

3.1

(a) Sample space:

	C, 0, $p=1/4$	C, 1, $p=1/2$	C, 2, $p=1/4$
M, 0, $p=1/4$	(0,0)	(0,1)	(0,2)
M, 1, $p=1/2$	(1,0)	(1,1)	(1,2)
M, 2, $p=1/4$	(2,0)	(2,1)	(2,2)

	C, 0	C, 1	C, 2
M, 0	1/16	1/8	1/16
M, 1	1/8	1/4	1/8
M, 2	1/16	1/8	1/16

(b) Mapping

	C, 0	C, 1	C, 2
M, 0	0	1	2
M, 1	1	1	2
M, 2	2	2	2

(c) Probabilities:

$$P(X=0)=1/16,$$

$$P(X=1)=1/8+1/8+1/4=1/2;$$

$$P(X=2)=1-1/16-1/2=7/16.$$

3.4

(a) $S = \{0000, 0001, \dots, 1110, 1111\}$, each element has an equal probability $1/16$;

(b) Mapping: $0000 \rightarrow 0$, $0001 \rightarrow 1$, \dots , $1110 \rightarrow 14$, $1111 \rightarrow 15$;

(c) $P(X=0)=P(X=1)=\dots=P(X=15)=1/16$;

(d) $P(Y=0)=\dots=P(Y=7)=1/32$; $P(Y=8)=\dots=P(Y=15)=3/32$;

3.7

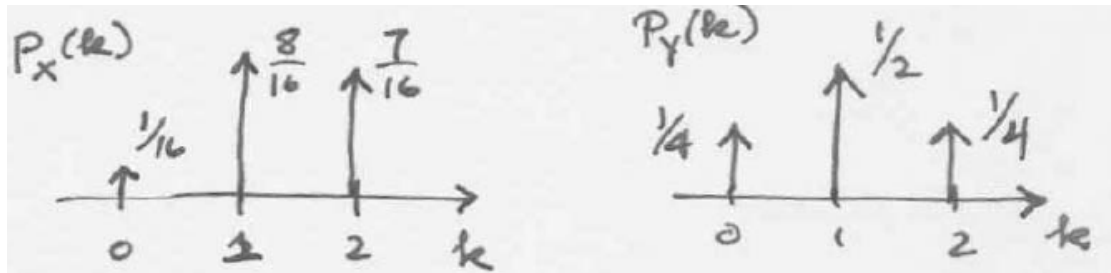
	B1	B1	B1	B1	B1	B1	B1	B1	B1	B50
B1	X	2	2	2	2	2	2	2	2	51
B1	2	X	2	2	2	2	2	2	2	51
B1	2	2	X	2	2	2	2	2	2	51
B1	2	2	2	X	2	2	2	2	2	51
B1	2	2	2	2	X	2	2	2	2	51
B1	2	2	2	2	2	X	2	2	2	51
B1	2	2	2	2	2	2	X	2	2	51
B1	2	2	2	2	2	2	2	X	2	51
B1	2	2	2	2	2	2	2	2	X	51
B50	51	51	51	51	51	51	51	51	51	X

$X=\{0,2\}$;

$P(X=2) = (9 \cdot 9 - 9) / (10 \cdot 10 - 10) = 4/5$; $P(X=51) = 1 - 4/5 = 1/5$.

3.11

(a)



(b)

	C, 0, $p=1/16$	C, 1, $p=3/8$	C, 2, $p=9/16$
M, 0, $p=1/4$	0	1	2
M, 1, $p=1/2$	1	1	2
M, 2, $p=1/4$	2	2	2

$$P(X=0) = 1/4 \cdot 1/16 = 1/64;$$

$$P(X=1) = 3/8 \cdot (1/4 + 1/2) + 1/16 \cdot 1/2 = 5/16;$$

$$P(X=2) = 1 - 1/64 - 5/16 = 43/64.$$

3.14

$$P(X \geq 8) = P(X=8) + \dots + P(X=15) = 1/2;$$

$$P(Y \geq 8) = P(Y=8) + \dots + P(Y=15) = 3/4.$$

3.23

$$(a) E[X] = \sum_{i=0}^{15} [P(X=i) \cdot i] = 15/2;$$

$$(b) E[X^2] = \sum_{i=0}^{15} [P(X=i) \cdot i^2] = 155/2;$$

Hence, $\text{Var}[X] = E[X^2] - E[X]^2 = 85/4$;

3.32

(a) $E[g(x)] = \sum_{i=1}^{15} P(X=i) = p_1 \sum_{i=1}^{15} 1/i = 0.1173$;

(b) $E[g(x)] = \sum_{i=1}^{15} P(X=i) = p_1 \sum_{i=1}^{15} (1/2)^{i-1} = 0.000946$;

(c) $E[g(x)] = \sum_{i=1}^{15} P(X=i) = p_1 \sum_{i=1}^{15} (1/2)^{i(i-1)/2}$;

3.35

(a) $P(X=1|X>0) = (1/2)/(1/2+7/16) = 8/15$; $P(X=2|X>0) = (7/16)/(1/2+7/16) = 7/15$;

(b) $P(X=1|M=1) = ((1/2)*(1/4)+(1/2)*(1/2))/(1/2) = 3/4$;

$P(X=2|M=1) = (1/2)*(1/4)/(1/2) = 1/4$.

(c)

	C00,1/4	C01,1/4	C01,1/4	C11,1/4
M00, 1/4				
M01, 1/4				
M10, 1/4	X=1, 1/16	X=1, 1/16	X=1, 1/16	X=2, 1/16
M11, 1/4	X=2, 1/16	X=2, 1/16	X=2, 1/16	X=2, 1/16

$P(X=1|M(1)=1) = 3/8$, $P(X=2|M(1)=1) = 5/8$,

$$(d) P(C=2|X=2) = 1 \cdot 9/16 / (1 \cdot 9/16 + 1/4 \cdot 1/16 + 1/4 \cdot 3/8) = 36/43$$

3.41

$$(a) P(X=2|1st\ draw=1) = 8/9, P(X=51|1st\ draw=1) = 1/9;$$

$$P(X=51|1st\ draw=50) = 1;$$

$$(b) E[X|1st\ draw=1] = 8/9 \cdot 2 + 1/9 \cdot 51 = 67/9;$$

$$E[X|1st\ draw=50] = 51;$$

$$(c) E[X] = E[X|1st\ draw=1] \cdot 9/10 + E[X|1st\ draw=50] \cdot 1/10 = 11.8 ;$$

$$(d) E[X^2] = (2^2 \cdot 8/9 + 51^2 \cdot (1/9)) \cdot 9/10 + 51^2 \cdot 1/10 = 523.4$$

$$\text{Var}[X] = E[X^2] - E[X]^2 = 384.16$$

3.50

$$(a) p_X(k+1) = C(n, k+1) p^{k+1} (1-p)^{n-k-1}$$

$$p_X(k) = C(n, k) p^k (1-p)^{n-k}$$

$$p_X(k+1)/p_X(k) = [n \cdots (n-k)] / [n \cdots (n-k+1)] \cdot k! / (k-1)! \cdot$$

$$p/(1-p) = (n-k)/(k+1) \cdot p/(1-p); \text{ It is easy to verify that the}$$

equation holds when $k = 0$;

$$(b) p_X(k+1)/p_X(k) = 1 + [(n+1)p-k]/[k(1-p)]. \text{ When}$$

$$0 \leq k < (n+1)p, \quad p_X(k+1)/p_X(k) < 1, \quad \text{otherwise}$$

$$p_X(k+1)/p_X(k) > 1. \text{ Therefore } k_{\max} = (n+1)p \text{ denotes}$$

the peak value. If $(n+1)p$ is an integer, $p_X(k+1) = p_X(k)$.