THROUGHPUT OF A MULTIHOP PACKET CDMA NETWORK WITH POWER CONTROL

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Abstract — In this paper we have modeled a slotted packet CDMA network with fixed repeaters to analyze the system throughput. Six repeaters forming a hexagon have been placed in the standard cellular network to reduce the total power consumption and intercell interference. In each slot, a repeater is either in transmit or receive mode. The effect of different mode selection timing schedules on throughput has been modeled. Repeaters and base station selection regions are defined in a way to limit the maximum interference from terminals. In this model, throughput is calculated as a function of number of packets transmitted to base station and interference, which is due to the data transmissions to repeaters, as well as the background noise. Chernoff bound approach has been used as a general method of finding the upper limit for interference in a network partitioned in any shape. It is shown that for the same cell size and power consumption, the network with repeaters can dramatically increase the system capacity.

I. INTRODUCTION

In an ideal mobile communication system, the minimum amount of transmit power that can provide satisfactory service will always be used as an efficient method to overcome the radio path effects. It has been shown that this can be achieved by breaking down the distance between two communicating points into smaller segments [1], [2]. In [2], direct sequence spread spectrum was assumed and optimum transmission ranges in a multihop packet radio network was investigated. The paper is concerned with equal transmit power for all terminals. In [3], for the same structure, a comparison between different routing protocols and the effect of power control has been studied. In all these papers, expected progress per hop has been used as a performance measure. Although this measure is valid for comparing different routing schemes, but the exact performance of the system is of critical importance.

In this paper, we have located repeaters in the cellular structure to increase both the system capacity and efficiency of radio transmissions towards the boundaries of the cell. At these regions due to considerable losses in the channel, only relatively low data rates can normally be achieved. Layering the cell and covering each layer with a suitable number of repeaters can highly extend the coverage region and at the same time reduce the intercell interference both on downlink and uplink. We calculate the lower bound of the throughput in a cell with one layer of repeaters and compare the results with the case of not using any repeaters. The extension to multilayer cells is straightforward.

II. SYSTEM MODEL

A. Network Structure and Assumptions

The network assumed in this paper is a single cell with hexagon or circular boundary. A base station is located at the center of the cell and 6 repeaters, constructing a hexagon, are located at distance R_c from base station. Terminals are spatially distributed in the cell according to a two dimensional Poisson point process with the density per unit area, λ_t. Therefore, the probability of finding k terminals in a region R_a with area D_a is:

$$\Pr[k \ in \ R_a] = e^{-\lambda_t D_a} \frac{(\lambda_t D_a)^k}{k!} \qquad (1)$$

In the uplink, terminals are the only sources of data and base station is the final destination in the cell. We investigate throughput of this structure in the uplink.

B. Connectivity and uplink transmission scenario

Two frequency bands are assumed to carry the information on the uplink and downlink. Code division multiple access (CDMA) is used as the access technique. Terminals and repeaters are synchronized and transmit packets as slotted ALOHA. The slot duration is equal to the transmission time of a packet. In each frequency band, repeaters receive data in some time slots and retransmit them in the same frequency band but in different time slots. Therefore, packets undergo some delay due to the store-and-forward mechanism of repeaters. Each terminal sends and receives on one channel (spreading sequence) but repeaters may send and receive on several channels, dynamically allocated by base station. We assume that repeaters can always be assigned sufficient number of channels to send their data.

In each slot, when a repeater is in receive mode the related terminals independently transmit data to that repeater with probability p_t. We assume that transmission of a terminal is independent from slot to slot. The percentage of slots that repeaters are in transmit mode, is shown by p_r. These parameters are fixed and are not effected when a packet is lost. We assume that base station knows all the traffic information of repeaters and terminals and can fully control and schedule their transmissions.

C. Interference Model

The model we use here for channel interference is that of summing the interference powers and treating the total interference as Gaussian noise. We assume that the level of interference is constant over the transmission of a packet. The noise at a detector is due to constant background thermal noise with power spectral density $N_0/2$ and interference from all channels which terminals and repeaters transmit data to. If the received signal from the desired channel has power P_0 and the total interference from other channels is summed up to Y, then SNR (E_b/N_{0eff}) at the detector in the case of DS/BPSK with rectangular chip pulse is equal to [7]

$$\mu = \frac{E_b}{N_{0eff}} = (\frac{2Y}{3LP_0} + \frac{1}{\mu_0})^{-1} \qquad (2)$$

where L is the processing gain and $\mu_0 = E_b/N_0$ is the SNR at the detector in the absence of interferers.

The probability of packet success is dependent on the coding scheme. We denote the probability of packet success conditioned on the SNR μ as $p_s(\mu)$.

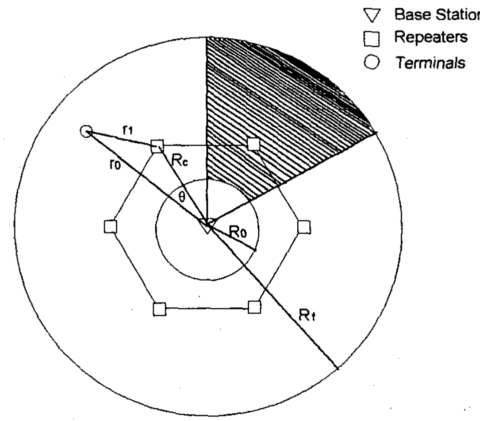

Figure 1 – Interference from terminals in layer 1.

Let $g(r)$ be the power of a given signal at a distance r from the transmitter. Usually the following class of propagation laws is considered for $g(r)$

$$g(r) = \frac{1}{r^\alpha} \qquad \alpha > 2.$$

D. Routing Scheme

In this part, we determine the repeaters and base station selection regions. In the uplink, we assume power is controlled in a way that at the detector, the received signal at base station and repeaters are P_0 and P_1, respectively. Since the structure has circular symmetry, we separate the selection region of neighboring repeaters as denoted in Figure 1.

Since traffic, interference and error rate are affected nonlinearly by selection region of repeaters and base station, it is difficult to find an analytical solution for defining the boundary between these regions. Different structures for defining the regions can be considered. In figure 1 we have used a circle with radius R_0 as the boundary between these two regions. In general, we denote the interior region including the base station as layer 0 and the exterior region as layer 1. By parameter definition according to figure 1 we can write interference at base station due to a terminal i at layer 1 as

$$I_{1i}(r_0, \theta) = (1 + \frac{R_c^2}{r_0^2} - \frac{2R_c \cos\theta}{r_0})^{\alpha/2}. \qquad (3)$$

Interference at repeaters due to terminals transmitting in layer 0 and layer 1 can be written as a function of (r_0, θ) in the same way.

III. Performance Analysis

Our goal is to find the throughput of the system, which is defined as the expected number of packets successfully received at base station in each slot. The method that we describe in this section is quite general and is applicable both to repeaters and base station.

A. Throughput

The number of successfully received packets at a receiver, N_s, is a binomial random variable with parameters p_s and N, the number of packets transmitted to that receiver. p_s is a function of random variables N and I, interference due to transmissions to other receivers. Then the system throughput, γ, is given by

$$\begin{aligned} \gamma &= E(N_s) = E(E(N_s|N,I) \qquad (4) \\ &= E(Np_s(N,I)) \\ &= \sum_{n=1}^{\infty} nE(p_s(n,I))\Pr(N=n) \end{aligned}$$

where the second expression is obtained by using the fact that N and I are independent.

For the best very long codes, the function p_s approaches a dirac delta at some value of μ, μ_c. Therefore, we can write p_s as

$$p_s(N,I) = \begin{cases} 1 & N + \frac{I}{P_x} < K(\mu_c) \\ 0 & N + \frac{I}{P_x} > K(\mu_c) \end{cases} \qquad (5)$$

where

$$K(\mu) = \frac{3L}{2}\left(\frac{1}{\mu} - \frac{1}{\mu_0}\right)$$

and P_x is the received fixed power on desired channels.

Substituting (5) into (4) we obtain the throughput as

$$\gamma = \sum_{n=1}^{\lfloor K(\mu_c) \rfloor} n\Pr(I/P_x < K(\mu_c) - n)\Pr(N=n) \qquad (6)$$

B. Data Rate

Since terminals transmit data independently of each other, the set of transmitting terminals also form a Poisson process with density per unit area $\lambda_t p_t$. Let S_0 and S_1 denote one sixth of the area of layer 0 and layer 1, respectively. Therefore, in each slot, we can write the number of packets transmitted from terminals to base station and to each repeater as Poisson random variables with parameters λ_0 and λ_1, respectively. λ_0 and λ_1 are defined as

$$\begin{aligned} \lambda_0 &= 6\lambda_t p_t S_0 \qquad (7) \\ \lambda_1 &= \lambda_t p_t S_1. \end{aligned}$$

Repeaters are in receive mode for $(1-p_r)/p_r$ slots until they retransmit data to base station in transmit mode. Therefore for the ideal case of perfect reception at repeaters, the number of retransmitted packets from each repeater will be a Poisson random variable with parameter

$$\lambda_2 = (1-p_r)\lambda_1/p_r. \qquad (8)$$

We assume that in general, this result is valid when we replace λ_1 with repeater throughput γ_R.

Now, if we denote the number of repeaters transmitting to base station in one slot by k_r, probability distribution for number of packets transmitted to base station is given by

$$\Pr(N=n|\ k_r \text{ repeaters transmit}) = e^{-\lambda_0 + k_r\lambda_2}\frac{(\lambda_0 + k_r\lambda_2)^n}{n!} \qquad (9)$$

For uniform traffic in each slot, $k_r = 6p_r$.

C. Interference

To evaluate equation (6), we need to obtain $\Pr\{I > t\}$ where t is an arbitrary threshold. For base station, interference is only due to terminals transmitting in the layer 1 and have Poisson distribution. For repeaters, interference is due to terminals in layer 0 and layer 1 and also from other repeaters. Distribution for interference from other repeaters can easily be obtained from the distribution of number of transmitted packets. For calculating the interference due to terminals, using the exact method as in [2] would sometimes be very difficult to calculate numerically. Therefore we use Chernoff bound to find the upper limit for $\Pr\{I > t\}$. By this method, the lower bound for throughput will be obtained.

By Chernoff bound we know

$$\Pr\{I > t\} = E\{u(I-t)\} \leq E\{\exp[\rho(I-t)]\} \quad \text{for all } \rho \geq 0 \quad (10)$$

If D_I denotes the region containing the interferer terminals and S_I denotes the area of this region, we can write

$$\Pr\{I > t\} = \sum_{k=0}^{\infty} e^{-\lambda_I} \frac{{\lambda_I}^k}{k!} \Pr\{I > t | k \text{ in } D_I\} \quad (11)$$

which "k in D_I" is the event that there are k transmitting terminals in region D_I and λ_I is the average number of transmitting terminals in this region. Let I_i be the received power from the ith interferer terminal. Now, given that there are k terminals in region D_I, and using the fact that the distribution of their locations is that of k independent and identically distributed points with uniform distribution, we can write

$$\begin{aligned}\Pr\{I > t | k \text{ in } S_I\} &\leq E\{e^{\rho(I-t)} | k \text{ in } S_I\} \quad (12)\\ &= e^{-\rho t}(E\{e^{\rho I_i}\})^k\end{aligned}$$

Substituting (12) in (11) and summing up the series results

$$\begin{aligned}\Pr\{I &> t\}\\ &\leq \min_{\rho>0}\{\exp(-\rho t + \lambda_I E\{e^{\rho I_i}\} - \lambda_I)\} \quad (13)\\ &= \exp(\min_{\rho>0}\{-\rho t + \lambda_I E\{e^{\rho I_i}\} - \lambda_I\}).\end{aligned}$$

The value $\rho_{\min}$ that minimizes the right side of equation (??), is the solution of

$$\lambda_I E\{I_i e^{\rho_{\min} I_i}\} = t \quad (14)$$

which has nonzero answer when $\lambda_I E\{I_i\} < t$.

Now by using (14), we find the lower bound for interference in equation (6)

$$\Pr(I < t) > \begin{cases} 0 & n > T \\ 1 - \exp(-\rho_{\min} t + \lambda_I E\{e^{\rho_{\min} I_i}\} - \lambda_I) & n < T \end{cases} \quad (15)$$

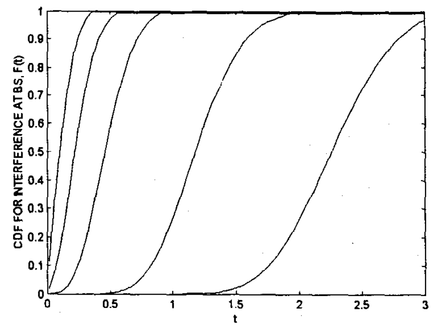

Figure 2 – Cumulative density function for interference at BS due to transmission of all terminals in layer 1. $\lambda_t = 5, R_0 = 2/3, R_1 = 2, p_t = .05, .1, .2, .5, 1$ which curves to right are related to higher p_t.

where

$$\begin{aligned} t &= P_0\,(K\,(\mu_c) - n) \\ T &= K(\mu_c) - \lambda_I E\{I_i\}/P_x. \end{aligned}$$

T shows the effect of interference on the maximum number of packets that can be transmitted to base station simultaneously.

IV. Numerical Results

For numerical calculations, we consider a circular cell with repeaters located at distance $R_c = 1$ from base station. We also consider the propagation law with $\alpha = 4$. In Figure 2 cumulative density function for interference at base station has been shown. For designing optimum selection region, different values of R_0 with different shape for boundaries need to be considered and the resulting interference both on repeaters and base station should be compared. In Figures 3, 4 and 5, throughput of each repeater, total system throughput and average of total transmitted power as a function of probability of transmission p_t have been shown. The effect of adding repeaters to the structure is a minor reduction in throughput while achieving a significant reduction in power consumption for the total system.

V. Conclusions

Multihop packet radio models published so far, have used the concept of expected progress to analyze sys-

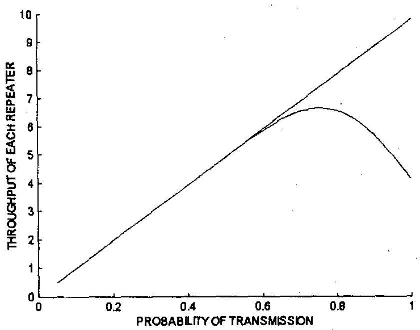

Figure 3 – Throughput of each repeater in comparison with the ideal case of $K_c = \infty$. System parameters are $K_c = 30, \lambda_t = 5, R_0 = 1/2, R_1 = 2$.

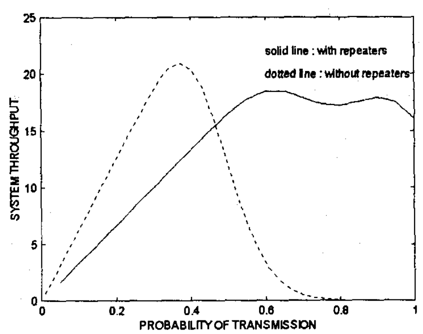

Figure 4 – Throughput of the new structure in comparison to the standard case of not using repeaters in the structure. System parameters are $K_c = 30, \lambda_t = 5, R_0 = 1/2, R_1 = 2$.

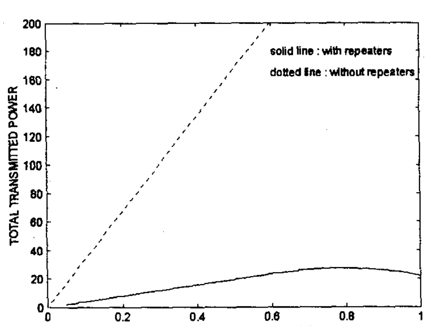

Figure 5 – Power consumption in the new structure in comparison to the standard case of not using repeaters in the structure. System parameters are $K_c = 30, \lambda_t = 5, R_0 = 1/2, R_1 = 2$.

tem performance. This method although is very useful and helps in finding a metric for comparing different structures and routing protocols, fails to show the real performance of the system. In this paper, we have modeled the simple case of multihop network in a cellular structure. Unlike previous works, the number of repeater's channels is not the fixed value one. Instead we have considered multichannel output for repeaters and have modeled the number of active channels for a repeater by a Poisson random process. The method described here can be applied to any multihop network structures when the repeaters are fixed. The results show that the simple case of including 6 repeaters in the standard structure can highly improve system throughput when system power consumption is kept the same.

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