Finite Frequency H_∞ Filtering for Switching LPV Systems

Heng Wang, He-Hua Ju and Yu-Long Wang

***Abstract*— This paper studies the problem of finite frequency H_∞ filter design for switching linear parameter varying (LPV) systems with disturbance frequency affected by system parameters. A set of switching filters are designed, each suitable for a specific parameter subregion and a specific finite frequency performance index, the frequency range of each performance index depends on the parameter subregion. The hysteresis switching logic is adopted, and the design problem is finally formulated into linear matrix inequality (LMI) which can be computed efficiently with LMI Control Toolbox. A numerical example is given to illustrate the effectiveness of the proposed method.**

I. INTRODUCTION

State estimation is a very important issue in control area, which has been widely studied in the past decades. Recently, the H_∞ filtering approach, has received considerable attention due to its wide applicability when robustness is imposed, where the main objective is to minimize the H_∞ norm from disturbances to the estimation error [1]-[2]. On the other hand, LMI techniques have been applied to filtering problems [3]-[6], which can be solved effectively using the LMI control toolbox. In [7]-[10], the parameter dependent Lyapunov method is adopted, and through introducing appropriate slack matrix variables, the conservatism is reduced greatly.

At the other hand, many systems encountered in practice exhibit switching between several subsystems that is dependent on various environmental factors [11]. These switching systems have numerous applications in control of mechanical systems, the automotive industry, aircraft and air traffic control, switching power converters, and many other fields. There are plenty of results discussing the stability and stabilizability of switching systems when designing switching controllers, see survey papers [12]-[16] and the references therein.

Moreover, the frequency ranges of disturbances are usually finite, and for some practical systems, the frequency range of disturbances changes with the varying of the system parameters [17]. For these systems, it is conservative to design full frequency filters to estimate system states. In [18], finite frequency H_∞ filtering problem for linear systems is investigated, where the disturbances are considered in finite frequency domain and less conservatism is achieved for some systems. However, for switched control systems, the finite frequency filtering approach presented in [18] is conservative to some extent since the finite frequency performance indexes considered there are in definite frequency ranges. To solve this problem, a switched finite frequency filtering approach is proposed in this paper.

This paper deals with the finite frequency H_∞ filtering problem for LPV systems with hysteresis switching behavior. The novelty of this paper is that a set of switching filters are designed through satisfying a set of different finite frequency performance indexes, each finite frequency performance index corresponds to a specific parameter subregions, and the designed problem is formulated into solving a set of LMI problem. This paper is organized as follows. Section 2 presents the problem under consideration. Section 3 illustrates the finite freqeuncy H_∞ filter design approach in details. Section 4 shows the effectiveness of the proposed design method via an example. Some concluding remarks are given in Section 5.

For a matrix A, $A^T, A^*, A^\perp$ denote its transpose, complex conjugate transpose and orthogonal complement, respectively. I denotes the identity matrix with an appropriate dimension. For a symmetric matrix, $A > (\geq)0$ and $A < (\leq)0$ denote positive (semi)definiteness and negative (semi) definiteness. The Hermitian part of a square matrix M is denoted by $\mathrm{He}(M) := M + M^*$. The symbol $\mathbf{H}_n$ stands for the set of $n \times n$ Hermitian matrices. The symbol $\star$ within a matrix represents the symmetric entries. $\sigma_{max}(G)$ denotes maximum singular value of the transfer matrix G.

II. PROBLEM FORMULATION

Consider the following LPV system

$$\begin{aligned}\dot{x}(t) &= A(\rho)x(t) + B(\rho)d(t)\\ y(t) &= C(\rho)x(t) + D(\rho)d(t)\\ z(t) &= L(\rho)x(t)\end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state, $d(t) \in \mathbf{L}_2^{n_d}[0,\infty)$ is exogenous disturbance whose frequency range $\mathcal{I}$ is finite [18] e.g., $\mathcal{I} = [\underline{\omega}, \overline{\omega}]$, $y(t) \in \mathbf{R}^{n_y}$ is the measured output, and $z(t) \in \mathbf{R}^{n_z}$ is the vector to be estimated. All matrices are of compatible dimensions and are continuous functions of the parameter ρ. It is assumed that ρ is in a compact set $\mathcal{P} \subset \mathbf{R}^s$, and it does not depend explicitly on the time variable but can be measured online. The parameter ρ can vary slowly due to changes in temperature, wind, pressure, humidity, atmosphere, or operating points [6][19].

Suppose that the parameter set $\mathcal{P}$ is covered by a finite number of closed subsets $\{\mathcal{P}_i\}_{i\in Z_N}$ by means of a family of switching surfaces, where the index set $Z_N = 1, 2, \ldots, N$, and $\mathcal{P} = \cup\mathcal{P}_i$. The adjacent parameter subsets are separated

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by switching surfaces, and they have either overlapped or disjointed interiors [20].

The finite frequency H_∞ filtering problem considered here is to design a family of finite frequency filters, each generates an estimate $\hat{z}(t)$ of $z(t)$ which is given by $\hat{z}(t) = \mathcal{F}_i \cdot y(t)$. The filter $\mathcal{F}_i$ is supposed to be in the following form

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{f,i}\hat{x}(t) + B_{f,i}y(t) \\ \hat{z}(t) &= C_{f,i}\hat{x}(t) + D_{f,i}y(t), i \in Z_N \end{aligned} \tag{2}$$

each suitable for a specific parameter subset $\mathcal{P}_i$. The vector $\hat{x}(t)$ is the filter state vector, and $A_{f,i}, B_{f,i}$, $C_{f,i}$, $D_{f,i}$ are filter parameters of appropriate dimensions to be determined. The order of the filter n_f is restricted to be less than or equal to the system n_p.

Each filter $\mathcal{F}_i$ corresponding to the parameter subregion is designed to satisfy an finite frequency H_∞ performance index, and the frequency ranges of these performance indexes are different from each other.

Remark 1: It is reasonable to design each filter $\mathcal{F}_i, i = 1, \ldots, N$ in different frequency domain for some practical systems. For example, for flight control systems, the disturbances on the aircraft are usually caused by the change of the aircraft weight, the temperature, density, pressure of the atmosphere and the wind. When the flight velocity or height varies such that the system parameter ρ exceeds some extent, the characteristic of disturbance varies, and the frequency of disturbance changes correspondingly [17].

The switching occurs when the parameter trajectory hits one of the switching surfaces. A switching signal is defined as a piecewise constant function σ. It is assumed that σ is continuous from the right everywhere, and only limited number of switches happen in any finite time interval.

The dynamics of (1) and (2) can be rewritten as the following augmented system:

$$\begin{aligned} \dot{\xi}(t) &= \bar{A}_\sigma(\rho)\xi(t) + \bar{B}_\sigma(\rho)d(t) \\ e(t) &= \bar{C}_\sigma(\rho)\xi(t) + \bar{D}_\sigma(\rho)d(t) \end{aligned} \tag{3}$$

where $e(t) = z(t) - \hat{z}(t)$ is the estimation error, and $\xi(t) = \begin{bmatrix} x(t)^T & \hat{x}(t)^T \end{bmatrix}^T$, and

$$\begin{aligned} &\begin{bmatrix} \bar{A}_\sigma(\rho) & \bar{B}_\sigma(\rho) \\ \bar{C}_\sigma(\rho) & \bar{D}_\sigma(\rho) \end{bmatrix} \\ = &\left[\begin{array}{cc|c} A(\rho) & 0 & B(\rho) \\ B_{f,\sigma}C(\rho) & A_{f,\sigma} & B_{f,\sigma}D(\rho) \\ \hline L(\rho) - D_{f,\sigma}C(\rho) & -C_{f,\sigma} & D_{f,\sigma}D(\rho) \end{array}\right] \end{aligned} \tag{4}$$

Note that the resulting filtering system is a switching LPV system, which could have discontinuity at switching surfaces due to the use of multiple finite frequency filters.

Define $G_{edi}(j\omega)$ as the transfer function from disturbance input $d(t)$ to the estimation error $e(t)$, the finite frequency H_∞ filtering problem is to find a guaranteed estimation performance index $\gamma_i > 0$, $i = 1, \ldots, N$ such that

$$\sup \|G_{edi}(j\omega)\|_\infty < \gamma_i \ \forall \omega \in [\underline{\omega}_i, \overline{\omega}_i] \tag{5}$$

where $G_{edi}(s) = \bar{C}_i(\rho)(j\omega I - \bar{A}_i(\rho))^{-1}\bar{B}_i(\rho) + \bar{D}_i(\rho)$.

The main task of this work can be reformulated as to design a family of filters $\mathcal{F}_i$ such that the augmented error system (3) is stable and satisfies the finite frequency performance index (5) for $i = 1, \ldots, N$.

III. Finite frequency H_∞ filter design via multiple parameter-dependent Lyapunov functions

In order to analyze the stability of the switching augmented LPV system (3), the following Lyapunov functions is considered

$$V_\sigma = \xi(t)^T X_\sigma(\rho)\xi(t) \tag{6}$$

where the value of the switching signal σ represents the active operating region $\mathcal{P}_i$ and thus determine the corresponding matrix function $X_i(\rho)$.

As pointed in [16][20], for a switching LPV system to be stable, the value of the discontinuous Lyapunov function V_σ is not necessary decreasing along the parameter trajectory, here it is required that the value of V_σ decrease in the active parameter region $\mathcal{P}_i$ provided proper switching logic is adopted. In the following section, the hysteresis switching is adopted for the filter design.

A. Preliminaries

In this subsection, we consider the case when hysteresis switching logic is adopted, and it is assumed that any two adjacent parameter subsets are overlapped. As pointed out in [20], there are two switching surfaces between any two adjacent parameter subsets, denote $\mathcal{S}_{ij}$ as the switching surface which specifies the one-directional move from $\mathcal{P}_i$ to $\mathcal{P}_j$. The switching event occurs when the parameter trajectory hits one of the switching surfaces $\mathcal{S}_{ij}$ or $\mathcal{S}_{ji}$.

For the filtering error system (3), it is assumed that the matrix function $X_i(\rho)$ is related to the Lyapunov function when the ith filter is active. If on the switching surface $\mathcal{S}_{ij}$, we have

$$X_i(\rho) \geq X_j(\rho) \tag{7}$$

which guarantee that the Lyapunov function of (3) nonincreases when switching from $\mathcal{P}_i$ to $\mathcal{P}_j$

Lemma 1: (Generalized KYP Lemma [21]) Given system (A, B, C, D), let a symmetric matrix Π of appropriate dimension be given, the following statements are equivalent:
i) The finite frequency inequality

$$\begin{bmatrix} G^*(j\omega) & I \end{bmatrix} \Pi \begin{bmatrix} G(j\omega) \\ I \end{bmatrix} < 0, \ \forall \ \omega \in [\underline{\omega}_i, \overline{\omega}_i] \tag{8}$$

where $G(j\omega) = C(j\omega I - A)^{-1}B + D$ is the transfer function.
ii) There exist Hermitian matrices $P, Q \in \mathbf{H}_n$ satisfying $Q > 0$, and

$$\begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^* \Xi \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} + \begin{bmatrix} C & D \\ 0 & I \end{bmatrix}^* \Pi \begin{bmatrix} C & D \\ 0 & I \end{bmatrix} < 0 \tag{9}$$

where

$$\Xi = \begin{bmatrix} -Q & P + j\varpi_c Q \\ P - j\varpi_c Q & -\underline{\omega}_i\overline{\omega}_i Q \end{bmatrix}$$
$$\varpi_c = (\underline{\omega}_i + \overline{\omega}_i)/2 \tag{10}$$

Define $J \in \mathbf{R}^{(2n+n_z)}, \bar{H} \in \mathbf{R}^{(2n+n_z)\times(n_d+n_z)}$, and $\bar{L} \in \mathbf{R}^{(2n+n_z)\times n}$ as

$$J := \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix}, \bar{H} := \begin{bmatrix} 0 & 0 \\ C^* & 0 \\ D^* & I \end{bmatrix}, \bar{L} := \begin{bmatrix} -I \\ A^* \\ B^* \end{bmatrix}$$

we have the following lemmas.

Lemma 2: Given $\Pi = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$, let Hermitian matrix variables $P, Q \in \mathbf{H}_n$ and $Q > 0, R \in \mathbf{R}^{n\times(2n+n_z)}$. The following statements are equivalent:

i) The condition in (9) holds and

$$N^*(J\Xi J^* + \bar{H}\Pi\bar{H}^*)N < 0 \tag{11}$$

ii) There exists $W \in \mathbf{R}^{n\times n}$ such that

$$J\Xi J^* + \bar{H}\Pi\bar{H}^* + \mathrm{He}(\bar{L}WR) < 0 \tag{12}$$

Proof: Similar to that of [18], it is omitted. ■

Based on Lemmas 1-2, we have the following Lemma 3, which is essential for later developments.

Lemma 3: Consider system (3), given $\Pi = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$, let Hermitian matrix variables $P, Q \in \mathbf{H}_n$ and $Q > 0, R \in \mathbf{R}^{n\times(2n+n_z)}$. The following finite frequency performance index

$$\sup \|G_{edi}(j\omega)\|_\infty < \gamma_i \ \forall \omega \in [\underline{\omega}_i, \overline{\omega}_i], \tag{13}$$

holds, if the following condition holds

$$\begin{bmatrix} -Q_i & P_i + j\omega_c Q_i - W_i & 0 & 0 \\ \star & \Delta_i(\rho) & W_i\bar{B}_i^*(\rho) & \bar{C}_i^*(\rho) \\ \star & \star & -\gamma_i^2 I & \bar{D}_i^*(\rho) \\ \star & \star & \star & -I \end{bmatrix} < 0, \tag{14}$$

where $\rho \in \mathcal{P}_i$, $\Delta_i(\rho) = -\underline{\omega}_i\overline{\omega}_i Q_i + \bar{A}_i^*(\rho)W_i + W_i\bar{A}_i(\rho)$,

Proof: Applying Lemma 1 and Lemma 2, it can be obtained that condition (12) provides a sufficient condition for condition (8). Note that for system (3), when the switching signal $\sigma = i$, the transfer function from $d(t)$ to $e(t)$ is $G_{edi}(j\omega) = \bar{C}_i(\rho)(j\omega I - \bar{A}_i(\rho))^{-1}\bar{B}_i(\rho) + \bar{D}_i(\rho)$, then the performance (8) in Lemma 1 becomes

$$G_{edi}^*(j\omega)G_{edi}(j\omega) - \gamma^2 I < 0 \ \forall\ \omega \in [\underline{\omega}_i, \overline{\omega}_i] \tag{15}$$

provided that $\Pi = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$. It can be easily obtained that condition (15) is equivalent to

$$\sigma_{max}(G_{edi}(j\omega)) < \gamma, \ \forall\ \omega \in [\underline{\omega}_i, \overline{\omega}_i] \tag{16}$$

which is just (13).

Note that when choosing $R = \begin{bmatrix} 0 & I & 0 \end{bmatrix}$, after some matrix manipulation,condition (12) can be converted into

$$\begin{bmatrix} -Q_i & P_i + j\omega_c Q_i - W_i & 0 \\ \star & \Delta_i(\rho) & W_i\bar{B}_i^*(\rho) + \bar{C}_i^*(\rho)\bar{D}_i(\rho) \\ \star & \star & \bar{D}_i^*(\rho)\bar{D}_i(\rho) - \gamma_i^2 I \end{bmatrix} < 0 \tag{17}$$

$\Delta_i(\rho) = -\underline{\omega}_i\overline{\omega}_i Q_i + \bar{A}_i^*(\rho)W_i + W_i\bar{A}_i(\rho) + \bar{C}_i^*(\rho)\bar{C}_i(\rho)$, and condition (11) becomes

$$\begin{bmatrix} -Q_i & 0 \\ 0 & \bar{D}_i^*(\rho)\bar{D}_i(\rho) - \gamma_i^2 I \end{bmatrix} < 0 \tag{18}$$

which is readily satisfied if condition (17) holds, since it exists on the diagonal of (17).

Apply Schur Complements lemma, (17) is equivalent to (14), which means that the finite frequency performance index (13) is satisfied if (14) holds, this completes the proof. ■

Remark 2: Note that for inequality (14), ρ is in the compact set $\mathcal{P}_i$, the matrix variable P_i in inequality (14) may be different from each other for different ρ. Similarly, matrix variable Q_i may also be different from each other for different $\rho \in \mathcal{P}_i$.

The following Lemma 4 is essential for later developments.

Lemma 4: Consider system (3) with system matrices $\bar{A}_\sigma(\rho)$, $\bar{B}_\sigma(\rho)$,$\bar{C}_\sigma(\rho)$,$\bar{D}_\sigma(\rho)$, when $\sigma = i$, the following statements are equivalent:

i) There exist matrix variables W_i, $A_f(\rho)$, $B_f(\rho)$, $C_f(\rho)$, $D_f(\rho)$, and matrix variable $W_i = \begin{bmatrix} W_{11}^i & W_{12}^i \\ W_{12}^{i^T} & W_{22}^i \end{bmatrix}$ such that

$$\begin{bmatrix} -Q_i & P_i + j\omega_c Q_i - W_i & 0 & 0 \\ \star & \Delta_i(\rho) & W_i\bar{B}_i^*(\rho) & \bar{C}_i^*(\rho) \\ \star & \star & -\gamma_i^2 I & \bar{D}_i^*(\rho) \\ \star & \star & \star & -I \end{bmatrix} < 0 \tag{19}$$

hold, where $\Delta_i(\rho) = -\underline{\omega}_i\overline{\omega}_i Q_i + \bar{A}_i^*(\rho)W_i + W_i\bar{A}_i(\rho)$, $\bar{A}_i(\rho)$, $\bar{B}_i(\rho)$,$\bar{C}_i(\rho)$, $\bar{D}_i(\rho)$ are defined in (4).

ii) There exist matrix variables W_{ei}, $A_{fe}(\rho)$, $B_{fe}(\rho)$, $C_{fe}(\rho)$, $D_{fe}(\rho)$, matrix variables $W_{ei} = \begin{bmatrix} Y_i & -N_i \\ -N_i & N_i \end{bmatrix}$ such that

$$\begin{bmatrix} -Q_{ei} & P_{ei} + j\omega_c Q_{ei} - W_{ei} & 0 & 0 \\ \star & \Delta_{ei}(\rho) & W_{ei}\bar{B}_{ei}^*(\rho) & \bar{C}_{ei}^*(\rho) \\ \star & \star & -\gamma_i^2 I & \bar{D}_i^*(\rho) \\ \star & \star & \star & -I \end{bmatrix} < 0 \tag{20}$$

hold, where $\Delta_{ei}(\rho) = -\underline{\omega}_i\overline{\omega}_i Q_{ei} + \bar{A}_{ei}^*(\rho)W_{ei} + W_{ei}\bar{A}_{ei}(\rho)$, $\bar{A}_{ei}(\rho) = \begin{bmatrix} A(\rho) & 0 \\ B_{fe,i}C(\rho) & A_{fe,i} \end{bmatrix}$, $\bar{B}_{ei}(\rho) = \begin{bmatrix} B(\rho) \\ B_{fe,i}D(\rho) \end{bmatrix}$, and matrix $\bar{C}_{ei}(\rho) = \begin{bmatrix} L(\rho) & -C_{fe,i} \end{bmatrix}$

Proof: Define $W_i = \begin{bmatrix} W_{11}^i & W_{12}^i \\ W_{12}^{i^T} & W_{22}^i \end{bmatrix}$ with $W_{12}^i, W_{22}^i \in$

$\mathbb{R}^{n\times n}$ being nonsingular. Then we have

$$\begin{aligned} W_{ei} &= \begin{bmatrix} I & 0 \\ 0 & -W_{12}^iW_{22}^{i^{-1}} \end{bmatrix} W_i \begin{bmatrix} I & 0 \\ 0 & -W_{12}^iW_{22}^{i^{-1}} \end{bmatrix}^T \\ &= \begin{bmatrix} Y_i & -N_i \\ -N_i & N_i \end{bmatrix} \end{aligned} \tag{21}$$

with $Y_i = W_{11}^i$ and $N_i = W_{12}^iW_{22}^{i^{-1}}W_{12}^{i^T}$. Let

$$T = \begin{bmatrix} I & 0 \\ 0 & -W_{12}^iW_{22}^{i^{-1}} \end{bmatrix},\ \mathcal{T} = diag\{T, I\} \tag{22}$$

Then multiplying the left hand sides of (19) by full rank matrix $\mathcal{T}$ while multiplying the right hand side of (19) by $\mathcal{T}^T$ produce (20), where $W_{ei} = TW_iT^T$ and

$$\begin{aligned} \bar{A}_{e,i}(\rho) &= \begin{bmatrix} I & 0 \\ 0 & -W_{12}^iW_{22}^{i^{-1}} \end{bmatrix} \bar{A}_i(\rho) \begin{bmatrix} I & 0 \\ 0 & -W_{12}^iW_{22}^{i^{-1}} \end{bmatrix}^{-1} \\ &= \begin{bmatrix} A(\rho) & 0 \\ B_{fe,i}C(\rho) & A_{fe,i} \end{bmatrix}, \\ \bar{B}_{e,i}(\rho) &= \begin{bmatrix} B(\rho) \\ B_{fe,i}D(\rho) \end{bmatrix}, \bar{C}_{e,i}(\rho) = \begin{bmatrix} L(\rho) & -C_{fe,i} \end{bmatrix} \end{aligned}$$

with

$$\begin{aligned} A_{fe,i} &= W_{12}^{i^{-T}}W_{22}^iA_{f,i}W_{22}^{i^{-1}}W_{12}^{i^T}, \\ B_{fe,i} &= -W_{12}^{i^{-T}}W_{22}^iB_{f,i}, \\ C_{fe,i} &= -C_{f,i}W_{22}^{i^{-1}}W_{12}^{i^T}, \end{aligned}$$

this completes the proof. ∎

Remark 3: From Lemma 4, it can be concluded that inequalities in (19) are equivalent to (20) with the structure of matrix W_i in (19) being replaced by W_{ei} in (20), and filter parameters $(A_{f,\sigma}, B_{f,\sigma}, C_{f,\sigma}, D_{f,\sigma})$ in (3) are replaced by a new realization $(A_{fe,\sigma}, B_{fe,\sigma}, C_{fe,\sigma}, D_{f,\sigma})$.

Summarizing the above arguments, we have the following theorem.

B. Finite frequency H_∞ filtering with hysteresis switching

Summarizing the above arguments, we have the following theorem.

Theorem 1: Consider system (3) with the parameter set $\mathcal{P}$ and its overlapped covering$\{\mathcal{P}_i\}_{i\in Z_N}$, if there exist matrix variables $Y_i, N_i, \mathcal{A}_{f,i}, \mathcal{B}_{f,i}, i \in Z_N$ such that for any $\rho \in \mathcal{P}_i$, the following inequalities hold

$$\begin{bmatrix} -Q_1^i & -Q_2^i & \phi_1 & \phi_2 & 0 & 0 \\ \star & -Q_3^i & \phi_3 & \phi_4 & 0 & 0 \\ \star & \star & \phi_5 & \phi_6 & \phi_7 & L^*(\rho) - C^*(\rho)D_{f,i}^* \\ \star & \star & \star & \phi_8 & \phi_9 & -C_{f,i}^* \\ \star & \star & \star & \star & -\gamma_i^2 I & (D_{f,i}D(\rho))^* \\ \star & \star & \star & \star & \star & -I \end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix} -2Y_i & 2N_i & \phi_{10} & -\mathcal{A}_{f,i} + X_2^i(\rho) & Y_i & -N_i \\ * & -2N_i & \phi_{11} & \mathcal{A}_{f,i} + X_3^i(\rho) & -N_i & N_i \\ * & * & -X_1^i(\rho) & -X_2^i(\rho) & 0 & 0 \\ * & * & * & -X_3^i(\rho) & 0 & 0 \\ * & * & * & * & -X_1^i(\rho) & -X_2^i(\rho) \\ * & * & * & * & * & -X_3^i(\rho) \end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix} X_1^i(\rho) & X_2^i(\rho) \\ \star & X_3^i(\rho) \end{bmatrix} \geq \begin{bmatrix} X_1^j(\rho) & X_2^j(\rho) \\ \star & X_3^j(\rho) \end{bmatrix}, \text{for } \rho \in \mathcal{S}_{ij} \tag{25}$$

where

$$\begin{aligned} \phi_1 &= P_1^i - j\varpi_c Q_1^i - Y_i \\ \phi_2 &= P_2^i - j\varpi_c Q_2^i + N_i \\ \phi_3 &= P_2^{i^*} - j\varpi_c Q_2^{i^*} + N_i \\ \phi_4 &= P_3^i - j\varpi_c Q_3^i - N_i \\ \phi_5 &= -\underline{\omega}\overline{\varpi}Q_1^i + Y_iA(\rho) - \mathcal{B}_{f,i}C(\rho) + (Y_iA(\rho) - \mathcal{B}_{f,i}C(\rho))^* \\ \phi_6 &= -\underline{\omega}\overline{\varpi}Q_2^i - \mathcal{A}_{f,i} + (-N_iA(\rho) + \mathcal{B}_{fi}C(\rho))^* \\ \phi_7 &= Y_iB(\rho) - \mathcal{B}_{fi}D(\rho) \\ \phi_8 &= -\underline{\omega}\overline{\varpi}Q_3^i + \mathcal{A}_{f,i} + \mathcal{A}_{f,i}^* \\ \phi_9 &= -N_iB(\rho) + \mathcal{B}_{f,i}D(\rho) \\ \phi_{10} &= Y_iA(\rho) - \mathcal{B}_{f,i}C(\rho) + X_1^i(\rho) \\ \phi_{11} &= -N_iA(\rho) + \mathcal{B}_{f,i}C(\rho) + X_2^{i^*}(\rho) \end{aligned}$$

then the augmented system (3) is stable with switching filters (2) satisfying the specification

$$\sup \|G_{ed}(j\omega)\|_\infty < \gamma, \forall\omega \in [\underline{\omega}_i, \overline{\omega}_i], i \in Z_N \tag{26}$$

for each $i \in Z_N$, where $\gamma = \max\{\gamma_i\}_{i\in Z_N}$. The filter parameters $A_{fe,i}, B_{fe,i}$ can be obtained as $A_{fe,i} = N_i^{-1}\mathcal{A}_{f,i}$, $B_{fe,i} = N_i^{-1}\mathcal{B}_{f,i}$.

Proof: Applying Lemma 3, it can be obtained that performance index (26) is satisfied if inequality (14) holds for any $\rho \in \mathcal{P}_i, i \in Z_N$

Applying Lemma 4, it can be concluded that conditions (20) is equivalent to (14) with $W_{ei} = \begin{bmatrix} Y_i & -N_i \\ -N_i & N_i \end{bmatrix}$. After some matrix manipulation, condition (20) can readily be converted to (23). Thus, performance index (26) is satisfied if inequality (23) holds.

Next, we prove that the augmented system (3) is stable with the hysteresis switching from $\mathcal{P}_i$ to $\mathcal{P}_j$. Applying Theorem 3.1 of [22], we have the augmented system (3) is stable if the following inequality hold

$$\begin{bmatrix} -W_i - W_i^T & W_i^T\bar{A}(\rho) + X_i(\rho) & W_i^T \\ \star & -X_i(\rho) & 0 \\ \star & \star & -X_i(\rho) \end{bmatrix} < 0, i \in Z_N \tag{27}$$

which is equivalent to

$$\begin{bmatrix} \bar{A}^T(\rho)X_i(\rho) + X_i(\rho)\bar{A}(\rho) & 0 \\ \star & -X_i(\rho) \end{bmatrix} < 0, i \in Z_N \tag{28}$$

where $X_i(\rho)$ is the Lyapunov matrix function.

Similar to [20], assumed that the sequence of finite switching time over the interval $[0, \eta]$ is $t_0, t_1, \ldots, t_N$ with $t_0 = 0$, from the (1, 1) element of the inequality (28) and compactness of the subset $\mathcal{P}_i$, it can be deduced that there exists a scalar $\lambda > 0$ such that

$$\bar{A}^T(\rho)X_i(\rho) + X_i(\rho)\bar{A}(\rho) < -\lambda X_i(\rho)$$

for any $\rho \in \mathcal{P}_i$. Then at any $t \in [t_k, t_{k+1})$, we get

$$V_\sigma(\xi(t)) \leq e^{-\lambda(t-t_k)}V_\sigma(\xi(t_k))$$

With the switching logic (7), we have $V_\sigma(\xi(t_k)) \leq V_\sigma(\xi(t_k^-))$. Therefore,

$$\begin{aligned} V_\sigma(\xi(t)) &\leq e^{-\lambda(t-t_k)}V_\sigma(\xi(t_k^-)) \\ &\leq e^{-\lambda(t-t_k)}e^{-\lambda(t_k-t_{k-1})}V_\sigma(\xi(t_{k-1})) \\ &\vdots \\ &\leq e^{-\lambda t}V_\sigma(\xi(0)) \end{aligned}$$

The stability of system (3) is then guaranteed.

Moreover, assume that matrix variable W_i in (27) is symmetric, similar to that of Lemma 4, W_i can be chosen to be $\begin{bmatrix} Y_i & -N_i \\ -N_i & N_i \end{bmatrix}$ without introducing conservatism, with $W_i = \begin{bmatrix} Y_i & -N_i \\ -N_i & N_i \end{bmatrix}$, after some matrix manipulation, condition (27) can readily be converted into (24).

Since condition $X_{ei}(\rho) \geq X_{ej}(\rho)$ can be rewritten as (25), it can be concluded that if conditions (23)-(25) hold for any $\rho \in \mathcal{P}_i$ and $i \in Z_N$, then finite frequency performance index (26) for each frequency range $\mathcal{I}_i$ is then satisfied. ■

Remark 4: It should be pointed out that inequalities (23)-(25) are all linear matrix inequalities which can be solved through LMI toolbox of MATLAB.

The matrix variables that needed for determining the H_∞ filter with hysteresis switching can be obtained by solving the following optimization problem.

$$\begin{aligned} \min \quad & \gamma \\ s.t. \quad & (23)-(25) \end{aligned} \tag{29}$$

through which matrix variables $C_{fe,i}, \mathcal{A}_{fe,i}, \mathcal{B}_{fe,i}, N_i$ can be determined, then the filter parameters are obtained as $A_{fe,i} = N_i^{-1}\mathcal{A}_{fe,i}$, $B_{fe,i} = N_i^{-1}\mathcal{B}_{fe,i}$.

IV. Example

In this section, an example is given to illustrate the effectiveness of our filter design methods. The considered system of form (1) is described by

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -0.1996 & \rho \\ -1.8704 & -0.1457 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} d(t) \\ y(t) &= \begin{bmatrix} 10 & 0.2 \end{bmatrix} x(t) + 0.5d(t) \\ z(t) &= \begin{bmatrix} 0 & 0.5 \end{bmatrix} x(t) \end{aligned} \tag{30}$$

where ρ is a real parameter satisfying $0.0235 \leq \rho \leq 0.2235$, and the frequency of disturbance is affected by the parameter ρ which is assumed to be $d(t) = 0.01\sin(100\rho t)$.

For switching LPV filter design with hysteresis switching logic, the parameter space is divided into two overlapped subsets: $[0.0235, 0.1235], [0.0835, 0.2235]$, corresponding to these two parameter subsets, the frequency ranges of disturbance can be seen as $\omega \in [2, 14]$ and $\omega \in [9, 23]$, respectively. Solve the optimization problem (29), we obtain the optimal value for the performance index γ is found to be 0.5109, and the switched filters are

$$A_{f,1} = \begin{bmatrix} -21.1935 & -1.6199 \\ -24.0363 & -1.9051 \end{bmatrix}, B_{f,1} = \begin{bmatrix} 2.1444 \\ 2.1475 \end{bmatrix},$$
$$C_{f,1} = \begin{bmatrix} 86.8020 & 7.0309 \end{bmatrix}, D_{f,1} = 0.6542$$

$$A_{f,2} = \begin{bmatrix} -400.5493 & -29.4971 \\ -31.7959 & -2.3883 \end{bmatrix}, B_{f,2} = \begin{bmatrix} 40.0139 \\ 3.1079 \end{bmatrix},$$
$$C_{f,2} = \begin{bmatrix} 33.6099 & 2.7096 \end{bmatrix}, D_{f,2} = 0.3750$$

To see the simulation results, we assume that the parameter ρ varies with time as shown in Fig. 1, from which it can be concluded that using the switching from filter 1 to filter 2 occurs at $t = 10s$, and the switching from filter 2 to filter 1 occurs at $t = 25s$. Fig. 2 shows the system state, state estimation and state estimation error $e(t) = z(t) - \hat{z}(t)$, from which it can be seen that with the LPV switching filters designed in this paper, the filtering error is nearly zero in spite of the effect of noise and switching.

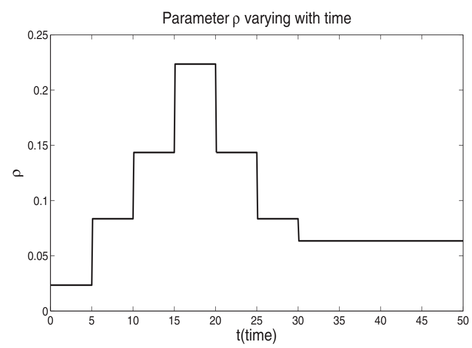

Fig. 1. Parameter ρ.

V. Conclusion

In this paper, we have investigated the problem of finite frequency H_∞ filtering for switching LPV systems with hysteresis switching logic. To estimate the states while attenuate the finite frequency disturbance effects, a family of filters are designed, each corresponds to a specific parameter subregion and a specific finite frequency disturbance attenuation performance index. The design problem is formulated into solving a set of linear matrix inequalities which can be computed by the LMI Control Toolbox. The simulation example has shown the advantage of filtering approach proposed in this paper.

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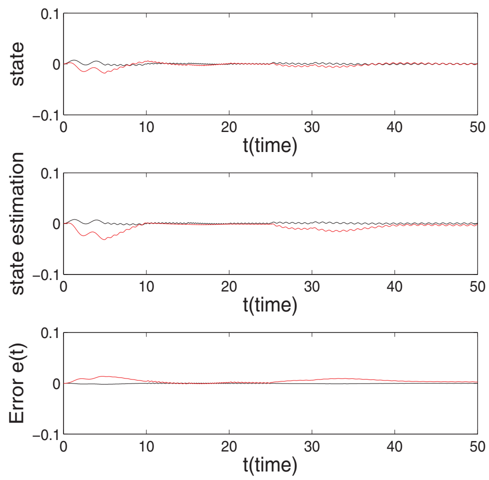

Fig. 2. Upper: state $x(t)$; Middle: estimation $\hat{x}(t)$; Lower: estimation error $e(t)$. The switching behavour occurs at $t = 10s$ and $t = 25s$.

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