

机密★启用前

答案:

$$(1) m = \sqrt{1 + (m-1) \sqrt{1 + m \sqrt{1 + (m+1) \sqrt{1 + \dots}}}}$$

$$(2) \text{约分得: } \frac{x^2-1}{k(x-1)} = \frac{x+1}{k}$$

$$\text{由(1)得: } x+1 = \sqrt{1 + (x+1-1) \sqrt{1 + (x+1) \sqrt{1 + (x+1+1) \sqrt{1 + \dots}}}}$$

$$= \sqrt{1 + x \sqrt{1 + (x+1) \sqrt{1 + (x+2) \sqrt{1 + \dots}}}}$$

$$= \frac{x+1}{k}$$

$$\therefore x+1 = \frac{x+1}{k}$$

$$\text{① } k=1 \quad \therefore k(x-1) = 1 \cdot (1-1) = 0 \quad \therefore \text{不成立}$$

$$\text{② } k=-1 \quad (x+1=0) \therefore k(x-1) = -1 \cdot (-1-1) = 2 \quad \therefore \text{成立}$$

$$\therefore x=k \quad \therefore x=k=-1$$

$$\therefore \text{原式} = \frac{-1+1}{-1} = 0$$

(3) 由(1)得:

$$\sqrt{1 + (y-1) \sqrt{1 + y \sqrt{1 + (y+1) \sqrt{1 + \dots}}}} = \sqrt{1 + z \sqrt{1 + (z+1) \sqrt{1 + (z+2) \sqrt{1 + \dots}}}}$$

$$1 + xy = z + 1$$

$$xy = z$$

$$\therefore AB, BC \text{ 的长度都为整数, } 4 < AC < 5, AB = x, BC = y \quad \therefore x + y = 6, AC = 2\sqrt{5}$$

$$\therefore \angle B = 90^\circ \quad \therefore \triangle ABC \text{ 为直角三角形 在 } Rt\triangle ABC \text{ 中: } AB^2 + BC^2 = AC^2 \quad x^2 + y^2 = (2\sqrt{5})^2$$

$$\therefore (x+y)^2 = x^2 + 2xy + y^2 = 36 \quad \therefore x^2 + y^2 = 36 - 2xy = (2\sqrt{5})^2 = 20$$

$$\therefore z = xy \quad \therefore 36 - 2z = 20 \quad \therefore z = 8$$