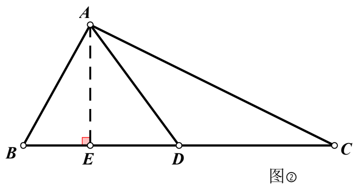
2.

(注:看懂的用蓝笔订正，没看懂的用红笔订正)

(1)$AB^2+AC^2=2(AD^2+CD^2)$

(2)



成立

证:作$AE\perp BC$交BC于E

则$AC^2=AE^2+CE^2=AE^2+(DC+DE)^2=AE^2+DC^2+DE^2+2\cdot DC\cdot DE$

$\because$ D 为 BC 中点 $\quad\therefore BD=CD\quad\therefore BE=BD-DE=DC-DE$

$\therefore AB^2=AE^2+BE^2=AE^2+(DC-DE)^2=AE^2+DC^2+DE^2-2\cdot DC\cdot DE$

$\therefore AB^2+AC^2=AE^2+DC^2+DE^2+2\cdot DC\cdot DE+AE^2+DC^2+DE^2-2\cdot DC\cdot DE$

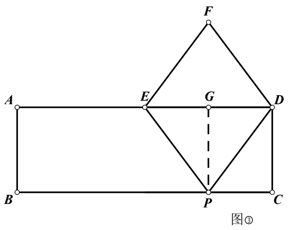
$=2(AE^2+DC^2+DE^2)$

又$\because AD^2=AE^2+DE^2$

$\therefore AB^2+AC^2=2(AD^2+CD^2)$

$\therefore$成立

(3)



当PE^2+PD^2取得最小值时,$C_{EPDF}=20$

作$EG=DG$,连接PG

当PE^2+PD^2取得最小值时:

由(2)得:$PE^2+PD^2=2(PG^2+DG^2)$

$\because AD=12,E$为AD中点,G为ED中点 $\quad\therefore DG=\frac{1}{2}\cdot\frac{1}{2}\cdot AD=\frac{1}{4}\cdot 12=3$

又$\because \mathrm{PG}=CD=AB,AB=4(PG\perp BC$时$PG$取最小值$)\quad\therefore PG=4$

$\therefore DP=\sqrt{DG^2+PG^2}=\sqrt{3^2+4^2}=5$

$\because G$为ED中点,$PG\perp ED$

$\therefore\Delta EPD$为等腰三角形

$\therefore PE=PD=5=>PE+PD=10$

又$\because F$是P关于AD的对称点

$\therefore PE+PD=FE+FD$

$\therefore C_{EPDF}=\mathrm{PE}+\mathrm{PD}+\mathrm{FE}+\mathrm{FD}=2(PE+PD)=2\cdot 10=20$