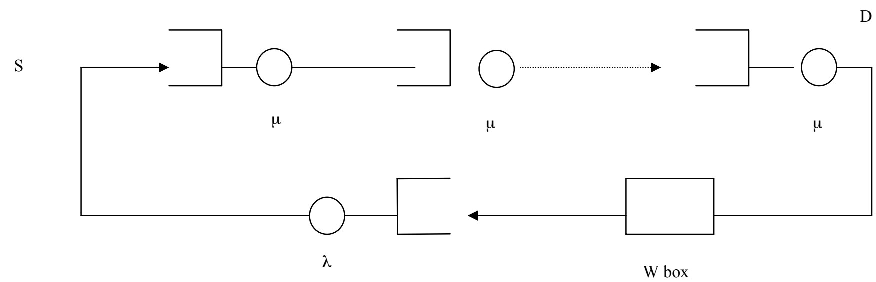
ECE 642

HW#8 Solutions

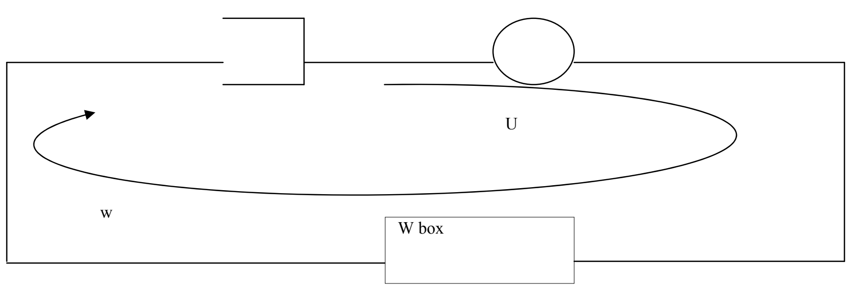
Problem 5.11

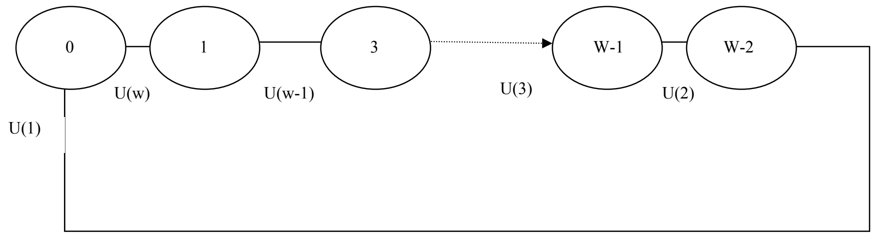


What happens in the window control with Acks, at the end is that the transmitter sends all the packets until the window is exhausted. The receiver does not acknowledge each packet but acknowledge at the end of the window. This process in turn reduces the transmission overhead but also reduces the throughput.This concept has been used on the VC's equivalent queue model as to aid congestion control. To demonstrate the window control with acknowledgement. At the end, we assume a sliding window size (w). The last packet in the window of (w) is acknowledged only. One keeps 'C' counter at the source, this counter decrements whenever a packet is sent to the VC. The counter is reset to (w) when the number of packets in the w box reaches ($w-1$). When the counter reaches '0', packets are blocked from entering the network. The Wbox in the model keeps ($w-1$) packets stored in it until the wth packet arrives. On the arrival of the wth packet it resets the counter at 'c' to (w), demonstrating the phenomena of ack at the end of the window.

Problem 5.12

For the case λ goes to infinity the C-count queue instantaneously empties its contents of w-packets into the closed VC as soon as it receives them from the w box. Thus we can drop the C-count queue from the model without effecting its performance and the model can be shown as following:





(a) The state diagram for the w-box model is as above. It is apparent from the state diagram that for the w-box, it has up to w-1 state after which it fires and enters state '0'. The balance equations are:

$$u(w)p_0 = u(1)p_{w-1}$$
$$u(w-1)p_1 = u(w)p_0$$
$$u(1)p_{w-1} = u(2)p_{w-2}$$

(b)

$$\begin{aligned}\frac{p_j}{p_0} &= \frac{p_1}{p_0}\frac{p_2}{p_1}\cdots\frac{p_j}{p_{j-1}} \\ &= \frac{u(w)}{u(w-1)}\frac{u(w-1)}{u(w-2)}\cdots\frac{u(w-j+1)}{u(w-j)} \\ &= \frac{u(w)}{u(w-j)}\end{aligned}$$

(c)

$$\frac{u(w)}{u(w-j)} = \frac{w[(w-j)+(M-1)]}{[w+(M-1)](w-j)}$$

Hence

$$\begin{aligned}\sum_{j=0}^{w-1} p_j &= \sum_{j=0}^{w-1} p_0 \frac{u(w)}{u(w-j)} \\ &= p_0\left[\frac{w}{w+(M-1)}(w+(M-1)T_w)\right] \\ &= 1\end{aligned}$$

Cohere

$$T_w = \sum_{j=1}^{w}\frac{1}{j} = \sum_{j=0}^{w-1}\frac{1}{w-j}$$

Thus

$$\begin{aligned}p_j &= \frac{w+(M-1)}{w[w+(M-1)T_w]}\frac{w[(w-j)+(M-1)]}{[w+(M-1)](w-j)} \\ &= \frac{(w-j)+(M-1)}{(w-j)[w+(M-1)T_w]}\end{aligned}$$

which is just equation (5-16).

(d) Throughput γ:

$$\begin{aligned}\gamma &= \sum_{n=0}^{w} u(n)p_n \\ &= \sum_{n=1}^{w}\left(\frac{n\mu}{n+M-1}\right)\left(\frac{n+M-1}{n(w+(M-1)T_w)}\right) \\ &= \frac{w\mu}{w+(M-1)T_w}\end{aligned}$$

The expected number of packets in system is calculated as follows:

$$\begin{aligned}E(n) &= \sum_{n=1}^{w} np_n \\ &= \sum_{n=1}^{w} n\left[\frac{n+w-1}{n[w+(M-1)T_w)]}\right] \\ &= \frac{\gamma}{\mu}\left[(M-1)+\left(\frac{1+w}{2}\right)\right]\end{aligned}$$

The normalized average delay is:

$$\mu E(T) = \frac{E(n)}{\gamma/\mu} = (M-1)+\frac{1+w}{2}$$

Sliding window control(ack each): $\mu E(T) = M-1+N$ (5-10) and $\gamma = \frac{N\mu}{M-1+N}$ (5-9), when $N = \frac{1+w}{2}$, both schemes have the same delay. From (5-18), $\frac{\gamma}{\mu} = \frac{1}{1+(M-1)T_w/w}$; From (5-9), $\frac{\gamma}{\mu} = \frac{1}{1+(M-1)/N}$. But $\frac{T_w}{w} > \frac{1}{N} = \frac{2}{w+1}$, thus sliding window control (ack each) has higher throughput.

Problem 5.13

For the case $\lambda = \mu$, the normalized throughput is as follows:

$$\frac{\gamma}{\mu} = \frac{w}{w+MT_w}$$

Also

$$\mu E(T) = \frac{E(n)}{\gamma/\mu} = \left[\left(M+\frac{w+1}{2}\right)\right]\left(\frac{M}{M+1}\right)$$

Then the power R can be calculated as:

$$R = \frac{\gamma/\mu}{\mu E(T)} = \frac{w}{(w+MT_w)} * \frac{2(M+1)}{(2M+1+w)M}$$

$$\therefore \quad \frac{dR}{dw} = Mw+2M^2T_w+MT_w-2w^2 = 0$$

Resulting

$$w = 2M-1$$

For the case $\lambda \to \infty$:

$$\frac{\gamma}{\mu} = \frac{w}{w+(M+1)T_w}$$

Also

$$uE(T) = \frac{E(n)}{\gamma/\mu} = (M-1)+\frac{w+1}{2}$$

Problem 5.20

$\mu = 3$ packets/sec

1. Average end-to-end delay from node 1 to 3 via node 2

$$\begin{aligned} E(T_{13})_2 &= \sum_{i=1}^{M=2} \frac{1}{\mu_i - \lambda_i} \\ &= \frac{1}{3 - \frac{1}{2}} + \frac{1}{3 - \frac{17}{12}} = 1.03\,sec \end{aligned}$$

2. Average end-to-end delay from node 1 to 3 via node 5 and 2

$$E(T_{13})_{5,2} = \frac{1}{3 - \frac{3}{2}} + \frac{1}{3 - \frac{7}{4}} + \frac{1}{3 - \frac{17}{12}} = 2.098\,sec$$

Problem 5.21

Net arrival rate into the system is $\gamma = 2 + 2 + 2 = 6$ packets/sec

(a) Given average time delay is like follows:

$$E(T) = \frac{1}{\gamma} \sum_{i=1}^{M} \frac{\lambda_i}{\mu_i - \lambda_i}$$

$\mu = 3$ packets/sec, $\gamma = 6$ packets/sec, and $M = 6$ links

$$E(T) = \frac{1}{6} \left[\frac{1/2}{3 - 1/2} + \frac{2/3}{3 - 2/3} + \frac{2(7/4)}{3 - 7/4} + \frac{17/12}{3 - 17/12} + \frac{17/6}{3 - 17/6} \right] = 3.53\,sec$$

(b) Load on link 1 is:

$$\lambda * \frac{3}{4} = 2 * \frac{3}{4} = \frac{3}{2}$$

Load on link 2 is:

$$\lambda * \frac{1}{4} = 2 * \frac{1}{4} = \frac{1}{2}$$

Load on link 5 is:

$$\frac{1}{2}[2 + \frac{3}{2} + \frac{7}{4}] = \frac{21}{8}$$

$$E(T) = \frac{1}{6} \left[\frac{3/2}{3 - 3/2} + \frac{1/2}{3 - 1/2} + \frac{2(7/4)}{3 - 7/4} + \frac{21/8}{3 - 21/8} + \frac{17/12}{3 - 17/12} \right] = 1.9825\,sec$$

Delay in case 2 is much less than case 1 with changing the probabilities.