ECE 528 – Introduction to Random Processes in ECE

Lecture 3: Conditional Probability & Independent Events

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Note

- These slides cover material partially presented in class. They are provided to help students to follow the textbook. The material here are partly taken from the book by A Leon-Garcia, Probability and Random Processes for Electrical Engineering, 3rd edition, whom I am thankful.
- There are many other topics which have been covered in class using the blackboard as step-by-set derivation and detailed discussions were need.

Additional Stuff Covered

- Chapter 2
- Problems 2.4 and 2.29 and 2.54
- The Balls and Boxes
- Tree diagram for Conditional probability
- BSC using conditional
- More on Bayes' Rule
- Application and use cases
- Covid-19 Testing and Bayes' Rule
- From Bernoulli to Binomial to Poisson
- Derivation of Poisson distribution

Conditional Probability

- Are events *A* & *B* interrelated?
- If we know that *B* occurred, how does probability of *A* change?

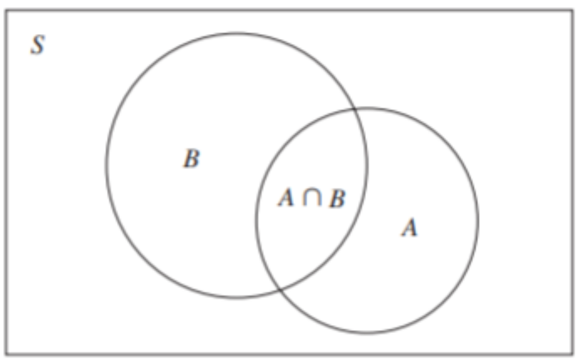


FIGURE 2.9
If *B* is known to have occurred, then *A* can occur only if $A \cap B$ occurs.

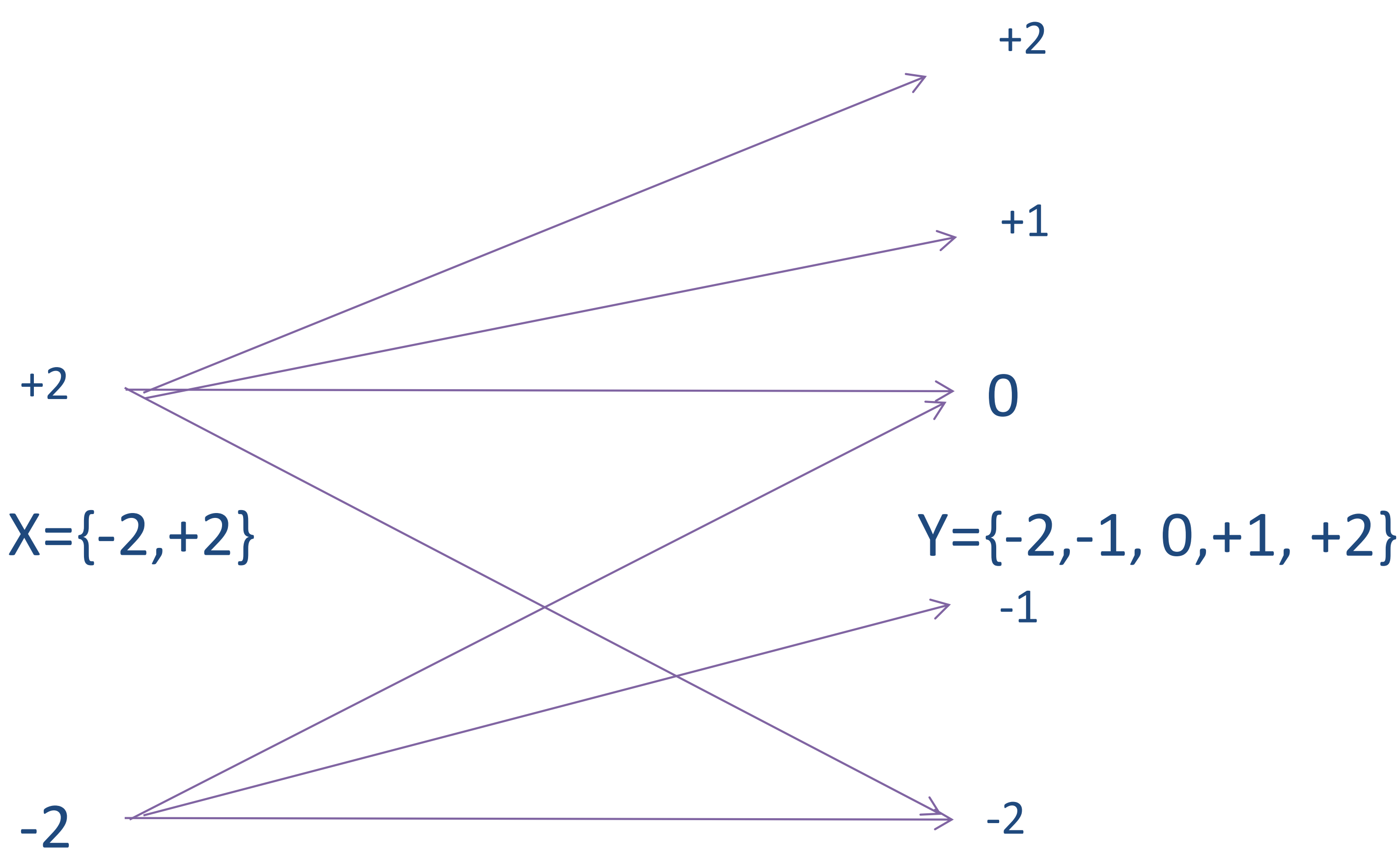
$$\frac{n_{A\cap B}}{n_B} = \frac{n_{A\cap B}/n}{n_B/n} \rightarrow \frac{P[A\cap B]}{P[B]}$$

$$P[A\,|\,B] = \frac{P[A\cap B]}{P[B]} \text{ for } P[B] > 0.$$

Problem 2.4

- A binary communication system transmits a signal X that is either a voltage signal +2 or a voltage signal -2. A malicious channel reduces the magnitude of the received signal by the number of heads it counts in two tosses of a coin. Let Y be the resulting signal.
- (a) Find the sample space.
- (b) Find the set of outcomes corresponding to the event "transmitted signal was definitely +2."
- (c) Describe in words the event corresponding to the outcome

Binary Communication System



- X={-2, +2} and Y ={-2, -1, 0, +1, +2}

Problem 2.4 Solution

- a)

X \ Y	-2	-1	0	1	2
+2	--	--	(2,0)	(2,1)	(2,2)
-2	(-2,-2)	(-2,-1)	(-2,0)	--	--

- b) "X definitely + 2" : {(2,1),(2,2)}

- c) {Y=0} = {(2,0),(-2,0)}

  Observed output is Zero. Cannot determine Input

Problem 2.29

- Let M be the number of message transmissions in Problem 2.7. Find the probabilities of the events A, B,C,C…… Assume the probability of successful transmission is 1/2.
- Problem 2.7: Let M be the number of message transmissions in Experiment E6. (a) What is the set A corresponding to the event “M is even”? (b) What is the set B corresponding to the event “M is a multiple of 3”? (c) What is the set C corresponding to the event “6 or fewer transmissions are required”? (d) Find the sets and describe the corresponding events in words

Problem 2.29 Solution

- Each Transmission is equivalent to tossing a fair coin. If outcome is heads, the transmission is successful. If tails, another transmission is required. Let's find the probability that j transmissions are required:

$$P[j] = \left(\frac{1}{2}\right)^j$$

$$P[A] = P[j \text{ even}] = \sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{2k} = \sum_{k=1}^{\infty}\left(\frac{1}{4}\right)^{k} = \sum_{k=0}^{\infty}\left(\frac{1}{4}\right)^{k} - 1 = \frac{1}{1-\frac{1}{4}} - 1 = \frac{1}{3}.$$

$$P[B] = P[j \text{ multiple of } 3] = \sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{3k} = \frac{1}{1-\frac{1}{8}} - 1 = \frac{1}{7}.$$

Problem 2.29 Solution

$$P[C] = \sum_{k=1}^{6}\left(\frac{1}{2}\right)^{k} = \frac{1}{2}\sum_{k=0}^{5}\left(\frac{1}{2}\right)^{k} = \frac{1}{2}\frac{1-(\frac{1}{2})^6}{1-\frac{1}{2}} = \frac{63}{64}.$$

$$P[C^c] = 1 - P[C] = \frac{1}{64}.$$

$$P[A \cap B] = \sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{6k} = \frac{1}{1-\frac{1}{64}} - 1 = \frac{1}{63}$$ since a multiple of 2 and 3 is a multiple of 6.

$$P[A - B] = P[A] - P[A \cap B] = \frac{1}{3} - \frac{1}{63} = \frac{20}{63}$$ since

$$A = (A-B) \cup (A \cap B)] \text{ and } (A-B) \cap (A \cap B) = \phi.$$

$$P[A \cap B \cap C] = \left(\frac{1}{2}\right)^{6} = \frac{1}{64}$$ since $A \cap B \cap C = \{6\}$.

Problem 2.54

- A lot of 100 items contains k defective items. M items are chosen at random and tested. (a) What is the probability that m are found defective? This is called the hypergeometric distribution. (b) A lot is accepted if 1 or fewer of the M items are defective. What is the probability that the lot is accepted?

Problem 2.54 Solution

2.54a The number of ways of choosing M out of 100 is $\binom{100}{M}$. This is the total number of equiprobable outcomes in the sample space.

We are interested in the outcomes in which m of the chosen items are defective and $M-m$ are nondefective.

The number of ways of choosing m defectives out of k is $\binom{k}{m}$.

The number of ways of choosing $M-m$ nondefectives out of 100 k is $\binom{100-k}{M-m}$.

The number of ways of choosing m defectives out of k *and* $M-m$ non-defectives out of $100-k$ is

$$\binom{k}{m}\binom{100-k}{M-m}$$

$$\begin{aligned}P[m \text{ defectives in } M \text{ samples}] &= \frac{\#\text{ outcomes with } k \text{ defective}}{\text{Total } \#\text{ of outcomes}} \\ &= \frac{\binom{k}{m}\binom{100-k}{M-m}}{\binom{100}{M}}\end{aligned}$$

This is called the Hypergeometric distribution.

(b) $P[\text{lot accepted}] = P[m=0 \text{ or } m=1] = \dfrac{\binom{100-k}{M}}{\binom{100}{M}} + \dfrac{k\binom{100-k}{M-1}}{\binom{100}{M}}.$

The number of ways of choosing m defectives out of k is $\binom{k}{m}$.

The number of ways of choosing $M-m$ nondefectives out of $100-k$ is $\binom{100-k}{M-m}$.

The number of ways of choosing m defectives out of k <u>*and*</u> $M-m$ non-defectives out of $100-k$ is

$$\binom{k}{m}\binom{100-k}{M-m}$$

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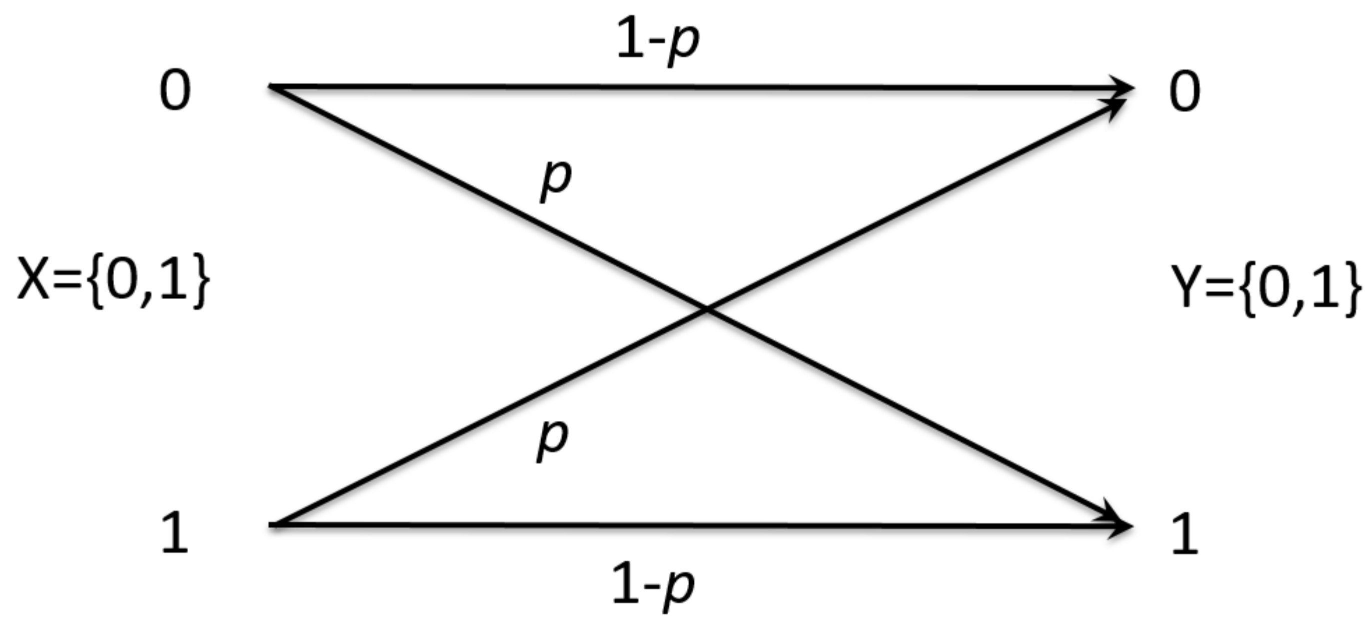
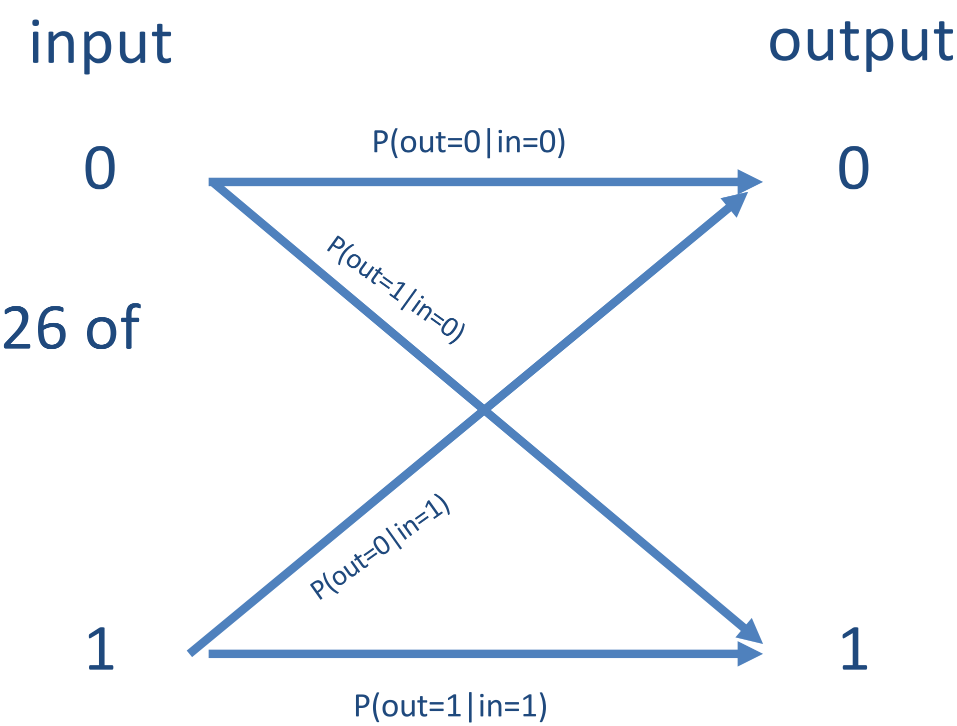
Binary Communication Channel



- Binary Symmetric Channel (BSC) model and noisy channel
- binary {0,1}
- symmetric means prob (0→1)= prob (1→0)

Example 1: Binary (Symmetric) Channel

- Given the binary symmetric channel depicted in figure, find P(input = j | output = i); i,j = 0,1. Given that P(input = 0) = 0.4, P(input = 1) = 0.6.

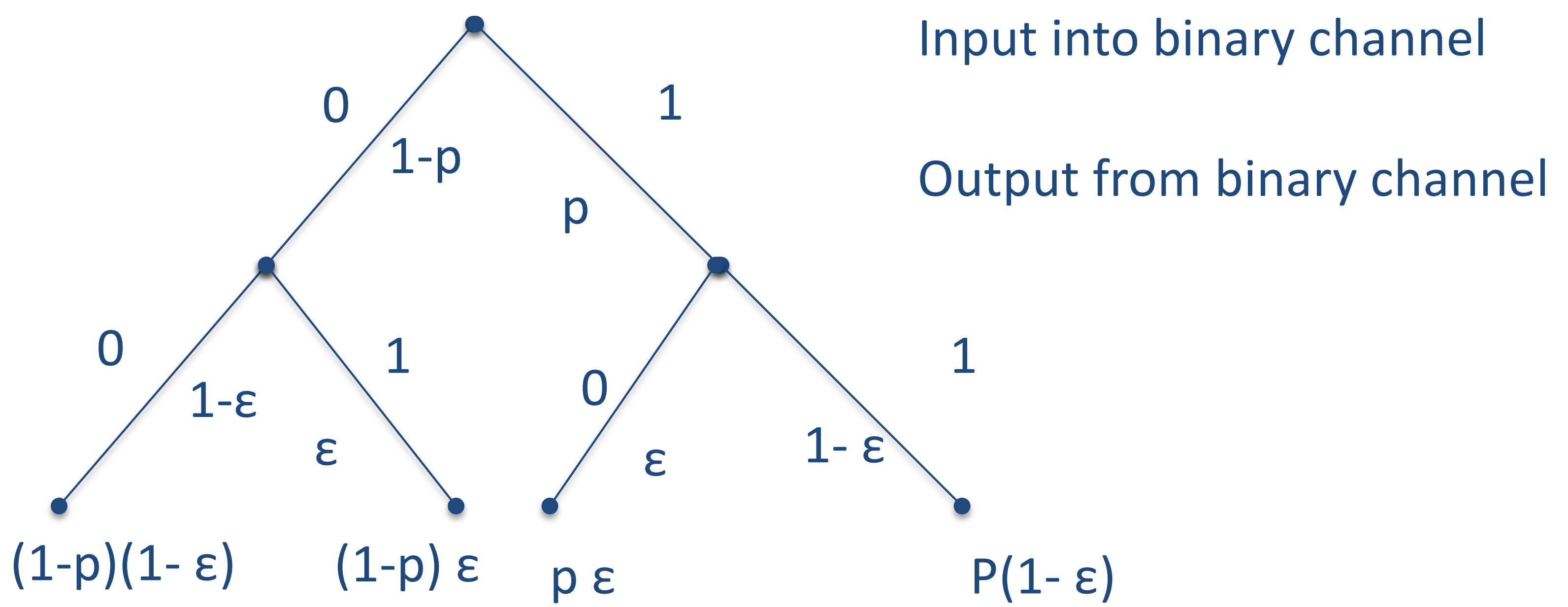


Solution:

Refer to examples 2.23 and 2.26 of Garcia's textbook

Binary Tree Diagram

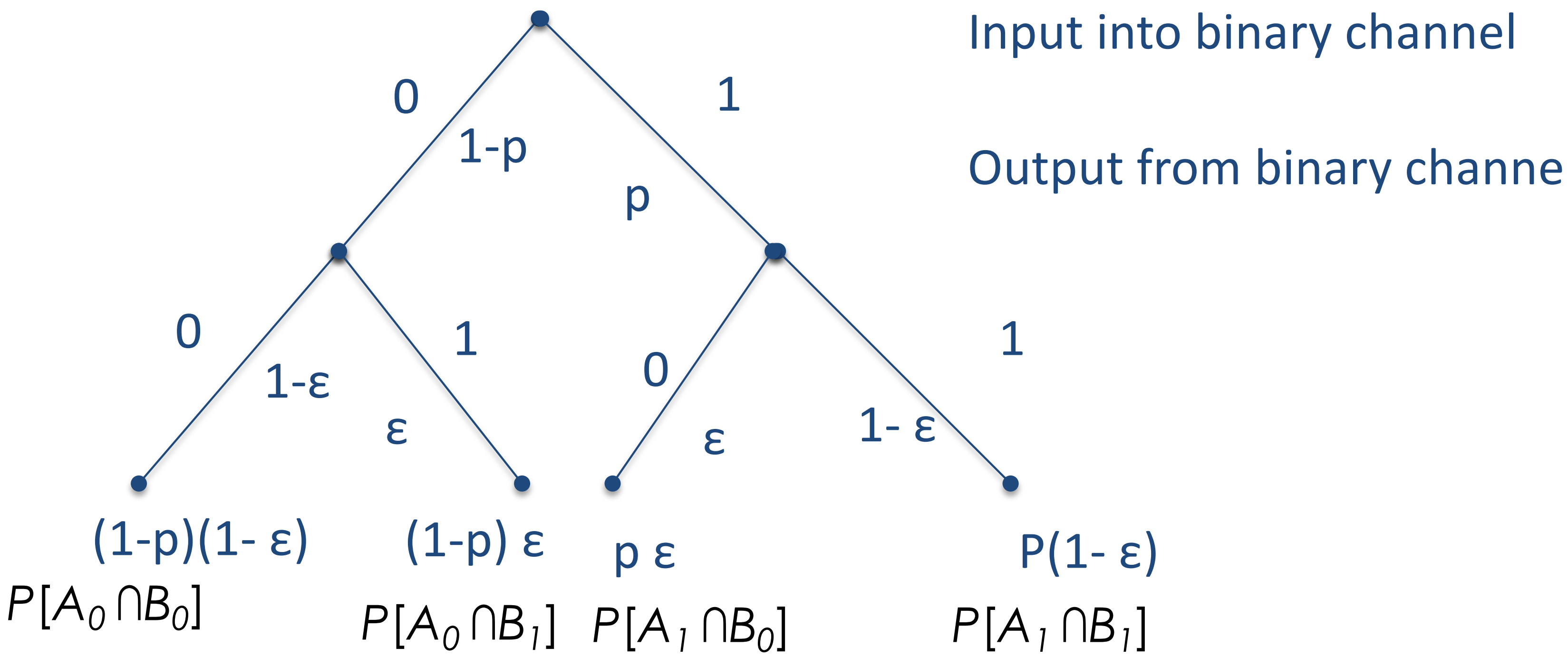
Let A_i be the event "input was i," and let B_j be the event "receiver decision was i."
Find the probabilities for $P[A_i \cap B_j]$ **i=0, 1 and j=0,1.**



$P[A_0 \cap B_0]$ $P[A_0 \cap B_1]$ $P[A_1 \cap B_0]$ $P[A_1 \cap B_1]$

Binary Tree Diagram (2)

Let A_i be the event "input was i," and let B_j be the event "receiver decision was i."
Find the probabilities for $P[A_i \cap B_j]$ **i=0, 1 and j=0,1.**



Find which input is more probable given that the receiver has output 1.
Assume that, a priori, the input is equally likely to be 0 or 1.

$$P[B_1] = P[B_1 \mid A_0]\, P[A_0] + P[B_1 \mid A_1]\, P[A_1]$$

Probability of Joint Occurrence

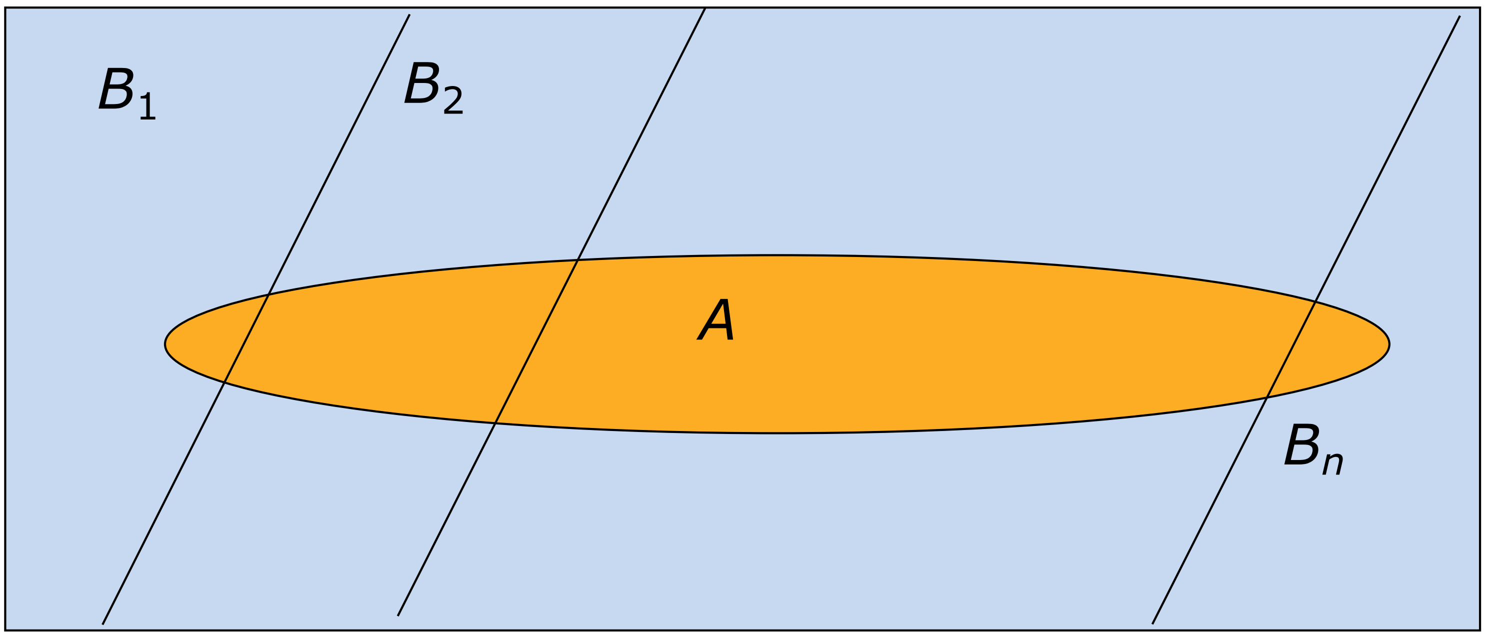
$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} \text{ for } P[B] > 0.$$

$$\begin{aligned} P[A \cap B] &= P[A \mid B]\, P[B] \\ &= P[B \mid A]\, P[A] \end{aligned}$$

$$\begin{aligned} P[A \cap B \cap C] &= P[A \mid B \cap C]\, P[B \cap C] \\ &= P[A \mid B \cap C]\, P[B \mid C]\, P[C] \end{aligned}$$

Theorem on Total Probability

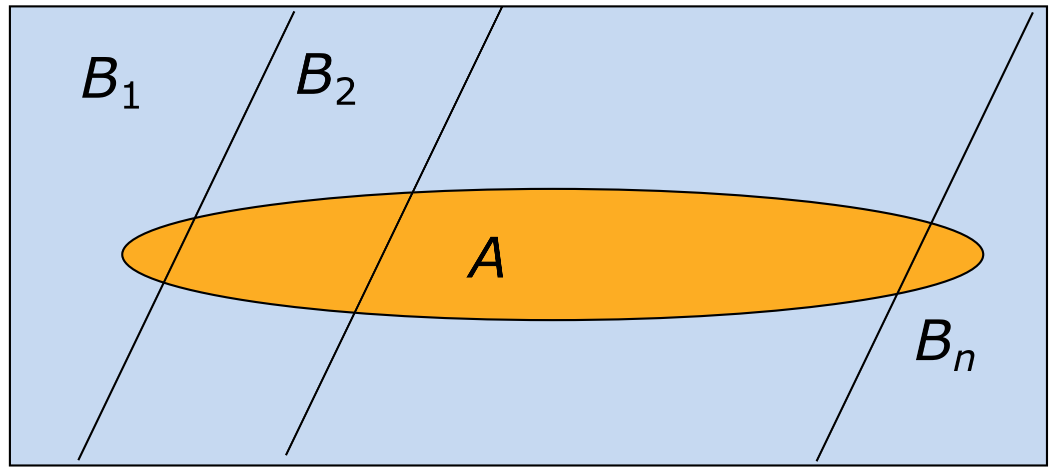
$$P[A]=\sum_{i=1}^{n}P[A\cap B_i]=\sum_{i=1}^{n}P[A\,|\,B_i]P[B_i]$$



Bayes' Rule

Suppose A occurs, what is the probability of B_j?

$$P\left[B_j \mid A\right] = \text{???}$$



$$P\left[B_j \mid A\right] = \frac{P\left[B_j \cap A\right]}{P[A]} = \frac{P\left[A \mid B_j\right]P\left[B_j\right]}{\sum_{i=1}^{n} P[A \mid B_i]P[B_i]}$$

Event Independence

- Intuition: *Knowledge that* A *occurred does not change the probability of* B.

$$P[A \cap B] = P[A]P[B]$$

$$P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{P[A]P[B]}{P[B]} = P[A]$$

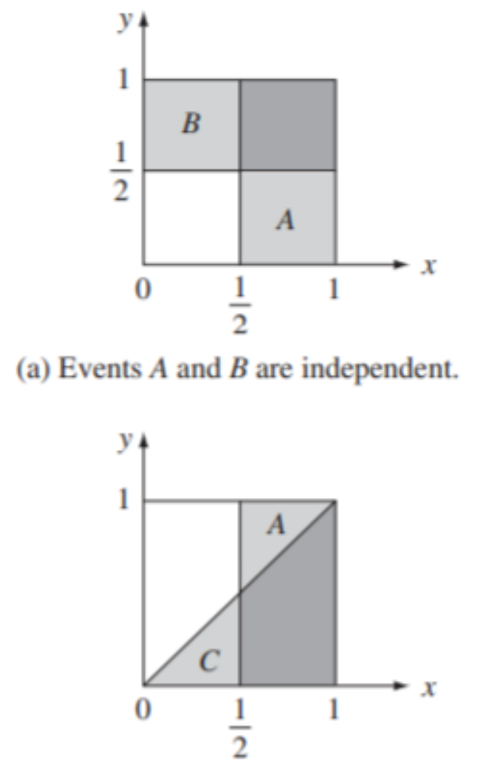
$$P[B|A] = \frac{P[A \cap B]}{P[A]} = \frac{P[A]P[B]}{P[A]} = P[B]$$

Example: Random Pair in Unit Square

$A = \{x > y\},\; B = \{x > 0.5\}\;\; C = \{y < 0.5\}$

$$P[A \mid B] =$$

$$P[B \mid C] =$$



(b) Events A and C are not independent.

FIGURE 2.13
Examples of independent and nonindependent events.

Independence of Three Events

- Definition: *A*, *B*, & *C* are independent if they are pairwise independent

$$P[A \cap B] = P[A]P[B], \quad P[A \cap C] = P[A]P[C],$$
$$\text{and} \quad P[B \cap C] = P[B]P[C]$$

- and if knowledge of 2 of them does not affect the probability of the 3rd

$$P[C \mid A \cap B] = \frac{P[A \cap B \cap C]}{P[A \cap B]} = P[C]$$

- Therefore, *A*, *B*, & *C* are independent if probability of ∩ of pairs & triplets = product of individual probabilities:

$$P[A \cap B \cap C] = P[A \cap B]P[C] = P[A]P[B]P[C]$$

Independence of Multiple Events

- Similarly, $A_1, \ldots, A_n$ are independent if for $k = 2, \ldots, n$:

$$P[A_1 \cap A_k] = P[A_1]\, P[A_k]$$

Exercise: Random Pair in Unit Square

$A = \{x < 0.5\}$, $B = \{y > 0.5\}$

$F = \{x < 0.5 \text{ and } y < 0.5\} \cup \{x > 0.5 \text{ and } y > 0.5\}$

Sequences of Independent Experiments

- Definition: Two experiments are independent if all of their respective events are independent.

- Suppose that a random experiment E involves performing n subexperiments: E_1, E_2, E_3, …, E_n.

- S is Cartesian product of individual sample spaces:

  $S = S_1 \times S_2 \times S_3 \times \ldots \times S_n$

- An outcome of random experiment consists of an n-tuple

$$\xi = (\xi_1, \xi_2, \ldots, \xi_n) \text{ where } \xi_i \text{ is an outcome of } E_i$$

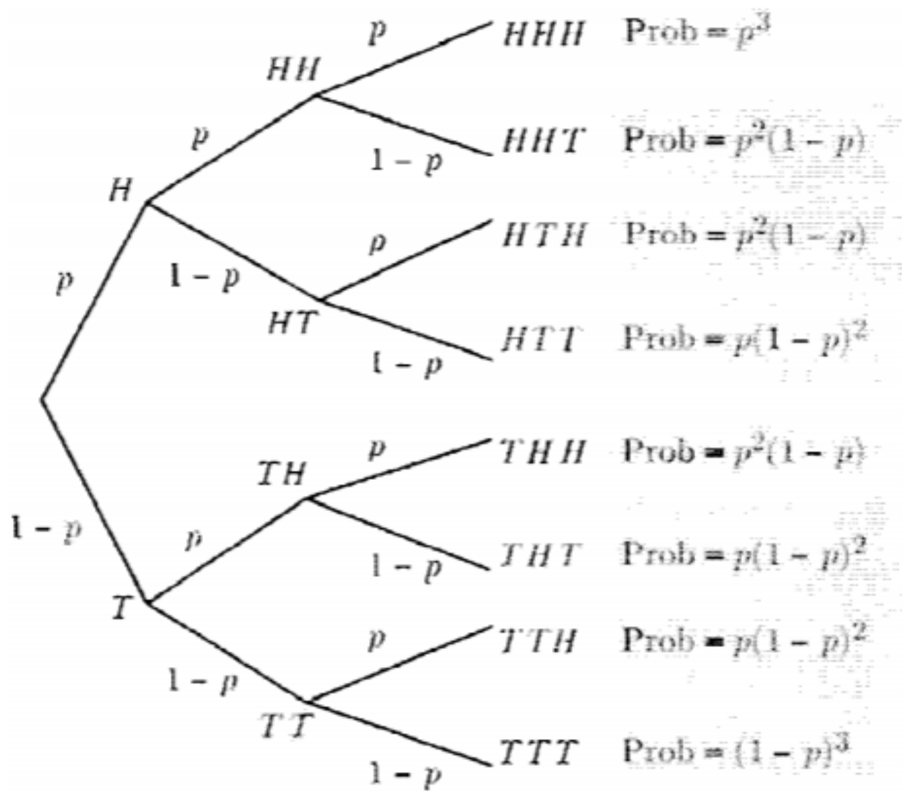
- If the subexperiments are independent, and if A_k only concerns the outcome E_k, then probabilities of events involving intersections of A_k are given by:

$$P\left[A_1 \cap A_2 \cap \cdots \cap A_n\right] = P\left[A_1\right]P\left[A_2\right]\ldots P\left[A_n\right]$$

Example: Independent Coin Tosses

- Toss a fair coin three times.
- *Assume* tosses are independent.

$$P[\{(h,t,h)\}] =$$



Example: Sequence of Bernoulli Trials

- Bernoulli trial involves performing an experiment once and noting whether an event A occurred.
  - "Success" or "1" if A occurs;
  - "Failure" or "0" otherwise
  - Suppose $P[A] = p$
- Perform n independent Bernoulli trials, what is probability of k successes in n trials?

Example: Sequence of Bernoulli Trials II

- The probability of a sequence with exactly *k* 1s in *n* trials:

$$p_n(k) = \binom{n}{k} p^k (1-p)^{n-k} \quad \text{for} \quad k = 0, \ldots, n,$$

- The number of distinct sequences with *k* 1s and (*n* – *k*) 0s is:

$$p_n(k) = N_n(k) p^k (1-p)^{n-k}.$$

- The probability of *k* successes in *n* trials is:

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Approximating Binomial Probabilities

- If n is large and p is small, then for $\alpha = np$

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \approx \frac{\alpha^k}{k!} e^{-\alpha} \quad \text{for } k = 0, 1, \ldots$$

$$\frac{p_{k+1}}{p_k} = \frac{\binom{n}{k+1} p^{k+1} q^{n-k-1}}{\binom{n}{k} p^k q^{n-k}} = \frac{k!(n-k)!p}{(k+1)!(n-k-1)!q}$$

$$= \frac{(n-k)p}{(k+1)q} = \frac{(1-k/n)\alpha}{(k+1)(1-\alpha/n)} \to \frac{\alpha}{k+1} \quad \text{as } n \to \infty$$

See the derivation in class how p_k approaches Poisson

Interpreting the Test Result for Covid-19

- NEW SLIDES FOR LECTURE 3
- Interpreting the result of a test for covid-19 depends on:
  - The accuracy of the test
  - The pre-test probability or estimated risk of disease before testing

Recent Federal Reserve Inflation Policy

- On August 27, 2020 Jerome Powel announced a major change to the current inflation policy
- The old policy kept the inflation below target of 2%
- The new policy keeps the **average inflation rate is at 2%**
- It means FED will let the inflation rate vary and go above 2% target before considering raising the interest rate
- So the new policy makes the inflation as a random variable

More on Bayes' Theorem

- Bayes' Theorem is a way of finding a probability when we know certain other probabilities.
  - The formula is: $P(A|B) = P(A)\,P(B|A)/P(B)$
  - Which tells us: how often A happens given that B happens, ie, $P(A|B)$
  - When we know: how often B happens given that A happens, ie, $P(B|A)$
  - and how likely A is on its own, ie, $P(A)$
  - and how likely B is on its own, ie, $P(B)$
- Let's say
  - P(Fire) means how often there is fire, and
  - P(Smoke) means how often we see smoke, then:
  - P(Fire|Smoke) means how often there is fire when we can see smoke
  - P(Smoke|Fire) means how often we can see smoke when there is fire
  - So the formula kind of tells us "forwards" P(Fire|Smoke) when we know "backwards" P(Smoke|Fire)

California WiFi

- Last few days (Sept 2020) California WiFi (Wild Fires) have killed more than 36 people
- Wild fires are rare (1%)
- But smoke is fairly common (10%) and 90% of wild fires make smoke
- We can then discover the probability of wild fires when there is Smoke:

$$P(\text{Wild Fire}|\text{Smoke}) = P(\text{Fire})\, P(\text{Smoke}|\text{Fire})/P(\text{Smoke})$$

$$= 1\% \times 90\%/10\% = 9\%$$

Example: Picnic Day

- You are planning a picnic today, but the morning is cloudy
- 50% of all rainy days start off cloudy!
- But cloudy mornings are common (about 40% of days start cloudy)
- And this is usually a dry month (only 3 of 30 days tend to be rainy, or 10%)
- **What is the chance of rain during the day?**
- Use Rain to mean rain during the day, and Cloud to mean cloudy morning.
- The chance of Rain given Cloud is written $P(Rain|Cloud)$
- $P(Rain|Cloud) = P(Rain)\ P(Cloud|Rain)/P(Cloud)$
- $P(Rain)$ is Probability of Rain = 10%
- $P(Cloud|Rain)$ is Probability of Cloud, given that Rain happens = 50%
- $P(Cloud)$ is Probability of Cloud = 40%
- $P(Rain|Cloud) = 0.1 \times 0.5/0.4 = .125$
- Or a 12.5% chance of rain. Not too bad! So, let’s go for it!

COVID-19 Pandemic

- Original Covid-19
- Mutated Virus (second wave) lingers longer (is smart!)
- Covid-19 Testing
- Antibody
- Interpreting the test result for Covid-19

Antibody Tests and Potential Shortcomings

- We have Developed tests that detect antibodies in the blood of people who have previously been infected with the COVID-19.
- These serology tests can provide important data on how COVID-19 is spreading through a population.
- There is also hope that the presence of certain antibodies may signify immunity to future infection
- Antibody shortcomings (just to be aware)
  - They may detect ineffective antibodies
  - They do not indicate if an infection is still active
  - They fail to detect infection if administered before antibodies develop

Testing Measures

- Two statistical measures of the testing performance widely used in medicine are **Sensitivity** and **specificity**
- Sensitivity measures the probability of testing Positive for those who are infected with Covid
  - **That is, P(Test+|Covid+)**
  - Those infected correctly identified as having Covid
- Specificity measures the probability of testing negative for those who are not infected with Covid
  - **That is, P(Test-|Covid-)**
  - Those healthy correctly identified as not having Covid

COVID-19: Probability of at least one Positive

- Initial screening for Covid-19 tests antibody in the blood
- Even if an individual is Covid-negative, the testing probability of p = 0.1 gives a positive result
- Suppose 100 people are tested who are all Covid-negative.
- **What is probability that at least one will show positive on the test?**
- Using complement rule:

  P(at least one positive) = 1 - P(all negative)
- Assuming that each individual is independent (independent events), then we can write P(all negative)= $(1\text{-}p)^{100}$

  P(at least one positive) =1- 0.99^{100} = 0.9999

Health Status and Testing

- A person is Positive (has Covid) given Tests Positive P(Test+|Covid+)
- A person is Negative given Tests Negative P(Test-|Covid-)
- A person is Positive but Tests Negative P(Test-|Covid+) (False Negative)
- A person is Negative but Tests Positive P(Test+|Covid-) (False Positive)
- **High Sensitivity** (more accurate) → Low rate of False Negative
- **High Specificity** → Low rate of False Positive
- For 80% sensitivity, among 100 people with Covid-19, 80 would have positive tests
- For 90% specificity, among 100 people without Covid-19, 90 would have negative tests

Covid-19 Testing Illustration (1)

- Infection rate of 10%
  - ie, Covid- = Green Box=90% (90), Covid+ =Red Box=10% (10)
- Testing Sensitivity of 80 % (relates to False Negative of 20%)
  - Among 10 people with Covid-19, 8 would be identified as positive and 2 as negative (False Negative)
- Testing Specificity of 90 % (relates to False Positive of 10%)
  - Among 90 people without Covid-19, 81 would be identified as negative and 9 as positive (False Positive)
- Legend:

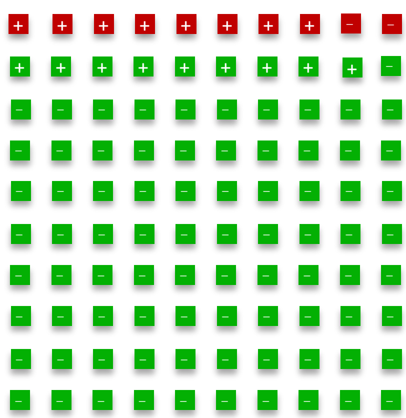
+ = Test +   - = Test -

= Covid - (not infected)   **= Covid + (infected)**

Cases: - + + -

Covid-19 Testing Illustration (2)

- What is the probability of having Covid given Test was positive?
- What is the probability of not having Covid given Test was negative?



Testing\Status	Covid +	Covid -	Total
Test +	8	9	17
Test -	2	81	83
Total	10	90	100

Covid-19 Testing Illustration (3)

- What is probability that a person is infected with Covid? From the Table we find P(Covid+)=10/100 = 0.10
- Now we ask, given that a person has tested positive, what is the probability that the person indeed has Covid, ie, P(Covid+|Test+)?
  - This is 8/17 (from the Table), called positive predictive value
- What is the probability of not having Covid if test is negative?
  - This is 81/83 (from the Table), called negative predictive value
- Later, we will use the Conditional Probability and the Bayes' Rule for calculations to answer the same questions.

Using the Bayes' Rule to Compute Covid Testing

- We want to know the chances of having Covid when test indicates positive, ie, P(Covid|Test+)
- Also we want to know the chances of not having Covid when test indicates negative, ie, *P*(Covid-|Test-)
- Let's try to formulate this using what we have learned:

Use iPad to show the calculation

Conditional Probability and Bayes' Rule (1)

What is the probability of having Covid given test was positive, ie, P(Covid+ |Test+)?

- Denote Event A for Covid infection with outcomes Covid+ and Covid-
- Denote Event B for Testing with outcomes Test+ and Test-
- A and B are two events in sample space

Note that we use the conditional probability of event B *given that event A has occurred ie,* P (B|A), to calculate P (A|B) by $P[A|B] = P[A \cap B] / P[B]$

P(Covid+ and Test+)=P[Covid+ ∩Test+]= P(Covid+)xP(Test+|Covid+)

=0.1x0.8=0.08

P[*Test+*] is probability of test indicating Positive to anyone (w or w/o Covid)

P[*Test+*]= P(Covid+)xP(Test+|Covid+)+P(Covid-)xP(Test+|Covid-)

P[*Test+*]= 0.1x0.8+0.9x0.1 = 0.17

P(Covid+ |Test+)=0.08/0.17=0.47

Conditional Probability and Bayes' Rule (2)

What is the probability of not having Covid given test was negative, ie, P(Covid- |Test-)?

P[not A/*not B*] is P(Covid-|Test-)

$$P[\text{not } A|\text{not } B] = [\text{not } A \cap \text{not } B]/P[\text{not } B]$$

P(Covid- and Test-) = P[Covid- ∩ Test-] = P(Covid-)xP(Test-|Covid-)
=0.9x0.9=0.81

P[*Test*-] is probability of test indicating Negative to anyone (w or w/o Covid)

P[*Test*-]= P(Covid-)xP(Test-|Covid-)+P(Covid+)xP(Test-|Covid+)

P[*Test*-]= 0.9x0.9+0.1x0.2 =0.83

P(Covid-|Test-)=0.81/0.83=0.976

Conditional Probability and Bayes' Rule (3)

- Now assume an Infection rate of 30%
- Keep the Testing Sensitivity and Specificity as before 80% and 90% respectively
- Using the method just described we compute $P(\text{Covid+} \mid \text{Test+})$ and $P(\text{Covid-} \mid \text{Test-})$
- $P(\text{Covid+} \mid \text{Test+}) = P(\text{Covid+} \cap \text{Test+})/P(\text{Test+}) = P(\text{Covid+}) \text{x} P(\text{Test+} \mid \text{Covid+})/P(\text{Test+}) = 0.3\text{x}0.8 = 0.24/P(\text{Test+})$
- $P[\mathit{Test+}] = P(\text{Covid+}) \text{x} P(\text{Test+} \mid \text{Covid+}) + P(\text{Covid-}) \text{x} P(\text{Test+} \mid \text{Covid-})$
- $P[\mathit{Test+}] = 0.3\text{x}0.8 + 0.7\text{x}0.1 = 0.31$
- $P(\text{Covid+} \mid \text{Test+}) = 0.24/0.31 = 0.774$

Similarly we compute $P(\text{Covid-} \mid \text{Test-}) = 0.7\text{x}0.9 / (0.7\text{x}0.9 + 0.3\text{x}0.2) = 0.912$

Dependency on Infection Rate*

- The accuracy of screening tests is highly dependent on the infection rate
- For infection rate of 10% we had
  - $P(\text{Covid+}|\text{Test+})=0.47$ $P(\text{Covid-}|\text{Test-})=0.976$
- For infection rate of 30% we had
  - $P(\text{Covid+}|\text{Test+})=0.774$ $P(\text{Covid-}|\text{Test-})=0.912$

*: Sara Lewin Fraser, "Coronavirus Antibody Tests Have a Mathematical Pitfall," *Scientific American*, July 1, 2020

Generalization - Bayes' Rule for Covid-19 Calculation

- We can write a special version of the Bayes' formula

P(A|B) = P(A)P(B|A)/{P(A)P(B|A)+P(not A)P(B|not A)}

This is for "A" with two cases (A and not A)

- Extending to "A" with three (or more) cases

When "A" has 3 or more cases we include them all in the denominator:

P(A1|B) = P(A1)P(B|A1)/{P(A1)P(B|A1) + P(A2)P(B|A2) + P(A3)P(B|A3) + ...}

Note that we have been using the following:

$$P[A \cap B] = P[A|B]\, P[B] = P[B|A]\, P[A]$$

Poisson Distribution

- Derivation of Poisson distribution

$$\Pr[X = k] = \frac{\alpha^k}{k!} e^{-\alpha}$$

Wrote 3 assumptions and derived the probabilities in class using iPad