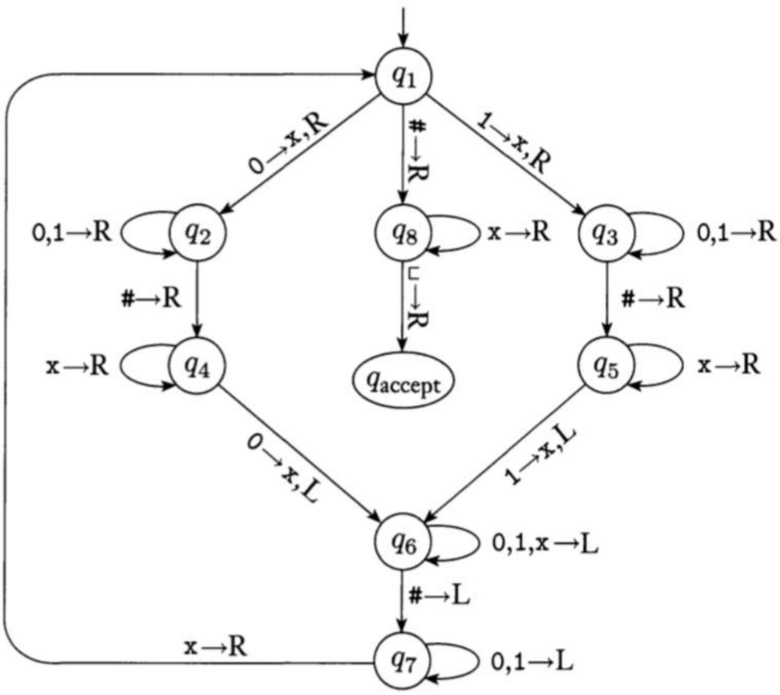
Assignment 5

1. Turing machine [6pts]

   a)

   The following is a formal description of $M_1 = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$, the Turing machine that we informally described (page 139) for deciding the language $B = \{w\#w \mid w \in \{0,1\}^*\}$.

   - $Q = \{q_1, \ldots, q_{14}, q_{\text{accept}}, q_{\text{reject}}\}$,
   - $\Sigma = \{0,1,\#\}$, and $\Gamma = \{0,1,\#,x,\sqcup\}$.
   - We describe δ with a state diagram (see the following figure).
   - The start, accept, and reject states are q_1, q_{accept}, and q_{reject}.



To simplify the figure, we don't show the reject state or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol.

Now there are two inputs. Please write down the sequence of configurations. [4pts]

(1)**10#0#**

(2)**01#00**

b) Let A be the language, for any $\omega \in A$, M halt and accept; for any $\omega \notin A$, M halt and reject. Is A a Turing-recognizable language? Is A a Turing-decidable language? [2pts]

2. Prove that $\forall a, b \in R$ where b > 0 (a, b are constants) [2pts]
   $(\boldsymbol{n} + \boldsymbol{a})^{\boldsymbol{b}} = \boldsymbol{O}(\boldsymbol{n}^{\boldsymbol{b}})$

3. Rank the following functions by order of growth. that is, find an arrangement g1, g2, g3, g4 of the functions satisfying $g1 = o(g2), g2 = o(g3), g3 = o(g4)$.[2pts]
   $\mathbf{g1} = \boldsymbol{n}^{\mathbf{1.01}}$ $\mathbf{g2} = \mathbf{10}\boldsymbol{n}\, \boldsymbol{log}_{\mathbf{2}} \sqrt{\boldsymbol{n}}$ $\mathbf{g3} = \mathbf{0.0001}\boldsymbol{n}$ $\mathbf{g4} = \mathbf{666}\, \boldsymbol{log}_{\mathbf{2}}\, \boldsymbol{n}^{\mathbf{3}}$