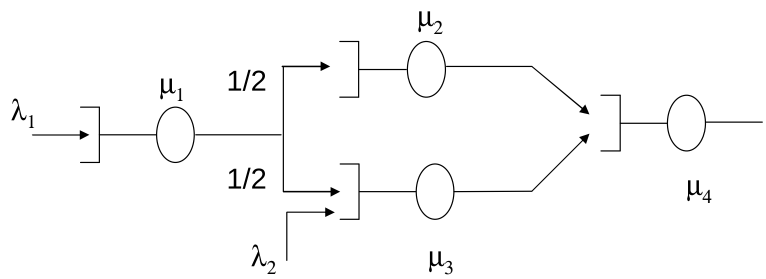
ECE 642 – Fall 2006
HW Set 11 Solutions

Problem 1



Let $N_1(t)$, $N_2(t)$ and $N_3(t)$ be the number of packets in queues 1, 2 and 3, respectively at time t. We have:

- $N_1(t) \propto N_2(t)$ are independent
- $N_1(t) \propto N_3(t)$ are independent

The input traffic to queue 2 is: $\lambda_1/2$; and the input traffic to queue 3 is: $\lambda_1/2 + \lambda_2$.

$$P\{N_1(t) = k, N_2(t) = m, N_3(t) = n\} = (1 - \rho_1)\rho_1^k * (1 - \rho_2)\rho_1^m * (1 - \rho_3)\rho_3^n$$

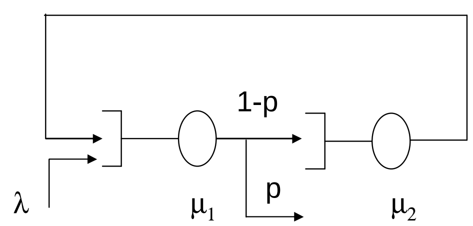
for $k, m, n > 0$.

$$\rho_1 = \lambda_1/\mu_1$$

$$\rho_2 = \frac{\lambda_1}{2\mu_2}$$

$$\rho_3 = \frac{\lambda_1/2 + \lambda_2}{\mu_3}$$

Problem 2



Let λ_1 and λ_2 be the input traffic to queues 1 and 2, respectively. We have the following relations:

$$\lambda_1 = \lambda_2 + \lambda$$

$$\lambda_2 = (1-p)\lambda_1$$

So we can get:

$$\lambda_1 = \frac{\lambda}{p}$$

$$\lambda_1 = \frac{(1-p)\lambda}{p}$$

The expected number of packets in the first queue is:

$$E(N_1) = \frac{\rho_1}{1-\rho_1}$$

The expected number of packets in the second queue is:

$$E(N_2) = \frac{\rho_2}{1-\rho_2}$$

where $\rho_1 = \lambda_1/\mu_1$ and $\rho_2 = \lambda_2/\mu_2$. So the average delay over the network is:

$$E(T) = \frac{E[N_1 + N_2]}{\lambda} = \frac{1}{\lambda}\left[\frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2}\right]$$