ECE 528: Introduction to Random Processes in ECE

Fall 2019

Solutions to Homework Set No. 1

Problem1 was solved in class

1.6 **a)** In the first draw the outcome can be black (b) or white (w). If the first draw is black, then the second outcome can be b or w. However if the first draw is white, then the run only contains black balls so the second outcome must be b. Therefore $\mathcal{S} = \{bb, bw, wb\}$.

b) In this case all outcomes can be b or w. Therefore $\mathcal{S} = \{bb, bw, wb, ww\}$.

c) In part a) the outcome ww cannot occur so $f_{ww} = 0$. In part b) let N be a larger number of repetitions of the experiment. The number of times the first outcome is w is approximately $N/3$ since the run has one white ball and two black balls. Of these $N/3$ outcomes approximately $1/2$ are also white in the second draw. Thus $N/9$ if the outcome result is ww, and thus $f_{ww} = \frac{1}{9}$.

d) In the first experiment, the outcome of the first draw affects the probability of the outcomes in the second draw. In the second experiment, the outcome of the first draw does not affect the probability of the outcomes in the second draw.

1.7 When the experiment is performed, either A occurs or it doesn't (i.e. B occurs); thus $N_A(n) + N_B(n) = n$ in n repetitions of the experiment, and

$$f_A(n) + f_B(n) = \frac{N_A(n)}{n} + \frac{N_B(n)}{n} = 1.$$

Thus $f_B(n) = 1 - f_A(n)$.

1.8 If A, B, or C occurs, then D occurs. Furthermore since A, B, or C cannot occur simultaneously, in n repetitions of the experiment we have

$$N_D(n) = N_A(n) + N_B(n) + N_C(n)$$

and dividing both sides by n

$$f_D(n) = f_A(n) + f_B(n) + f_C(n).$$

1.9

$$\begin{aligned}
< X >_n &= \frac{1}{n}\sum_{j=1}^{n} X(j) \qquad n > 0 \\
&= \frac{n-1}{n}\frac{1}{n-1}\left\{\sum_{j=1}^{n-1} X(j) + X(n)\right\} \\
&= \left(1 - \frac{1}{n}\right) < X >_{n-1} + \frac{1}{n}X(n) \\
&= < X >_{n-1} + \frac{X(n) - < X >_{n-1}}{n}
\end{aligned}$$