Solutions to Homework 2

Problem 3.3

a.
$$\sin(2\pi ft - \pi) + \sin(2\pi ft + \pi) = 2\sin(2\pi ft - \pi)$$
 [or $2\sin(2\pi ft + \pi)$ or $-2\sin(2\pi ft)$]

b.
$$\sin(2\pi ft) + \sin(2\pi ft - \pi) = \sin(2\pi ft) - \sin(2\pi ft) = 0$$

Problem 3.4

| Ν | С | | D | | Е | | F | | G | | Α | | В | | С |
|---|------|----|------|----|-----|----|------|----|------|----|------|----|------|----|------|
| F | 264 | | 297 | | 330 | | 352 | | 396 | | 440 | | 495 | | 528 |
| D | | 33 | | 33 | | 22 | | 44 | | 44 | | 55 | | 33 | |
| W | 1.25 | | 1.11 | | 1 | | 0.93 | | 0.83 | | 0.75 | | 0.67 | | 0.63 |

Problem 3.12

Refer to Lecture notes. Retaining the vertical resolution of 483 lines, each horizontal line occupies 52.5 µsec. A horizontal resolution of H lines results in a maximum of H/2 cycles per line, thus the bandwidth of 5 MHZ allows:

$$5 \text{ MHZ} = (H/2) / 52.5 \,\mu\text{sec}$$

H = 525 lines

Now if we assume the same horizontal resolution of H = 450 lines, then for a bandwidth of 5 MHZ, the duration of one line is:

$$5MHZ = (450 / 2) / T$$

T = 45 µsec

Allowing 11 µsec for horizontal trace, each line occupies 56 µsec. The scanning frequency is:

In other words, number of vertical lines X 30 scans/sec gives you the number of lines/sec. So, when you have video line scan duration as 56 μ sec, you have 1/56 μ sec = 17857 lines/sec, or vertical resolution (number of lines) of 17857/30=595 lines.

Problem 3.13

- **a.** (30 pictures/s) (480 × 500 pixels/picture) = 7.2×10^6 pixels/s Each pixel can take on one of 32 values and can therefore be represented by 5 bits: R = 7.2×10^6 pixels/s × 5 bits/pixel = 36 Mbps
- **b.** We use the formula: $C = B \log_2 (1 + SNR)$ $B = 4.5 \times 10^6$ Hz = bandwidth, and SNR dB = 35 = 10 log₁₀ (SNR), hence SNR = $10^{3.5}/10 = 103.5$, and therefore

C =
$$4.5 \times 10^6 \log_2 (1 + 103.5) = 4.5 \times 10^6 \times \log_2 (3163)$$

C = $(4.5 \times 10^6 \times 11.63) = 52.335 \times 10^6 \text{ bps}$

c. Allow each pixel to have one of ten intensity levels and let each pixel be one of three colors (red, blue, and green) for a total of $10 \times 3 = 30$ levels for each pixel element.

Problem 3.14

Problem 3.15

Using Shannon's Formula:
$$C = B \log_2 (1 + SNR)$$

We have $W = 300 \text{ Hz}$
(SNR) $dB = 3$, therefore, $SNR = 10^{0.3}$
 $C = 300 \log_2 (1 + 10^{0.3}) = 300 \log_2 (2.995) = 474 \text{ bps}$

Problem 3.16

Using Nyquist's equation: C = 2B log₂M We have C = 9600 bps

a. $log_2M = 4$, because a signal element encodes a 4-bit word Therefore, $C = 9600 = 2B \times 4$, and B = 1200 Hz

b. $9600 = 2B \times 8$, and B = 600 Hz

Problem 3.17

$$N = 1.38 \times 10^{-23} \times (50 + 273) \times 10{,}000 = 4.5 \times 10^{-17} \text{ watts}$$

Problem 3.18

- **a.** Using Shannon's formula: $C = 3000 \log_2 (1+400000) = 56 \text{ Kbps}$
- **b.** Due to the fact there is a distortion level (as well as other potentially detrimental impacts to the rated capacity, the actual maximum will be somewhat degraded from the theoretical maximum. A discussion of these relevant impacts should be included and a qualitative value discussed.

Problem 3.22

a. Output waveform: $\sin{(2\pi f_1 t)} + 1/3 \sin{(2\pi (3f_1)t)} + 1/5 \sin{(2\pi (5f_1)t)} + 1/7 \sin{(2\pi (7f_1)t)}$ where $f_1 = 1/T = 1 \text{ kHz}$

Output power = 1/2 (1 + 1/9 + 1/25 + 1/49) = 0.586 watt

b. Output noise power = $8 \text{ kHz} \times 0.1 \mu \text{Watt/Hz} = 0.8 \text{ mWatt}$ SNR = 0.586/0.0008 = 732.5 (SNR) db = 28.65