# Theoretical task 1

due 9:00 January 28 (Thursday).

In Gaussian classifier we assume some fixed class prior probabilities $p(y)$ and class-conditional probabilities to be $p(x|y) = N(\mu_y, \Sigma_y)$. Classification is performed by posterior class probability maximization

$$\widehat{y} = \arg\max_y p(y|x) \tag{1}$$

1. Write the decision rule of (1) in terms of $p(y)$ and $p(x|y)$. Maximize the logarithm of class score rather than initial class score.

2. In nearest mean classifier each class $\omega_c$ is associated some mean vector $\mu_c \in \mathbb{R}^D$ and $x$ is assigned a class for which the distance to its mean is minimal. Prove that Gaussian classifier reduces to nearest mean classifier when $\Sigma_1 = \Sigma_2 = ... = \Sigma_C = I$ (identity matrix) and prior class probabilities are equal: $p(y) = \frac{1}{C}\,\forall y$.

3. Prove that decision boundaries between classes for Gaussian classifiers will be quadratic polynomial surfaces.

4. Prove that under restriction of common covariance matrices $\Sigma_y = \Sigma\,\forall y$ decision boundaries between classes for Gaussian classifiers will be linear hyperplanes.