Logics for Artificial Intelligence:
Assignment 1

Leonard Botha

August 7, 2015

1. (a) Prove that $Z \cap (X \cup Y) = (Z \cap X) \cup (Z \cap Y)$

$$\begin{aligned} \text{Take } \alpha \in Z \cap (X \cup Y) &\Leftrightarrow \alpha \in Z \text{ and } \alpha \in X \cup Y \\ &\Leftrightarrow \alpha \in Z \text{ and } (\alpha \in X \text{ or } \alpha \in Y) \\ &\Leftrightarrow (\alpha \in Z \text{ and } \alpha \in X) \text{ or } (\alpha \in Z \text{ and } \alpha \in Y) \\ &\Leftrightarrow \alpha \in Z \cap X \text{ or } \alpha \in Z \cap Y \\ &\Leftrightarrow \alpha \in (Z \cap X) \cup (Z \cap Y) \end{aligned}$$

   (b)

   (c)

   (d) Prove that $X \times (Y \cup Z) = (X \times Y) \cup (X \times Z)$

$$\begin{aligned} \text{Take } (x, y) \in X \times (Y \cup Z) &\Leftrightarrow x \in X \text{ and } y \in Y \cup Z \\ &\Leftrightarrow x \in X \text{ and } y \in Y \text{ or } x \in X \text{ and } y \in Z \\ &\Leftrightarrow (x, y) \in (X \times Y) \cup (X \times Z) \end{aligned}$$

2. Interpretations are represented by an ordered pair, (p, q), where the first element is the truth value of p and the second the truth value of q.

   (a) $\text{Mod}(p \vee \neg p) = \{(T, T), (T, F), (F, T), (F, F)\}$

   (b) $\text{Mod}(p \vee q) = \{(T, T), (T, F), (F, T)\}$

   (c) $\text{Mod}(p \vee \neg q) = \{(T, T), (T, F), (F, F)\}$

   (d) $\text{Mod}(\neg p \vee q) = \{(T, T), (F, T), (F, F)\}$

(e) $\mathrm{Mod}(\neg p \vee \neg q) = \{(T, F), (F, T), (F, F)\}$

(f) $\mathrm{Mod}(p) = \{(T, T), (T, F)\}$

(g) $\mathrm{Mod}(q) = \{(T, T), (F, T)\}$

(h) $\mathrm{Mod}(p \leftrightarrow q) = \{(T, T), (F, F)\}$

(i) $\mathrm{Mod}(\neg(p \leftrightarrow q)) = \{(T, F), (F, T)\}$

(j) $\mathrm{Mod}(\neg q) = \{(T, F), (F, F)\}$

(k) $\mathrm{Mod}(\neg p) = \{(F, T), (F, F)\}$

(l) $\mathrm{Mod}(p \wedge q) = \{(T, T)\}$

(m) $\mathrm{Mod}(p \wedge \neg q) = \{(T, F)\}$

(n) $\mathrm{Mod}(\neg p \wedge q) = \{(F, T)\}$

(o) $\mathrm{Mod}(\neg p \wedge \neg q) = \{(F, F)\}$

(p) $\mathrm{Mod}(p \wedge \neg p) = \emptyset$

3. (a)

4.

5.

6.

References