Solutions to Homework Set No. 3

ECE 642

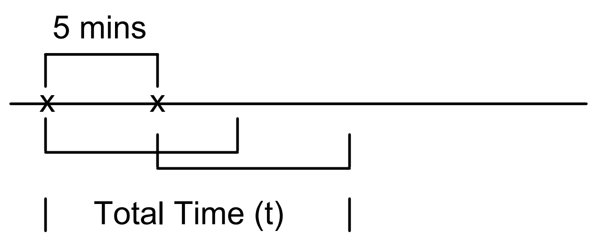
Problem 1 (3.1 B-G)

Solved in class
Average wait time for customer to receive their order is 5 minutes, then
E(T_1) = Average delay for carry out customers= 5 minutes
E(T_2) = Average delay for eat-in customers= 20+5=25 minutes
E(T) = Average delay for all customers regardless of type= 0.5 E(T1) + 0.5 E(T2)=15 minutes

Using Little's Formula N= λ*E(T)=(5 customers/min)(15 minutes)=75 customers

Problem 2 (3.5 B-G)

The timeline for the students is as follows:



In order to calculate the E[t] it is necessary to notice that the first student may not be in the professor's office more than 5 minutes. Therefore, the expected times must be weighted with the probabilities that he will be in less than or more than 5 minutes.

$$E[t] = (5 + E[2^{\text{nd}} \text{ Stud Time}]) \cdot \text{P}[1^{\text{st}} \text{ Stud Time} \le 5 \text{ mins}] +$$
$$\{E[1^{\text{st}} \text{ Student Time} \| 1^{\text{st}} \text{ Student Time} \ge 5 \text{ mins}] + E[2^{\text{nd}} \text{ Student Time}]\} \cdot P[1^{\text{st}} \text{ Student Time} > 5 \text{ mins}]$$

Noting that the exponential distribution is memoryless, the expected time left is still the mean of the exponential distribution.

$$E[2^{\text{nd}} \text{ Student Time}] = 30 \text{ mins}$$
$$P[1^{\text{st}} \text{ Student Time} \le 5 \text{ mins}] = 1 - e^{-5/30} = 1 - e^{-1/6}$$
$$P[1^{\text{st}} \text{ Student Time} > 5 \text{ mins}] = 1 - P[1^{\text{st}} \text{ Student Time} \le 5 \text{ mins}] = 1 - (1 - e^{-1/6}) = e^{-1/6}$$
$$E[1^{\text{st}} \text{ Student Time} \| 1^{\text{st}} \text{ Student Time} \ge 5 \text{ mins}] = E[1^{\text{st}} \text{ Student Time}] + 5 \text{ mins} = 35 \text{ mins}$$

$$E[t] = (5 \text{ mins} + 30 \text{ mins})\left[1 - e^{-1/6}\right] + (35 \text{ mins} + 30 \text{ mins})\left[e^{-1/6}\right] = 35 \text{ mins} + 30\, e^{-1/6}$$

$$\boxed{E[t] = 60.394 \text{ mins}}$$

Problem 3 (3.6 B-G)

This problems was done in class- Remember Alice, Bob and Charles

a) ¼, b) 5/4 Minutes, c) no change

Problem 4 (3.7 B-G)

We note that the waiting time, $0<W \leq T/2$. Half of the packets have system time of $T/2 + W$ and waiting time in queue of W. Then

Avg System time = $0.5 (T/2) + 0.5 (T/2 + W) = (T + W)/2$
Avg Waiting time = $W/2$

So the system time is between $T/2$ and $3T/2$.

Var of Waiting time = $0.5 (W/2)^2 + 0.5 (W/2)^2 = W^2/4$

So the variance of waiting time is between 0 and $T^2/16$.