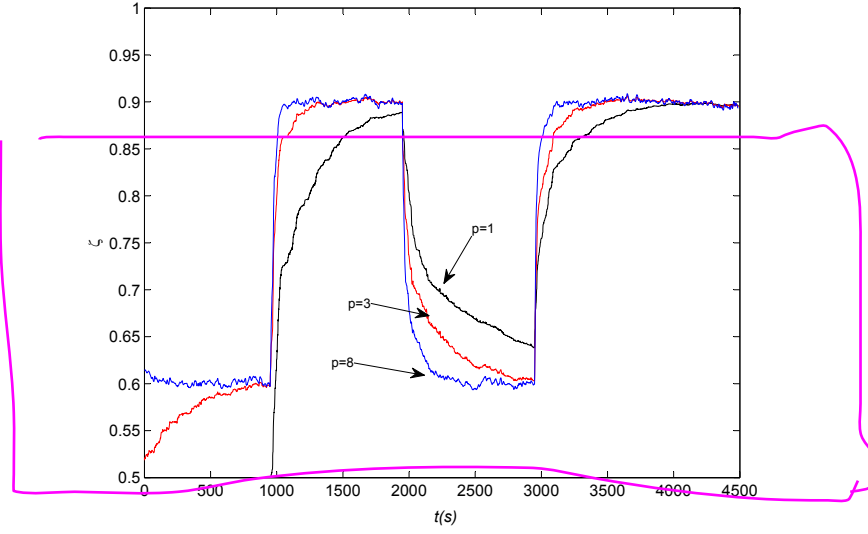
**Table 3** Real Simulation Time

Innovation length	Time
$p = 1$	0.313650s
$p = 3$	0.270484s
$p = 8$	0.360650s
$p = 12$	0.497493s
$p = 14$	0.534856s
$p = 16$	0.601536s
$p = 32$	0.894214s

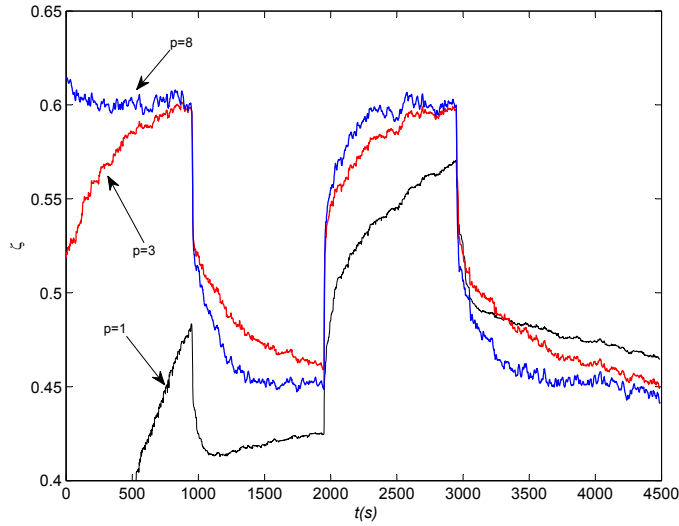
converted wind power is based on power coefficient  $C_p(\lambda(t), \beta(t))$ , where  $\lambda(t)$  is tip-speed ratio and  $\beta(t)$  is pitch angle. The aerodynamic torque applied on rotor shaft is given as[21]

$$T_a(t) = \frac{1}{2\omega_r(t)} \rho A v_r^3(t) C_p(\lambda(t), \beta(t)). \quad (26)$$

The drive train is consisted of low-speed shaft and high-speed shaft, the inertias and friction coefficient of both side are  $J_r, J_g$  and  $B_r, B_g$ . The two shafts are interconnected by gears, and the gear ratio is  $N_g$ , combined with torsion stiffness  $K_{dt}$  and torsion damping  $B_{dt}$ . This results



**Fig. 7**  $\zeta$  with Hydraulic Leakage Fault



**Fig. 8**  $\zeta$  with High Air Content Fault

in a torque applied to generator  $T_g(t)$  with a speed  $\omega_g(t)$ . The drive train model is given as[22]

$$J_r \dot{\omega}_r(t) = T_a(t) + \frac{B_{dt}}{N_g} \omega_g(t) - (B_{dt} + B_r) \omega_r(t), \quad (27)$$

$$J_g \dot{\omega}_g(t) = \frac{B_{dt}}{N_g} \omega_r(t) - T_g(t) - \left( \frac{B_{dt}}{N_g^2} + B_g \right) \omega_g(t). \quad (28)$$

Electric power is generated by the generator, while the generator torque is adjusted by the reference  $T_{g,ref}(t)$ . The actual torque from converter is described as a first order system with time constant  $\tau_g$  and time delay  $t_{g,d}$ [11],

$$\dot{T}_g(t) = -\frac{1}{\tau_g} T_g(t) + \frac{1}{\tau_g} T_{g,ref}(t - t_{g,d}). \quad (29)$$

The electric power produced by the generator is described as follows, where the  $\eta$  is efficiency of generator, which is considered to be a constant[11],

$$P_g(t) = \eta_g \omega_g(t) T_g(t). \quad (30)$$

The purpose of this section is to explain how the pitch system is modeled. The pitch system shown in Fig.9 adjusts the pitch of a blade by rotating it according to wind velocity. In general, one turbine is composed of three pitches, each pitch is regulated by a hydraulic actuator, which are controlled separately by three valves.

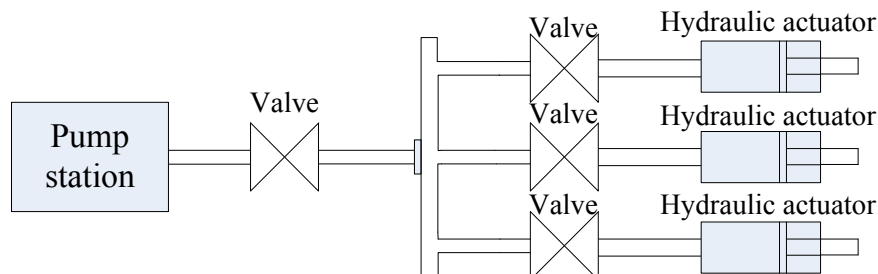


Fig. 9 Hydraulic Pitch System

The hydraulic pitch is modeled as a second order system,

$$\frac{\beta(s)}{\beta_{ref}(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\ddot{\beta}(t) = -2\zeta\omega_n \dot{\beta}(t) - \omega_n^2 \beta(t) + \omega_n^2 \beta(t) + \omega_n^2 \beta_{ref}(t), \quad (31)$$

where  $\beta(t)$  is the pitch angle,  $\beta_{ref}(t)$  is the reference to the pitch angle,  $\omega_n$  is the natural frequency of the pitch model,  $\zeta$  is the damping ratio of the pitch model.

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