

Logics for Artificial Intelligence:

Assignment 1

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1. (a) Prove that $Z \cap (X \cup Y) = (Z \cap X) \cup (Z \cap Y)$

$$\begin{aligned}\text{Take } \alpha \in Z \cap (X \cup Y) &\Leftrightarrow \alpha \in Z \text{ and } \alpha \in X \cup Y \\ &\Leftrightarrow \alpha \in Z \text{ and } (\alpha \in X \text{ or } \alpha \in Y) \\ &\Leftrightarrow (\alpha \in Z \text{ and } \alpha \in X) \text{ or } (\alpha \in Z \text{ and } \alpha \in Y) \\ &\Leftrightarrow \alpha \in Z \cap X \text{ or } \alpha \in Z \cap Y \\ &\Leftrightarrow \alpha \in (Z \cap X) \cup (Z \cap Y)\end{aligned}$$

(b)

(c)

- (d) Prove that $X \times (Y \cup Z) = (X \times Y) \cup (X \times Z)$

$$\begin{aligned}\text{Take } (x, y) \in X \times (Y \cup Z) &\Leftrightarrow x \in X \text{ and } y \in Y \cup Z \\ &\Leftrightarrow x \in X \text{ and } (y \in Y \text{ or } y \in Z) \\ &\Leftrightarrow (x, y) \in (X \times Y) \cup (X \times Z)\end{aligned}$$

2. Interpretations are represented by an ordered pair, (p, q) , where the first element is the truth value of p and the second the truth value of q .

- (a) $\text{Mod}(p \vee \neg p) = \{(T, T), (T, F), (F, T), (F, F)\}$
- (b) $\text{Mod}(p \vee q) = \{(T, T), (T, F), (F, T)\}$
- (c) $\text{Mod}(p \vee \neg q) = \{(T, T), (T, F), (F, F)\}$
- (d) $\text{Mod}(\neg p \vee q) = \{(T, T), (F, T), (F, F)\}$

- (e) $\text{Mod}(\neg p \vee \neg q) = \{(T, F), (F, T), (F, F)\}$
- (f) $\text{Mod}(p) = \{(T, T), (T, F)\}$
- (g) $\text{Mod}(q) = \{(T, T), (F, T)\}$
- (h) $\text{Mod}(p \leftrightarrow q) = \{(T, T), (F, F)\}$
- (i) $\text{Mod}(\neg(p \leftrightarrow q)) = \{(T, F), (F, T)\}$
- (j) $\text{Mod}(\neg q) = \{(T, F), (F, F)\}$
- (k) $\text{Mod}(\neg p) = \{(F, T), (F, F)\}$
- (l) $\text{Mod}(p \wedge q) = \{(T, T)\}$
- (m) $\text{Mod}(p \wedge \neg q) = \{(T, F)\}$
- (n) $\text{Mod}(\neg p \wedge q) = \{(F, T)\}$
- (o) $\text{Mod}(\neg p \wedge \neg q) = \{(F, F)\}$
- (p) $\text{Mod}(p \wedge \neg p) = \emptyset$

3. There are no formulas that are not logically equivalent to one of the formulas in question 2. This is obvious as the union over the models in question 2 is exactly the power set of all possible interpretations for (p,q).
- 4.
- 5.
- 6.

References