ECE 642
Lecture 11 - Excerpts

Part I Open queueing networks

For an M queue VC, the state probability vector is as follows:

$$p(\underline{n}) = \prod_{i=1}^{M}(1-\rho_i)\rho_i^{n_i}$$

Given the service time at a the ith queue in the M-hop VC is $1/\mu_i$, the average end-to-end time delay is calculated as follows:

$$E(T) = \sum_{i=1}^{M}\frac{1}{\mu_i - \lambda_i} = \sum_{i=1}^{M}\frac{1/\mu_i}{1-\rho_i}$$

for ρ defined as:

$$\rho = \frac{\lambda_i}{\mu_i}$$

We obtain the network-wide delay by invoking Little's formula in the network:

$$E(n_i) = \lambda_i T_i = \lambda_i \frac{1}{\mu_i - \lambda_i}$$

$$E(n) = \sum_{i=1}^{M} E(n_i) = \sum_{i=1}^{M}\frac{\lambda_i}{\mu_i - \lambda_i}$$

$$E(T) = \frac{E(n)}{\gamma} = \frac{1}{\gamma}\sum_{i=1}^{M}\frac{\lambda_i}{\mu_i - \lambda_i}$$

Part II Closed queueing networks

The state probability vector is defined as:

$$\begin{aligned} p(\underline{n}) &= p(n_1, n_2 \ldots n_M) \\ &= \prod_{i=1}^{M}(\frac{\lambda_i}{\mu_i})^{n_i} p(\underline{0}) \end{aligned}$$

- n_i: number of packets in queue i
- $1/\mu_i$: average service time in queue i
- λ_i: flows over link (queue) i

Define $g(N, M) = \frac{1}{p(\underline{0})}$, the state probability vector is given by:

$$p(\underline{n}) = \left[\prod_{i=1}^{M} (\frac{\lambda_i}{\mu_i})^{n_i}\right] \frac{1}{g(N, M)}$$

Part III Buzen's algorithm

1. $g(n, m) = g(n, m-1) + \rho_m g(n-1, m)$ with $\rho_m = \frac{\lambda_m}{\mu_m}$ where $m = 1, 2, \ldots M$.
2. The initial condition is define as follows:

   $$g(n, 1) = \rho_1^n$$

   where $n = 0, 1, 2, \ldots N$, and

   $$g(0, m) = 1$$

   where $m = 1, 2 \ldots M$.

Let's consider a 3-hop VC as an example, assume:

$$\rho_1 = \rho_2 = \ldots = \rho_M = \frac{\lambda_1}{\mu} = 1$$

$$\rho_{M+1} = \frac{1}{\rho}$$

The g(n,m) matrix is shown as follows:

n/m	1	2	3	4
0	1	1	1	1
1	1	2	3	$(3 + \frac{1}{\rho})$
2	1	3	6	$6 + (3 + \frac{1}{\rho})$
3	1	4	10	$10 + (6 + (3 + \frac{1}{\rho}))$
4	1	5	15	$15 + (10 + (6 + (3 + \frac{1}{\rho})))$

When $\rho = 1$, redo the matrix above:

n/m	1	2	3	4
0	1	1	1	1
1	1	2	3	4
2	1	3	6	10
3	1	4	10	20
4	1	5	15	35

After obtaining this matrix by using Buzen's algorithm, the throughput and delay is calculated as follows:

$$P(n_i \geq k) = \rho_i^k \frac{g(N-k, M)}{g(N, M)}$$

$$\begin{aligned} P(n_i = k) &= P(n_i \geq k) - P(n_i \geq k+1) \\ &= \rho_i^k \left[\frac{g(N-k, M)}{g(N, M)}\right] - \rho_i^{k+1} \left[\frac{g(N-k-1, M)}{g(N, M)}\right] \\ &= \frac{\rho_i^k}{g(N, M)} [g(N-k, M) - \rho_i g(N-k-1, M)] \end{aligned}$$

For each VC, the expected number of packets is calculated as:

$$\begin{aligned} E(n_i) &= \sum_{k=0}^{N} k P(n_i = k) \\ &= \sum_{k=0}^{N} \frac{k \rho_i^k}{g(N, M)} [g(N-k, M) - \rho_i g(N-k-1, M)] \\ &= \sum_{k=1}^{N} \rho_i^k \left[\frac{g(N-k, M)}{g(N, M)}\right] \end{aligned}$$

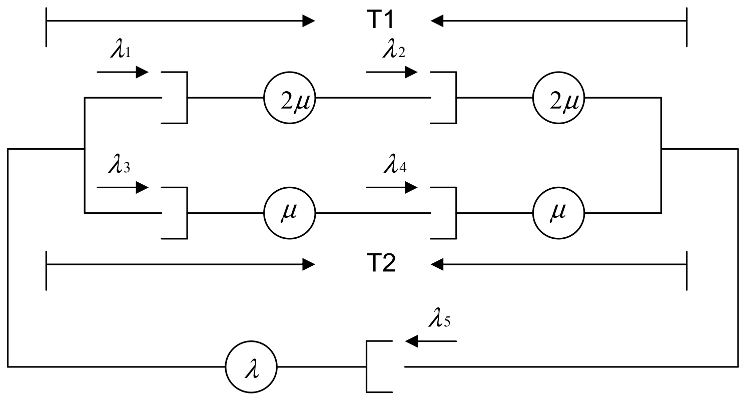
The throughput is then calculated:

$$\begin{aligned} \gamma_i &= \mu_i P(n_i \geq 1) \\ &= \mu_i \rho_i \left[\frac{g(N-1, M)}{g(N, M)}\right] \end{aligned}$$

The normalized throughput is:

$$\frac{\gamma_i}{\mu_i} = \rho_i \left[\frac{g(N-1, M)}{g(N, M)}\right]$$

Part IV Example



Given $\lambda_1 = \lambda_2$, $\lambda_3 = \lambda_4$, and $q_{51} = q_{53} = 1/2$, we know that:

$$\rho_1 = \rho_2 = \frac{\lambda_1}{2\mu}$$

$$\rho_3 = \rho_4 = \frac{\lambda_3}{\mu} = \frac{\lambda_1}{\mu}$$

If $\rho_1 = \rho_2 = \frac{\lambda_1}{2\mu} = 1$, then:

$$\rho_1 = \rho_2 = \frac{\lambda_1}{\mu} = 2$$

$$\begin{aligned} \rho_5 &= \frac{\lambda_5}{\lambda} = \frac{2\lambda_1}{\lambda} \\ &= \frac{2\lambda_1}{\mu}\frac{\mu}{\lambda} = \frac{4}{\rho} \end{aligned}$$

By using Buzen's algorithm, we can get the matrix like this:

n/m	1	2	3	4	5
0	1	1	1	1	1
1	1	2	4	6	$6 + 4/\rho$
2	1	3	11	23	$23 + (6 + 4/\rho)$
3	1	4	26	72	$72 + (23 + (6 + 4/\rho))$
4	1	5	57	201	$201 + (72 + (23 + (6 + 4/\rho)))$

The throughput is calculated like this:

$$\frac{\gamma_i}{\mu_i} = \rho_i \left[\frac{g(N-1, M)}{g(N, M)} \right]$$

The expected number of packets in system is calculated like this:

$$E(n_i) = \sum_{k=1}^{N} \rho_i^k \left[\frac{g(N-k, M)}{g(N, M))} \right]$$

The expected time delay is calculated like this:

$$E(T_1) = \frac{E(n_1) + E(n_2)}{\gamma_1}$$

$$E(T_2) = \frac{E(n_3) + E(n_4)}{\gamma_3}$$