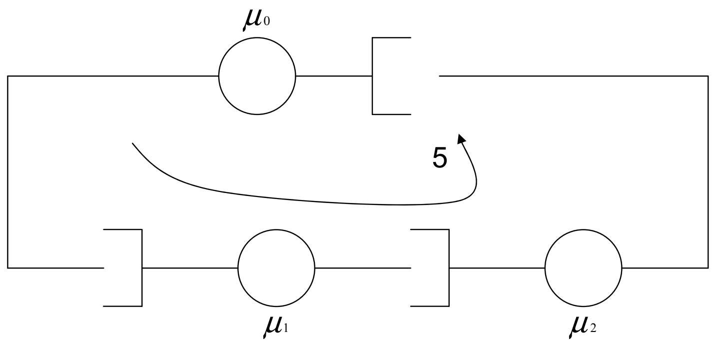
ECE 642
HW Set 10 Solutions

Problem 1



$\mu_1 = \mu_2 = 2$ and $\mu_0 = 1$. Calculate the percentage of time that bottom line is full of 5 packets?

- Method 1: Applying the Buzen's algorithm since the servers have different average service time. For $\rho_0 = 1$ and $\rho_1 = \rho_2 = \frac{1}{2}$, we can get Buzen's table like following:

n/m	1	2	3
0	1	1	1
1	1	3/2	2
2	1	7/4	11/4
3	1	15/8	26/8
4	1	31/16	57/16
5	1	63/32	120/32

By applying the formula below:

$$p(n_i = k) = \frac{\rho_i^k}{g(N,M)} \left[g(N-k,M) - \rho_i g(N-k-1,M)\right]$$

we can get the state probability for the queue with service rate μ_0:

$$p_0 = \frac{32}{120}\left(\frac{120}{32} - \frac{57}{16}\right) = 0.05$$

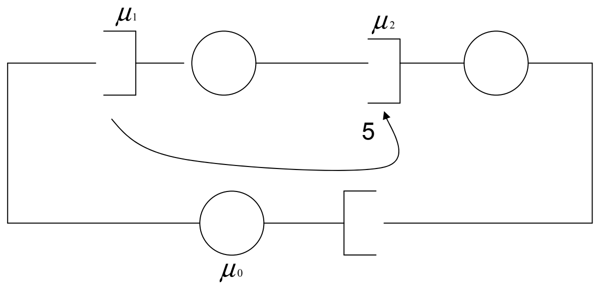
$$p_1 = \frac{32}{120}\left(\frac{57}{16} - \frac{26}{8}\right) = 0.0833$$

$$p_2 = \frac{32}{120}\left(\frac{26}{8} - \frac{11}{4}\right) = 0.133$$

$$p_3 = \frac{32}{120}\left(\frac{11}{4} - 2\right) = 0.2$$

$$p_4 = p_5 = \frac{32}{120}(2-1) = 0.266$$

- Method 2: Flip the graph and applying the Norton's algorithm.



By applying two formulas below:

$$\frac{1}{p_0} = \sum_{n=0}^{N} \rho^n \frac{(M-1+n)!}{(M-1)!n!}$$

$$\frac{p_n}{p_0} = \rho^n \frac{(M-1+n)!}{(M-1)!n!}$$

we can get the state probability for the upper line as following:

$$p_0 = 0.266$$

$$p_1 = 0.266$$

$$p_2 = 0.2$$

$$p_3 = 0.133$$

$$p_4 = 0.0833$$

$$p_5 = 0.05$$

Problem 2

According to the flipped graph above, we can apply the formulas for mean values:
For N=1:

1.
$$\mu\overline{t_i}(1) = 1 + \overline{n_i}(0) = 1$$
$$\Rightarrow \quad \overline{t_i}(1) = 1/\mu$$

2.
$$\gamma(1) = \frac{1}{\sum_{i=1}^{M} \overline{t_i}(1)} = \frac{\mu}{M}$$
$$\Rightarrow \quad \overline{t_i}(1) = \frac{1}{M}$$

3.
$$\overline{n_i}(1) = [\gamma(1)/\mu]\mu\overline{t_i}(1) = \frac{1}{M}$$

For N=2

1.

$$\mu\overline{t_i}(2) = 1 + \overline{n_i}(1) = 1 + \frac{1}{M} = \frac{M+1}{M}$$

2.

$$\gamma(2) = \frac{2}{\sum_{i=1}^{M} \overline{t_i}(2)} = \frac{2\mu}{M+1}$$

3.

$$\overline{n_i}(2) = [\gamma(2)/\mu]\mu\overline{t_i}(2) = \frac{2}{M}$$

So for N=N,

$$\overline{n_i}(N) = \frac{N}{M} = \frac{5}{2} = 2.5$$

$$\gamma(N) = \frac{N\mu}{M+N-1} = \frac{5 \cdot 2}{2+5-1} = 1.667$$