Homework set #6

Problem 1- (2-14, Schwartz)

1) L = 960 bits

$\lambda_T = 10/8$

C = 2400 bps

$\mu = 2400/960 = 2.5$

$\rho = 0.5$

$E[n] = \frac{\rho}{1-\rho} = 1$

E[n]=λ E[T] (Little's formula)

E[T]= E[n]/λ =1/2.5=0.8 sec

E[w] =E[T] - 1/μ

=0.4 sec

2) λ =1/5

$\lambda_T = 10/5 = 2$

E[n]=0.8/(1-0.8)=4 ρ=2/2.5=0.8

E[T]=4/2 =2 sec

E[w]=2-1/2.5=1.6 sec

3) λ =1/8

$\lambda_T = 16/8 = 2$

E[n]=0.8/(1-0.8)=4 ρ=2/2.5=0.8

E[T]=4/2 =2 sec

E[w]=2-1/2.5=1.6 sec

4) C=9600 bits/sec

L=960 bits μ=10 packets /sec

a) λ =40/8=5 pkts/sec ρ=5/10=0.5

E[n]=0.5/(1-0.5)=1

$E[T]=1/5=0.2$ sec

$E[w]=0.2-1/10=0.1$ sec

b) $\lambda_{each}=1/5$

$\lambda_T=40/5=8$ pkts/sec

$\rho=8/10=0.8$

$E[n]=0.8/(1-0.8)=4$

$E[T]=4/8=0.5$ sec

$E[w]=0.5-1/10=0.4$ sec

c) L=1600 bits

$\mu=6$ packets /sec

$\lambda_T=40/8=5$ pkts/sec

$\rho=5/6=0.833$

$E[n]=0.833/(1-0.833)=5$

$E[T]=5/5=1$ sec

$E[w]=1-1/6=0.833$ sec

d) $\lambda_T = 40/5 = 8$ pkts/sec

$\mu = 6$ packets /sec

$\rho=8/6=1.33$

N=5

$$E[n] = \sum_{n=0}^{N} n \frac{(1-\rho)}{(1-\rho^{N+1})}\rho^n$$

N=5	E[n]=3.2928	E[T]=0.4116	E[w]=0.244
N=10	E[n]=7.4668	E[T]=0.9331	E[w]=0.766
N=100	E[n]=96.9697	E[T]=12.1212	E[w]=11.94

Problem 2- (2-18, Schwartz)

1) M/M/∞ queue $\lambda_n = \lambda$, $\mu_n = n\mu$, $\rho = \lambda/\mu$

$$P_n = \frac{\prod_{i=0}^{n-1} \lambda_i}{\prod_{i-1}^{n} \mu_i} P_0 = \frac{\rho^n}{n!} P_0$$

$$\sum_{n=0}^{\infty} P_n = 1$$

$$P_0 = e^{-\rho}$$

$$E[n] = \sum_{n=0}^{\infty} n P_n = \rho$$

$E[T]=E[n]/\gamma =1/\mu$

$$\gamma = \sum_{n=0}^{\infty} \lambda_n P_n = \sum_{n=0}^{\infty} \mu_n P_n = \lambda$$

2) Queuing with discouragement
$\lambda_n = \lambda/(n+1)$, $\mu_n = \mu$, $\rho = \lambda/\mu$

$$P_n = \frac{\prod_{i=0}^{n-1} \lambda_i}{\prod_{i-1}^{n} \mu_i} P_0 = \frac{\rho^n}{n!} P_0$$

$$P_0 = e^{-\rho}$$

$$E[T] = \frac{\rho}{\mu(1 - e^{-\rho})}$$

Problem 3- (2-21, Schwartz)

This is a M/M/2/2 queue

The Buffer length = 2 (for 2 packets in both servers with no waiting room)

1) We use Erlang-B formula with N = 2

$$P_B = \frac{\rho^N / N!}{\sum_{l=0}^{N} \rho^l / l!}$$

$$= \frac{1/2}{[1+1+0.5]} = 0.2$$

$$= \frac{(1)^N / (2!)}{\sum_{l=0}^{N} (1)^l / l!}$$

2)

$$E[n] = \sum_{n=0}^{\infty} nP_n = \rho(1-P_B)$$

= 0.8 packets

3) γ = average throughput

$$= \lambda(1 - P_B)$$

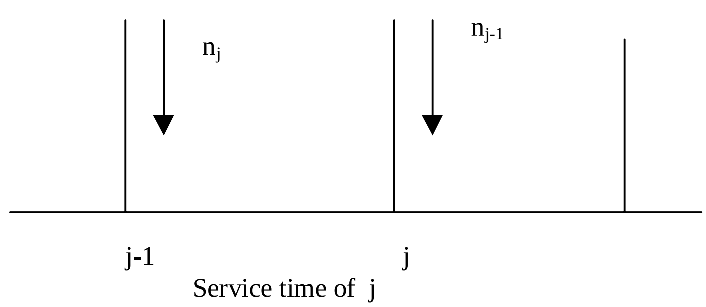
$$\therefore \frac{\gamma}{\mu} = E[n] = 0.8$$

4)E[T]=E[n]/λ=0.8

ρ=1

E[T]/μ = 0.8/(λ/μ)=0.8/ρ=0.8

Problem 4- (2-23, Schwartz)



ν_j = # of customers arriving during service interval ' j '

$n_j = n_{j-1} - 1 + \nu_j \qquad n_{j-1} > 1 , =1 \qquad (1)$

$\quad = \nu_J \qquad n_{j-1} = 0$

Define function:

$$u(x) = \begin{cases} 1 & x \geq 1 \\ 0 & x \lhd 1 \end{cases}$$

Now, (1) can be written as:

$n_j = n_{j-1} - u(n_{j-1}) + \nu_j \qquad (2)$

Assume $j \rightarrow \infty$ and take the expected value of both sides of equation (2):

$E[n_j]=E[n_{j-1}]-E[u(n)]+E[\nu_j] \Rightarrow E[\nu_j]=E[u(n)]$, since as $j \rightarrow \infty$ we have $E[n_j]=E[n_{j-1}]=E[n]$

$$E[u(n)] = \sum_{n=0}^{\infty} P_n = P(n > 0) = \rho$$

Squaring Equation (2) yields:

$n_j^2 = [n_{j-1} - U(n_{j-1}) + \nu_J]^2$

Simplifying as $j \rightarrow \infty$, and considering:

$E[u^2(n)]=E[u(n)]=\rho=E[\nu]$

$E[u(n)\, n]=E[n]$

we get:

$2E[n]=2E[n]\rho+\rho - 2\rho^2 - E[\nu^2] \qquad (3)$

Putting:

$\sigma_\nu^2=E[\nu^2]-E^2[\nu]$

$\quad =E[\nu^2]-\rho^2$

into equation (3):

$2E[n](1-\rho)=\rho - \rho^2+\sigma_\nu^2$

$\qquad =\rho(1-\rho)+\sigma_\nu^2$

Finally, this gives:

$$E[n] = \frac{\rho}{2} + \frac{\sigma_{\nu}{}^{2}}{2(1-\rho)}$$

Assume $f_\tau(\tau)$=General Service time pdf

$$E[\nu] = \sum_{k=0}^{\infty} KP(\nu = K)$$

$$\sigma^2{}_{\nu} = \sum_{k=0}^{\infty} (K - E[\nu])^2 P(\nu = K)$$

$$= \sum_{k=0}^{\infty} (K-\rho)^2 \int_0^{\infty} \frac{(\lambda\tau)^k}{k!} e^{-\lambda\tau} f_\tau(\tau) d\tau$$

$$= \int_0^{\infty} \sum_{k=0}^{\infty} (K-\rho)^2 \frac{(\lambda\tau)^k}{k!} e^{-\lambda\tau} f_\tau(\tau) d\tau$$

$$= \int_0^{\infty} f_\tau(\tau)[\sum_{k=0}^{\infty} (K^2 \frac{(\lambda\tau)^k}{k!} e^{-\lambda\tau} - \sum_{k=0}^{\infty} 2K\rho \frac{(\lambda\tau)^k}{k!} e^{-\lambda\tau} + \rho^2 \sum_{k=0}^{\infty} \frac{(\lambda\tau)^k}{k!} e^{-\lambda\tau}]d\tau$$

$$= \int_0^{\infty} f_\tau(\tau)[\rho^2 - 2\rho\lambda\tau + \tau + (\lambda\tau)^2]d\tau$$

$$= \lambda^2 \int_0^{\infty} f_\tau(\tau)[\tau - \rho/\lambda]^2 d\tau + \int_0^{\infty} \lambda E[\tau] d\tau$$

$$= \lambda^2 \sigma^2 + \rho$$

Which is Equation 2.76

Problem 5- (2-24, Schwartz)

Type 1 L_1 = 48 bits (fixed length control bits)

Type 2 L_2 = 960 bits ($1/\mu_2$ -average packet length)

$\sigma_2^2=2(1/\mu_2)^2$

C=9600 bps

$\rho=0.5$

a) For control packets:

L_1=48 bits

μ_1 =9600/48 =200 packets /sec

$E[\tau_1]=0.005$ sec $\lambda_1=0.2\lambda$

b) For data packets:

L_2=960 bits

μ_2 =9600/960 =10 packets /sec

$E[\tau_2]=0.1$ $\lambda_2=0.8\lambda$

$\rho=\lambda_1/\mu_1+\lambda_2/\mu_2=0.2\lambda*5*10^{-3}+0.8\lambda*0.1=0.081\lambda$

The traffic intensity ρ = 0.5

λ=0.5/0.081=6.17 packets /sec

$E[\tau^2]=\lambda_1/\lambda\, E[\tau_1^2]+\lambda_2/\lambda\, E[\tau_2^2]$ $E[\tau_1^2]=0$

$=0.8*3*(0.1)^2=0.024$ sec

$$E[w]=\frac{\lambda E[\tau^2]}{2(1-\rho)}$$

= <u>0.148 sec</u>

b)

$$E[T_0]=\frac{\lambda E[\tau^2]}{2}$$

= <u>0.074 sec</u>

$E[W_1]=E[T_0]/(1-\rho_1)$

= 0.074/(1-0.00617) = <u>74.5 msec</u>

$E[W_2]=E[T_0]/[(1-\rho_1)(1-(\rho_1+\rho_2))]$

= 0.074/(1-0.00617)(1- 0.4998) = <u>0.1489 sec</u>