Physics, Backwards: Hamiltonian Reconstruction

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Brown Physics DUG Scialogue

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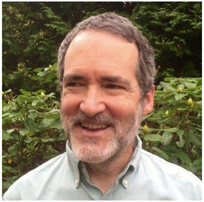
Collaborators



Brad Marston

Stephen Carr

Alex Jacoby (Princeton)

Preliminaries

- Project in quantum many-body physics, condensed matter theory

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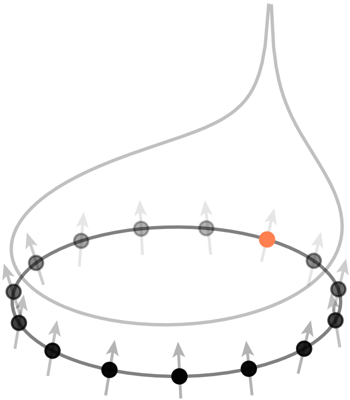
Usually:	$H \longrightarrow |\psi\rangle$
Hamiltonian reconstruction:	$|\psi\rangle \longrightarrow H$

Quantum Many Body Systems

- Ignore translational freedom (lattice)



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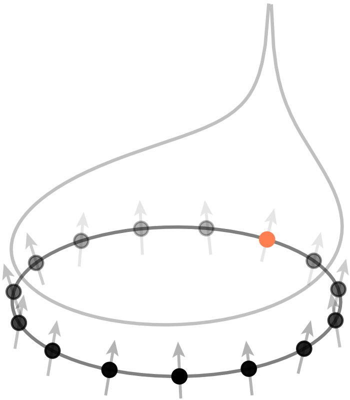

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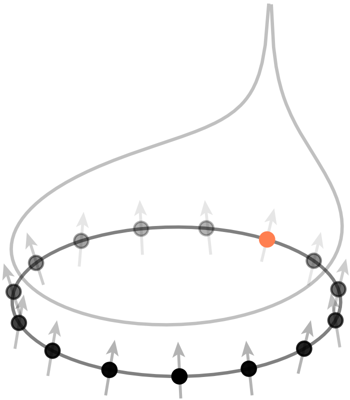

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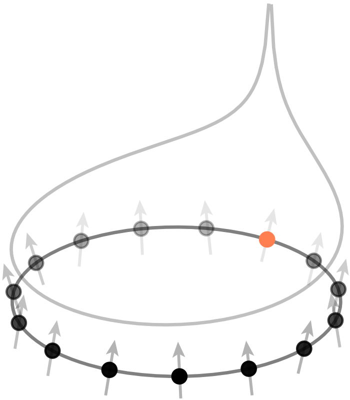
- Model of interest: Haldane-Shastry model



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Big Picture

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- Due to Qi and Ranard (2019)
- Fix some state $|\psi\rangle$, $\langle\psi| O_i |\psi\rangle = \langle O_i\rangle$
- Correlation matrix:

$$M_{ij} = \langle O_i O_j\rangle_\psi - \langle O_i\rangle_\psi \langle O_j\rangle_\psi$$

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- Hamiltonian $H = \sum_i h_i O_i$

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- $\mathbf{h} = [h_1, \ldots, h_n]$ are the coefficients of $H = \sum_i h_i O_i$!
- The Hamiltonian is in the nullspace of M_{ij}

Our Work

Caveat

How do we pick O_i?

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- (My task) validate with the Haldane-Shastry model

Interlude: The Research Workflow

- Pen-and-paper calculations and numerical work

Visualization of matrix

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- Going back to analytical calculations to understand results

Visualization of matrix

Over and Undercomplete Bases

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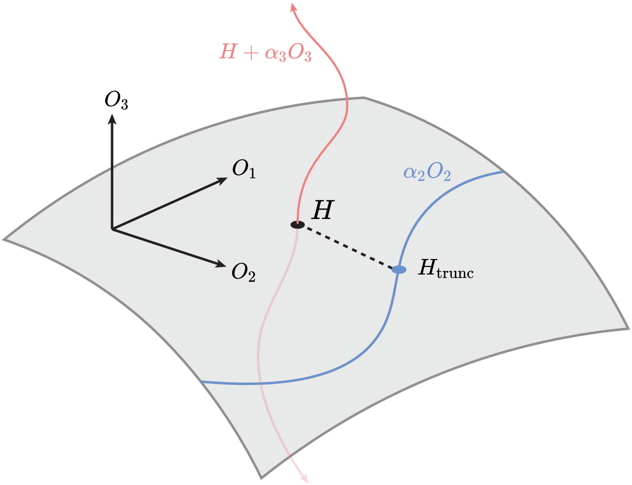
Over and Undercomplete Bases



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- E.g., HS has total spin as conserved quantity

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- New zero shows up, as in the conserved quantity case above

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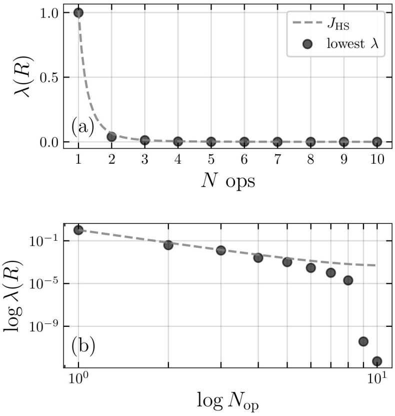

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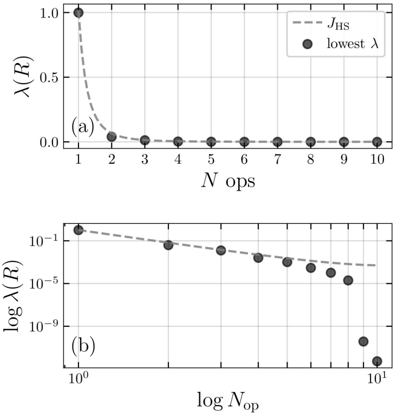

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- $\lambda_0 \propto$ magnitude of largest truncated operator in Hamiltonian

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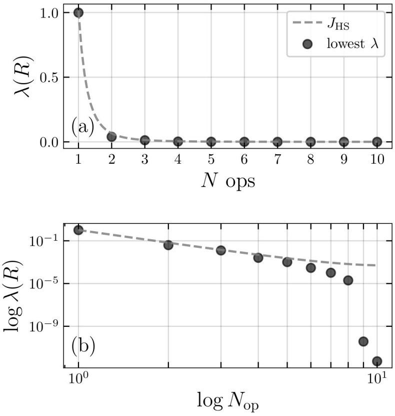

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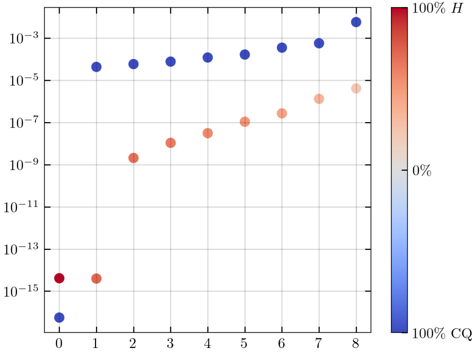

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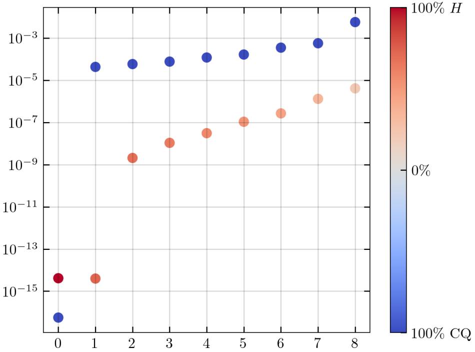

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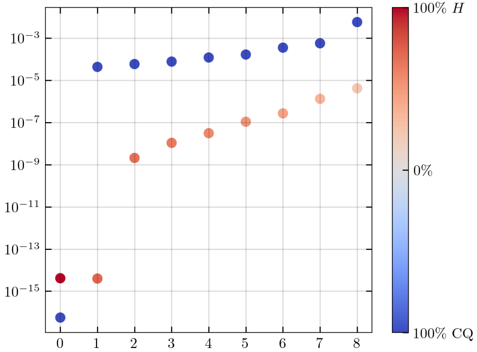
- Finite-temperature simulations?



The End

Thank you! Questions?