3.1

(a) Sample space:

	C, 0, p=1/4	C, 1, p=1/2	C, 2, p=1/4
M, 0, p=1/4	(0,0)	(0,1)	(0,2)
M, 1, p=1/2	(1,0)	(1,1)	(1,2)
M, 2, p=1/4	(2,0)	(2,1)	(2,2)

	C, 0	C, 1	C, 2
M, 0	1/16	1/8	1/16
M, 1	1/8	1/4	1/8
M, 2	1/16	1/8	1/16

(b) Mapping

	C, 0	C, 1	C, 2
M, 0	0	1	2
M, 1	1	1	2
M, 2	2	2	2

(c) Probabilities:

P(X=0)=1/16,

P(X=1)=1/8+1/8+1/4=1/2;

P(X=2)=1-1/16-1/2=7/16.

3.4

(a) S = {0000,0001,⋯,1110,1111}, each element has an equal probability 1/16;

(b) Mapping: 0000->0, 0001->1, ⋯, 1110->14, 1111->15;

(c) P(X=0)=P(X=1)=⋯=P(X=15)=1/16;

(d) P(Y=0)=⋯=P(Y=7)=1/32; P(Y=8)=⋯=P(Y=15)=3/32;

3.7

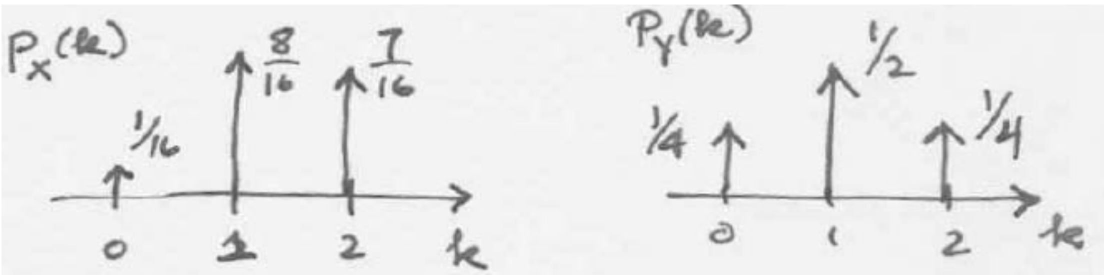
	B1	B1	B1	B1	B1	B1	B1	B1	B1	B50
B1	X	2	2	2	2	2	2	2	2	51
B1	2	X	2	2	2	2	2	2	2	51
B1	2	2	X	2	2	2	2	2	2	51
B1	2	2	2	X	2	2	2	2	2	51
B1	2	2	2	2	X	2	2	2	2	51
B1	2	2	2	2	2	X	2	2	2	51
B1	2	2	2	2	2	2	X	2	2	51
B1	2	2	2	2	2	2	2	X	2	51
B1	2	2	2	2	2	2	2	2	X	51
B50	51	51	51	51	51	51	51	51	51	X

X={0,2};

P(X=2) = (9*9-9)/(10*10-10) = 4/5; P(X=51) = 1-4/5 = 1/5.

3.11

(a)



(b)

	C, 0, p=1/16	C, 1, p=3/8	C, 2, p=9/16
M, 0, p=1/4	0	1	2
M, 1, p=1/2	1	1	2
M, 2, p=1/4	2	2	2

P(X=0) = 1/4*1/16=1/64;

P(X=1) = 3/8*(1/4+1/2)+1/16*1/2=5/16;

P(X=2) = 1-1/64-5/16=43/64.

3.14

P(X>=8) = P(X=8) + ··· + P(X=15) = 1/2;

P(Y>=8) = P(Y=8) + ··· + P(Y=15) = 3/4.

3.23

(a) E[X] = sum_{i=0}^15 [P(X=i)*i] = 15/2;

(b) E[X^2] = sum_{i=0}^15 [P(X=i)*i^2] = 155/2;

Hence, Var[X] = E[X^2] – E[X]^2 = 85/4;

3.32

(a) E[g(x)]=sum_{i=11}^15 P(X=i) = p1*sum_{i=11}^15 1/i = 0.1173;

(b) E[g(x)]=sum_{i=11}^15 P(X=i) = p1*sum_{i=11}^15 (1/2)^(i-1) = 0.000946;

(c) E[g(x)]=sum_{i=11}^15 P(X=i) = p1*sum_{i=11}^15 (1/2)^(i*(i-1)/2);

3.35

(a) P(X=1|X>0) = (1/2)/(1/2+7/16) = 8/15; P(X=2|X>0) = (7/16)/(1/2+7/16) = 7/15;

(b) P(X=1|M=1) = ((1/2)*(1/4)+(1/2)*(1/2))/(1/2) = 3/4;

P(X=2|M=1) = (1/2)*(1/4)/(1/2) = 1/4.

(c)

	C00,1/4	C01,1/4	C01,1/4	C11,1/4
M00, 1/4				
M01, 1/4				
M10, 1/4	X=1, 1/16	X=1, 1/16	X=1, 1/16	X=2, 1/16
M11, 1/4	X=2, 1/16	X=2, 1/16	X=2, 1/16	X=2, 1/16

P(X=1|M(1)=1) = 3/8, P(X=2|M(1)=1) = 5/8,

(d) P(C=2|X=2) = 1*9/16/(1*9/16+1/4*1/16+1/4*3/8)=36/43

3.41

(a) P(X=2|1st draw=1) = 8/9, P(X=51|1st draw=1) = 1/9;

P(X=51|1st draw=50) = 1;

(b)E[X|1st draw=1] = 8/9*2 + 1/9*51=67/9;

E[X|1st draw=50] = 51;

(c)E[X] = E[X|1st draw=1]*9/10+E[X|1st draw=50]*1/10=11.8；

(d)E[X^2]=(2^2*8/9 + 51^2*(1/9))*9/10 + 51^2*1/10=523.4

Var[X] = E[X^2] - E[X]^2 = 384.16

3.50

(a) p_X(k+1) = C(n,k+1) p^(k+1) (1-p)^(n-k-1)

p_X(k) = C(n,k) p^(k) (1-p)^(n-k)

p_X(k+1)/p_X(k) = [n*···*(n-k)] / [n*···*(n-k+1)] * k!/(k-1)! * p/(1-p) = (n-k)/(k+1) * p/(1-p); It is easy to verify that the equation holds when k = 0;

(b) p_X(k+1)/p_X(k) = 1 + [(n+1)p-k]/[k*(1-p)]. When 0<=k<(n+1)*p, p_X(k+1)/p_X(k) < 1, otherwise p_X(k+1)/p_X(k) > 1. Therefore k_{max} = (n+1)*p denotes the peak value. If (n+1)*p is an integer, p_X(k+1) = p_X(k).