HW SET No. 6

1. Consider a statistical multiplexer (or a data concentrator) in which the input packets from terminals connected to it are merged in order of arrival in a buffer and are then read out first come—first served over an out going transmission link. An infinite buffer M/M/1 model is to be used to represent the concentrator.
   Find the mean delay E(T) and the average wait time E(W) in each of the following cases.
   a) Ten terminals are connected to the statistical multiplexer. Each generates, on the average, one 960-bit packet, assumed to be distributed exponentially, every 8 sec. A 2400-bits/sec outgoing line is used.
   b) Repeat if each terminal now generates a packet every 5 sec, on the average.
   c) Repeat a) above if 16 terminals are connected.
   d) 40 terminals are now connected & a 9600-bits/sec outline is used. Repeat a) & b) in this case. Now increase the average packet length to 1600 bits. What is the average buffer occupancy if a packet is generated every 8 sec at each terminal? What would happen if each terminal were allowed to increase it's packet generation rate to 1 per 5 sec, on the average? (Hint: It might now be appropriate to use a finite M/M/1 model with your own choice of buffer size.)

2. Refer to the multiple (or ample) server and queue with discouragement. Show that the state probability distribution and the average queue occupancy are given in by the following equations: $p_n / p_0 = (\lambda / \mu)^n / n!$ …………………(a)

   $p_0 = e^{-\rho}$ where $\rho = \lambda / \mu$ …………….(b)

   $E(n) = \sum_{n=0}^{\infty} n p_n = \rho = \lambda / \mu$ ………………..(c) respectively

   in both cases. However, the average time delay and throughput are different in the two cases. Calculate these two quantities in both cases and compare.

3. A queuing system has two outgoing lines, used randomly by packets requiring service. Each transmits at a rate of μ packets/sec. When both lines are transmitting (serving) packets are blocked from entering—i.e. there is no buffering in the system. Packets are exponentially distributed in length, arrivals are Poisson, with average λ.. $\rho = \lambda / \mu = 1$.
   i. Find the blocking probability P_B of this system.
   ii. Find the average number E(n) in the system.
   iii. Find the normalized throughput γ / μ, with γ the average throughput in packets/sec.
   iv. Find the average delay E(T) through the system, in units of $1/\mu$. Alternatively find E(T)/1/μ.)

4. Refer to the derivation of P-K formulas.
   i. Derive $E(n) = \frac{\rho}{2} + \frac{\sigma_v^{\ 2}}{2(1-\rho)}$, the general expression for the average number of customers in the queue.
   ii. For the case of Poisson arrivals, calculate $E(v)$ and $\sigma_v^{\ 2}$ and show that the following equations result.

$$E(v) = \lambda E(\tau) \equiv \rho$$

$$\sigma_v^{\ 2} = \rho + \lambda^2 \sigma^2$$

5. Two types of packets are transmitted over a data network. Type1, control packets, are all 48 bits long; type 2, data packets, are 960 bits long on the average. The transmission links all have a capacity of 9600bps. The data packets have a variance $\sigma_2^{\ 2} = 2(1/\mu_2)^2$, with $1/\mu_2$ as the average packet length in seconds. The type1 control packets constitute 20 percent of the total traffic. The overall traffic utilization over a transmission link is ρ=0.5.
   i. FIFO (non-priority service) is used. Show the average waiting time for either type of packet is $E(W) = 148 m\sec$.
   ii. Non pre-emptive priority is given to the control packets (type1). Show that the wait time of the data packets (type 2) is increased slightly to $E(W_2) = 149 m\sec$.