# Logics for Artificial Intelligence
# Assignment 1

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Due date: Friday 7 August 2015 at 11:00pm

**Instructions:**

1. Your assignment should be a single document, preferably a pdf file, with the questions answered in the order given.
2. Upload a zip file (with your student number as the file name) containing your assignment to Vula by Friday 7 August at 11pm.
3. You may discuss the assignment questions with others, but you need to submit and complete your own assignment. It must be your own work.
4. You may consult any source. All sources consulted must be referenced in your assignment. This includes sources from the Internet.
5. If you are unsure about anything, send me an email ASAP to resolve the matter.

**Questions**

1. Do the following problems in the notes on Sets, Logic and Relations:
   (a) Problem 2.4.3 on page 27.
   (b) Problems 2.13.4 and 2.13.5 on page 31.
   (c) Problem 2.21.2 on page 38. Note that $2^X$ is another way to refer to the powerset of $X$. That is, $2^X = \mathscr{P}(X) = \{Y \mid Y \subseteq X\}$.
   (d) Problem 2.22.3 on page 42.
   (e) Problem 4.2.1 and 4.2.2 on page 61.
2. Consider the language $\mathcal{L}$ generated from the set of atoms $\mathcal{P} = \{p, q\}$. For each of the following formulas $\alpha$, write down its set of models $Mod(\alpha)$.
   (a) $p \vee \neg p$
   (b) $p \vee q$

(c) $p \vee \neg q$

(d) $\neg p \vee q$

(e) $\neg p \vee \neg q$

(f) $p$

(g) $q$

(h) $p \leftrightarrow q$

(i) $\neg(p \leftrightarrow q)$

(j) $\neg q$

(k) $\neg p$

(l) $p \wedge q$

(m) $p \wedge \neg q$

(n) $\neg p \wedge q$

(o) $\neg p \wedge \neg q$

(p) $p \wedge \neg p$

3. Consider again the language $\mathcal{L}$ generated from the set of atoms $\mathcal{P} = \{p, q\}$. Is there any formula in $\mathcal{L}$ which is not logically equivalent to one of the formulas in question 2 above? If there is, provide an example of such a formula. If there isn't such a formula, prove it. Hint: Use your answers to question 2 as a guide.

4. Do Problem 2.9 in Chapter 2 of Ben-Ari.

5. Given $\alpha, \beta \in \mathcal{L}$, prove the following:

   (a) $Mod(\alpha \wedge \beta) = Mod(\alpha) \cap Mod(\beta)$.

   (b) $Mod(\neg\alpha) = W - Mod(\alpha)$.

   (c) If $\mathsf{K}$ is satisfiable and $\alpha$ is valid, then $\mathsf{K} \cup \{\alpha\}$ is satisfiable.

   (d) $\alpha \equiv \beta$ if and only if both $\alpha \models \beta$ and $\beta \models \alpha$.

   (e) $\alpha \equiv \beta$ if and only if $\models \alpha \leftrightarrow \beta$.

6. Prove that entailment has the following properties:

   (a) $\models$ is reflexive, i.e., for all $\alpha \in \mathcal{L}$, $\alpha \models \alpha$.

   (b) $\models$ is monotonic, i.e., for any $\mathsf{K} \subseteq \mathcal{L}$, if $\mathsf{K} \models \alpha$, then $\mathsf{K} \cup \{\beta\} \models \alpha$.

   (c) $\models$ is explosive, i.e., if $p, \neg p \in \mathsf{K}$, then $\mathsf{K} \models \gamma$ for every $\gamma \in \mathcal{L}$.