Typical Solution for Project No. 1 (Part II)

ECE 642
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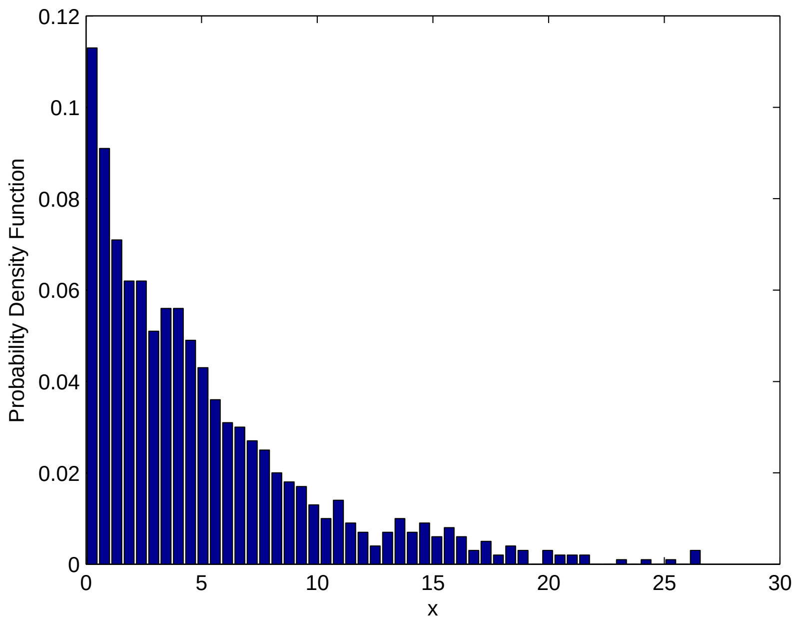
I. This part of the project is to generate exponential random variables from uniform random variables with mean 5.0

Procedure:

- Generate uniform random variables between 0 and 1.
- Let $u = F_x(u) = 1 - e^{-\mu x}$ where μ is the mean
- $\therefore 1 - u = e^{-\mu x}$ which implies $x = -\frac{1}{\mu} ln(1 - u)$
- If u is a random variable, then $1 - u$ is also a random variable. Hence, we can use $x = -\frac{1}{\mu} ln(\mu)$.

Here taking $\mu = 5.0$ we get

- $Mean = 5.2799$
- $Variance = 28.5540$



Theoretical values are:

- $Mean = 5.0$
- $Variance = 25.0$

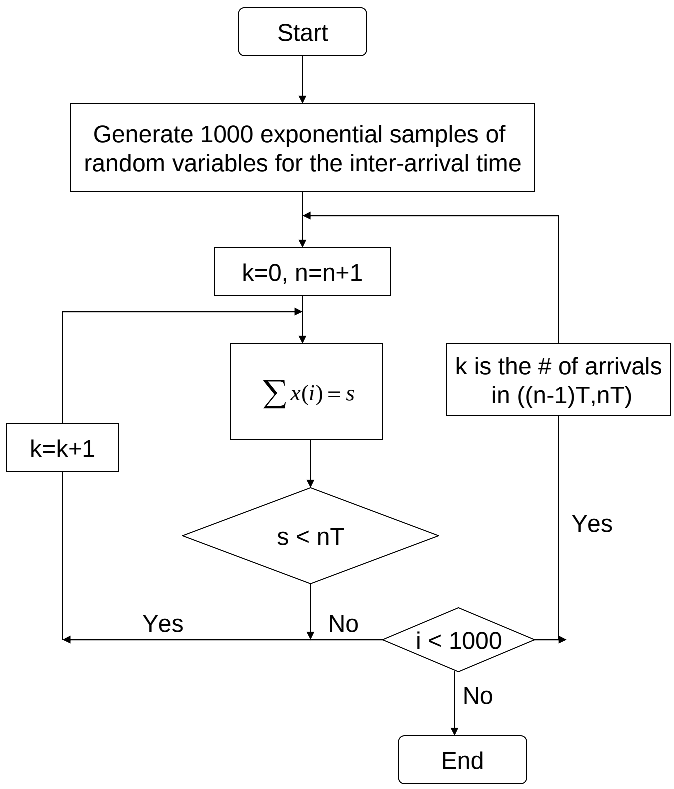
MATLAB CODE:

```
%This is to generate 1000 samples of uniform random variable and transform
%them to samples of the exponential random variable
u=rand(1,1000); %u represents the uniform random variable
x=-5.0*log(u); %x represents the exponential random variable with mean 5.0
m=mean(x) %Calculate mean of the r.v.
var=cov(x) %Calculate variance of the r.v.
[p,q]=hist(x,50); %pdf of the exponential random variable
bar(q,p/1000);
xlabel('x');
ylabel('Probability Density Function');
```

II. This part of the project finds the Poissons random variable for the number of packet arrivals and plot the probability mass function.

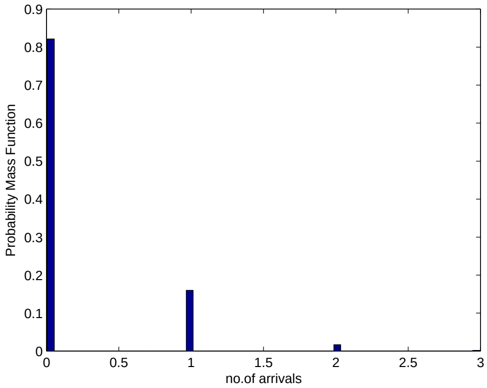
We know that if the inter-arrival times are exponential then the arrivals are Poisson. Therefore, in a given time interval the number of arrivals will give the Poisson distribution.

To understand the working of the program to calculate the Poissons random variable, we use the following flow chart:



- The following are the values obtained by running the Matlab code, and the time unit is 1 second.

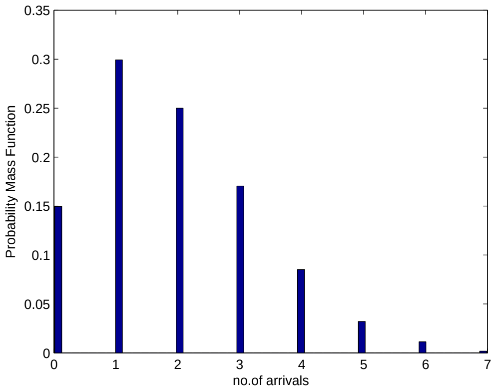
- Mean(For the exponential r.v.)= 4.9832
- Variance (for the exponential r.v.)= 25.7551
- Mean (for the Poissons r.v.)= 0.2086
- Variance (for the Poissons r.v.)= 0.2173



MATLAB CODE:

```
%Now we generate the exponential samples of random variable which represent
%the inter arrival time.
u=rand(1,1000); %u represents the uniform random variable
x=-5.0*log(u); %x represents the exponential random variable with mean 5.0
sum=0; k=zeros(1,10000);
for i=1:1000
    sum = sum+x(i);
    k(ceil(sum)) = k(ceil(sum))+1;
end
number = ceil(sum);
clear sum;
k=k(1:number);
z=sum(k);
m1=mean(k) %Calculating the mean & variance of the distribution%
var1=cov(k)
[p,q]=hist(k,50); %p:number of elements in each container; q: the position of
 %the bin center; function: bins the elements of k into 50 equally spaced
 %containers%
bar(q,p/number); %draw bar graph
xlabel('no.of arrivals');
ylabel('Probability Mass Function');
```

- The following are the values obtained by running the Matlab code, and the time unit is 10 second.
  - Mean(For the exponential r.v.)= 5.2799
  - Variance (for the exponential r.v.)= 28.5540
  - Mean (for the Poissons r.v.)= 1.8939



– Variance (for the Poissons r.v.)= 1.9204

MATLAB CODE:

```
%Now we generate the exponential samples of random variables which represent
%the inter arrival time.
u=rand(1,1000); %u represents the uniform random variable
x=-5.0*log(u); %x represents the exponential random variable with mean 5.0
sum=0; k=zeros(1,10000);
for i=1:1000
    sum = sum+x(i);
    k(ceil(sum/10)) = k(ceil(sum/10))+1;
end
number = ceil(sum/10);
clear sum;
k=k(1:number);
z=sum(k);
m1=mean(k) %Calculating the mean & variance of the distribution%
var1=cov(k)
[p,q]=hist(k,50); %p:number of elements in each container; q: the position of
 %the bin center; function: bins the elements of k into 50 equally spaced
 %containers%
bar(q,p/number); %draw bar graph
xlabel('no.of arrivals');
ylabel('Probability Mass Function');
```