George Mason University

Department of Electrical and Computer Engineering

ECE 528: Introduction to Random Processes in ECE — Fall Semester

Dr. Bijan Jabbari — Homework

Homework Set 7

Due One week from date assigned

1. Consider the random variable N taking values $n \geq 0$ with probabilities $p_1, p_2, p_3, ...$ and the indicator function $u(x) = 1, x \geq 1$ and 0 otherwise.

$$u(x) = \begin{cases} 1, & x \geq 1 \\ 0, & otherwise \end{cases}$$

   (a) Find $E[u(n)]$

   (b) Find $E[nu[n]]$

2. In the class, we derived the distribution of a linear function of a random variable when the scaling was positive. In other words, consider the random variable X which has Cumulative Distribution Function (CDF) $F_X(x)$, and suppose further that the random variable Y is a linear function of X, that is,

$$Y = aX + b$$

   where a and b are constants. We obtained the cdf $G_Y(y)$ and the pdf $g_Y(y)$ of Y in terms of $F_X(x)$ and $f_X(x)$ respectively when $a > 0$. Now do this for any general case of a (i.e., for both cases of $a > 0$ and $a < 0$) and derive a single formula for the pdf $g_Y(y)$.

3. Apply the above problem to the case where the random variable X has a standard negative exponential distribution, ($f_X(x) = e^{-x}$ for $x \geq 0$, $f_X(x) = 0$ for $x < 0$, $E[X] = \mu = 1$) and Y = aX + b with a and b positive constants. What is the CDF of Y? Can you think of what a might represents?

4. Apply the above problem to the case where X is a standard normally distributed random variable, and Y = aX + b. What is the CDF of Y? What do a and b represent?

5. Consider the random variable X which has a uniform distribution over the interval [0, 1], that is, the pdf of X, $f_X(x)$ is equal to 1 for $0 \leq x \leq 1$, and is 0 otherwise and let $Y = e^X$. In this case, Y is not a linear function of X, but the method used in problem 2 can be used to obtain the distribution function and probability density function of Y.

6. Consider a non-negative Random Variable, X with CDF of $F_X(x)$ and show that:

$$E[X] = \int_0^{+\infty} [1 - F_X(x)]dx$$