ECE 462 **Data and Computer Communications**
Solutions to Homework 9

Problem 1

(a)

The delay at each station to process the poll message T_p is computed as

$$T_p = \frac{poll\ message\ length}{C} = \frac{48}{512 \times 10^3} = 93.75\ \mu\text{sec}$$

Assume the propagation delay per km T_L = 5 μsec

Now, time required for the poll message to travel from first node to the last node in the system (τ) is calculated as

$$\tau = 2 \times 10 \times N \times T_L = 2 \times 10 \times 10 \times 5 \mu\text{sec} = 1\ \text{msec}$$

$$\tau' = \frac{\tau}{2} \times (N+1) = 5.5\ \text{msec}$$

L is the total time required to traverse all the nodes

$L = N \times T_p + N \times T_s + \tau' = 10 \times 93.75 \mu\text{sec} + 10 \times 0.1 m\text{sec} + 5.5 m\text{sec}$ = 7.4375 msec

Frame transmission time M is computed as

$$M = \frac{FrameLength}{C} = 1.953\ \text{msec}$$

ρ is the traffic arrival intensity

$\rho = N \times \lambda \times M = 10 \times 20 \times 1.953$ msec = 0.3906

The cycle duration T_c is calculated as shown below

$$T_c = \frac{L}{1-\rho} = 12.2046\ \text{msec}$$

(b)

$L = N \times T_s + \tau$ = 1.9 msec

$T_c = \dfrac{L}{1-\rho}$ = 3.118 msec

Problem 2

Assume the propagation delay per km T_L = 5 μsec

In this system, the total time required to traverse all the nodes L is computed as

$L = \tau + N \times T_S$

Note that in this scenario, we have a ring topology 'τ' is the time required to travel the entire length of the ring.

Assuming the synchronization time T_S = 0

L = τ = T_L = 5 μsec (Since ring length is 1 km)

$M = \dfrac{FrameLength}{C} = \dfrac{10^3}{4 \times 10^6}$ = 250 μsec

$\rho = N \times \lambda \times M$ = 0.10

$T_c = \dfrac{L}{1-\rho}$ = 5.55 μsec

Problem 3

(a)
The maximum throughput achievable in an ALOHA system = 0.18394
The capacity of the link = 10^5 frames/sec
The maximum throughput achievable = $10^5 \times 0.18394$ = 18394 frames/sec
The aggregate arrival rate = $100 \times \lambda$ = 18394
Where λ is the average input to the system from individual user.
Therefore λ = 183.94 frames/sec

(b)
The maximum un-normalized throughput for the network = 100×183.94 = 18394 frames/sec

(c)
Average number of retransmissions = $\dfrac{0.5 - 0.18394}{100 \times 10\mu\sec}$ = 316.1 frames/sec/station

Average number of retransmissions in the network = 316.06×100 = 31606 frames/sec

Problem 4

(a)
Probability (successful transmission of frame) = Probability (no packets when the frame arrives at the server)
Probability (“0” packets) = *p* = $e^{-\lambda \times \tau}$ = e^{-2} = 0.1353
Where λ is the arrival rate of frames
and τ is the frame transmission time

(b)
A geometric random variable can be used to indicate the successful transmission of frames in an ALOHA system.
If a packet has been transmitted successfully after “*k*” failures
Probability (successful transmission after “k” retransmissions) = $(1 - p)^k * p$
Where *p* = $e^{-\lambda \times \tau}$ = e^{-2} = 0.1353

(c)
If the number of attempts required to transmit the frame successfully are “*k*”; then
Probability (successful transmission after “k” retransmissions) = $(1 - p)^{k-1} * p$
Therefore the expected number of attempts = $\frac{1}{p}$ = 7.389

Problem 5

(a)
Probability of the system being idle for slotted aloha is given by $\frac{S}{G}$ = e^{-G}

The percentage of the system being idle = 10 % = e^{-G} = 0.1
Therefore *G* = ln(10) = 2.303

(b)
For *G* = 2.303
Throughput *S* = $G \times e^{-G}$
Therefore *S* = 0.2303

(c)
Yes. The channel is optimally loaded when *G* = 1. As *G* in the given scenario is 2.303, the system has been overloaded.

Problem 6

Please note that the problem should have stated "A small slotted ALOHA system has only *k* users, each of whom………."

Probability of success in each time slot is when one user transmits and the other *k-1* users are silent.

Probability (one user transmitting) = $\frac{1}{k}$

Probability (*k-1* users are not transmitting) = $\left(1-\frac{1}{k}\right)^{k-1}$

Therefore, the probability for successful transmission of any one user among the *k* users is given as

Probability (one successful transmission among k users) = $k \times \left(\frac{1}{k}\right) \times \left(1-\frac{1}{k}\right)^{k-1}$

Now, Throughput is determined as shown below
Throughput (S) = Probability (successful transmission for k users)

Therefore, Throughput = S = $k \times \left(\frac{1}{k}\right) \times \left(1-\frac{1}{k}\right)^{k-1}$

"S" for different values of *k* is given in the table below

***k* (# of users)**	**S (Throughput)**
2	0.5
3	0.44
4	0.422
5	0.410
10	0.387
∞	e^{-1}