Homework Set #7

Problems 5.7, 5.8, 5.9 & 5.10 from M. Schwartz, Chapter 5

1. Refer to the figure below
    a. Show that the probability of state of the upper queue for the case of the Norton equivalent model of the M-node virtual circuit with sliding window control of figure a) below is given by the following 2 equations: $\frac{p_n}{p_0} = \rho^n \binom{M-1+n}{n}$ ……..(a)
    $\frac{1}{p_0} = \sum_{n=0}^{N} \rho^n \binom{M-1+n}{n}$ ………………(b). Start with the state dependent service characteristics u(n).
    b. Using the above equations (a) & (b), show that the normalized average throughput of the sliding-window control with a window size of N is given by $\frac{\gamma}{\mu} = \sum_{n=0}^{N} u(n) p_n / \mu = \frac{B(N-1)}{B(N)}$,
    $$B(N) = [1 + M\rho + M\binom{M+!}{2}\rho^2 + ... + \frac{M(M+1)...(M+N-1)\rho^N}{N!}]$$
    c. Plot γ/μ as a function of ρ for M=3 and N=1,2,3,4. Compare and explain the results.
    d. Let ρ=γ/μ→∞. Show that $\frac{\gamma}{\mu} = \frac{N}{(N+M-1)}$. Check with the results of part c. show that this simple result for the limiting throughput of the sliding window control can be contained directly from the figure b) below without any calculation. Show that the average time delay across the virtual circuit in this case is given by $E(T) = \frac{N}{\gamma} = \frac{[M-1+N]}{\mu}$ as $\lambda \to \infty$
    e. Check the expression in b) by calculating $\lambda(1-P_B)$

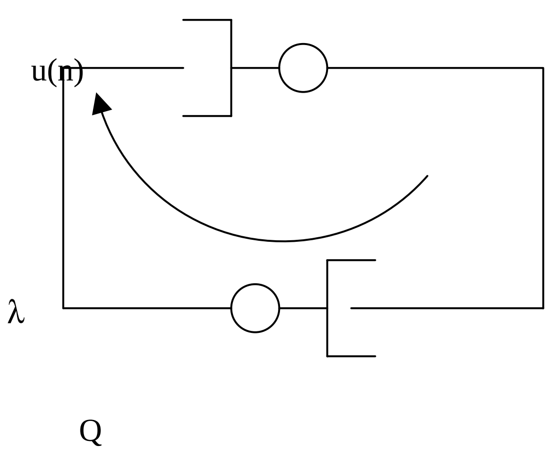

Figure 1a): The equivalent model using Norton's theorem

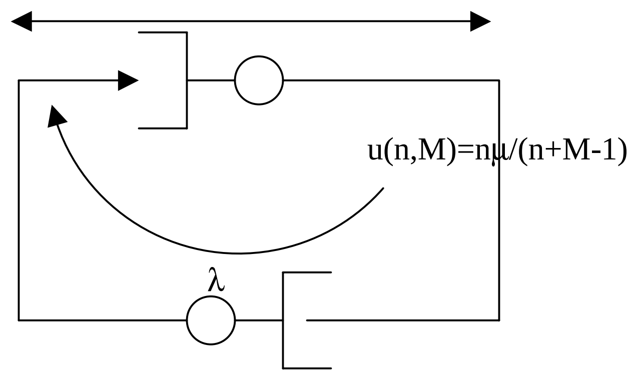

Figure 1b): Norton equivalent cyclic queue network

2. Consider the throughput –time delay performance equations as given below
   a. . $\gamma = u(N) = \dfrac{N\mu}{[N+(M-1)]}$
   b. $E(T) = \dfrac{N}{\gamma} = \dfrac{[M-1+N]}{\mu}$

   for the sliding window control in the case of λ→∞. As N increases both throughput and time delay increases. Show that the value of N that maximizes the ratio of throughput to time delay $\left(\dfrac{\gamma}{\mu}\right)/\mu E(T)$ is N=M-1. This measure of performance of the congestion control mechanism is called "power". Why would you want to maximize the ratio? Explain. Locate this point on the sliding-window performance curve. Does it seem an appropriate operating point? Explain.

3. Refer to the figure below for the sliding-window control. Take the case $\lambda = \mu$.
   a. Show that the throughput of the controlled virtual circuit and the time delay across it are given by the following equations
      i. $\gamma = \dfrac{N\mu}{(N+M)} \quad \lambda = \mu$
      ii. $\mu E(T) = \left[\dfrac{M}{M+1}\right](M+N) \quad \lambda = \mu$
   b. show that the window size N that maximizes the "power" defined as $\left(\dfrac{\gamma}{\mu}\right)/\mu E(T)$ is given by N=M. Compare with the results of the previous problem #2.

4. Plot $\mu E(T)$ versus $\dfrac{\gamma}{\mu}$ for the sliding window control for M=4 hops, for the two cases λ→∞ and λ=μ. Compare with the case for M=3 and λ→∞ (Graph drawn in class). Locate the N=M point on both the curves. This is the point that maximizes the "power" defined in the previous problem#3. Does this appear to be an operating point? Explain