Homework Set No. 4

ECE 642
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1-4. Problems 2.8, 2.9, 2.10, 2.11 from M. Schwartz chapter 2 (see below)

5. Show that P_n and P_0 are the same for queues with discouragement and $M/M/\infty$

6-8. Following problems from Bertsekas-Gallager chapter 3: 3.8, 3.9, 3.10

Problem 1

Consider the $M/M/1$ queue analysis. Show that the stationary state probability p_n is given by : $p_n = \rho^n p_0$ where $\rho = \lambda/\mu$ in two ways.

1. Show that this solution for p_n satisfies the following equation governing the queue operation.
   $(\lambda + \mu)p_n = \lambda p_{n-1} + \mu p_{n+1} \; n \geq 1 \cdots$ (a)
2. Show that the balanced equation $\lambda p_n = \mu p_{n+1}$ or $p_{n+1} = \rho p_n$ satisfies the above equation (a). Then iterate n times.

Calculate p_0 for the finite $M/M/1$ queue and show that p_n is given by the following equation:
$p_n = (1-\rho)\rho^n/(1-\rho^{N+1})$

Problem 2

Show that the blocking probability P_B of the finite $M/M/1$ queue is given by $P_B = p_N$ by equating the net arrival rate $\lambda(1-P_B)$ to the average departure rate $\mu(1-p_0)$ and solving for P_B.

Problem 3

Consider a finite $M/M/1$ queue capable of accommodating N packets (customers). Calculate the values of N required for the following situations:

1. $\rho = 0.5 \; P_B = 10^{-3}, 10^{-6}$
2. $\rho = 0.8 \; P_B = 10^{-3}, 10^{-6}$

Compare the results obtained.

Problem 4

The probability p_n that an infinite $M/M/1$ queue is in state n is given by: $p_n = (1-\rho)\rho^n$, $\rho = \lambda/\mu$.

a Show that the average queue occupancy is given by $E(n) = \sum_n np_n = \rho(1-\rho)$

b Plot P_n as a function of n for $\rho = 0.8$

c Plot $E(N)$ versus ρ